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OPTIMIZATION OF EFFICIENCY OF
A CESIUM DIODE CONVERTER

by

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OPTIMIZATION OF EFFICIENCY OF A CESIUM DIODE CONVERTER

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INTRODUCTION

The efficiency of the converter is computed with the least possible assumptions. The current density in the diode is assumed to be given by the Richardson-Dushman equation. Space-charge effects, magnetic-field effects, and transport effects are therefore neglected. The interelectrode spacing is assumed to be very small, and hence the electrical resistance of the plasma is neglected. Also, for small spacing, the thermal conduction across the plasma appears to contribute to the losses in the diode and so is included. Thermal emissivity and the interelectrode heat-transfer coefficient are varied, and the effect on efficiency is investigated. The following treatment on the subject is very similar to that of Alfred Schock.¹

LIST OF SYMBOLS USED

A	Constant in Richardson-Dushman Equation, 120 amp/(cm ²)(deg ²)
e	Electronic charge, 1.6×10^{-19} coulomb
h	Interelectrode-conduction heat-transfer coefficient, watt/(cm ²)(°K)
J	Current density, amp/cm ²
k	Boltzmann constant, 1.38×10^{-23} joule/°K
q _e	Electron cooling watt/cm ² of cathode
q _c	Conductive cooling across the lead wire, watt/cm ² of cathode
q _r	Radiative cooling, watts/cm
q _p	Conductive cooling across the plasma; watt/cm ²
q _t	Total cathode cooling watt/cm ²
T _a	Anode temperature °K
T	Cathode temperature, °K

¹Alfred Schock, Optimization of Emission-limited Thermionic Generators, J. Appl. Phys. 32, 1564 (1961).

V	Load voltage
V_w	Lead wire voltage drop
$(V_w)_{\text{opt}}$	Optimized lead wire voltage
K	Thermal conductivity, watt/(cm)(°K)
P	Power, watt/cm ²
η	Efficiency
$\bar{\epsilon}$	Effective thermal emissivity
ϕ_a	Anode work function, volt
ϕ_c	Cathode work function, volt
ϕ_c^*	Critical cathode work function, volt
σ	Stefan-Boltzmann constant, 5.669×10^{-12} watt/(cm ²)(°K ⁴)

Superscripts:

*	Optimized with respect to ϕ_c
"	Optimized with respect to V_w
'	Makes the assumption: $V = \phi_c - \phi_a - V_w$

DEFINITIONS

$$\beta \equiv \phi_c - \phi_a$$

$$\gamma \equiv kT_c/e$$

$$R \equiv \exp(e\phi_c/kT_c) \left[\frac{\sigma\bar{\epsilon}T_c^4 + h(T_c - T_a)}{AT_c^2} \right]$$

$$\alpha \equiv \phi_c + 2\gamma + R$$

$$S \equiv \frac{\pi}{\sqrt{3}} \frac{k}{e} \sqrt{T_c^2 - T_a^2}$$

$$Q^{*2} \equiv 1 + 2 \frac{R^*}{\gamma}$$

$$\zeta \equiv \alpha/\beta$$

$$\psi \equiv B^2/\beta^2$$

$$J \equiv AT_c^2 \exp(-e\phi_c/kT_c)$$

ANALYSIS

The efficiency of the converter is defined to be given by

$$\eta \equiv \frac{JV}{q_e + q_r + q_c + q_p} \quad (1)$$

where

$$q_e = J \left(\phi_c + \frac{2kT_c}{e} \right)$$

$$q_r = \sigma \bar{\epsilon} T_c^4$$

$$q_c = J \left[\frac{\pi^2 k^2 (T_c^2 - T_a^2)}{6e^2 V_w} - \frac{V_w}{2} \right]^*$$

$$q_p = h(T_c - T_a)$$

Rearranging Eq. (1). and substituting for q_e , q_r , q_c , and q_p , we have

$$\frac{JV}{\eta} = J \left(\phi_c + \frac{2kT_c}{e} \right) + J \left[\frac{\pi^2 k^2 (T_c^2 - T_a^2)}{6e^2 V_w} - \frac{V_w}{2} \right] + \sigma \bar{\epsilon} T_c^4 + h(T_c - T_a) \quad (2)$$

Division of Eq. (2) by J and substitution for J lead to

$$\begin{aligned} \frac{V}{\eta} = & \phi_c + \frac{2kT_c}{e} + \frac{\pi^2 k^2 (T_c^2 - T_a^2)}{6e^2 V_w} - \frac{V_w}{2} \\ & + \frac{\sigma \bar{\epsilon} T_c^4}{AT_c^2} \exp\left(\frac{e\phi_c}{kT_c}\right) \\ & + \frac{h(T_c - T_a)}{AT_c^2} \exp\left(\frac{e\phi_c}{kT_c}\right) \end{aligned} \quad (3)$$

*Derived in Ref. (1). Here, T_a is not assumed to be equal to zero as in Ref. (1).

The efficiency η is calculated at the maximum power point which is attained when $V = \phi_c - \phi_a - V_w$. It is shown later that η can be improved further, if the output voltage is optimized, keeping the lead wire voltage drop at the optimum value. This can be done by suitably adjusting the load resistance.

Substituting for $V = \phi_c - \phi_a - V_w$, and changing the notation for η to η' , we have

$$\begin{aligned} \frac{\phi_c - \phi_a - V_w}{\eta'} &= \phi_c + \frac{2kT_c}{e} + \frac{\pi^2 k^2 (T_c^2 - T_a^2)}{6e^2 V_w} - \frac{V_w}{2} \\ &+ \exp(e\phi_c/kT_c) \left[\frac{\sigma \bar{\epsilon} T_c^4 + h(T_c - T_a)}{AT_c^2} \right] \end{aligned} \quad (4)$$

Equation (4) is differentiated with respect to V_w and $\partial\eta'/\partial V_w$ is equated to zero in order to optimize η' in terms of V_w :

$$\frac{1}{2} + \frac{\pi^2 k^2 (T_c^2 - T_a^2)}{6e^2} \frac{1}{(V_w)_{opt}^2} - \frac{1}{\eta''} = 0$$

Rearrangement gives

$$\frac{\pi^2 k^2 (T_c^2 - T_a^2)}{e^2 (V_w)_{opt}^2} = \frac{6 - 3\eta''}{\eta''}$$

Therefore,

$$(V_w)_{opt} = \left[\frac{\eta''}{(2 - \eta'')} \right]^{1/2} \left(\frac{\pi k \sqrt{T_c^2 - T_a^2}}{\sqrt{3} e} \right)$$

Substituting for $(V_w)_{opt}$ in Eq. (4), we have

$$\begin{aligned} \phi_c \left(\frac{1 - \eta''}{\eta''} \right) - \frac{\phi_a}{\eta''} - \frac{2kT_c}{e} - \left(\frac{2 - \eta''}{2\eta''} \right) \left[\frac{\eta''}{3(2 - \eta'')} \right]^{1/2} \left(\frac{\pi k \sqrt{T_c^2 - T_a^2}}{e} \right) \\ - \frac{\pi^2 k^2 (T_c^2 - T_a^2)}{6e^2} \left[\frac{3(2 - \eta'')} {\eta''} \right]^{1/2} \left(\frac{e}{\pi k \sqrt{T_c^2 - T_a^2}} \right) = R \end{aligned} \quad (5)$$

which, when rearranged and substitutions made for the various symbols, becomes

$$\frac{\beta}{\eta''} - \alpha = S \left(\frac{2 - \eta''}{\eta''} \right)^{1/2} \quad (6)$$

Squaring and rearranging gives

$$\frac{\beta^2}{\eta''^2} - \frac{2\beta\alpha + 2S^2}{\eta''} + (\alpha^2 + S^2) = 0 \quad (7)$$

The solution of $1/\eta''$ from Eq. (7) gives

$$\frac{1}{\eta''} = \frac{(2\beta\alpha + 2S^2) \pm 2[S^4 + 2S^2\beta\alpha - S^2\beta^2]^{1/2}}{2\beta^2},$$

that is,

$$\frac{1}{\eta''} = \left(\frac{\alpha}{\beta} + \frac{S^2}{\beta^2} \right) \pm \left[\left(\frac{S^2}{\beta^2} \right)^2 + 2 \left(\frac{S^2}{\beta^2} \right) \left(\frac{\alpha}{\beta} \right) - \frac{S^2}{\beta^2} \right]^{1/2} \quad (8)$$

Substitution for α/β and S^2/β^2 , yields

$$1/\eta'' = (\xi - \psi) \pm [\psi^2 + 2\xi\psi - \psi]^{1/2} \quad (9)$$

The + sign is considered to determine the value of η'' , as it would give the lesser of the two possible values of efficiency.

Equation (5) is now differentiated with respect to V , where $V = \phi_c - \phi_a - V_w$. Here we note that V_w is kept constant at $(V_w)_{opt}$, and ϕ_a is constant. Hence, the differentiation is the same as when differentiated with respect to ϕ_c , and the optimum value of $\phi_c = \phi_c^*$ is calculated. Here ϕ_c^* is interpreted as being equal to $\phi_c + \Delta V$, and ΔV is now the change in the load voltage to get maximum efficiency. We know (see Fig. 1) that negative ΔV would not cause any change in J ; hence, η can no longer be improved from the value η'' . But if ΔV is +ve, i.e., $\phi_c^* > \phi_c$, then η'' can be improved to η^* by suitably changing the load voltage to

$$\phi_c = \phi_a - (V_w)_{opt} + \Delta V$$

Fig. 1

Log of Current Density (J) vs
Load Voltage (V) for Idealized
Cesium Diode Converter

Rewriting Eq. (5), we have

$$\frac{\phi_c}{\eta''} - \frac{\phi_a}{\eta''} - \left(\frac{2 - \eta''}{\eta''} \right)^{1/2} S = \phi_c + 2V + \exp(e\phi_c/kT_c) \left[\frac{\sigma \epsilon T_c^4 + h(T_c - T_a)}{AT_c^2} \right] \quad (5')$$

Now Eq. (5') is differentiated with respect to ϕ_c , and $\frac{\partial \eta''}{\partial \phi_c}$ is equated to zero; the resulting η'' is called η^* :

$$\frac{1}{\eta^*} = 1 + \frac{R^*}{\gamma} \quad (10)$$

Using η^* for the value of η'' in Eq. (5'), we can solve for the value of ϕ_c^* . Equation (5') now becomes

$$\left(1 + \frac{R^*}{\gamma} \right) (\phi_c^* - \phi_a) - \left[1 + \frac{2R^*}{\gamma} \right]^{1/2} S = \alpha^* \quad (11)$$

Equation (11) gives the value of ϕ_c^* . Equations (6), (9), (10), and (11) are evaluated for a wide range of values of the several variables.

Variable	Range
$\bar{\epsilon}$	0.1, 0.2, 0.3, and 1.0
h	0, 0.005, and 0.01
T_c	1200 to 2600 at an interval of 200
ϕ_c	2.0 to 4.0 at an interval of 0.5
T_a	600 only
ϕ_a	1.0, 1.6, and 2.2

COMPUTING PROCEDURE

Several quantities were evaluated with the computer LGP-30, and the following will give the sequence of evaluation.

All the independent variables were read in with the initial, incremental and final values. Also, the first guess of the root, R^* for Eq. (11), the tolerance, and the number of iterations in root-finding [for Eq. (11)] were also read in. The numerical values of physical constants like σ , k , etc., were used directly.

1. γ is evaluated as being equal to kT_c/e
2. T_c^4
3. $\bar{\epsilon}\sigma T_c^4 = q_r$
4. $h(T_c - T_a) = q_p$
5. $\frac{(q_r + q_p)}{120T_c^2} \equiv M$
6. $\frac{\pi}{\sqrt{3}} \frac{k}{e} \sqrt{T_c^2 - T_a^2} = 1.565 \times 10^{-4} \sqrt{T_c^2 - T_a^2} \equiv S$
7. A suitable program for finding the root R^* in the nonlinear Eq. (11) is installed, where

$$\gamma \ln [R^*/M] = \phi_c^* ;$$
 and the value of ϕ_c^* that satisfies the Eq. (11) is found.
8. Now ϕ_c is compared with ϕ_c^* just evaluated, and one of the two sets of calculations is performed according to the satisfaction of their respective conditions.

If $\phi_c \geq \phi_c^*$, then the computations 9 through 26 are performed.

9. $\phi_c + 2\gamma + M \exp(\phi_c/\gamma) = \alpha$
10. $\phi_c - \phi_a = \beta$
11. $\alpha/\beta = \xi$
12. $S^2/\beta^2 = \psi$
13. $(\xi + \psi) + [\psi^2 + 2\xi\psi - \psi]^{1/2} = 1/\eta_1$
14. $(\xi + \psi) - [\psi^2 + 2\xi\psi - \psi]^{1/2} = 1/\eta_2$
15. $120 T_c^2 \exp(-\phi_c/\gamma) = J$
16. $J(\phi_c + 2\gamma) = q_e$

$$17. \left[\frac{\eta_1}{2 - \eta_1} \right]^{1/2} S = (V_w)_{\text{opt } 1}$$

$$18. \left[\frac{\eta_2}{2 - \eta_2} \right]^{1/2} S = (V_w)_{\text{opt } 2}$$

$$19. J \left[\frac{S^2}{2 V_{w1}} - 1/2 V_{w1} \right] = q_{c1}$$

$$20. J \left[\frac{S^2}{2 V_{w2}} - 1/2 V_{w2} \right] = q_{c2}$$

$$21. q_r + q_e + q_p + q_{c1} = q_{t1}$$

$$22. q_r + q_e + q_p + q_{c2} = q_{t2}$$

$$23. \phi_c - \phi_a - (V_w)_{\text{opt } 1} = V_1$$

$$24. \phi_c - \phi_a - (V_w)_{\text{opt } 2} = V_2$$

$$25. J V_1 = P_1$$

$$26. J V_2 = P_2$$

In the above, we find that η_2 is always greater than η_1 , and is not considered. Hence, η_2 , V_{w2} , P_2 , q_{t2} , V_2 , and q_{c2} are unnecessary. They are computed, however.

If $\phi_c < \phi_c^*$, then the computations 27 through 34 are performed.

$$27. 1 + \frac{R^*}{\gamma} = \frac{1}{\eta^*}$$

$$28. 120 T_c^2 \exp(-\phi_c^*/\gamma) = J^*$$

$$29. J^*(\phi_c + 2\gamma) = q_e^*$$

$$30. \left[\frac{\eta_3}{2 - \eta_3} \right]^{1/2} S = (V_w)_{\text{opt}}^*$$

$$31. J^* \left[\frac{P^2}{2 V_w^*} - \frac{(V_w)_{\text{opt}}^*}{2} \right] = q_c^*$$

$$32. q_r + q_e^* + q_p + q_c^* = q_t^*$$

$$33. \phi_c^* - \phi_a - (V_w)_{\text{opt}}^* = V^*$$

$$34. J^* \times V^* = P^*$$

Now, if $\phi_c \geq \phi_c^*$, the following quantities are printed in order:

$\bar{\epsilon}$, h, T_c , T_a ,

ϕ_c , ϕ_a , q_r , J

ϕ_c^* , q_e , η_1 , η_2

$(V_w)_{opt\ 1}$, $(V_w)_{opt\ 2}$, P_1 , P_2

q_{t1} , q_{t2} , V_1 , V_2

q_{c1} , q_{c2}

If $\phi_c < \phi_c^*$, the following quantities are printed in order:

$\bar{\epsilon}$, h, T_c , T_a

ϕ_c , ϕ_a , η^* , $(V_w)_{opt}^*$,

P^* , V^* , q_c^* , q_t^* ,

q_e^* , J^* , ϕ_c^* .

The program is repeated with a proper increment in the quantities in the following order:

$\bar{\epsilon}$, T_c , h, T_a , ϕ_c , ϕ_a .

The nonlinear Eq. (11) is first put in the form, $R^* = f(R^*)$, to find the root R^* ; and ϕ_c^* is evaluated therefrom.

The term $\exp(\phi_c/\gamma)$ is evaluated as $\exp(\phi_c/\gamma - 20) \times 4.85165 \times 10^8$ to avoid overflow.

DISCUSSION

The elaborate data obtained from the computer is very useful in obtaining the performance of a particular system with specified values for ϕ_c , ϕ_a , h, $\bar{\epsilon}$, T_c , and T_a . A graphic summary extending Schock's work⁽¹⁾ appears in Figs. 2 through 25. Maximum efficiency (η_{max}) vs cathode temperature (T_c) is plotted in Figs. 2 through 13 for a range of values of ϕ_c , ϕ_a , $\bar{\epsilon}$, and h. In Figs. 14 through 25 the effects of ϕ_c^* , V^* , $(V_w)_{opt}^*$, and J^* vs T_c caused by variations of $\bar{\epsilon}$, h, and ϕ_a are shown on plots. Interpolation is also possible within the range of computation.

A sizable effect in η_{max} is observed when h is varied. In Fig. 6, for example, at $T_c = 1800$, $\phi_a = 1.6$, $\bar{\epsilon} = 0.1$, $T_a = 600$, and $\phi_c = 3.0$, we have the following values of h and η_{max} :

h	$\eta_{\max}$
0	17.5
0.005	12.0
0.01	8.5

However, for $\phi_c = 2.5$ and with the other conditions the same, variations in h have much smaller influence. Comparison of Figs. 6 and 9 shows that the effect of h becomes less predominant when $\bar{\epsilon}$ increases.

An illustrative example indicates further use of the computed results. Consider a thermionic converter with

$$\phi_c = 3.0, \phi_a = 1.6, h = 0.005, \bar{\epsilon} = 0.1, T_a = 600,$$

and T_c varying from 1800°K to 2400°K. Figure 26 shows the variation of efficiency $\eta_{\max}$, critical cathode work function ϕ_c^* , optimum load voltage V , optimum lead wire voltage drop $(V_w)_{\text{opt}}$, and current J with cathode temperature T_c .

The order of magnitude of h considered in this analysis is that estimated from the evaporation rate in a film-boiling liquid metal anode experiment,² where the temperatures are approximately $T_c = 1800^\circ\text{K}$ and $T_a = 600^\circ\text{K}$, the vapor pressure is about one millibar, and the spacing between the cathode and liquid metal anode is estimated very roughly to be 10^{-1} mm.

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²A. J. Ulrich, Thermionic Energy Conversion Diode Using a Film Boiling Liquid Metal Anode, ANL-6465 (Dec 1961), and J. Appl. Phys. (in press).

Fig. 2. Maximum Efficiency (η_{max}) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}K$; $v_a = 1.0$ volt; $\beta = 0.1$; $h = 0-0.01$ watt (cm^2) ($^{\circ}K$); and $\phi_c = 2.0-4.0$ volts

Fig. 3. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^\circ\text{K}$; $\phi_a = 1.0$ volt; $\bar{\epsilon} = 0.2$; $h = 0-0.01$ watt/(cm²)(°K); and $\phi_c = 2.0-4.0$ volts

Fig. 4. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}\text{K}$; $\phi_a = 1.0$ volt; $\bar{\epsilon} = 0.3$; $h = 0-0.01$ watt/(cm²)($^{\circ}\text{K}$); and $\phi_c = 2.0-4.0$ volts

Fig. 5. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^\circ\text{K}$; $\phi_a = 1.0$ volt; $\bar{\epsilon} = 1.0$; $h = 0-0.01$ watt/(cm²)(°K); and $\phi_c = 2.0-4.0$ volts

Fig. 6. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}\text{K}$; $\phi_a = 1.6$ volts; $\bar{c} = 0.1$; $h = 0-0.01$ watt $(\text{cm}^2)(^{\circ}\text{K})$; and $\phi_c = 2.0-4.0$ volts

Fig. 7. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^\circ\text{K}$; $\phi_a = 1.6$ volts; $\bar{\epsilon} = 0.2$; $h = 0-0.01$ watt/(cm²)(°K) and $\phi_c = 2.0-4.0$ volts.

Fig. 8. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}\text{K}$; $\phi_a = 1.6$ volts; $\bar{\epsilon} = 0.3$; $h = 0-0.01$ $\text{watt}/(\text{cm}^2)(^{\circ}\text{K})$; and $\phi_c = 2.0-4.0$ volts

Fig. 9. Maximum Efficiency (η_{max}) vs Cathode Temperature (T_c) for $T_a = 600^\circ\text{K}$; $\phi_a = 1.6$ volts; $\bar{\epsilon} = 1.0$; $h = 0-0.01$ watt/(cm²)(°K); and $\phi_c = 2.5-4.0$ volts

Fig. 10. Maximum Efficiency (η_{max}) vs Cathode Temperature (T_c) for $T_a = 600^\circ\text{K}$; $\phi_a = 2.2$ volts; $\bar{\epsilon} = 0.1$; $h = 0-0.01$ watt/(cm²)(°K); and $\phi_c = 2.5-4.0$ volts

Fig. 11. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}\text{K}$; $\gamma_a = 2.2$ volts; $\bar{\gamma} = 0.2$; $h = 0-0.01$ watt (cm²)($^{\circ}\text{K}$); and $\phi_c = 2.5-4.0$ volts

Fig. 12. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}\text{K}$; $\phi_a = 2.2$ volts; $\bar{\gamma} = 0.3$; $h = 0-0.01$ watt (cm²)($^{\circ}\text{K}$); and $\phi_c = 2.5-4.0$ volts

Fig. 13. Maximum Efficiency ($\eta_{\max}$) vs Cathode Temperature (T_c) for $T_a = 600^\circ\text{K}$; $b_a = 2.2$ volts; $\bar{\epsilon} = 1.0$; $h = 0-0.01$ watt/(cm²)(°K); and $\phi_c = 2.5-4.0$ volts

Fig. 14. Critical Cathode Work Function (ϕ_c^*) vs Cathode Temperature (T_c) for $T_a = 600^\circ\text{K}$; $\phi_a = 1.6$ volts; $h = 0.005$ watt/(cm²)(°K); and $\bar{\epsilon} = 0.1, 0.2, 0.3, 1.0$

Fig. 15. Critical Cathode Work Function (ϕ_c^*) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}\text{K}$; $\bar{\epsilon} = 0.1$; $\phi_a = 1.6$ volts; and $h = 0, 0.005, 0.01$ watt/(cm^2)($^{\circ}\text{K}$)

Fig. 16. Critical Cathode Work Function (ϕ_c^*) vs Cathode Temperature (T_c) for $T_a = 600^{\circ}\text{K}$; $h = 0.005$ watt/(cm^2)($^{\circ}\text{K}$); $\bar{\epsilon} = 0.1$; and $\phi_a = 1.0, 1.6, 2.2$ volts

Fig. 17.

Load Voltage (V^*) vs Cathode Temperature (T_c) for $\phi_c \leq \phi_c^*$ volts; $T_a = 600^{\circ}\text{K}$; $\phi_a = 1.6$ volts; $h = 0.005$ watt/(cm^2)($^{\circ}\text{K}$); and $\bar{\epsilon} = 0.1, 0.2, 0.3, 1.0$

Fig. 18.

Load Voltage (V^*) vs Cathode Temperature (T_c) for $\phi_c \leq \phi_c^*$ volts; $T_a = 600^{\circ}\text{K}$; $\bar{\epsilon} = 0.1$; $\phi_a = 1.6$ volts; and $h = 0, 0.005, 0.01$ watt/(cm^2)($^{\circ}\text{K}$)

Fig. 19.

Load Voltage (V^*) vs Cathode Temperature (T_c) for $\phi_c \leq \phi_c^*$ volts; $T_a = 600^{\circ}\text{K}$; $h = 0.005 \text{ watt}/(\text{cm}^2)(^{\circ}\text{K})$; $\bar{e} = 0.1$; and $\phi_a = 1.0, 1.6, 2.2$ volts

Fig. 20.

Optimized Lead Wire Voltage $[(V_w)_{\text{opt}}^*]$ vs Cathode Temperature (T_c) for $\phi_c \leq \phi_c^*$ volts; $T_a = 600^{\circ}\text{K}$; $\phi_a = 1.6$ volts; $h = 0.005 \text{ watt}/(\text{cm}^2)(^{\circ}\text{K})$; and $\bar{e} = 0.1, 0.2, 0.3, 1.0$

Fig. 22.

Optimized Lead Wire Voltage $[(V_w)^*_{opt}]$ vs Cathode Temperature (T_c) for $\phi_c \leq \phi_c^*$ volts; $T_a = 600^{\circ}\text{K}$; $h = 0.005$ watt/(cm^2)($^{\circ}\text{K}$); $\bar{\epsilon} = 0.1$; and $\phi_a = 1.0, 1.6, 2.2$ volts

Fig. 21.

Optimized Lead Wire Voltage $[(V_w)^*_{opt}]$ vs Cathode Temperature (T_c) for $\phi_c \leq \phi_c^*$ volts; $T_a = 600^{\circ}\text{K}$; $\bar{\epsilon} = 0.1$; $\phi_a = 1.6$ volts; and $h = 0, 0.005, 0.01$ watt/(cm^2)($^{\circ}\text{K}$)

Fig. 23. Current Density (J^*) vs Cathode Temperature (T_c) for $i_c \leq i_c^*$ volts; $T_a = 600^\circ\text{K}$; $i_a = 1.6$ volts; $h = 0.005$ watt/(cm^2)($^\circ\text{K}$); $\bar{e} = 0.1, 0.2, 0.3, 1.0$

Fig. 24. Current Density (J^*) vs Cathode Temperature (T_c) for $i_c \leq i_c^*$ volts; $T_a = 600^\circ\text{K}$; $\bar{e} = 0.1$; $i_a = 1.6$ volts; and $h = 0, 0.005, 0.01$ watt/(cm^2)($^\circ\text{K}$)

Fig. 25. Current Density (J^*) vs Cathode Temperature (T_c) for $\phi_c \leq \phi_c^*$ volts; $T_a = 600^\circ\text{K}$; $h = 0.005 \text{ watt}/(\text{cm}^2)(^\circ\text{K})$; $\bar{\epsilon} = 0.1$; and $\phi_a = 1.0, 1.6, 2.2$ volts

Fig. 26. Characteristics of an Illustrative Converter as a Function of Cathode Temperature for $\phi_c = 3.0$ volts; $\phi_a = 1.6$ volts; $T_a = 600^{\circ}\text{K}$; $\bar{\epsilon} = 0.1$; $h = 0.005$ watt/(cm²)($^{\circ}\text{K}$)