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ON THE CALCULATION OF
REACTIVITY WORTHS IN
FISSION REACTORS

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On the Calculation of Reactivity Worths in Fission Reactors*

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ABSTRACT

Two categories of approximations that can contribute to the reactivity worth discrepancy are considered: approximations in the theoretical formulation and its interpretation, and approximations in the actual calculations. Several pairs of expressions for the determination of reactivity corresponding to two experimental methods are given. One expression in each pair is for the experimental determination of reactivity and the other is a perturbation theory expression for the calculation of exactly the same reactivity.

Errors that can be caused in the calculation of reactivity worths as a result of inconsistent reduction of the continuous energy variable to the group approximations are investigated. Fine structure effects are investigated for resolved resonances and coarse structure effects are

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examined for smoothly varying cross sections.

Three methods for the calculation of group constants are considered: the flux method, the bilinear method, and the consistent method.

It is found that the fine structure details of the importance function can have significant effects on the calculated contribution of the resonance energy range to the reactivity worth. The flux method does not account correctly for these fine structure details. It is estimated that computational techniques in common use can overpredict the reactivity worth of single resolved resonances of both fertile and fissile isotopes by more than 30%.

Coarse structure considerations are expected to be important for the calculation of the reactivity contribution of processes involving a large energy transfer. For the range of problems studied the consistent method is found to be the most accurate method for reactivity calculations.

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INTRODUCTION

Several integral parameters were calculated by many groups for a series of 13 fast reactor benchmark assemblies as part of the Cross Section Evaluation Working Group (CSEWG) program for data testing.¹ The calculated results for three of these integral parameters; Keff, spectral indices (i.e., reaction rate ratios), and central reactivity worths, were compared with the corresponding experimental results. The largest discrepancy between the calculational and the experimental results is found for the central reactivity worths. This discrepancy has long been known and observed on many assemblies in many laboratories.² The ratio for the calculation to experiment (C/E) for the fissile and fertile isotopes is usually well above unity.^{1,2} We observe that of the three integral parameters considered only the calculation of the reactivity worth is based on perturbation theory formulation. The common practice is to use only the first order approximation to the exact formulation. When the comparison between calculations and measurements is carried out for the purpose of data testing, however, it is desirable to reduce uncertainties not originating from the nuclear data. Such uncertainties may result from approximations in the theoretical formulation and in the computational methods. These approximations need to be explored also as a part of a program to resolve the discrepancy between the measurement and the calculation of reactivities.

In the first part of this work we review the exact relationship between the theoretical formulations for the calculational and the experimental determination of reactivities. We also point to

approximations used in practical applications of these formulations. In the second part of the work we examine the approximations introduced while performing the calculations in practice (using an approximate or an exact expression). The emphasis is given to the treatment of the energy variable, and in particular, to the reduction of the continuous energy variable to the group approximation. This includes the generation of the group parameters.

Particular attention is given to the question of fine-structure effects. These are the effects associated with the details of the energy dependence in the resonance energy range. Our theoretical derivations and numerical examples are restricted to simple resolved narrow resonances.

Coarse structure effects were investigated in the past.³⁻⁶ We review the question of the generation of group constants representing smooth cross sections, starting from the point-wise data.⁵ This is done by an intercomparison of three methods for cross section averaging. These are the flux method, the bilinear method, and the consistent method.⁵ The question of collapsing a given set of group constants to a set of constants for a smaller number of groups had been investigated and reported in the literature.^{3,4,6}

Although discussed in the context of reactivity coefficients, or reactivity worths, for fast reactors the derivations are applicable equally well to all kinds of reactivities and for all reactor types. Moreover, the discussions concerning group constants averaging for reactivity calculations are relevant to the subject

of group cross sections for the calculation of bilinear parameters in reactor theory in general.

The numerical results presented in this work were obtained from numerical experiments based on simplified models. They are to be considered as illustrations of expected effects rather than results for realistic problems.

A COMPARISON OF EXPRESSIONS FOR PERTURBATION THEORY CALCULATION, AND EXPERIMENTAL DETERMINATION OF REACTIVITIES

Consider a reactor in which some perturbation, the reactivity effect of which is to be determined, is induced. For convenience let us assume that the unperturbed reactor is critical. We shall examine now definitions of several reactivities and compare the expressions for the calculation and for the experimental determination of these reactivities.

The Static Reactivity, ρ_k

The static reactivity, defined as

$$\rho_k = (k-1)/k \quad (1)$$

is the reactivity most commonly calculated. In Eq. (1) k is the eigenvalue of the Boltzmann equation for the perturbed reactor:

$$G \phi_k(x) + \frac{1}{k} F \phi_k(x) = 0 \quad , \quad (2)$$

where $(G + F)$ is the Boltzmann operator for the perturbed reactor and F is its fission component. Let H designate the Boltzmann operator for the unperturbed reactor,

$$H \phi_0(x) = 0 \quad (3)$$

and let δH stand for the perturbation from criticality. Then,

$$(G + F) \phi(x) = (H + \delta H) \phi(x) \quad (4)$$

Multiplying Eqs. (2) and (4) by the importance function, the solution of the adjoint equation for the critical reactor:

$$H^+ \phi_0^+(x) = 0 \quad (5)$$

integrating over the phase space, and making use of the mathematical properties of the adjoint we find:

$$\rho_k = \frac{\langle \phi_0^+, \delta H \phi_k \rangle}{\langle \phi_0^+, F \phi_k \rangle} \quad (6)$$

In the above we use the following notations:

$$(x) \equiv (\vec{r}, E, \vec{\Omega}) \quad (7)$$

and

$$\langle f(x), g(x) \rangle \equiv \int_{\text{phase space}} dx f(x) g(x) \quad (8)$$

Equation (6) is an exact expression for ρ_k , defined in Eq. (1), provided the adjoint function used is the exact solution of the adjoint equation for the unperturbed reactor, and the flux is the exact solution of the perturbed reactor flux (Boltzmann) equation. Equation (6) will be correct to first order when the flux of the unperturbed reactor is substituted for the flux of the perturbed reactor. The association of the adjoint function with the unperturbed reactor and of the flux with the perturbed

reactor can be interchanged. The common practice in reactivity calculations is to use the first order perturbation theory expression, with both the flux and the adjoint coming from the solution of the equations for the unperturbed system.

The Asymptotic Period Reactivity

Central reactivity worths are usually determined experimentally by the Asymptotic Period (AP) method, or by other methods intercalibrated against the AP method. The reactivity is then inferred from the measured period, τ , using the Inhour equation. Let us examine the relationship between the reactivity thus determined experimentally, between the perturbation theory formulation for this reactivity, and between the perturbation theory expression for ρ_k .

In a reactor running on the asymptotic period, the flux can be expressed as

$$\phi(x,t) = \phi_{\omega}(x) e^{\omega t} \quad (9)$$

where $\omega = 1/\tau$ and $\phi_{\omega}(x)$ is the eigenfunction of the following Boltzmann equation:

$$\left[G - \frac{\omega}{V}\right] \phi_{\omega}(x) + \left[F_p + \sum_i \frac{\lambda_i}{(\lambda_i + \omega)} F_i\right] \phi_{\omega}(x) = 0 \quad (10)$$

The operators F_p and F_i are the fission operator components corresponding to the prompt neutrons and the i th delayed neutron group, respectively. Upon rearrangement Eq. (10) becomes

$$[G + F]\phi_\omega(x) = \left[\frac{\omega}{v} \phi_\omega(x) + \sum_i \frac{\omega}{(\lambda_i + \omega)} F_i \phi_\omega(x) \right] \quad (11)$$

Multiplying Eq. (11) by $\phi_0^+(x)$ [the solution of Eq. (5)] integrating over phase space, and making use of Eq. (4) and the properties of the adjoint, we find:

$$\frac{\langle \phi_0^+, \delta H \phi_\omega \rangle}{\langle \phi_0^+, F \phi_\omega \rangle} = \frac{\bar{\Lambda}_\omega}{\tau} + \sum_i \frac{(\bar{\beta}_i)_\omega}{(1 + \lambda_i \tau)} \quad (12)$$

The effective prompt neutron generation time, $\bar{\Lambda}_\omega$, and the effective delayed neutron fraction $(\bar{\beta}_i)_\omega$ are defined as:

$$\bar{\Lambda}_\omega \equiv \frac{\langle \phi_0^+, 1/v \phi_\omega \rangle}{\langle \phi_0^+, F \phi_\omega \rangle} \quad (13)$$

$$(\bar{\beta}_i)_\omega \equiv \frac{\langle \phi_0^+, F_i \phi_\omega \rangle}{\langle \phi_0^+, F \phi_\omega \rangle} \quad (14)$$

and

$$\bar{\beta}_\omega \equiv \sum_i (\bar{\beta}_i)_\omega \quad (15)$$

Equation (12) together with Eqs. (5), (11), (13), and (14) provide an exact relationship between the measured period and a perturbation theory expression for a reactivity. Notice, however, the following: (1) The left-hand side of Eq. (12), to be designated as ρ_ω , is different, in general, from ρ_k of Eq. (6). That is, in principle, $\rho_\omega \neq \rho_k$ unless $\rho \rightarrow 0$. The two expressions will be the same, however, in first order perturbation theory. (2) The value of the effective kinetic parameters, defined in

Eqs. (13) - (15) are different from the corresponding physical value, and may also be different from the values measured for the same parameters by the various experimental techniques.

The Source Multiplication Reactivity

As another example for the relation between a measured and a calculated reactivity, let us consider the Source Multiplication and Rate of linear power rise (SMR) method for reactivity measurement.⁷ In this method one measures the response of the critical (unperturbed) reactor, and the response of the perturbed (subcritical) reactor to an external neutron source. After initial transients die out, the response of a given detector in the critical reactor will increase linearly with time with the rate R. The response of the same detector to the same source in the subcritical reactor will reach, asymptotically, the value M.

The perturbed reactor flux equation is:

$$(G + F) \phi_s(x) = (H + \delta H) \phi_s(x) = S(x) \quad (16)$$

where $S(x)$ is the distribution of the external source. Following the previous procedures we find that

$$\rho_s \equiv \frac{\langle \phi_0^+, \delta H \phi_s \rangle}{\langle \phi_0^+, F \phi_s \rangle} = \frac{\langle \phi_0^+, S \rangle}{\langle \phi_0^+, F \phi_s \rangle} \quad (17)$$

The relation between ρ_s and the experimentally measured parameters, R and M, can be obtained from the time-dependent Boltzmann equation for the unperturbed reactor. Thus expressing the flux time evolution (and similarly for the delayed neutrons precursors) as:⁷

$$\phi(x,t) = \phi_{\text{ext}}(x) + \phi_o(x) R t \quad (18)$$

we find

$$\frac{R}{v} \phi_o(x) + \sum_i \frac{R}{\lambda_i} F_i \phi_o(x) = S(x) \quad (19)$$

After multiplying by $\phi_o^+(x)$, integrating over phase space, and dividing by $\langle \phi_o^+, F \phi_s \rangle$ Eq. (19) becomes:

$$\rho_s = \frac{R}{M} \left[\bar{\Lambda}_s + \sum_i \frac{(\bar{\beta}_i)_s}{\lambda_i} \right] \quad (20)$$

where

$$\bar{\Lambda}_s \equiv \frac{\langle \phi_o^+, 1/v \phi_o \rangle}{\langle \phi_o^+, F \phi_{so} \rangle} = \frac{\langle \phi_o^+, 1/v \phi_{so} \rangle}{\langle \phi_o^+, F \phi_{so} \rangle} \left[\frac{\langle \phi_o^+, 1/v \phi_o \rangle}{\langle \phi_o^+, 1/v \phi_{so} \rangle} \right] \quad (21)$$

$$(\bar{\beta}_i)_s \equiv \frac{\langle \phi_o^+, F_i \phi_o \rangle}{\langle \phi_o^+, F \phi_{so} \rangle} \quad (22)$$

and $\phi_{so}(x)$ is the solution of Eq. (16) normalized so that

$$\langle 1, \phi_{so} \rangle = \langle 1, \phi_o \rangle \quad (23)$$

Equation (20) and Eq. (17) provide expressions for the same reactivity, ρ_s . Equation (20) provides the relation between the measured parameters and the reactivity, whereas Eq. (17) is the perturbation theory expression for the calculation of the reactivity.

We conclude this subject by making the following observations:

1. Different experimental techniques can measure reactivities of different definition. In principle, $\rho_w \neq \rho_s \neq \rho_k$ unless $\rho = 0$.

2. It is possible to derive (and use) perturbation theory formulae that give exact expressions for the reactivities that are determined experimentally. Several alternative expressions for the calculational and experimental determination of reactivities are summarized below:

AP Method

$$\rho_w = \frac{\langle \phi_o^+, \delta H \phi_w \rangle}{\langle \phi_o^+, F \phi_w \rangle} = \frac{\bar{\Lambda}_w}{\tau} + \sum_i \frac{(\bar{\beta}_i)_w}{(1 + \lambda_i \tau)} \quad (24)$$

$$\left(\frac{\rho_w}{\bar{\Lambda}_w} \right) = \frac{\langle \phi_o^+, \delta H \phi_w \rangle}{\langle \phi_o^+, 1/v \phi_w \rangle} = \frac{1}{\tau} + \left(\frac{\bar{\beta}_w}{\bar{\Lambda}_w} \right) \sum_i \frac{(\bar{\beta}_i)_w / \bar{\beta}_w}{(1 + \lambda_i \tau)} \quad (25)$$

$$\left(\frac{\rho_w}{\bar{\beta}_w} \right) = \frac{\langle \phi_o^+, \delta H \phi_w \rangle}{\sum_i \langle \phi_o^+, F_i \phi_w \rangle} = \frac{1/\tau}{(\bar{\beta}_w / \bar{\Lambda}_w)} + \sum_i \frac{(\bar{\beta}_i)_w / \bar{\beta}_w}{(1 + \lambda_i \tau)} \quad (26)$$

SMR Method

$$\rho_s = \frac{\langle \phi_o^+, \delta H \phi_s \rangle}{\langle \phi_o^+, F \phi_s \rangle} = \frac{R}{M} \left(\bar{\Lambda}_s + \sum_i \frac{(\bar{\beta}_i)_s}{\lambda_i} \right) \quad (27)$$

$$\left(\frac{\rho_s}{\bar{\Lambda}_s} \right) = \frac{\langle \phi_o^+, \delta H \phi_s \rangle}{\langle \phi_o^+, 1/v \phi_s \rangle} = \frac{R}{M} \left[1 + \left(\frac{\bar{\beta}_s}{\bar{\Lambda}_s} \right) \sum_i \frac{(\bar{\beta}_i)_s / \bar{\beta}_s}{\lambda_i} \right] \quad (28)$$

$$\left(\frac{\rho_s}{\bar{\beta}_s}\right) = \frac{\langle \phi_0^+, \delta H \phi_s \rangle}{\sum_i \langle \phi_0^+, F_i \phi_s \rangle} = \frac{R}{M} \left(\frac{1}{(\bar{\beta}_s / \bar{\Lambda}_s)} + \sum_i \frac{(\bar{\beta}_i)_s / \bar{\beta}_s}{\lambda_i} \right) \quad (29)$$

3. Approximations in the formulation for the comparison between measured and calculated reactivities can be introduced by either one of the following actions: (a) The use of adjoint and/or flux obtained from the solution of equations that are inconsistent with the exact definition. (b) The approximation of the flux for the perturbed reactor by the unperturbed reactor flux (or vice versa); or a similar type of approximation for the adjoint function. (c) The use of effective kinetic parameters that are inconsistent with their definition in the expressions used for translating the measured parameters to the reactivity.

It should be emphasized, though, that the various expressions for reactivity are correct to first order. The magnitude of possible errors introduced by these approximations decrease as the reactivity becomes smaller. In the limit $\rho \rightarrow 0$ these errors will be negligibly small.

THE CALCULATION OF REACTIVITY WORTHS

Let us suppose that the perturbation theory expression for the calculation of reactivity is consistent with the experimentally determined reactivity. The question we shall be concerned with now is how to go about calculating the perturbation theory expression numerically. There are several practical difficulties

in evaluating these expressions, including the description of spatial effects (heterogeneties), of anisotropy effects and of the energy variable. We restrict this work to the energy variable and consider the particular question of the reduction of the continuous energy variable to the group (multi-group or few-group) approximation. This for the purpose of the calculation of reactivity coefficients, and bilinear expressions in general.

Thus consider a general bilinear integral of the form:

$$B \equiv \int dE \phi^+(E) P \phi(E) \quad (30)$$

where P represents the perturbation operator or, in general, any term of the Boltzmann operator. In the group approximation B is calculated from an expression of the form

$$B = \sum_{g=1}^G \phi_g^+ P_g \phi_g \equiv \sum_{g=1}^G B_g \quad (31)$$

The question is how to define the group quantities so as to preserve the value of B . A cross section averaging method that will assure that the value of B_g (and hence B) calculated using the group quantities will be identical to its value calculated using the continuous energy integration is the following:

$$P_g \equiv \int_{E_g^-}^{E_g^+} dE \phi^+(E) P \phi(E) / \phi_g^+ \phi_g \quad (32)$$

ϕ_g and ϕ_g^+ are, respectively, the total fine group flux and the average fine group importance function:

$$\phi_g \equiv \int_{E_g^-}^{E_g^+} dE \phi(E) \quad (33)$$

and

$$\phi_g^+ = \frac{1}{\Delta E_g} \int_{E_g^-}^{E_g^+} dE \phi^+(E) , \quad (34)$$

where

$$\Delta E_g \equiv E_g^+ - E_g^- . \quad (35)$$

These group fluxes and adjoints are, respectively, the solutions of the appropriate group Boltzmann equation:

$$\mathcal{D}_g \phi_g(x) = 0 \quad (36)$$

and group adjoint equation:

$$A_g \phi_g^+(x) = 0 . \quad (37)$$

The group constants included in the $\mathcal{D}_g$ operator are to be obtained using the flux as the weighting spectrum, i.e.,

$$\mathcal{D}_g = \int_{E_g^-}^{E_g^+} dE \mathcal{D} \phi(E) / \phi_g . \quad (38)$$

Whereas the corresponding group constants for the adjoint equations are to be obtained using the importance function as the weighting spectrum:

$$A_g = \int_{E_g^-}^{E_g^+} dE A \phi^+(E) / \phi_g^+ \Delta E_g . \quad (39)$$

We shall refer to this method as the consistent method for cross section averaging.⁵ It is consistent with the continuous energy formulation in the sense that it preserves the numerical value and physical meaning of both the bilinear expressions, the neutron flux, and the importance function.

We shall compare this consistent method with two other well known cross section averaging methods: the flux method and the bilinear method. In these two methods one uses a single set of group-constants for the calculation of bilinear expressions, fluxes and adjoints. These group constants are obtained by using the flux or the bilinear function, as the weighting function in the flux and bilinear methods, respectively. Table I lists the weighting functions utilized for the different purposes in the three cross section averaging methods.

Fine Structure Effects

Consider a fine group g that spans the energy interval $E_g^- \leq E < E_g^+$ in the resolved resonance region. The term fine group structure used here means the first group structure for which group constants are prepared from the point wise data. We shall calculate the relative contribution of the fine group resonance cross sections to the reactivity worth using two methods: the consistent method and the flux method. For this intercomparison study we assume knowledge of the detailed energy dependence of the cross sections, flux, and importance function within the fine group.

Thus consider a fine group g in which there is a single resonance corresponding to one of the major constituents of the reactor (see Fig. 1). Let us also make the following simplifying assumptions:

1. The resonance satisfies the NR approximation.
2. The resonance cross section can be expressed by the single level Breit-Wigner formula at zero degrees.

3. There are no interference effects in the scattering cross section. Scattering is isotropic.

4. In the energy range of interest

$$(E - E_0)/E_0 \ll 1, \quad (40)$$

where E_0 is the resonance energy (see Fig. 1). With this assumption we can take E to be a constant in the range of integration and consider only the variation with $(E - E_0)$.

The flux fine-structure in the vicinity of a resonance obeying the NR approximation can be obtained⁸ from simple neutron conservation considerations:

$$\phi(E) = \frac{\phi_{as} \Sigma_p}{\Sigma_t(E)} \propto \frac{1}{\Sigma_t(E)} \quad (41)$$

where ϕ_{as} and Σ_p are the value of the flux and total macroscopic cross section, respectively, if the resonance cross section were to be replaced by the smooth background cross sections.

Using the principle of importance conservation we find for the fine structure of the importance function in the vicinity of the resonance:

$$\phi^+(E) = \phi_{as}^+ \frac{\Sigma_s(E)}{\Sigma_t(E)} + \phi_f^+ \frac{\nu \Sigma_f(E)}{\Sigma_t(E)} \quad (42)$$

where ϕ_{as}^+ and ϕ_f^+ are the average value of the importance function for a neutron that has undergone a scattering collision at the resonance energies and for a fission born neutron, respectively.

Replacing the energy variable by

$$x \equiv 2(E - E_0)/\Gamma \quad (43)$$

we have the following expressions for the cross sections:

$$\sigma_s(x) = \sigma_o \frac{\Gamma_n}{\Gamma} \frac{1}{(1+x^2)} \quad (44)$$

$$\sigma_Y(x) = \sigma_o \frac{\Gamma_Y}{\Gamma} \frac{1}{(1+x^2)} \quad (45)$$

$$\sigma_f(x) = \sigma_o \frac{\Gamma_f}{\Gamma} \frac{1}{(1+x^2)} \quad (46)$$

$$\Sigma_s(x) = \Sigma_p \left(\frac{(s^2 + x^2)}{(1+x^2)} \right) \quad (47)$$

$$v\Sigma_f(x) = \Sigma_p \left(\frac{\Gamma^2}{(1+x^2)} \right) \quad (48)$$

$$\Sigma_t(x) = \Sigma_p \left(\frac{(t^2 + x^2)}{(1+x^2)} \right) \quad (49)$$

where,

$$\Sigma_p = N_R \sigma_{p,R} + N_m \sigma_m \quad (50)$$

and $\sigma_{p,R}$ is the microscopic potential scattering cross section of the resonance isotope, the atomic density of which is N_R . $N_m \sigma_m$ is the effective macroscopic total cross section of all the isotopes excluding the resonance isotope.

$$t^2 = \left(1 + \frac{N_R \sigma_o}{\Sigma_p} \right) \quad (51)$$

$$s^2 = \left(1 + \frac{\Gamma_n}{\Gamma} \frac{N_R \sigma_o}{\Sigma_p} \right) \quad (52)$$

and

$$f^2 = v \frac{\Gamma_f}{\Gamma} \frac{N_R \sigma_o}{\Sigma_p} \quad (53)$$

With the above expressions for the cross sections Eqs. (41) and (42) can be expressed in the following manner:

$$\phi(x) = \phi_{as} \left(\frac{(1 + x^2)}{(t^2 + x^2)} \right) \quad (54)$$

and

$$\phi^+(x) = \phi_{as}^+ \left(\frac{(a^2 + x^2)}{(t^2 + x^2)} \right) \quad (55)$$

with

$$a^2 \equiv s^2 + (\phi_f^+ / \phi_{as}^+) f^2 \quad (56)$$

Let us examine now the relative shape of $\phi(x)$ and $\phi^+(x)$ by considering three special cases:

a. A Capture Resonance, $\Gamma_\gamma = \Gamma$

As $\Gamma_n = \Gamma_f = 0$, we find that $f^2 = 0$, $a^2 = s^2 = 1$, and therefore

$$\phi^+(x) = \phi_{as}^+ \left(\frac{(1 + x^2)}{(t^2 + x^2)} \right) \propto \phi(x) \quad (57)$$

and

$$\phi^+(x) \phi(x) \propto \phi^2(x) \quad (58)$$

In this special case the importance function has the same relative depression in the resonance region as exhibited by the flux.

Therefore the weighting function appropriate for obtaining the fine group cross sections for the calculation of bilinear parameters (in the consistent and bilinear methods) is the square of the weighting function used in the flux method!

b. A scattering Resonance, $\Gamma_n = \Gamma$

Here $s^2 = t^2$, $f^2 = 0$, $a^2 = s^2 = t^2$, and therefore

$$\phi^+(x) = \phi_{as}^+ = \text{constant} \quad (59)$$

c. A Fission Resonance, $\Gamma_f = \Gamma$

Here $s^2 = 1$, and

$$a^2 = t^2 + \frac{N_R \sigma_o}{\Sigma_p} \left(v \frac{\phi_f^+}{\phi_{as}^+} - 1 \right) \quad (60)$$

For most problems we expect $v\phi_f^+/\phi_{as}^+ > 1$ and so $a^2 > t^2$. In such a case $\phi^+(x) > \phi_{as}^+$ as $x \rightarrow 0$. That is, we expect that the importance function will have a peak at a fission resonance while the flux will still have the usual resonance depression. The relative behavior of the importance function fine structure in the vicinity of the three resonances considered is shown schematically in Fig. 2.

At this point we can already conclude that fine structure details of the importance function may have significant contributions to the calculated reactivities. In particular, for problems in which the contribution of the resolved resonances to the reactivity is significant, we can expect to obtain noticeable errors in reactivity calculations based on group constants generated using the flux as the weighting function. It is interesting to note here that a replacement of $1/\Sigma_t(E)$ by $1/E\Sigma_t(E)$ as the weighting function was found⁹ to have noticeable effects on the calculated neutronics properties of certain LMFBR problems.

As an illustration for the magnitude and direction of errors in calculated reactivities that can be introduced by the fine structure effects, we have numerically worked out two examples. The first example is for the 273.7 eV capture-scattering resonance of U-238. The second example is for the 32.3 eV fission resonance of P_u -239. The resonance parameters used for the two examples are summarized in Table 2. The P_u -239 resonance does not satisfy the NR approximation, but rather falls in between the NR and the NRIM approximations. Nevertheless, we have treated this resonance as a NR, expecting that the conclusions from this approximate treatment will be, qualitatively, the same as from a more accurate model.

Figures 3 and 4 show the relative distribution of the flux, the importance function, and the bilinear function ($\phi^+\phi$) for the two examples, respectively. For the data in these figures, as well as for the rest of the numerical calculations, we assumed the following:

1. The effective microscopic potential scattering cross section is 30 barns, that is about three times that of the U-238 potential scattering cross section.

2. The nonresonance absorption cross sections are zero. The step change in the asymptotic values of ϕ and ϕ^+ across the resonance is neglected.

3. In the fission resonance problem $\phi_{as}^+ = \phi_f^+$.

The distributions shown in Figs. 3 and 4 are in general agreement with the expectations based on the special case analysis (see Fig. 2). The bilinear function is lower than the flux in the capture resonance (Fig. 3) but higher than the

flux in the fission resonance (Fig. 4). Accordingly we expect that the value of a bilinear expression for the resonance absorption cross section calculated using the flux method will be overestimated for a capture resonance problem, but will be underestimated for a fission resonance problem.

The contribution of the resonance absorption cross section in fine group g to the numerator of the perturbation theory expression for reactivity is proportional to:

$$B_g = \int_0^{x_g} dx \phi^+(x) \sigma_a(x) \phi(x) , \quad (61)$$

where x_g is the value of x corresponding to the fine group limit.

The value of B_g is essentially proportional to the reactivity worth of the resonance in group g . This is so because the denominator of the perturbation theory expression for reactivity [e.g., Eq. (6)] is not sensitive to the cross section averaging method. The calculations of the fission rate by the flux and consistent method, are identical. The average importance of the fission neutrons is not sensitive to the averaging method as long as $\chi(E)$ or $\phi^+(E)$ do not vary much within the group limits.

The group expressions for B_g in the consistent and in the flux methods are, respectively:

$$B_g^C = \sigma_{a,g}^{\phi^+} \phi_g^+ \phi_g \quad (62)$$

and

$$B_g^F = \sigma_{a,g}^{\phi} (\phi_g^+)^{\phi} \phi_g , \quad (63)$$

where $\sigma_{a,g}^{\phi}$ and $\sigma_{a,g}^{\phi^+}$ are the group absorption cross section calculated using, respectively, the flux and the bilinear function

as the weighting function. ϕ_g and ϕ_g^+ are defined, respectively, by Eqs. (33) and (34), and $(\phi_g^+)^{\phi}$ is the group adjoint calculated with flux averaged group constants:

$$(\phi_g^+)^{\phi} = \frac{\sum_{s,g}^{\phi}}{\sum_{t,g}^{\phi}} \phi_{as}^+ + \nu_g \frac{\sum_{f,g}^{\phi}}{\sum_{t,g}^{\phi}} \phi_f^+ \quad (64)$$

We have calculated the ratio B_g^F/B_g^C for the absorption cross section of the resonance isotope for the two examples. In these numerical experiments the results from the consistent method stand for the experimental reactivities whereas the results from the flux method stand for the calculated reactivities. Therefore the ratio B_g^F/B_g^C corresponds to the C/E ratio for the contribution of the group to the reactivity worth of the resonance in question.

The results for this C/E ratio are plotted for the U-238 and the P_u-239 problems in Figs. 5 and 6, respectively, as a function of the group width. It is observed that the C/E ratio deviation from unity increases, up to some asymptotic value, as the group width increases. In the capture resonance $C/E \geq 1.0$; whereas in the fission resonance $C/E \leq 1.0$, when only the contribution of the absorption cross section to reactivity is considered. These opposite trends are expected in view of the difference in the relative shape of the importance function in the two problems, as discussed before.

The total contribution of the fission resonance to reactivity has two components: The fission contribution, which is positive, and the absorption contribution, which is negative. That is:

$$\rho \propto \phi_f^+ \vee \int_0^x g \, dx \, \sigma_f(x) \phi(x) - \int_0^x g \, dx \, \phi^+(x) \sigma_a(x) \phi(x) \quad . \quad (65)$$

The rightmost integral is the same one expressed in Eq. (61). The contribution of the fission term in Eq. (65) is the same (assuming ϕ_f^+ is the same) when calculated by the consistent and by the flux methods. The C/E ratio for the total reactivity worth of the 32.3 eV P_u -239 resonance (neglecting scattering effects) is shown in Fig. 6 (as the solid line). In calculating it we assumed, as before, that $\phi_f^+ = \phi_{as}^+$. It is observed that $C/E \geq 1.0$ also for the fission resonance and approaches a value of 1.52, in this example, as the group width increases. Thus, $C/E \geq 1.0$ for both the U-238 capture resonance and for the P_u -239 fission resonance considered. Moreover, when compared as a function of group width in units of the practical resonance width (Γ_p , marked in Figs. 5 and 6), C/E have a similar shape for the two resonances. Its value for the U-238 resonance is 1.02, 1.25, and 1.31 for group widths of Γ_p , $10\Gamma_p$, and $100\Gamma_p$, respectively. The corresponding values for the P_u -239 are, 1.07, 1.44, and 1.51.

It is interesting to note that present codes for the preparation of group constants for fast reactor calculations employ group widths of the order of 0.01 lethargy units¹⁰ or larger. The C/E ratio for the narrow resonances is significantly different from 1.0 even for the narrowest of these groups. It is clear that errors of this magnitude should be eliminated from the calculations if accurate reactivity values or meaningful testing of the nuclear data are looked for.

The elimination or reduction of the fine structure errors may be achieved in one of two ways, within the framework of perturbation theory calculations:

1. By the employment of the consistent method for the generation of group constants.
2. By the use of a very fine group structure for the calculation. With a group width of the order of the practical resonance width (0.0023 and 0.0088 lethargy units in the first and second examples, respectively) the C/E values for the two examples considered approach unity (see Figs. 5 and 6). Such calculations can be performed, for example, using Monte-Carlo codes.

Bilinear expressions in use in reactor calculations employ adjoint functions of different physical meaning. The fine-structure of the adjoint function will depend, in general, on its physical meaning (or the detector response it represents). Hence, a consistent calculation of different bilinear parameters will require, in general, group constants obtained by averaging the same basic data with different bilinear weighting functions. We can visualize problems in which the flux method will overestimate the value of a bilinear expression for one physical parameter, and it will underestimate the value of the bilinear expression for a different physical parameter. In both cases the same basic cross sections are used, and the same system is considered.

Finally let us recall that stationary functionals, involving bilinear expressions, are sometimes employed in reactor theory to

obtain accurate calculation of reaction rates or their ratios. Unfortunately, errors that can be introduced in the calculation of bilinear expressions by the flux method can, in certain problems, make the stationary functional calculations less accurate than the straightforward calculations.

Coarse Structure Effects

We review here results from a series of numerical experiments performed in 1967⁵ concerning the following two subjects:

1. Smooth cross section averaging for reactivity calculations, and
2. A comparison between the three methods (flux, bilinear, and consistent) for cross section averaging for reactivity (or bilinear parameters in general) calculations.

The numerical experiments were performed for the GODIVA reactor, by comparing results obtained from a model based on the continuous energy variable with results obtained using the few-group approximation. For the continuous energy model we have used, Stacey's expressions¹¹ for the flux and importance function, and Schmidt's¹² pointwise (tabulated) cross sections. The energy range considered is $1 \text{ keV} \leq E \leq 10 \text{ MeV}$ and it was represented by 335 energy points. Several few group structures have been examined, with the 6-group Los Alamos structure chosen as the reference.

The flux and importance function of the continuous energy model were calculated from the expressions:¹¹

$$\phi(E) = \frac{\chi(E)}{\Sigma(E)} + \alpha [\Sigma, \Sigma_{in}, \chi, P] \frac{P(E)}{\Sigma(E)} \quad (66)$$

and

$$\phi^+(E) = \frac{\nu \Sigma_f(E)}{\Sigma(E)} + \beta [\Sigma, \nu \Sigma_f, \Sigma_{in}, P] \frac{\Sigma_{in}(E)}{\Sigma(E)} \quad (67)$$

where

$$\alpha = \int_0^\infty dE (\Sigma_{in}/\Sigma) \chi(E) / [1 - \int_0^\infty dE (\Sigma_{in}/\Sigma) P(E)] \quad (68)$$

and

$$\beta = \int_0^\infty dE (\nu \Sigma_f/\Sigma) P(E) / [1 - \int_0^\infty dE (\Sigma_{in}/\Sigma) P(E)] \quad (69)$$

The effective cross section is:

$$\Sigma = \Sigma_t - \Sigma_e + \Sigma_L [\Sigma_{tr}, B] \quad (70)$$

where the leakage cross section is given by:

$$\Sigma_L(E) = \Sigma_{tr}(E) \left(\frac{(\Sigma_{tr}/B)}{\tan^{-1}(\Sigma_{tr}/B)} - 1 \right) \quad (71)$$

and B is the geometric buckling of the system (inversely proportional to the square of the radius).

Schmidt's data is used for the total (t), transport (tr), elastic (e), and inelastic (in) cross sections. The spectrum of the fission neutrons (χ), of the inelastically scattered neutrons (P), and the number of neutrons per fission (ν), are calculated from the following expressions:

$$\chi(E) = \frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{\theta^{3/2}} \exp(-E/\theta) \quad (72)$$

$$P(E) = \frac{E}{T^2} \exp(-E/T) \quad (73)$$

and

$$v(E) = 2.43 + bE \quad (74)$$

Equations (66) and (67) were used to represent both "exact" (in the scope of the model used) and approximate weighting functions for the generation of group constants. The approximate functions were obtained by changing the value of one of the following four parameters: R (radius), b , θ , and T . The nominal values of these parameters, defining the exact functions, were:

$$R_0 = 8.7 \text{ cm}$$

$$b_0 = 0.1346 \text{ MeV}^{-1}$$

$$\theta_0 = 1.32 \text{ MeV}$$

and

$$T_0 = 0.33 \text{ MeV}$$

Figure 7 shows the exact flux and importance function for GODIVA. The 6 group importance functions calculated using the flux and the consistent cross section averaging methods are also shown in Fig. 7. It is observed that the group importance function can be highly distorted when flux averaged group constants are used. The distortion is usually large in energy ranges where the slope of the flux differs considerably from the slope of the importance function. The distortion usually increases also as the group width becomes larger. Similar observations were reported also by other researchers.^{3,4,6}

The reactivity worth of hydrogen and U-235 in GODIVA were calculated using both the continuous energy formulation and the

group formulations. The group calculations were performed with group constants generated by each one of the three averaging methods and for a variety of approximate weighting functions. The continuous energy expressions used for the calculation of the reactivities are:

$$\rho^H \propto - \int_0^{\infty} dE \phi^+(E) \sigma_t^H(E) \phi(E) + \int_0^{\infty} dE \frac{\sigma_e^H(E)}{E} \phi(E) \int_0^E dE' \phi^+(E') \quad (75)$$

and

$$\begin{aligned} \rho^{25} \propto & - \int_0^{\infty} dE \phi^+(E) [\sigma_t^{25}(E) - \sigma_e^{25}(E)] \phi(E) \\ & + \int_0^{\infty} dE \sigma_{in}^{25}(E) \phi(E) \int_0^E dE' \phi^+(E') P(E') \\ & + \int_0^{\infty} dE \nu \sigma_f^{25}(E) \phi(E) \int_0^{\infty} dE' \phi^+(E') \chi(E') \quad . \end{aligned} \quad (76)$$

Figure 8 shows the effect of the cross section averaging method and of the distortion in the weighting functions on the reactivity worth of hydrogen in GODIVA. The results shown in the figure are the hydrogen reactivity worth as calculated in the 6 group approximation relative to the exact reactivity, i.e., the reactivity calculated from Eq. (75) using the exact spectra of the model employed. The dashed line sections in Figs. 8(a) and 8(c) were obtained by extrapolation, as the calculated data for $R/R_0 = 0.5$ and for $b/b_0 = 0$ were found to be inconsistent with the rest of the results.

The following observations can be made referring to the data of Fig. 8:

1. The consistent and bilinear methods give identical results when the exact spectra are used for the weighting functions. This result is also identical with the exact result, that is, with the result obtained using the continuous energy variable and the exact spectra.

2. The flux method underestimates the reactivity worth of hydrogen by 29%, when the exact spectra are used for the weighting functions. This error is due mainly to the distortion in the 6 group adjoint distribution caused by the flux method (see Fig. 7).

3. The error in the calculated reactivity usually increases as the weighting functions become more distorted relative to the exact spectra for the problem.

4. For all series of approximate (distorted) weighting functions considered, the reactivity calculated using the consistent method is the closest to the exact reactivity. The bilinearly calculated reactivity is found to be more sensitive to the weighting functions than the consistent method.

5. For certain categories of approximate weighting spectra the reactivity calculated by the flux method is the least sensitive to the degree of distortion in the spectra. This is, for example, the case when the value of ν is changed [Fig. 8(c)] since a change in $\nu(E)$ leaves the flux unchanged, as seen from Eq. (66). Such a change will, however, effect the importance function distribution [Eq. (67)]. On the other hand we expect the flux to be more sensitive to variations in θ than the importance function. Both the flux and the importance functions are expected to be effected by a change in the leakage (R).

The error in the hydrogen reactivity worth introduced by the flux method is found to depend strongly on the group structure. Thus, for example, this error increases to 92% when a two-group structure is employed.

The reactivity worth of U-235 in GODIVA was found to be insensitive to the cross section averaging method in the model employed for the present investigation.

We conclude that coarse structure effects are expected to be important for the calculation of reactivity coefficients of low-absorption light scatterers. The reactivity worth of such isotopes depends on the difference between the importance function at various energies rather than on the absolute value of the importance function. The errors in importance function differences are considerably larger than errors in the absolute value of the importance function.

The errors in reactivity worth calculations introduced by coarse structure effects when the flux method is used will decrease, in general, as the number of groups increases. The number of groups required to eliminate these errors is usually within the range of the computing capability available at present.

SUMMARY

Two categories of approximations that can contribute to the discrepancy between calculated and measured reactivity worths were considered in this work:

1. Approximations made in the formulation or interpretation of the expressions for the perturbation theory calculation or the experimental determination of reactivity. These approximations are due, essentially, to inconsistent definitions of reactivities.

2. Approximations made in the actual calculations.

The approximations in the basic formulation include the following:

1. The use of fluxes (or adjoints) from the solution of a Boltzmann equation that is different from the one called for by the theoretical formulation.

2. The substitution of the unperturbed reactor flux for the perturbed reactor flux (or vice versa).

3. The use of effective kinetic parameters that are inconsistent with the theoretical formulation.

Errors caused by these approximations become smaller as the perturbation diminishes. Exact relationships between expressions for the experimental determination of reactivity and between perturbation theory expressions for the same reactivity are given for two experimental methods: the asymptotic period method and the source multiplication method for absolute reactivity determination.

Errors that can be caused in practical calculations of reactivity worths by the process of the reduction of the continuous energy variable into the group approximation were investigated. Both fine structure and coarse structure effects were considered. The investigations were performed by comparing results from three different methods for averaging the pointwise nuclear data

to obtain group constants: The flux method, the bilinear method, and the consistent method.

The fine structure effects on reactivity calculations were studied by investigating the behavior of the importance function in the vicinity of resolved resonances that satisfy the NR approximation. Numerical calculations were performed for the 273.7 eV U-238 capture resonance and for the 32.3 eV P_u -239 fission resonance. Following is a summary of results from these investigations:

1. The fine structure of the importance function in the vicinity of a resonance depends strongly on the resonance type: it sinks in a capture resonance, it can peak in a fission resonance, and it remains constant in a scattering resonance. (The flux, however, sinks in all the resonances, regardless of their type.)

2. The fine structure of the bilinear function ($\phi^+\phi$) can differ considerably from that of the flux. For a pure capture resonance, for example, the bilinear function is proportional to the square of the flux. The use of the flux fine structure as the weighting function for cross section averaging is expected, therefore, to lead to significant errors in the calculation of the contribution of resonance cross sections to reactivity.

3. The value of a bilinear expression which includes the absorption cross section will be overestimated for a capture resonance but underestimated for a fission resonance, when calculated by the flux method. The flux method is designed to preserve the values of reaction rates, but it can not preserve the values of bilinear parameters.

4. The C/E ratio calculated for the reactivity of a given resonance depends on the group width. This ratio is obtained in the numerical experiments as the ratio of the results of the flux method to the results of the consistent method. The deviation of the C/E ratio from unity increases up to some asymptotic value as the group width becomes larger.

The C/E ratios for the reactivity worths of the U-238 capture resonance and for the P_u -239 fission resonance are larger than unity. The C/E ratio for the U-238 resonance is found to be 1.02, 1.25, and 1.31 for group widths of 1, 10, and 100 practical widths, respectively. The corresponding C/E values for the P_u -239 resonance are, respectively, 1.07, 1.44, and 1.51.

Based on these results we make the following conclusions and observations:

1. Multi-group methods presently in use (based on the flux method) are not likely to account correctly for the contribution of the resolved resonance energy range to reactivity worths.

2. It is estimated that the reactivity worth of the single resonances of U-238 or P_u -239 considered can be overpredicted by about 30% or 40%, respectively, when calculated with the multi-group methods using 100 group flux averaged cross section set. It therefore seems reasonable to expect that the C/E discrepancy will be reduced when the fine structure effects will be accounted for accurately.

3. The contribution of the resonance energy range to the total (actually observed) C/E discrepancy is expected to depend on the neutron spectrum of the system. This contribution is expected to be more pronounced in soft spectrum fast reactors as compared

with hard spectrum fast reactors (e.g., GODIVA) on one hand, and with thermal reactors, on the other hand.

4. Fine structure details of the importance function in the thermal energy range of thermal reactors¹³ can have a non-negligible effect on certain reactivity worths.

5. The contribution of different fine structure effects to the total C/E of a given perturbation can be of a different direction. In the case of the reactivity worth of U-238 in thermal reactors, for example, the flux method is expected¹³ to underestimate the contribution from the thermal energy range, but to overestimate the contribution from the resonance energy range. Cancellation of errors can make the overall C/E discrepancy smaller than the error in the C/E for separate contributions.

6. It is possible to eliminate or to reduce the errors in reactivity worth calculations (based on perturbation theory expressions) caused by the flux method in one of the following ways:

a. By employing the consistent method instead of the flux method.

b. By performing the calculations with the flux (or any other) method using group widths smaller than the practical width of the resonances.

7. The discussion and conclusions about the group calculation of reactivity worths are relevant also for the group calculation of other bilinear expressions in reactor theory.

8. The advantages of the stationary functional approach to the calculation of reaction rates or their ratios can be reduced or even negated by the use of the flux method to prepare the group constants for the calculation of bilinear expressions. Thus, the direct (using flux only) calculation of reaction rates can yield results of better accuracy and is, of course, simpler to perform.

9. Adjoint functions expressing the response of different detectors can have fine structure behavior different from that of the importance function. Consequently, the consistent calculation of bilinear expressions involving the same cross section but different adjoint functions can require the preparation of different sets of group constants (for the same cross sections and same reactor).

10. The fine structure details of the spatial and angular dependence of the flux and adjoint functions have to be treated in a consistent method, similar to the treatment of the energy variable, if the group calculations of bilinear expressions are to preserve their realistic values.

It is recommended that computer codes used and developed for the preparation of group constants be expanded to include options for the calculation of group constants required by the consistent method.

Coarse structure effects on reactivity calculations were examined by intercomparing reactivity worths calculated by the three methods for a problem involving smoothly varying cross sections. The numerical experiments were performed for GODIVA

and were based on a simple model for the continuous energy variable. Reactivity worths were calculated for hydrogen and for U-235. The following is a summary of results obtained for the hydrogen reactivity worth :

a. The consistent method gives, as expected, the exact reactivity worth with any group structure, when the weighting functions used for cross section averaging are exact. The same holds true for the bilinear method, in the model employed for this problem. However, in problems for which spatial and angular effects are important, the bilinear method is not expected to give the exact reactivities.

b. The flux method underestimates the hydrogen reactivity worth by 29% when the 6-group structure of Los Alamos is used. With a 2-group structure the underestimation reaches 92%.

c. When the weighting functions are not exact, the consistent method gives more accurate reactivities than the bilinear method.

The reactivity worth of U-235 was found to be insensitive to the method of calculation.

It is concluded that:

1. Coarse structure effects are expected to be important for low-absorption light scatterers, or in general, for processes involving a large energy transfer.

2. Present computational capability can handle problems with group structures fine enough to reduce errors in reactivity calculations caused by coarse structure effects to insignificant levels.

3. The consistent method is more accurate than the bilinear method, at least for the type of problems examined. Reactivities calculated with both the consistent and the bilinear methods are considerably less sensitive to the group structure than reactivities calculated with the flux method.

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REFERENCES

¹H. Alter, Benchmark Testing of Nuclear Data for Fast Reactors, in Proceedings of the National Topical Meeting on New Developments in Reactor Physics and Shielding, Conf-720901, pp. 1033-1047 (1972)

²R. Avery, Review of FBR Physics, in Proceedings of the National Topical Meeting on New Developments in Reactor Physics and Shielding, Conf-720901, pp. 771-787 (1972)

³T. A. Pitterle and C. W. Maynard, Bilinear Averaging for Diffusion-Theory Parameters, Trans. Am. Nucl. Soc. 8, 205-206 (1965).

⁴W. W. Little, Jr. and R. W. Hardie, Methods for Collapsing Fast-Reactor Neutron Cross Sections, Nucl. Sci. and Eng. 29, 402-407 (1967)

⁵E. Greenspan, Consistent vis Conventional Few-Group Reactivity Calculations, Trans. Am. Nucl. Soc. 10, 583 (1967).
A more complete summary of this work is available as the RPI Internal Memo: RPI-328-103 (1967).

⁶E. Kiefhaber, Evaluation and Interpretation of Integral Experiments in Fast Criticals, in Proceedings of the National Topical Meeting on New Developments in Reactor Physics and Shielding, Conf-720901, pp. 623-635 (1972).

⁷E. Greenspan, The Source Multiplication Method for Absolute Reactivity Determination, J. Nucl. Energy (to be published).

⁸G. I. Bell and S. Glasstone, Nuclear Reactor Theory (Van Nostrand Reinhold Company, 1970).

⁹W. M. Stacey, Jr., H. Henryson, II, B. A. Zolotar, R. N. Hwang, B. J. Toppel, C. G. Stenberg, and E. M. Pennington, Studies of Methods for Fast Neutron Multigroup Cross Section Generation and Their Effect Upon the Neutronics Properties of LMFBR Critical Assemblies, in Proceeding of the National Topical Meeting on New Developments in Reactor Physics and Shielding, Conf-720901, pp. 211-228 (1972).

¹⁰B. J. Toppel, A. L. Rago, and D. M. O'Shea, MC²-A code to Calculate Multigroup Cross Sections, ANL-7318 (1967).

¹¹W. M. Stacey, Jr., Spectrum and Importance Function in Spatially Independent Fast-Reactor Assemblies, Nucl. Sci. and Eng. 29, 259-263 (1967).

¹²J. J. Schmidt, Neutron Cross Sections for Fast Reactor Materials, Part II: Tables, KFK 120, Karlsruhe (1962).

¹³E. Greenspan and K. B. Cady, Some Applications of the Thermal-Neutron Importance Function to Unit-Cell Studies, Trans. Am. Nucl. Soc. 9, 243-244 (1966).

TABLE I. Weighting functions employed for cross section averaging in the flux, the bilinear, and the consistent methods.

Type of Calculations	Weighting Functions		
	Flux Method	Bilinear	Consistent
Bilinear	$\phi(E)$	$\phi(E) \phi^+(E)$	$\phi(E) \phi^+(E)$
Flux	$\phi(E)$	$\phi(E) \phi^+(E)$	$\phi(E)$
Adjoint	$\phi(E)$	$\phi(E) \phi^+(E)$	$\phi^+(E)$

TABLE II. Resonance parameters for the 273.7 eV U-238 and the 32.3 eV P_u-239 resonances

Isotope	E_0 (eV)	σ_0 (barns)	Γ (mV)	Γ_n (mV)	Γ_γ (mV)	Γ_f (mV)
U-238	273.7	4950	48	25	23	0
P _u -239	32.3	66.6	260	0	40	220

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Fig. 1. A schematic description of a fine group with a single resonance.

724362

Fig. 2. A schematic representation of the relative change of the importance function in the vicinity of a capture resonance (a), a scattering resonance (b), and a fission resonance (c).

724367

Fig. 3. The relative distribution of the flux, the importance function, and the bilinear function in the vicinity of the 272.7 eV ^{238}U resonance.

724368

Fig. 4. Relative distribution of the flux, the importance function, and the bilinear function in the vicinity of the 32.3 eV ^{239}Pu resonance.

724360

Fig. 5. The contribution of the 273.7 eV U-238 capture resonance to the perturbation integral as calculated for the fine group by the flux method relative to the consistent method.

724372

Fig. 6. The contribution of the 32.3 eV P_u -239 fission resonance to the perturbation integral as calculated for the fine group by the flux method relative to the consistent method.

724363

Fig. 7. Flux and importance functions distribution in GODIVA.

724365

Fig. 8. Six-groups reactivity worth of hydrogen in GODIVA relative to the exact reactivity as calculated using the consistent (C), the bilinear (B), and the flux (F) methods for cross section averaging.

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