CALCULATION OF RADIAL MAGNETIC FIELDS FOR AXISYMMETRIC SOLENOIDS WITH RECTANGULAR CROSS SECTION

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ORNL-TM-4198

Contract No. W-7405-eng-26

Thermonuclear Division

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[To be submitted for publication in J. Appl. Phys.]

APRIL 1973

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CALCULATION OF RADIAL MAGNETIC FIELDS FOR AXISYMMETRIC SOLENOIDS
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ABSTRACT

The ratio of maximum radial field to maximum axial field has been calculated numerically for axisymmetric solenoids of rectangular cross section and constant current density over a wide range of coil configurations. It is also shown that for a certain range of coil shapes the maximum modulus of the magnetic field occurs on the end faces of the windings. These considerations are pertinent in the design of superconducting magnets for two reasons: many superconductors have lower critical current in a field normal to the wide surface than in a field parallel to the surface and large radial fields can cause catastrophic flux jumps.

I. INTRODUCTION

For the proper design of a superconducting magnet, the maximum value of the magnetic field must be known. If the magnet uses high field superconducting ribbon, then it is also necessary to know the magnitude of the radial field component for two reasons. First, it is this field component which induces the diamagnetic screening currents which result in catastrophic flux jumps in the end pancakes of magnets constructed with high field ribbon conductors. Convincing proof of this instability has been given¹⁻⁶ and a number of methods which render it harmless have been devised.⁷⁻¹⁰ Magnetic damping has been successfully demonstrated in a large Nb_3Sn magnet.¹¹ It is less well known that the critical current for some ribbon conductors is lower with the field normal to the ribbon surface (perpendicular limit) than when it is parallel to the width (but in both cases perpendicular to the transport current). If this difference in critical current is large enough and the magnet has unusual geometry such as a large bore, short length construction, then it is possible that the perpendicular short sample limit will be reached before the parallel value. This limitation on magnet design must also be considered for magnets wound with rectangular NbTi conductor where it is well known that the critical current is less with the field normal to the surface than when it is parallel,^{12,13} even for material with aspect ratios of only 2/1. It is the purpose of the present paper to present calculations of $A \equiv B_r^{\text{max}}/B_z^{\text{max}}$ for axisymmetric rectangular cross section coils with constant current density and delineate which magnet shapes have $A > 1$ from those with $A < 1$. To the best of the authors' knowledge, the quantity $A(\alpha, \beta)$ has never been tabulated. We shall also show some

short sample data of recently available commercial high field ribbon superconductors which illustrate both types of anisotropy of critical current with field direction.

II. CALCULATION

The dimensionless ratio $A \equiv B_r^{\max}/B_z^{\max}$, as will be shown below, is a function of two normalized parameters, α and β only (for the definition of α and β , as well as other coil geometry parameters, refer to Fig. 1). Another dimensionless ratio, $K(\alpha, \beta) \equiv B_z^{\max}/B_0$, the ratio of the maximum axial field component to the central field, has been extensively tabulated.^{14,15} The value of B_0 for a given solenoid can be expressed as $B_0 = \langle j \rangle a_1 F(\alpha, \beta)$ where $\langle j \rangle$ is the average current density, a_1 is the inner bore radius, and the field factor, $F(\alpha, \beta)$, first introduced by Gauster,¹⁶ is a dimensionless function of α and β . The F factor has the convenient form

$$F(\alpha, \beta) = \frac{4\pi\beta}{10} \ln \frac{\alpha + (\alpha^2 + \beta^2)^{1/2}}{1 + (1 + \beta^2)^{1/2}}, \quad (1)$$

and also has been extensively tabulated.¹⁵ The maximum value of the radial field can be obtained from the expression,

$$B_r^{\max} = \langle j \rangle a_1 A(\alpha, \beta) K(\alpha, \beta) F(\alpha, \beta) \quad (2)$$

with $B_r^{\max}$ given in gauss when a_1 is in centimeters and j is in amperes per square centimeters. The calculation of $A(\alpha, \beta)$ and the corresponding location of the maximum radial field was undertaken as part of a general study of stability considerations in superconducting solenoids.

The problem of calculating magnetic field components produced by axisymmetric rectangular cross-section coils with constant current density is well known. In this calculation the cross section of the coil was divided into one, two, three, or four regions by cylindrical surfaces coaxial with the windings. The expressions for the field components and derivatives produced by infinitesimally thick solenoids, which involve elliptic integrals of the first, second, and third kind,¹⁷ were integrated in radial depth by numerical gaussian quadrature of the sixteenth order and the results summed over the regions. The elliptic integrals were evaluated by the iterative method of King¹⁸ and Bartky.¹⁹

The magnetic field components and derivatives (B_z , B_r , $\partial B_z/\partial r$, and $\partial B_r/\partial z$), as well as the absolute magnitude of the field, were calculated over a matrix of α and β values, evaluating the quantities at a specified set of points for each (α, β) pair. In some cases (i.e., for small α) the program was made to step through a matrix of α and ϵ values, where $\epsilon = \beta/(\alpha - 1)$.

It is a common misconception that the maximum magnitude of the magnetic field in axisymmetric solenoids with rectangular cross section and constant current density always occurs at $z = 0$, $r = a_1$ (see Fig. 1). In fact, for a certain region in the (α, β) plane, the maximum occurs on the end faces of the windings at $z = b$, $r = d - \delta$, where δ is a small quantity and depends on the particular coil geometry. This fact can be understood intuitively by considering an infinitely long, straight conductor of rectangular cross section with vertical and horizontal dimensions of $2h$ and $2w$, respectively. For such a configuration the maximum magnetic field occurs at the center of the narrow face of the conductor and is parallel to the face. If the cross section of the conductor is referred

to a rectangular coordinate system with origin at the center of the conductor, the maximum value of the y component of the field occurs at $y = 0$, $x = \pm w$; and the maximum value of the x component occurs at $x = 0$, $y = \pm h$. The ratio $B_y^{\max}/B_x^{\max}$ can be obtained easily by using the Biot-Savart law and integrating over the cross section. The result is

$$\frac{B_y^{\max}}{B_x^{\max}} = \frac{4 \frac{w}{h} \tan^{-1} \left(\frac{h}{2w} \right) + \ln \left[1 + \frac{4w^2}{h^2} \right]}{4 \tan^{-1} \left(\frac{w}{2h} \right) + \frac{w}{h} \ln \left[1 + \frac{4h^2}{w^2} \right]}. \quad (3)$$

If we fix the parameter $\epsilon = \beta/(\alpha - 1)$, then as α approaches unity, the ratio of maximum radial field to maximum axial field in a rectangular solenoid approaches a limit given by Eq. (3) with $w/h = 2\epsilon$. As a result, if the ratio of maximum radial and maximum axial field components varies continuously with α , there must be a finite region with $\alpha > 1$, $\epsilon > 0.5$ where $B_r^{\max}$ is greater than $B_z^{\max}$. Consequently, there must be a finite region where the maximum absolute magnitude of the field does not occur at $z = 0$, $r = a_1$.

III. RESULTS OF THE CALCULATION

In Fig. 2a the function $A(\alpha, \beta) \equiv B_r^{\max}/B_z^{\max}$ is shown plotted against β for different fixed values of α . As can be seen, for small α , the function reaches a maximum value greater than one. The region of very small β values is seen more clearly in Fig. 2b. The location of the point of maximum radial field always occurs on the end face of the windings, $z = b$. For all values of β , and α values greater than four, the maximum radial field occurs at a point slightly beyond the midpoint of the winding

thickness, while for α values less than three, the maximum occurs slightly before the midpoint of the end face. For the region $1 < \alpha \leq 5$, $0 < \beta \leq 10$, the displacement of the maximum from the midpoint is less than one-tenth of the winding depth.

Figure 3 is a graph of the function $A(\alpha, \beta)$ vs α for different fixed values of the parameter ϵ . The limiting values at $\alpha = 1$ were obtained by evaluating Eq. (3) and served as a check on the accuracy of the machine calculation. In all cases the points determined for $\alpha > 1$ could be extrapolated with smooth curves to the limiting values obtained from Eq. (3).

It was found that the magnitude of the field is a maximum either at the windings in the center ($z = 0$, $r = a_1$) or near the midpoint on the end faces ($z = \pm b$). A series of curves similar to those in Fig. 3 was calculated for the quantity $B^{\max}(z = \pm b)/B_z^{\max}$, the ratio of the maximum value of the modulus of B at the end faces to the maximum value of the axial component of B . From the intersections of these curves with the line $B^{\max}(z = \pm b)/B_z^{\max} = 1$, a curve of α vs ϵ (or equivalently β) was obtained which separates the α, ϵ (or α, β) plane into two regions. In one region, $B^{\max}$ is at $z = 0$, $r = a_1$, and in the other $B^{\max}$ occurs at $z = \pm b$. The regions where the maximum field magnitude occurs (i.e., at either $z = 0$, $r = a_1$, or $z = \pm b$) are shown in Fig. 4 for various values of α and ϵ . In the region where $B^{\max}$ is at $\pm b$, the coil has a "barrel hoop" appearance; the winding depth is small compared to the inner bore, and the axial length is greater than the radial depth. However, coils of the type to be used as elements in toroidal plasma-confinement devices may well be in this regime when energized as separate entities apart

from the adjacent coils or if adjacent coils quench in an operating toroidal system. This fact is an important consideration in building test facilities for prototype superconducting coils designed for use in toroidal magnet systems; it will certainly be necessary to energize the coils in an axial or predominately axial background field in order to duplicate the field distribution in the windings that would occur in an assembly of individual coils.

IV. SHORT SAMPLE CRITICAL CURRENT MEASUREMENTS

Another consideration in the testing of any magnet with a small value of both α and β is the fact that the radial component will be a large fraction of the central field. This may result in the limiting current carrying property of the superconductor being determined by the radial field rather than the axial field at the windings in the center of the coil. Figure 5 is a plot of the short sample critical current vs field for both the field normal and parallel to the width of the Nb_3Sn ribbon. The measuring techniques used have been discussed in detail.²⁰ This result where the short sample value is less when the field is normal to the broad face of a rectangular superconductor is the usual case. This is always found for rectangular shaped NbTi conductor as well.^{12,15} However, the opposite situation is also possible as seen in Fig. 6 which shows a short sample graph of the high field ribbon, V_3Ga produced by Sumitomo. For commercial Nb_3Sn produced by the diffusion process, no anisotropy has been reported,²¹ but Jergel and Hlasnik, in contrast, found an anisotropy of the usual case for diffusion produced Nb_3Sn .²²

V. CONCLUDING REMARK

As larger, higher field magnet systems of complex design are built, the considerations put forth in the present paper will become more pertinent. The range of α and β parameters where radial fields are as high or higher than axial components are within present designs for magnets which comprise the components of a large toroidal device of a size believed capable of demonstrating fusion feasibility.

VI. ACKNOWLEDGMENTS

The authors are grateful to W. F. Gauster and C. E. Parker for helpful discussions and wish to thank W. H. Wagner for assisting with the experiment.

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FIGURE CAPTIONS

- Fig. 1 Definition of parameters for axisymmetric solenoids of rectangular cross section with uniform current density.
- Fig. 2a The function $A(\alpha, \beta) \equiv B_r^{\max}/B_z^{\max}$ vs β for various values of α .
- Fig. 2b The same as Fig. 2a for small values of β .
- Fig. 3 The function $A(\alpha, \beta) \equiv B_r^{\max}/B_z^{\max}$ vs α for fixed values of ϵ .
- Fig. 4 Regimes where maximum field magnitude occurs for various values of α and ϵ .
- Fig. 5 Short sample critical current values at 4.2 K vs the field normal and parallel to the broad face of the ribbon. The Nb_3Sn superconductor was produced by the vapor deposition process by CSCC. It has dimensions of 6.35 mm wide by 0.2 mm thick. CSCC quotes $I_c(H_{\parallel})$ values of 200 A at 100 kG and using their normalized curve, this material should carry current of 320, 400, and 600 A at 75, 59, and 40 kG, respectively.
- Fig. 6 Short sample critical current values at 4.2 K vs both field directions for a V_3Ga superconductor produced by Sumitomo Company. It has dimensions of 12.7 mm wide by 0.12 mm thick. Sumitomo specifies that $I_c(H_{\parallel})$ is 500, 280, 150 A at 50, 100, and 150 kG, respectively.

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FIGURE 1

FIGURE 2a.

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FIGURE 2b

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FIGURE 3

FIGURE 4

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FIGURE 5

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FIGURE 6