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LARGE ANGLE PROTON-PROTON
ELASTIC SCATTERING AT
INTERMEDIATE MOMENTA**

BY

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ABSTRACT

Differential cross sections for proton-proton elastic scattering are presented at thirteen incident proton momenta between 1.5 and 5.5 GeV/c. The data cover the center of mass angular region from 30° to 90° with data point spacing of 2° . Typical statistical errors of $\pm 4\%$ have been obtained.

The experiment was performed at the Argonne National Laboratory Zero Gradient Synchrotron. The particle detection system consisted of a combination of wire spark chambers and scintillation counters.

The results of the experiment indicate that the angular distributions in this intermediate energy region are quite smooth, varying monotonically with both momentum and angle. No strong evidence is observed in this momentum region for secondary diffraction-like behavior which has been observed at higher momenta.

Among the more interesting results of the experiment is the determination of the location of a suspected discontinuity in the fixed angle cross section as a function of the four momentum transfer. Also, evidence is presented for an additional discontinuity. A calculation which predicts the masses at which di-baryon resonances might be expected to occur is outlined and a possible interpretation of the discontinuities in terms of these resonances is offered.

Attempts to interpret the data in terms of several models are described. These include both diffractive models and a simple Regge pole model. In the latter case, an effective Regge trajectory has been extracted

from the data.

Finally, strong evidence is found for rapid changes in the nature of the large angle differential cross section in the region of 1.5 and 4.0 GeV/c. This conclusion is based on the observed discontinuities in the fixed angle cross section and on the momentum dependence of parameters used in fitting the large angle data.

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I. INTRODUCTION

We present the results of a wire spark chamber experiment designed to measure the large angle differential cross sections in proton-proton elastic scattering at intermediate momenta. Data were collected at thirteen incident proton momenta between 1.5 and 5.5 GeV/c and in the c.m. angular region $30^\circ \leq \theta_{c.m.} \leq 90^\circ$. The experiment was performed during the period from January through April of 1969 in the external proton septum beam at the Argonne National Laboratory Zero Gradient Synchrotron.

The motivation for this experiment was provided by the results of several other experiments performed in recent years. In at least four of these somewhat unusual or unexpected behavior of the p-p system was indicated. Briefly described below are the results of these four experiments.

The measurements of Heal and Longo¹ have indicated the existence of structure in the polarization parameter, $P(t)$, in p-p elastic scattering at intermediate momenta. They observed a dip in the polarization near $t = -.7 \text{ (GeV/c)}^2$ where $P(t)$ was actually consistent with zero. This dip persisted throughout the higher energy region ($2.0 \leq P_L \leq 3.7 \text{ GeV/c}$) of the experiment. Their results at 2.05 and 2.4 GeV/c are shown in Figure 1. Also shown in this figure are the measurements of Booth et al.² at 3.75 and 5.15 GeV/c. As shown, the latter results confirm the existence of a dip in this region, but indicate that it is shallower and $P(t)$ is nonzero at the higher momenta. Other polarization experiments³ are consistent with these results.

FIGURE 1.

In an experiment by Akerlof et al.⁴ the differential cross sections at 90° in the c.m. were measured as a function of incident proton momentum between about 5.0 and 13.0 GeV/c. They found that the differential cross sections between about 5.0 and 8.0 GeV/c could be well represented by a single exponential in $-t$, and between about 8.0 and 13.0 GeV/c, by a different exponential in $-t$. Their results are shown in Figure 2. The pronounced "break" is shown to occur at $-t \approx 6.5 \text{ (GeV/c)}^2$. This finding enhanced speculation about the exact nature of the 90° cross section in the region $1.0 \leq -t \leq 4.0$. As shown in Figure 3, this region had not been investigated in detail. Extrapolation of the exponential fit to the data of Akerlof et al., the solid line in Figure 3, and the data from cyclotron experiments⁵, indicated another possible discontinuity in the region of $-t \sim 1 \text{ (GeV/c)}^2$.

Precision measurements of the large-angle p-p elastic scattering angular distributions between 8.0 and 21 GeV/c by Allaby et al.^{6,7,8,9} have revealed diffraction-like structure in the region of $-t \approx 1.5 \text{ (GeV/c)}^2$. Their results at 19.2 GeV/c are shown in Figure 4. This structure was not observed in the intermediate energy experiments of Clyde¹⁰ or Ankenbrandt et al.¹¹, but these investigations were not as detailed as those of Allaby et al.

A thorough investigation of the total cross section for proton-proton scattering between 1.2 and 7.8 GeV/c by Bugg et al.¹² indicated a slight bump which occurred at about 3.0 GeV/c. This is shown in Figure 5. Several attempts to interpret this bump in terms of N^* production or di-baryon resonances have been offered^{12,13}, but the structure is too

FIGURE 2

FIGURE 3

FIGURE 4

FIGURE 5

small to permit any definitive statements.

The goal of this experiment was to make detailed measurements of the p-p angular distribution in the momentum region from 1.5 to 5.5 GeV/c. It was intended to obtain statistical accuracies of about $\pm 4\%$ with datum points spaced at two degree intervals in the c.m. for angles between 30° and 90° . In making these measurements we sought; (1) to observe manifestations in the angular distributions of the structure in the polarization and total cross section, (2) to determine the nature of the $\theta_{c.m.} = 90^\circ$ cross section in the area of the indicated break near $-t = 1 \text{ (GeV/c)}^2$, and (3) to determine if secondary diffraction-like structure persists at lower momenta.

In addition, sufficiently precise results provide the possibility of discriminating against some of the profusion of different models which have been proposed in order to "explain" the p-p scattering data.

In summary, the major results of this experiment are: (1) the establishment of the suspected break in the fixed angle cross section near $-t \sim .8 \text{ (GeV/c)}^2$ and evidence for another discontinuity near $-t = 3 \text{ (GeV/c)}^2$, (2) the evidence for rapid changes in the nature of the large angle ($\theta_{c.m.} > 60^\circ$) cross section near 1.5 and 4.0 GeV/c; and, (3) the verification that some of the predictions of a simple Regge model are obeyed at intermediate momenta and the extraction of an effective trajectory from the data.

In Chapter II we briefly review the general theory applicable to the proton-proton interaction, and outline several of the more promising interpretative models. The experimental equipment and procedures are

described in Chapter III and the method of analysis is detailed in Chapter IV. The experimental results appear in Chapter V and their interpretation in terms of the various models comprise Chapter VI.

II. THEORY

The formal discussion of an interaction involving two spin- $\frac{1}{2}$ particles in both the initial and final states leads to a description in terms of sixteen non-independent amplitudes. Application of the appropriate invariance principles results in a set of independent amplitudes.

We adopt the helicity representation of Jacob and Wick¹⁴ and write the proton-proton interaction as

$$P_{\lambda_1} + P_{\lambda_2} \rightarrow P_{\lambda'_1} + P_{\lambda'_2}$$

where λ_1 and λ'_1 represent the helicities in the initial and final states respectively ($\lambda = \pm \frac{1}{2}$). As shown by Goldberger et al.¹⁵, invariance under isotopic spin-space rotations, parity transformation and time reversal leads to the following relations between the elements of the T matrix

(i) Parity invariance

$$\langle \lambda'_1 \lambda'_2 | T^J(E) | \lambda_1 \lambda_2 \rangle = \langle -\lambda'_1 - \lambda'_2 | T^J(E) | -\lambda_1 - \lambda_2 \rangle$$

(ii) Time reversal invariance

$$\langle \lambda'_1 \lambda'_2 | T^J(E) | \lambda_1 \lambda_2 \rangle = \langle \lambda_1 \lambda_2 | T^J(E) | \lambda'_1 \lambda'_2 \rangle$$

(iii) Total spin conservation

$$\langle \lambda'_1 \lambda'_2 | T^J(E) | \lambda_1 \lambda_2 \rangle = \langle \lambda'_2 \lambda'_1 | T^J(E) | \lambda_2 \lambda_1 \rangle$$

The interaction is described by a spin-space matrix ϕ . Due to the rotational invariance of the S matrix, the elements of ϕ can be written

$$\langle \lambda_1' \lambda_2' | \phi | \lambda_1 \lambda_2 \rangle = \frac{1}{K} \sum_J (2J+1) \langle \lambda_1' \lambda_2' | T^J(E) | \lambda_1 \lambda_2 \rangle d_{\lambda\lambda'}^J(\theta)$$

where K is the momentum in the c.m. and $d_{\lambda\lambda'}^J$ are the reduced rotation matrices with

$$\begin{aligned} \lambda &= \lambda_1 - \lambda_2 \\ \lambda' &= \lambda_1' - \lambda_2' \end{aligned}$$

For the proton-proton system, the five independent amplitudes can be written

$$\begin{aligned} \phi_1 &= \langle ++ | \phi | ++ \rangle = \frac{1}{K} \sum_J (2J+1) \langle ++ | T^J(E) | ++ \rangle d_{00}^J(\theta) \\ \phi_2 &= \langle ++ | \phi | -- \rangle = \frac{1}{K} \sum_J (2J+1) \langle ++ | T^J(E) | -- \rangle d_{00}^J(\theta) \\ \phi_3 &= \langle +- | \phi | +- \rangle = \frac{1}{K} \sum_J (2J+1) \langle +- | T^J(E) | +- \rangle d_{11}^J(\theta) \\ \phi_4 &= \langle +- | \phi | -+ \rangle = \frac{1}{K} \sum_J (2J+1) \langle +- | T^J(E) | -+ \rangle d_{-11}^J(\theta) \\ \phi_5 &= \langle ++ | \phi | +- \rangle = \frac{1}{K} \sum_J (2J+1) \langle ++ | T^J(E) | +- \rangle d_{10}^J(\theta) \end{aligned}$$

where $\pm$ denotes $\pm \frac{1}{2}$. These amplitudes are, in general, complex so that a total of nine experiments¹⁶ are necessary in order to completely specify the amplitudes to within an overall relative phase (the real and imaginary part of the five amplitudes subject to the unitarity condition). Of these, only two types of distribution have been experimentally investigated.

These are:

- (i) Differential cross section

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} [|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2 + |\phi_4|^2 + 4|\phi_5|^2]$$

- (ii) The elastic polarization

$$P = -\text{Im} \left[\phi_5^* (\phi_1 + \phi_2 + \phi_3 - \phi_4) \right] / (d\sigma/d\Omega)$$

The Pauli principle imposes the following restrictions on the nucleon-nucleon helicity amplitudes

$$\phi_1(\pi - \theta) = (-1)^{I+1} \phi_1(\theta)$$

$$\phi_2(\pi - \theta) = (-1)^{I+1} \phi_2(\theta)$$

$$\phi_3(\pi - \theta) = (-1)^I \phi_3(\theta)$$

$$\phi_5(\pi - \theta) = (-1)^I \phi_5(\theta)$$

From this, we deduce the following conditions on the proton-proton helicity amplitudes in states of definite total angular momentum

$$\langle ++ | T^J(E) | \pm\pm \rangle = (-1)^J \langle ++ | T^J(E) | \pm\pm \rangle$$

$$\langle +- | T^J(E) | +- \rangle = (-1)^J \langle +- | T^J(E) | +- \rangle$$

$$\langle ++ | T^J(E) | +- \rangle = (-1)^J \langle ++ | T^J(E) | +- \rangle$$

The complex spin structure of the proton-proton system, together with the relatively sparse amount of experimental information available,

has resulted in a rather limited amount of theoretical analysis of this interaction. This is especially true of the large angle, low and intermediate energy region. At higher energies, several theories exist and most predict significant simplification of the description. Regge analysis, for example, predicts that at large s and small $|t|$ ¹⁷,

$$\phi_1 = \phi_3$$

$$\phi_2 = -\phi_4$$

leading to a description in terms of a single amplitude for each of the three possible helicity flip combinations (0, 1 or 2 helicity flips).

Other models have been proposed for application in the "high energy domain" and have met with some success. These models invariably neglect the spin of the particles resulting in considerable simplification. Notable among these models are the diffraction model of Chou and Yang¹⁸ and the Hybrid Model (diffraction plus Regge poles) of Chiu and Finkelstein.¹⁹ In comparison with the differential cross section data of Allaby *et al.* as discussed earlier, both models have offered an explanation of the observed shoulder near $t = -1.5 \text{ (GeV/c)}^2$ in Figure 4.

We describe below several phenomenological approaches to the interpretation of the existing data. We consider both diffractive type models and a simple Regge-pole model proposed for the large angle region.

A. Diffraction

Underlying all diffraction models of high energy scattering is the supposition that the scattering is dominated by inelastic processes;

that is, that the elastic channel is but one of the many channels which are phase-space available to the system. This is in contrast to potential scattering, for example, where the elastic process is analysed independent of the other (inelastic) channels which may be available to the system. The relationship between the elastic and inelastic channels is then dependent upon the particular diffraction model being applied.

The applicability of diffractive models is dependent upon the degree of inelasticity of the process. There are several ways to observe this; the most direct of which is to compare the total and total elastic cross sections. This is shown in Figure 6 for the case of proton-proton scattering between laboratory momenta of about 1 and 25 GeV/c. The data shown in this figure are taken from references 12 and 20 through 29. Above about 5 GeV/c, the total cross section becomes quite insensitive to the energy, though a slight slope persists to the highest energies presently available. The elastic cross section decreases significantly faster indicating that the ratio of σ_{el}/σ_{tot} is continuing to decrease. It has been predicted that at 200 GeV/c, this ratio may have decreased to about 1%.

The flattening of the total cross section at higher energies is a property of all processes which have been investigated, including protons, kaons and pions on protons and neutrons. In terms of the operation of a diffractive mechanism, the flattening of the total cross section implies; (1) that since the number of inelastic channels available increases rather rapidly with energy, the flat cross section

FIGURE 6

indicates a decrease of excitation of a particular channel; (2) the absence of significant structure in the total cross section at high energies suggests some kind of a "statistical" limitation on the excitation of a particular channel.

A second way to observe the inelasticity of a scattering process is to measure the ratio of the real to the imaginary part of the forward scattering amplitude. When the number of available inelastic channels is large, that is, when the inelastic cross section is large compared to the elastic cross section, unitarity implies the dominance of the imaginary part of the forward scattering amplitude.³⁰ This is most easily seen from the unitarity condition of the T matrix,

$$\text{Im } \langle f | T | i \rangle = \frac{K}{4\pi/s} \sum_n \langle f | T^\dagger | n \rangle \langle n | T | i \rangle$$

where $\sqrt{s}$ is the total energy in the c.m. and where the summation on n includes all four momenta conserving intermediate states.

In forward elastic scattering $|i\rangle = |f\rangle$, to within a possible phase. The unitarity condition reduces to

$$\text{Im } T_{ii} = \frac{K}{4\pi/s} \sum_n T_{ni}^* T_{ni}$$

The states $|n\rangle$ are complete and compatible with $|i\rangle$. This same requirement must be satisfied by the original final states $|f\rangle$ so that the summation on $|n\rangle$ is equivalent to a summation on $|f\rangle$

$$\text{Im } T_{ii} = \frac{K}{4\pi/s} \sum_f |T_{fi}|^2$$

$$\begin{aligned}
&= \frac{K}{4\pi/s} \left[|T_{11}|^2 + \sum_{r \neq 1} |T_{r1}|^2 \right] \\
&= \frac{K}{4\pi/s} \left[(\text{Re } T_{11})^2 + (\text{Im } T_{11})^2 + \sum_{r \neq 1} |T_{r1}|^2 \right]
\end{aligned}$$

The last term on the right represents the summed production cross sections for scattering into all inelastic final states. The optical theorem relates the total cross section and $\text{Im } T_{11}$

$$\sigma_{\text{tot}} = \frac{4\pi}{K} \text{Im } T_{11}$$

If σ_{tot} becomes energy independent at high energy, then $\text{Im } T_{11}$ is fixed, and, if the inelastic contribution is increasing with energy, as new channels open the sum must be increasing. Thus, the real part of the forward elastic scattering amplitude must be decreasing at about the same rate as the increase in the inelastic contribution.

The experimentally measured ratio of the real to the imaginary part of the forward scattering amplitude is shown in Figure 7, the data being taken from references 24,28 and 32-36. The ratio is quite large and positive for the lower momenta, drops through zero near 1.6 GeV/c and as indicated, reaches a minimum of $\sim -.3$ for momenta above 3.0 GeV/c. There is some indication that at higher momenta, the ratio is approaching zero quite slowly. In the intermediate energies of interest herein, the ratio is not "small" as is expected in a region where the scattering is primarily diffractive in nature.

Described below are three models based upon diffractive mechanisms. The first model is proposed to provide an energy independent fit to the

REAL $f(0^\circ)$ / IMAGINARY $f(0^\circ)$ Vs P_L FOR p-p SCATTERING

FIGURE 7

differential cross section. The second model is variously referred to as the optical model, impact parameter model, diffraction model or absorption model, the name being associated with the tenets on which the development of the model is based. This model is generally formulated for application to the small momentum transfer region. Finally, a description of the statistical model for large angle scattering is presented.

1. Krisch's Model

One of the more successful attempts to apply a diffraction model to high energy elastic p-p scattering is due to Krisch.^{37, 38, 39} This model results in an energy independent parameterization of the differential cross section which provides a good qualitative fit to the available data over a cross section range of 12 orders magnitude.

In addition to the factors motivating the diffractive description in general, i.e., flattening of the total cross section and the small ratio of the real to the imaginary part of the forward scattering amplitude, another observation is made; that is the basic similarity of the diffraction peaks observed in all high energy scattering data. This is taken as evidence that the dominating inelastic processes are identical for all scattering problems. It is assumed that, in the region of interaction, three "basic" quanta can be exchanged by the incoming particles, resulting in an excited region of space from which particle emission can occur. This is the mechanism by which the dominant inelastic processes proceed.

The emitted particles are assumed to obey the inhomogeneous Klein-Gordon equation

$$(\square + m^2) \phi = 0$$

Being a scalar equation, the spin of the emitted particles is neglected. The function ρ is the source for the particle production and is taken to be a function of the incident particle space-time separation R , the 4-vector from the center of the excited region to the production point, r , and the energy, $\gamma = \frac{E}{m}$. It is expected that ρ will be a strongly damped function of both distances. The function ρ is taken to be factorable in all variables and gaussian in R .

$$\rho(R, r, \gamma) = \frac{\gamma^2}{A^3} \rho(r) e^{i\omega_0} e^{-\sum \frac{3}{2} (R_i/A_i)^2}$$

The three exponentials correspond respectively to three (concentric) regions of excitation, each with an associated quantum. Spherical symmetry of ρ in R -space is assumed, but Lorentz contraction of the excited region in the direction of motion (z) is considered.

$$(\vec{A}, A_t) = \left((A, A, A/\gamma), \beta A/\gamma \right)$$

The size of the region of interaction is dependant upon the mass of the associated quantum in the usual way,

$$A_i^2 \propto 1/m_i$$

The choice of the quanta and their number is not totally ad-hoc. It is necessary to allow for the exchange of strangeness and baryon number. It is also desirable to allow exchange of an $S = B = 0$ particle. Thus the quantum associated with outer ring ($A \approx .92F$) is

taken to be the pion, the lightest strongly interacting particle. The central region ($A \approx .48f$) has as its quantum the lightest particle with strangeness, the kaon. The core ($A \approx .32f$) provides baryon number exchange through the exchange of baryons.

Solution of the field equation and calculation of the elastic scattering amplitude, f , as a function of the c.m. momentum yields

$$f(P) = f_{\pi}(P) + f_K(P) + f_B(P)$$

the three terms again being the contributions from the three indicated inelastic processes. Some care is required in calculating the individual contributions to the scattering amplitude since there is absorption in the pion production area of the excitation region. This results in a net reduction in the cross section for kaon and baryon production.

The scattering amplitude for each of the three types of process is calculated to be

$$f_1(P) = \frac{p\sigma_1}{4\pi} e^{-(A_1/2)^2 (\beta P_1)^2},$$

where $P_1 = |\vec{P}| \sin \rho$ is the transverse momentum and the differential cross section is

$$\frac{d\sigma}{d\Omega} = |f(P)|^2 = |f_{\pi}(P) + f_K(P) + f_B(P)|^2.$$

For the outer region

$$\sigma_{\pi} = \pi A_{\pi}^2,$$

the geometric cross section. Though calculable in principle (the absorp-

tion in the π region is calculable), the K and B region cross sections are generally treated as parameters.

When the constants are determined by fitting the data ($\log d\sigma/d\Omega$ vs. $\beta^2 P_1^2$), it is found that there are three nearly linear regions. Thus, away from the regions where the linear sections meet, we can neglect the cross terms in squaring the sums of the three amplitudes and write

$$\frac{d\sigma}{d\Omega} = \sum_{i=1}^3 A_i e^{-a_i (\beta P_1)^2}$$

More recently this model has been modified in order to account for significant deviations between measured and predicted angular distributions. The data of Allaby et al. at momenta between 8 and 21 GeV/c were consistently flatter near 90° than was expected on the basis of Krisch's predictions. It has been suggested⁴⁰ that this is due to particle identity effects in the p-p system. In order to remove the identity effect, a quantity, $(d\sigma^+/dt)$, was defined as the "true" cross section. The measurable cross section $(d\sigma/dt)$ is written in terms of the true cross section as

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma^+}{dt} \right)_f + \left(\frac{d\sigma^+}{dt} \right)_b$$

where f and b denote the forward ($\theta_{c.m.} \leq 90^\circ$) and backward ($\theta_{c.m.} > 90^\circ$) directions respectively. The addition of these two cross sections requires that no interference terms contribute. Defining the quantity

$$I = 1 + \frac{(d\sigma^+/dt)_b}{(d\sigma^+/dt)_f},$$

the measured cross section becomes

$$\frac{d\sigma}{dt}^{\dagger} = I \left(\frac{d\sigma}{dt} \right)_f.$$

The model in the original form is expected to apply to $d\sigma^{\dagger}/dt$, i.e.

$$\frac{d\sigma}{dt}^{\dagger} = \sum_{i=1}^3 A_i e^{-a_i (\beta P_1)^2}.$$

Then in any linear region, away from a break, we have

$$\frac{d\sigma}{dt}^{\dagger} = A_1 e^{-a_1 (\beta P_1)^2}.$$

Evaluation of the quantity I then permits the measured cross section to be transformed into an energy independent form

$$\frac{d\sigma}{dt} = I \sum_{i=1}^3 A_i e^{-a_i (\beta P_1)^2}.$$

To evaluate I we note that, by definition, $(d\sigma^{\dagger}/dt)_f = (d\sigma^{\dagger}/dt)_b$ at $\theta_{c.m.} = 90^\circ$ and $(d\sigma^{\dagger}/dt)_b = 0$ at $\theta_{c.m.} = 0^\circ$. Between these two limits

$$\left(\frac{d\sigma}{dt} \right)_f > \left(\frac{d\sigma}{dt} \right)_b.$$

This is inconsistent with the parameterization within the model in terms of $(\beta P_1)^2$ since at any angle, the quantity $(\beta P_1)^2$ will be equal in the two terms. To correct this, Kriech writes

$$P_1^2 = P^2 - P_L^2$$

where $P_L = |\vec{P}| \cos \theta_{c.m.}$, and makes the replacement

$$P_L^2 = P_L |P_L|.$$

This gives the forward and backward cross sections the proper behavior as they pass through 90° . The only justification for this step is the success of the result it produces. In the linear regions we then find

$$I = 1 + e^{-2a_1(\beta p_1)^2}$$

In spite of the "ad-hoc" assumptions the results are quite impressive, qualitatively. To within factors of two or three, the fit is good over about twelve orders of magnitude.

2. Optical Models, Absorption Models, and the Impact Parameter Representation

The optical model formalism has arisen from several different approaches to scattering descriptions. In its classical form, it appears as a result of the eikonal approximation in potential scattering theory in nuclear physics. Somewhat more generally, it is the small angle, peripheral interaction approximation to the usual partial wave analysis of low or intermediate energy nuclear or particle scattering theory. In both formulations applicability of the model is strictly valid only in the region of peripheral interaction, the small angle diffraction peak region, although in practice this limitation has not hindered its application to the intermediate and large angle region.

The original semi-classical development of the optical model is due to Fernbach et. al.⁴¹ They describe the scattering of neutrons from nuclei as the scattering of a wave from a sphere possessing a complex index of refraction. In potential scattering language, this corresponds

to taking the interaction (optical) potential to be complex. The real and imaginary parts of the index are directly related to the corresponding parts of the potential. The index itself is calculable from known information about the scattering cross section. Assuming an incident plane wave of unit amplitude and zero phase shift, the traversal of the opaque region results in a decrease in amplitude and a phase shift. The transmitted wave is represented by an amplitude and phase shift

$$a = Ae^{i\phi}.$$

In the shadow of the scatterer, the wave is no longer plane with respect to the incident direction, that is, it differs from a plane wave, but differs only by the noted attenuation and phase shift. Thus the scattered wave can be written as a plane wave

$$(1 - a) e^{i\vec{K} \cdot \vec{r}}.$$

The usual formalism for scattering of a wave by an opaque body then yields the scattering amplitude

$$f(\theta) = K \int_0^R [1 - a(b)] J_0(Kb \sin \theta) b db$$

and

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2$$

Also, for an opaque sphere, the elastic and inelastic cross sections⁴¹ are given by

$$\sigma_{el} = 2\pi \int_0^R |1 - A(b) e^{i\eta(b)}|^2 b db$$

$$\sigma_{inel} = 2\pi \int_0^R (1 - A(b)^2) b db$$

The quantum mechanical complex potential scattering problem leads to virtually the same result in the eikonal approximation. The approximation yields the same limitations on the resulting amplitude, that is, the eikonal approximation assumes that the particle in traversing the interaction region suffers little change in direction, the primary effect being to attenuate the wave.

Variations of this form of the optical model have been applied to high energy proton-proton scattering at all angles by several authors (see for example, references 21, 29, and 42) with at least limited success. The major difficulty is that, although the model yields respectable fits to the data ($d\sigma/d\Omega$, σ_{el} and σ_{inel}) with specific parameterizations of A and η as functions of b , the resulting value of R becomes energy dependent. This is rather unappealing from a physical standpoint since R is closely connected with the size of the region of interaction.

An analogous result can be obtained from the usual partial wave decomposition of the asymptotic wave function⁴³ for spinless particles

$$\psi(r, \theta) \underset{r \rightarrow \infty}{\sim} e^{i\vec{K} \cdot \vec{r}} + f(\theta) \frac{e^{iKr}}{r}$$

where the first and second terms are respectively the contribution from the unscattered plane incident wave and the spherical waves scattered by the target. Since the total angular momentum is a constant of the motion,

one may treat the scattering in each angular momentum state individually

$$f(\theta) = \frac{1}{2iK} \sum_{L=0}^{\infty} (2L+1)(1-\eta_L) P_L(\cos \theta).$$

The quantity η_L is the fraction of the incident wave found in the outgoing wave and thus contains all details of the interaction. For pure elastic scattering $|\eta_L| = 1$ by unitarity;

$$\eta_L = e^{2i\phi_L}$$

where ϕ_L is real. More generally, to allow for absorption of the incident wave

$$\eta_L = A_L e^{i\phi_L}$$

where both A_L (≤ 1) and ϕ_L are real. The individual outgoing waves are depleted by the fraction A_L of the incident wave absorbed (inelastically scattered) in the interaction. Also

$$\begin{aligned} \sigma_{el} &= r \left(\frac{d\sigma}{d\Omega} \right)_{el} d\Omega = r |f(K, \theta)|^2 d\Omega \\ &= \frac{1}{4\pi K^2} \sum_{L=0}^{\infty} (2L+1) |\eta_L - 1|^2 \\ \sigma_{inel} &= \sigma_{tot} - \sigma_{el} = \frac{4\pi}{K} \text{Im } f(K, \theta) - \sigma_{el} \\ &= \frac{1}{4\pi K^2} \sum_{L=0}^{\infty} (2L+1) (1 - |\eta_L|^2) \end{aligned}$$

In high energy scattering, one expects many partial waves to contribute.

This prompts the replacement of the sum on L to an integral over L .

Also, for $\sin \frac{\theta}{2} \ll 1$ and $L \gg 1$

$$P_L(\cos\theta) \approx J_0(2L \sin\frac{\theta}{2}) \approx J_0(L \sin\theta)$$

Then within these limitations

$$f(K, \theta) \approx \frac{1}{2iK} \int_0^\infty L dL J_0(L \sin\theta) (1 - \eta_L)$$

Assuming in addition that we can form a packet of definite angular momentum and impact parameter,

$$Kb \approx L$$

Thus we have the elastic scattering amplitude

$$f(K, \theta) \approx iK \int_0^\infty b db J_0(Kb \sin\theta) [\eta(b) - 1]$$

as obtained before.

One of the major disadvantages of this traditional optical model approach is the assumption that the particles are structureless, and the neglect of spin. Although it is generally expected that spin dependence will be less important at higher energies, the existence of a nonvanishing polarization as discussed earlier, indicates that at least at the intermediate energies of interest herein, a physically meaningful model cannot ignore spin.

Addition of spin dependent amplitudes to the semi-classical model of Fernbach et al.⁴¹ is, at best, difficult. The formalism of addition of at least a single spin-flip amplitude to the quantum mechanical form exists, though for more than one amplitude this becomes prohibitively complex. In the eikonal approximation to the quantum mechanical potential

scattering problem one is forced to choose, or at least accept the existence of a potential which fully describes the interaction.⁴⁴ Simple choices of this potential tend to result in essential singularities in the scattering amplitude.

A systematic manner of eliminating these objections is provided by an alternate "optical-type" model. This is the result of the impact parameter representation as developed by Blankenbecler and Goldberger.⁴⁵ This approach neither mentions nor infers the existence of a local potential. The physical interpretation of the results is however, identical to models described above.

The impact-parameter formalism was developed in order to exploit the fact that the eikonal approximation, at high energies, automatically satisfies the unitarity condition (in spite of the fact that it is an approximation). The basis for the representation is the assumed existence of a Mandelstam representation for the scattering amplitude.

The impact parameter representation of the nucleon-nucleon scattering amplitude is given by

$$M(s,t,u) = \int_0^\infty b db \{ J_0(b/\sqrt{-t}) H_3(b,s) + J_0(b/\sqrt{-u}) H_2(b,s) \}$$

where H_1 can be expressed as a superposition of hyperbolic Bessel functions

$$H_1 = \int_1^\infty \frac{dx}{x} K_0(b/x) A_1(x,s)$$

Normalization is such that

$$\frac{d\sigma}{d\Omega} = \frac{k_{cm}^2}{s} |M(s,t,u)|^2$$

The desired forms of the Fourier-Bessel transforms, H_2 and H_3 , are dictated by requiring the asymptotic elastic unitarity condition to be satisfied

$$\text{Im } H_{\pm}(b,s) = \frac{1}{\sqrt{s} p(s)} |H_{\pm}(b,s)|^2$$

with

$$H_{\pm}(b,s) = H_3(b,s) \pm H_2(b,s)$$

Thus the impact parameter representation problem formally reduces to finding the functions H_i which satisfy the unitarity condition at large momentum (large compared to the inverse of the range of the interaction).

The detailed analytic properties of H_i have been discussed by Blankenbecler and Goldberger.⁴⁵ In determining H_1 , it is assumed that it is a smoothly varying function of b and has a long range ($pb_{\text{max}} \gg 1$). Little more can be said about H_1 until the data are investigated. In the proton-proton case, the important observations about the data are: (1) $\sigma_{\text{tot}} \approx 40$ mb and is nearly energy independent, (2) $\frac{d\sigma}{dt} \approx e^{-\alpha t}$ over 2 or 3 orders of magnitude at a particular energy, and is nearly structureless, (3) $\sigma_{\text{el}}/\sigma_{\text{tot}} \ll .25$, (4) $\left(\frac{d\sigma}{dt}\right)_{t \rightarrow 0} \approx \left(\frac{\sigma_{\text{tot}}}{4\pi}\right)^2$, that is, the amplitude is dominated by the imaginary part, and (5) the diffraction peak "shrinks" with increasing energy. These facts have lead to several parameterizations of H_1 (e.g., ref. 45 and other references cited therein) and reasonable success in explaining the diffraction peak portion of the angular distribution have resulted.

3. Statistical and Statistical Thermodynamic Model

In considering diffraction scattering or scattering in the presence of many available channels, one of the more appealing approaches is to consider the final state as a statistical distribution of the available final states. This was first suggested by Fermi⁴⁶ several years ago, and has developed into a rather sophisticated analog to the statistical thermodynamic theory of many-macroscopic-body systems. This latter development is due primarily to the work of Hagedorn^{47,48} and Hagedorn and Ranft⁴⁹ after a suggestion by Cocconi.⁵⁰

The original statistical model⁴⁶ assumes that in the collision of two high energy nucleons (for instance) the total energy will be concentrated in a small region of space surrounding the interaction point. The energy will rapidly be distributed, according to statistical laws, among the available degrees of freedom in the excited region. These degrees of freedom are the various final state particles which can be created, and which are consistent with the applicable conservation laws. It is assumed that some sort of static equilibrium state is attained and the system then dissipates into the observable final state.

The result is that, based upon the assumed equilibrium statistical distribution of competing channels, the probability of the various final state channels can be calculated. The probability for the formation of a particular n particle state, $S(n)$, is proportional to the phase space volume available for that state:

$$S(n) = \left[\Omega / (2\pi\Lambda)^3 \right]^{n-1} dQ(W) / dW$$

where $Q(W)$ is the momentum space volume (W is the total energy). The quantity Ω is the interaction volume in which the total energy is initially concentrated. The interaction volume, due to relativistic contraction is energy dependent

$$\Omega = \Omega_0 (2mc^2/W)$$

with

$$\Omega_0 = 4\pi R^3/3$$

The radius R is arbitrarily taken to be about a pion Compton wave length.

The problem is then to calculate $Q(W)$ for a particular energy. In addition, in describing the pp interaction the multiplicative factor preceding $dQ(W)/dW$ must be modified to include the effects of identical particles and the increased multiplicity of states due to proton spin. Also, angular momentum conservation requires that the equilibrium situation be assumed separately for each angular momentum state of the incoming protons. One then can compute the probability of the various final states for each angular momentum state.

The n -particle phase space-volume takes the usual (non-invariant) form

$$\frac{dQ(W)}{dW} = \int d\vec{k}_1, d\vec{k}_2, \dots, d\vec{k}_n \delta^3\left(\sum_{i=1}^n \vec{k}_i\right) \delta\left(\sum_{i=1}^n E_i - W\right)$$

with

$$E_i = \sqrt{\vec{k}_i^2 + m_i^2}$$

In application to higher energy processes or to heavy particle annihilation reactions, the phase space volume can be suitably altered so as to be covariant.

There are two implicit conditions on the statistical model which should be emphasized. First, it is a high energy model, that is, it assumes that the available phase space includes many particles so that the application of statistics is meaningful. The second and somewhat related condition is that the model is best applied in the case of central collisions where a high concentration of the total energy in a small volume is achieved. In the peripheral interactions, Ω becomes large, the energy density is small and the number of available channels is then limited.

The statistical model can be applied to predict, for instance, multiplicity of produced particle in inelastic final states, or, more important in this context, the differential cross section for large angle elastic scattering.⁵¹

$$\left(\frac{d\sigma}{d\Omega}\right)_{el} = \frac{P_{e1}}{\sum_b P_b}$$

where P_{e1} is the probability of forming an $n = 2$ (2 proton) final state and the summation is over the probabilities of all other final states. Several numerical approaches^{52,53} to the calculation of the ratio of the probabilities have been tried, the result being

$$\left(\frac{d\sigma}{d\Omega}\right)_{pp} \approx e^{-3.3(W-2)}$$

This results in a relatively good fit to the large angle data in the

intermediate energy region. Attempts to interpret this result have, however, met with little success.

If the proton-proton scattering is purely a statistical problem, then the partial wave amplitudes are uncorrelated and their phases and magnitudes are randomly distributed. This implies fluctuations in the large angle angular distribution due to interference effects.⁵⁴ The lack of such structure observed in experiments indicates that the statistical mechanism alone cannot fully describe the scattering process. In addition, at higher energies resonances are produced which decay into many channels, some of which consist of lighter resonances and particles, which in turn may decay into multiparticle systems. The statistical model as applied to these processes becomes a rather cumbersome iterative problem.

These failures or disadvantages of the statistical model have prompted investigation of statistical thermodynamic models which are more closely tied to physics and yet maintain their basis in statistical theory.

The basis of the thermodynamic model lies in the observed mass distribution in high energy collisions. The multi-channel decay schemes of heavy resonances, instead of being a hindrance, becomes an integral part of the basis of the model.^{47,49} Since there is every indication that the mass spectrum resulting from the high energy collision will continue to become more complex with the formation of heavier resonances with more decay channels, it appears more straightforward to treat all of these systems as excited hadrons. By definition,⁴⁹ an excited hadron

is a statistical equilibrium of an undetermined number of all kinds of excited hadrons, each of which is, in turn, considered to be an excited hadron. This is the basic postulate of the thermodynamic model of Hagedorn, and combined with kinematics, results in a highly successful description of high energy scattering.

Inherent in this description is an asymptotic bootstrap resulting from the following self consistency condition.⁴⁸

If: $\rho(m)dm$ = mass spectrum of hadrons, that is, the number of excited hadrons with mass between m and $m + dm$
 and: $\sigma(E)dE$ = number of states with energy between E and $E + dE$
 in the thermodynamic description of the scattering process,

then, if the thermodynamic system is to describe the excited hadron system

$$\lim_{x \rightarrow \infty} [\rho(x) - \sigma(x)] = 0$$

Hagedorn shows that this implies an exponential growth of the mass spectrum and the existence of a maximum "temperature" which governs all high energy phenomena. For sufficiently high energies and large momentum transfers, as this maximum temperature is approached, the details of the interaction and the structure of the interacting hadrons will cease to influence the results of the interaction. The actual determination of this maximum temperature could most accurately be done by investigating the presently unavailable asymptotic mass spectrum of hadrons since the distribution is shown to be

$$\rho(m)dm \rightarrow \frac{C}{5/2} e^{m/T_0} dm$$

Since this determination is not possible as yet, other methods can be used, one of which is indicated below.

As noted, it is predicted that the temperature T_0 will replace the details of the structure of the interacting particles. In particular, it is expected that T_0 will govern the distribution in transverse momentum of the outgoing particles. That only the transverse distribution is affected follows from the fact that this distribution will not be affected by kinematical effects of relative motion of different portions of Ω which are highly varying from collision to collision. Specifically, the transverse momentum distribution is unaffected by Lorentz transformations along the collision axis.

As a result, (as can be rigorously shown within the context of this model) the resulting transverse momentum distribution is

$$W(P_\perp) \sim C P_\perp^{3/2} e^{-P_\perp/T_0}$$

at sufficiently high energies. We can then write the differential cross section as product of: (i) $W(P_\perp)$, (ii) the probability that the final state is an $n = 2$ (p-p system) and (iii) kinematic factors.

Since $P_\perp = |\vec{P}| \sin \theta$ is slowly varying at large angles, we expect

$$\left(\frac{d\sigma}{d\Omega}\right)_{\theta \approx \pi/2} \approx A e^{-P_\perp/T_0}$$

where A , at a particular energy is nearly independent of angle.

That the large angle p-p differential cross section could be fit

with a functional form of this type was noted by Orear⁵⁵ somewhat before the model was proposed. In Orear's work, this form provided a good fit to the data over a calculated range of $W(P_A)$ of nine orders of magnitude. The resulting maximum temperature was found to be

$$T_0 = 158 \pm 3 \text{ MeV}$$

B. Regge Poles

The Regge pole model, developed for application to potential scattering problems,⁵⁶ has undergone piecemeal modification in extension to high energy scattering^{57,58} in order to produce predictions consistent with experiment. Although the relativistic generalization of the original Regge-hypothesis has never been proved, the model, in its pure form, met with considerable success in qualitatively explaining some of the rather subtle characteristics of high energy scattering phenomena.

The Regge-pole model attempts to describe a direct channel reaction in terms of the analytic properties of the crossed-channel amplitude. In the usual manner we write the asymptotic s-channel scattering amplitude (neglecting spin) as a sum over appropriate Regge poles

$$A(s,t) = \sum_k \beta_k(t) \frac{1 + \tau e^{i\pi\alpha_k(t)}}{\sin\alpha_k(t)} \left(\frac{s}{s_0}\right)^{\alpha_k(t)}$$

where $\alpha_k(t)$ is the trajectory of the Regge pole k , β_k is the residue function and τ is the signature factor for the trajectory ($\tau = \pm 1$). The quantity s_0 is a scale factor typically taken to be 1 (GeV/c)^2 .

The poles k are those consistent with the various conservation laws for the internal quantum numbers.

The t-channel description of the s-channel reaction $pp \rightarrow p$ is the elastic scattering of the $\bar{p}$ from p. The t-channel quantum numbers are $B = 0$, $S = 0$, $I = 0$ or $I = 1$. Thus the candidates for inclusion in the Regge pole expansion include those particles with these internal quantum numbers. The principal trajectories to be included are then the ρ and A_2 together with the vacuum poles P and P' . (See Ref. 59 and Ref. cited therein). In the asymptotic limit, it is expected that this simple form of the model, including only the vacuum may be valid in the small momentum transfer region. In this limit, spin probably is not important and for small momentum transfers, the residues may be nearly constant.

At intermediate momenta and large angles however, rigorous application of the Regge model is prohibitively complex. Spin cannot be neglected so a single Reggeized amplitude is probably not sufficient. The individual residues also would not be expected to lack t dependence. There is no a-priori motivation, then, for application of the model to the data from this experiment. Pinsky⁶⁰ has shown, however, that some of the predictions of a simple Regge model are indeed obeyed by the large angle p-p data at high momenta. He has shown that the cross section parameterization

$$\frac{d\sigma}{d\Omega} = \frac{1}{s} K(t) \left(\frac{\nu}{s_0} \right)^{2\alpha_{\text{eff}}(t)},$$

with

$$\nu = \frac{s - u}{2}$$

is meaningfully related to the experimental data. Here $K(t)$ includes the contributions of the residue and signature factor for the single "effective" trajectory, $\alpha_{\text{eff}}(t)$. One nontrivial prediction of this model is that if one plots $\log (S d\sigma/d\eta)$ vs. $\log v$, at constant values of $-t$, straight lines will result. The slopes of these lines are twice the values of $\alpha_{\text{eff}}(t)$ at that t .

Pinsky has applied this model successfully to the high energy data of Allaby et al., verifying that this parameterization does describe the large angle p-p data. Interpretation of this result in terms of the contributions from different trajectories have not, as yet, been offered.

III. EXPERIMENTAL EQUIPMENT AND PROCEDURES

A. Chamber Description

The wire spark chambers used in this experiment were designed and fabricated by the Indiana University Spark Chamber Group. A total of twelve chambers were used, being equally divided among three sizes (nominal active areas of 8" x 8", 17" x 23", and 28" x 36").

The chambers consisted of two parallel planes of wires separated by a $\frac{1}{2}$ " G-10 phenolic (fiberglass) frame. The planes were made up of parallel wires (.005" diameter, tempered beryllium copper) with a separation of approximately 1 mm (26 wires/inch). A typical chamber is shown in schematic form in Figure 8.

The frames were 4" in width (6" in the case of the larger chambers) and were cut from sheets of phenolic. The planes of wires were automatically wound on a rotating platform, glued directly to the frames and one end was soldered to the bus from which the chambers were pulsed. Each plane of wires completely covered the nominal active area (determined by the frame) and extended 1" on either side onto the frame. The edge wires, from about $\frac{1}{4}$ " into the active area, were not connected to the bus. This produced a less abrupt change in the field characteristics in the region of the edge of the phenolic where edge sparking typically presents problems. In cases where edge sparking still appeared, more of the end wires were disconnected from the bus. The actual active area was thus determined somewhat by the behavior of the chamber.

Directly over the wires, on each face of the chamber, a sheet of

FIGURE 8

.002" aclar was attached with a silicone rubber adhesive. This material is highly impervious to the diffusion of water vapor which would contaminate the chamber. This served as the initial gas seal. A sheet of wire cloth (~200 mesh stainless steel) was placed over the aclar. The mesh shielded the plane and allowed a more uniform field within the chamber.

Finally, strips of lucite (1" x $\frac{1}{8}$ " thick) were placed around the border of the active region of the chamber and covered with a sheet of .009" mylar. This step provided a secondary gas seal.

During the experiment, the chambers were continuously purged with a mixture of 90% neon and 10% helium gas. In order to suppress spurious sparks, a small amount of isopropyl alcohol was introduced into the gas mixture before it entered the chamber. This was done by bubbling ~3% (by volume) of the total gas flowing through the system through an alcohol bath. The chamber operation was quite sensitive to the amount of alcohol included in the mixture and was visually monitored at regular intervals.

The spark chamber trigger system consisted of a master spark gap driver/amplifier which received the trigger command from the fast logic and which in turn provided an output to each of the slave gaps. Two chambers were associated with each slave gap. The master gap signal caused slave gap breakdown and application of the 7.5 KV high voltage pulse to the chamber. A 100V clearing field of opposite polarity to the pulse was maintained at all times in order to reduce the chamber memory time to about 1.5 μ sec.

The magnetostrictive readout system consisted of an aluminum bar

($\frac{1}{2}$ " x $\frac{1}{2}$ ") with a groove milled its full length on one side. The magnetostrictive ribbon (.016" x .004" remanur-P) was attached at one end, laid in the groove and extended into a box at the other end where the pickup coil and preamplifier were located. Reflections of the magnetostrictive signal were damped at each end of the ribbon by placing it between strips of tape. In some cases, in order to increase the size of the output signal two ribbons were used together. A schematic of the preamplifier is shown in Figure 9.

At each side of the active chamber wires of each plane, and parallel to the chamber wires in that plane, fiducial wires were epoxied directly to the chamber surface. Each magnetostrictive "wand" laid across the two fiducials and the chamber wires between the bus and lucite strip border. The preamp output consisted of two fiducial pulses with one (or possibly more) spark pulses between them. The pulse sizes varied somewhat from preamp to preamp but were typically 3 ± 1 volts.

B. Beam Description

The experiment was performed in the secondary beam #1 of the external proton beam at the Argonne National Laboratory zero gradient synchrotron. The beam transport system is shown in Figure 10. The beam enters the figure from the left, after extraction from the ring and initial focusing. The septum magnet, XB5, provided the initial intensity selection for the secondary beam. After passing through the Piccioni bending magnet, XB6, the 12.5 GeV/c protons struck the target TAl ($\frac{1}{4}$ " x $\frac{1}{4}$ " x $\frac{1}{4}$ " long Cu). Quadrupoles XQ11 and XQ12 provided the first focus for the secondary beam at

MAGNETOSTRICTIVE READOUT PREAMPLIFIER

FIGURE 9

POSITION CODE	EQUIPMENT TYPE
XB5	Septum Bending Magnet 2 I 60
XB6	Piccioni Bending Magnet 3.5 II 39.5 (BM-101)
XBA11, XBA12, XBA13	Bending Magnets 18 VI 72 (BM-105)
XQA11, XQA12, XQA13, XQA14	Quad. Magnets 10 Q 36 (QM-102)
TA-1	Copper Target .320" x .320" x 4"
TG	Hydrogen Target
A	Collimator, 1st Focus

FIGURE 10

the collimator A. The bending magnet XBAl1 together with the lead collimator provided the momentum selection for the secondary beam. The beam passed through the final two bending magnets XBAl2 and XBAl3 to the liquid hydrogen target, TG. Quadrupoles XQAl3 and XQAl4 provided the second focus at the target.

In the region of the copper target the beam cross section was basically elliptical. Segmented wire ion chamber photographs of the beam at this point indicated its horizontal width to be $\sim .25''$ and its vertical dimension to be $\sim .60''$ (FWHM). In the liquid hydrogen target area, the beam spot was more nearly circular with a diameter of approximately $.75''$.

The beam in which this experiment was performed is the same as that used in an optical spark chamber experiment performed during the summer of 1968 by the Indiana University Group. In preparation for this earlier experiment, beam studies were conducted.⁶¹ The momentum spread of the beam was found to be 1% (FWHM) at 3.0 GeV/c. The uncertainty in the central momentum was determined to be less than 1%. These figures were obtained by making time-of-flight measurements between the momentum slit (A in Figure 3) and the liquid hydrogen target area, a distance of about 52.2 feet. These measurements were performed at several momenta for protons, pions and deuterons.

During this experiment the beam spill lasted 600 ns and had a repetition rate of approximately 1000 per hour.

A gas threshold Cerenkov counter (located between the collimator and bending magnet XBAl2) was used to distinguish protons from pions in the beam. This counter was filled with Freon 12. The counter pressure

(index of refraction) was adjusted at each momentum so that the threshold for Cerenkov radiation production was well above the proton velocity and below the pion velocity. The pressure curve for the Cerenkov counter obtained with a 3.0 GeV/c beam is shown in Figure 11. The ordinate in this figure is the percentage of the total charged particle beam not counted by the Cerenkov counter. The sharp drop in anticounting rate between ~ 15 psia and 25 psia is due to the counting of pions in the beam.

The beam intensity varied from 30,000 particles ($\sim 15,000$ protons) per burst at the lower momenta to 70,000 particles ($\sim 50,000$ protons) per burst at the higher momenta.

C. Experimental Layout

A schematic of the layout of the equipment is shown in Figure 12. The beam enters from the left and is incident on the 12" $\times$ 1.75" diameter liquid hydrogen target. The solid figures represent plastic scintillation counters and the open figures are the spark chambers. The beam particles are detected by counters B_1 (2" $\times$ 2" $\times$ 1/8") and B_2 (1" $\times$ 1" $\times$ 1/8"). Counter AH was $\frac{1}{4}$ " thick and had a 1" diameter hole through which the beam passed. This served as the beam halo anticounter, and typically intercepted 5 to 10% of the beam. A beam proton had the logic signature $B_1 B_2 \overline{AH} \overline{CG}$. The Cerenkov counter (CG) is not shown in this figure.

Counters A_1 , A_2 , A_3 and A_4 (A_3 and A_4 were located above and below the target and are not shown in Figure 12) were anticounters which covered most of the solid angle not subtended by the detecting system. The

PRESSURE CURVE AT 3.00 GeV/c
BEAM CERENKOV COUNTER (FREON-12)

$\overline{BIB2\ AH\ CG} / \overline{BIB2\ AH} \ (%)$

90
80
70
60
50
40

10

20

30

40

50

60

70

80

PRESSURE - (PSIA)

FIGURE 11

FIGURE 12

scattered particles were detected by at least one counter in each of the two arrays (F_1, F_2, F_3 and S_1, S_2, S_3) located behind the spark chambers in the two arms. All counters except B_1, B_2 and AH , were made of $\frac{1}{2}$ " thick polystyrene.

The logic signature which resulted in a trigger of the spark chambers was: $B_1 B_2 \overline{AH} \overline{CG} F S \overline{A}$ where F, S , and A represent the inclusively ORed result of the individual array.

The laboratory polar angle acceptance of the system was from about 5° to 43° in the F (fast) arm and from 24° to 75° in the S (slow) arm. This corresponds to a c.m. angular acceptance of at least $30^\circ \leq \theta_{c.m.} \leq 90^\circ$ at all momenta. A discussion of the resulting detection efficiency will appear in Chapter IV.

Elastic scatters in which counter F_1 is struck correspond to small center of mass angles (near $\theta_{c.m.} = 30^\circ$). This is the region in which the cross section is relatively large at any particular energy. In order to increase the large angle statistics without recording an exorbitant number of small angle scatters, part of the run time at most momenta was used with F_1 not included in the logic.

D. Fast Electronics

A block diagram of the fast electronics used in the experiment is shown in Figure 13.

At the top of the drawing are shown the input signals. The initial discriminators were gated on by a signal from the accelerator which indicates the beginning of the spill. The beam logic signature ($B_1 B_2 \overline{AH}$) was formed at coincidence module C2 at the left of the drawing.

FIGURE 13

Logical separation of protons from other beam particles occurred at C4 where the Cerenkov signal was placed in anticoincidence with the beam. The pile up gate (PG) $\bar{B}$ (not busy) output was also required to be in coincidence at this point. This suppressed some of the events with multiple beam tracks. If the beam logic signature is satisfied at C2, the $\bar{B}$ output of PG becomes logically false for a preset period (1.5 sec) corresponding to the chamber memory. A second beam particle cannot satisfy the C4 coincidence within this period.

The signal output of the phototube viewing the Cerenkov counter was quite small. This required the discriminator (D1) setting to be relatively low resulting in a noisy signal C5. In order to estimate the accidental coincidences due to these spurious beam counts the C5 signal was deliberately mistimed by about 30 nsec. with respect to $B_1 B_2 \bar{A}\bar{H}$ and scaled in coincidence (C3) as $B_1 B_2 \bar{A}\bar{H} \bar{C}$. In this manner it was determined that less than 1% of the beam particles were vetoed by spurious signals from the Cerenkov counter.

At the top center of the figure is shown the ORing of the fast and slow arm counters individually and on the right, the ORing of the anticounter output. The final coincidence (C6) of the proper beam signature together with at least one fast arm counter and one slow arm counter and no anticounter signals determined whether the event was acceptable and would be recorded. A positive decision results in a trigger signal for the spark gap preamplifier, the incrementing of the M' accumulated scaler (scaling the number of recorded events), a signal to the wire spark chamber master control initiating the data acquisition and digitiser

scanning cycles, gating off the fast logic for the preset system dead time, and finally, gating on the tagging units which then were allowed to store information as to which of the F and S scintillation counters had detected passage of a particle.

E. Slow Electronics

Figure 14 shows the various elements of the slow logic system involved in the experiment, that is, the actual digitising and recording system for the wire spark chambers.

The lines entering the diagram at the top emanate from the fast logic and the magnetostrictive readout preamplifiers. As indicated in Section D, the proper logic signal led to (1) opening of the tagging gate so that the tagging units could accept information indicating which scintillation counters were struck by a particle, (2) incrementing of the accumulator scaler M' , which indicated the number of accepted triggers, (3) accumulating the B' scaler, which contained the total number of beam particles realised in the burst, (4) accumulating the ZIS pulse number (NP) scaler and finally (5) signaling the master control to hold all wire chamber scalars in reset for a preset period of time to avoid hearing the noise generated by the spark chamber pulse and following this delay, to begin the scan and readout cycles. At the top right of the diagram are shown the inputs from the 24 magnetostrictive readout preamplifiers (2/chamber) feeding into the digitiser.

The master control unit signaled the tape transport to come to recording speed and the contents of the data input units (fixed data,

FIGURE 14

tagging unit, accumulator scaler and wire spark chamber scalers) were sequentially strobed onto magnetic tape. The tape transport operated in a continuous - write mode until the master control indicated that the end of scan had been reached.

The logic fixed the system dead time to approximately 80 ns. (60 ns. in the early portion of the experiment). At the end of this delay, another system trigger could occur, generating another master control "start" signal. This reset all scalers (except the accumulator scalers) and the tagging unit. At the lower momenta, the data recording rate was approximately 10 per burst, determined primarily by the equipment itself, while at higher momenta the rate dropped to about 5 events per burst due to the low cross section and limited beam intensity.

A total of approximately 20,000 events were recorded on each magnetic tape. Before the beginning of each tape, the appropriate fixed data (run no., date, target condition [full or empty], and beam momenta) were updated, the system was manually reset to clear the accumulator scalers and the system restarted.

F. Monitoring

During the performance of the experiment, several methods were used to check the system operation. A scaler readout system which monitored the number of pulses appearing on each magnetostrictive wand preamplifier for each system trigger was visually observed. This permitted a constant check of the chamber and readout system operation.

The experiment was monitored off-line using an Advanced Scientific Instruments Model 6020 computer. A program which read the magnetic tape

and histogrammed the wand scaler information was run several times daily. This provided a more detailed check on chamber and readout system operation.

Finally, the main analysis program was run periodically. This too served as a system operation check and also gave some indication of the number of background events being accepted so that the run time at each momentum could be adjusted to meet the experimental goals.

IV. ANALYSIS

The analysis of the data from this experiment was performed on the Indiana University Pattern Recognition Project's Sigma 5 computer and it proceeded in four steps; (i) filtering and reconstruction, (ii) application of cuts and weighting of each event according to the geometric detection efficiency, (iii) background subtraction, and (iv) application of corrections to the data. These items will be discussed below. In addition, several methods have been used to check the internal consistency of the data. These will be briefly discussed in Section E.

A. Filtering and Reconstruction

A possible requirement for reconstruction of a recorded event is that there be two sparks in each of two arms and one spark in the remaining arm. This would permit location of the vertex from the intersection of the two trajectories which can be reconstructed in the arms with two sparks. The third trajectory is then determined by the vertex and remaining spark. Due to the redundancy of the system and the relatively high efficiency of the individual chambers, however, the minimum requirements for reconstruction were made somewhat more stringent.

The main analysis program first searched each digitised wand output for the proper fiducial location, and counted the number of sparks observed by that wand. If no fiducial was found, this gap was assumed, in the remainder of the reconstruction, to have no sparks. It was demanded that of the four chambers in the beam arm, at least three recorded the passage of one and only one particle. If this condition

was not met, the offending event was discarded and the total number of beam protons used in the calculation of the differential cross section was scaled down to account for this. The effect of this requirement was to eliminate events with possible multiple beam tracks and to select a more pure sample of the data by eliminating about 5 to 10% of the recorded events. Of these discarded events, about half failed due to lack of sparks and the remainder due to scattering of sparks around a fitted line. This latter condition indicated that at least two π^0 meson particles entered the system and different chambers were responding singly to different particles. This procedure was not used on the arms detecting the final state particles as this could introduce angular biases if chamber efficiencies are not known accurately.

It was required that at least one of the two final state detecting arms had at least two sparks so that vertex reconstruction was possible. In addition, the other arm must have at least a single spark. About 6% of the recorded events had 0 or 1 spark in one of the two arms, but the vast majority of these were reconstructable as there was 1 spark in one arm and 2 or more sparks in the other arm. Cases where there were no sparks in one arm were extremely rare and attributable to accidental coincidences in the F and S counters.

The next step was to reconstruct the spark locations in three-dimensions. The trajectory in each arm was then determined by the two chambers which were separated by the largest distance and which had one and only one spark. If neither final state arm had two chambers with a single spark, the event was discarded. This was an extremely rare occurrence.

The interaction vertex was determined by the intersection of the trajectories in the beam arm and the final state arm which had at least two chambers with one and only one spark. If both final state arms satisfied this condition, the beam arm and slow arm (which makes the larger angle with the beam and therefore determines a vertex position more independent of the chamber resolution) were used. The three-dimensional locations of all possible combinations of sparks in the other arm were determined and, using elastic scattering kinematics to predict the missing trajectory, the spark lying closest to this trajectory was found. The vector from the "known" vertex through this "saved" spark was then accepted as the true trajectory.

This procedure had the desirable effect of simplifying the analysis in that multiple tracks in a particular arm present many possible trajectories each of which must be investigated. This "saving" procedure permitted an immediate prediction of which of these trajectories should be used. The undesirable feature was that it tended to make randomly positioned sparks look like elastic scatters, that is, it tended to draw background events toward the kinematic curve for elastic scattering. As will be indicated in Section IV-D, this procedure did not present any significant problems.

Shown in Figures 15, 16 and 17 are the distributions of vertex locations which resulted from the reconstruction. These figures include all of our data at 3.0 GeV/c. As shown in Figures 16 and 17, the beam was not centered precisely on the centerline of the target. However, the target diameter was sufficiently large so that the target walls did not intercept the beam.

FIGURE 15

FIGURE 16

FIGURE 17

Since no magnets were used in the detection system the remainder of the analysis depended upon the two available kinematic constraints. These are the coplanarity and the opening angle restrictions. Figures 18 and 19 show the distributions in coplanarity

$$COP = \hat{P}_D \cdot (\hat{P}_F \times \hat{P}_S)$$

and opening angle deviation from elastic scattering kinematics

$$DEL = \theta_{S,Lab}^{Measured} - \theta_{S,Lab}^{Kinematic}$$

Here $\hat{P}_D$, $\hat{P}_F$ and $\hat{P}_S$ represent unit vectors in the direction of the beam, fast and slow protons respectively; $\theta_{S,Lab}^{Measured}$ is the slow proton laboratory angle as determined in the reconstruction and $\theta_{S,Lab}^{Kinematic}$ is the slow proton laboratory angle predicted by elastic scattering kinematics using the laboratory angle of the fast proton as determined in the reconstruction. These figures include the same data as the target distributions. As shown, the elastic peak is quite well defined and sits on a relatively small and smooth background.

Figures 20 and 21 show the distributions in angle of the reconstructed beam trajectories. Figure 20 is the azimuthal angle of the beam (angle between the trajectory and the beam direction) and Figure 21 is the dip angle (angle between the trajectory and the projection of the trajectory in the x - y [horizontal] plane). The beam direction in the region of the target is seen to deviate slightly from the nominal center line and is slightly skewed, possibly indicating some upstream interference.

FIGURE 18

FIGURE 19

FIGURE 20

FIGURE 21

B. Target and Fiducial Volume Cuts

Figure 22 shows the kinematic curves for p-p elastic scattering at several momenta. Also shown are the maximum and minimum limits of the detection system (determined by the scintillation counter sizes). The 45° line represents $\theta_{c.m.} = 90^\circ$, the symmetry axis for the kinematic curves, and these curves extend to $\theta_{c.m.} = 30^\circ$ to the right. As shown, the detection system was capable of accepting scatters for $\theta_{c.m.} > 90^\circ$. Since the large angle region is characterized by relatively small cross sections, and the kinematic curve is symmetric about 90° , those events occurring with $\theta'_{c.m.} > 90^\circ$ were folded back to $\theta_{c.m.} = 180^\circ - \theta'_{c.m.}$ to improve statistics. Typical geometric detection efficiencies, obtained by a Monte-Carlo generation of events, are shown in Figure 23. The rapid increase at large angles is the result of this foldover. In practice, each acceptable event was weighted by the factor of 1 divided by the detection efficiency at the particular angle of the event.

As previously indicated, the fast arm counter array was such that it was possible to suppress small angle scatters to provide smaller statistical errors in the large angle cross sections. The folding of the F_1 - out data was done in a straightforward manner. The small angle detection efficiency for F_1 - out data is zero and at large angles it is identical to the detection efficiency shown in Figure 23. The recorded F_1 - out data were used only in the region where the two detection efficiencies were equal. That is, although the F_1 - out data extended typically from 90° to less than 50° in the c.m., these events included in the final results come only from the region of $\theta_{c.m.} > 65^\circ$. These

FIGURE 22

FIGURE 23

F_1 - cut events coming from $50^\circ \leq \theta_{c.m.} < 65^\circ$ are in an angular region where the detection efficiency is changing quite rapidly and improper weighting of these events would be a danger.

The fiducial volume cuts used in the determination of the detection efficiency and in the final analysis steps were: (i) to discard all events in which the reconstruction indicated the particle was not inside a 1" border around the active area of the spark chambers, (ii) to discard all events in which the reconstruction indicated that the P and/or S counters were struck within $\frac{1}{2}$ " or less of the counter edge and finally, (iii) to reject events for which the target support structure provided an azimuthal cut.

The vertex location in the target was also subject to cuts. The y and z cuts (Figures 16 and 17) were dependent upon the centering of the beam on the target and beam spot size. A three standard deviation cut was typically applied resulting in a cylindrical target of acceptance with a diameter of from 1.2" to 1.9" depending on momentum. The actual target length was 12" (including hemispherical end caps) but the upstream end could not be seen by most of the system because of a large brass target mounting structure surrounding this portion of the target. The effective target length was taken to be 8.5", extending from +5.0" to -3.5" from the target center.

C. Kinematic Restrictions and Background Subtraction

The final steps in the analysis consisted of binning the reconstructed events, applying the kinematic cuts and manually subtracting the background events from each bin.

As previously indicated, the two available kinematic restrictions are on the coplanarity and opening angle. First, all of the reconstructed events at a particular momentum were subjected to a three standard deviation coplanarity cut. Figure 24 shows the coplanarity distribution for all events at 3.0 GeV/c which satisfy the target and fiducial volume cuts.

The remaining events were sorted into the appropriate 2° bins in $\theta_{c.m.}$. For each bin, the distribution in the angular deviation from elastic scattering kinematics was obtained. Figure 25 shows typical DEK distributions at 3.0 GeV/c.

These histograms were used to determine the background level, by extrapolating the tails (from approximately ± 1.5 times the full width at half maximum) under the elastic peak. The resulting background subtraction varied considerably with both angle and momentum. Table I shows a brief summary of the subtraction in several $\theta_{c.m.}$ bins. Table II is a summary showing for each momentum, the number of recorded events, and the resulting number of elastic events.

The remaining events, after the background subtraction, were then used in calculating the differential cross section for each bin from

$$\frac{d\sigma}{d\Omega} = \frac{N_w}{B \Delta\Omega}$$

where N_w is the weighted number of events in that bin, B is the total number of beam protons ($E_1 E_2 \overline{A} \overline{B}$ accumulated) at the particular momentum $\Delta\Omega$ is the solid angle element ($2\pi \sin\theta_{c.m.} \Delta\theta = 4\pi \sin^2\theta_{c.m.} / 57.296$ for a 2° bin centered at $\theta_{c.m.}$) and where

FIGURE 24

DISTRIBUTION IN ANGULAR DEVIATION FROM ELASTIC SCATTERING
KINEMATICS AT 3.0 GeV/c (AFTER TARGET, GEOMETRY AND COPLANARITY CUTS)

FIGURE 25

TABLE I

BACKGROUND SUBTRACTION SUMMARY

P_t	$30 \leq a_{c.m.} < 32$		$48 \leq a_{c.m.} < 50$		$68 \leq a_{c.m.} < 70$		$88 \leq a_{c.m.} < 90$	
	TOTAL EVENTS	% BACKGROUND	TOTAL EVENTS	% BACKGROUND	TOTAL EVENTS	% BACKGROUND	TOTAL EVENTS	% BACKGROUND
1.50			2582	2.0	1593	1.1	1745	1.1
1.65			2409	1.2	1145	1.9	1265	1.4
1.80			2098	2.6	952	2.5	1100	1.3
2.00			1494	2.4	1153	2.2	1450	1.7
2.25	2632	2.9	993	3.3	802	3.5	1170	2.7
2.50	2911	2.5	932	2.9	746	3.6	1190	3.4
2.70	3048	3.7	818	4.4	732	7.4	1121	4.0
3.00	3663	4.4	852	5.6	782	4.2	1216	4.4
3.50	2133	5.1	621	4.8	612	12.4	778	10.8
4.00	3881	5.8	726	7.7	484	10.7	575	12.2
4.50	2888	5.4	472	5.1	177	22.6	347	17.3
5.00	1311	5.5	165	7.3	47	12.8	153	31.4
5.50	2981	7.4	324	10.2	108	30.6	166	32.5

TABLE II
DATA DISTRIBUTION

P_L	No. of Recorded Events	No. of Usable Elastic Events
1.50	159,285	40,961
1.65	180,889	44,361
1.80	158,598	37,478
2.00	181,876	41,925
2.25	169,990	36,742
2.50	181,457	34,813
2.70	202,104	33,237
3.00	231,628	35,601
3.50	265,431	25,271
4.00	364,351	24,970
4.50	335,640	16,411
5.00	184,792	6,553
5.50	399,664	10,095

$$P = 10^{30} \frac{1}{A_0 \rho L_T}$$

A_0 being Avogadro's number, ρ the density of liquid hydrogen (.07 gm/cm³) and L_T is the target length (8.5 in.). This yields the differential cross section in μb .

D. Checks

Several methods have been used to determine the internal consistency of the method of analysis and of the resulting data.

One of the more basic methods used to check the reconstruction was the manual reconstruction of many events. This verified that the "saving" technique described in Section IV-A was valid and did not draw a significant number of background events within the kinematic cuts. This was due, at least in part, to the high chamber efficiency and the small number of spurious sparks occurring in the chambers.

During the experiment, two special sets of data were recorded. The first involved looking only at beam particles which did not interact in the target, but passed directly through the four spark chambers in the fast arm. This served as a check on the alignment of these two arms.

Since the relative alignment of the beam and fast arms was quite well known, the calculation of $\theta_{c.m.}$ for each event was based upon the laboratory angle of the final state particle in the fast arm.

The second set of data recorded in order to check the results involved an empty liquid hydrogen target. This was done at several moments, and these tapes were subjected to the usual analysis procedure in

order to investigate the empty target background. In the reconstruction of these events, the vertex distribution in the beam direction prominently displayed the solid ends of the target as peaks, separated by a small background of scatters from gaseous hydrogen in the target. The beam spot size was shown to be sufficiently small so that no visible scattering from the cylindrical surface of the target was observed.

One method was used to check the fiducial volume of the system. This was to subject the recorded events to the usual reconstruction, and then to histogram the horizontal and vertical coordinates of the point of intersection of the reconstructed trajectory with the planes of the F and S counter arrays. In this manner, the edges of the counters (outside of which no intersections would occur ideally) were precisely located. In addition, the possibility of "holes" existing between the counters in each array was investigated by using the recorded tagging information (Chapter III, Sections D and E). By looking only at those events where two adjacent counters "saw" a particle, the actual overlap of the counters was determined to be approximately $1/8^{\circ}$ in all cases.

Two methods were used to investigate the manual background subtraction procedure and consistency. The first was to reverse the entire procedure, that is, to impose the DEL cut on the data, prior to binning, and to make the manual background subtraction on the coplanarity of the binned data. At the lower energies, the two methods gave consistent results. Above ~ 3.5 GeV/c however, the latter method proved superior. At the higher energies, some bins in the distribution fall quite close to counter edges. The DEL distributions in these bins would then tend

to be very asymmetric and the background level in the tails was questionable. The coplanarity distributions were however symmetric, and in these cases, the background was determined from the coplanarity distributions.

The second method used to check the background subtraction involved investigating the coplanarity (DEL) of the events lying outside the DEL (coplanarity) cut used in determining the background level. Peaking of these distributions would have indicated the possibility of elastic events not included in the corresponding cut.

E. Corrections to the Data

Discussed below are the various corrections which have been applied to the reported data. Table III briefly summarized these corrections as a function of beam momenta. Where appropriate, the angular dependence of the corrections has been taken into account. This dependence is not reflected in Table III.

1. Counter Inefficiencies

During the performance of an earlier experiment⁶¹, the efficiency of the individual counters was measured by placing each counter in the beam, and determining the ratio of counting rates

$$\frac{B_1 B_2 \overline{AH} C_1}{B_1 B_2 \overline{AH}} \cdot 10^2,$$

which gives the percent efficiency of the counter C_1 . All cross sections have been scaled up by 1% (±1%) to account for the counter inefficiencies.

2. Incident Beam Absorption

The recorded beam intensity ($B_1 B_2 \overline{AH} \overline{CD}$) denotes the proton flux

TABLE III
CORRECTIONS

P_T GeV/c	P-Arm Absorp- tion %	S-Arm Absorp- tion %	Incident Beam Absorption %	Cerenkov Ineffi- ciency %	Beam and Anti- counter Acci- dentals %	Counter Ineffi- ciency %	Chamber Ineffi- ciency %	Kaon Beam Contam- ination %	Recon- struction Ineffi- ciency %
1.50	3.9	1.8	4.0	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
1.65	4.1	1.8	4.1	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
1.80	4.2	1.8	4.1	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
2.00	4.2	1.8	4.1	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
2.25	4.3	1.8	4.0	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
2.50	4.3	1.8	3.9	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
2.70	4.3	1.8	3.7	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
3.00	4.3	1.8	3.6	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
3.50	4.4	1.7	3.6	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
4.00	4.6	1.7	3.6	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
4.50	4.7	1.7	3.6	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
5.00	4.9	1.7	3.6	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2
5.50	5.0	1.7	3.6	.3 ± .3	1 ± 1	1 ± 1	2 ± 2	2 ± 1	2 ± 2

at the upstream end of the target. Due to absorption, the intensity of the beam as a function of position in the target is⁶²

$$I = I_0 e^{-x/L_0}$$

where I_0 is the incident intensity (B_1 B_2 $\overline{AH}$ $\overline{OG}$), x is the distance from the upstream end of the target and L_0 is the collision length which is given by

$$L_0 = \frac{A}{N_0 \sigma}$$

with A = atomic number of the material (liquid hydrogen) through which the particle is passing ($A = 1$), N_0 is Avogadro's number and σ is the total cross section. Thus, L_0 is energy dependent.

Since $L_0 \gg 12$ inches, the maximum target position, then

$$\begin{aligned} I &\approx I_0(1 - x/L_0) \\ &\approx I_0(1 - xN_0\sigma) \end{aligned}$$

Since this expression is linear in x and $L_0 \gg x$, the average value of x is approximately 6.75 inches, the distance from the upstream end of the target to the center of the target cuts.

The energy dependence of this correction (σ) was included in the calculations.

In the same manner, a constant correction has been made for beam absorption in counter B_2 and in the .010 inch thick mylar window of the target through which the beam passed.

The combination of these three corrections is shown in Table III.

3. Nuclear Interaction of the Final State Protons

Each scattered proton, in reaching the F and S counters in order to produce a trigger, passed through liquid hydrogen in the target, 4 wire spark chambers and the surrounding air. In each chamber the particles traversed a total of .010" of mylar, .008" of alar, an average thickness of stainless steel (the wire mesh) of approximately .001" and an average thickness of copper (the wires in the chambers) of about .001". All of these thicknesses depend upon the scattering angle which has been suitably taken into account for each $\theta_{c.m.}$ bin.

The calculation of this correction proceeds in the same manner as that detailed for the beam. The figures for this correction given in Table II represent the bin $30^\circ \leq \theta_{c.m.} < 32^\circ$.

4. Beam Contamination

Corrections have been applied to the data to account for beam contamination arising from kaons and pions which escape the Cerenkov veto.

In order to minimize the density of material in the beam, the Cerenkov counter was not operated at a sufficiently high pressure to veto the beam kaons. The kaon component in the beam was measured to be approximately 2%. This was done by placing the Cerenkov counter in coincidence with the beam logic. The results are shown in Figure 26. The K threshold occurs near 75 psia at 5.0 GeV/c as shown in this figure.

No corrections for lepton contamination have been made since the threshold is considerably below the pressure at which the Cerenkov counter was operated. The final beam contamination correction to be applied

FIGURE 26

accounted for the Cerenkov inefficiency in vetoing pions. This correction is relatively small; being measured (in a negative beam) to be $\sim 0.3\%$.

5. Accidentals

A 1% correction has been applied to the data to account for accidental triggers due to chance satisfaction of beam logic requirements, and for chance anticounter vetoing of acceptable scatters.

The estimate of the effect of accidental anticounter vetoing of acceptable interactions depended upon (1) C , the random anticounter rate per burst, and (2) τ , the length (in time) of the anticounter logic signal. The fraction of the 600 nsec. pulse during which chance anticounter signals prevented satisfaction of the logic signature is

$$v = \frac{C\tau}{600} 10^3$$

Here, $\tau \approx 16$ nsec. and C is the sum of the random rates in the four anticounters. Due to the relatively high background in the experimental area, the anticounter rates were predominantly random. Taking C to be approximately 390,000/burst, a correction of slightly less than 1% was indicated.

The correction for accidentals in the beam was relatively small. This was determined by comparing the normal beam rate ($B_1 B_2 \overline{AH}$) and the corresponding rate when B_2 was deliberately mistimed by 36 nsec. ($B_1 \overline{B_2} \overline{AH}$). It was found that for a given total number of counts in B_1 , the number of beam counts with B_2 mistimed was about .2% of the number of normal beam counts.

6. Chamber Efficiency

Prior to the beginning of the experiment, the efficiency of the individual chambers was measured. This was done by placing the chambers in a relatively low intensity beam, defined by a beam telescope, and comparing the beam telescope rate with the chamber rate. This indicated an efficiency of about $90 \pm 5\%$ for each chamber, and it was nearly constant over the active area of the chamber.

Due to the manner in which the data analysis was performed, the determination of a system efficiency from the individual efficiencies was rather difficult. The individual efficiencies were instead used to substantiate system efficiency values arrived at in other ways.

During the analysis, the reconstruction program was run on samples of data where individual chambers and/or parts of chambers were eliminated. This resulted in the assignment of a 2% ($\pm 2\%$) correction to the data to account for events lost due to chamber inefficiency. This number is consistent with the individual chamber efficiency measurements.

7. Reconstruction Inefficiency

In addition to loss of elastic scatters due to chamber inefficiency, there is a loss due to inability to obtain a proper reconstruction of events. This is most serious in the case where one of the final state particle trajectories is determined by one of the two pairs of chambers which are closely spaced. The inherent scatter of sparks about the actual particle trajectory in this case can have the effect of producing a vertex located outside the target cuts.

In order to investigate this effect, the analysis of a sample of data at several moments was conducted with the restriction that the event was to be eliminated if only one of the two pairs of closely spaced chambers was used in the reconstruction. The beam was scaled down with the number of events discarded because of this condition. The resulting cross sections were typically 2% (12%) larger under these conditions, indicating that an insignificant number of events were lost because of this effect.

V. RESULTS

The differential cross sections are presented in Table IV for the 13 incident proton momenta of this experiment. The quoted errors neglect the normalization uncertainties specified in Chapter IV. They include only the combined statistical uncertainties in the signal (S) and background (B) as calculated from

$$\frac{\Delta(d\sigma/d\Omega)}{(d\sigma/d\Omega)} = \sqrt{\frac{S + 2B}{S}}$$

There is an overall normalization uncertainty of $\pm 7\%$ in the data. This is due primarily to the uncertainty in the corrections which have been applied to the data.

The results are shown graphically in Figure 27. The lines drawn through the data are hand drawn fits, intended to guide the eye. The angular distribution at each energy is quite smooth. We observe no strong evidence for secondary diffraction-like behavior as seen by Allaby *et al.*^{8,9} at higher momenta. The angular distribution at each energy is linear from $\theta_{c.m.} \approx 30^\circ$ where this data begins, to a c.m. angle corresponding to a constant value of $-t \approx .7 \text{ (GeV/c)}^2$, the t value near which structure in the polarization is observed. Above this angle, the distribution smoothly begins to flatten and approach 90° with a slope consistent with zero as required by the Pauli Principle.

Figure 28 presents a comparison of our data at 3.0 and 5.0 GeV/c with that from the two earlier experiments of Clyde⁵ and Ankenbrandt *et al.*¹¹ The agreement is, in general, within the combined statistical and normalization uncertainties. A systematic discrepancy appears to

TABLE IV
 PROTON-PROTON ELASTIC SCATTERING
 DIFFERENTIAL CROSS SECTIONS

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
1.50	47.6	4738.	.254	38180.	1.96
	49.6	4211.	.275	33930.	2.03
	51.6	3645.	.296	29380.	2.13
	53.6	3488.	.317	28110.	2.12
	55.6	3277.	.340	26410.	2.14
	57.6	2894.	.362	23000.	2.25
	59.6	2641.	.385	21280.	2.30
	61.6	2421.	.409	19510.	2.35
	63.6	2152.	.433	17340.	2.44
	65.6	2057.	.458	16570.	2.45
	67.6	1872.	.483	15080.	2.52
	69.6	1678.	.508	13520.	2.64
	71.6	1510.	.534	12170.	2.74
	73.6	1383.	.560	11150.	2.82
	75.6	1350.	.586	10880.	2.81
	77.6	1225.	.612	9869.	2.91
	79.6	1068.	.639	8608.	3.00
	81.6	1097.	.666	8839.	2.80
	83.6	1033.	.692	8328.	2.74
	85.6	1025.	.720	8256.	2.61
	87.6	1009.	.747	8132.	2.52
	89.6	1017.	.774	8199.	2.43

TABLE IV (CONTINUED)

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
1.65	43.7	5179.	.249	36140.	1.88
	45.6	4544.	.271	31700.	1.95
	47.6	4097.	.294	28580.	2.01
	49.6	3611.	.318	25190.	2.07
	51.6	3160.	.342	22050.	2.17
	53.6	2879.	.367	20080.	2.24
	55.6	2411.	.392	16820.	2.42
	57.6	2208.	.418	15400.	2.46
	59.6	2081.	.445	14520.	2.48
	61.6	1885.	.473	13150.	2.55
	63.6	1691.	.500	11800.	2.66
	65.6	1471.	.529	10270.	2.76
	67.6	1351.	.558	9425.	2.86
	69.6	1163.	.587	8115.	3.03
	71.6	1097.	.616	7654.	3.07
	73.6	1040.	.646	7257.	3.10
	75.6	923.1	.677	6440.	3.23
	77.6	856.3	.707	5974.	3.27
	79.6	797.0	.738	5560.	3.18
	81.6	802.5	.769	5599.	3.02
	83.6	765.4	.800	5340.	2.92
	85.6	713.9	.831	4981.	2.88
	87.6	720.9	.862	5030.	2.76
	89.6	655.5	.894	4573.	2.88

TABLE IV (CONTINUED)

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
1.80	39.7	5705.	.236	35000.	1.92
	41.7	5157.	.259	31630.	1.96
	43.7	4368.	.283	26800.	2.05
	45.7	3725.	.308	22850.	2.16
	47.7	3499.	.334	21470.	2.18
	49.7	3076.	.361	18870.	2.27
	51.7	2719.	.389	16680.	2.34
	53.6	2331.	.417	14300.	2.50
	55.6	2110.	.446	12940.	2.57
	57.6	1876.	.476	11510.	2.69
	59.6	1683.	.506	10320.	2.76
	61.6	1438.	.538	8823.	2.91
	63.6	1307.	.569	8018.	3.01
	65.6	1155.	.602	7087.	3.15
	67.6	1040.	.634	6383.	3.24
	69.6	955.1	.668	5859.	3.37
	71.6	903.9	.701	5545.	3.41
	73.6	788.4	.735	4837.	3.63
	75.6	762.6	.770	4678.	3.59
	77.6	689.7	.804	4231.	3.51
	79.6	639.8	.839	3925.	3.45
	81.6	623.1	.874	3822.	3.31
	83.6	594.1	.910	3645.	3.23
	85.6	569.4	.945	3493.	3.15
	87.6	600.6	.981	3685.	3.02
	89.6	564.5	1.02	3463.	3.07

TABLE IV (CONTINUED)

F_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\gamma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error $\pm$
2.00	37.7	6094.	.249	32110.	3.66
	39.7	4676.	.275	24640.	3.97
	41.7	4193.	.302	22100.	4.07
	43.7	3789.	.330	19970.	4.24
	45.7	3081.	.359	16240.	4.45
	47.7	2563.	.390	13500.	4.77
	49.7	2277.	.421	12000.	2.68
	51.7	1906.	.453	10050.	2.90
	53.7	1610.	.486	8485.	3.11
	55.7	1453.	.520	7656.	3.19
	57.7	1334.	.554	7028.	2.51
	59.6	1146.	.590	6039.	2.68
	61.6	1037.	.626	5463.	2.74
	63.6	945.0	.663	4980.	2.81
	65.6	833.2	.701	4390.	2.92
	67.6	763.9	.739	4025.	3.02
	69.6	730.4	.777	3849.	3.04
	71.6	612.7	.816	3229.	3.30
	73.6	617.4	.856	3253.	3.19
	75.6	578.3	.896	3047.	3.13
	77.6	560.3	.937	2952.	2.99
	79.6	539.8	.977	2844.	2.84
	81.6	518.5	1.02	2732.	2.75
	83.6	489.8	1.06	2581.	2.70
	85.6	466.7	1.10	2459.	2.68
	87.6	459.2	1.14	2420.	2.70
	89.6	465.0	1.18	2450.	2.69

TABLE IV (CONTINUED)

t_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
2.25	31.7	7270.	.210	32470.	2.04
	33.7	6179.	.237	27590.	2.14
	35.7	5659.	.265	25270.	2.16
	37.7	4645.	.294	20740.	2.31
	39.7	3831.	.325	17110.	2.46
	41.7	3386.	.357	15120.	2.53
	43.7	2813.	.390	12560.	2.68
	45.7	2326.	.424	10390.	2.88
	47.7	1863.	.460	8319.	3.13
	49.7	1560.	.497	6965.	3.30
	51.7	1412.	.535	6308.	3.39
	53.7	1115.	.574	4979.	3.79
	55.7	991.7	.614	4429.	3.93
	57.7	875.9	.655	3911.	3.11
	59.7	774.6	.696	3459.	3.22
	61.7	707.9	.739	3161.	3.31
	63.7	656.1	.783	2930.	3.35
	65.6	562.4	.827	2512.	3.60
	67.6	557.7	.872	2491.	3.44
	69.6	470.3	.918	2100.	3.72
	71.6	434.3	.964	1940.	3.88
	73.6	405.7	1.01	1812.	3.70
	75.6	398.8	1.06	1781.	3.50
	77.6	375.8	1.10	1678.	3.38
	79.6	362.3	1.15	1618.	3.22
	81.6	354.8	1.20	1585.	3.12
	83.6	371.6	1.25	1659.	2.96
	85.6	354.7	1.30	1584.	3.01
	87.6	336.8	1.35	1504.	3.09
	89.6	342.1	1.40	1528.	3.05

TABLE IV (CONTINUED)

F_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
2.50	31.8	6424.	.243	24840.	1.92
	33.8	5047.	.274	19510.	2.10
	35.7	4352.	.306	16830.	2.19
	37.7	3492.	.340	13500.	2.36
	39.7	2979.	.375	11520.	2.47
	41.7	2462.	.412	9518.	2.66
	43.7	1925.	.451	7444.	2.92
	45.7	1634.	.490	6317.	3.09
	47.7	1362.	.532	5267.	3.30
	49.7	1185.	.574	4582.	3.42
	51.7	977.4	.618	3779.	3.71
	53.7	834.9	.663	3228.	3.93
	55.7	705.1	.709	2726.	4.21
	57.7	613.6	.756	2373.	4.44
	59.7	565.6	.805	2157.	3.33
	61.7	480.7	.854	1858.	3.63
	63.7	466.0	.904	1802.	3.62
	65.7	405.8	.956	1569.	3.91
	67.7	381.9	1.01	1477.	3.84
	69.7	361.9	1.06	1399.	3.87
	71.6	347.9	1.11	1345.	3.81
	73.6	336.9	1.17	1303.	3.56
	75.6	321.6	1.22	1243.	3.35
	77.6	302.4	1.28	1169.	3.29
	79.6	302.8	1.33	1171.	3.14
	81.6	297.4	1.39	1150.	3.08
	83.6	286.7	1.44	1108.	3.09
	85.6	286.6	1.50	1108.	3.07
	87.6	266.4	1.56	1030.	3.16
	89.6	284.0	1.61	1098.	3.05

TABLE IV (CONTINUED)

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
2.70	31.8	5354.	.270	18670.	1.91
	33.8	4434.	.304	15460.	2.03
	35.8	3526.	.340	12300.	2.20
	37.8	2837.	.377	9895.	2.37
	39.8	2304.	.416	8035.	2.56
	41.8	1779.	.457	6204.	2.83
	43.7	1532.	.500	5343.	2.98
	45.7	1214.	.544	4235.	3.24
	47.7	940.8	.590	3281.	3.64
	49.7	822.9	.637	2870.	3.74
	51.7	646.2	.686	2254.	4.30
	53.7	610.5	.735	2129.	4.23
	55.7	511.5	.787	1784.	4.48
	57.7	472.8	.839	1649.	4.64
	59.7	399.4	.892	1393.	4.85
	61.7	366.7	.947	1279.	3.89
	63.7	363.7	1.00	1268.	3.78
	65.7	313.4	1.06	1093.	3.98
	67.7	317.3	1.12	1107.	3.98
	69.7	287.2	1.18	1002.	4.14
	71.7	275.3	1.23	960.0	3.97
	73.6	258.9	1.29	902.9	3.71
	75.6	248.8	1.35	867.9	3.42
	77.6	260.1	1.42	906.9	3.19
	79.6	237.1	1.48	826.9	3.16
	81.6	230.9	1.54	805.3	3.16
	83.6	220.6	1.60	769.5	3.19
	85.6	236.0	1.66	823.0	3.06
	87.6	214.8	1.73	749.1	3.19
	89.6	221.5	1.79	772.5	3.17

TABLE IV (CONTINUED)

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
3.00	31.8	4482.	.310	13610.	1.77
	33.8	3546.	.349	10770.	1.95
	35.8	2848.	.391	8650.	2.10
	37.8	2196.	.434	6670.	2.27
	39.8	1770.	.479	5374.	2.43
	41.8	1323.	.526	4017.	2.73
	43.8	1037.	.575	3149.	3.02
	45.8	816.3	.626	2479.	3.38
	47.8	713.7	.678	2168.	3.47
	49.8	586.3	.732	1781.	3.73
	51.7	520.1	.788	1579.	3.84
	53.7	421.0	.845	1278.	4.16
	55.7	385.7	.904	1171.	4.20
	57.7	360.7	.964	1096.	4.32
	59.7	331.4	1.02	1006.	4.42
	61.7	294.4	1.09	894.0	4.64
	63.7	274.2	1.15	832.7	3.58
	65.7	247.2	1.22	750.7	3.71
	67.7	219.5	1.28	666.5	3.87
	69.7	217.1	1.35	659.4	3.81
	71.7	209.2	1.42	635.4	3.69
	73.7	198.4	1.49	602.5	3.38
	75.7	189.0	1.56	574.1	3.07
	77.6	185.1	1.63	562.2	2.94
	79.6	177.8	1.70	539.9	2.99
	81.6	168.8	1.77	512.6	3.01
	83.6	168.2	1.84	510.9	3.03
	85.6	166.0	1.91	504.1	3.02
	87.6	161.0	1.98	489.0	3.05
	89.6	149.6	2.05	454.4	4.24

TABLE IV (CONTINUED)

P_2 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
3.50	31.8	3201.	.2399	7982.	1.93
	33.8	2375.	.427	5922.	2.16
	35.8	1837.	.477	4582.	2.38
	37.8	1406.	.529	3507.	2.64
	39.8	1046.	.584	2609.	3.00
	41.8	830.2	.642	2070.	3.21
	43.8	664.8	.701	1658.	3.51
	45.8	507.9	.763	1267.	4.02
	47.8	457.2	.827	1140.	4.01
	49.8	366.2	.893	913.3	4.32
	51.8	296.1	.961	738.4	4.75
	53.8	267.3	1.03	666.7	4.97
	55.8	232.1	1.10	578.8	5.15
	57.8	221.7	1.18	552.9	5.31
	59.8	198.4	1.25	494.8	5.35
	61.7	167.7	1.33	418.3	5.82
	63.7	145.3	1.40	362.3	6.46
	65.7	124.9	1.48	311.5	6.71
	67.7	124.3	1.56	309.9	4.64
	69.7	113.3	1.65	282.6	4.89
	71.7	99.69	1.73	248.6	4.96
	73.7	91.43	1.81	228.0	4.71
	75.7	83.04	1.90	207.1	4.41
	77.7	81.04	1.98	202.1	4.32
	79.7	81.54	2.07	203.4	4.16
	81.7	76.02	2.15	189.6	4.33
	83.6	75.51	2.24	188.3	4.32
	85.6	74.77	2.33	186.5	4.16
	87.6	68.21	2.42	170.1	4.81
	89.6	69.16	2.50	172.5	4.23

TABLE IV (CONTINUED)

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
4.00	31.9	2257.	.449	4768.	1.75
	33.9	1671.	.505	3529.	1.97
	35.9	1192.	.564	2518.	2.22
	37.9	841.1	.626	1777.	2.61
	39.8	653.7	.691	1381.	2.84
	41.8	516.6	.759	1091.	3.14
	43.8	403.1	.829	851.6	3.45
	45.8	320.6	.902	677.2	3.76
	47.8	265.2	.977	560.2	4.00
	49.8	235.6	1.06	497.6	4.17
	51.8	189.8	1.14	400.9	4.60
	53.8	171.6	1.22	362.4	4.90
	55.8	132.0	1.30	278.7	5.33
	57.8	124.9	1.39	263.8	5.31
	59.8	107.5	1.48	227.1	5.74
	61.8	91.65	1.57	193.6	6.19
	63.8	77.68	1.66	164.1	6.37
	65.8	72.69	1.75	153.5	6.62
	67.7	64.18	1.85	135.6	7.26
	69.7	53.76	1.94	113.6	5.36
	71.7	45.67	2.04	96.48	5.53
	73.7	41.11	2.14	86.83	5.55
	75.7	38.21	2.24	80.70	5.33
	77.7	35.02	2.34	73.97	5.41
	79.7	34.34	2.44	72.54	4.98
	81.7	32.06	2.54	67.72	5.10
	83.7	32.71	2.65	69.08	4.81
	85.6	30.84	2.75	65.14	5.04
	87.6	27.81	2.85	58.74	5.24
	89.6	29.37	2.96	63.03	5.03

TABLE IV (CONTINUED)

F_1 (GeV/c)	$\theta_{o.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
4.50	31.9	1543.	.519	2824.	2.02
	33.9	1085.	.584	1986.	2.38
	35.9	798.2	.652	1461.	2.60
	37.9	569.3	.724	1042.	3.03
	39.9	418.7	.799	766.5	3.36
	41.9	301.1	.877	551.1	3.82
	43.9	245.0	.958	448.5	4.12
	45.9	201.6	1.04	369.0	4.52
	47.9	168.6	1.13	308.6	4.74
	49.8	143.3	1.22	262.3	4.97
	51.8	120.7	1.31	221.0	5.28
	53.8	98.64	1.41	180.5	6.04
	55.8	75.13	1.50	137.5	6.48
	57.8	71.35	1.60	130.6	6.65
	59.8	58.30	1.71	106.7	7.14
	61.8	51.94	1.81	95.08	7.88
	63.8	43.95	1.92	80.45	8.06
	65.8	36.62	2.02	67.04	8.68
	67.8	31.80	2.13	58.21	9.22
	69.8	30.19	2.24	55.26	10.75
	71.8	25.76	2.36	47.14	7.90
	73.7	25.64	2.47	46.98	8.23
	75.7	22.59	2.59	41.35	8.67
	77.7	19.11	2.70	34.98	9.36
	79.7	19.57	2.82	35.83	8.23
	81.7	18.42	2.94	33.71	8.75
	83.7	17.31	3.05	31.68	7.52
	85.7	16.56	3.17	30.31	7.37
	87.7	17.69	3.29	32.39	6.68
	89.6	17.19	3.41	31.47	7.03

TABLE IV (CONTINUED)

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
5.00	32.0	1058.	.590	1707.	3.00
	34.0	731.8	.663	1181.	3.49
	35.9	521.7	.741	842.0	3.96
	37.9	341.7	.822	551.5	4.81
	39.9	252.8	.907	408.0	5.32
	41.9	202.1	.996	326.2	6.06
	43.9	162.9	1.09	262.9	6.51
	45.9	117.1	1.18	189.0	7.54
	47.9	94.25	1.28	152.1	8.08
	49.9	73.66	1.38	118.9	8.70
	51.9	61.86	1.49	99.84	9.80
	53.9	53.37	1.60	86.15	9.90
	55.8	39.73	1.71	64.12	11.29
	57.8	36.12	1.82	58.30	12.17
	59.8	30.04	1.94	48.49	12.69
	61.8	22.67	2.05	36.56	15.11
	63.8	21.59	2.17	34.85	15.81
	65.8	15.73	2.30	25.38	16.78
	67.8	14.65	2.42	23.64	16.84
	69.8	13.68	2.55	22.08	17.76
	71.8	12.47	2.67	20.13	18.98
	73.8	12.65	2.80	20.42	14.57
	75.8	12.06	2.93	19.47	16.71
	77.7	10.55	3.07	17.02	15.44
	79.7	9.689	3.20	15.64	14.14
	81.7	10.10	3.33	16.31	16.15
	83.7	11.54	3.47	18.62	12.70
	85.7	9.821	3.60	15.85	13.80
	87.7	9.926	3.73	16.02	11.99
	89.7	9.234	3.87	14.90	13.50

TABLE IV (CONTINUED)

P_1 (GeV/c)	$\theta_{c.m.}$ (degrees)	$d\sigma/d\Omega$ ($\mu\text{b}/\text{Sr}$)	$-t$ (GeV/c) ²	$d\sigma/dt$ $\mu\text{b}/(\text{GeV}/c)^2$	Error %
5.50	32.0	684.3	.661	987.3	2.05
	34.0	452.8	.744	653.4	2.40
	36.0	309.5	.830	446.5	2.79
	38.0	215.7	.922	311.2	3.24
	40.0	155.1	1.02	223.7	3.69
	42.0	123.4	1.12	178.1	4.08
	43.9	95.63	1.22	138.0	4.58
	45.9	73.04	1.33	105.4	5.16
	47.9	53.54	1.44	77.25	6.11
	49.9	40.49	1.55	58.42	6.49
	51.9	34.20	1.67	49.34	6.91
	53.9	25.27	1.79	36.46	8.28
	55.9	25.01	1.91	36.08	8.44
	57.9	19.68	2.04	28.40	9.48
	59.9	14.93	2.17	21.54	10.23
	61.8	13.74	2.30	19.82	11.17
	63.8	12.10	2.43	17.45	11.57
	65.8	10.95	2.57	15.81	12.14
	67.8	7.533	2.71	10.87	14.17
	69.8	7.295	2.85	10.53	15.83
	71.8	9.582	2.99	13.83	14.56
	73.8	7.195	3.14	10.38	18.04
	75.8	6.285	3.28	9.069	20.84
	77.8	6.869	3.43	9.911	19.00
	79.7	6.164	3.58	8.893	18.82
	81.7	5.799	3.73	8.367	17.94
	83.7	4.880	3.88	7.041	20.33
	85.7	4.553	4.03	6.570	17.76
	87.7	4.680	4.18	6.752	15.52
	89.7	5.350	4.33	7.720	13.24

FIGURE 27

FIGURE 28

exist between the data from this experiment and that of Clyde at 3.0 GeV/c near 60° in the c.m.s. We observe a slight shoulder faintly reminiscent of the structure observed by Allaby *et al.* in this region while Clyde's data remain relatively flat from 90° to an unusually small angle at this momentum. With respect to this shoulder which occurs near $t = -1 \text{ (GeV/c)}^2$, we do observe a similar structure in the higher momenta data, at nearly the same value of $-t$. Considering the point-to-point uncertainty however, this structure is not statistically significant.

The differential cross section at 90° as a function of $-t$ is presented in Figure 29. This figure summarizes most of the existing 90° data between $-t = .5$ and $-t = 10 \text{ (GeV/c)}^2$. We show the break observed by Akerlof *et al.*⁴ near $-t = 6.5 \text{ (GeV/c)}^2$. The dashed line drawn through the low energy data⁶ is a hand drawn fit. As shown, our data are quite consistent with Akerlof *et al.* at 5.0 and 5.5 GeV/c, and with the low energy data at 1.5 and 1.65 GeV/c. Our data indicate that the suspected break at small $-t$ occurs quite close to $-t = .8 \text{ (GeV/c)}^2$. In addition, we observe another less pronounced but well defined break near $-t = 3 \text{ GeV/c}$.

The solid lines appearing in Figure 29 have resulted from linear regression type fits to the data. The parameters for these fits are given in Table V. In particular, we note that a fit to the data for $.8 \leq t \leq 6 \text{ (GeV/c)}^2$ results in a significantly poorer chi-square per data point than is obtained by using two exponentials.

The existence of the break near $-t = 3 \text{ (GeV/c)}^2$ had been graphically implied by Allaby *et al.*⁸ some time ago. In this reference, they

FIGURE 29

TABLE V
 PARAMETERS IN LINEAR REGRESSION FITS
 TO THE DIFFERENTIAL CROSS SECTION AT 90°*

$d\sigma/dt = A e^{Bt} (\mu b)$				
REGION FIT	# DATA POINTS	A	B	$\chi^2/\text{pt.}$
$.85 \leq t \leq 2.75$	8	2.65×10^4	1.99	1.3
$3.0 \leq t \leq 6.0$	21	8.35×10^3	1.63	1.2
$7.5 \leq t \leq 12$	23	23.4	.76	.9
$.85 \leq t \leq 6$	36	1.91×10^4	1.81	12.6

*These fits include the data from this experiment and from that of Akerlof et. al.⁴.

presented a plot of $d\sigma/dt$ vs. $-t$ with points of constant $\theta_{c.m.} = 90^\circ, 80^\circ$, and 60° connected respectively by straight lines. They observed that the break at $t \approx -6.5 \text{ (GeV/c)}^2$ was equally as apparent at the other angles. The situation near $t = -3 \text{ (GeV/c)}^2$ is shown in Figure 30 where we have plotted data from Akerlof et al.⁴, Clyde⁵, Allaby et al.¹⁰, Arkenbrandt et al.¹¹ and some of the data from this experiment. Points of constant $\theta_{c.m.} = 90^\circ, 80^\circ$ and 70° are connected. As shown, a change in slope occurs near $t = -3 \text{ (GeV/c)}^2$ regardless of the angle. In the plot shown by Allaby et al., this same result was indicated, that is, they did show a break near $-t = 3 \text{ (GeV/c)}^2$.

FIGURE 30

VI. INTERPRETATION

We have attempted to interpret the results of this experiment in terms of the various models described in Chapter II. In so doing, it was hoped that the detailed angular distribution data would be sufficiently restrictive so as to allow physical interpretation of the structure observed in the data, notably the breaks in the 90° cross section.

The results of these attempts are described in the following sections.

A. Optical Models

We have compared the predictions of the classical optical model of Fernbach et al.⁴¹ with the results of this experiment. The scattering amplitude, elastic cross section and absorption cross section are given by

$$f(\theta) = K \int_0^\infty (1 - ae^{i\varphi}) J_0(Kb \sin \theta) b db$$

$$\sigma_{el} = 2\pi \int_0^\infty |1 - ae^{i\varphi}|^2 b db$$

$$\sigma_{ab} = 2\pi \int_0^\infty (1 - a^2) b db$$

with θ = c.m.s. scattering angle, b is the impact parameter, and a and φ are the amplitude and phase shift of the scattered wave corresponding to an incident wave of unit amplitude and zero phase shift. Note that we have neglected all proton spin effects. Following the approach of Cork et al.⁴²

we have further limited the model to correspond to;

1. Purely Absorbing Disc

$$\left. \begin{array}{l} \varphi = 0 \\ a = c \end{array} \right\} \quad 0 \leq b \leq R$$

$$\left. \begin{array}{l} \varphi = 0 \\ a = 1 \end{array} \right\} \quad R < b$$

2. Absorbing Disc with Short Range Phase Shift

$$\left. \begin{array}{l} \varphi = c_1 \\ a = c_2 \end{array} \right\} \quad 0 \leq b \leq R_1$$

$$\left. \begin{array}{l} \varphi = 0 \\ a = c_3 \end{array} \right\} \quad R_1 \leq b \leq R_2$$

$$\left. \begin{array}{l} \varphi = 0 \\ a = 1 \end{array} \right\} \quad R_2 < b$$

3. Tapered Absorption with Short Range Phase Shift

$$a = 1 - A \left[1 - \left(\frac{b}{R_2} \right)^2 \right]$$

$$\left. \begin{array}{l} \varphi = c_1 \\ A = c_2 \end{array} \right\} \quad 0 \leq b \leq R_1$$

$$\left. \begin{array}{l} \varphi = 0 \\ A = c_3 \end{array} \right\} R_1 < b \leq R_2$$

$$\left. \begin{array}{l} \varphi = 0 \\ A = 0 \end{array} \right\} R_2 < b$$

The purely absorbing disc gave acceptable fits to the data only at 1.5 GeV/c with $R \approx .9$ f. Above this, two regions (model B) were required. Good fits were obtained between 1.65 and 3.00 GeV/c to the angular distribution, and the elastic and absorption cross sections with

$$R_1 = .6 \text{ f.}$$

$$R_2 = 1.2 \text{ f.}$$

Figure 31 shows the resulting fits to the angular distribution at 1.80 and 2.50 GeV/c and Table VI gives the parameters obtained for the fits. Above 3.0 GeV/c, the model began to exhibit structure not present in the data. This structure was of the form of a dip which appeared at large angles. This is characteristic of models with sharp cutoffs. Tapering of the edges, as in model c, eliminates this objectionable behavior. Using model c, the fits were extended to higher momenta. As an example of the results obtained, the fits at 1.50 and 2.70 GeV/c are shown in Figure 32. In order to fit the data at all energies with model c, it was necessary to allow the radii to vary

$$.5 \leq R_1 \leq .9 \text{ f.}$$

$$1.1 \leq R_2 \leq 1.5 \text{ f.}$$

FIG. 7

FIGURE 31

TABLE VI

OPTICAL MODEL PARAMETERS

$$f(\theta) = K \int_0^{.6} (1-a_1 e^{i\gamma_1}) J_0(Kb \sin \alpha) b db + K \int_{.6}^{1.2} (1-a_2) J_0(Kb \sin \alpha) b db$$

P_L	ϕ_1	a_1	a_2	$\chi^2(d\sigma/d\Omega)$	$\chi^2(\sigma_{el})$	$\chi^2(\sigma_{ab})$
1.50	2.42	.41	.56	103/ 22 pts.	2	156
1.65	3.15	.33	.59	45/ 24	4	71
1.80	3.14	.30	.65	24/ 26	.4	29
2.00	1.73	.60	.63	29/ 27	2	4
2.25	1.63	.61	.63	40/ 30	4	1
2.50	1.64	.64	.62	40/ 30	6	5
2.70	1.50	.62	.60	86/ 30	12	1
3.00	1.40	.61	.59	366/ 30	24	2

FIGURE 32

The results described above are quite consistent with those of Cork, et al. As noted in Chapter II, one of the more objectionable features of this type of model is that it is necessary to allow the radii to vary in fitting data over a sizeable energy range. We have tried to extend these models in order to remove this objection. Our first attempt was to modify model b by adding an additional region. This resulted in good fits at all energies, but uniqueness problems arose, and at 5 and 5.50 GeV/c, unwanted structure again began to appear due to the sharp cutoff.

The second attempt consisted of trying to get one step closer to reality by introducing a single spinflip amplitude into model b. This was of the form

$$g(\theta) = c \int_0^{\infty} e^{i\omega} J_1(b/\sqrt{-t}) b db$$

suggested, in principle, by the hadronic form factor approach of Durand and Lipes⁶⁶ in which weak LS coupling gives rise to an analogous amplitude. The same parameterization was used as in model b. By adding this term it was hoped that we could fit not only the angular distribution and total cross section, but also the polarization. This too was rather unsuccessful in that the angular distributions required a small spin flip contribution ($C \sim .01$) while the polarization demanded a much larger value of C ($C \sim .2$).

The conclusions which can be drawn from this application of the classical optical model and its impact parameter representation generalization are quite clear. Although this simple model is capable of providing very good fits to the data (the angular distribution, total elastic cross

section and total absorption cross section) the physical interpretations of the results are, at best, nebulous.

This conclusion is not meant to indict optical type models in general, but is intended to question the appropriateness of application to processes which they are inherently unable to fully describe. This is true of the p-p system with its complex spin structure.

B. Krisch's Model

We have attempted to apply Krisch's Model ^{37,38,39,40} to our data, that is, to investigate the differential cross section as a function of the proposed universal parameter $(\beta P_L)^2$. As pointed out in Chapter II, this parameterization is intended to remove the energy dependence of the differential cross section.

As pointed out by Allaby et al.⁹, application of Krisch's Model in its modified form, when examined on a more revealing scale, was not totally satisfactory in removing the energy dependence in their data at higher momenta. The corresponding investigation at the lower momenta of this experiment leads to a concordant conclusion. Figure 33 shows some of the modified data $(d\sigma^\dagger/dt)$ from this experiment, from Clyde¹⁰ and from Allaby et al.⁹ plotted as a function of $(\beta P_L)^2$. The line appearing on this figure uses the parameters reported by Krisch in fitting the data. We note the violent disagreement of our low energy data with the proposed fit. At the higher momenta ($P_L > 4.0$ GeV/c) the agreement improves considerably but tends to oscillate somewhat around the line. This is essentially consistent with the results at higher momenta.

FIGURE 33

In conclusion, it should again be pointed out that at some momentum, the angular distribution ceases to be diffractive in nature. At this point, Krisch's Model should not be expected to be valid. In his fits, Krisch placed the minimum momentum cutoff arbitrarily at 5.0 GeV/c. The deviation of our low energy data from the model is probably a manifestation of the non-diffractive nature of the data.

C. Statistical Model

We have attempted to fit the large angle data with an Orear type function

$$\frac{d\sigma}{d\Omega} = A e^{-P_1/g}$$

which arises, for instance, from the statistical thermodynamic model. A and g are free parameters.

Allaby *et al.*,⁹ using this type of fit, noted that the observed marked change in the nature of the angular distribution near 8 GeV/c was reflected in a marked change in g, going from one constant value at lower momenta to another possibly constant value at higher momenta.

In fitting the data with this functional form, there is some uncertainty in the meaning of the term "large-angle". For $80^\circ < \theta_{c.m.} < 90^\circ$, the differential cross section and P_1 are both nearly independent of angle, so that the parameter g is not well determined. The attitude taken in our application of the model was to assume that the data between $\sim 75^\circ$ and 90° could be fit with this form. The fit was then extended to smaller angles, requiring that there be no significant changes

in the parameters and that the quality of the fit be maintained (no serious degradation in the average chi-square/data point). This typically resulted in fits to the data from near 60° to 90° .

The results of these fits are shown in Figure 34 where g is plotted as a function of momentum. The parameter values are summarized in Table VII. This figure includes our data together with that of other experiments.^{9,10,11} Rapid changes in g are noted to appear near 1.5, 4.0 and as Allaby observed, 8.0 GeV/c. It should be noted that these momentum values correspond to the positions of the breaks in the 90° cross section shown in Chapter V.

D. Effective Regge Trajectory

We have applied a simple Regge model to our data. Pinsky⁶³ has argued that high energy p-p elastic scattering can be represented at large angles in the framework of the Regge pole model with the cross section being given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{s} K(t) \left(\frac{s}{s_0} \right)^{2\alpha(t)}$$

with

$$v = \frac{s - u}{2}$$

and $K(t)$ contains the contribution of the residue and signature factor. One prediction of this relation is that if one makes a log-log plot of $s \, d\sigma/d\Omega$ versus v at a constant value of $-t$, a straight line with a slope of 2α should result. Pinsky⁶³ and Allaby et al.⁹ have shown this

FIGURE 34

TABLE VII
STATISTICAL MODEL PARAMETERS

$$\frac{d\sigma}{d\Omega} = A e^{-P} 1/g$$

P_L	A	$\bar{g}$ (MeV)	$\chi^2 / \# \text{DATA POINTS}$
1.50	3.74×10^6	75	10/13
1.65	2.95	80	16/15
1.80	1.46	91	8/16
2.00	3.28×10^5	118	12/13
2.25	1.53	136	19/18
2.5	2.66×10^4	198	7/15
2.7	2.22	206	17/18
3.00	2.51	201	10/19
3.50	7.90	160	15/23
4.00	4.93×10^5	125	7/15
4.50	1.43	145	6/22

to be true for the high energy p-p scattering data of Allaby et. al.

In Figure 35, we show the results of this approach as applied to our data ($P_L \geq 2.7$ GeV/c) Figure 36 is a comparison of our results with those of Allaby et. al. Both sets of data are individually consistent with the prediction, but there is a significant difference in the slopes of the lines. In Figure 37 we show the effective trajectories extracted from the two sets of data. The values of α_{eff} from the data of Allaby et. al. are those reported in Reference 9. We observe that the characteristics of the two trajectories are somewhat similar, being linear in the small $-t$ region and flattening at large $-t$.

The linear region of the effective trajectory obtained from the data from this experiment ($.7 \leq |t| \leq 2$ (GeV/c)²) has a slope of about 1.2 and a intercept of slightly more than one. It is interesting to note that if one extrapolates straight line fits to the two seemingly linear regions of α_{eff} derived from the higher energy data, the α_{eff} from this experiment seems to be flattening at each end, consistent with these lines.

E. Breaks in the 90° Cross Section

The combination of the data from this experiment, together with that of Akerlof et. al.⁴ and cyclotron experiments⁵ makes possible a detailed investigation of the observed structure in the $\theta_{\text{c.m.}} = 90^\circ$ cross section. This combination of data indicates the existence of three well defined and rapid changes in slope at $-t \approx .8, 3, \text{ and } 6.5$ (GeV/c)². These discontinuities are related to the discontinuities in the $(\beta P_\perp)^2$ plot of Krisch. For the data at $\theta_{\text{c.m.}} = 90^\circ$;

$$(\beta P_\perp)^2_{90^\circ} = \frac{1}{2} \frac{t_{90^\circ}^2}{2m^2 - t_{90^\circ}}$$

FIGURE 36

FIGURE 37

The observed breaks in the 90° cross section at $-t = .8, 3$ and 6.5 (GeV/c)^2 should then be reflected in Kirsch's plot at $(sP_1)^2 \approx .12, .95$, and 2.6 (GeV/c)^2 . The latter corresponds quite well to one of the two discontinuities reported by Kirsch. Both of the two small $-t$ breaks have not been reported to have been observed in application of this model. In Figure 38, we show the 90° data from this experiment and some of that from the experiment of Akerlof *et al.*⁴ plotted as $d\sigma^+/dt$ versus $(sP_1)^2$. The solid line is that reported as the energy independent fit. As shown, the 90° break at $-t \approx 3 \text{ (GeV/c)}^2$ corresponds quite well to the joining of the upper two linear regions of Kirsch's Model. The small $-t$ break has no such counterpart. This is not an indication of an inconsistency however, since the small $-t$ breaks in the 90° cross section occurs at low momenta where Kirsch's Model is not expected to offer a description of the data.

While discussing the comparison of the observed structure in the 90° cross section ($d\sigma/dt$ versus $-t$) and in the Kirsch plot ($d\sigma/dt$ or $d\sigma^+/dt$ versus sP_1^2) it should be pointed out that the straight line fits to the data which are typically used are mutually inconsistent since the scale factor between the two parameters is nonconstant. The Kirsch Model does predict essentially linear regions joined smoothly together. In the case of the 90° cross section, the straight line description is not, in itself, significant. (It is undoubtedly true that a fit of some other form would provide an equally good representation of the data.) The significant point seems to be that in the regions of the breaks in the straight line segments, there is a rapid change in the nature of the cross section reflecting perhaps the opening of new channels, of the

FIGURE 38

dominance of a different amplitude or some other alteration of the scattering mechanism.

The existence of the third (and possibly more) discontinuity in the $\theta_{c.m.} = 90^\circ$ cross section has been predicted by at least three models. Fleming et. al.⁶⁴ have proposed a semi-empirical model which was motivated by the successes of the Krisch Model, but which attempted to circumvent some of its theoretical difficulties. In addition, this model asymptotically yields the Orear exponential behavior in $P_\perp$. Basically they propose that the sum of three exponentials in $(\beta P_\perp)^2$ suggested by Krisch be replaced by an infinite sum of exponentials in the same parameter. The leading behavior of the cross section is still given by the Krisch type equation.

The most serious impairment of the Krisch Model is its violation of the Cerulus-Martin Bound⁶⁵, that is as $K \rightarrow \infty$ and for $\theta \neq 0, \pi$

$$\frac{d\sigma}{dt} > e^{-c(\theta) K \ln K}$$

where $c(\theta)$ is a positive function. This limiting behavior does not admit any finite series of gaussians in $(\beta P)^2$. The fact that the Orear equation does not violate this bound and provides a fit over many orders of magnitude makes the asymptotic reduction to this form most desirable. The result of this proposed model is the prediction of an infinite number of breaks, two of which are those predicted by Krisch, and another which is expected to occur near $t = -3 (\text{GeV}/c)^2$.

I has been suggested by Libby⁶⁶ that the interaction energy between colliding protons can be calculated from the high energy differential cross section data. It is found that a repulsive core begins

to dominate the scattering for separations of less than about .3 f. The interaction energy is calculated from the differential cross sections at constant $\theta_{c.m.}$. The result is a curve of E (interaction energy) versus r , the nucleon separation. Depending upon the energy range of the data used in the computation, four E versus r curves result. They are similar in appearance, being "well-shaped"; the energy becoming negative (attractive) for $r < .8$ f., reaching a minimum and rising with a sharp repulsive core behavior. The four wells obtained have different depths and different nucleon separations at the minimum interaction energy. It is suggested that these four wells, which correspond to the four linear sections of the fixed angle cross section, belong to specific excited states of the two baryon system.

The third model suggesting the existence of additional breaks is due to Libby and Predazzi.⁶⁷ They have presented a combination of experimental and theoretical evidence for the existence of a Regge trajectory for the two baryon system. The result is the prediction of a small t ($\approx -.8$ (GeV/c)² or less) break, a large t break ($t \approx -6.5$ (GeV/c)²) and two intermediate breaks ($t \approx -1.7$ and -4.1 (GeV/c)²). These correspond to di-baryon masses of about 2.2, 2.6, 3.4 and 3.9 GeV/c. The approach used to theoretically verify the existence of these systems was to minimize the two proton interaction energy using Fermi's "Golden Rule No. 2".⁶⁸

In a further attempt to interpret the breaks in the 90° cross section we have performed a Regge calculation which predicts the energies at which di-baryon resonances might occur. We assume exchange

degeneracy and duality. The resonance amplitude might be expected to be represented by the amplitude for exchange of t channel Regge poles. We assume that the residue functions are essentially independent of t and then partial wave project the Reggeized s -channel amplitudes. In the spirit of the calculation, maxima in the imaginary parts of these amplitudes are taken to represent di-baryon resonances. We have not looked separately at the real parts of the amplitudes, in order to attempt to obtain Schmid-loops, because the difficulty in extracting this portion of the amplitude is considerably greater and would require a totally different analysis. The calculation is outlined below.

The states from which the partial waves are projected must be states of definite total angular momentum, J , and definite parity. Conservation of g -parity and isotopic spin also indicates that total spin, S , must be conserved in the transition. An acceptable representation for the partial wave decomposition is then the usual description in terms of the singlet ($S = 0$) and triplet ($S = 1$) states. These are of the form $|J M; L S\rangle$ with parity $(-1)^L$.

Total spin conservation implies that there will be a single transition amplitude, f_0^J , in the $S = 0$ state, and no transitions between the singlet and triplet states. Parity conservation indicates that there is a single amplitude, f_1^J , for $S = 1$, $J = L$ transitions. The parity of the $J = L \pm 1$ states of the triplet are the same, so we expect three amplitudes to describe the scattering in the $-(-1)^J$ parity state. These will involve orbital angular momentum transitions $J + 1 \rightarrow J + 1$, $J - 1 \rightarrow J - 1$ and $J + 1 \leftrightarrow J - 1$.

The five s-channel helicity amplitudes do not suffice for this purpose as they are not unique parity states. However, from the helicity amplitudes, we can form four independent states of definite parity and total spin¹⁵ which are denoted below as $|J M; P S\rangle$. The first two

$$\begin{aligned} |J M; (-1)^J 0\rangle &= \sqrt{\frac{1}{2}} (|J; ++\rangle - |J; --\rangle) \\ |J M; (-1)^J 1\rangle &= \sqrt{\frac{1}{2}} (|J; +- \rangle - |J; -+ \rangle), \end{aligned}$$

are the usual singlet and triplet $J = L$ states ($|J M; L S\rangle = |J M; J 0\rangle$, $|J M; J 1\rangle$ respectively). The remaining two states in the $|J M; P S\rangle$ representation are taken to be

$$\begin{aligned} |J M; -(-1)^J 1\rangle &= \sqrt{\frac{1}{2}} (|J; ++\rangle + |J; --\rangle) \\ |J M; -(-1)^J 1\rangle &= \sqrt{\frac{1}{2}} (|J; +- \rangle + |J; -+ \rangle), \end{aligned}$$

which, as indicated have the same parity and total spin, but are independent. (This indicates only that the $|J M; P S\rangle$ representation does not completely specify the states.) These states are mixtures of $|J M; L S\rangle = |J M; J + 1 1\rangle$ and $|J M; J - 1 1\rangle$ states. Although they are not physically very meaningful, their use simplifies the description in terms of the transition amplitudes which are given by

$$f^J = \langle J M; \alpha p s | T^J(E) | J M; \alpha' p s \rangle$$

where α and α' denote the missing compatible observable (which is the orbital angular momentum) in the states $|J M; P S\rangle$.

The resulting transition amplitudes are

1. Singlet ($J = L, S = 0$)

$$f_0^J = \langle J; ++ | T^J(E) | J; ++ \rangle - \langle J; ++ | T^J(E) | J; -- \rangle$$

2. Triplet ($J = L, S = 1$)

$$f_1^J = \langle J; +- | T^J(E) | J; +- \rangle - \langle J; +- | T^J(E) | J; -+ \rangle$$

3. Triplet ($J = L \pm 1, S = 1$)

$$f_{11}^J = \langle J; ++ | T^J(E) | J; ++ \rangle + \langle J; ++ | T^J(E) | J; -- \rangle$$

$$f_{12}^J = 2 \langle J; ++ | T^J(E) | J; +- \rangle$$

$$f_{22}^J = \langle J; +- | T^J(E) | J; +- \rangle + \langle J; +- | T^J(E) | J; -+ \rangle$$

The latter three amplitudes are combinations of the three possible orbital angular momentum transitions in the triplet state.

The object is then to find suitable combinations of the helicity amplitudes such that a partial wave projection will yield these five amplitudes for singlet and triplet state transitions. The proper choice¹⁵ is

$$f_1 = E(\phi_1 - \phi_2)$$

$$f_2 = E(\phi_1 + \phi_2)$$

$$f_3 = E \frac{\phi_3}{1+z} - \frac{\phi_4}{1-z}$$

$$f_4 = E \frac{\phi_3}{1+z} + \frac{\phi_4}{1-z}$$

$$f_5 = \frac{2z}{1-z^2} \phi_5$$

with

$$z = \cos \theta_{c.m.}$$

From these amplitudes, we may make the partial wave projections

$$f_0^J = \frac{P}{2E} \int_{-1}^1 f_1(s, z) P_J(z) dz$$

$$f_1^J = \frac{P}{2E} \int_{-1}^1 f_2(s, z) P_J(z) dz$$

$$f_{22}^J = \frac{P}{2J+1} \frac{P}{2E} \int_{-1}^1 \left(f_3(s, z) P_J(z) + f_4(s, z) \times \right. \\ \left. \frac{JP_{J+1}(z) + (J+1)P_{J-1}(z)}{2J+1} \right) dz$$

$$f_{12}^J = \frac{P}{2E} \int_{-1}^1 f_5(s, z) \sqrt{\frac{J(J+1)}{2J+1}} (P_{J+1}(z) - P_{J-1}(z)) dz$$

$$f_{22}^J = \frac{1}{2J+1} \frac{P}{2E} \int_{-1}^1 \left(f_4(s, z) P_J(z) + f_3(s, z) \times \right. \\ \left. \frac{JP_{J+1}(z) + (J+1)P_{J-1}(z)}{2J+1} \right) dz$$

The goal is then to introduce a Regge parameterisation for the helicity amplitudes, and to make the partial wave projections. Assuming duality, the resulting partial wave amplitudes can be interpreted in terms of s-channel (di-baryon) resonances.

The choice of parameterisation of the helicity amplitude parallels the earlier work of Rarita et al.⁶⁹ In extending their parameterisation away from the diffraction peak we were forced to alter the form of the helicity amplitudes due to the restrictions imposed by the assumed crossing symmetry and by the Pauli principle. As in Rarita et al. the P, P' and ω poles are assumed to dominate in the cross-channel exchange. The duality assumption permits the neglect of the Pomeron in the parameterisation, which is assumed responsible for the diffraction peak. The effects of the other trajectories which can be exchanged in the

t-channel (e.g. φ, ρ, A_2) are believed to be sufficiently small so that they can be neglected.⁶⁹

The helicity amplitudes are then written

$$\phi_1 = \frac{\pi E_0}{4\gamma} \left\{ \frac{e^{-i\pi\alpha_{p'}(t)} + 1}{\sin\pi\alpha_{p'}(t)} \left(\frac{E_L}{E_0}\right)^{\alpha_{p'}(t)} \eta_{p'}^2 + \frac{e^{i\pi\alpha_w(t)} - 1}{\sin\pi\alpha_w(t)} \left(\frac{E_L}{E_0}\right)^{\alpha_w(t)} \eta_w^2 + (t \leftrightarrow u) \right\}$$

where the last term is identical to the first with the interchange of t and u , as required by crossing symmetry.

$$\phi_3 = \frac{\pi E_0}{4\gamma s} \left[- \frac{e^{i\pi\alpha_w(t)} + 1}{\sin\pi\alpha_w(t)} \left(\frac{E_L}{E_0}\right)^{\alpha_w(t)} \sqrt{\frac{-u}{4s^2}} \right]$$

From the Pauli principle

$$\phi_4(\theta) = -\phi_3(\pi - \theta)$$

The other two amplitudes, ϕ_2 and ϕ_5 are taken to be negligible. We assume the trajectories to have the same intercept and slope, and make the partial wave projections by numerical integration using only the imaginary parts of the complex amplitudes. This results in the reduction to a set of (imaginary parts of) amplitudes representing the scattering in states of definite angular momentum, as functions of the incident momentum. The location of maxima in these amplitudes is then taken as evidence for the formation of resonances in the p - p system. Typical results are shown in Figure 39. In Figure 40, we again show the $\theta_{c.m.} = 90^\circ$ cross section.

FIGURE 39

FIGURE 40

with the locations of these maxima indicated by arrows. As shown, in the immediate vicinity of the sharp discontinuities at $-t \approx .7 \text{ (GeV/c)}^2$, we observe maxima in three of the amplitudes. We find maxima approximately equally spaced about the discontinuity near $-t \approx 6.5 \text{ (GeV/c)}^2$. In the intermediate region, maxima are also found, spaced equally on either side of the observed break at $-t \approx 3 \text{ (GeV/c)}^2$. A possible interpretation⁷⁰ of these latter two breaks is then that they are a result of the compound effects of these maxima which simulate a discontinuity at an effective t which is essentially midway between the originating effects.

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