STUDY OF SCTI CONTROL SYSTEM

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ABSTRACT

The Sodium Components Test Installation (SCTI) is a 35-Mw design capacity fossil-fired test installation with a primary and secondary system; a sodium-heated steam generator; a pressure-reducing system desuperheater, to simulate the turbine; a condenser; and a feedwater cooling system.

Stability studies of the control systems associated with this complex were performed on analog equipment. The greatest difficulty was encountered in attempting to describe the IHX and once-through steam generator in simple mathematical terms, especially since very little could be found in the literature on once-through steam generators.

It was discovered that a single-node representation of the IHX was ample for purposes of obtaining controller settings and stability determinations. However, a 3-node simplified model of the steam generator proved inadequate in studying low-flow conditions.

The complexity of the problems required that the control system be divided into two main sections for analog study, so as to limit the number of analog machines to no more than three in any one setup. The two sections were: (1) the primary system and IHX; and (2) the secondary system, steam generator, and feedwater-steam control complex. Other areas were analyzed by use of Bode plots.

The individual systems were found to be stable, and controller settings were obtained. In view of the low-flow inadequacy of the model for the once-through steam generator, a somewhat more complex mathematical model was developed and has been included in this report.

I. INTRODUCTION

This report has been prepared to document the extensive analytical work required to design the control system for the Sodium Components Test Installation (SCTI), constructed for the Atomic Energy Commission at the Atomic International field laboratory. The exacting test requirements, along with the fact that tests were initiated and controlled from both heat source and steam end, made the control system analytical construction somewhat complex. Added complexity resulted when a literature search and consultation with an expert in the field showed very little yet published on practical representation of an analog model of a once-through steam generator.

In filling this gap as to transient analysis of either intermediate heat exchangers or once-through steam generators, researchers found both difficult and time-consuming the developing of analytical mathematical models for analog use. But since the general approach described herein is applicable with minor modifications to most typical systems, it is hoped this detailed document will be justified in helping to avoid duplicated effort.

The SCTI is a 35-Mw design capacity fossil-fired test installation with a primary and secondary system; a sodium-heated steam generator; a pressure-reducing system desuperheater, to simulate the turbine; a condenser; and a feed-water cooling system.

The system is designed to perform, on a sodium-to-sodium intermediate heat exchanger and on a sodium once-through steam generator, the same drastic transient tests that might be needed if the heat source were a nuclear reactor. The full number of design cycles for each test can be performed on each test unit, which may be stressed to rupture if improperly designed. Therefore to secure success, the control system, protective system, and transient loops are more complex than would be normal in an actual plant. Both load-following and load-forcing transient tests must be conducted.

The analog simulation was made mainly to represent the system with sufficient accuracy so that precisely stipulated transients could be performed over a wide enough range of operating conditions to ensure proper control system design. The second objective was to obtain approximate settings of proportional bond, rate, and reset for each of the many controllers, in order to expedite initial startup and checkout in the field.

Because only three large PACE analog computers were available for the analysis, the system was divided into two major sections for analog study and a third section for direct computation analysis.

The analog divisions consisted of: (1) the fossil-fired furnace, the primary sodium system, and a single-node representation of the intermediate heat exchanger (IHX); and (2) the secondary sodium system, the steam generator, and the feedwater and steam-control portions of the steam loop. The direct computation analysis was made by using the Bode plot frequency response method to study the stability of (3) the rest of the steam loop. No allowance was made for the coupling between individually analyzed regions. This, however, placed a more stringent requirement on the control system, since coupling generally causes a damping effect upon transients.

After completion of the analysis, a multinode representation of the IHX, using all three Pace computers, was analyzed to determine the degree of output error existent in the first analysis, which used a single-node IHX representation. It was found that for the stated objectives, the single-node representation was adequate.

The model of the once-through steam generator, which was divided into the three nodular regions of preheat, boiling, and superheat, proved oversimplified for low feedwater flow because of the greater complexity of the heat-transfer coefficients. Upon learning of this, a German student, Reiner Schmit, who was studying with us under the auspices of Interatom, contributed freely of his time to develop a more complex model of the steam generator during the preheat in boiling regions. This model is included in the report but was never fully checked out on the analog. The balance of the report covers detailed representation and analysis of each of the four areas described.

II. PRIMARY SODIUM LOOP ANALYSIS

A. DESCRIPTION

The primary loop of the Sodium Component Test Installation consists of a gas-fired furnace for heating the sodium, the necessary piping and pump for transfer of the sodium, and the intermediate heat exchanger for transferring the heat to the secondary loop. In addition, as shown in Figure 1, there are supplemental loops within the primary loop which may be described as: a bypass for the furnace (cold bypass), a bypass for the heat exchanger (hot bypass), and a loop for emergency or auxiliary cooling.

B. DESIGN SPECIFICATIONS

Functionally, the primary loop is designed to supply 35 Mw of heat to the secondary loop under steady state and transient conditions. The maximum heating capability of the loop is 1×10^6 lb/hr of sodium heated from 900 to 1300°F. Transient design specifications require the following capabilities:

- 1) At a constant sodium flow rate of 1.14×10^6 lb/hr, the sodium temperature into the primary side of the IHX may be reduced at the rate of $-35^\circ\text{F}/\text{sec}$ for 10 sec.
- 2) At a constant sodium temperature of 1200°F the sodium flow rate into the primary side of the Intermediate Heat Exchanger (IHX) may be reduced from 100% (1×10^6 lb/hr) to 10% in 12 sec.
- 3) At a constant primary sodium flow rate of 1×10^6 lb/hr, the sodium temperature into the primary side of the IHX may be increased from 1200 to 1300°F at the rate of $+11^\circ\text{F}/\text{sec}$ for 9 sec.
- 4) At a constant sodium inlet condition into the primary side of the IHX of 0.6×10^6 lb/hr and 1200°F the secondary loop flow may be reduced from $\sim 0.46 \times 10^6$ lb/hr to zero in 10 sec. (This rate corresponds to loss of the secondary pump.)

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Figure 1. Primary Sodium Control System Schematic

C. METHOD OF OPERATION

Achievement of the various transients is accomplished by operating the primary loop complex as follows.

Transient (a). The temperature and flow rate of the sodium are established with the sodium flowing from the pump to the economizer, furnace, IHX, and back into the pump. The transient is obtained by controlled mixing of cold bypass sodium with the hot sodium from the furnace at the IHX inlet. As the sodium flow through the furnace is reduced, the firing rate is reduced. Sodium flow through the furnace is maintained at an established minimum flow by allowing sodium to flow through the cooler if required. When the temperature of the sodium leaving the IHX drops below the desired value, the hot bypass loop opens and maintains the desired temperature by mixing of hot sodium into the cold stream.

Transient (b). The temperature and flow rate of the sodium are established with the sodium flowing from the pump. The transient is obtained by reducing the sodium flow through the IHX at a controlled rate. As in transient (a) the firing rate of the furnace is reduced as the sodium flow through the furnace is reduced; however, a minimum flow through the furnace is insured by use of the cooler loop. As before the hot bypass loop is used as necessary to maintain the furnace inlet sodium temperature near the desired temperature.

Transient (c). The temperature and flow rate of the sodium are established with the furnace heating 1×10^6 lb/hr of sodium from 900 to 1300°F. One quarter of the furnace flow is directed through the cooler where it is cooled to 900°F; the balance of the sodium flow through the furnace (0.75×10^6 lb/hr) is mixed with 0.25×10^6 lb/hr of sodium from the cold bypass loop to give 1×10^6 lb/hr of sodium at 1200°F at the IHX inlet. The transient is obtained by reducing the cold bypass sodium at a controlled rate while increasing the flow of hot sodium so as to give an increase of +11°F per sec temperature rise at the inlet to the IHX. During this transient the flow through the IHX and furnace is held constant with the increased demand for hot sodium being obtained by reducing the flow through the cooler.

Transient (d). The temperature and flow rate of the sodium are established with the furnace heating 0.9×10^6 lb/hr of sodium from 1100 to 1300°F. Two-thirds of the furnace flow (0.6×10^6 lb/hr) is directed through the hot

bypass; the balance of the furnace flow (0.3×10^6 lb/hr) is mixed with 0.3×10^6 lb/hr of sodium from the cold bypass to give 0.6×10^6 lb/hr of sodium at 1200°F at the IHX inlet. Initially the flow through the cooler is essentially zero, except for leakage flow. The flow through the IHX is cooled to 900°F by the secondary loop sodium and then mixed into the hot bypass sodium to give 1100°F for the sodium temperature entering the furnace.

The transient is obtained by reducing the secondary sodium flow to zero, which results in the primary sodium temperature leaving the IHX rising to 1200°F. As the IHX exiting temperature rises, the hot bypass is cut back and sodium is directed through the cooler and cooled to 900°F. The sodium flow from the cooler is mixed with that from the IHX to give a final temperature of 1100 to 1111°F for the sodium entering the furnace. At the end of the transient, the sodium flow through the furnace is about 0.536×10^6 lb/hr with the sodium being heated from 1111 to 1300°F. This flow is then divided so that about 0.286×10^6 lb/hr from the hot bypass mixes with 0.314×10^6 lb/hr from the cold bypass at 1111°F to give the constant IHX inlet condition. The balance of the furnace flow, about 0.25×10^6 lb/hr, passes through the cooler. The hot bypass flow has been reduced to essentially zero.

D. DERIVATION OF SYSTEM EQUATIONS

In consideration of the above transient, where many valves must operate simultaneously over their entire range from full-open to full-closed and the furnace has to be controlled over a load reduction of 10 to 1, the entire primary loop complex was programmed on the analog computer. The computer program consisted of the transient heat transfer equation describing the various components, control valve and controller characteristics, pump characteristics, and hydraulic characteristics of all the loops. The derivations of these equations are described in detail as follows: (Refer to Appendix D for nomenclature).

1. Heat Transfer Equations

a. Furnace Transient

An incremental section of the radiant region of the furnace may be considered in the following manner.

$$\text{assuming } \frac{dt_{fwr}}{d\theta} = \frac{dt_{fsr}}{d\theta} \quad \text{and} \quad t_{fsr} = \frac{t_{fso} + t_{fsl}}{2}$$

then using a single node approximation

$$\begin{aligned} \frac{dt_{fsr}}{d\theta} &= \frac{U_{fr} \rho_{fr}}{2 (\rho_s A_s Cp_s + \rho_w A_w Cp_w)} (t_{fgl} - t_{fsr}) \\ &= - \left[\frac{W_{123} Cp_s}{(\rho_s A_s Cp_s + \rho_w A_w Cp_w)} \right] \left[\frac{t_{fso} - t_{fsl}}{(\Delta X_{fr})} \right] \end{aligned}$$

Considering the fuel as natural gas composed of 86% by volume methane (CH_4) and 14% volume ethane (C_2H_6), with a heating value of 1000 Btu/ft^3 , the theoretical flame temperature is:

$$t_{fgf} = 80 + \frac{1000}{0.0471 + \frac{0.216}{16.95} R} ,$$

where R is the air-to-go ratio.

The effective radiant temperature in the radiant zone is considered to be

$$t_{fg1} = t_{fgf} \left[1 - \frac{450}{581} \right] + t_{fg2} \left[\frac{450}{581} \right] .$$

To determine the gas temperature at the entrance to the convection region, an energy balance on the gas is taken, which gives

$$t_{fg2} = t_{fgf} - \frac{U_{fr} P_{fr} (\Delta X_{fr}) (t_{fg1} - t_{fsr})}{(2) (w_{fa} + w_{ff})^* (0.297)} .$$

The convection region was considered in the following manner.

where

$$q_{fgc} = U_{fc} P_{fc} (\Delta X) \Delta t_{lm, fc} ,$$

assuming

$$\frac{dt_{fw}}{d\theta} = \frac{dt_{fsc}}{d\theta}$$

and

$$t_{fsc} = \frac{t_{fs1} + t_{fs2}^*}{2}$$

Then, using a single-node approximation:

$$\begin{aligned} \frac{dt_{fsc}}{d\theta} &= \left[\frac{U_{fc} P_{fc}}{\rho_s A_s C_{p_s} + \rho_w A_w C_{p_w}} \right] \Delta t_{lm_{fc}} \\ &= - \left[\frac{W_{123} C_{p_s}}{\rho_s A_s C_{p_s} + \rho_w A_w C_{p_w}} \right] \left[\frac{t_{fs1} - t_{fs2}^*}{(\Delta X_{fc})} \right] \end{aligned}$$

where

$$\Delta t_{lm_{fc}} = \frac{(t_{fg2} - t_{fs1}) - (t_{fg3} - t_{fs2}^*)}{\ln \frac{t_{fg2} - t_{fs1}}{t_{fg3} - t_{fs2}^*}}$$

and ,

$$\frac{dt_{fs2}^*}{d\theta} = \frac{1}{\tau_{fs2}} (t_{fs2} - t_{fs2}^*)$$

To determine the gas temperature at the exit of the convection region, an energy balance on the gas is taken, which gives:

$$(W_{fa} + W_{ff})^* (0.297) (t_{fg2} - t_{fg3}) = U_{fc} P_{fc} (\Delta X_{fc}) \Delta t_{lm_{fc}}$$

The economizer region was considered in the following manner.

where

$$q_{fge} = U_{fe} P_{fe} (\Delta X) \Delta t_{m_{fe}} ,$$

assuming

$$\frac{dt_{fwe}}{d\theta} = \frac{dt_{fse}}{d\theta}$$

and

$$t_{fse} = \frac{t_{fs2} + t_{fs3}}{2} .$$

Then, using a single node approximation,

$$\begin{aligned} \frac{dt_{fse}}{d\theta} &= \left[\frac{U_{fe} P_{fe}}{\rho_s A_s C_{p_s} + \rho_w A_w C_{p_w}} \right] \Delta t_{m_{fe}} \\ &= - \left[\frac{W_{123} C_{p_s}}{\rho_s A_s C_{p_s} + \rho_w A_w C_{p_w}} \right] \left[\frac{t_{fs3} - t_{fs2}}{(\Delta X_{fe})} \right] , \end{aligned}$$

where

$$\Delta t_{m_{fe}} = \frac{t_{fg3} + t_{fg4} - t_{fs2} - t_{fs3}}{2} .$$

To determine the gas temperature at the exit of the economizer region, an energy balance on the gas is taken, which gives

$$(W_{fa} + W_{ff})^* (0.297) (t_{fg3} - t_{fg4}) = U_{fe} P_{fe} (\Delta X_{fe}) \Delta t_{m_{fe}} .$$

From the physical dimensions of the furnace and economizer:

$$\frac{U_{fr} P_{fr}}{2} = 870 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$\rho_s A_s C_{p_s} + \rho_w A_w C_{p_w} = 53.1 \frac{\text{Btu}}{\text{lb}^{\circ}\text{F}}$$

$$\frac{W_o C_{p_s}}{\Delta X_{fr}} = 14,500 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$\frac{(W_{fa} + W_{ff})_o (0.297)}{\Delta X_{fr}} = 2,670 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$U_{fc,o} P_{fc} = 694 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$\frac{W_o C_{p_s}}{\Delta X_{fc}} = 2780 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$\frac{(W_{fa} + W_{ff})_o (0.297)}{\Delta X_c} = 512 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$U_{fe,o} P_{fe} = 3510 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$\frac{W_o C_{p_s}}{\Delta X_{fe}} = 39,200 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$\frac{(W_{fa} + W_{ff})_o (0.297)}{\Delta X_e} = 7,220 \frac{\text{Btu}}{\text{Hr ft}^{\circ}\text{F}}$$

$$(\rho_s A_s C_{p_s} + \rho_w A_w C_{p_w})_e = 42.3 \frac{\text{Btu}}{\text{lb}^{\circ}\text{F}}$$

Then the equations are

$$\frac{dt_{fsr}}{d\theta} - 16.4 (t_{fg1} - t_{fsr}) = -272.6 \left[\frac{W_{123}}{W_o} \right] (t_{fso} - t_{fs1})$$

$$t_{fgf} = 80 + \frac{1000}{0.0471 + 0.01275 (R)}$$

$$t_{fg1} = 0.225 t_{fgf} + 0.775 t_{fg2}$$

$$t_{fg2} = t_{fgf} - \frac{0.325 (t_{fg1} - t_{fsr})}{\left(\frac{W_{fa} + W_{ff}}{W_{fa} + W_{ff}} \right)_o^*}$$

$$\frac{dt_{fsc}}{d\theta} - 13.05 \left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6} \Delta t_{lm} = -52.2 \left(\frac{W_{123}}{W_o} \right) (t_{fs1} - t_{fs2}^*)$$

$$\Delta t_{lm} = \frac{(t_{fg2} - t_{fs1}) - (t_{fg3} - t_{fs2}^*)}{\ln \left[\frac{t_{fg2} - t_{fs1}}{t_{fg3} - t_{fs2}^*} \right]}$$

$$\frac{dt_{fs2}^*}{d\theta} = \frac{\left(\frac{W_{123}}{W_o} \right) (t_{fs2} - t_{fs2}^*)}{3.2 \times 10^{-3}}$$

$$\left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.4} (t_{fg2} - t_{fg3}) = 1.35 (\Delta t_{lm})$$

$$\frac{dt_{fse}}{d\theta} = 83 \left[\frac{(W_{fa} + W_{ff}^*)}{(W_{fa} + W_{ff})_o} \right]^{0.6} \Delta t_m = -924 \left(\frac{W_{123}}{W_o} \right) (t_{fs3} - t_{fs2})$$

$$\Delta t_m = \frac{t_{fg3} + t_{fg4} - t_{fs2} - t_{fs3}}{2}$$

$$\left[\frac{(W_{fa} + W_{ff}^*)}{(W_{fa} + W_{ff})_o} \right]^{0.4} (t_{fg3} - t_{fg4}) = 0.487 [\Delta t_m] \quad .$$

b. Intermediate Heat Exchanger

An incremental section of the intermediate heat exchanger may be considered in the following manner.

Assuming

$$\frac{\partial t_{xc}}{\partial \theta} = \frac{\partial t_{xsp}}{\partial \theta}$$

and the wall heat capacity ($\rho_{wx} C_{pwx} A_{wx} \Delta X_x$) is equally divided between, and varies as the primary and secondary coolant.

In addition, let

$$t_{xss} = \frac{t_{xss0} + t_{xssi}}{2}, \quad t_{xsp} = \frac{t_{xspo} + t_{xspi}^{**}}{2}$$

and

$$q_x = U_x P_x (\Delta X_x) (t_{xsp} - t_{xss})$$

where

$$\frac{dt_{xspi}^{**}}{d\theta} = \frac{1}{\tau_{xspip}} \left[t_{xspi} - t_{xspi}^{**} \right]$$

Using a single node approximation,

$$\frac{dt_{xsp}}{d\theta} + U_x \alpha_{xp} (t_{xsp} - t_{xss}) = \beta_{xp} \left(\frac{W_{14}}{W_o} \right) (t_{xspi}^{**} - t_{xspo})$$

$$\frac{dt_{xss}}{d\theta} - U_x \alpha_{xs} (t_{xsp} - t_{xss}) = -\beta_{xs} \left(\frac{W_5}{W_o} \right) (t_{xss0} - t_{xssi})$$

where

$$\alpha_{xp} = \frac{P_x}{\rho_s A_{xp} C_{p_s} + \frac{\rho_{xw} C_{p_{xw}} A_{xw}}{2} + \rho_{xc} C_{p_{xc}} A_{xc}}$$

$$\alpha_{xs} = \frac{P_x}{\rho_s A_{xs} C_{p_s} + \frac{\rho_{xw} C_{p_{xw}} A_{xw}}{2}}$$

$$\beta_{xp} = \frac{W_o C_{p_s}}{\left(\rho_s A_{xp} C_{p_s} + \frac{\rho_{xw} C_{p_{xw}} A_{xw}}{2} + \rho_{xc} C_{p_{xc}} A_{xc} \right) (\Delta X_x)}$$

$$\beta_{xs} = \frac{W_o C_{p_s}}{\left(\rho_s A_{xs} C_{p_s} + \frac{\rho_{xw} C_{p_{xw}} A_{xw}}{2} \right) (\Delta X_x)}$$

From the physical dimensions of the intermediate heat exchanger,

Heat transfer area

$$P_x = 60.5 \text{ ft}^2/\text{ft}$$

Tube side coefficient

$$h_{xt} = 7400 \left(\frac{W_5}{W_o} \right)^{0.4} \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F}$$

Wall coefficient

$$\frac{k_{xw}}{l_{xw}} = 3830 \text{ Btu/hr ft}^2 \text{ } ^\circ\text{F}$$

Shell side coefficient

$$h_{xs} = 2940 \left(\frac{W_{14}}{W_o} \right)^{0.6}$$

Overall value of heat transfer coefficient.

In the analog studies, the IHX was represented by using an average Δt rather than a Δt_{lm} . This required a modification of the overall heat transfer coefficient so as to agree with the predicted performance of the Alco IHX. Therefore

$$U_x = \frac{10,000}{\frac{1.51}{\left(\frac{W_5}{W_o} \right)^{0.4}} + 2.93 + \frac{3.81}{\left(\frac{W_{14}}{W_o} \right)^{0.6}}}$$

Also

$$\rho_s A_{xs} C_{p_s} = 16.1 \text{ Btu/ft } ^\circ\text{F}$$

$$\rho_s A_{xp} C_{p_s} = 96.5 \text{ Btu/ft } ^\circ\text{F}$$

$$\rho_{xw} A_{xw} C_{p_{xw}} = 38.3 \text{ Btu/ft } ^\circ\text{F}$$

$$\rho_{xc} A_{xc} C_{p_{xc}} = 57.1 \text{ Btu/ft } ^\circ\text{F}$$

Then

$$\alpha_{xs} = 1.72 \frac{\text{ft}^2 \text{ } ^\circ\text{F}}{\text{Btu}} \text{ (modified to 1.36 due to use of } \Delta t \text{ avg)}$$

$$\beta_{xs} = 366 \frac{1}{\text{hr}}$$

$$\alpha_{xp} = 0.35 \frac{\text{ft}^2 \text{ } ^\circ\text{F}}{\text{Btu}} \text{ (modified to 0.277, due to use of } \Delta t \text{ avg)}$$

$$\beta_{xp} = 74.6 \frac{1}{\text{hr}}$$

Then the equations are,

$$\frac{dt_{xsp}}{d\theta} + 0.277 U_x (t_{xsp} - t_{xss}) = 74.6 \left(\frac{W_{14}}{W_o} \right) (t_{xspi}^{**} - t_{xspo})$$

$$\frac{dt_{xss}}{d\theta} - 1.36 U_x (t_{xsp} - t_{xss}) = -366 \left(\frac{W_5}{W_o} \right) (t_{xss0} - t_{xssi})$$

$$t_{xss} = \frac{t_{xss0} + t_{xssi}}{2}$$

$$t_{xsp} = \frac{t_{xspo} + t_{xspi}^{**}}{2}$$

$$\frac{dt_{xspi}^{**}}{d\theta} = \frac{\left(\frac{W_{14}}{W_o} \right)}{1.362 \times 10^{-3}} (t_{xspi} - t_{xspi}^{**}) \quad (\text{See Appendix D})$$

$$U_x = \frac{10,000}{\left(\frac{W_5}{W_o} \right)^{0.4} + 2.93 + \frac{3.81}{\left(\frac{W_{14}}{W_o} \right)^{0.6}}}$$

c. Sodium Cooler

An incremental section of the cooler may be considered in the following manner.

Assuming

$$\frac{dt_{cw}}{d\theta} = \frac{dt_{cs}}{d\theta}$$

and

$$t_{cs} = \frac{t_{cso} + t_{csi}}{2} ,$$

then using a single-node approximation gives

$$\begin{aligned} \frac{dt_{cs}}{d\theta} + \left[\frac{U_c P_c}{\rho_s A_c C_{p_s} + \rho_{cw} A_{cw} C_{p_{cw}}} \right] \Delta t_{mc} \\ = - \left[\frac{W_3 C_{p_s}}{\rho_s A_c C_{p_s} + \rho_{cw} A_{cw} C_{p_{cw}}} \right] \left[\frac{t_{cso} - t_{csi}}{\Delta X_c} \right] \end{aligned}$$

where

$$\Delta t_{mc} = \frac{t_{cso} + t_{csi} - t_{cao} - t_{cai}}{2}$$

and

$$U_c = 25 \left(\frac{W_{ca}}{W_{cao}} \right)^{0.566} .$$

In addition, an energy balance between the heat transferred and that absorbed by the air gives

$$W_{ca} C_{p_{ca}} (t_{cao} - t_{cai}) = U_c P_c (\Delta X_c) \Delta t_{mc} .$$

From the physical dimensions of the cooler,

$$\rho_s A_c C_{p_s} = 3.27 \frac{\text{Btu}}{\text{ft } ^\circ\text{F}}$$

$$\rho_{cw} A_{cw} C_{p_{cw}} = 16.8 \frac{\text{Btu}}{\text{ft } ^\circ\text{F}}$$

$$\frac{U_c P_c}{\rho_s A_s C_{p_s} + \rho_{cw} A_{cw} C_{p_{cw}}} = 99.1 \left(\frac{W_{ca}}{W_{ca,o}} \right)^{0.566} \frac{1}{\text{hr}}$$

$$\frac{W_o C_{p_s}}{(\rho_s A_s C_{p_s} + \rho_{cw} A_{cw} C_{p_{cw}}) (\Delta X_c)} = 424 \frac{1}{\text{hr}}$$

$$\frac{U_c P_c (\Delta X_c)}{\left(\frac{W_{ca}}{W_{ca,o}} \right)^{0.566} \left(W_{ca,o} \right) C_{p_{ca}}} = 0.686 \text{ (dimensionless)}$$

The equations then reduce to

$$\frac{dt_{cs}}{d\theta} + 99.1 \left(\frac{W_{ca}}{W_{ca,o}} \right)^{0.6} \Delta t_{cm} = -424 \left(\frac{W_3}{W_o} \right) (t_{cso} - t_{csi})$$

$$\left(\frac{W_{ca}}{W_{ca,o}} \right)^{0.434} (t_{cao} - t_{cai}) = 0.686 (\Delta t_{cm})$$

$$\Delta t_{cm} = \frac{t_{cso} + t_{csi} - t_{cao} - t_{cai}}{2} .$$

2. System Controller Equations

a. Furnace Gas Flow Rate

The command signal to the fuel gas valve is a function of the furnace outlet sodium temperature set point, furnace inlet sodium temperature, and the sodium flow rate through the furnace. This is described by the following equation.

$$\left[\frac{W_{ff}}{(W_{fa} + W_{ff})_o} \right] = \left[\frac{0.04819}{400} \left(\frac{W_{123}}{W_o} \right) (t'_{fso} - t_{fs3}) \right] + \gamma$$

The time delay between the command signal to the fuel gas valve and the fuel gas entering the furnace is described as follows.

$$\frac{d \left(\frac{W_{ff}}{(W_{fa} + W_{ff})_o} \right)^*}{d\theta} = \frac{1}{\tau_{ff}} \left[\left(\frac{W_{ff}}{(W_{fa} + W_{ff})_o} \right) - \left(\frac{W_{ff}}{(W_{fa} + W_{ff})_o} \right)^* \right]$$

The γ term in the equation just above is the output from a 3-node controller whose signal is a function of the furnace sodium outlet temperature. The controller was represented by the following equation.

$$\gamma = \frac{1}{K_{3vii}} \left[K_{1vll} \int (t'_{fso} - t^*_{fso}) d\theta + [t'_{fso} - t^*_{fso}] + K_{2vll} \frac{d}{d\theta} [t'_{fso} - t^*_{fso}] \right]$$

The thermocouple time delay in measuring the furnace sodium outlet temperature is represented in the following manner.

$$\frac{dt^*_{fso}}{d\theta} = \frac{1}{\tau_{fso}} [t_{fso} - t^*_{fso}]$$

where

$$\tau_{fso} = \frac{2.285 \times 10^{-4}}{\left(\frac{W_{123}}{W_o} \right)}$$

b. Furnace Air Flow Rate

The command signal to the furnace air fans is a function of the demand furnace fuel flow rate plus a corrective signal obtained by comparing the demand furnace air flow to the actual furnace air flow.

The controller will be adjusted so as to provide 15% excess air at full power and 50% excess air at 10% power. This is described by the following equation.

$$\left(\frac{W_{fa}}{(W_{fa} + W_{ff})_O} \right) = 0.03178 + 18.833 \left(\frac{W_{ff}}{(W_{fa} + W_{ff})_O} \right) - \eta$$

The time delay between the command signal to the furnace air fans and the air entering the furnace is described by the following equation.

$$d \left(\frac{W_{fa}}{(W_{fa} + W_{ff})_O} \right)^* = \frac{1}{\tau_{fa}} \left[\left(\frac{W_{fa}}{(W_{fa} + W_{ff})_O} \right) - \left(\frac{W_{fa}}{(W_{fa} + W_{ff})_O} \right)^* \right]$$

The η term in the equation just above is the output from a three-mode controller whose signal is a function of the error between the desired furnace air flow and the actual furnace air flow. This correction is required since at the lower furnace loads the efficiency of the furnace will increase which will require an adjustment in the fuel and air flow rates. The three-mode controller is described by the following equation.

$$\eta = \frac{1}{K_{3v10}} \left[K_{1v10} \int \epsilon \, d\theta + [\epsilon] + K_{2v10} \frac{d\epsilon}{d\theta} \right]$$

where

$$\epsilon = \frac{W_{fa}^*}{(W_{fa} + W_{ff})_O} - 0.03178 - 18.833 \left(\frac{W_{ff}^*}{(W_{fa} + W_{ff})_O} \right)$$

c. IHX Primary Inlet Sodium Temperature Controller (TRC-100)

The primary sodium temperature entering the IHX is controlled by mixing hot sodium from the furnace and cold sodium which has bypassed the furnace. The sodium flows are controlled by valves which are mechanically linked together so that as one valve is closed the other valve opens. The

positioning of the valves is controlled by a three-mode controller which operates off the error between the desired set point temperature and the actual temperature of the primary sodium entering the IHX. The controller may be represented by the following equation:

$$\phi_{v4} = \frac{1}{K_{3v4}} \left[K_{1v4} \int (t'_{xspi} - t^*_{xspi}) d\theta + (t'_{xspi} - t^*_{xspi}) + K_{2v4} \frac{d}{d\theta} (t'_{xspi} - t^*_{xspi}) \right] .$$

The measured sodium temperature in the above equation will be delayed due to the response time of the thermocouple. Then:

$$\frac{dt^*_{xspi}}{d\theta} = \frac{1}{\tau_{xspit}} \left[t_{xspi} - t^*_{xspi} \right] .$$

The output from the three-mode controller will be a valve position command signal. The actual valve position will be delayed due to valve response time, therefore:

$$\frac{d\phi^*_{v4}}{d\theta} = \frac{1}{\tau_{v4}} \left[\phi_{v4} - \phi^*_{v4} \right] .$$

The pressure drop associated with the valve position and sodium flow is †

$$\Delta P_{v4} = \frac{\left(\frac{W_4}{W_o} \right)^2}{\left[e^{-4.96 + 6.30 (\phi^*_{v4})} \right] \left[\phi^*_{v4} \right]} .$$

Since the other valve controlling the hot sodium is complimentary to the valve controlling the cold sodium,

$$\phi_{v1}^* = 1.0 - \phi_{v4}^*$$

and

$$\Delta P_{v1} = \frac{\left(\frac{W_1}{W_o}\right)^2}{\left[e^{-4.96 + 6.30 (\phi_{v1}^*)} \right] \left[\phi_{v1}^* \right]}$$

The temperature of the sodium after mixing of the cold and hot streams is described by the following equation

$$t_{xspi} = \frac{\left(\frac{W_1}{W_o}\right) t_{fso} + \left(\frac{W_4}{W_o}\right) t_{fs3}}{\left(\frac{W_1 + W_4}{W_o}\right)}$$

d. IHX Primary Sodium Flow Rate Controller

The primary sodium flow rate through the IHX is controlled by a valve located on the outlet side of the exchanger. The positioning of the valve is controlled by a three-mode controller which operates off the error between the desired flow rate and the actual flow rate. The controller was represented by the following equation:

$$\phi_{v6} = \frac{1}{K_{3v6}} \left[K_{1v6} \int \left(\frac{W'_{14}}{W_o} - \frac{W_{14}^*}{W_o} \right) d\theta + \left(\frac{W'_{14}}{W_o} - \frac{W_{14}^*}{W_o} \right) + K_{2v6} \frac{d}{d\theta} \left(\frac{W'_{14}}{W_o} - \frac{W_{14}^*}{W_o} \right) \right]$$

The actual flow rate signal is delayed due to the response time of the measuring device, therefore

$$\frac{d\left(\frac{W_{14}^*}{W_o}\right)}{d\theta} = \frac{1}{\tau_{xsp}} \left[\frac{W_{14}}{W_o} - \frac{W_{14}^*}{W_o} \right] .$$

In addition the actual valve position is delayed relative to the command signal from the controller due to the response time of the valve, therefore

$$\frac{d\phi_{v6}^*}{d\theta} = \frac{1}{\tau_{v6}} \left[\phi_{v6} - \phi_{v6}^* \right] .$$

The pressure drop associated with the valve position and sodium flow is

$$\Delta P_{v6} = \frac{\left[\frac{W_{14}}{W_o} \right]^2}{\left[e^{-4.96 + 6.30 \phi_{v6}^*} \right] \left[\phi_{v6}^* \right]} .$$

e. Hot Bypass Sodium Flow Rate Controller

The sodium flow rate which by-passes the IHX is used primarily for compensating any drop in the sodium temperature leaving the IHX. By mixing hot sodium directly from the furnace with cold sodium from the IHX, the sodium temperature entering the furnace can be maintained constant. The amount of by-passed sodium is regulated by a valve controlled by a three-mode controller which operates off the error between the desired furnace inlet sodium temperature and the actual temperature. The controller may be represented as follows:

$$\begin{aligned} \phi_{v2} = & \frac{1}{K_{3v2}} \left[K_{1v2} \int (t'_{fs3} - t_{fs3}^*) d\theta + [t'_{fs3} - t_{fs3}^*] \right. \\ & \left. + K_{2v2} \frac{d}{d\theta} [t'_{fs3} - t_{fs3}^*] \right] . \end{aligned}$$

The measured sodium temperature in the above equation will be delayed due to the response time of the thermocouple. Then

$$\frac{dt_{fs3}^*}{d\theta} = \frac{1}{\tau_{fs3}} \left[t_{fs3} - t_{fs3}^* \right] .$$

The output from the three-mode controller will be a valve position command signal. The actual valve position will be delayed due to valve response time. Therefore:

$$\frac{d\phi_{v2}^*}{d\theta} = \frac{1}{\tau_{v2}} \left[\phi_{v2} - \phi_{v2}^* \right] .$$

The pressure drop associated with the valve position and sodium flow is

$$\Delta P_{v2} = \frac{\left(\frac{W_2}{W_o} \right)^2}{\left[e^{-6.02 + 6.02 \phi_{v2}^*} \right] \left[\phi_{v2}^* \right]} .$$

The temperature of the sodium after mixing of the cold and hot stream is described by the following equation:

$$t_{fs3} = \frac{\left(\frac{W_{14}}{W_o} \right) t_{xspo}^* + \left(\frac{W_3}{W_o} \right) t_{cso} + \left(\frac{W_2}{W_o} \right) t_{fso}}{\left(\frac{W_{1234}}{W_o} \right)}$$

where t_{xspo} is determined from:

$$\frac{dt_{xspo}^*}{d\theta} = \frac{1}{\tau_{xspo}} [t_{xspo} - t_{xspo}^*]$$

and

$$\tau_{xspo} = \frac{2.58 \times 10^{-3}}{\left(\frac{W_{14}}{W_o}\right)} .$$

f. Cooler Sodium Flow Rate Control

The sodium cooler is used primarily as a protection device for the furnace. With the cooler, the furnace is assured of a minimum load equal to the cooler rating, even though the system demand should go to zero. For other transients, reduction of flow through the cooler increases the mixed mean temperature of the hot sodium in the main loop which temperature would otherwise have to be attained by increasing the furnace firing at an inadvisable rate.

In order to protect the furnace, a minimum sodium flow rate through the furnace was established which was consistent with the minimum firing rate at which the furnace could operate. This minimum flow rate or a larger flow rate was used as the set point for the cooler sodium flow. As long as the furnace flow rate exceeded the minimum flow, the cooler sodium flow rate control valve remained closed. When the furnace flow rate decreased, the specified minimum flow was maintained by opening of the sodium cooler control valve.

The cooler sodium flow rate controller is described by the following equation:

$$\begin{aligned} \phi_{v3} = & \frac{1}{K_{3v3}} \left[K_{1v3} \int \left(\frac{W'_{123}}{W_o} - \frac{W_{123}^*}{W_o} \right) d\theta + \left(\frac{W'_{123}}{W_o} - \frac{W_{123}^*}{W_o} \right) \right. \\ & \left. + K_{2v3} \frac{d}{d\theta} \left(\frac{W'_{123}}{W_o} - \frac{W_{123}^*}{W_o} \right) \right] . \end{aligned}$$

The actual flow rate signal is delayed due to the response time of the measuring device; therefore,

$$\frac{d \left(\frac{W_{123}}{W_o} \right)^*}{d\theta} = \frac{1}{\tau_{123}} \left[\frac{W_{123}}{W_o} - \frac{W_{123}^*}{W_o} \right] .$$

In addition, the actual valve position is delayed relative to the command signal from the controller due to the response time of the valve; therefore,

$$\frac{d\phi_{v3}^*}{d\theta} = \frac{1}{\tau_{v3}} \left[\phi_{v3} - \phi_{v3}^* \right] .$$

The pressure drop associated with the valve position and sodium flow is

$$\Delta P_{v3} = \frac{\left(\frac{W_3}{W_o} \right)^2}{\left[e^{-6.02 + 6.02 \phi_{v3}^*} \right] \phi_{v3}^*} .$$

g. Cooler Sodium Flow Temperature Control Equations

The temperature control for the cooler maintains the outlet sodium temperature at the control point by varying the amount of air flow across the cooler. Due to the sodium flow varying from zero to full-rated-cooler flow, the control of the air flow rate is accomplished by dampers and a variable speed motor. The controller for the temperature control is described as follows.

$$\begin{aligned} \left(\frac{W_{ca}}{W_{ca,o}} \right) &= \frac{1}{K_{3v7}} \left(K_{1v7} \int (t'_{cso} - t_{cso}^*) d\theta + [t'_{cso} - t_{cso}^*] \right. \\ &\quad \left. + K_{2v7} \frac{d}{d\theta} [t'_{cso} - t_{cso}^*] \right) \end{aligned}$$

The actual temperature signal is delayed due to the response time of the thermocouple; therefore,

$$\frac{dt_{cso}^*}{d\theta} = \frac{1}{\tau_{cs}} \left[t_{cso} - t_{cso}^* \right] \quad .$$

In addition, the actual cooler air flow rate is delayed relative to the command signal from the controller, due to the response time of the dampers and variable speed motor, therefore,

$$\frac{d \left(\frac{W_{ca}}{W_{ca,o}} \right)^*}{d\theta} = \frac{1}{\tau_{ca}} \left[\left(\frac{W_{ca}}{W_{ca,o}} \right) - \left(\frac{W_{ca}}{W_{ca,o}} \right)^* \right] \quad .$$

3. Other Required System Equations

a. System Pressure Drops

The primary sodium system is shown schematically in Figure 2 with the various piping runs sectionalized to facilitate pressure drop calculations. Results of the pressure drop calculation for the various sections are shown in Table 1, where

$$\Delta P = \Delta P_i \left(\frac{W_i}{W_o} \right)^2$$

and

$$W_o = 1 \times 10^6 \text{ lb/hr} \quad .$$

Since the primary sodium system consists of four loops, interconnected to a single line across the pump, pressure drops may be determined for each loop and equated to the pressure drop developed across the pump. The resulting equations are shown below.

1) Main Primary Loop

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TABLE 1
PRIMARY SODIUM PIPING PRESSURE DROP CALCULATIONS

Primary Sodium Piping Section (i)	Pressure Drop For Particular Section ΔP_i (psi)	Sodium Flow Rate in Particular Section W_i (lb/hr)
A	1.42	W_{123}
B	$0.905 + \frac{1}{\left[e^{-4.96 + 6.30 \phi_{v1}^*} \right] \left[\phi_{v1}^* \right]}$	W_1
C	2.9	W_{123}
D	$4.28 + \frac{1}{\left[e^{-6.02 + 6.02 \phi_{v2}^*} \right] \left[\phi_{v2}^* \right]}$	W_2
E	0.67	W_{14}
G	$0.87 + \frac{1}{\left[e^{-4.96 + 6.30 \phi_{v6}^*} \right] \left[\phi_{v6}^* \right]}$	W_{14}
H	1.46	W_{1234}
J	0.354	W_{1234}
K	0.154	W_{1234}
L	$1.58 + \frac{1}{\left[e^{-4.96 + 6.30 \phi_{v4}^*} \right] \left[\phi_{v4}^* \right]}$	W_4
M	1.12	W_{123}
N	1.22	W_{123}
P & Q	$17.20 + \frac{1}{\left[e^{-6.02 + 6.02 \phi_{v3}^*} \right] \left[\phi_{v3}^* \right]}$	W_3
Cooler	68.4	W_3
Furnace	6.95	W_{123}
Economizer	2.51	W_{123}
IHX	6.25	W_{14}

$$\Delta P = \Delta P_i \left(\frac{W_i}{W_o} \right)^2 \quad W_o = 1 \times 10^6 \text{ lb/hr.}$$

2) Cooler Loop

$$\begin{aligned}
 & 104 + 0.873 \left(\frac{W_{1234}}{W_o} \right) - 13.518 \left(\frac{W_{1234}}{W_o} \right)^2 - 13.22 \left(\frac{W_{123}}{W_o} \right)^2 \\
 & = 2.9 \left(\frac{W_{23}}{W_o} \right)^2 + \left[85.6 + \frac{1}{\left[e^{-6.02 + 6.02 \phi_{v3}^*} \right] \left[\phi_{v3}^* \right]} \right] \left[\frac{W_3}{W_o} \right]^2 .
 \end{aligned}$$

3) Hot Bypass Loop

$$\begin{aligned}
 & 104 + 0.873 \left(\frac{W_{1234}}{W_o} \right) - 13.518 \left(\frac{W_{1234}}{W_o} \right)^2 - 13.22 \left(\frac{W_{123}}{W_o} \right)^2 \\
 & = 2.9 \left(\frac{W_{23}}{W_o} \right)^2 + \left[4.28 + \frac{1}{\left[e^{-6.02 + 6.02 \phi_{v2}^*} \right] \left[\phi_{v2}^* \right]} \right] \left[\frac{W_2}{W_o} \right]^2 .
 \end{aligned}$$

4) Cold Bypass Loop

$$\begin{aligned}
 & 104 + 0.873 \left(\frac{W_{1234}}{W_o} \right) - 13.518 \left(\frac{W_{1234}}{W_o} \right)^2 - 13.22 \left(\frac{W_{123}}{W_o} \right)^2 \\
 & = \left(1.58 + \frac{1}{\left[e^{-4.96 + 6.30 \phi_{v4}^*} \right] \left[\phi_{v4}^* \right]} \right) \left(\frac{W_4}{W_o} \right)^2 \\
 & + \left(7.79 + \frac{1}{\left[e^{-4.96 + 6.30 \phi_{v6}^*} \right] \left[\phi_{v6}^* \right]} \right) \left(\frac{W_{14}}{W_o} \right)^2 - 13.22 \left(\frac{W_{123}}{W_o} \right)^2 .
 \end{aligned}$$

With the above equations, sodium flow in the various loops may be determined as a function of the various valve positions.

b. Piping Transport Time Delays

An incremental section of sodium piping is shown below.

Taking heat balances gives,

$$W_s Cp_s t_s + U_{spw} P_{spw} (\Delta X) (t_{spw} - t_s) = \rho_{sp} A_{sp} Cp_s (\Delta X) \frac{dt_s}{d\theta} + W_s Cp_s t_s + W_s Cp_s \frac{dt_s}{dx} \Delta X$$

and

$$U_{spw} P_{spw} (\Delta X) (t_{spw} - t_s) + \rho_{spw} A_{spw} Cp_{spw} (\Delta X) \frac{dt_{spw}}{d\theta} = 0$$

Simplifying gives,

$$a (t_{spw} - t_s) = \frac{dt_s}{d\theta} + V_s \frac{dt_s}{dx}$$

$$b (t_{spw} - t_s) + \frac{dt_{spw}}{d\theta} = 0$$

where

$$a = \frac{U_{spw} P_{spw}}{\rho_{sp} A_{sp} Cp_s} \quad \text{and} \quad b = \frac{U_{spw} P_{spw}}{\rho_{spw} A_{spw} Cp_{spw}}$$

Letting

$$V_s = V(o) + \Delta V(\theta)$$

$$t_{spw} = t_{spw}(o, x) + \Delta t_{spw}(\theta, x)$$

$$t_s = t_s(o, x) + \Delta t_s(\theta, x)$$

and taking the La Place transform gives

$$\frac{\Delta t_s(S, L)}{\Delta t_s(S, o)} = e^{-\frac{SL}{V(o)} \left[\frac{s + a + b}{s + a} \right]} .$$

Assuming

$$t_{spw}(o, x) = t_s(o, x) = \frac{d}{dx} \left[t_s(o, x) \right] = 0 .$$

Assuming a first-order time delay of

$$G(S) = \frac{1}{\tau S + 1}$$

the value of τ may then be determined by referring to the equations developed from the actual system. That is

$$\tau = \frac{L}{V_s} \left[1 + \frac{a}{b} \right]$$

and the assumed response is

$$\frac{\Delta t_s(\theta, L)}{\Delta t_s(\theta, o)} = \frac{e^{-\frac{\theta}{\tau}}}{\tau} ;$$

then

$$1 + \frac{a}{b} = 1 + \frac{\rho_{spw} A_{spw} C_{p_{spw}}}{\rho_{sp} A_{sp} C_{p_s}} = 1.522 \quad .$$

1) Time Delay – High Temperature Mixing Tee to IHX

Length of piping $L = 31.38 \text{ ft}$

Velocity of sodium $V_s = \frac{W_{14}}{A_{sp} \rho_{sp}} = 3.5 \times 10^4 \left(\frac{W_{14}}{W_o} \right) \frac{\text{ft}}{\text{hr}} \quad .$

then

$$\tau_{xspip} = \frac{1.362 \times 10^{-3}}{\left(\frac{W_{14}}{W_o} \right)} \text{ hr} \quad .$$

2) Time Delay – IHX to Low Temperature Mixing Tee

Length of piping $L = 59.25 \text{ ft}$

Velocity of sodium $V_s = \frac{W_{14}}{A_{sp} \rho_{sp}} = 3.5 \times 10^4 \left(\frac{W_{14}}{W_o} \right) \frac{\text{ft}}{\text{hr}}$

then

$$\tau_{xspo} = \frac{2.58 \times 10^{-3}}{\left(\frac{W_{14}}{W_o} \right)} \text{ hr} \quad .$$

3) Time Delay – Furnace Outlet Sodium Temperature

Sodium flow through tubes = $\frac{\left(\frac{W_{123}}{W_o} \right) 1 \times 10^6}{183} \frac{\text{lb}}{\text{hr}}$

Volume of headers = 0.0615 ft^3

Volume of pipe wall = 0.00557 ft^3

then

$$\tau_{fso} = \frac{2.285 \times 10^{-4}}{\left(\frac{W_{123}}{W_o}\right)} .$$

4) Time Delay — Furnace Economizer to Furnace Convection Region

Length of piping $L = 73.67 \text{ ft}$

Velocity of sodium $V_s = \frac{W_{123}}{A_{sp} \rho_{sp}} = 3.5 \times 10^4 \left(\frac{W_{123}}{W_o}\right) \frac{\text{ft}}{\text{hr}}$

$$\tau_{fs2} = \frac{3.2 \times 10^{-3}}{\left(\frac{W_{123}}{W_o}\right)} .$$

c. Butterfly Valve Characteristics

Manufacturer's data pertaining to butterfly valve characteristics were obtained in the form of valve C_v vs position. The C_v for the valve is defined by the equation

$$C_v = \frac{Q}{\sqrt{\Delta P \left(\frac{\rho_w}{\rho}\right)}} .$$

The manufacturer's data for an 8-in. valve flowing water with a ΔP of 1.0 psi are given below.

Valve position (°F)	10	20	30	40	50	60	70	80	90
$C_v \text{ (gpm/ } \sqrt{\Delta P})$	85	150	260	420	700	1200	1950	3000	4000

The above equation when converted to sodium flow at 900°F is:

$$\frac{C_v}{2190} = \frac{\left(\frac{W}{W_o}\right)}{\sqrt{\Delta P}} .$$

The equation used to represent an 8-in. valve in this analog study was obtained by converting the manufacturer's data to sodium flow. See Figure 3.

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Figure 3. C_v Approximation for 8-in. Valve

The equation obtained for a ΔP of 1.0 psi, and plotted in Figure 4, is

$$\frac{W}{W_0} = \frac{C_v}{2190} = \left[e^{-2.48 + 3.15 \phi} \right] \left[\phi \right]^{1/2}$$

7593-1036

Figure 4. Control Characteristics of 8-in. Butterfly Valve

This can then be used to obtain the equation describing the pressure drop across the 8-in. valve as a function of sodium flow and valve position as follows:

$$\Delta P = \frac{\left(\frac{W}{W_o}\right)^2}{\left[e^{-4.96 + 6.30 \phi}\right] \left[\phi\right]} .$$

In a similar manner, manufacturer's data for 6-in. valves flowing water with a ΔP of 1.0 psi are:

Valve position (°F)	10	20	30	40	50	60	70	80	90
C_v (gpm/ $\sqrt{\Delta P}$)	50	90	150	250	410	650	1150	1750	2300

The above data converted to sodium flow at 900°F are shown in Figure 5 along with the equation used to represent a 6-in. valve in the analog study

The equation obtained for a ΔP of 1.0 psi is

$$\frac{W}{W_o} = \left[e^{-3.01 + 3.01 (\phi)}\right] \left[\phi\right]^{1/2} .$$

This equation was then used to obtain the equation describing the pressure drop across a 6-in. valve as a function of sodium flow and valve position, as follows:

$$\Delta P = \frac{\left(\frac{W}{W_o}\right)^2}{\left[e^{-6.02 + 6.02 (\phi)}\right] \left[\phi\right]} .$$

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Figure 5. Control Characteristics of 6-in. Butterfly Valve

d. Sodium Pump Characteristics

A head-vs-flow curve supplied by the pump manufacturer was used to obtain the approximate pump characteristic equation. The fitted equation obtained was:

$$\Delta P = 104 + 0.873 \left(\frac{W_{1234}}{W_o} \right) - 11.55 \left(\frac{W_{1234}}{W_o} \right)^2 .$$

In addition to the described representation for system studies, a multinode model of the IHX (Appendix B) was developed and put on the analog computer. The response of the multinode model was then determined for the various transients and compared to the single-node model response curves. This was primarily done for the purpose of insuring that the single-node representation of the IHX used in the system studies was satisfactory for determining the system controller characteristics.

For completion of the necessary information needed for reproduction of this effort, a detailed explanation of analog conversion methods, as applied to this particular control system, is given in Appendix A.

E. DYNAMIC ANALYSIS AND RESULTS WITH OPTIMUM CONTROLLER SETTINGS

1. Results

Results of the analog study on the primary system are shown in Figures 5 through 16.

a. Transient (a) — IHX Primary Inlet Cold Shock

The command signal to the IHX primary inlet temperature controller was ramped from 1200 to 900°F in about 9 sec, as shown in Figure 6A. The error between the command signal and the delayed (1 sec) sodium temperature entering the IHX reaches a maximum of about 40°F at the end of the transient, and the IHX primary outlet temperature rises as shown in Figure 6B. This rise, however, is due to use of a single node representation for the IHX. In reality, a rise in IHX sodium outlet temperature for this transient is a physical impossibility. The direct use of a more elaborate IHX representation in the system studies was not possible because of analog equipment limitations. In actuality, this erroneous characteristic imposes a more severe control problem than that

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Figure 6A. IHX Primary Inlet Cold Shock Transients

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Figure 6B. IHX Primary Inlet Cold Shock Transients

which the actual system will impose. Reference to the multinode IHX analysis shows the IHX outlet temperature drops as expected.

The furnace inlet and outlet temperatures are also shown in Figure 6B. They are within the allowable changes of $\pm 50^{\circ}\text{F}$ and in reality should be less due to pipe, valve and pump heat capacities which were not considered.

The transient achieved was $30^{\circ}\text{F}/\text{sec}$ for 9 sec, whereas the specific rate was $35^{\circ}\text{F}/\text{sec}$ for 10 sec. Due to secondary effects which was not considered in the analog study, the final adjustment of the controllers to achieve the specified rate of temperature change will be done on the actual system.

b. Transient (b) — IHX Primary Flow Transient

The command signal to the IHX primary flow control was ramped from 1.14×10^6 lb/hr to 0.114×10^6 lb/hr in about 8 sec as shown in Figure 7A. It is seen that actual sodium flow rate followed command, with the other flows changing as expected. The various temperatures shown on Figure 7B reacted as expected. The IHX primary outlet dropped to about the secondary inlet sodium temperature. The furnace outlet temperature error was about 55°F and then started to return to the control point. It is felt that this can be improved on the actual system with adjustment of the controller constants. The balance of the temperatures varied up to 60°F for a short period of time and then returned to their control points. The IHX primary inlet sodium temperature developed oscillations of about $\pm 20^{\circ}\text{F}$ primarily due to the valve being nearly closed and being operated in a region of high sensitivity. Since the analog studies had no "dead band", a "dead band" on the actual system will alleviate the oscillations.

c. Transient (c) — IHX Primary Inlet Hot Shock

The command signal to the IHX primary inlet temperature controller was ramped from 1200 to 1300°F in about 9 sec, as shown in Figure 8A. It is seen that flows changed as expected, with flows adjusting to changes in the furnace outlet temperature (Figure 8B). The other various temperatures reacted as expected. The IHX primary outlet temperature rose due to the higher inlet temperature and the economizer or furnace inlet temperature also rose since there was no means of reducing the temperature. This was expected and was acceptable as long as the rise in temperature did not exceed 50°F . The IHX primary inlet sodium temperature again oscillated about $\pm 10^{\circ}\text{F}$ primarily due to being in a region of high sensitivity. A "dead band" on the actual controller will alleviate this condition.

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Figure 7A. IHX Primary Flow Transients

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Figure 7B. IHX Primary Flow Transients

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Figure 8A. IHX Primary Inlet Hot Shock Transients

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Figure 8B. IHX Primary Inlet Hot Shock Transients

d. Transient (d) — IHX Primary Outlet Hot Shock

The secondary flow was reduced to zero in about 8 sec, as shown in Figure 9A. The flows varied as expected. Similarly, the temperatures shown in Figure 9B varied as expected with oscillations in the cooler outlet, primary inlet and economizer inlet temperatures $\sim \pm 10$ or 15°F . A "dead band" on the controllers will alleviate these oscillations. The IHX primary outlet temperature and economizer or furnace inlet temperature increased but this was expected. The IHX primary outlet temperature should go to 1200°F but the analog only shows 1175°F . The cause for this was never determined. The economizer inlet or furnace inlet temperatures increased about 15°F which was expected since there was no means of reducing the temperature due to the cooler operating at maximum load. The rise of 15°F was acceptable. The furnace outlet temperature oscillated about $\pm 10^\circ\text{F}$, however, the use of a "dead band" on the controllers will alleviate these oscillations.

2. Conclusions

In general, the system performed as expected with no severe oscillations or divergent characteristics. The furnace was adequately protected during all transients and the furnace control system was satisfactory.

The system as described and controlled on the analog was more critical than the actual physical system. In the physical system the effect of sodium mixing and piping, valves and pump heat capacities would tend to reduce the small rapid oscillations which the analog indicated. The piping transport time delays would also tend to reduce the rapid oscillations. For example in the analog representation any change in furnace outlet temperature was immediately felt at the IHX mixing valve, hot bypass and cooler inlet, whereas in the actual physical system a time delay and damping would occur. This would make the control of the system less critical. However, piping transport delays between equipment and the sensor used to control the equipment were included in the analog model. These were required to insure stability of the specific control loop.

The comparison of the multinode results with the single node representation are shown in Figures 10 through 13. The comparisons indicated that in all cases the single node approximation was more than satisfactory to determine system controller characteristics.

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Figure 9A. IHX Primary Outlet Hot Shock Transients

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Figure 9B. IHX Primary Outlet Hot Shock Transients

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Figure 10. Transient (a) Temperatures for Multi- and Single Node Approximations

Figure 11. Transient (b) Temperatures for Multi- and Single Node Approximations

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Figure 12. Transient (c) Temperatures for Multi- and Single Node Approximations

Figure 13. Transient (d) Temperatures for Multi- and Single Node Approximations

3. Determination of Controller Settings

The results of the various analog studies conducted on the control system were evaluated and the results transposed into a form suitable for setting the actual controllers. The recommended initial controller settings are tabulated and included.

Controller constants (proportional band, rate, reset, and reset limit) were determined on a basis which satisfactorily controlled the various transients required of the SCTI. Values existed as various potentiometer settings. It was then necessary to transpose these analog potentiometer settings into such a form that the results could be used to initially set the various controllers. Final adjustments and confirmation of the instrument ranges should be made in the field.

The controller equations which were used in the analog study along with the related calculations necessary to transpose the analog results were as follows.

a. TRC (Temperature Recorder Controller)-100

$$\phi_{v4} = \frac{1}{K_{3v4}} \left[K_{1v4} \int (t'_{xspi} - t^*_{xspi}) d\theta + [t'_{xspi} - t^*_{xspi}] + K_{2v4} \frac{d(t'_{xspi} - t^*_{xspi})}{d\theta} \right]$$

from analog studies

$$\phi_{v4} = \frac{\Delta t}{K_{3v4}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta t}{\phi_{v4}} = K_{3v4} = 500 (0.1905) = 95.3$$

from controller

input range 400 to 1400°F

output range 0 to 90° Rotation or 0 to 1.0

then

$$\frac{\text{input}}{\text{output}} = \frac{1000}{1.0} = 1000$$

then

$$\text{P. B.} = \frac{95.3 (100)}{1000} = 9.53\%$$

$$\text{Reset} = K_{1v4} = 0.2970 \text{ repeats/sec} = 17.8 \text{ repeats/min}$$

$$\text{Rate} = K_{2v4} = 0.3723 \text{ sec} = 0.0062 \text{ min.}$$

b. TRC-103

$$\phi_{v2} = \frac{1}{K_{3v2}} \left[K_{1v2} \int (t'_{fs3} - t^*_{fs3}) d\theta + [t'_{fs3} - t^*_{fs3}] + K_{2v2} \frac{d(t'_{fs3} - t^*_{fs3})}{d\theta} \right]$$

from analog studies

$$\phi_{v2} = \frac{\Delta t}{K_{3v2}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta t}{\phi_{v2}} = K_{3v2} = 5000 (0.1985) = 993$$

from controller

input range 400 to 1400°F

output range 0 to 90° Rotation or 0 to 1.0

then

$$\frac{\text{input}}{\text{output}} = \frac{1000}{1.0} = 1000$$

then

$$\text{P. B.} = \frac{993 (100)}{1000} = 99.3\%$$

$$\text{Reset} = K_{1v2} = 0.0996 \text{ repeats/sec} = 5.98 \text{ repeats/min.}$$

$$\text{Reset Limit} = \frac{0.35 (100) (50) (12)}{1000} + 3.0 = 24 \text{ psi}$$

c. FRC (Flow Recorder Controller)-101

$$\phi_{v3} = \frac{1}{K_{3v3}} \left[K_{1v3} \int \left(\frac{W'_{123}}{W_o} - \frac{W^*_{123}}{W_o} \right) d\theta + \left(\frac{W'_{123}}{W_o} - \frac{W^*_{123}}{W_o} \right) + K_{2v3} \frac{d \left(\frac{W'_{123}}{W_o} - \frac{W^*_{123}}{W_o} \right)}{d\theta} \right]$$

from analog studies

$$\phi_{v3} = \frac{\Delta \left(\frac{W_{123}}{W_o} \right)}{K_{3v3}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta \left(\frac{W_{123}}{W_o} \right)}{\phi_{v3}} = K_{3v3} = 2 (0.190) = 0.38$$

from Controller input range 0 to 4000 gpm
 output range 0 to 90° Rotation or 0 to 1.0
 since $W_o = 1 \times 10^6$ lbs/hr or 2500 gpm then
 input range 0 to 1.6

then

$$\frac{\text{input}}{\text{output}} = \frac{1.6}{1.0} = 1.6$$

then

$$\text{P. B. } \frac{0.38 (100)}{1.6} = 23.8\%$$

$$\text{Reset} = K_{1v3} = 0.5997 \text{ repeats/sec} = 36 \text{ repeats/min.}$$

d. FRC-100

$$\phi_{v6} = \frac{1}{K_{3v6}} \left[K_{1v6} \int \left(\frac{W'_{14}}{W_o} - \frac{W^*_{14}}{W_o} \right) d\theta + \left[\frac{W'_{14}}{W_o} - \frac{W^*_{14}}{W_o} \right] + K_{2v6} \frac{d \left(\frac{W'_{14}}{W_o} - \frac{W^*_{14}}{W_o} \right)}{d\theta} \right]$$

from analog studies

$$\phi_{v6} = \frac{\Delta \left(\frac{W_{14}}{W_o} \right)}{K_{3v6}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta \left(\frac{W_{14}}{W_o} \right)}{\phi_{v6}} = K_{3v6} = 2 (0.7085) = 1.417$$

from Controller input range 0 to 4000 gpm
 output range 0 to 90° Rotation or 0 to 1.0
 since $W_o = 1 \times 10^6$ lb/hr or 2500 gpm then
 input range 0 to 1.6

then

$$\frac{\text{input}}{\text{output}} = \frac{1.6}{1.0} = 1.6$$

then

$$\text{P. B.} = \frac{1.417}{1.6} (100) = 88.5\%$$

$$\text{Reset} = K_{1v6} = 0.249 \text{ repeats/sec} = 14.95 \text{ repeats/min.}$$

e. TRC-101

$$\frac{W_{ca}}{W_{ca,o}} = \frac{1}{K_{3v7}} \left[K_{1v7} \int (t'_{cso} - t^*_{cso}) d\theta + [t'_{cso} - t^*_{cso}] + K_{2v7} \frac{d(t'_{cso} - t^*_{cso})}{d\theta} \right]$$

from analog studies

$$\left(\frac{W_{ca}}{W_{ca,o}} \right) = \frac{\Delta t}{K_{3v7}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta t}{\left(\frac{W_{ca}}{W_{ca,o}} \right)} = K_{3v7} = 5000 (0.020) = 100$$

from Controller input range 400 to 1400°F
 output range 0 to 90° Rotation
 since $W_{ca,o} = 403,000$ lb/hr at 90° open
 output range 0 to 1.0

then

$$\frac{\text{input}}{\text{output}} = \frac{1000}{1.0} = 1000$$

then

$$P. B. = \frac{100 (100)}{1000} = 10\%$$

$$\text{Reset} = K_{1v7} = \frac{0.500}{10} = 0.05 \text{ repeats/sec} = 3 \text{ repeats/min}$$

$$\text{Reset Limit} = \frac{0.200 (100) (50) (12)}{1000} + 3.0 = 15 \text{ psi .}$$

f. FCp (Flow Comput)-101

$$\eta = \frac{1}{K_{3v10}} \left[K_{1v10} \int \epsilon d\theta + (\epsilon) + K_{2v10} \frac{d\epsilon}{d\theta} \right]$$

$$\epsilon = \frac{W_{fa}^*}{(W_{fa} + W_{ff})_O} - 0.03178 - 18.833 \left(\frac{W_{ff}^*}{(W_{fa} + W_{ff})_O} \right)$$

$$\frac{W_{fa}}{(W_{fa} + W_{ff})_O} = 0.03178 + 18.833 \left(\frac{W_{ff}}{(W_{fa} + W_{ff})_O} \right) - \eta$$

from analog studies

$$\eta = \frac{\epsilon}{K_{3v10}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\epsilon}{\eta} = K_{3v10} = \frac{0.10}{10} = 0.01$$

from Controller input range 3 to 15 psi
 output range 0 to 90° rotation

when input is 3 psi, $W_{ff} = 0$ and $W_{fa} = 200,000 \frac{\text{lb}}{\text{hr}}$

input is 15 psi, $W_{ff} = 10,000 \frac{\text{lb}}{\text{hr}}$ and $W_{fa} = 0$

when output is 0 or valve closed $W_{fa} = 0$

output is 90° or valve open $W_{fa} = 200,000 \frac{\text{lb}}{\text{hr}}$

since $(W_{fa} + W_{ff})_O = 186,000 \text{ lb/hr}$

Input range +1.041 to -1.041

Output range 0 to 1.072

then

$$\frac{\text{input}}{\text{output}} = \frac{1.041 + 1.041}{1.072} = 1.94$$

then

$$\text{P. B.} = \frac{0.01 (100)}{1.94} = 0.5\% \text{ or } 1/2\%$$

$$\text{Reset} = K_{1v10} = 0.050 \text{ repeats/sec} = 3 \text{ repeats/min.}$$

g. TCp (Temperature Computer)-100

$$\gamma = \frac{1}{K_{3v11}} \left[K_{1v11} \int (t'_{fso} - t^*_{fso}) d\theta + [t'_{fso} - t^*_{fso}] + K_{2v11} \frac{d(t'_{fso} - t^*_{fso})}{d\theta} \right]$$

$$\frac{W_{ff}}{(W_{fa} + W_{ff})_o} = \left[\frac{0.04819}{400} \left(\frac{W_{123}^*}{W_o} \right) (t'_{fso} - t^*_{fs3}) \right] + \gamma$$

from analog studies

$$\gamma = \frac{\Delta t}{K_{3v11}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta t}{\gamma} = K_{3v11} = 25,000 (0.657) = 16,400$$

from Controller	input range	400 to 1400°F
	output range	0 to 10,000 lb/hr for W_{ff}
	since $(W_{fa} + W_{ff})_o$	= 186,000 lb/hr
	output range	0 to 0.0538

then

$$\frac{\text{input}}{\text{output}} = \frac{1000}{0.0538} = 18,600$$

then

$$\text{P. B.} = \frac{16,400}{18,600} (100) = 88\%$$

$$\text{Reset} = K_{1v11} = 10 (0.020) = 0.20 \text{ repeats/sec} = 12 \text{ repeats/min}$$

$$\text{Reset Limit} = \frac{0.023 (100) (50) (12)}{1000} + 3.0 = 4.38 \text{ psi}$$

$$\text{Rate} = K_{2v11} = \frac{10}{0.3968} = 25.2 \text{ sec} = 0.42 \text{ min.}$$

The summary of the analog results and these calculations is shown in Table 2.

These controller settings are recommended for the initial startup of the SCTI.

TABLE 2
SCTI PRIMARY SODIUM SIDE RECOMMENDED CONTROLLER SETTINGS

Controller	Term	Range	Analog Computer Potentiometer No.	Potentiometer Setting	Analog Multiplication Factor	Recommended Controller Setting
TRC-100	Prop. Band	0-90° Rot. 400-1400°F	CP18	0.1905	$K_{3v4}/500$	9.5%
	Reset		CQ15	0.2970	K_{1v4}	17.8 repeats/min
	Rate		CQ18	0.3723	K_{2v4}	0.006 min
TRC-103	Prop. Band	0-90° Rot. 400-1400°F	CP38	0.1985	$K_{3v2}/2$	100%
	Reset		CQ35	0.0996	K_{1v2}	6.0 repeats/min
	Reset Limit		CP31 & 32	0.3500		24 psi
FRC-101	Prop. Band	0-90° Rot. 0-4000 gpm	CP28	0.1900	$K_{3v3}/2$	24%
	Reset		CQ25	0.5997	K_{1v3}	36 repeats/min
FRC-100	Prop. Band	0-90° Rot. 0-4000 gpm	CP08	0.7085	$K_{3v6}/2$	89%
	Reset		CQ05	0.249	K_{1v6}	15 reports/min
TRC-101	Prop. Band	0-403,000 lb/hr 400-1400°F	BQ45	0.020	$K_{3v7}/5000$	10%
	Reset		BQ46	0.500	$10 K_{1v7}$	3 repeats/min
	Reset Limit		BQ40 & 41	0.200		15 psi
FCp-101	Prop. Band	0-100% 0-100%	CQ45	0.100	$10 K_{3v10}$	1/2%
	Reset		CP43	0.050	K_{1v10}	3 repeats/min
TCp-100	Prop. Band	0-10,000 lb/hr at 70° R open 400-1400°F	CP48	0.657	$K_{3v11}/25,000$	88%
	Reset		CP45	0.020	$K_{1v11}/10$	12 repeats/min
	Reset Limit		CP41 & 42	0.023		4.38 psi
	Rate		CQ46	0.3968	$10/K_{2v11}$	0.42 min

This valve is questionable and special attention should be paid to the controller in the field

III. DYNAMIC ANALYSIS OF CONTROL OF STEAM GENERATOR SYSTEM

A. INTRODUCTION

This part of the report covers the analysis of the steam generator and the pressure, temperature and flow control of its associated feedwater, sodium and steam control elements. As in Part II, the system equations are derived and presented with the analog conversion criteria and implementation referred to in Appendix A and Nomenclature to Appendix D.

The same transient tests were conducted as had been made on the primary and IHX portions of the system, and on other transients associated solely with control of steam or feedwater. Since the tests may be conducted either with the IHX in the system or bypassed, the bypassed condition was assumed. This mode of operation required more stringent control capability, since an inherent function of the IHX is that of dampening the effects of any transient which originates in the primary system.

Since some reactors act as base load plants which do not have to absorb load transients, they require a different mode of system control than those required to adjust to rapid load transients. The SCTI has been constructed with the option of either "load-forcing" control for the base plant mode of operation or "load-following" control for absorption of load transients.

B. DESIGN SPECIFICATION FOR STEAM SIDE

The secondary, or steam side of the SCTI, is designed to dissipate the 35 Mw (minus loop losses) of heat supplied by the fossil-fired boiler in the primary loop. Secondary sodium is supplied at up to 1175°F to generate steam at 1050°F and 2200 psia. Also capability of absorbing the following transients is required.

- 1) A load-following transient with a simulated step change in power demand. The system must be capable of taking up to a 25% increase or decrease in steam flow demand at maximum throttle valve speed with a constant sodium inlet temperature and a maximum steam pressure transient of ± 200 psi.

- 2) A load-following transient in which a power demand ramp occurs with no malfunction of the control system. The system must take up to a 10% minimum increase or decrease in steam flow demand at constant primary sodium inlet temperature between 20% and 100% rated power.
- 3) A load-following transient under simulated turbine trip conditions with no malfunction of the control system. This requires that steam flow be reduced to zero at maximum throttle valve closing rate.
- 4) A load-forcing transient in which a scram is simulated (by burner flameout) without flow reduction. This will require a decrease of primary sodium inlet temperature to the intermediate heat exchanger at $-35^{\circ}\text{F}/\text{sec}$ for 10 sec at full flow.
- 5) A load-forcing transient with a power increase that requires an increase of the primary sodium inlet temperature to the intermediate heat exchanger at $+11^{\circ}\text{F}/\text{sec}$ for 9 sec at full flow.
- 6) A load-forcing normal scram condition transient requiring a reduction to 10% of flow in 12 sec at constant intermediate heat exchanger inlet temperature.
- 7) A load-forcing transient with a normal scram in the primary loop without secondary flow reduction. This entails a decrease in primary flow to the intermediate heat exchanger of 100% to 20% in 12 sec at constant inlet temperature with secondary sodium flow constant at 100%.

C. OPERATION UNDER STEADY-STATE AND TRANSIENT CONDITIONS

Necessary controls for the steady-state and transient conditions imposed on the steam side are effected as follows.

- 1) The steam flow rate is controlled by a pressure control valve (PRC200).
 - a) For the load-following mode of control, the feedwater flow rate is controlled by pump speed using a standard 3-element control system. The three elements are steam flow, feedwater flow,

and stream pressure in lieu of an unavailable feedwater level signal, which is usually the third element.

- b) For the load-forcing mode of control, the feedwater flow rate is controlled by varying pump speed. Power signals with temperature trim from the secondary sodium loop, steam pressure signals, and steam flow signals are summed and transmitted to a feedwater pump speed controller simultaneously with a feedwater flow signal. Reset action takes place in the speed controller when the measured flow signal approaches the demand flow signal.
- c) For the load-following mode of control, secondary sodium flow is controlled with a butterfly valve operating on signals from a flow recorder controller. A sodium demand flow signal is generated in FRC 106 steam flow signals and trimmed from steam temperature or may be overridden by a hand-control station. The demand flow signal is compared with a measured flow signal to generate the valve signal. Reset action occurs when the measured flow signal approaches the demand flow signal.
- d) For the load-forcing mode of control, secondary flow is slaved to primary.

D. DERIVATION OF SYSTEM EQUATIONS

Equations were written describing controlling actions of components described in Part C with transient effects described in Part B of this introduction. Analog equations were then written, and the program was run to obtain the desired control settings.

The conversion of subcooled water to superheat steam is effected in three primary stages described in the model by the preheat, boiling, and superheat regions. A detailed description of the entire conversion process was not attempted due to the lack of detailed heat transfer data. However, a detailed description would not be necessary until general response characteristics have been evaluated.

The thermodynamics of the steam generator are based on a set of heat balance equations for each of the three regions used to describe the steam

generator. The axial length, or the heat transfer area, for each region is variable and determined by the boundary conditions defining each region.

A major assumption made in preparing the model was that of constant saturation temperature over the length of the boiling region. At full load and flow conditions the design pressure drop through this region is 6.2 psia, giving a change in saturation temperature from one end of the region to the other of 0.39°F when operating in the region of 2200 psia. Because of the relative flatness of the saturation temperature curve at high pressures as shown in Figure 14, this assumption is considered reasonable down to operating pressures of 500 psia.

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Figure 14. Saturation Temperature-Pressure Characteristics

Another major assumption is that of negligible flashing or condensing of steam in the boiling region during transients. Calculations indicate these effects contribute a negligible error in the results, due to the high operating pressure.

The equations representing the steam generator thermal kinetics, feedwater and secondary sodium loop hydraulics, pump dynamics, and steam generator controls are derived from the schematic shown in Figure 15. System schematics representing the control arrangements to be studied for different design transients are shown in Figures 16, 17, and 18. Figures 16 and 17 illustrate load-following control; and Figure 18, load-forcing control.

E. STEAM GENERATOR THERMAL KINETICS

The thermal kinetics of the once-through heat exchanger are based on heat balances for each axial phase region of the steam generator. This division into preheat, boiling, and superheat regions permits an independent description of the heat transfer mechanism for each region. A simplified description of heat transfer in the boiling and preheat regions has been used on the basis of available data from the steam generator design.

Heat balances for each axial region are described for a 3-node radial heat flow model to describe the transfer of heat from the sodium through the tube wall to the water or steam. The length of each axial region, or the heat transfer area for each region, is a variable in the heat balance equations, and is determined by the boundary conditions defining the region.

The assumptions used in arriving at the equations are indicated below in the development for each region.

1. Preheat Region

a. Assumptions

- 1) Feedwater inlet temperature = 600°F
- 2) Feedwater outlet temperature = 650°F
- 3) Average feedwater temperature = 625°F
- 4) Sub-cooled nucleate boiling exists over the entire region in steady-state and transient operations.
- 5) Heat transfer coefficient $\sim \Delta T_p^2$.

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Figure 15. Control Schematic

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Figure 16. Steam Flow Transients (1) and (3)

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Figure 17. Steam Flow Transient (2)

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Figure 18. Load-Forcing Transients

b. Model

c. Heat Balances

1) Sodium side

Heat stored = input heat - output heat

$$A_n \rho_n C_n X_p \frac{d\bar{T}_{np}}{dt} = W_n C_n (T_{np} - T_{no}) - h_{np} P_{np} X_p (\bar{T}_{np} - T_{wnp})^*$$

2) Tube Wall

heat stored = input heat - output heat

$$A_w \rho_w C_w X_p \frac{d\bar{T}_{wp}}{dt} = h_{np} P_{np} X_p (\bar{T}_{np} - T_{wnp}) - h_{sp} P_{sp} X_p (T_{wsp} - \bar{T}_w)$$

3) Water Side

input heat = output heat

$$h_{sp} P_{sp} X_p (T_{wsp} - \bar{T}_w) = W_s C_s (T_s - T_w)$$

No heat storage term appears as all the heat transferred is used in raising the water to saturation temperature. Variations in heat transfer rate, $(h_{sp} P_{sp} (T_{wsp} - T_w))$ or feedwater flow (W_s) are compensated for by variations in X_p to maintain the heat balance. Additional fixed relationships are:

$$T_{wp} = \frac{T_{wnp'} + T_{wsp}}{2}$$

$$\frac{2\pi k_w X_p}{L_n (R_o/R_l)} (T_{wnp} - T_{wsp}) = h_{sp} P_{sp} X_p (T_{wsp} - \bar{T}_w)$$

$$\Delta T_p = (T_{wsp} - \bar{T}_w)$$

defining the tube wall temperature profile and heat flux.

The heat balances for the preheat region developed above are typical for all three heat transfer regions. The length of the preheat (X_p) and boiling (X_b) regions are based on the water side balance for these regions with the remainder of the steam generator becoming the superheat region. These equations are not shown here, but are contained in the Summary in Appendix A. The assumptions used for the boiling and superheat regions are presented below.

2. Boiling Region

a. Assumptions

- 1) Water inlet temperature = 650°F
- 2) Steam outlet temperature = 650°F
- 3) Average steam/water temperature = 650°F
- 4) Steam quality at region outlet = 100%
- 5) Heat transfer on steam/water side consists of convective and boiling components acting in parallel.
- 6) Convective heat transfer $\sim W_s^{0.8}$
- 7) Boiling heat transfer $\sim \Delta T_b^2$

8) Negligible compressibility of steam portion of two-phase mixture during pressure transients.

9) Negligible flashing and condensing during pressure transients.

3. Superheat Region

a. Assumptions

1) 100% quality steam at 650°F at entry

2) Convective heat transfer $\sim W_s^{0.8}$

F. HEAT TRANSFER COEFFICIENTS

Heat transfer coefficients on the sodium and steam side were based on data from steam generator design calculations¹ at full load conditions. These calculations indicated subcooled nucleate boiling heat transfer over the length of the preheat region, so that a single heat transfer mechanism could be assumed near full load conditions. For the transients to be studied, this assumption may predict high heat transfer rates as the preheat region is expected to be in a combination of convective and boiling heat transfer at reduced load and flow conditions. The use of more than one heat transfer mechanism in this region would further complicate the model, so that the addition of another node to this region has been delayed for future studies. An elaboration upon this fact has been made in Appendix C by Reiner Schmidt of Interatom.

The boiling region heat transfer consists of a combination of convective and boiling heat transfer acting in parallel over the entire boiling region. This assumption is in keeping with the process described in the design calculations, and considerably simplifies the description of the boiling process. This simplification is necessary however without detailed heat transfer data for each definable stage in converting saturated water to 100% quality steam in its passage through a heated tube.

The superheat region is considered the most adequately described due to the small variation in heat transfer coefficient over the length of the superheat region.

The heat transfer characteristics for the steam generator given in the design calculations were as follows.

1. Sodium Side

$$N_u = 0.212 \left(D_e R_e P_r \right)^{0.6}$$

or

$$h_n = h_{no} \left(\frac{V_n}{V_{no}} \right)^{0.6} = h_{no} \left(\frac{W_n}{W_{no}} \right)^{0.6}$$

This relationship is identical for all three regions of the steam generator, as convective heat transfer exists over the entire length of the steam generator on the sodium side.

2. Water/Steam Side

a. Preheat Region

$$h_{sp} = C \left(\Delta T_p \right)^2$$

or

$$h_{sp} = h_{spo} \left(\frac{\Delta T_p}{\Delta T_{po}} \right)^2$$

b. Boiling Region

$$h_{sb} = h_c + h_b$$

$$h_c = 0.019 C_w G_t R_e^{-0.2} P_r^{-2/3}$$

$$= 0.019 C_w \left(\frac{V}{A} \right) \left(\frac{D_e V \rho}{\mu} \right)^{-0.2} \left(\frac{C_p \mu}{k} \right)^{-2/3} = K_c V^{0.8}$$

or

$$h_c = h_{co} \left(\frac{V_s}{V_{so}} \right)^{0.8} = h_{co} \left(\frac{W_s}{W_{so}} \right)^{0.8}$$

$$h_b = C (\Delta T_b)^2$$

or

$$h_b = h_{bo} \left(\frac{\Delta T_b}{\Delta T_{bo}} \right)^2$$

and

$$h_{sb} = h_{co} \left(\frac{W_s}{W_{so}} \right)^{0.8} + h_{bo} \left(\frac{\Delta T_b}{\Delta T_{bo}} \right)^2$$

G. SUMMARY OF STEAM GENERATOR HEAT BALANCES

1. Superheat Region

$$a. \dot{\bar{T}}_{ns} = 0.2116 \left[\frac{W_n/W_n(0)}{X_s/X_t} \right] (\bar{T}_{ns} - \bar{T}_{ns}) - 2.008 \left(\frac{W_n}{W_n(0)} \right)^{0.6} (\bar{T}_{ns} - T_{wns})$$

$$b. \dot{\bar{T}}_{ws} = 2.850 \left(\frac{W_n}{W_n(0)} \right)^{0.6} (\bar{T}_{ns} - T_{wns}) - 0.2994 \left(\frac{W_s}{W_s(0)} \right)^{0.8} (T_{wss} - \bar{T}_s)$$

$$c. \dot{\bar{T}}_s = 9.465 \left(\frac{W_s}{W_s(0)} \right)^{0.8} (T_{wss} - \bar{T}_s) - 4.086 \left[\frac{W_s/W_s(0)}{X_s/X_t} \right] (\bar{T}_s - T_s)$$

$$d. T_{wns} = 2 \bar{T}_{ws} - T_{wss}$$

$$e. \quad T_{wss} = \frac{T_{wns} + 0.242 \left(W_s / W_s(0) \right)^{0.8} \bar{T}_s}{1 + 0.242 \left(W_s / W_s(0) \right)^{0.8}}$$

$$f. \quad \frac{X_s}{X_t} = 1 - \frac{X_b}{X_t} - \frac{X_p}{X_t}$$

2. Boiling Region

$$a. \quad \dot{\bar{T}}_{nb} = 0.2116 \left[\frac{W_n / W_n(0)}{X_b / X_t} \right] (\bar{T}_{nb} - \bar{T}_{nb}) - 2.002 \left(\frac{W_n}{W_n(0)} \right)^{0.6} (\bar{T}_{nb} - T_{wnb})$$

$$b. \quad \dot{\bar{T}}_{wb} = 2.850 \left(\frac{W_n}{W_n(0)} \right)^{0.6} (\bar{T}_{nb} - T_{wnb}) \\ - 9.473 \left[0.169 \left(\frac{W_s}{W_s(0)} \right)^{0.8} + 0.831 \left(\frac{\Delta T_b}{\Delta T_b(0)} \right)^2 \right] (T_{wsb} - T_s)$$

$$c. \quad \frac{X_b}{X_t} = \frac{3.005 \left(W_s / W_s(0) \right)}{\left[0.169 \left(\frac{W_s}{W_s(0)} \right)^{0.8} + 0.831 \left(\frac{\Delta T_b}{\Delta T_b(0)} \right)^2 \right] (T_{wsb} - T_s)}$$

$$d. \quad T_{wnb} = 2 \bar{T}_{wb} - T_{wsb}$$

$$e. \quad T_{wsb} = \frac{T_{wnb} + 11.38 \left[0.169 \left(\frac{W_s}{W_s(0)} \right)^{0.8} + 0.831 \left(\frac{\Delta T_b}{\Delta T_b(0)} \right)^2 \right] T_s}{1 + 11.38 \left[0.169 \left(\frac{W_s}{W_s(0)} \right)^{0.8} + 0.831 \left(\frac{\Delta T_b}{\Delta T_b(0)} \right)^2 \right]}$$

$$f. \quad T_{nb} = 2 \bar{T}_{ns} - T_{ns}$$

$$g. \quad \Delta T_b = T_{wsb} - T_s$$

3. Preheat Region

$$a. \dot{\bar{T}}_{np} = 0.2116 \left[\frac{W_n / W_n(0)}{X_p / X_t} \right] (\bar{T}_{np} - \bar{T}_{np}) - 1.977 \left(\frac{W_n}{W_n(0)} \right)^{0.6} (\bar{T}_{np} - T_{wnp})$$

$$b. \dot{\bar{T}}_{wp} = 2.850 \left(\frac{W_n}{W_n(0)} \right)^{0.6} (\bar{T}_{np} - T_{wnp}) - 6.061 \left(\frac{\Delta T_p}{\Delta T_p(0)} \right)^2 (T_{wsp} - \bar{T}_w)$$

$$c. \frac{X_p}{X_t} = \frac{0.9311 (W_s / W_s(0))}{(\Delta T_p / \Delta T_p(0))^2 (T_{wsp} - \bar{T}_w)}$$

$$d. T_{wnp} = 2 \bar{T}_{wp} - T_{wsp}$$

$$e. T_{wsp} = \frac{T_{wnp} + 8.62 \left(\Delta T_p / \Delta T_p(0) \right)^2 \bar{T}_w}{1 + 8.62 \left(\Delta T_p / \Delta T_p(0) \right)^2}$$

$$f. T_{np} = 2 \bar{T}_{nb} - T_{nb}$$

$$g. T_{no} = 2 \bar{T}_{np} - T_{np}$$

$$h. \Delta T_p = T_{wsp} - \bar{T}_w$$

The model represented by the equations is considered to be a reasonable first approximation of the system; however, several features of secondary interest should be included in further studies. Some of these are as follows.

- 1) A saturation temperature variable with absolute pressure in the boiling region
- 2) A further breakdown of the various heat transfer mechanisms in the boiling and preheat regions

- 3) The substitution of actual instrumentation amplifiers, controllers and valve operators in the simulation to accurately describe the dynamics of these components
- 4) The addition of flashing and condensing effects due to region boundary changes during transients
- 5) The addition of steam compressibility effects
- 6) The addition of preheaters, pressure reduction valves, and boost pump dynamics in the feedwater system for evaluation of interactions between controllers
- 7) The addition of a complete secondary loop description to evaluate interaction between secondary and feedwater loop controllers

Some of these improvements are incorporated in the model described in Appendix C.

c. Superheat Region

$$Nu = 0.019 Re^{0.8} Pr^{0.4}$$

$$h_{ss} = 0.019 \frac{k}{De} \left(\frac{De V \rho}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.4}$$

or

$$h_{ss} = h_{ss} \circ \left(\frac{V}{V_o} \right)^{0.8} = h_{sso} \left(\frac{W_s}{W_{so}} \right)^{0.8}$$

3. Full Load Heat Transfer Coefficients

$$h_{no} = 4020 \text{ Btu/hr-ft}^2\text{°F}$$

$$h_{spo} = 8500 \text{ Btu/hr-ft}^2\text{°F}$$

$$h_{co} = 2250 \text{ Btu/hr-ft}^2\text{°F}$$

$$h_{bo} = 11,100 \text{ Btu/hr-ft}^2\text{°F}$$

$$h_{sso} = 422 \text{ Btu/hr-ft}^2\text{°F}$$

H. FEEDWATER HYDRAULICS

In developing a mathematical representation of the feedwater loop hydraulics, several simplifications were made to isolate the effects of the controlled components of interest. From Figure 19, which is a line diagram of the complete feedwater system, the system has been reduced to the following:

consisting of the steam generator (ΔP_b), first pressure reduction valve (ΔP_{v200}), feedwater flow control valve (ΔP_{v201}), feedwater pump (ΔP_p), and the lumped remainder of the feedwater loop (ΔP_l). Because this remainder consists of several components with independent controllers, the probable interaction of these controllers with the steam generator controllers is eliminated from this stage of the study for purposes of simplification. With approximate steam generator controller settings to be obtained from the proposed study, the addition of these components and controllers to the model could be meaningful in later studies, but would unnecessarily mask the effects of interest at this time.

1. Flow Dynamics

In a given pipe or flow channel,

$$F = \Delta P A = M a = M \frac{dv}{dt}$$

or

$$\frac{dv}{dt} = \frac{\Delta P A}{M}$$

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Figure 19. Steam and Feedwater
P & I Diagram, SCTI

but

$$V = \frac{W}{\rho A} \text{ , and } \frac{dV}{dt} = \frac{1}{\rho A} \frac{dW}{dt}$$

so that

$$\frac{1}{\rho A} \frac{dW}{dt} = \frac{\Delta P A}{M} = \frac{\Delta P A}{\frac{L A \rho}{g}} = \frac{g \Delta P}{L \rho}$$

$$\frac{dW}{dt} = \frac{\rho A g \Delta P}{L \rho} = \frac{g}{\frac{L}{A}} \Delta P$$

where

F = force

ΔP = pressure drop per unit area

A = flow cross-sectional area

M = fluid mass

V = fluid velocity

W = weight flow rate

ρ = fluid density

L = length of pipe or channel

g = acceleration of gravity,

the sum of all L/A for the loop was calculated from preliminary layouts to be

$$\sum \left(\frac{L}{A} \right) = 3265 \text{ ft}^{-1} \text{ .}$$

Normalizing flow rate to full-flow conditions,

$$W_{so} = 1.16 \times 10^5 \text{ lbs/hr} = 32.2 \text{ lb/sec}$$

$$\frac{d \left(W_s / W_{so} \right)}{dt} = 0.0441 \Delta P \quad (\Delta P \text{ in } P_{sl})$$

From the diagram,

$$\Delta P = \Delta P_p - \Delta P_{v201} - \Delta P_b - \Delta P_{v200} - \Delta P_\ell \quad .$$

2. Pump Head Characteristics

From feedwater pump head curves, the characteristic equation describing head as a function of pump speed and fluid flow is

$$\Delta P_p = K_1 N^2 - K_2 W_s^2$$

Normalizing to full flow and speed conditions,

$$\frac{\Delta P_p}{\Delta P_{po}} = K_1 \left(\frac{N}{N_o} \right)^2 - K_2 \left(\frac{W_s}{W_{so}} \right)^2 \quad .$$

From steady state conditions,

$$K_1 = 1 + K_2 \quad ,$$

and from pump performance curves,

$$K_2 = 0.33 \quad .$$

So that,

$$\frac{\Delta P_p}{\Delta P_{po}} = 1.33 \left(\frac{N}{N_o} \right)^2 - 0.33 \left(\frac{W_s}{W_{so}} \right)^2 \quad .$$

3. Feedwater Valve Characteristics

The feedwater valve (FRC-201V) characteristics are approximated by a Fisher 3-in. double-ported throttle plug valve³ shown on Figure 20⁴ for the valve coefficient as a function of valve travel.

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Figure 20. Feedwater Valve (FRC-201V) Characteristics

From the valve pressure drop and flow relationship given for this type valve,⁵

$$\Delta P_{v201} = \frac{W_s^2}{C_{v201}^2}$$

and from the approximate expression shown on Figure 20 for the valve coefficient as a function of valve position,

$$C_{v201} = 109 \left(\frac{X_{v201}}{X_{v201(t)}} \right)^2,$$

an expression for the pressure drop ratio as a function of valve position is developed.

Nondimensionalizing both relationships:

$$\frac{\Delta P_{v201}}{\Delta P_{v201_o}} = \left(\frac{W_s}{W_{so}} \right)^2 \left(\frac{C_{v201(t)}}{C_{v201}} \right)^2$$

$$\left(\frac{C_{v201(t)}}{C_{v201}} \right)^2 = \left(\frac{X_{v201(t)}}{X_{v201}} \right)^4$$

so that

$$\frac{\Delta P_{v201}}{\Delta P_{v201_o}} = \left(\frac{W_s}{W_{so}} \right)^2 \left(\frac{X_{v201(t)}}{X_{v201}} \right)^4$$

where

$X_{v201(t)}$ = total valve travel.

4. Pressure Reduction Valve Characteristics

The first steam pressure-reduction valve (PRC-200V) is the only valve of the three pressure-reduction stations considered in this study, and is approximated by a Fisher single-ported throttle plug valve³ with valve coefficient characteristics falling between a 2-1/2- and 3-in. valve⁶ as shown in Figure 21. The pressure drop to flow relationship for this type valve is:⁷

$$(2.32 \Delta P_{v200})^{0.44} = \frac{W_s (1 + 0.00065 \Delta T_{sh})}{C_{v200} P_{sh}^{0.56}}$$

where

ΔT_{sh} = °F superheat of steam at valve inlet.

To simplify this expression, the following approximations were made:

$$(2.32 \Delta P_{v200})^{0.44} = (K_x \Delta P)^{0.5}$$

$$P_{sh}^{0.56} = (K_y P_{sh})^{0.5}$$

$$1 + 0.00065 \Delta T_{sh} = K_t \text{ (which assumes almost constant superheat steam)}$$

Evaluating the K's at full load conditions,

$$K_x = 1.51$$

$$K_y = 2.45$$

$$K_t = 1.261$$

$$(1.51 \Delta P_{v200})^{0.5} = \frac{W_s K_t}{C_{v200} (2.45 P_{sh})^{0.5}}$$

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Figure 21. Pressure Control Valve (PRC-200 V) Characteristics

From Figure 21, the valve coefficient is approximated by

$$C_{v200} = 285 \left(\frac{X_{v200}}{X_{v200}(t)} \right)^2$$

so that squaring the pressure drop equation, and substituting for the valve coefficient,

$$\Delta P_{v200} = K_{v200} \frac{W_5^2}{C_{v200}^2} .$$

Normalizing

$$\frac{\Delta P_{v200}}{\Delta P_{v200}(t)} = \left(\frac{W_s}{W_{so}} \right)^2 \left(\frac{X_{v200}(t)}{X_{v200}} \right)^4$$

where

$X_{v200}(t)$ = total valve travel.

5. Steam Generator Pressure Drop

The phase changes occurring on the steam side of the generator give rise to a large variation in flow velocity resulting in pressure drop variations dependent on heat transfer rates over the length of the generator tubes. Interaction between the flow and heat transfer dynamics is introduced through the variable region lengths, or areas. Because each region contains fluid in a different phase, velocity and friction factors are different, and are accounted for by considering pressure drop dynamics for each region, independently.

The forces acting on a typical element of coolant in a generator tube are

$$F = F_x + F_f + F_a$$

$$F_x = A \Delta X \bar{\rho}$$

$$F_f = \pi D \Delta X k_f \bar{V}^2$$

$$F_a = M_a = \frac{d(M\bar{V})}{dt}$$

$$= M \frac{d\bar{V}}{dt} + \bar{V} \frac{dM}{dt} \quad ,$$

where

F = total force,

F_x = static head force,

F_f = flow friction,

F_a = flow acceleration,

$\bar{\rho}$ = average density,

$\bar{V}$ = average velocity, and

k_f = flow friction coefficient.

From the continuity equation,

$$\overline{W} = \overline{\rho} A \overline{V}$$

$$\overline{V} \frac{dM}{dt} = \frac{\overline{W}}{A\overline{\rho}} \left(\frac{1}{g} dW \right) = \left(\frac{\overline{W}^2}{Ag} \right) d \left(\frac{1}{\overline{\rho}} \right) ,$$

which is the steady-state force of acceleration due to the density gradient along the flow path.

The total acceleration force is:

$$F_a = M \frac{d\overline{V}}{dt} + \overline{V} \frac{dM}{dt} = \left(\frac{A\Delta X \overline{\rho}}{g} \right) \frac{d\overline{V}}{dt} + \frac{(\overline{A\overline{\rho}\overline{V}})^2}{Ag} \left(\frac{1}{\rho_o} - \frac{1}{\rho_{in}} \right)$$

Summing all the forces acting on the element,

$$F = A\Delta X \overline{\rho} + \pi D \Delta X k_f \overline{V}^2 + \left(\frac{A\Delta X \overline{\rho}}{g} \right) \frac{d\overline{V}}{dt} + \frac{(\overline{A\overline{\rho}\overline{V}})^2}{Ag} \left(\frac{1}{C_o} - \frac{1}{C_{in}} \right)$$

Dividing by the flow area to obtain the external pressure differential:

$$\frac{F}{A} = \Delta P = \Delta X \overline{\rho} + \left(\frac{4\Delta X k_f}{D} \right) \overline{V}^2 + \left(\frac{\Delta X \overline{\rho}}{g} \right) \frac{d\overline{V}}{dt} + \frac{(\overline{\rho} \overline{V})^2}{g} \left(\frac{1}{\rho_o} - \frac{1}{\rho_{in}} \right)$$

Driving Force = Static Head + Friction + Transient Acceleration
+ Steady State Acceleration

The density terms in the equations for the preheat and boiling regions are constants because the region lengths are determined on the basis of constant boundary temperature. The only variables in these equations are then the region lengths and the average fluid velocity. The transient acceleration term is not used in the calculation of generator pressure drop because this effect is already included in the loop inertia determined in III - A above.

6. Feedwater Loop Pressure Drop

The pressure drop in the remaining components of the feedwater loop has been arbitrarily made a function of flow to simplify this portion of the study and permit an isolated study of steam generator dynamics and controls. It will be noted from Figure 22 that this is a major simplification in the overall dynamics of the system.

$$\Delta P_l = K_l W_s^2$$

And in normalized form:

$$\frac{\Delta P_l}{\Delta P_{l0}} = \left(\frac{W_s}{W_{s0}} \right)^2$$

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Figure 22. Feedwater Hydraulics

7. Pump Speed Dynamics

The feedwater pump for the SCTI is a variable speed pump used to control feedwater flow in conjunction with the feedwater flow control valve. Course control of flow will be performed through pump outlet pressure control and fine, or trim, control with the valve operating to match flow with flow demand.

The acceleration torque applied to the pump impeller is as follows.

$$F = \Delta P A = M_a = M \frac{dV}{dt}$$

$$\Delta P = \Delta P_p - \Delta P_{v201} - \Delta P_b - \Delta P_{v200} - \Delta P_\ell$$

In a given length of pipe or flow channel,

$$M = \frac{L A \rho}{g}$$

$$V \equiv \frac{W}{\rho A} \quad \text{and,} \quad \frac{dV}{dt} \equiv \frac{1}{\rho A} \frac{dW}{dt}$$

From the first equation above,

$$\frac{dV}{dt} = \frac{\Delta P A}{M}$$

$$\therefore \frac{1}{\rho A} \frac{dW}{dt} = \frac{\Delta P A}{M} = \frac{\Delta P A}{\frac{L A \rho}{g}} = \frac{g \Delta P}{L \rho} = \frac{g \Delta P}{L \rho}$$

and,

$$\frac{dW}{dt} = \frac{A g \Delta P}{L} = \left(\frac{g}{\frac{L}{A}} \right) \Delta P$$

$$\frac{L}{A} = \sum \left[\left(\frac{L}{A} \right)_\ell + \left(\frac{L}{A} \right)_{hph} + \left(\frac{L}{A} \right)_b + \left(\frac{L}{A} \right)_{\ell ph} \right] \quad .$$

With

T_m = motor torque on impeller,

T_i = fluid torque on impeller,

I = motor and impeller inertia,

η = impeller efficiency, and

ΔX_{st} = speed control scoop tube travel,

$$T = T_m - T_i = I \frac{dN}{dt}$$

$$T_m = T_{mo} + K_a \Delta X_{st}$$

$$T_i = \frac{\Delta P_p W_s}{\eta N \rho}$$

so that,

$$\frac{dN}{dt} = \frac{1}{I} (T_m - T_i) = \frac{1}{I} \left(T_{mo} + K_a \Delta X_{st} - \frac{\Delta P_p W_s}{\eta N \rho} \right) .$$

In non-dimensional form,

$$\begin{aligned} \frac{d(N/N_o)}{dt} = \frac{1}{N_o I} \left[T_{mo} + K'_a \left(\frac{\Delta X_{st}}{\Delta X_{st_o}} \right) \right. \\ \left. - \left(\frac{\Delta P_{po} W_{so}}{N_o \rho} \right) \frac{(\Delta P_p / \Delta P_{po}) (W_s / W_{so})}{\eta (N/N_o)} \right] . \end{aligned}$$

From pump characteristic curves the impeller efficiency is

$$\eta = f(N, W_s) .$$

At a given speed, the shape of the efficiency curve as a function of flow is non-linear. With speed changes, the efficiency curve is shifted along the flow axis in a manner that can be represented by the following.

$$W_{sp} = W_{so} + K_w \left(\frac{N}{N_o} - 1 \right) ,$$

where

$$W_{sp} = \text{peak efficiency flow rate.}$$

Normalizing,

$$\frac{W_{sp}}{W_{so}} = 1 + \frac{K_w}{W_{so}} \left(\frac{N}{N_o} - 1 \right) .$$

From pump curves,

$$\frac{K_w}{W_{so}} \approx 1.0 , \text{ so that}$$

$$\frac{W_{sp}}{W_{so}} = 1 + \left(\frac{N}{N_o} - 1 \right) = \frac{N}{N_o} .$$

The ratio of peak efficiency flow to reference flow is directly proportional to the ratio of pump speed to reference pump speed.

At a given pump speed,

$$\eta = f \left(\frac{W_s}{W_{sp}} \right) = f \left(\frac{W_s / W_{so}}{W_{sp} / W_{so}} \right) = f \left(\frac{W_s / W_{xo}}{N / N_o} \right) .$$

This function is shown in Figure 23.

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Figure 23. Feedwater Pump Efficiency

I. SUMMARY OF FEEDWATER HYDRAULICS

1. Feedwater Pump Head

$$\frac{\Delta P_p}{\Delta P_p(o)} = 1.33 \left(\frac{N}{N(o)} \right)^2 - 0.33 \left(\frac{W_s}{W_{s(o)}} \right)^2$$

2. Feedwater Loop Pressure Drop

Feedwater control valve

$$a. \frac{\Delta P_{v201}}{\Delta P_{p(o)}} = 0.0137 \left[\frac{\left(W_s / W_{s(o)} \right)^2}{\left(X_{v201} / X_{v201(o)} \right)^4} \right]$$

Boiler

$$b. \frac{\Delta P_b}{\Delta P_{p(o)}} = 0.0039 \left(\frac{X_p}{X_t} \right) + 0.0021 \left(\frac{X_b}{X_t} \right) + 0.0004 \left(\frac{X_s}{X_t} \right) \\ + \left[0.0060 \left(\frac{X_p}{X_t} \right) + 0.0090 \left(\frac{X_b}{X_t} \right) + 0.0212 \left(\frac{X_s}{X_t} \right) + 0.0016 \right] \left(\frac{W_s}{W_{s(o)}} \right)^2 \\ - 0.0037$$

Pressure control valve

$$c. \frac{\Delta P_{v200}}{\Delta P_{p(o)}} = 0.1825 \left[\frac{\left(W_s / W_{s(o)} \right)^2}{\left(X_{v200} / X_{v200(o)} \right)^4} \right]$$

Loop fixed components and piping

$$d. \frac{\Delta P_c}{\Delta P_{p(o)}} = 0.7894 \left(\frac{W_s}{W_{s(o)}} \right)^2$$

$$e. \frac{\Delta P_\ell}{\Delta P_{p(o)}} = \frac{\Delta P_{v201}}{\Delta P_{p(o)}} + \frac{\Delta P_b}{\Delta P_{p(o)}} + \frac{\Delta P_{v200}}{\Delta P_{p(o)}} + \frac{\Delta P_c}{\Delta P_{p(o)}}$$

3. Feedwater Flow

$$\left(\frac{\dot{W}_s}{W_{s(o)}} \right) = 94.39 \left(\frac{\Delta P_p}{\Delta P_{p(o)}} - \frac{\Delta P_l}{\Delta P_{p(o)}} \right)$$

4. Steam Pressure

$$P_{sh} = 99 + 2190 \left[\frac{\Delta P_p}{\Delta P_{p(o)}} - \frac{\Delta P_{v201}}{\Delta P_{p(o)}} - \frac{\Delta P_b}{\Delta P_{p(o)}} \right]$$

5. Feedwater Pump Speed

$$a. \left(\frac{\dot{N}}{N_{(o)}} \right) = 0.0413 + 0.0826 \left(\frac{\Delta X_{st}}{X_{st}} \right) - 0.0276 \left[\frac{(\Delta P_p / \Delta P_{p(o)}) (W_s / W_{s(o)})}{\eta (N / N_{(o)})} \right]$$

$$b. \eta = f \frac{(W_s / W_{s(o)})}{(N / N_{(o)})}$$

6. Pump Outlet Pressure

$$P_p = 99 + 2190 \left(\frac{\Delta P_p}{\Delta P_{p(o)}} \right)$$

J. SECONDARY SODIUM LOOP HYDRAULICS

The secondary sodium loop is more accurately represented than the feed-water loop, due to single-phase fluid throughout the loop, and the fewer neglected components.

The loop is represented by:

which includes the effects of the tube side of the intermediate heat exchangers (ΔP_{ihx}), the shell side of the steam generator (ΔP_{bn}), the secondary loop pump (ΔP_{pn}), and the flow control valve (ΔP_{v106}). In normal plant operation, control of secondary loop sodium flow will be a function of primary loop flow; however, for purposes of this study, control and thermodynamic interaction between these loops are omitted so that the feedwater control can be investigated independently.

1. Flow Dynamics

The development of the flow dynamics is identical with that for the feedwater loop, so that,

$$\frac{dW_n}{dt} = g \left(\frac{A}{L} \right) \Delta P \quad ,$$

and the sum of all L/A calculated from preliminary piping and equipment layout is

$$\sum \left(\frac{L}{A} \right) = 144.5 \text{ ft}^{-1}$$

$$W_{no} = 855 \times 10^3 \text{ lb/hr} = 237.5 \text{ lb/sec}$$

and,

$$\begin{aligned} \frac{d(W_n/W_{no})}{dt} &= \frac{g \Delta P}{\sum (L/A) W_{no}} = \frac{(32.2 \text{ ft/sec}) (144 \text{ in.}^2/\text{ft}^2)}{(237.5 \text{ lb/sec}) (144.5/\text{ft})} \Delta P \\ &= 0.1352 \Delta P \end{aligned}$$

$$\Delta P = \Delta P_{pn} - \Delta P_{v106} - \Delta P_{ihx} - \Delta P_{bn} \quad .$$

2. Pump Head Characteristics

The secondary loop pump will be a constant speed pump³ so that pump head characteristics are a function of flow alone. From estimated pump performance curves,

$$\Delta P_{pn} = K_o - K_1 \left(\frac{W_n}{W_{no}} \right)$$

From pump data, the normalized head equation can be written as:

$$\frac{\Delta P_{pn}}{\Delta P_{pno}} = 1.138 - 0.138 \left(\frac{W_n}{W_{no}} \right)$$

3. Sodium Flow Control Valve

The secondary loop flow control valve (FRC-106V) characteristics are based on the data for an 8-in. Continental butterfly valve shown in Figure 4.

An approximation of the valve coefficient curve was taken as:

$$C_{v106} = 1210 \left(\frac{X_{v106}}{X_{v106(t)}} \right)^2$$

The pressure drop-flow relationship for this type of valve is:

$$\Delta P_{v106} = G \frac{W_n^2}{C_{v106}^2}$$

$$\frac{\Delta P_{v106}}{\Delta P_{v106(t)}} = \frac{\left(\frac{W_n}{W_{no}} \right)^2}{\left(\frac{C_{v106}}{C_{v106(t)}} \right)^2} = \frac{\left(\frac{W_n}{W_{no}} \right)^2}{\left(\frac{X_{v106}}{X_{v106(t)}} \right)^4}$$

From system constants, the dynamic sodium flow equation becomes:

$$\left(\frac{\dot{W}_n}{W_n(o)} \right) = 4.536 \left(\frac{\Delta P_n}{\Delta P_n(o)} - \frac{\Delta P_{ln}}{\Delta P_n(o)} \right) .$$

4. Loop Pressure Drop

The pressure drop around the remainder of the loop is a fixed relationship to flow as single phase sodium flows through the tubes of the intermediate heat exchanger and shell side of the steam generator, and the flow channel geometries are fixed.

Sodium flow control valve

$$a. \frac{\Delta P_{v106}}{\Delta P_n(o)} = 0.1865 \left[\frac{\left(W_n / W_n(o) \right)^2}{\left(X_{v106} / X_{v106(o)} \right)^4} \right]$$

IHX

$$b. \frac{\Delta P_{ihx}}{\Delta P_n(o)} = 0.2210 \left(\frac{W_n}{W_n(o)} \right)^2$$

Boiler

$$c. \frac{\Delta P_{bn}}{\Delta P_n(o)} = 0.3670 \left(\frac{W_n}{W_n(o)} \right)^2$$

Loop fixed components and piping

$$d. \frac{\Delta P_{cn}}{\Delta P_n(o)} = 0.2255 \left(\frac{W_n}{W_n(o)} \right)^2$$

$$e. \frac{\Delta P_{ln}}{\Delta P_n(o)} = \frac{\Delta P_{v106}}{\Delta P_n(o)} + \frac{\Delta P_{ihx}}{\Delta P_n(o)} + \frac{\Delta P_{bn}}{\Delta P_n(o)} + \frac{\Delta P_{cn}}{\Delta P_n(o)}$$

K. CONTROL SYSTEM

1. Instrumentation

The instrumentation used with the control system has been assumed to have first-order response characteristics throughout, and is represented by:

$$\frac{dX^*}{dt} = \frac{1}{\tau_x} (X - X^*) \quad ,$$

where

X = instantaneous parameter value

X^* = instrumentation channel output

τ_x = instrumentation channel time constant.

The parameters instrumented for control of the feedwater system are:

P_{sh} = superheat steam pressure at steam generator outlet

W_s = steam or feedwater flow

P_p = feedwater pump outlet pressure

W_n = secondary loop sodium flow

T_{no} = sodium temperature at steam generator outlet

X_{v200} = pressure reduction valve position.

2. Controllers

The controllers to be used in the feedwater control system consist of standard commercial type equipment containing 3-mode action. A general representation of this type of controller would be:

$$X = K_p \left[K_i \int V \, dt + V + K_2 \frac{dV}{dt} \right]$$

where

X = controller output

V = controller input

K_p = proportional gain - proportional control

K_i = interaction gain - reset control

K_r = rate gain - rate control

One of the objectives of the study to be conducted with this model will be to determine the need for the type of control to be used with each controller, and the approximate gains to be used in putting the test loop into operation. The most simple form of control will be investigated initially for most controllers, namely proportional and integral, and rate will be added as the need becomes apparent to avoid unnecessary control complication. In the case of the three valve controllers, the sodium temperature controller and the pump speed controllers, in the system, the need for integral control is inherent to provide smooth action, so that integral (reset) action will be included at the start.

In arriving at preliminary controller settings for the study, assumptions have been made for each controller on the degree of control action that may be desired for a given error condition.

a. PRC - 200

Assume that a 200-psi error in superheat steam pressure produces a full valve travel, and the instrumented pressure range is 3000 psi.

$$PB_{200} = \frac{2 (200)}{3000} (100) = 13.3\%$$

and,

$$K_{p200} = \frac{100}{PB_{200}} = \frac{100}{13.3} = 7.5$$

so that proportional control would be expressed by

$$\frac{\Delta X_{v200}}{X_{v200}(t)} = 7.5 \frac{\Delta P_{sh}}{P_{sho}}$$

where

$X_{v200}(t)$ = Full valve travel

P_{sho} = 3000 psi.

Arbitrarily assuming the integral gain to be:

$$K_i = 1 \frac{\text{repeat}}{\text{min}} = 0.0167 \frac{\text{repeats}}{\text{sec}}$$

then the controller equation becomes

$$\frac{\Delta X_{v200}}{X_{v200}(t)} = 7.5 \left[1 + 0.0167 \int dt \right] \left(\frac{\Delta P_{sh}}{P_{sho}} \right)$$

b. FRC - 201

Assume a 10-gpm feedwater flow error produces a full valve travel, and the instrumented flow range is 400 gpm.

$$PB_{201} = \frac{2(10)}{400} (100) = 5\%$$

$$K_{p201} = \frac{100}{5} = 20$$

and making the same assumption on integral gain as in a. above,

$$\frac{\Delta X_{v201}}{X_{v201}(t)} = 20 \left[1 + 0.0167 \int dt \right] \left(\frac{\Delta W_s}{W_{so}} \right)$$

where

$X_{v201}(t)$ = full valve travel

$W'_{so} = 400$ gpm.

c. FRC - 106

In this case, a 100-gpm sodium flow error is assumed to produce a full valve travel, and the instrumented flow range is 4000 gpm.

$$PB_{106} = \frac{2(100)}{4000}(100) = 5\%$$

and with the integral gain as before

$$\frac{\Delta X_{v106}}{X_{v106}(t)} = 20 \left[1 + 0.0167 \int dt \right] \left(\frac{\Delta W_n}{W'_{no}} \right) ,$$

where

$X_{v106}(t)$ = full valve travel

$W'_{no} = 400$ gpm.

d. TRC - 200

A 4°F error is assumed to require a 1% correction in feedwater flow with an instrumented range of 1500°F.

$$PB_{200}(t) = \left(\frac{2(4)}{1500} \right) \left(\frac{100}{1} \right) 100 = 53.3\%$$

$$K_{p200}(t) = \frac{100}{53.3} = 1.875$$

and with the 1 repeat/min integral gain assumed,

$$\frac{\Delta W_s}{W'_{so}} = 1.875 \left[1 + 0.0167 \int dt \right] \left(\frac{\Delta T_{no}}{T_{no}} \right)$$

where

$$W'_{so} = 400 \text{ gpm}$$

$$T_{no} = 1500^\circ\text{F}.$$

$$\text{e. } \underline{PC_p - 202}$$

Assuming a full range speed change for a 300-psi error at the pump outlet, and an instrumentation range of 3000 psi,

$$PB_{202} = \frac{2 (300)}{3000} (100) = 20$$

$$K_{p202} = \frac{100}{20} = 5,$$

so that

$$\frac{\Delta N'_o}{N_o} = 5 \left[1 + 0.0167 \int dt \right] \frac{\Delta P_p}{P_{po}}$$

$$\text{f. } \underline{PC_p - 203}$$

Assuming a 100% change in speed demand produces a full travel of the scoop tube,

$$PB_{203} = \frac{2 (100)}{100} (100) = 200$$

$$K_{p203} = \frac{100}{200} = 0.5$$

then

$$\frac{\Delta X_{st}}{X_{st}} = 0.5 \left[1 + 0.0167 \int dt \right] \frac{\Delta N_o}{N_o} .$$

3. Operators

The valve and scoop tube operators have been assumed to have first order response characteristics as a first approximation, and are represented by the same equation used for the instrumentation.

L. SUMMARY OF EQUATIONS FOR INSTRUMENTATION AND CONTROLLERS

1. Temperature

$$a. \dot{T}_{no}^* = \frac{1}{\tau_t} (T_{no} - T_{no}^*)$$

$$b. \dot{T}_{ni}^* = \frac{1}{\tau_t} (T_{ni} - T_{ni}^*)$$

$$c. \dot{T}_{sh} = \frac{1}{\tau_t} (T_{sh} - T_{sh}^*)$$

2. Flow

$$a. \left(\frac{\dot{W}_s}{W_{so}} \right)^* = \frac{1}{\tau_{ws}} \left[\left(\frac{W_s}{W_{so}} \right) - \left(\frac{W_s}{W_{so}} \right)^* \right]$$

$$b. \left(\frac{\dot{W}_s}{W_{no}} \right)^* = \frac{1}{\tau_{wn}} \left[\left(\frac{W_n}{W_{no}} \right) - \left(\frac{W_n}{W_{no}} \right)^* \right]$$

$$c. \left(\frac{\dot{W}}{W_o} \right)^* = \frac{1}{\tau_w} \left[\left(\frac{W}{W_o} \right) - \left(\frac{W}{W_o} \right)^* \right] .$$

3. Pressure

$$\dot{P}_{sh}^* = \frac{1}{\tau_{ps}} (P_{sh} - P_{sh}^*) ,$$

4. PRC - 200V

$$a. \left(\frac{X_{v200}}{X_{v200,o}} \right) = 0.600 + \left(\frac{\Delta X_{v200}}{X_{v200,o}} \right)^*$$

$$b. \left(\frac{\dot{\Delta X}_{v200}}{X_{v200,o}} \right)^* = \frac{1}{\tau_{200}} \left[\left(\frac{\Delta X_{v200}}{X_{v200,o}} \right) - \left(\frac{\Delta X_{v200}}{X_{v200,o}} \right)^* \right]$$

5. FRC - 106V

$$a. \left(\frac{X_{v106}}{X_{v106,o}} \right) = 1 + \left(\frac{\Delta X_{v106}}{X_{v106,o}} \right)^*$$

$$b. \left(\frac{\dot{\Delta X}_{v106}}{X_{v106,o}} \right) = \frac{1}{\tau_{106}} \left[\left(\frac{\Delta X_{v106}}{X_{v106,o}} \right) - \left(\frac{\Delta X_{v106}}{X_{v106,o}} \right)^* \right]$$

6. PRC - 200

$$a. \frac{\Delta P_{sh}^*}{P_{sh,o}} = \frac{P_{sh}^D}{P_{sh,o}} - \frac{P_{sh}^*}{P_{sh,o}}$$

$$b. \frac{\Delta X_{v200}^*}{X_{v200,o}} = \left(\frac{X_{v200}^D}{X_{v200,o}} \right) - \left(\frac{X_{v200}}{X_{v200,o}} \right)^*$$

$$c. \left(\frac{X_{v200}}{X_{v200,o}} \right) - \left(\frac{X_{v200}}{X_{v200,o}} \right)_{ic} = K_{14} \left(\frac{K_{24}^{**}}{S} + 1 \right) \left(\frac{\Delta P_{sh}^*}{P_{sh,o}} \right)$$

$$d. \left(\frac{X_{v200}}{X_{v200,o}} \right) - \left(\frac{X_{v200}}{X_{v200,o}} \right)_{ic} = K_{14} \left(\frac{K_{24}^*}{5} + 1 \right) \left(\frac{\Delta X_{v200}^*}{X_{v200,o}} \right)$$

7. FRC - 106

$$a. \left(\frac{\Delta W_n}{W_{no}} \right)^1 = \left(\frac{W_{nd}}{W_{no}} \right)^* - \left(\frac{W_n}{W_{no}} \right)^*$$

$$b. \left(\frac{X_{v106}}{X_{v106,0}} \right) - \left(\frac{X_{v106}}{X_{v106,o}} \right)_{ic} = K_{13} \left(\frac{K_{23}}{S} + 1 \right) \left(\frac{\Delta W_n}{W_{no}} \right)$$

$$c. \left(\frac{W_{nd}}{W_{no}} \right)^* = \left(\frac{W_{sd}}{W_{so}} \right) + \left(\frac{\Delta W_{nd}}{W_{no}} \right)$$

8. FRC - 200

$$a. \frac{\Delta P_{sh}^*}{P_{sho}} = \frac{P_{shd}}{P_{sho}} - \frac{P_{sh}^*}{P_{sho}}$$

$$b. \left(\frac{\Delta W_{sd}}{W_{so}} \right)_1 = -K_{12} \left(\frac{K_{22}}{S} + 1 + K_{32}S \right) \left(\frac{\Delta P_{sh}^*}{P_{sho}} \right)$$

9. TRC - 200

$$a. \Delta T_{sh}^* = T_{shd} - T_{sh}^*$$

$$b. \frac{\Delta W_{nd}}{W_{no}} = -K_{11} \left(\frac{K_{21}}{S} + 1 + K_{31}S \right) \Delta T_{sh}^*$$

10. TRC - 201

$$a. \Delta T_{no}^* = T_{nod} - T_{no}^*$$

$$b. \left(\frac{\Delta W_{sd}}{W_{so}} \right)_2 = -K_{15} \left(\frac{K_{25}}{S} + 1 + K_{35}S \right) \Delta T_{no}^*$$

11. $\frac{NC_p - 1}{}$

$$a. \frac{N_d}{N_o} = K_{16} \left(\frac{K_{26}}{S} + 1 \right) \left[\left(\frac{W_{sd}}{W_{so}} \right) - \left(\frac{W}{W_o} \right)^* \right]$$

$$b. \left(\frac{W_{sd}}{W_{so}} \right) = \left(\frac{W_{sd}}{W_{so}} \right) + \left(\frac{\Delta W_{sd}}{W_{so}} \right)_1 + \left(\frac{\Delta W_{sd}}{W_{so}} \right)_2$$

$$+ \left(\frac{K_{ts}}{1 + ts} \right) \frac{W_n^* (T_{ni}^* - T_{no}^*)}{W_{no} (T_{ni} - T_{no})_o}$$

$$c. \frac{W_{sd}}{W_{so}}, \text{ programmed by FS-12 for steam flow transient 1}$$

$$\frac{W_{sd}}{W_{so}} = \left(\frac{W_s}{W_{so}} \right)^*, \text{ for steam flow transient 2 and sodium flow transients}$$

M. CONTROLLER SETTINGS

The results of the various analog studies conducted on the control system were evaluated and the results transposed into a form suitable for setting the actual controllers. The recommended initial controller settings are tabulated and included.

The controller constants (proportional band, rate, reset, and reset limit) were determined on a basis which satisfactorily controlled the various transients required of the SCTI. Values existed as various potentiometer settings. It was then necessary to transpose these analog potentiometer settings into such a form that the results could be used to initially set the various controllers. Final adjustments and confirmation of the instrument ranges have been made in the field.

The controller equations which were used in the analog study along with the related calculations necessary to transpose the analog results were as follows:

1. PRC - 200

$$\Delta P_{sh} = P_{sho} - P_{sh}^*$$

$$\frac{X_{v200}}{X_{v200,o}} = \frac{1}{K_{3v200}} \left[K_{1v200} \int (\Delta P_{sh}) d\theta + [\Delta P_{sh}] + K_{2v200} \frac{d(\Delta P_{sh})}{d\theta} \right]$$

from analog results

$$\frac{X_{v200}}{X_{v200,o}} = \frac{\Delta P_{sh}}{K_{3v200}} \quad ;$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta P_{sh}}{\left(\frac{X_{v200}}{X_{v200,o}} \right)} = K_{3v200} = 5000 (0.20) = 1000$$

from controller

$$\text{input range} = 0-3000 \text{ psi}$$

$$\text{output range} = 0-1.0 \quad ;$$

then

$$\frac{\text{input}}{\text{output}} = \frac{3000}{1.0} = 3000$$

then

$$PB = \frac{1000 (100)}{3000} = 33\%$$

$$\text{Reset} = K_{1v200} = 0.100 \text{ resets/sec} = 6 \text{ resets/min}$$

2. FRC - 200

$$\Delta P_{sh} = P_{sho} - P_{sh}^*$$

$$\left(\frac{\Delta W_{sd}}{W_{so}} \right)_1 = \frac{1}{K_{3ws1}} \left[K_{1ws1} \int (\Delta P_{sh}) d\theta + [\Delta P_{sh}] + K_{2ws1} \frac{d(\Delta P_{sh})}{d\theta} \right]$$

from analog results

$$\left(\frac{\Delta W_{sd}}{W_{so}} \right)_1 = \frac{\Delta P_{sh}}{K_{3ws1}}$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta P_{sh}}{\left(\frac{\Delta W_{sd}}{W_{so}} \right)_1} = K_{3ws1} = 12,500 (0.10) = 1,250$$

from controller

$$\text{input range} = 0\text{-}3000 \text{ psi}$$

$$\text{output range} = 0\text{-}130,000 \text{ lb/hr}$$

$$\text{Since } W_{so} = 116,000 \text{ lb/hr}$$

then output range 0-1.12

then

$$\frac{\text{input}}{\text{output}} = \frac{3000}{1.12} = 2680$$

then

$$PB = \frac{1,250 (100)}{2680} = 46.6\%$$

$$\text{Reset} = K_{lws1} = 10 (0.1) = 1.0 \text{ repeats/sec} = 60 \text{ repeats/min.}$$

3. FRC - 106

$$\left(\frac{\Delta W_n}{W_{no}} \right) = \left(\frac{W_{nd}}{W_{no}} \right) - \left(\frac{W_n}{W_{no}} \right)^*$$

$$\left(\frac{W_{nd}}{W_{no}} \right) = \left(\frac{W_{sd}}{W_{so}} \right) + \left(\frac{\Delta W_{no}}{W_{no}} \right)$$

$$\frac{X_{v106}}{X_{v106,o}} = \frac{1}{K_{3v106}} \left[K_{1v106} \int \left(\frac{\Delta W_n}{W_{no}} \right) d\theta + \left[\frac{\Delta W_n}{W_{no}} \right] + K_{2v106} \frac{d \left(\frac{\Delta W_n}{W_{no}} \right)}{d\theta} \right],$$

from analog results

$$\frac{X_{v106}}{X_{v106,o}} = \frac{\left[\frac{\Delta W_n}{W_{no}} \right]}{K_{3v106}} ;$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\left(\frac{\Delta W_n}{W_{no}} \right)}{\left(\frac{X_{v106}}{X_{v106,o}} \right)} = K_{3v106} = 10 (0.10) = 1.0$$

from controller

input range = 0-4000 gpm

output range = 0-90° Rotation or 0-1.0

since $W_{no} = 4000$ gpm input range 0-1.0 ;

then

$$\frac{\text{input}}{\text{output}} = \frac{1.0}{1.0} = 1.0 ,$$

then

$$PB = \frac{1.0 (100)}{1.0} = 100\% .$$

$$\text{Reset} = K_{1v106} = 0.166 \text{ repeats/sec} = 9.96 \text{ repeats/min.}$$

4. TRC - 200

$$\Delta T_{sh} = T_{shd} - T_{sh}^*$$

$$\left(\frac{\Delta W_{nd}}{W_{no}} \right) = \frac{1}{K_{3wn}} \left[K_{1wn} \int (\Delta T_{sh}) d\theta + [\Delta T_{sh}] + K_{2wn} \frac{d(\Delta T_{sh})}{d\theta} \right]$$

from analog results

$$\frac{\Delta W_{nd}}{W_{no}} = \frac{\Delta T_{sh}}{K_{3wn}} ;$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta T_{sh}}{\left(\frac{\Delta W_{nd}}{W_{no}} \right)} = K_{3wn} = 10,000 (0.5) = 5,000 ;$$

from controller

input range = 400-1400°F

output range = 0-4000 gpm

Since $W_{no} = 4,000$ gpm - output range 0-1.0 ;

then

$$\frac{\text{input}}{\text{output}} = \frac{1000}{1.0} = 1000 \quad ,$$

then

$$PB = \frac{5000}{1000} (100) = 500\% \quad .$$

$$\text{Reset} = K_{1wn} = 0.200 \text{ repeats/sec} = 12 \text{ repeats/min}$$

$$\text{Rate} = K_{2wn} = \frac{1}{0.225} = 4.44 \text{ sec} = 0.074 \text{ min.}$$

5. TRC - 201

$$\Delta T_{no} = T_{nod} - T_{no}^*$$

$$\left(\frac{\Delta W_{sd}}{W_{so}} \right)_2 = \frac{1}{K_{3ws2}} \left[K_{1ws2} \int (\Delta T_{no}) d\theta + [\Delta T_{no}] + K_{2ws2} \frac{d(\Delta T_{no})}{d\theta} \right]$$

from analog results

$$\left(\frac{\Delta W_{sd}}{W_{so}} \right)_2 = \frac{\Delta T_{no}}{K_{3ws2}} \quad ;$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\Delta T_{\text{no}}}{\left(\frac{\Delta W_{\text{sd}}}{W_{\text{so}}}\right)_2} = K_{3\text{ws}2} = 10,000 (0.1) = 1000 \quad ;$$

from controller

$$\text{input range} = 400-1400^\circ\text{F}$$

$$\text{output range} = 0-130,000 \text{ lb/hr} \quad ,$$

$$\text{Since } W_{\text{so}} = 116,000 \text{ lb/hr} - \text{output range } 0-1.12 \quad ;$$

then

$$\frac{\text{input}}{\text{output}} = \frac{1000}{1.12} = 894 \quad ;$$

then

$$\text{PB} = \frac{1000}{894} (100) = 112\% \quad .$$

$$\text{Reset} = K_{1\text{ws}2} = 0.010 \text{ repeats/sec} = 0.60 \text{ repeats/min}$$

$$\text{Rate} = K_{2\text{ws}2} = \frac{1}{0.10} = 10 \text{ sec} = 0.167 \text{ min}$$

6. NCp - 1

$$\begin{aligned} \frac{N_d}{N_o} = \frac{1}{K_{3n}} & \left[K_{ln} \int \left[\left(\frac{W_{sd}}{W_{so}} \right) - \left(\frac{W_s}{W_{so}} \right)^* \right] d\theta \right. \\ & \left. + \left[\left(\frac{W_{sd}}{W_{so}} \right) - \left(\frac{W_s}{W_{so}} \right)^* \right] + K_{2n} \frac{d \left[\left(\frac{W_{sd}}{W_{so}} \right) - \left(\frac{W_s}{W_{so}} \right)^* \right]}{d\theta} \right] \end{aligned}$$

$$\left(\frac{W_{sd}}{W_{so}}\right) = \left(\frac{W_{sd}}{W_{so}}\right) + \left(\frac{\Delta W_{sd}}{W_{so}}\right)_1 + \left(\frac{\Delta W_{sd}}{W_{so}}\right)_2 + (\text{filtered rate})$$

from analog results

$$\frac{N_d}{N_o} = \frac{\left[\left(\frac{W_{sd}}{W_{so}}\right) - \left(\frac{W_s}{W_{so}}\right)^*\right]}{K_{3n}};$$

then

$$\frac{\text{input}}{\text{output}} = \frac{\left[\left(\frac{W_{sd}}{W_{so}}\right) - \left(\frac{W_s}{W_{so}}\right)^*\right]}{\left(\frac{N_d}{N_o}\right)} = K_{3n} = 10 (0.3) = 3.0 ,$$

from controller

input range = 0-130,000 lb/hr

output range = 0-6960 rpm

since $W_{so} = 116,000$ lb/hr and $N_o = 6960$ rpm ;

then

input range = 0-1.12,

output range = 0-1.0 ,

then

$$\frac{\text{input}}{\text{output}} = \frac{1.12}{1.0} = 1.12 .$$

then

$$PB = \frac{3.0}{1.12} (100) = 268\%$$

$$\text{Reset} = K_{in} = 0.1125 \text{ repeats/sec} = 6.75 \text{ repeats/min.}$$

7. Filtered Rate

The transformed equation for the filtered rate is

$$\frac{K\tau s}{1 + \tau s} \left[\left(\frac{W_n}{W_{no}} \right)^* \left(\frac{T_{ni}^* - T_{no}^*}{(T_{ni} - T_{no})_o} \right) \right]$$

Then by referring to the NCp - 1 controller, the output due to the filtered rate is

$$\left(\frac{W_{sd}}{W_{so}} \right) = \frac{K\tau s}{1 + \tau s} \left[\left(\frac{W_n}{W_{no}} \right)^* \left(\frac{T_{ni}^* - T_{no}^*}{(T_{ni} - T_{no})_o} \right) \right]$$

Then

$$K = 2 (0.500) = 1.0 \text{ repeats/sec from the analog results,}$$

or

$$K = 60 \text{ repeats/min,}$$

and the time constant is

$$\tau = \frac{1}{0.20} = 5 \text{ sec} = 0.083 \text{ min.}$$

The summary of the analog results and these calculations are shown in Table 3. These controller settings are recommended for the initial startup of the SCTI.

TABLE 3

SCTI STEAM SIDE RECOMMENDED CONTROLLER SETTINGS

Controller	Term	Range	Analog Computer Potentiometer No.	Potentiometer Setting	Analog Multiplier Factor	Recommended Controller Setting
NCp-1	Prop. Band	0-130,000 lb/hr 0-6960 rpm	AP16	0.300	$K_{3n}/10$	268%
	Reset		AP12	0.1125	K_{ln}	6.8 repeats/min
TRC-201	Prop. Band	400-1400°F 0-130,000 lb/hr	AP44	0.100	$K_{3ws2}/10,000$	112%
	Reset		AP04	0.010	K_{1ws2}	0.60 repeats/min
	Rate		AP80	0.100	$1/K_{2ws2}$	0.167 min
FRC-200	Prop. Band	0-3000 psi 0-130,000 lb/hr	AP34	0.100	$K_{3ws1}/12,500$	47%
	Reset		AP70	0.100	$K_{1ws1}/10$	60 repeats/min
TRC-200	Prop. Band	400-1400°F 0-4000 gpm	BQ14	0.500	$K_{3wn}/10,000$	500%
	Reset		BQ12	0.200	K_{1wn}	12 repeats/min
	Rate		BP15	0.225	$1/K_{2wn}$	0.074 min
FRC-106	Prop. Band	0-4000 gpm 0-90° Rot.	BQ34	0.100	$K_{3v106}/10$	100%
	Reset		BQ32	0.166	K_{1v106}	10 repeats/min
PRC-200	Prop. Band	0-3000 psi 0-1.0	AP74	0.200	$K_{3v200}/5000$	33%
	Reset		AP29	0.100	K_{1v200}	6 repeats/min
Filtered	Reset	0-130,000 lb/hr 0-4000 gpm	AP46	0.500	$K/2$	60 repeats/min
Rate	Time Const		AP07 & 08	0.200	$1/\tau$	0.083 min

REFERENCES

1. Alco Products, Inc., "30 Megawatt Heat Exchanger and Steam Generator for Sodium Cooled Reactor System," APAE No. 112, Vol. I (January 31, 1962)
2. R. Hankel, "Stress and Temperature Distributions" Nucleonics, Vol. 18, No. 11 (November 1960)
3. Personal communication from J. McCarty, Dept 717 (February 1962)
4. Fisher Governor Co., Catalog AL-9, p 4
5. Fisher Governor Co., Catalog AL-9, p 42
6. Fisher Governor Co., Catalog AL-9, p 8
7. Fisher Governor Co., Catalog AL-9, p 40

IV. AUXILIARY LOOP CONTROL ANALYSIS

A. THEORY AND INTRODUCTION

Analysis of control systems, because of the complex nature of the actual control loop, is normally performed with the aid of representative block diagrams. In this analysis the method of block diagram representation was used to simulate the feedback control of the steam and feedwater auxiliary loops.

Figure 24 shows a block diagram of a system with negative feedback. G and H are the transfer functions of the forward and feedback components, respectively, of the system.

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Figure 24. Steam and Feedwater Auxiliary Feedback System

The open and closed loop transfer functions in terms of the Laplace transform variable are:

$$\text{open loop} = \frac{B(S)}{E(S)} = G(S) H(S) \quad ,$$

$$\text{closed loop} = \frac{C(S)}{R(S)} = 1 + \frac{G(S)}{G(S) H(S)} \quad .$$

The characteristic equation is the denominator of the closed loop transfer function, which is $1 + G(S) H(S)$. The stability of a closed loop system depends upon the characteristic equation. However, the stability can also be determined from the open loop transfer function. In this case a Bode diagram is obtained by plotting the magnitude and phase of the open loop transfer function as a function of frequency on semilog paper. The magnitude is expressed in decibels, and the angle in degrees.

From the Bode diagram the stability of the closed loop system can then be determined by the gain and phase margin. The ASME has recommended that for a stable system a phase margin >30 degrees and a gain margin >8 db is required. Phase margin is the amount of phase shift that would produce instability. Gain margin is the factor by which the gain must be changed to produce instability.

All level control loops covered in this report are similar. Both temperature control loops are also similar but differ from the level control loops in that the temperature is determined by a thermocouple in place of a transmitter.

B. ANALYSIS

Because of the repetitious nature of this study, the analysis will be shown only for the high pressure feedwater heater level control system. The transfer functions and Bode diagrams for the remaining auxiliary control loops covered in this report may be found in Table 4 and Figures 28-32, respectively.

The block diagram for the high pressure feedwater heater level control system is shown in Figure 25.

Figure 25. High Pressure Feedwater Heater Level Control System

The forward and reverse components are: G_1 = feedwater heater, H_1 = transmitter, H_2 = pneumatic tubing, H_3 = controller, H_4 = pneumatic tubing, H_5 = valve with operator, and H_6 = condensate flow through valve. Q_s and Q_c are the steam and condensate flow respectively.

The transfer functions of the forward and reverse components of the system are derived below.

C. FEEDWATER HEATER

For a 1-in. drop in level, there is a change in area of 10.8 in.² at the normal operating level.

$$\Delta \text{ area} = 10.8 \Delta \text{ level}$$

$$\Delta \text{ volume} = \Delta \text{ area (length of chamber)}$$

$$\Delta \text{ volume} = 10.8 \Delta \text{ level (180)}$$

$$\Delta \text{ volume} = 1944 \Delta \text{ level}$$

$$\frac{dV}{dt} = (Q_s - Q_c) (V_s) (1728)$$

$$S (1944 \Delta L) = (Q_s - Q_c) (V_s) (1728)$$

$$\frac{\Delta L}{(Q_s - Q_c)} = \frac{V_s (1728)}{S (1944)}$$

$$V_s = 0.924664 \text{ ft}^3/\text{lb at operating temperature}$$

$$\frac{\Delta L}{(Q_s - Q_c)} = \frac{(0.924664) (1728)}{S (1944)}$$

$$\frac{\Delta L}{(Q_s - Q_c)} = \frac{0.0218}{S}$$

$$G_1 (S) = \frac{0.0218}{S}$$

D. TRANSMITTER

The following equations are from A Simplified Technique of Control System Engineering, by G. K. Tucker and D. M. Wells, p 282.

$$\text{Damping factor} = 0.505$$

$$\text{Natural frequency} = 87.5 \text{ cpm} = 9.15 \text{ rad/sec.}$$

$$H_1 (S) = \frac{\frac{\Delta O}{\Delta I} \left(\frac{100}{PB} \right)}{\left(\frac{S}{W_n} \right)^2 + \frac{2\zeta}{W_n} S + 1}$$

$$H_1 (S) = \frac{\frac{\Delta O}{\Delta I} \left(\frac{100}{PB} \right)}{\left(\frac{S}{9.15} \right)^2 + 0.1104 S + 1}$$

The level of the condensate should never be less than 1/2 in. from the bottom of the heater. Since the normal operating level is 2-1/2 in. from the bottom of the heater, the maximum level change is ± 2 in. Therefore, the proportional band of the transmitter must be set at $4/14 (100) = 28.5\%$.

$$\frac{\Delta O}{\Delta I} = \frac{15 - 3}{14} = \frac{12}{14} = 0.856$$

$$H_1 (S) = \frac{0.856 \left(\frac{100}{28.5} \right)}{\left(\frac{S}{9.15} \right)^2 + 0.1104 S + 1}$$

$$H_1 (S) = \frac{3}{\left(\frac{S}{9.15} \right)^2 + 0.1104 S + 1}$$

E. PNEUMATIC TUBING

The following facts and values are from Control Engineers Handbook, by Truxal, p 16-11.

$$H (S) = \frac{e^{-\tau_p s}}{(1 + \tau_l s)}$$

$$\tau_l = 17.7 (10^{-10}) \frac{L^2}{D^2}$$

$$\tau_p = \frac{L'}{\sqrt{K_g RT}}$$

where

K = ratio of specific heats of air

g = gravitational constant

R = gas constant of air

T = absolute temperature of fluid

D = ID in inches

L = line length in inches

L' = pipe length in feet

The length of pneumatic tubing between the transmitter and controller is approximately 300 ft. The length of pneumatic tubing between the controller and valve is approximately 286 ft.

Let L' = 300 ft,

$$\tau_l = \frac{17.7 (10^{-10}) \left[\left(\frac{300}{12} \right) \right]^2}{(0.186)^2} = 0.664$$

$$\tau_p = \frac{300}{\sqrt{(1.4) (32.2) (53.3) (528)}} = 0.266$$

$$H_2 (S) = \frac{e^{-0.266S}}{(1 + 0.664 S)} \quad .$$

Let L' = 286 ft,

$$\tau_l = \frac{17.7 (10^{-10}) \left[\left(\frac{286}{12} \right) \right]^2}{(0.186)^2} = 0.642$$

$$\tau_p = \frac{286}{\sqrt{(1.4) (32.2) (53.3) (528)}} = 0.254$$

$$H_4 (S) = \frac{e^{-0.254S}}{(1 + 0.642S)} \quad .$$

F. CONTROLLER

The following statements are from A Simplified Technique of Control System Engineering, p 282.

Damping factor = 1.0

Natural frequency = 300 cpm = 31.4 rad/sec.

$$H_3 (S) = \frac{\frac{\Delta O}{\Delta I} \left(\frac{100}{PB} \right) \left[1 + \frac{K_r}{60} S + K_d S \right]}{\left(\frac{S}{W_n} \right)^2 + \frac{2}{W_n} S + 1}$$

where

K_r = repeats/min

K_d = rate time in sec

$$H_3 (S) = \frac{\frac{\Delta O}{\Delta I} \left(\frac{100}{PB} \right) \left[1 + \frac{K_r}{60} S + K_d S \right]}{\left(\frac{S}{31.4} + 1 \right)^2}$$

The controller has the following range:

0 to 500% proportional band

0 repeats/min

0 rate

$$H_3 (S) = \frac{\frac{\Delta O}{\Delta I} \left(\frac{100}{PB} \right)}{\left(\frac{S}{31.4} + 1 \right)^2}$$

By setting the proportional band at 50%, the maximum condensate level change would be reduced from ± 2 to ± 1 in,

$$\frac{\Delta O}{\Delta I} = \frac{12}{12} = 1$$

$$H_3 (S) = \frac{(1) \left(\frac{100}{50} \right)}{\left(\frac{S}{31.4} + 1 \right)^2}$$

$$H_3 (S) = \frac{2}{\left(\frac{S}{31.4} + 1 \right)^2}$$

G. VALVE WITH OPERATOR

The following statements are from A Simplified Technique of Control System Engineering, p 282.

Damping factor = 1.0

Natural frequency = 60 cpm = 6.28 rad/sec

$$H_5 (S) = \frac{1}{\left(\frac{S}{W_n}\right)^2 + \frac{2\zeta}{W_n} S + 1}$$

$$H_5 (S) = \frac{1}{\left(\frac{S}{6.28} + 1\right)^2}$$

H. CONDENSATE FLOW THROUGH VALVE

At 100% load,

flow = 63,600 lb/hr = 191 gpm.

From the valve characteristics in Figure 26,

$$H_6 (S) = \frac{(215 - 155)}{70 - 50} = 3 \frac{\text{gpm}}{\% \text{ valve opening}}$$

$$H_6 (S) = 3 \frac{\text{gpm}}{\% \text{ valve opening}} \left(\frac{63,600 \text{ lb/hr}}{191 \text{ gpm}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ sec}} \right)$$

$$H_6 (S) = \frac{(3) (63,600)}{(191) (3600)}$$

$$H_6 (S) = 0.278$$

At 20% load,

flow = 12,100 lb/hr = 35.6 gpm.

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Figure 26. Valve (LIC-200V) Characteristics

From the valve characteristics in Figure 26,

$$H_6 (S) = \frac{(44 - 28)}{17.5 - 7.5} = 1.6 \frac{\text{gpm}}{\% \text{ valve opening}}$$

$$H_6 (S) = \frac{1.6 \text{ gpm}}{\% \text{ valve opening}} \left(\frac{12,100 \text{ lb/hr}}{35.6 \text{ gpm}} \right) \left(\frac{1 \text{ hr}}{3600 \text{ sec}} \right)$$

$$H_6 (S) = \frac{(1.6) (12,100)}{(35.6)(3600)}$$

$$H_6 (S) = 0.151$$

The open loop transfer functions at 100% and 20% load respectively are as follows:

$$G(S) H(S) \text{ at 100\%} = \frac{0.0364 (e^{-0.254S}) (e^{-0.266S})}{S \left(\frac{S}{31.4} + 1 \right)^2 (1 + 0.642S) \left(\frac{S}{6.28} + 1 \right)^2 \left[\left(\frac{S}{9.15} \right)^2 + 0.1104S + 1 \right] (1 + 0.664S)}$$

$$G(S) H(S) \text{ at 20\%} = \frac{0.01975 (e^{-0.254S}) (e^{-0.266S})}{S \left(\frac{S}{31.4} + 1 \right)^2 (1 + 0.642S) \left(\frac{S}{6.28} + 1 \right)^2 \left[\left(\frac{S}{9.15} \right)^2 + 0.1104S + 1 \right] (1 + 0.664S)}$$

From the Bode diagram, Figure 27, the following information is obtained.

At 100% load, gain margin = 26 db

phase margin = 80°

At 20% load, gain margin = 31 db

phase margin = 85°.

The system is stable, since the gain and phase margin criteria are met.

I. RESULTS

The results of the analysis for the SCTI steam and feedwater auxiliary loop control systems are as follows.

1) High pressure feedwater heater level control

at 100 and 20% load,

controller (LIC-200)

50% proportional band

transmitter (LIC-200T)

28.5% proportional band

2) High pressure feedwater heater temperature control

at 100% load

controller (TRC-201)

50% proportional band

12 repeats per min

0 rate

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Figure 27. Log Magnitude and Phase Diagram for High Pressure Feedwater Heater Level Control System (a)

at 20% load,	191% proportional band
	12 repeats per min
	0 rate
- 3) Intermediate pressure feedwater heater level control
- at 100 and 20% load
controller (LIC-202)	42% proportional band
transmitter (LIC-202T)	34% proportional band
- 4) Intermediate pressure feedwater heater level control
- at 100% load,
controller (TRC-203)	38% proportional band
	12 repeats per min
	0 rate
- at 20% load,
	191% proportional band
	12 repeats per min
	0 rate
- 5) High pressure flash tank level control
- at 100 and 20% load
controller (LIC-205)	16% proportional band
transmitter (LIC-205T)	37% proportional band
- 6) Low pressure flash tank level control
- at 100 and 20% load
controller (LIC-203)	16% proportional band
transmitter (LIC-203T)	37% proportional band

Other reference books used for the work of Section IV are: "Control System Analysis and Synthesis" by D'Azzo and Houpis, "Level Trols," Bulletin F-4B by Fisher Governor Co; "Valve Sizing Coefficients and Capacity Charts," Bulletin AL-9 by Fisher Governor Co; and "Flow of Fluids Through Valves, Fittings, and Pipes," Technical Paper No. 409, by Crane Co.

The component transfer functions for the auxiliary control loops are given in Table 4. The Bode diagrams for the five additional loops listed in the table are shown in Figures 28 through 32. The steam and feedwater P & I diagram is Figure 19.

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Figure 28. Log Magnitude and Phase Diagram for High Pressure Feedwater Heater Temperature Control System (b)

TABLE 4

COMPONENT TRANSFER FUNCTIONS FOR AUXILIARY CONTROL LOOPS

Component	H. P. Feedwater Heater Temperature Control System	I. P. Feedwater Heater Temperature Control System	I. P. Feedwater Heater Level Control System	H. P. Flash Tank Level Control System	L. P. Flash Tank Level Control System
Heater or Tank			$\frac{0.0279}{S}$ at 100% load $\frac{0.0275}{S}$ at 20% load	$\frac{0.0346}{S}$	$\frac{0.00732}{S}$
Controller	$\frac{0.0048 (5S + 1)}{S \left(\frac{S}{31.4} + 1 \right)^2}$	$\frac{0.01262 (5S + 1)}{S \left(\frac{S}{31.4} + 1 \right)^2}$	$\frac{2.38}{\left(\frac{S}{31.4} + 1 \right)^2}$	$\frac{6.25}{\left(\frac{S}{31.4} + 1 \right)^2}$	$\frac{6.25}{\left(\frac{S}{31.4} + 1 \right)^2}$
Transmitter			$\frac{2.52}{\left(\frac{S}{9.15} \right)^2 + 0.1104S + 1}$	$\frac{1}{\left(\frac{S}{9.15} \right)^2 + 0.1104S + 1}$	$\frac{1}{\left(\frac{S}{9.15} \right)^2 + 0.1104S + 1}$
Pneumatic Tubing	$\frac{e^{-0.254S}}{(1 + 0.642S)}$	$\frac{e^{-0.254S}}{(1 + 0.642S)}$	$\frac{(e^{-0.254S})(e^{-0.266S})}{(1 + 0.642S)(1 + 0.664S)}$	$\frac{(e^{-0.254S})(e^{-0.266S})}{(1 + 0.642S)(1 + 0.664S)}$	$\frac{(e^{-0.254S})(e^{-0.266S})}{(1 + 0.642S)(1 + 0.664S)}$
Valve with Operator	$\frac{1}{(0.159S + 1)^2}$	$\frac{1}{(0.159S + 1)^2}$	$\frac{1}{(0.159S + 1)^2}$	$\frac{1}{(0.159S + 1)^2}$	$\frac{1}{(0.159S + 1)^2}$
Heat Exchange	$\frac{13.52}{(7.65S + 1)}$ at 100% load $\frac{69.946}{(39.43S + 1)}$ at 20% load	$\frac{16.85}{(9.64S + 1)}$ at 100% load $\frac{65.6}{(37.2S + 1)}$ at 20% load			
Thermocouple	$\frac{1}{(13.39S + 1)(6.85 + 1)}$	$\frac{1}{(13.39S + 1)(6.85 + 1)}$			
Steam or Condensate Flow Through Valve	0.667 at 100% load 0.222 at 20% load	0.183 at 100% load 0.125 at 20% load	0.216 at 100% load 0.1048 at 20% load	0.49 at 100% load 0.572 at 20% load	0.548 at 100% load and 20% load

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Figure 29. Log Magnitude and Phase Diagram for Intermediate Pressure Feedwater Heater Temperature Control System (c)

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Figure 30. Log Magnitude and Phase Diagram for Intermediate Pressure Feedwater Heater Level Control System (d)

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Figure 31. Log Magnitude and Phase Diagram for High Pressure Flash Tank Level Control System (e)

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Figure 32. Log Magnitude and Phase Diagram for Low Pressure Flash Tank Level Control System (f)

APPENDIX A ANALOG SIMULATION OF SCTI SODIUM SYSTEM

I. GENERAL DISCUSSION

The analog computer study was performed to (a) establish controller configurations compatible with the physical system and with practical hardware; (b) optimize parameters (proportional gain, reset, and rate) for adopted controls; and (c) obtain dynamic characteristics of the controlled system during specified operational transients.

The magnitude of the study as well as limited availability of analog equipment restricted development of detail. The study was performed by separately analyzing these major subsections of the total system: the main sodium system with the IHX, and the steam system with secondary flow control. Where inter-effects of these systems were indicated, approximations were made.

II. ANALOG SIMULATION

After the system was described by the design engineer, a sequence of preparations was necessary before the analog study could be made.

A. TRANSFORMATION OF SYSTEM EQUATIONS TO MACHINE EQUATIONS

In order to simplify the application of the problem to analog computation, it was necessary to modify the system equations to a form that the computer can accept. Many of the more complicated mathematical expressions were reduced to relatively simple mathematical expressions and constants were evaluated.

Scale factors included time and amplitude considerations.

The system equations were transformed to machine equations by introducing linear relationships between the problem variables (pressures, flows, temperatures, etc) and the machine variables (time and voltage). The linear relations, representing time and amplitude scaling, are the scale factors. These were chosen to give a convenient rate of solution, and to restrict values of dependent variables within the limitations of analog equipment.

1. Time Scale

The time scale was not modified; i. e., 1:1 time correspondence between the physical system and the simulated system was in effect. This time scale was adopted because time constants of instrumentation and transmission lags were in the order of magnitude of 1 and 10 sec.

2. Amplitude Scale

The amplitude scaling was necessary to satisfy the physical requirements of the computer. Linear operation of the amplifiers is limited in the range from 100 v to -100 v. If only a small portion of this range is used (i. e., for small signals), then computer drift and noise become appreciable relative to computing voltages.

Gains have been distributed in many instances between two or more amplifiers, so the gain of any single amplifier did not exceed 50. Any gain in excess of this value makes amplifier stability difficult to achieve.

The following typical example will illustrate the technique of transforming the system equation to an equation compatible with computer mechanization.

Example:

a) System Equation

$$\dot{\bar{T}}_{ws} = 2.850 (W_r/W_{no})^{0.6} (\bar{T}_{ns} - T_{wns}) - 0.2993 (W_s/W_{so})^{0.8} (T_{wss} - \bar{T}_s)$$

$$T_{wss} = \bar{T}_s + \Delta T_s$$

Combining these two equations, the system equation becomes:

$$\dot{\bar{T}}_{ws} = 2.850 (W_n/W_{no})^{0.6} (\bar{T}_{ns} - T_{wns}) - 0.2993 (W_s/W_{so})^{0.8} (\Delta T_s)$$

b) Scale Factors

$$T_{\max} < 2000^{\circ}\text{F} \quad \alpha_t T_{\max} < 100 \text{ v}$$

$$\alpha_t T_{\max} < \alpha_t (2000^{\circ}\text{F}) \equiv 100 \text{ v}$$

$$\alpha_t = \frac{1}{20} \text{ v per } ^{\circ}\text{F}$$

or:

$$1 \text{ v} = 20^{\circ}\text{F}$$

$$W_n/W_{no} < 200\% \quad \alpha_w (W_n/W_{no}) < 100 \text{ v}$$

$$\alpha_w (W_n/W_{no}) < \alpha_w (200) \equiv 100 \text{ v}$$

$$\alpha_w = \frac{1}{2} \times 100 = 50 \text{ v per } 100\%$$

or:

$$1 \text{ v} = 2\%$$

$$\Delta T \approx 150^{\circ} \text{ (from steady-state calculations)}$$

$$\text{Scale for } \frac{\Delta T}{2} \approx 75 \text{ v}$$

or

$$\frac{1^{\circ}}{2} \text{ per v.}$$

c) Substituting Scale Factors Into the System Equation

$$\begin{aligned} \frac{\dot{T}_{ws}}{20} &= \frac{2.850 (100)}{4 (50)^{0.6}} \frac{\left[4 (50 W_w / W_{no})^{0.6} \right]}{100} \left[\frac{\bar{T}_{ns}}{20} - \frac{T_{wns}}{20} \right] \\ &- \frac{(0.2993) (100)}{4 (50)^{0.8}} \frac{\left[4 (50 W_s / W_{so})^{0.8} \right]}{100} \left(\frac{1}{10} \right) \frac{\Delta T_s}{2} \\ \frac{\dot{T}_{ws}}{20} &= (0.6812) \frac{\left[4 (50 W_n / W_{no})^{0.6} \right]}{100} (10) \left[\frac{\bar{T}_{ns}}{20} - \frac{T_{wns}}{20} \right] \\ &- (0.0327) \frac{\left[4 (50 W_s / W_{so})^{0.8} \right]}{100} \left(\frac{\Delta T_s}{2} \right) \end{aligned}$$

Note that whenever a product is formed, an additional scale factor of 100 is required. This is an inherent requirement of analog multiplication, to provide full-scaled output when both inputs are at full scale.

B. BLOCK DIAGRAM

The block diagram of Figure 33 was prepared from the machine equations. Using the symbols shown in Table 5 to represent available computing units, their interconnections were drawn and potentiometer dial settings were established. Several common combinations are shown and described in Table 6.

The drawing of the block diagram (or flow sheet) of the physical system was most helpful in interconnecting small blocks so as to simulate the complete system. Appropriate computing elements were then identified with various computations indicated by the equations.

Several arrangements of symbols were available for solving the problem, and one is likely to need fewer or simpler units than another. The initial mechanization was progressively modified and simplified until reliable simulations were attained to achieve the required problem solution. The ultimate circuit was not necessarily the simplest, and by no means is the mechanization unique; but it represents a circuit which evolved with available equipment.

TABLE 5
ANALOG SYMBOLS

POTENTIOMETER, MANUAL ADJUST

e_i — P20 — e_o $e_o = P_{20} e_i$

POTENTIOMETER, SERVO MULTIPLIER

e_i — $\frac{1}{2} \frac{\omega_n}{\omega_{no}}$ — M12C — e_o $e_o = \left(\frac{1}{2} \frac{\omega_n}{\omega_{no}} \right) e_i$

AMPLIFIER, HIGH GAIN

e_i — 23 — e_o $e_o = (-A) e_i$
 $A \approx 10^8$

AMPLIFIER, INVERTER

e_i — 02 — e_o $e_o = -e_i$

AMPLIFIER, SUMMER

e_1 — 70 — e_o $e_o = -(e_1 + e_2)$

AMPLIFIER, INTEGRATOR (W/IC)

e_i — 45 — e_o $e_o = \int e_i dt + (P_{45}) 100V$
P45
+100

AMPLIFIER, DIFFERENTIATION

e_i — 1.0 20K — 46 — e_o $e_o \approx -\frac{d}{dt} e_i$

SERVO MULTIPLIER, DRIVE AND F. B. POT

e_i — SM — 2 — e_{fb} $e_{fb} = e_i = (\theta) 100V$

DIODE FUNCTION GENERATOR

e_i — 15 — 19 — e_o $e_o = \pm f(e_i)$
3

LIMITER, SIGN.
(USING THERMIONIC OR SOLID STATE)

e_i — G — e_o $e_o = \begin{cases} -e_i & \text{if } e_i \geq 0 \\ 0 & \text{if } e_i < 0 \end{cases}$

LIMITER, $\pm E$ (USING ZENER DIODE)

e_i — G — e_o $e_o = \begin{cases} +E & \text{if } e_i < -E \\ -e_i & \text{if } -E < e_i < +E \\ -E & \text{if } e_i > +E \end{cases}$

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Figure 33. Representation of $\dot{T}_{ws}$ Sealed to Computer Circuit

A typical example will illustrate the transition from the scaled computer equation to the computer circuit.

Example: Using the equation of the previous illustration,

$$\begin{aligned} \frac{\dot{T}_{ws}}{20} = & 0.6812 \left[\frac{4 (50 W_n / W_{no})^{0.6}}{100} \right] (10) \left[\frac{T_{ns}}{20} - \frac{T_{wns}}{20} \right] \\ & - (0.0327) \left[\frac{4 (50 W_s / W_{so})^{0.8}}{100} \right] \left(\frac{1}{10} \right) \left[\frac{\Delta T_s}{2} \right] , \end{aligned}$$

it is seen that the generation of $\dot{T}_{ws}$ may be represented diagrammatically as here shown.

TABLE 6
CONFIGURATIONS FOR VARIOUS MACHINE EQUATIONS

Figure 34 represents a general program for generating $\overline{T}_{ws}$ when the contributing parameters are available. If the indicated mathematical operations are performed, then the desired variable $\overline{T}_{ws}$ will be simulated. The representation of this equation is by no means unique, but constitutes one satisfactory arrangement.

In this illustration, each operation is indicated by a box with the appropriate description of the operation to be performed at the location of the box. The analog computer art has adopted symbols to represent the various mathematical operations which have been designated literally in Figure 33. Some of the pertinent symbols are shown in Table 5. In Table 6 are combinations of operations which might simplify interpretation of the overall schematic which is

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Figure 34. Analog Computer Variables for Generation of $\bar{T}_{ws}$

to follow. The symbols as shown are not universally accepted in the field but will suffice to support this discussion.

Figure 34 shows the analog computer schematic for the generation of $\bar{T}_{ws}$ as described. Some points which merit explanation are as follows.

Variables other than $\bar{T}_{ws}$ are generated by other equations of the system.

The signs as shown are arbitrary, but consistent with the equations when variables are combined.

Initial conditions appear at the appropriate amplifier, although not explicitly in the system equation. The I. C. for $\bar{T}_{ws}$ is provided by Potentiometer 5 with the +100 v reference applied.

Coefficients are set on the potentiometers (Q09 and Q03). Note that Q03 is set at 0.0570 instead of 0.0327. It was found during the computer setup that the generation of $2.3 (50 W_n / W_{no})^{0.6}$ was more convenient than $4 (50 W_n / W_{no})^{0.6}$ therefore, the coefficients of the system equations were modified to accommodate this change.

At the input of Amplifier 52, two X10 inputs were found convenient for generating the scaled difference between T_{wns} and $\overline{T}_{ns}$.

The inputs to the system are represented by simple potentiometer settings.

C. COMPUTER MECHANIZATION

The required equivalent relations were established through the use of computer elements that perform the following mathematical operations: addition, multiplication by a constant, integration, multiplication of two functions, function generation, and combinations of these.

The computer elements were combined to synthesize the machine equations by relating the machine variables. The computer elements were "patched" together on the problem board which is inserted into the computer.

Known or assumed parameters or characteristics are set in by adjustment of various computer components. Generally, there will be several unknown variables involved; and there may be a choice as to how corresponding voltages in the computer are allowed to affect one another.

D. ANALOG SOLUTION

The function of the analog computer was to solve simultaneously the numerous equations describing the dynamic operation of the physical system. After the problem was mechanized on the machine, disturbances were simulated and the results presented in understandable form as the machine output, which was then compared with desired answers if such comparison was required. Judgment was exercised to determine whether the answers were acceptable or unacceptable. If the latter, adjustments were made and new solutions were generated for perusal.

E. ACCURACY

Modern analog machines exhibit linearity and precision (repeatability) of the order of 0.1%, drift and calibration errors excluded. Drift is usually small enough to be ignored except when integrations are operated "open loop" for part or all of the solution. As a result, the usual machine is limited by operational component calibrations and drifts, which to a great degree are under the control of the user. In consideration of these inaccuracies and the errors arising from the preparation and readout procedures, accuracy in the order of 2% is plausible.

F. ANALOG REQUIREMENTS

Equipment requirements for simulation of the complete SCTI system exceeded the inventory at Atomics International. The requirements of the main sodium system, hydraulics, and associated controls required the full capacity of three of the four available computers. The mechanization involved approximately 150 amplifiers, 125 potentiometers, and thirty 5-product servo multipliers.

The simulation of the steam system was somewhat less demanding on equipment and required only two computer consoles involving approximately 135 amplifiers, 100 potentiometers, and twenty 5-product servo multipliers.

Because of the evolution of the control configuration, very little effort was directed toward optimization of circuits, minimization of computing elements, or simplification of representation, although in the latter study the controller configurations were somewhat standardized as a result of the first study. If the complete system is reconstructed for continuous analysis of experiments while operating the SCTI, more thought will be given to optimization and simplification.

III. MAIN SODIUM SYSTEM

A. DESCRIPTION

A schematic of the main sodium system is shown in Figure 35. The distinct elements of this subsystem as studies are hydraulics, intermediate heat exchanger, furnace, cooler, and controls. The latter include: IHX inlet temperature, IHX sodium flow, furnace gas flow rate, furnace air flow rate, furnace sodium inlet temperature, cooler sodium flow rate, and cooler temperature.

The mathematical relationships describing the operation of each element were complemented with interrelationships to obtain the mathematical description of the subsystem operation. Hand analysis was limited because the complexities of the problem presented formidable obstacles (nonlinearities, differential equations of multiple orders, etc). The exploratory nature of the study and the numerous controls suggested the use of analog rather than digital instrumentation.

7593-1070

Figure 35. Main Sodium System

B. SYSTEM EQUATIONS

The system equations are a summary of the equations obtained for the analog study. The coefficients have been evaluated, and the variables are in symbolic form. In each equation the first term or, generally, the term on the left of the equation, represents the variable to be generated by the equation.

1. Hydraulic System Equations

The hydraulic system equations define the generation of flows as a function of the position of a number of valves. The flow in four of the lines has been identified as W_1 , W_2 , W_3 , and W_4 . Flows in other lines have been defined as combinations of these four.

The flows in the system are related to the positions of five valve openings which have been designated as θ_1 , θ_2 , θ_3 , θ_4 , and θ_6 . Nominal settings of these valves are modified by automatic controls during operation.

The hydraulic system equations, which include a supplementary pressure equation, are as follows.

$$\begin{aligned}\Delta P_a &= 104 + 0.873 (W_1 + W_2 + W_3 + W_4)/W_o \\ &- 13.518 \left[(W_1 + W_2 + W_3 + W_4)/W_o \right]^2 \\ &- 13.22 \left[(W_1 + W_2 + W_3)/W_o \right]^2 . \\ \dot{W}_1/W_o &= \alpha \left\{ \Delta P_a - \left[0.905 + (Y_{v1}/\phi_{v1}^*) \right] (W_1/W_o)^2 \right. \\ &- \left. \left[7.79 + (Y_{v6}/\phi_{v6}^*) \right] \left[(W_1 + W_4)/W_o \right]^2 \right\} . \\ \dot{W}_2/W_o &= \alpha \left\{ \Delta P_a - 2.9 \left[(W_2 + W_3)/W_o \right]^2 \right. \\ &- \left. \left[4.28 + (Y_{v2}/\phi_{v2}^*) \right] (W_2/W_o)^2 \right\} . \\ \dot{W}_3/W_o &= \alpha \left\{ \Delta P_a - 2.9 \left[(W_2 + W_3)/W_o \right]^2 \right. \\ &- \left. \left[85.6 + (Y_{v3}/\phi_{v3}^*) \right] (W_3/W_o)^2 \right\} . \\ \dot{W}_4/W_o &= \alpha \left\{ \Delta P_a - \left[1.58 + (Y_{v4}/\phi_{v4}^*) \right] (W_4/W_o)^2 \right. \\ &- \left[7.79 + (Y_{v6}/\phi_{v6}^*) \right] \left[(W_1 + W_4)/W_o \right]^2 \\ &+ 13.22 \left[(W_1 + W_2 + W_3)/W_o \right]^2 \left. \right\} .\end{aligned}$$

2. Intermediate Heat Exchanger (IHX)

The operation of the intermediate heat exchanger is defined by equations relating heat transfer by thermal conductivity and convection in a counterflow heat exchanger. Simplified inlet-outlet relations defined both primary and secondary channels of the heat exchanger. The summarized equations are as follows.

$$U_x = 10,000 \left\{ (1.51/W_s/W_o)^{0.4} + 2.93 + 3.81/[(W_1 + W_4)/W_o]^{0.6} \right\}^{-1} .$$

$$\dot{T}_{xss} = \frac{1.36}{3600} U_x (T_{xsp} - T_{xss}) - \frac{366}{3600} \left(\frac{W_5}{W_o} \right) (T_{xss o} - T_{xssi}) .$$

$$\dot{T}_{xsp} = 2 \frac{74.6}{3600} \left(\frac{W_1 + W_4}{W_o} \right) (T_{xspi}^{**} - T_{xsp}) - \frac{0.277}{3600} U_x (T_{xsp} - T_{xss}) .$$

$$\dot{T}_{xspi}^{**} = \frac{1}{0.001362 \times 3600} (W_1/W_o) (T_{xspi} - T_{xspi}^{*}) .$$

$$\dot{T}_{xspi}^{**} = \frac{1}{0.0075 \times 3600} [(W_1 + W_4)/W_o] (T_{xspi}^{*} - T_{xspi}^{**}) .$$

$$\dot{T}_{xsp}^{*} = \frac{1}{\frac{0.0075}{2} \times 3600} [(W_1 + W_4)/W_o] (T_{xsp} - T_{xsp}^{*}) .$$

$$T_{xspo} = 2 T_{xsp}^{*} - T_{xspi}^{**} .$$

$$\dot{T}_{xspo}^{*} = \frac{1}{0.0003 \times 3600} [(W_1 + W_4)/W_o] (T_{xspo} - T_{xspo}^{*}) .$$

$$T_{fs3} = \frac{[(W_1 + W_4)/W_o] T_{xspo}^{*} + (W_3/W_o) T_{co} + (W_2/W_o) T_{csi}}{(W_1 + W_4)/W_o} .$$

$$T_{xspi} = \frac{(W_1/W_o) T_{csi} + (W_4/W_o) T_{fs3}}{(W_1 + W_4)/W_o} .$$

3. Furnace

The furnace equations define the interrelationships between temperatures and flows to establish temperatures in the environment of the furnace. The temperatures are derived from coolant flow through the furnace, and are modified by air and gas flow in the furnace. Theoretical approximations include mean and log-mean temperatures.

The summary of furnace equations follows.

$$R = \frac{W_a^* / (W_a + W_f)_o}{W_f^* / (W_a + W_f)_o} \quad .$$

$$T_{fgf} = 80 + 1000 / (0.0471 + 0.01275R) \quad .$$

$$T_{fg2} = T_{fgf} - \frac{0.325 (T_{fg1} - T_{fsr})}{\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o}} \quad .$$

$$T_{fg1} = 0.225 T_{fgf} + 0.775 T_{fg2} \quad .$$

$$\dot{T}_{fsr} = \frac{16.4}{3600} (T_{fg1} - T_{fsr}) - \frac{272.6}{3600} \left(\frac{W_1 + W_2 + W_3}{W_o} \right) (T_{fso} - T_{fs1}) \quad .$$

$$T_{csi} = 2 T_{fsr} - T_{fsi} \quad .$$

$$\dot{T}_{fse} = \frac{83}{3600} \left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6} \Delta T_m - \frac{924}{3600} \frac{(W_1 + W_2 + W_3)}{W_o} (T_{fs3} - T_{fs2}) \quad .$$

$$\Delta T_m = \frac{1}{2} (T_{fg3} + T_{fg4} - 2 T_{fse}) \quad .$$

$$T_{fg4} = T_{fg3} - 0.4870 \Delta T_m \frac{\left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6}}{\left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]} .$$

$$T_{fs2} = 2 T_{fse} - T_{fs3} .$$

$$\dot{T}_{fs2}^* = \frac{1}{0.0032 \times 3600} \left(\frac{W_1 + W_2 + W_3}{W_o} \right) (T_{fs2} - T_{fs2}^*) .$$

$$T_{fsc} = \frac{13.05}{3600} \left[\frac{(W_{fa} + W_{ff})}{(W_{fa} + W_{ff})_o} \right]^{0.6} \Delta T_{ln} \\ - \frac{52.2}{3600} \frac{(W_1 + W_2 + W_3)}{W_o} (T_{fs1} - T_{fs2}^*) .$$

$$\Delta T_{ln} = \frac{(T_{fg2} - T_{fs1}) - (T_{fg3} - T_{fs2}^*)}{\ln \left(\frac{T_{fg2} - T_{fs1}}{T_{fg3} - T_{fs2}^*} \right)} .$$

$$\dot{T}_{fg3} = \alpha \left\{ - 1.35 \Delta T_{ln} \left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6} \right. \\ \left. + \frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} (T_{fg2} - T_{fg3}) \right\} .$$

4. Cooler

The cooler equations relate the temperatures in the cooler to the flow of air which is set manually and maintained automatically by a controller. The equations define the temperature-flow characteristics for the cooler under the influence of Controller V7 (see Figure 35).

The equations governing the dynamics of the cooler with its controller are as follows.

$$T_{cso} = 2 T_{cs} - T_{csi} \quad .$$

$$\dot{T}_{cso}^* = \frac{1}{\tau_{cso}} (T_{cso} - T_{cso}^*) \quad .$$

$$\epsilon_{v7} = T_{cso}' - T_{cso}^* \quad .$$

$$\frac{\Delta W_{ca}}{W_{cao}} = \frac{1}{\alpha_{3v7}} \left(K_{1v7} \int \epsilon_{v7} dt + \epsilon_{v7} \right) \quad .$$

$$\frac{\Delta W_{ca}^*}{W_{cao}} = \frac{1}{\tau_{ca}} \left(\frac{\Delta W_{ca}}{W_{cao}} - \frac{\Delta W_{ca}^*}{W_{cao}} \right) \quad .$$

$$\frac{W_{ca}^*}{W_{cao}} = \frac{\bar{W}_{ca}}{W_{cao}} + \frac{\Delta W_{ca}^*}{W_{cao}} \quad .$$

$$T_{cao} + T_{cai} = 0.686 \frac{(W_{ca}^*/W_{ca,o})^{0.6}}{(W_{ca}^*/W_{cao})} \Delta T_{cm} + T_{cai} \quad .$$

$$\Delta T_{cm} = T_{cs} - \frac{1}{2} (T_{cao} - T_{cai}) \quad .$$

$$\dot{T}_{cs} = 2 \frac{424}{3600} \frac{W_3}{W_o} \left(T_{cs} - T_{csi} \right) - \frac{99.1}{3600} \left(\frac{W_{ca}}{W_{cao}} \right)^{0.6} \Delta T_{cm} \quad .$$

5. Controls

Besides the cooler controller indicated in the foregoing equations, other controls operating on temperatures and flows were used in the main sodium system. The operation of controllers will be generalized and discussed in detail in a later section of this report. It will suffice here to summarize the equations.

$$\frac{W_{sh}}{W_o} = \frac{(W_1 + W_2 + W_3)}{W_o} \quad .$$

$$\frac{\dot{W}_{sh}^*}{W_o} = \frac{1}{\tau_{sh}} \left(\frac{W_{sh}}{W_o} - \frac{W_{sh}^*}{W_o} \right) \quad .$$

$$\epsilon_{sh} = \frac{W'_{sh}}{W_o} - \frac{W_{sh}^*}{W_o} \quad .$$

$$\Delta \phi_{v3} = \frac{1}{\alpha_{v3}} \left(K_{1v3} \int \epsilon_{sh} dt + \epsilon_{sh} + K_{2v3} \dot{\epsilon}_{sh} \right) \quad .$$

$$\phi_{v3} = \Phi_{v3} + \Delta \phi_{v3} \quad .$$

$$\dot{\phi}_{v3}^* = \frac{1}{\tau_{v3}} (\phi_{v3} - \phi_{v3}^*) \quad .$$

$$\dot{Y}_{v3} = -6.04 Y_{v3} \dot{\phi}_{v3}^* \quad .$$

$$T_{xspi}^{***} = \frac{1}{\tau_{xpi}} (T_{xspi} - T_{xspi}^{***}) \quad .$$

$$\epsilon_{\text{xspi}} = T'_{\text{xspi}} - T_{\text{xspi}}^{***} \quad .$$

$$\Delta\phi_{v4} = \frac{1}{\alpha_{v4}} \left(K_{1v4} \int \epsilon_{\text{xspi}} dt + \epsilon_{\text{xspi}} + K_{2v4} \dot{\epsilon}_{\text{xspi}} \right) \quad .$$

$$\phi_{v4} = \Phi_{v4} + \Delta\phi_{v4} \quad .$$

$$\dot{\phi}_{v4} = \frac{1}{\tau_{v4}} (\phi_{v4} - \phi_{v4}^*) \quad .$$

$$\dot{Y}_{v4} = -6.30 \phi_{v4}^* Y_{v4} \quad .$$

$$Y_{v1} = \frac{37.338}{Y_{v4}} \quad .$$

$$\frac{W_{\text{sp}}}{W_o} = \frac{(W_1 + W_4)}{W_o} \quad .$$

$$\frac{\dot{W}_{\text{sp}}^*}{W_o} = \frac{1}{\tau_{\text{sp}}} \left(\frac{W_{\text{sp}}}{W_o} - \frac{W_{\text{sp}}^*}{W_o} \right) \quad .$$

$$\epsilon_{\text{sp}} = \frac{W'_{\text{sp}}}{W_o} - \frac{W_{\text{sp}}^*}{W_o} \quad .$$

$$\Delta\phi_{v6} = \frac{1}{\alpha_{v6}} \left(K_{1v6} \int \epsilon_{\text{sp}} dt + \epsilon_{\text{sp}} + K_{2v6} \dot{\epsilon}_{\text{sp}} \right) \quad .$$

$$\phi_{v6} = \Phi_{v6} + \Delta\phi_{v6} \quad .$$

$$\phi_{v6}^* = \frac{1}{\tau_{v6}} (\phi_{v6} - \phi_{v6}^*) \quad .$$

$$\dot{Y}_{v6} = -6.30 Y_{v6} \phi_{v6}^* \quad .$$

$$T_{fs3}^* = \frac{1}{\tau_{fs3}} (T_{fs3} - T_{fs3}^*) \quad .$$

$$\epsilon_{fs3} = T_{fs3}' - T_{fs3}^* \quad .$$

$$\Delta \phi_{v2} = \frac{1}{\alpha_{v2}} \left(K_{1v2} \int \epsilon_{fs3} dt + \epsilon_{fs3} + K_{2v2} \epsilon_{fs3}' \right) \quad .$$

$$\phi_{v2} = \Phi_{v2} + \Delta \phi_{v2} \quad .$$

$$\phi_{v2}^* = \frac{1}{\tau_{v2}} (\phi_{v2} - \phi_{v2}^*) \quad .$$

$$\dot{Y}_{v2} = -6.02 Y_{b2} \phi_{v2}^* \quad .$$

$$\dot{T}_{fso}^* = \frac{1}{\tau_{fso}} (T_{fso} - T_{fso}^*) \quad .$$

$$\epsilon_{fso} = T_{fso}' - T_{fso}^* \quad .$$

$$\delta = \frac{1}{\alpha_{3v11}} \left(K_{1v11} \int \epsilon_{fso} dt + \epsilon_{fso} + K_{2v11} \epsilon_{fso}' \right) \quad .$$

$$\frac{W_{ff}}{(W_{fa} + W_{ff})_o} = \frac{0.04819}{400} \frac{(W_1 + W_2 + W_3)}{W_o} (\epsilon_{fso}) + \delta \quad .$$

$$\frac{W_{fa}}{(W_{fa} + W_{ff})_o} = 0.03178 + 18.833 \frac{W_{ff}}{(W_{fa} + W_{ff})_o} - \eta \quad .$$

$$\eta = \frac{1}{\alpha_{3v10}} \left(K_{1v10} \int \epsilon dt + \epsilon + K_{2v10} \epsilon' \right) \quad .$$

$$\epsilon = \frac{W_{fa}^*}{(W_{fa} + W_{ff})_0} - 0.03178 - 18.833 \frac{W_{ff}^*}{(W_{fa} + W_{ff})_0} \quad .$$

$$\frac{\dot{W}_{fa}^*}{(W_{fa} + W_{ff})_0} = \frac{1}{\tau_{fa}} \left[\frac{W_{fa}}{(W_{fa} + W_{ff})_0} - \frac{W_{fa}^*}{(W_{fa} + W_{ff})_0} \right] \quad .$$

$$\frac{\dot{W}_{ff}^*}{(W_{fa} + W_{ff})_0} = \frac{1}{\tau_{ff}} \left[\frac{W_{ff}}{(W_{fa} + W_{ff})_0} - \frac{W_{ff}^*}{(W_{fa} + W_{ff})_0} \right] \quad .$$

C. INITIAL CONDITIONS, RANGE OF VARIABLES, AND STEADY STATE VALUES

Table 7 represents available information concerning the variables in the system equations. Maximum values have been approximated, and the steady-state solution values were computed as a check case for the analog study. The initial condition values as listed above were used as voltage inputs to the computer model.

TABLE 7
INITIAL CONDITIONS AND RANGE OF VARIABLES
IN SYSTEM EQUATIONS

(Temperatures are scaled for $T/50-1 \text{ v} = 50 \text{ R}$. T^* designates a delayed temperature; T^{**} , a double delayed temperature; and T' , a temperature setpoint.)

		(S. S.)	(Max)	(I. C.)
T_{ai}	Cooler	100	--	2.00
T_{ao}	Cooler	819	1200	16.39
T_c	Cooler	--	1400	21.80
T_{fc}	Cooler	--	1400	21.35
T_{fci}	Cooler, inlet	1300	1400	26.00

TABLE 7

INITIAL CONDITIONS AND RANGE OF VARIABLES
IN SYSTEM EQUATIONS (CONTINUED)

		(S. S.)	(Max)	(I. C.)
T_{fco}	Cooler, outlet	900	1400	17.60
T_{fe}	Economizer	--	1400	18.41
T_{fei}	Economizer, inlet	900	1400	18.00
T_{fho}	Furnace, outlet	1300	1400	26.00
T_{fhl}	Furnace	1194	1400	23.88
T_{fh2}	Furnace	941	1400	18.82
T_{fr}	Furnace	--	1400	24.94
T_{gei}	Economizer			
T_{geo}	Economizer	1269	2000	25.39
T_{gh1}	Furnace	3013	4000	60.25
T_{gh2}	Furnace	2882	3000	57.63
T_{gh3}	Furnace	1495	2500	29.91
T_{pxi}	IHX, primary inlet	1200	1400	24.00
T_{pxo}	IHX, primary outlet	900	1400	18.00
T_{sp}	IHX	--	1400	21.00
T_{ss}	IHX	--	1400	19.63
T_{sxi}	IHX, secondary inlet	--	1400	15.54
T_{sxo}	IHX, secondary outlet	--	1400	23.73

(Flows are scaled for 50 w/w_o. And W_o = 10⁶ psi; W_a + W_f = 186,000 lb/hr)

W_1/W_o	Main	2.0	0.7500	37.50
W_2/W_o	Cooler bypass	1.0	0.012	0.60
W_3/W_o	Cooler	0.5	0.2380	11.90

TABLE 7
INITIAL CONDITIONS AND RANGE OF VARIABLES
IN SYSTEM EQUATIONS (CONTINUED)

		(S. S.)	(Max)	(I. C.)
W_4/W_o	IHX bypass	2.0	0.2500	12.50
W_{14}/W_o	IHX	2.0	1.0000	50.00
W_{1234}/W_o	(TOTAL)	2.0	1.250	62.50
W_{123}/W_o	Furnace	2.0	1.000	50.00
$W_a/(W_a + W_f)_o$	Furnace air			
$W_f/(W_a + W_k)_o$	Furnace fuel			
$(W_a + W_f)/(W_a + W_k)_o$	Furnace air and fuel	--	0.9877	49.38
W_{ca}/W_{cao}	Cooler pump	--	0.43119	
W_{ss}/W_{sso}		2.0	0.7332	36.66
W_{sp}/W_{spu}		2.0	1.000	50.00

E. MACHINE EQUATIONS

The machine equations are the results of transforming the system equations for use on the computer. The constants differ from the system equations, but the equations are algebraically unchanged. The change in coefficients represents the consolidation of scale factors applied to the variables.

TABLE 8
MACHINE EQUATIONS

$\Delta P_a = 100 (0.1040) (10) + (0.0175) (50) (W_1 + W_2 + W_3 + W_4)/W_o$ $- (0.5407) \frac{[50 (W_1 + W_2 + W_3 + W_4)/W_o]^2}{100}$ $- \frac{1}{[1/0.5288 - 1] + 1} \frac{[50 (W_1 + W_2 + W_3)/W_o]^2}{100}$
--

TABLE 8
MACHINE EQUATIONS (CONTINUED)

$$\begin{aligned}
 50 \dot{W}_1/W_o &= 50 \alpha / \phi_{v1}^* \phi_{v6}^* \left\{ \left(\frac{100 \phi_{v1}^*}{100} \right) \left(\frac{100 \phi_{v6}^*}{100} \right) \Delta P_a \right. \\
 &\quad - \left(\frac{100 \phi_{v6}^*}{100} \right) \left(\frac{1}{0.1250} \right) \left[\left(0.0045 \right) \left(100 \phi_{v1}^* \right) \right. \\
 &\quad \left. \left. + \left(Y_{v1}/2 \right) \right] \left(\frac{50 W_1/W_o}{100} \right)^2 - \left(\frac{100 \phi_{v1}^*}{100} \right) \left[\left(0.0779 \right) \left(100 \phi_{v6}^* \right) \right. \right. \\
 &\quad \left. \left. + 2 \left(\frac{Y_{v6}}{2} \right) \right] (4) \left[\frac{50 (W_1 + W_4)/W_o}{100} \right]^2 \right\} . \\
 50 \dot{W}_2/W_o &= 50 \alpha / \phi_{v2}^* \left\{ \left(\frac{100 \phi_{v2}^*}{100} \right) \left[\Delta P_a - \frac{1}{10 (0.8621)} \right] \left[\frac{50 (W_2 + W_3)/W_o}{100} \right]^2 \right. \\
 &\quad \left. - \left(\frac{1}{0.0500} \right) \left[(0.00856) (100 \phi_{v2}^*) + (Y_{v2}/5) \right] \left[\frac{50 W_2/W_o}{100} \right]^2 \right\} . \\
 50 \dot{W}_3/W_o &= 50 \alpha / \phi_{v3}^* \left\{ \left(\frac{100 \phi_{v3}^*}{100} \right) \left[\Delta P_a - \frac{1}{10 (0.8621)} \right] \left[\frac{50 (W_2 + W_3)/W_o}{100} \right]^2 \right. \\
 &\quad \left. - \left(\frac{1}{0.0500} \right) \left[(0.1712) (100 \phi_{v3}^*) + (Y_{v3}/5) \right] \left[\frac{50 W_3/W_o}{100} \right]^2 \right\} . \\
 50 \dot{W}_4/W_o &= 50 \alpha / \phi_{v4}^* \phi_{v6}^* \left\{ \left(\frac{100 \phi_{v4}^*}{100} \right) \left(\frac{100 \phi_{v6}^*}{100} \right) \left[\Delta P_a \right. \right. \\
 &\quad \left. \left. + \left(\frac{1}{1 + 0.8990} \right) \left[\frac{50 (W_1 + W_2 + W_3)/W_o}{100} \right]^2 \right] \right\} .
 \end{aligned}$$

TABLE 8

MACHINE EQUATIONS (CONTINUED)

$$\begin{aligned}
& - \left(\frac{100 \phi_{v6}^*}{100} \right) \left[\frac{1}{0.1250} \right] \left[(0.0079) (100 \phi_{v4}^*) \right. \\
& \left. - (Y_{v4}/2) \right] \left(\frac{50 W_4/W_o}{100} \right)^2 \\
& - \left(\frac{100 \phi_{v4}^*}{100} \right) \left(\frac{1}{0.1250} \right) \left[(0.03895) (100 \phi_{v6}^*) \right. \\
& \left. - (Y_{v6}/2) \right] \left[\frac{50 (W_1 + W_4)/W_o}{100} \right]^2 \Bigg\} .
\end{aligned}$$

Pump speed is scaled for 50 N/N_o (1 ν = 2%).

(N/N_o) Furnace, (N/N_o) Cooler 0 to 1.3

Valve openings are scaled for 100 φ (1 ν = 1% open)

φ_{v1}, φ_{v2}, φ_{v3}, φ_{v4}, φ_{v6}, φ_{v7}, φ_{v10}, φ_{v11} 0 to 1

Intermediate Heat Exchanger (IHX)

$$\begin{aligned}
10^4 (6.56/2V_x) = \frac{1}{2} \Bigg\{ & \left[9.9 (W_5/W_o)^{-0.4} \right] + 100 (0.0192) (10) \\
& + \left[25 \left(\frac{W_1 + W_4}{W_o} \right)^{-0.6} \right] \Bigg\} .
\end{aligned}$$

$$\begin{aligned}
\frac{\dot{T}_{xss}}{50} = \frac{2 \times 2 \times 366}{3600} \Bigg\{ & \frac{1.36 \times 6.56 \times 10^4}{366 \times 8 \times 100} 100 \frac{(T_{xspo}/50 - T_{xss}/50)}{(6.56/2) (10^4/V_x)} \\
& - \left(\frac{50 W_5/W_o}{100} \right) \left[\frac{T_{xss}}{50} - 100 \left(\frac{T_{xssi}}{5000} \right) \right] \Bigg\} .
\end{aligned}$$

$$\frac{\dot{T}_{xsp}}{50} = \frac{2 \times 2 \times 74.6}{3600} \left\{ \frac{50 (W_1 + W_4)/W_o}{100} \left(\frac{T_{xspi}^*}{50} - \frac{T_{xsp}}{50} \right) - \frac{(0.277) (6.56) (100)}{74.6 \times 8} \frac{100 T_{xsp}/50 - T_{xss}/50}{\frac{1}{2} \times 6.56 (10^4/V_x)} \right\} .$$

$$\frac{\dot{T}_{xspi}^*}{50} = \frac{2}{(0.001362) (3600)} \left[\frac{50 (W_1 + W_4)/W_o}{100} \right] \left(\frac{T_{xspi}}{50} - \frac{T_{xspi}^*}{50} \right) .$$

$$\frac{\dot{T}_{xspi}^{**}}{50} = \frac{2}{(0.0075) (3600)} \left[\frac{50 (W_1 + W_4)/W_o}{100} \right] \left(\frac{T_{xspi}^*}{50} - \frac{T_{xspi}^{**}}{50} \right) .$$

$$\frac{\dot{T}_{xsp}^*}{50} = \frac{2}{0.0075} \times \frac{2}{3600} \left[\frac{50 (W_1 + W_4)/W_o}{100} \right] \left(\frac{T_{xsp}}{50} - \frac{T_{xsp}^*}{50} \right) .$$

$$\frac{T_{xspo}}{50} = 2 \frac{T_{xsp}^*}{50} - \frac{T_{xspi}^{**}}{50} .$$

$$\frac{\dot{T}_{xspo}^*}{50} = \frac{2}{0.0003 \times 3600} \left[\frac{50 (W_1 + W_4)/W_o}{100} \right] \left(\frac{T_{xspo}}{50} - \frac{T_{xspo}^*}{50} \right) .$$

$$\frac{T_{fs3}}{50} = \frac{\left[\frac{50 (W_1 + W_4)}{W_o} \right] \frac{T_{xspo}^*}{50} + \left(\frac{50 W_3}{W_o} \right) \left(\frac{T_{co}}{50} \right) + \left(\frac{50 W_2}{W_o} \right) \left(\frac{T_{csi}}{50} \right)}{50 (W_1 + W_4)/W_o} .$$

$$\frac{T_{xspi}}{50} = \frac{\left(\frac{50 W_1}{W_o} \right) \frac{T_{csi}}{50} + \left(\frac{50 W_4}{W_o} \right) \frac{T_{fs3}}{50}}{50 \frac{(W_1 + W_4)}{W_o}} .$$

Furnace

$$2R = 2 \frac{\frac{50 W_{fa}^*}{(W_{fa} + W_{ff})_o}}{10 \left[\frac{500 W_{ff}^*}{(W_{fa} + W_{ff})_o} \right]} 100 \quad .$$

$$\frac{T_{fgf}}{50} = 100 \left(\frac{0.80}{50} \right) + \frac{100 (0.2000)}{100 (0.0471) + (0.6375) (2R)} 100 \quad .$$

$$\frac{T_{fg2}}{50} = \frac{T_{fgf}}{50} - (0.3250) \frac{\frac{1}{2} \left(\frac{T_{fg1}}{50} - \frac{T_{fsr}}{50} \right)}{50 \frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o}} 100 \quad .$$

$$\frac{T_{fg1}}{50} = (0.2250) \frac{T_{fgf}}{50} + (0.7500) \frac{T_{fg2}}{50} \quad .$$

$$\begin{aligned} \frac{T_{fsr}}{50} = \frac{2 \times 2 \times 272.6}{3600} \left[\frac{16.4}{2 \times 2 \times 272.6} \left(\frac{T_{fg1}}{50} - \frac{T_{fsr}}{50} \right) \right. \\ \left. - 50 \left(\frac{W_1 + W_2 + W_3}{100} \right) \left(\frac{T_{fsr}}{50} - \frac{T_{fs1}}{50} \right) \right] \quad . \end{aligned}$$

$$\frac{T_{csi}}{50} = 2 \frac{T_{fsr}}{50} - \frac{T_{fs1}}{50} \quad .$$

$$\begin{aligned} \frac{T_{fse}}{50} = \frac{2 \times 2 \times 924}{3600} \left\{ \frac{83 \times 2}{2 \times 2 \times 924} \frac{50}{100} \left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6} \frac{\Delta T_m}{50} \right. \\ \left. - \frac{50 \frac{(W_1 + W_2 + W_3)}{W_o}}{100} \left[\frac{T_{fsr}}{50} - \frac{T_{fs2}}{50} \right] \right\} \quad . \end{aligned}$$

$$\frac{\Delta T_m}{50} = \frac{1}{2} \left[\frac{T_{fg3}}{50} - \frac{2 T_{fse}}{50} + \frac{T_{fg4}}{50} \right] .$$

$$\frac{T'_{fg4}}{50} = \frac{T_{fg3}}{50} - (0.4870) \frac{\Delta T_m}{50} \frac{\left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6}}{\left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]} .$$

$$\frac{T_{fs2}}{50} = 2 \frac{T_{fse}}{50} - \frac{T_{fs3}}{50} .$$

$$\frac{\dot{T}_{fs2}^*}{50} = \frac{2}{0.0032 \times 3600} \frac{50 \frac{(W_1 + W_3)}{W_o}}{100} \left(\frac{T_{fs2}}{50} - \frac{T_{fs2}^*}{50} \right) .$$

$$\begin{aligned} \frac{\dot{T}_{fsc}}{50} = \frac{2 \times 2 \times 52.2}{3600} & \left\{ \frac{13.05 \times 2}{2 \times 2 \times 52.2} \frac{50}{100} \left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6} \frac{\Delta T_{ln}}{50} \right. \\ & \left. - \frac{50 \frac{(W_1 + W_2 + W_3)}{W_o}}{100} \left(\frac{T_{fsc}}{50} - \frac{T_{fs2}^*}{50} \right) \right\} . \end{aligned}$$

$$\frac{\Delta T_{ln}}{50} = \frac{(0.2500) \left[\frac{1}{2} \left(\frac{T_{fg2}}{50} - \frac{T_{fs1}}{50} \right) - \frac{1}{2} \left(\frac{T_{fg2}}{50} - \frac{T_{fs2}}{50} \right) (10) \frac{1}{10} \right]}{\frac{1}{14.7372} 50 \log_{10} \left[\frac{\frac{T_{fg2}}{100} - \frac{T_{fs1}}{100}}{\frac{T_{fg2}}{100} - \frac{T_{fs2}^*}{100}} \right]} .$$

$$\dot{T}_{fg3} = 10 \frac{50}{100} \frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \left(\frac{T_{fg2}}{50} - \frac{T_{fg3}}{50} \right) - \left[10 + (0.3500) (10) \right] \frac{50}{100} \left[\frac{(W_{fa} + W_{ff})^*}{(W_{fa} + W_{ff})_o} \right]^{0.6} \frac{\Delta T_{ln}}{50} .$$

Cooler

$$\frac{T_{cso}}{50} = 2 \frac{T_{cs}}{50} - \frac{T_{csi}}{50} .$$

$$\frac{\dot{T}_{cso}^*}{50} = \frac{1}{10 \tau_{cso}} \left[(10) \frac{T_{cso}}{50} - (10) \frac{T_{cso}^*}{50} \right] .$$

$$\frac{\epsilon_{v7}}{50} = \frac{T'_{cso}}{50} - \frac{T_{cso}^*}{50} .$$

$$100 \frac{\Delta W_{ca}}{W_{cao}} = \frac{1}{10 \left(\frac{\alpha_{3v7}}{5000} \right)} \left[10 K_{1v7} \int \frac{\epsilon_{v7}}{50} dt + 10 \frac{\epsilon_{v7}}{50} \right] .$$

$$100 \frac{\Delta W_{ca}^*}{W_{cao}} = \frac{1}{10 \tau_{ca}} \left[(10) 100 \frac{\Delta W_{ca}}{W_{cao}} - (10) 100 \frac{\Delta W_{ca}^*}{W_{cao}} \right] .$$

$$50 \frac{W_{ca}^*}{W_{cao}} = \frac{1}{2} \left\{ 100 \left(\frac{\bar{W}_{ca}}{W_{cao}} \right) - 100 \frac{\Delta W_{ca}^*}{W_{cao}} \right\} .$$

$$\frac{1}{2} \left[\frac{T_{cao} + T_{cai}}{50} \right] = \frac{0.686}{2} \left(\frac{\Delta T_{cm}}{50} \right) \left[\frac{50 \left(\frac{W_{ca}^*}{W_{cao}} \right)^{0.6}}{50 \left(\frac{W_{ca}}{W_{cao}} \right)} \right] + 100 \left(\frac{T_{cai}}{5000} \right) .$$

$$\frac{\Delta T_{cm}}{50} = \frac{T_{cs}}{50} - \frac{1}{2} \left(\frac{T_{cao}}{50} - \frac{T_{csi}}{50} \right) .$$

$$\begin{aligned} \frac{\dot{T}_{cs}}{50} = & \frac{2 \times 2 \times 424}{3600} \left\{ \frac{50 W_3/W_o}{100} \left(\frac{T_{cs}}{50} - \frac{T_{csi}}{50} \right) \right. \\ & \left. - \frac{2 (99.1)}{2 \times 2 \times 424} \frac{50 (W_{ca}/W_{cao})^{0.6}}{100} \frac{\Delta T_{cm}}{50} \right\} . \end{aligned}$$

$$\frac{50 W_{sp}}{W_o} = \frac{50(W_1 + W_4)}{W_o} .$$

$$\frac{50 W_{sp}^*}{W_o} = \frac{1}{10 (0.1000)} (10) \left[\frac{50 W_{sp}}{W_o} - \frac{50 W_{sp}^*}{W_o} \right] .$$

$$50 \epsilon_{sp} = \frac{50 W'_{sp}}{W_o} - \frac{50 W_{sp}^*}{W_o} .$$

$$\begin{aligned} 100 \Delta \phi_{v6} = & \frac{1}{10 (\alpha_{v6}/2)} \left[K_{1v6} \int 10 (50 \epsilon_{sp}) dt \right. \\ & \left. + 10 (50 \epsilon_{sp}) + (10) (K_{2v6}) (50 \epsilon'_{sp}) \right] . \end{aligned}$$

$$100 \phi_{v6} = 100 (\Phi_{v6}) + 100 \Delta \phi_{v6} .$$

$$100 \phi_{v6}^* = \left(\frac{1}{0.6300} \right) (0.6300) \left[100 \phi_{v6} - 100 \phi_{v6}^* \right] .$$

$$\frac{\dot{Y}_{v6}}{2} = - \left(0.6300 \right) \left(100 \dot{\phi}_{v6} \right) (10) \left(\frac{Y_{v6}}{2} \right) \frac{1}{100} .$$

$$\frac{\dot{T}_{fs3}}{50} = \frac{1}{10 (0.1000)} (10) \left[\frac{T_{fs3}}{50} - \frac{T_{fs3}^*}{50} \right] \quad .$$

$$\frac{\epsilon_{fs3}}{50} = \frac{T_{fs3}'}{50} - \frac{T_{fs3}^*}{50} \quad .$$

$$100 \Delta \phi_{v2} = \frac{1}{\frac{\alpha_{v2}}{5000}} \left[K_{1v2} \int \frac{\epsilon_{fs3}}{50} dt + \frac{\epsilon_{fs3}}{50} + 10 \left(\frac{K_{2v2}}{10} \right) \frac{\epsilon_{fs3}}{50} \right] \quad .$$

$$100 \phi_{v2} = 100 (\Phi_{v2}) + 100 \Delta \phi_{v2} \quad .$$

$$100 \phi_{v2}^* = \left(\frac{1}{0.6020} \right) (0.6020) \left[100 \phi_{v2} - 100 \phi_{v2}^* \right] \quad .$$

$$\frac{\dot{Y}_{v2}}{5} = (0.6020) \left(100 \dot{\phi}_{v2} \right) (10) \left(\frac{\left(\frac{Y_{v2}}{5} \right)}{100} \right) \quad .$$

Controls

$$\frac{50 W_{sh}}{W_o} = \frac{50 (W_1 + W_2 + W_3)}{W_o} \quad .$$

$$\frac{50 \dot{W}_{sh}^*}{W_o} = \frac{1}{10 \tau_{sh}} (10) \left[\frac{50 W_{sh}}{W_o} - \frac{50 W_{sh}^*}{W_o} \right] \quad .$$

$$50 \epsilon_{sh} = \frac{50 W_{sh}'}{W_o} - \frac{50 W_{sh}^*}{W_o} \quad .$$

$$10 \Delta \phi_{v3} = \frac{1}{10 \left(\frac{\alpha_{v3}}{2} \right)} \left[K_{1v3} \int 50 \epsilon_{sh} dt + 50 \epsilon_{sh} + K_{2v3} 50 \dot{\epsilon}_{sh} \right] \quad .$$

$$100 \phi_{v3} = 100 \Phi_{v3} + 10 (10 \Delta \phi_{v3}) \quad .$$

$$100 \dot{\phi}_{v3}^* = (0.6020) (100 \phi_{v3} - 100 \phi_{v3}^*) \left(\frac{1}{0.602} \right) \quad .$$

$$\frac{\dot{Y}_{v3}}{5} = -(0.6020) (100 \dot{\phi}_{v3}) (10) \frac{\left(\frac{Y_{v3}}{5} \right)}{100} \quad .$$

$$\frac{T_{xspi}^{***}}{50} = \frac{1}{10 (0.1000)} \left[10 \frac{T_{xspi}}{50} - 10 \frac{T_{xspi}^{***}}{50} \right] \quad .$$

$$\frac{\epsilon_{xspi}}{50} = \frac{T_{xspi}'}{50} - \frac{T_{xspi}^{***}}{50} \quad .$$

$$10 \Delta \phi_{v4} = \frac{1}{10 \left(\frac{\alpha_{v4}}{500} \right)} \left[K_{1v4} \int 10 \frac{\epsilon_{xspi}}{50} dt + 10 \frac{\epsilon_{xspi}}{50} \right. \\ \left. + 10 K_{2v4} \frac{\epsilon_{xspi}}{50} \right] \quad .$$

$$100 \phi_{v4} = 100 \Phi_{v4} + 10 (10 \Delta \phi_{v4}) \quad .$$

$$100 \dot{\phi}_{v4} = \frac{1}{(0.6300)} (0.6300) \left[100 \phi_{v4} - \phi_{v4}^* \right] \quad .$$

$$\frac{\dot{Y}_{v4}}{2} = (10) (0.6300) (100 \dot{\phi}_{v4}) \frac{\left(\frac{Y_{v4}}{2} \right)}{100} \quad .$$

$$\frac{Y_{v1}}{2} = 100 (0.0093) \frac{1}{10 \left(\frac{Y_{v4}}{2} \right)} 100 \quad .$$

$$\frac{\dot{T}_{fso}}{50} = \frac{1}{10 (0.1000)} 10 \left[\frac{T_{fso}}{50} - \frac{T_{fso}^*}{50} \right] .$$

$$\frac{\epsilon_{fso}}{50} = \frac{T'_{fso}}{50} - \frac{T_{fso}^*}{50} .$$

$$50 \delta = \frac{1}{10 \left(\frac{\alpha_{v11}}{25,000} \right)} \left[\frac{K_{1v11}}{10} \int 10 \frac{\epsilon_{fso}}{50} dt + \frac{\epsilon_{fso}}{50} + \frac{1}{\frac{10}{K_{2v11}}} (10) \frac{\dot{\epsilon}_{fso}}{50} \right] .$$

$$\frac{500 W_{ff}}{(W_{fa} + W_{ff})_o} = (0.6024) (10) \left[100 \frac{\epsilon_{fso}}{50} \right] 50 \frac{(W_1 + W_2 + W_3)}{\frac{W_o}{100}} + 10 (50 \delta) .$$

$$\frac{50 W_{fa}}{(W_{fa} + W_{ff})_o} = 100 (0.01589) + (0.1883) (10) \frac{(500 W_{ff})}{(W_{fa} + W_{ff})_o} - 50 \eta .$$

$$50 \eta = \frac{1}{10 K_{v10}} \left[K_{1v10} \int 10 (50 \epsilon) dt + 10 (50 \epsilon) \right] .$$

$$50 \epsilon = \frac{50 W_{fa}^*}{(W_{fa} + W_{ff})_o} - 100 (0.0159) - (0.1883) (10) \frac{500 W_{ff}^*}{(W_{fa} + W_{ff})_o} .$$

$$\frac{50 \dot{W}_{fa}^*}{(W_{fa} + W_{ff})_o} = \frac{1}{10 (0.1000)} (10) \left[\frac{50 W_{fa}}{(W_{fa} + W_{ff})_o} - \frac{50 W_{fa}^*}{(W_{fa} + W_{ff})_o} \right] .$$

$$\frac{50 \dot{W}_{ff}}{(W_{fa} + W_{ff})_o} = \frac{1}{10 (0.1000)} (10) \left[\frac{50 W_{ff}}{(W_{fa} + W_{ff})_o} - \frac{50 W_{ff}^*}{(W_{fa} + W_{ff})_o} \right] .$$

E. COMPUTER SCHEMATICS

Figures 36 through 41, the schematics of the main sodium system elements, are identified as follows: Figure 36, Hydraulics; Figure 37, Intermediate Heat Exchanger; Figure 38, Furnace; Figure 39, Cooler; Figure 40, Furnace Sodium Flow and IHX Controls; and Figure 41, Furnace Temperature Controls.

The complete system was separated arbitrarily into the subsystems for convenience of representation and discussion. No fixed criterion defines the individual schematics; and the computer identification merely indicates on which console most of the circuiting is located.

F. POTENTIOMETER SETTINGS

The "pot sheets" of Tables 9, 10, and 11 describe the potentiometers and their settings for computer consoles A, B, and C, respectively. Each setting may be identified with a coefficient of the machine equations, and the pot numbers may be identified on the schematic diagrams.

G. DIODE FUNCTION GENERATOR (DFG) SETUP

Diode function generators were used to generate fractional powers of different flows. Table 12 indicates the identity of functions generated by four DFG's. The input to each DFG is a flow of a given sign. The DFG was required to modify the flow to the indicated fractional power, and the output was either maintained at the same sign as the input, or it was reversed.

The diode function generator incrementally modifies the input function to provide the output. For each function, twenty separate points as well as sign and zero value must be established by resistor-diode networks. By selecting diode bias voltages and appropriate resistor networks, the break-points and slopes of segments of a monotonic curve may be synthesized. Table 12 indicates the break points (B. P. voltage) and the DFG output voltage at that break-point for each of the four DFG's.

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Figure 36. Hydraulics Schematic

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Figure 38. Furnace Schematic

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7593-1075

Figure 40. Schematic of Furnace Sodium Flow and IHX Controls

Figure 41. Schematic of Furnace Temperature Controls

7593-1076

TABLE 9

PACE 31-R POT SHEET
MAIN SODIUM SYSTEM (Machine 1)

PROBLEM _____
BY _____

PACE 31-R POT SHEET

____ DATE
A COMPUTER

POTENTIOMETER		
NO.	SET	DESCRIPTION
01		
02		
03		
04		
05		
06		
07	0045	$\frac{1}{100} \times \frac{1}{2} \times 0.905$
08	3750*	I. C. 50 W ₁ /W ₀
09		
10		
11		
12		
13		
14		
15	0079	$\frac{1}{100} \times \frac{1}{2} \times 1.58$
16	1250*	I. C. 50 W ₄ /W ₀
17		
18		
19		
20		

POTENTIOMETER		
NO.	SET	DESCRIPTION
21		
22		
23	0086	$\frac{1}{100} \times \frac{1}{5} \times 4.28$
24	0060*	I. C. 50 W ₂ /W ₀
25		
26		
27		
28		
29		
30		
31	1712	$\frac{1}{100} \times \frac{1}{5} \times 85.60$
32	1190*	I. C. 50 W ₃ /W ₀
33		
34		
35	1000	1/10
36		
37		
38	1250	1/8
39	0390	$\frac{1}{100} \times \frac{1}{2} \times 7.79$
40		

POTENTIOMETER		
NO.	SET	DESCRIPTION
41	1040	$\frac{1}{100} \times \frac{1}{10} \times 104$
42		
43		
44		
45		
46		
47		
48		
49		
50	1250	1/8
51		
52		
53		
54	8990	(1/0.5288)-1
55		
56		
57		
58	1250	1/8
59		
60	0175	0.873 x 1/50

POTENTIOMETER		
NO.	SET	DESCRIPTION
61	5407	$\frac{1}{50} \times 2 \times 13.518$
62		
63		
64		
65		
66	0500	1/20
67		
68		
69		
70	8621	1/10 (25/2.9)
71		
72		
73		
74	0500	1/20
75		
76		
77		
78		
79		
80		

1/17/ 2-24:5

*Add 0.0050 to compensate direction-limiting diode

TABLE 10
 PACE 231-R POT SHEET
 MAIN SODIUM SYSTEM (Machine 2)

12/13/62 DATE
 B COMPUTER

P-SERIES		POT NO.	Q-SERIES	
DESCRIPTION	SET		SET	DESCRIPTION
		00		
		01		
		02		
		03	1487	$2 \times 2 \times \frac{1}{0.0075} \times \frac{1}{3600}$
		04		
$\frac{1}{10} \times 0.0293 \times 6.56$	0192	05		
		06		
		07		
		08	0743	$2 \times \frac{1}{3600} \times \frac{1}{0.0075}$
I.C. $\frac{1}{2} W_5/W_0$	3668	09	0471	0.0471
		10		
		11		
		12		
		13	2153	$\frac{1}{3600} \times 2 \times \frac{1}{0.00258}$
		14		
I.C. TFS _E /5000	1841	15	1736	$\frac{1}{3600} \times \frac{1}{0.0032}$
I.C. TFS ₂ /5000	1882	16	4870	0.4870
I.C. TFS _C /5000	2135	17	0580	$\frac{1}{3600} \times 2 \times 2 \times 52.2$
		18	0449	$\frac{1}{2} \times \frac{1}{2} \times 83/924$
		19	1027	$\frac{1}{3600} \times 2 \times 2 \times 924$
		20	1554	I.C. T _{XSS} I/5000
		21		
		22		
		23	4080	$\frac{1}{3600} \times 2 \times \frac{1}{0.01362}$
		24	3047	$\frac{1.36 \times 6.56 \times 10000}{2 \times 2 \times 2 \times 100 \times 366}$

P-SERIES		POT NO.	Q-SERIES	
DESCRIPTION	SET		SET	DESCRIPTION
$[\frac{1}{2} \times 8 \times \frac{1}{2.3026}] - 1$	7372	25		
I.C. TFG ₃ /5000	2991	26	2500	1/4
$[\frac{1}{2} \times 13.05/52.2] - 1$	2500	27	3500	1.35 - 1
		28	1000	1/10
		29		
I.C. TXSS/5000	1963	30	4067	$\frac{1}{3600} \times 2 \times 2 \times 366$
I.C. TXSP/5000	2102	31		
		32		
		33	0829	$\frac{1}{3600} \times 2 \times 2 \times 74.6$
		34	3045	$\frac{0.277 \times 6.56 \times 10000}{2 \times 2 \times 2 \times 100 \times 74.6}$
		35	32.50	$2 \times \frac{1}{2} \times 1.3250$
0.2250	2250	36	7750	0.7750
		37	1504	$\frac{1}{2} \times \frac{1}{2} \times \frac{16.4}{272.6}$
I.C. TFS _E /5000	2494	38	3029	$\frac{1}{3600} \times 2 \times 2 \times 272.6$
$1000 \times \frac{1}{50} \times \frac{1}{1000}$	2000	39	0130	$\frac{1}{100} \times 80/50$
1/T _{C50}	1000	40	2593	LIMITER
1/T _{C50}	1000	41	2593	LIMITER
		42	4372	I.C. $\frac{1}{2} W_{CA}/W_{CRO}$
		43	3430	$\frac{1}{2} \times 0.686$
		44	0200	I.C. T _{CAI} /5000
1/10TV ₇	1000	45	0200	K _{3V7} /5000
1/10TV ₇	1000	46	5000	10K _{1V7}
		47	6375	$\frac{1}{2} \times 100 \times 0.01275$
I.C. T _{CS} /5000	2200	48	4711	$\frac{1}{3600} \times 2 \times 2 \times 424$
$\frac{1}{2} \times \frac{1}{2} \times 2 \times 99/424$	1169	49	1800	I.C. T _{C50} /5000

TABLE 11

PACE 231-R POT SHEET
MAIN SODIUM SYSTEM (Machine 3)

12/13/62 DATE
C COMPUTER

P-SERIES		POT NO.	Q-SERIES	
DESCRIPTION	SET		SET	DESCRIPTION
I.C. $\gamma_{V6}/200$	0737	00	1000	TIMER CKT
		01		
		02	6300	$\frac{1}{10} \times 6.3$
$0.63 - 1$	5873	03		
		04		
I.C. $\frac{1}{2} W_{1/4}/W_0$	5000	05	5520	K_{IV6}
$1/10 T_{SH}$	1000	06	1000	$1/10 T_{SH}$
		07	2000	RAMP CKT
$K_{3V6}/2$	7085	08	0000	K_{2V6}
I.C. Φ_{V6}	3596	09	4113	$\frac{1}{3600} \times \frac{1}{2000229} \times 2$
I.C. $\gamma_{V4}/200$	2520	10		
LIMITER	9999	11		
LIMITER	9999	12	6300	$\frac{1}{10} \times 6.3$
$0.63 - 1$	5873	13		
		14		
I.C. $T_{PXI}/5000$	2400	15	2970	K_{IV4}
$1/10 T_{XSPI}$	1000	16	1000	$1/10 T_{XSPI}$
$\frac{1}{4} \times \frac{1}{100} \times e^{3.62}$	0093	17	0159	$\frac{1}{2} \times 9.03178$
$K_{3V4}/500$	1905	18	3723	K_{2V4}
I.C. Φ_{V4}	1650	19	1883	$\frac{1}{10} \times \frac{1}{10} \times 18.83$
I.C. $\gamma_{V3}/500$	3449	20		
		21		
		22	6020	$\frac{1}{10} \times 6.02$
$0.604 - 1$	6611	23		
		24		

P-SERIES		POT NO.	Q-SERIES	
DESCRIPTION	SET		SET	DESCRIPTION
		25		
		26		
		27	1000	$1/T_{FA}$
		28		
		29	1000	$1/T_{FA}$
I.C. $\gamma_{V2}/2500$	8232	30		
LIMITER	3700	31		
LIMITER	3700	32	6020	$\frac{1}{10} \times 6.02 \times \frac{1}{T_{V2}}$
$0.602 - 1$	6611	33		
		34		
I.C. $T_{FS3}/5000$	1800	35	1000	K_{IV2}
$1/10 T_{FS3}$	1000	36	1000	$1/T_{FS3}$
I.C. $T_{FS3}^*/5000$	1800	37	2600	I.C. $T_{FS0}/5000$
$K_{3V2}/5000$	1985	38	0000	K_{2V2}
$1/100$	6100	39	0360	I.C. $\Phi_{V2} \times 100$
LIMITER	0238	40	1000	$1/10 T_{FF}$
LIMITER	0238	41	1883	$\frac{1}{10} \times \frac{1}{10} \times 18.83$
K_{IV10}	0500	42	0159	$\frac{1}{2} \times 9.03178$
		43		
		44		
$K_{IV11}/10$	0200	45	1000	$10 K_{3V10}$
		46	3968	$10/K_{2V11}$
		47		
$K_{3V11}/25000$	6567	48	6024	$\frac{1}{4} \times 500 \times 0.04819$
$\frac{1}{2} \left[\frac{W_{FA}}{(W_{FA} + W_{FE})_0} \right]_{\min}$	0241	49	0241	$\frac{1}{2} \left[\frac{W_{FA}}{(W_{FA} + W_{FE})_0} \right]_{\min}$

TABLE 12

FRACTIONAL POWERS FROM DIODE FUNCTION GENERATORS

BP Voltage	Break- Points	DFG No. 1	DFG No. 2	DFG No. 3	DFG No. 4
-100	-9			7.50	
90	-9			7.83	
80	-7			8.20	
70	-6			8.65	
60	-5			9.20	
50	-4			9.90	
40	-3			10.82	
30	-2			12.14	
20	-1			14.28	
10	-0			18.85	
0	-	-	-	-	-
+ 10	+0	19.54	65.66		19.54
20	+1	28.85	43.35		28.85
30	+2	36.80	33.97		36.80
40	+3	43.74	28.71		43.74
50	+4	50.00	25.00		50.00
60	+5	55.78	22.41		55.78
70	+6	61.19	20.43		61.19
80	+7	66.29	18.86		66.29
90	+8	71.14	17.57		71.14
100	+9	75.79	16.49		75.79
				<u>Input Sign</u>	<u>Output Sign</u>
DFG1:	$50 (W_a + W_f) / (W_a + W_f)_o^{0.6}$			-	+
DFG2:	$25 (W_{sp} / W_{spo})^{-0.6}$			+	+
DFG3:	$9.9 (W_{ss} / W_{sso})^{-0.4}$			+	-
DFG4:	$50 (W_{ca} / W_{cao})^{0.6}$			-	+

H. DISCUSSION OF CONTROLS CIRCUITS

The basic configuration of the controllers is typified by the cooler sodium flow controller (Figure 42). This controller will be described in some detail. Deviations from the basic configuration are used for the other controllers. See Figure 1: "Primary Sodium Side Control System Schematic," may be used in reference.

Figure 42. Cooler Sodium Flow Controller

1. Normalization

The furnace sodium flow W_{123} is generated (see Figure 1) by summing three flows, thus,

$$W_{123} = W_1 + W_2 + W_3 \quad .$$

Normalized to W_0 , the above relationships may be written

$$\frac{W_{123}}{W_0} = \frac{W_1}{W_0} + \frac{W_2}{W_0} + \frac{W_3}{W_0} \quad .$$

The flow W_{123}/W_0 is compared to a demand value, W'_{123}/W_0 , and control is exercised on an error signal to modify the valve position.

2. Instrumentation Delay

The physical comparison of W_{123}/W_0 to W'_{123}/W_0 entails an instrumentation time delay, τ_{123} , and the relationship describing the delay may be approximated by

$$\frac{\dot{W}_{123}}{W_0} = \left(\frac{1}{\tau_{123}} \right) \left(\frac{W'_{123}}{W_0} - \frac{W^*_{123}}{W_0} \right) ,$$

where W_{123}/W_0 is the flow at the point of measurement, and W^*_{123}/W_0 is the flow at the point of comparison with the demand setpoint W'_{123}/W_0 . Then τ_{123} is the time delay through the pipe line between the flow line and the controller. (The asterisk * designates delayed entity and is delay-time in seconds. Throughout the system, time delays were set for either 1 or 1/10 sec.)

3. Error Signal

The delayed variable W^*_{123}/W_0 is compared with the set-point W'_{123}/W_0 to generate the error signal, ϵ_{123} , which is the input to the controller; i. e.,

$$\frac{W'_{123}}{W_0} - \frac{W^*_{123}}{W_0} = \epsilon_{123} .$$

4. Controller Modes

A 3-mode controller provides for proportional, integral (reset), and rate control to modify the valve position. For linear operation, the mathematical statement of this control action is

$$\Delta\phi_{v3} = \frac{1}{\alpha_{v3}} \left[K_{1v3} \int \epsilon_{123} dt + \epsilon_{123} + K_{2v3} \frac{d}{dt} \epsilon_{123} \right] ,$$

where $\Delta\phi_{v3}$ is the incremental change in the opening of Valve 3, α_{v3} is the gain of the proportional band, K_{1v3} is the reset adjustment, and K_{2v3} is the rate adjustment.

The above relationship can be simplified to

$$\Delta\phi_{v3} = \Delta\phi_{v3a} + \Delta\phi_{v3b} + \Delta\phi_{v3c} \quad ,$$

where

$$\Delta\phi_{v3a} = \left[\frac{K_{1v3}}{\alpha_{v3}} \right] \left[\int \epsilon_{123} dt \right] \quad , \quad (\text{integral}),$$

$$\Delta\phi_{v3b} = \left[\frac{1}{\alpha_{v3}} \right] \dot{\epsilon}_{123} \quad (\text{proportional}), \text{ and}$$

$$\Delta\phi_{v3c} = \left[\frac{K_{2v3}}{\alpha_{v3}} \right] \frac{d}{dt} \epsilon_{123} \quad (\text{rate}) .$$

Disregarding, for the moment, the effect of nonlinearities such as valve position limits and integration limits, the controller operation will be described for small changes in valve position.

5. Valve Position Demand

The three modes of control act on the relative values of set-point and measured value to change the position of the valve from its steady-state (or operating position) Φ_{v3} . The change command from the controller, $\Delta\phi_{v3}$, will open or close the valve by moving it from the position Φ_{v3} . The position at any time may be described by

$$\phi_{v3} = \Phi_{v3} + \Delta\phi_{v3}$$

$$0 < \phi_{v3} < 1.0 \quad ,$$

where

$$\phi_{v3} = 0 \text{ when the valve is closed,}$$

and

$$\phi_{v3} = 1 \text{ when the valve is fully open.}$$

In reality, ϕ_{v3} as defined above is merely the valve position demand. A second controller time delay, here defined as τ_{v3} , is associated with the command for the valve to change position and the actual position of the valve. The actual position of the valve may thus be designated as ϕ_{v3}^* to distinguish it from the command, and the relationship between command and position may be written

$$\dot{\phi}_{v3} = \left(\frac{1}{\tau_{v3}} \right) (\phi_{v3} - \phi_{v3}^*)$$

6. Valve Nonlinearity

The valve position and an exponential parameter

$$Y_{v3} = e^{7.54 - 6.04 \phi_{v3}^*}$$

modify the flow W_3 , which in turn reflects a change in W_{123} to maintain $W_{123}^* = W_{123}^1$. The exponential parameter relationship has been modified as follows in the simulation:

$$Y_{v3} = e^{+7.54 - 6.04 \phi_{v3}^*}$$

$$\dot{Y}_{v3} = -6.04 \left(e^{+7.54 - 6.04 \phi_{v3}^*} \right) \dot{\phi}_{v3}^*$$

$$\dot{Y}_{v3} = -6.04 Y_{v3} \dot{\phi}_{v3}^*$$

7. Controller Nonlinearities

The three nonlinearities possible in a basic controller system shall be designated as (a) valve position limit, (b) reset check, and (c) reset limit.

These three limits may be used in any combination, and their effects are not mutually exclusive. A simple explanation of these nonlinearities follows.

a. Valve Position Limit — Valve position limits simulate mechanical stops on valve travel in the physical system. One limit restricts the valve from moving to theoretical "more than closed" position; another limiter restricts the valve motion to prevent "more than fully open" positions. The mathematical description of valve position limits may be written

$$\phi_{v3} = \left| \begin{array}{ll} 0 \text{ (Fully closed)} & \text{at } \phi_{v3} < 0 \\ \Phi_{v3} + \Delta\phi_{v3} & \text{at } 0 < \phi_{v3} < 1 \\ 1 \text{ (fully open)} & \text{at } \phi_{v3} > 1 \end{array} \right|$$

b. Reset Check — The reset check is a relay circuit which checks the "windup" of the integral control at the level it has attained at the instant the valve is fully open or fully closed. Otherwise, the error signal which may still exist will cause the integral control to attain such a level that the control system will contribute to system instability.

The mathematical statement of the reset check is

$$\Delta\phi_{v3a} = \left| \begin{array}{ll} \frac{K_{1v3}}{\alpha_{v3}} \int_0^{\text{const}} \epsilon_{123} dt & \text{at } \phi_{v3}^* < 0 \\ \frac{K_{1v3}}{\alpha_{v3}} \int \epsilon_{123} dt & \text{at } 0 < \phi_{v3}^* < 1 \\ \frac{K_{1v3}}{\alpha_{v3}} \int_0^{\text{const}} \epsilon_{123} dt & \text{at } \phi_{v3}^* > 1 \end{array} \right|$$

c. Reset Limit — The reset limits may be set to limit the excursion of the integral control. In effect, this is a band-pass limiter similar to the valve position limiter, although its limits may have different levels. Its operation may be clarified by examination of the characteristic shown in Figure 43.

Figure 43. Reset Limiter

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Assuming valves are in the operating region, any error voltage will be modified by the reset coefficient K_{lv3} and will be integrated until a preset limit is attained. The error voltage and the coefficient define the rate at which the integral control circuit will reach the limit. The sign of the error will indicate which limit.

Once the limit is attained, the effect of the integral control is maintained constant until the error changes sign. (Note that a decrease in error will have no effect on the integral control once it has reached the limit).

The mathematical description of this control mode is as follows.

$$\Delta\phi_{v3a} = \begin{array}{|c|c|c|} \hline E_a & & \text{at } K_{lv3} \int_0^t \epsilon dt < E_a \\ \hline K_{lv3} \epsilon dt & & \text{at } E_a < K_{lv3} \int_0^t \epsilon dt < E_b \\ \hline E_b & & \text{at } K_{lv3} \int_0^t \epsilon dt > E_b \\ \hline \end{array}$$

8. Intereffects of Nonlinearities

The intereffects of the nonlinearities should be considered when all the control modes and all the nonlinearities are present. However, the discussion may be restricted to the reset limiters since the valve position limiter operation is independent of the other two.

The reset limits are also in the circuit at all times. Two situations should be considered:

- a) Reset limits are attained prior to the valve reaching its extreme position; and
- b) Reset limits are not attained when the valve has reached the extreme position.

In the former case the reset check is a redundant operation, since $\Delta\phi_{v3a}$ cannot change after the reset limit has been reached. To make the mode operative again, it will be necessary for the combined proportional and rate modes to overcome the limit value of the reset mode. The error value for this recuperation needs to be optimized.

If the valve reaches an extreme position before the reset attains its limit, or if the limit value is set too high (as may be the case when there is no limit), then the reset check circuit will hold the reset control contribution at the level it attained when the valve reached its extreme position. This value will not be constant and may be different (a) for the open and closed positions, (b) for different values of the coefficient K_{1v3} , and (c) for different values of the coefficient K_{2v3} .

When the reset mode is in the "hold" condition, it will again be necessary for the error to attain a value such that the combined contributions of the proportional and rate modes overcome the reset value. When this occurs, the reset check is removed, and the integral control again operates to reduce the error.

9. Summary of Controller Equations

The system equation for the furnace sodium flow rate controller may be summarized as follows.

$$\begin{aligned} \text{a) } \frac{W_{sh}}{W_0} &= \frac{(W_1 + W_2 + W_3)}{W_0} \\ \text{b) } \frac{\dot{W}_{sh}^*}{W_0} &= \left(\frac{1}{\tau_{sh}} \right) \left(\frac{W_{sh}/W_0}{W_{sh}^*/W_0} \right) \end{aligned}$$

$$c) \epsilon_{sh} = \frac{W'_{sh}}{W_0} - \frac{W^*_{sh}}{W_0}$$

$$d) \Delta\phi_{v3} = \Delta\phi_{v3a} + \Delta\phi_{v3b} + \Delta\phi_{v3c}$$

$$e) \alpha_{v3} \Delta\phi_{v3a} = \begin{cases} E_a & \text{at } K_{lv3} \int_0^t \epsilon dt < E_a \text{ and } 0 < \phi_{v3}^* < 1 \\ K_{lv3} \int_0^a \epsilon dt & \text{at } E_a < K_{lv3} \int_0^t \epsilon dt < E_b \text{ and } \phi_{v3}^* < 0 \\ K_{lv3} \int_0^a \epsilon dt & \text{at } E_a < K_{lv3} \int_0^t \epsilon dt < E_b \text{ and } 0 < \phi_{v3}^* < 1 \\ K_{lv3} \int_0^b \epsilon dt & \text{at } E_a < K_{lv3} \int_0^t \epsilon dt < E_b \text{ and } \phi_{v3}^* > 1 \\ K_b & \text{at } K_{lv3} \int_0^t \epsilon dt > E_b \text{ and } 0 < \phi_{v3}^* < 1 \end{cases}$$

Note: The above conditions apply only as the nonlinearity is attained from the normal operating level within the limits. In order for the system to make the transition back to normal operation after it has gone into the nonlinear region, the following conditions must be satisfied.

a) To recover from E_a , the error must change sign and

$$0 < \phi_{v3}^* < 1$$

b) To recover from $K_{1v3} \int_0^a \epsilon \, dt$, $\phi_{v3} > 1$

$$K_{1v3} \int_0^a \epsilon \, dt = \alpha_{v3} (\Delta\phi_{v3b} + \Delta\phi_{v3c}) \quad .$$

c) To recover from $K_{1v3} \int_0^b \epsilon \, dt$,

$$K_{1v3} \int_0^b \epsilon \, dt = \alpha_{v3} (\Delta\phi_{v3b} + \Delta\phi_{v3a}) \quad .$$

d) To recover from E_b ,

$$E_b = \alpha_{v3} (\Delta\phi_{v3b} + \Delta\phi_{v3c}) \quad .$$

$$e) \dot{\phi}_{v3}^* = \frac{1}{\tau_{v3}} (\phi_{v3} - \phi_{v3}^*) \quad ,$$

and

$$f) \dot{Y}_{v3} = -6.04 Y_{v3} \phi_{v3}^* \quad .$$

I. TRANSIENT STUDY TESTS

Three transient study tests were specified for the original study of the main sodium system. Subsequently, a fourth test was submitted and included.

Figure 44 illustrates the circuit required to instrument the tests and Table 13 is a program of switch and potentiometer settings for each of the four tests. In Figure 44,

- 1) QOOC setting establishes the range rate for the prescribed transient input;

W_{14} was required to be ramped from 100 to 10% in 12 sec, Q00C was set at

$$\frac{100}{12} = 0,0833 \quad .$$

TABLE 13
SWITCH AND POTENTIOMETER SETTINGS

Test	Check	1	2	3	4	Timer Limit
Q00C		0700*	0833	1100	1000	
Q07C		1000*	5131	0200	2019	
FS00	C	C	C	C	C	
FS03		R	R	C	R	
FS10		L	C	R	C	
FS12		C	C	C	R	
P09B	3668	4275	4275	3668	2019	$1/2 W_5/W_0$
P05C	5000	5700	5700	5000	3000	$1/2 W'_{14}/W_0$
P25C	5000	1250	1250	5000	2680	$1/2 W'_{123}/W_0$
P35C	1800	1800	1800	1800	2200	$T'_{fs3}/5000$
Q37C	2600	2400	2400	2600	2600	$T'_{fso}/5000$
		<u>Q00C</u>		<u>Q07C</u>		
*Test 1A		1167		0600		
Test 1B		0857		0800		

Amplifier 80 was limited at 100 v. When 100 v was attained, W_{14} was required to change from 57.00 to 05.70 v; i. e., the change was

$$\Delta W_{14} = 57.00 - 05.70 = 51.30$$

$$100 (Q_{07}) = 51.30 \quad .$$

Q_{07} was set at 0.5130 and the output of amplifier 70 was 51.30 v.

Amplifier A09B generated the controller demand W'_{14}/W_0 . Thus the switches were set so 51.30 v change in this demand forced the appropriate controller action.

When FS00 was switched, the ramp was initiated, and the transient test was in effect.

APPENDIX B MULTINODE REPRESENTATION OF IHX

The responses of an IHX multi-node representation are used for comparison purposes to determine the accuracy of the IXH single-node representation used in the primary loop transient study. The multinode representation responses are determined by subjecting the multi-node representation to the programmed transients of the primary loop transient study.

I. PRIMARY INLET AND OUTLET

An incremental section of the typical primary inlet and outlet region may be considered in the following manner.

The energy balance on this increment yields

$$W_{14} C_{ps} t_{yspo} + \frac{dQ}{d\theta} P_{xp} \Delta X_{xp} = \rho_s C_{ps} A_{xp} \Delta X_{xp} \frac{\partial t_{xsp}}{\partial \theta} + W_{14} C_{ps} t_{xsp} .$$

Q from a unit area of a slab initially at a uniform temperature and cooled by convection is obtained by using the physical properties and dimensions of the inlet and outlet regions and the analytical information from Reference 1, at end of this appendix, on the slab heat loss. An approximate equation for Q is

$$Q = 13.05 (t_{xspw} - t_{xsp}) (1 - e^{-98\theta}) .$$

Differentiating Q with respect to time and considering only solutions for short time periods give:

$$\frac{dQ}{d\theta} = 13.05 (98) (t_{xspw} - t_{xsp}) \quad ;$$

also,

$$\frac{dQ}{d\theta} = - \rho_w C_{p_w} \ell_w \frac{d t_{xspw}}{d\theta} \quad ;$$

therefore

$$\frac{d t_{xspw}}{d\theta} = \frac{-13.05 (98) (t_{xspw} - t_{xsp})}{\rho_w C_{p_w} \ell_w} \quad .$$

Substituting the $dQ/d\theta$ equation into the energy balance equation and rearranging give:

$$\begin{aligned} \frac{d t_{xsp}}{d\theta} + \left[\frac{W_{14} C_{p_s} + 13.05 (98) (P_{xp}) (\Delta X_{xp})}{\rho_s C_{p_s} A_{xp} \Delta X_{xp}} \right] t_{xsp} \\ = \frac{W_{14} C_{p_s} t_{xspo} + 13.05 (98) (P_{xp}) (\Delta X_{xp}) (t_{xspo})}{\rho_s C_{p_s} A_{xp} \Delta X_{xp}} \quad ; \end{aligned}$$

and, as found above,

$$\frac{d t_{xspw}}{d\theta} = \frac{-13.05 (98) (t_{xspw} - t_{xsp})}{\rho_w C_{p_w} \ell_w} \quad .$$

From the physical dimensions of the intermediate heat exchanger, the following values are obtained.

<u>Term</u>	<u>Primary Inlet Region</u>	<u>Primary Outlet Region</u>
$A_{xp} =$	2.76 ft ²	2.76 ft ²
$P_{xp} =$	9.4 ft	9.4 ft
$\Delta X_{xp} =$	3.21 ft	3.75 ft

<u>Term</u>	<u>Primary Inlet Region</u>	<u>Primary Outlet Region</u>
$\rho_s =$	51 lb/ft ³	57 lb/ft ³
$Cp_s =$	0.30 Btu/lb-°F	0.30 Btu/lb°F
$\rho_w Cp_w \ell_w =$	6.54 Btu/ft ² °F	6.54 Btu/ft ² °F

Writing the outlet primary equations in node form, with the notation used in the analog study, the final equations are as follows.

$$\begin{aligned} \frac{d t_{xsp12}}{d\theta} + \left[\frac{W_{14} (0.30) + 45,000}{158.3} \right] t_{xsp12} \\ = \frac{W_{14} (0.30) (t_{xsp2}) + 45,000 t_{xspw}}{158.3} \end{aligned}$$

and

$$\frac{d t_{xspw1}}{d\theta} = 196 (t_{xsp12} - t_{xspw1}) ,$$

where

$$t_{xsp12} = \frac{t_{xsp1} + t_{xsp2}}{2} .$$

In the same manner, the primary inlet equations yield

$$\begin{aligned} \frac{d t_{xsp1011}}{d\theta} + \left[\frac{W_{14} (0.30) + 38,600}{135.5} \right] t_{xsp1011} \\ = \left[\frac{W_{14} (0.30) t_{xsp11} + 38,600 t_{xspw11}}{135.5} \right] , \end{aligned}$$

and

$$\frac{d t_{xspw11}}{d\theta} = 196 (t_{xsp11} - t_{xspw11}) \quad ,$$

where

$$t_{xsp1011} = \frac{t_{xsp10} + t_{xsp11}}{2} \quad .$$

II. SECONDARY INLET AND OUTLET

An incremental section for the secondary inlet and outlet region may be approximated in the following manner.

The energy balance on this section yields

$$W_5 C_p_s t_{xss0} + \frac{d Q_t}{d\theta} (\rho_{xst} \Delta X_{xst}) + \frac{d Q_h}{d\theta} (\rho_{xsh} \Delta X_{xsh})$$

$$\rho_s C_p_s A_{xs} (\Delta X_{xs}) \frac{\partial t_{xss}}{\partial \theta} + W_5 C_p_s t_{xss} \quad .$$

The head structure and physical dimensions are such that the expression for Q_h and consequently $d Q_h / d\theta$ may be considered the same as for Q and $dQ/d\theta$ in the primary inlet and outlet region. Thus

$$Q_h = 13.05 \left[1 - e^{-98\theta} \right] (t_{xsswh} - t_{xss}) \quad ,$$

and, for short time intervals,

$$\frac{d Q_h}{d\theta} \approx 13.05 (98) (t_{xsswh} - t_{xss}) \quad .$$

Using the physical properties and dimensions of the tube sheets and the analytical information (Conduction of Heat in Solids, by Carslaw and Jaeger, p 125) on the heat loss Q_t from a unit area of a slab initially at a uniform temperature and cooled by convection, an approximate equation for Q_t is,

$$Q_t = 6.54 \left[1 - e^{-38\theta} \right] (t_{xsswt} - t_{xss}) \quad .$$

Differentiating Q_t with respect to time and considering only solutions for short time periods give:

$$\frac{d Q_t}{d\theta} = 6.54 (38) (t_{xsswt} - t_{xss}) \quad .$$

The above equations when combined with those relating the heat loss to the average temperature of a slab give:

$$\frac{d t_{xsswh}}{d\theta} = \frac{-13.05 (98)}{\rho_{wh} C_{pwh} \ell_{wh}} \left[t_{xsswh} - t_{xss} \right] \quad ,$$

and

$$\frac{d t_{xsswt}}{d\theta} = \frac{-6.54 (38)}{\rho_{wt} C_{pwt} \ell_{wt}} \left[t_{xsswt} - t_{xss} \right] \quad .$$

Substituting the $dQ_h/d\theta$ and $dQ_t/d\theta$ equations into the energy balance equations gives,

$$\begin{aligned} \frac{dt_{xss}}{d\theta} + \left[\frac{W_5 C_{p_s} + 13.05 (98) (\rho_{xsh}) (\Delta X_{xsh}) + 6.54 (38) (\rho_{xst}) (\Delta X_{xst})}{\rho_s C_{p_s} A_{xs} \Delta X_{xs}} \right] t_{xss} \\ = \frac{W_5 C_{p_s} t_{xss0} + 6.54 (38) (\rho_{xst}) (\Delta X_{xst}) t_{xsswt} + 13.05 (98) (\Delta X_{xsh}) t_{xsswh}}{\rho_s C_{p_s} A_{xs} \Delta X_{xs}} ; \end{aligned}$$

and, as found above,

$$\frac{dt_{xsswh}}{d\theta} = \frac{-13.05 (98)}{\rho_{wh} C_{p_{wh}} \ell_{wh}} \left[t_{xsswh} - t_{xss} \right] ,$$

and

$$\frac{dt_{xsswt}}{d\theta} = \frac{-6.54 (38)}{\rho_{wt} C_{p_{wt}} \ell_{wt}} \left[t_{xsswt} - t_{xss} \right] .$$

From the physical dimensions of the intermediate heat exchanger secondary inlet and outlet,

$$\begin{aligned} \Delta X_{xsh} \rho_{xsh} &= 14.35 \text{ ft}^2 \\ \Delta X_{xst} \rho_{xst} &= 14.6 \text{ ft}^2 \\ A_{xs} \Delta X_{xs} &= 7.33 \text{ ft}^3 . \end{aligned}$$

Writing the secondary inlet equations in node form, with the notation used in the analog study, the final equations are:

$$\begin{aligned} \frac{dt_{xss12}}{d\theta} + \left[\frac{W_5 (0.30) + 21,980}{112} \right] t_{xss12} \\ = \left[\frac{W_5 (0.30) t_{xss1} + 3630 t_{xsswt1} + 18,350 t_{xsswh1}}{112} \right] , \end{aligned}$$

$$\frac{d t_{xsswt1}}{d\theta} = 76 (t_{xss12} - t_{xsswt1})$$

$$\frac{d t_{xsswh1}}{d\theta} = 196 (t_{xss12} - t_{xsswt1})$$

when

$$t_{xss12} = \frac{t_{xss1} + t_{xss2}}{2} .$$

Doing the same for the secondary outlet equations yields

$$\begin{aligned} \frac{d t_{xss1011}}{d\theta} + \left[\frac{W_s (0.30) + 21,980}{112} \right] t_{xss1011} \\ = \left[\frac{W_5 (0.30) t_{xss10} + 3630 t_{xsswt11} + 18,350 t_{xsswh11}}{112} \right] \end{aligned}$$

$$\frac{d t_{xsswt11}}{d\theta} = 76 (t_{xss1011} - t_{xsswt11})$$

$$\frac{d t_{xsswh11}}{d\theta} = 196 (t_{xss1011} - t_{xsswh11}) ,$$

where

$$t_{xss1011} = \frac{t_{xss10} + t_{xss11}}{2} .$$

III. HEAT TRANSFER REGION

An incremental section of the heat transfer region may be considered in the following manner.

$$q_x = U_x P_x (\Delta X_x) (t_{xsp} - t_{xss})$$

The energy balance on this increment yields

$$\begin{aligned} W_{14} Cp_s t_{xsp} &= W_{14} Cp_s t_{xsp} - W_{14} Cp_s \frac{\partial t_{xsp}}{\partial X} \Delta X_x \\ &+ \rho_{xc} Cp_{xc} A_{xc} (\Delta X_x) \frac{\partial t_{xc}}{\partial \theta} + \rho_s A_{xp} Cp_s (\Delta X_x) \frac{\partial t_{xsp}}{\partial \theta} \\ &+ U_x P_x (\Delta X_x) (t_{xsp} - t_{xss}) + \frac{\rho_{xw} Cp_{xw} A_{xw} (\Delta X_x)}{2} \frac{\partial t_{xw}}{\partial \theta} \end{aligned}$$

and

$$\begin{aligned} W_5 Cp_s t_{xss} &+ U_x P_x (\Delta X_x) (t_{xsp} - t_{xss}) \\ &= W_5 Cp_s t_{xss} + W_5 Cp_s \frac{\partial t_{xss}}{\partial X} \Delta X_x \\ &+ \rho_s A_{xs} Cp_s (\Delta X_x) \frac{\partial t_{xss}}{\partial \theta} + \frac{\rho_{xw} Cp_{xw} A_{xw}}{2} (\Delta X_x) \frac{\partial t_{xw}}{\partial \theta} \end{aligned}$$

Using the standard assumptions for this type of analysis, it is assumed that

$$\frac{\partial t_{xc}}{\partial \theta} = \frac{\partial t_{xsp}}{\partial \theta}$$

and the heat storage of the tube is equally divided between the primary and secondary flows. Then

$$\frac{\partial t_{xwa}}{\partial \theta} = \frac{\partial t_{xsp}}{\partial \theta}$$

and

$$\frac{\partial t_{xwb}}{\partial \theta} = \frac{\partial t_{xss}}{\partial \theta} .$$

Substituting the mathematics of the assumptions and rearranging, the primary and secondary energy balance equations reduce to

$$\begin{aligned} \frac{\partial t_{xsp}}{\partial \theta} + \left[\frac{U_x P_x}{\rho_s A_{xp} C_{p_s} + \rho_{xc} C_{p_{xc}} A_{xc} + \frac{\rho_{xw} C_{p_{xw}} A_{xw}}{2}} \right] (t_{xsp} - t_{xss}) \\ = \frac{W_{14} C_{p_s} \left(\frac{\partial t_{xsp}}{\partial X} \right)}{\left[\rho_s A_{xp} C_{p_s} + \rho_{xc} C_{p_{xc}} A_{xc} + \frac{\rho_{xw} C_{p_{xw}} A_{xw}}{2} \right]} , \text{ and} \end{aligned}$$

Using the physical dimensions of the intermediate heat exchanger (Table 14) and the nodal distributions as set forth for the three sections, the equations for the heat-transferring region are obtained. The selected nodal distributions are as follows.

- 1) Cross-axial flow section, considered as one node, where the primary sodium flow is in transition between regular baffle flow and existing flow, and which includes the first two baffles after the tube sheet;

- 2) Cross-flow section--considered as six nodes--where the major heat transfer occurs, and which includes all the baffled portion except that associated with the two baffles of the cross-axial flow section; and
- 3) Axial-flow section--considered as one node. This is the unbaffled heat transfer section where the sine wave is made in the heat transfer tubes.

Values of the various terms from the physical dimensions are given in Table 14.

TABLE 14
PHYSICAL DIMENSIONS OF INTERMEDIATE HEAT EXCHANGER

Constants	Cross Axial Flow Section (1 node, 2 baffles, and shroud)	Crossflow Section (6 nodes, 4 baffles, and shield plate)	Axial Flow Section (1 node, shrouds)
$A_{xp} \rho_s C_{p_s}$ (Btu/ft°F)	56.3	98.6	56.3
A_{xc} (ft ²)	0.214	0.545	0.224
$A_{xc} \rho_{xc} C_{p_{xc}}$ (Btu/ft°F)	16.7	42.8	17.6
$\frac{A_{xw} \rho_{xw} C_{p_{xw}}}{2}$ (Btu/ft°F)	6.43		
$A_{xs} \rho_s C_{p_s}$ (Btu/ft°F)	7.12		
ρ_x (ft)	60.2		
ΔX (ft)	1.75	2.67	4.83
Shell side thermal resistance (Btu/hr ft ² °F)	$0.000300 \left(\frac{1,140,000}{W_{14}} \right)^{0.6}$	$0.000243 \left(\frac{1,140,000}{W_{14}} \right)^{0.6}$	$0.000620 \left(\frac{1,140,000}{W_{14}} \right)^{0.6}$
Tube wall thermal resistance (Btu/hr ft ² °F)	0.000261		
Tube side thermal resistance (Btu/hr ft ² °F)	$0.000144 \left(\frac{855,000}{W_5} \right)^{0.4}$		
Overall heat transfer coefficient (Btu/hr ft ² °F)	U_{x1}	U_{x2}	U_{x3}

$$U_{x1} = \frac{10,000}{1.44 \left(\frac{855,000}{W_5} \right)^{0.4} + 2.61 + 3.00 \left(\frac{1,140,000}{W_{14}} \right)^{0.6}}$$

$$U_{x2} = \frac{10,000}{1.44 \left(\frac{855,000}{W_5} \right)^{0.4} + 2.61 + 2.43 \left(\frac{1,140,000}{W_{14}} \right)^{0.6}}$$

$$U_{x3} = \frac{10,000}{1.44 \left(\frac{855,000}{W_5} \right)^{0.4} + 2.61 + 6.20 \left(\frac{1,140,000}{W_{14}} \right)^{0.6}}$$

Writing the equations in node form, with the notation used in the analog study, the final equations are as follows.

IV. CROSSAXIAL FLOW SECTION

$$\frac{dt_{xsp23}}{d\theta} + (0.76 U_{x1}) (t_{xsp23} - t_{xss23}) = W_{14} (2.165 \times 10^{-3}) (t_{xsp3} - t_{xsp2})$$

$$\frac{dt_{xss23}}{d\theta} - (4.45 U_{x1}) (t_{xsp23} - t_{xss23}) = -1.26 \times 10^{-2} W_5 (t_{xss3} - t_{xss2})$$

$$t_{xsp23} = \frac{t_{xsp2} + t_{xsp3}}{2}$$

$$t_{xss23} = \frac{t_{xss2} + t_{xss3}}{2}$$

V. CROSSFLOW SECTION

$$\frac{dt_{xsp34}}{d\theta} + (0.408 U_{x2}) (t_{xsp34} - t_{xss34}) = W_{14} (7.62 \times 10^{-4}) (t_{xsp4} - t_{xsp3})$$

$$\frac{dt_{xss34}}{d\theta} - (4.45 U_{x2}) (t_{xsp34} - t_{xss34}) = -W_s (8.31 \times 10^{-3}) (t_{xss4} - t_{xss3})$$

$$t_{xsp34} = \frac{t_{xsp3} + t_{xsp4}}{2}$$

$$t_{xss34} = \frac{t_{xss3} + t_{xss4}}{2}$$

Therefore,

$$\frac{dt_{xsp45}}{d\theta} + (0.408 U_{x2}) (t_{xsp45} - t_{xss45}) = W_{14} (7.62 \times 10^{-4}) (t_{xsp5} - t_{xsp4})$$

$$\frac{dt_{xss45}}{d\theta} - (4.45 U_{x2}) (t_{xsp45} - t_{xss45}) = W_5 (8.31 \times 10^{-3}) (t_{xss5} - t_{xss4})$$

$$t_{xsp45} = \frac{t_{xsp4} + t_{xsp5}}{2}$$

$$t_{xss45} = \frac{t_{xss4} + t_{xss5}}{2} .$$

Therefore,

$$\frac{dt_{xsp56}}{d\theta} + (0.408 U_{x2}) (t_{xsp56} - t_{xss56}) = W_{14} (7.62 \times 10^{-4}) (t_{xsp6} - t_{xsp5})$$

$$\frac{dt_{xss56}}{d\theta} - (4.45 U_{x2}) (t_{xsp56} - t_{xss56}) = -W_s (8.31 \times 10^{-3}) (t_{xss56} - t_{xss5})$$

$$t_{xsp56} = \frac{t_{xsp5} + t_{xsp6}}{2}$$

$$t_{xss56} = \frac{t_{xss5} + t_{xss6}}{2}$$

$$\frac{dt_{xsp67}}{d\theta} + (0.408 U_{x2}) (t_{xsp67} - t_{xss67}) = W_{14} (7.62 \times 10^{-4}) (t_{xsp7} - t_{xsp6})$$

$$\frac{dt_{xss67}}{d\theta} - (4.45 U_{x2}) (t_{xsp67} - t_{xss67}) = -W_s (8.31 \times 10^{-3}) (t_{xss7} - t_{xss6})$$

$$t_{xsp67} = \frac{t_{xsp6} + t_{xsp7}}{2}$$

$$t_{xss67} = \frac{t_{xss6} + t_{xss7}}{2} .$$

Hence,

$$\frac{dt_{xsp78}}{d\theta} + (0.408 U_{x2}) (t_{xsp78} - t_{xss78}) = W_{14} (7.62 \times 10^{-4}) (t_{xsp8} - t_{xsp7})$$

$$\frac{dt_{xss78}}{d\theta} - (4.45 U_{x2}) (t_{xsp78} - t_{xss78}) = -W_5 (8.31 \times 10^{-3}) (t_{xss8} - t_{xss7})$$

$$t_{xsp78} = \frac{t_{xsp7} + t_{xsp8}}{2}$$

$$t_{xss78} = \frac{t_{xss7} + t_{xss8}}{2} ,$$

and

$$\frac{dt_{xsp89}}{d\theta} + (0.408 U_{x2}) (t_{xsp89} - t_{xss89}) = W_{14} (7.62 \times 10^{-4}) (t_{xsp9} - t_{xsp8})$$

$$\frac{dt_{xss89}}{d\theta} - (4.45 U_{x2}) (t_{xsp89} - t_{xss89}) = W_5 (8.31 \times 10^{-3}) (t_{xss9} - t_{xss8})$$

$$t_{xsp89} = \frac{t_{xsp8} + t_{xsp9}}{2}$$

$$t_{xss89} = \frac{t_{xss8} + t_{xss9}}{2} .$$

VI. AXIAL-FLOW SECTION

$$\frac{dt_{xsp910}}{d\theta} + (0.75 U_{x3}) (t_{xsp910} - t_{xss910}) = W_{14} (7.75 \times 10^{-4}) (t_{xsp10} - t_{xsp9})$$

$$\frac{dt_{xss910}}{d\theta} - (4.45 U_{x3}) (t_{xsp910} - t_{xss910}) = -W_5 (4.59 \times 10^{-3}) (t_{xss10} - t_{xss9})$$

$$t_{xsp910} = \frac{t_{xsp9} + t_{xsp10}}{2}$$

$$t_{xss910} = \frac{t_{xss9} + t_{xss10}}{2} \quad .$$

A representative set of curves showing the results of the analog study on the IHX multimode model is shown in Figures 45 and 46. Various temperature nodes were recorded and the results plotted as a function of time. This gave the temperature profile through the IHX as a function of time for the various transients. In general, the curves were as expected, with no unexplainable characteristics. The figures shown were obtained by drawing straight line through the various nodal points. This indicated in some cases discontinuous axial temperature gradients. In reality, a smooth curve should be drawn through the points; however, to obtain a true curve many more points would be required. Since the objective of the multinode IHX study was to insure the adequacy of the single-node representation used in the system studies, the primary interest lay in the entering and exiting sodium temperatures.

These sodium temperatures obtained from the multinode IHX analog model were obtained for each transient and compared with the single-node results obtained from the system studies (see Figures 10, 11, 12, and 13). The comparisons indicated that in all cases the single-node approximation was more than satisfactory to determine system-controller characteristics.

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Figure 45. Secondary Transient Temperature Multinode

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Figure 46. Primary Transient Temperature Multinode

APPENDIX C

EXPANDED STEAM GENERATOR THERMAL KINETICS

The 3-node model used in the thermal analysis is similar to that used in Section III of the main report. A fourth node, a nonboiling entrance region, was introduced tentatively. It was found, however, that in most practical cases this node does not exist. The criterion for the existence of the fourth node can be expressed by a function $\delta_0 \geq 0$; δ_0 is defined by Equation (25). If $\delta_0 > 0$, the set of equations listed in Part VIII has to include corresponding expressions for the nonboiling section, which can be derived from Section IV. A. 1. Corrections also are necessary in case of flooding, or $(X_s/X_t) < 0$ (see Equation 27). In case the steam outlet temperature T_{sh} should deviate by more than 200°F from its nominal value, serious errors in the superheat region would be introduced by the assumed average physical properties of the superheated steam.

One set of equations was developed which is valid for all regions in steady state and transient state. Under the restrictions imposed by the set of basic assumptions given in Section II, it represents the most general solution for the stated problem.

However, to obtain an analytical solution in either space or time, the time or the space variable was averaged out. The space average was taken over each region length X_j and the time average over an unspecified time period t . Steady state and space dependent parameters were obtained by letting t go to infinity. The results were used to calculate average heat transfer coefficients for each region, the regional lengths, etc; these parameters were used in the time-dependent transient solution. In a more elaborate study, t could be replaced by the transient period and the space-averaged parameters could be calculated more accurately by iteration.

I. MODEL

The once-through steam generator is split into three regions: preheat, boiling, and superheat. The boiling region reaches from zero to 100% quality; the bulk temperature in the boiling region is assumed to be the saturation temperature and constant. A small vapor void, however, will exist at the entrance to the boiling region. But the void fraction is so small that it can be neglected; the energy balance over the region is not affected by this error.

II. ASSUMPTIONS

The derivation below is based on the following assumptions.

- a) No axial (parallel to flow) conduction in walls
- b) No physical property changes in each section
- c) No kinetic energy changes appreciable for the thermal analysis
- d) Flat plate approximation for the perpendicular conduction in the wall
- e) Constant saturation temperature throughout the generator
- f) Nonslip condition between vapor and liquid phase
- g) The average wall temperature can be approximated by $\bar{T}_w(x, t)$, as shown in Figure 47.
- h) No heat losses to the outside
- i) Effect of shell side crossflow negligible on heat balance.

III. DERIVATION OF GENERAL EQUATION

A general equation for all regions, under the assumptions just given, can be derived.

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Figure 47. Steam Generator Tube Temperature Nomenclature

α) Balance on sodium side:

heat:

$$\begin{aligned}
 & C_n W_n \left[T_n(x, t) + \frac{\partial T_n(x, t)}{\partial x} \frac{dx}{2} \right] dt && \text{in} \\
 & - C_n W_n \left[T_n(x, t) - \frac{\partial T_n(x, t)}{\partial x} \frac{dx}{2} \right] dt && \text{- out} \\
 & = h_n (W_n) P_{en} dx \left[T_n(x, t) - T_{wn}(x, t) \right] dt && (1\alpha) \quad \text{= loss} \\
 & + \frac{\partial T_n(x, t)}{\partial t} dt C_n A_n dx \rho_n && \text{+ storage}
 \end{aligned}$$

divided by $dx dt C_n W_n$ gives:

$$(1\alpha) \frac{\partial T_n(x, t)}{\partial x} = \frac{h_n (W_n) P_{en}}{C_n W_n} \left[T_n(x, t) - T_{wn}(x, t) \right] + \frac{A_n \rho_n}{W_n} \frac{\partial T_n(x, t)}{\partial t}$$

β) Balance on water side

$$\begin{aligned}
 & CW \left[T(x, t) - \frac{\partial T(x, t)}{\partial x} \frac{dx}{2} \right] \left[1 - \frac{a(x, t) Pe}{A} \right] \frac{\zeta}{\zeta_{av}} dt && \text{in} \\
 & - CW \left[T(x, t) + \frac{\partial T(x, t)}{\partial x} \frac{dx}{2} \right] \left[1 - \frac{a(x, t) Pe}{A} \right] \frac{\zeta}{\zeta_{av}} dt && \text{- out} \\
 & + h(W, \Delta T) P_e dx \left[T_w(x, t) - T(x, t) \right] dt && (1\beta) \text{ + gain} \\
 & = \frac{\partial T(x, t)}{\partial t} dt C \zeta A dx \left[1 - \frac{a(x, t) Pe}{A} \right] && \text{= storage} \\
 & + \frac{\partial a(x, t)}{\partial x} dx \frac{Pe}{A} \frac{\zeta_v}{\zeta_{av}} \left\{ h_{fg} + C \left[T_s - T(x, t) \right] \right\} dt W && \text{+ conversion}
 \end{aligned}$$

divided by $CW \, dx \, dt \, \zeta_{av} \left(1 - \frac{a(x,t) Pe}{A} \right)$ gives:

$$= \frac{\partial T(x,t)}{\partial x} = \frac{-h(W, \Delta T)}{\left[1 - \frac{a(x,t) Pe}{A} \right]} \frac{\zeta_{av}}{\zeta} \frac{Pe}{CW} \left[T_w(x,t) - T(x,t) \right]$$

$$+ \frac{\partial T(x,t)}{\partial t} \frac{\zeta_{av} A}{W} + \frac{\frac{\partial a(x,t)}{\partial x}}{\left[1 - \frac{a(x,t) Pe}{A} \right]} \frac{Pe}{A} \frac{\zeta_r}{\zeta} \left[\frac{h+g}{C} + T_s - T(x,t) \right]$$

γ) Balance for wall

$$\left. \begin{aligned} & h_n(W_n) P_{en} \, dx \left[T_n(x,t) - T_{wn}(x,t) \right] \, dt \\ & - h(W, \Delta T) P_e \, dx \left[T_w(x,t) - T(x,t) \right] \, dt \\ & = \frac{\partial \bar{T}_w(x,t)}{\partial t} \, dt \, C_w A_w \, dx \, \zeta_w \end{aligned} \right\} \begin{array}{ll} \text{in} \\ \text{- out} \\ \text{= storage} \end{array} \quad (1\gamma)$$

divided by $P_{en} \, dx \, dt$ gives:

$$h_n(W_n) \left[T_n(x,t) - T_{wn}(x,t) \right]$$

$$- h(W, \Delta T) \left[T_w(x,t) - T(x,t) \right] \frac{P_e}{P_{en}}$$

$$= \frac{\partial \bar{T}_w(x,t)}{\partial t} C_w \zeta_w \frac{A_w}{P_{en}}$$

Using the definition

$$(2) \quad 2 \bar{T}_w(x,t) = T_{wn}(x,t) + T_w(x,t)$$

Equation (1) simplifies to:

$$(1) \left\{ \begin{array}{l} \alpha) \quad \frac{\partial T_n(x, t)}{\partial x} = H_n \left[T_n(x, t) + T_w(x, t) - 2 \bar{T}_w(x, t) \right] + \frac{1}{V_n} \frac{\partial T_n(x, t)}{\partial t} \\ \\ \beta) \quad \frac{\partial T(x, t)}{\partial x} = \frac{-H}{1 - \alpha(x, t)} \frac{\zeta_{av}}{\zeta} \left[T_w(x, t) - T(x, t) \right] + \frac{1}{V} \frac{\partial T(x, t)}{\partial t} \\ \\ \quad + \frac{\frac{\partial \alpha(x, t)}{\partial x}}{1 - \alpha(x, t)} \frac{\zeta_v}{\zeta} \left(t_s - T(x, t) \right) \\ \\ \gamma) \quad T_n(x, t) + T_w(x, t) - 2 \bar{T}_w(x, t) - f_h \left[T_w(x, t) - T(x, t) \right] \\ \\ \delta) \quad = t_w \frac{\partial \bar{T}_w(x, t)}{\partial t} . \end{array} \right.$$

For steady state and for long transients, the following additional equation can be used.

$$P_{en} h_n (W_n) \left[T_n(x, t) - T_{wn}(x, t) \right] = \frac{k_w}{\delta} P_{ew} \left[T_{wn}(x, t) - T_w(x, t) \right]$$

$$\text{with (2): } h_n (W_n) \left[T_n(x, t) + T_w(x, t) - 2 \bar{T}_w(x, t) \right] = h_w 2 \left[\bar{T}_w(x, t) - T_w(x, t) \right]$$

$$(3) \quad T_n(x, t) + T_w(x, t) - 2 \bar{T}_w(x, t) = f_k \left[\bar{T}_w(x, t) - T_w(x, t) \right]$$

Equation (3) is obtained from a continuity statement for the wall:
 "Input on the sodium side of the wall equals the transfer through the wall";
 f_k is given in IX. g, Simplified Symbols.

IV. STEADY STATE SOLUTION

For Long transients and steady state, the following averaging technique is used to get rid of the time variable.

$$(4) \quad F(x) = \frac{1}{t} \int_0^t F(x, t) dt ,$$

where: $F(x, t)$ is any function of x and t . Applying (4) on every time-dependent function in Equations (1) and (3) gives the following set of equations.

$$(5) \quad \left\{ \begin{array}{l} \alpha) \quad \frac{\partial T_n(x)}{\partial x} = Hn \left[T_n(x) + T_w(x) - 2 \bar{T}_w(x) \right] + \frac{1}{V_n t} \left[T_n(x, t) - T_n(x, 0) \right] \\ \beta) \quad -\frac{\partial T(x)}{\partial x} = \frac{-H}{1 - \alpha_t(x)} \frac{\zeta_{av}}{\zeta} \left[T_w(x) - T(x) \right] + \frac{1}{Vt} \left[T(x, t) - T(x, 0) \right] \\ \quad + \frac{\partial d_t(x)}{\partial x} \frac{\zeta_v}{1 - \alpha_t(x) \zeta} \left[t_s - T(x) \right] \\ \gamma) \quad T_n(x) + T_w(x) - 2 \bar{T}_w(x) - f_h \left[T_w(x) - T(x) \right] = \frac{t_w}{t} \left[\bar{T}_w(x, t) - \bar{T}_w(x, 0) \right] \\ \delta) \quad T_n(x) + T_w(x) - 2 \bar{T}_w(x) = f_k \left[\bar{T}(x) - T_w(x) \right] \end{array} \right.$$

For steady state let $t \rightarrow \infty$, and Equation (5) becomes:

$$(6) \quad \left\{ \begin{array}{l} \alpha) \quad \frac{1}{H_n} \frac{\partial T_n(x)}{\partial x} - T_n(x) + 2 \bar{T}_w(x) - T_w(x) = 0 \\ \beta) \quad \frac{1 - \alpha_t(x)}{H} \frac{\zeta}{\zeta_{av}} \frac{\partial T(x)}{\partial x} + T(x) + \frac{1}{H} \frac{\partial \alpha_t(x)}{\partial x} \frac{\zeta_v}{\zeta_{av}} \left[t_s - T(x) \right] = 0 \\ \gamma) \quad T_n(x) - 2 \bar{T}_w(x) + (1 - f_h) T_w(x) + f_h T(x) = 0 \\ \delta) \quad T_n(x) - (2 + f_k) \bar{T}_w(x) + (1 + f_k) T_w(x) = 0 \end{array} \right. .$$

A. SOLUTION FOR PREHEAT REGION

Assume;

$$(7) \left\{ \begin{array}{l} \alpha_t(x) \ll 1 \\ \frac{\partial \alpha_t(x)}{\partial x} = 0 \\ \zeta_{av} = \zeta \end{array} \right.$$

Then, with the use of Equation (7), Equation (6) simplifies to:

$$(8) \left\{ \begin{array}{l} \frac{1}{H_n} \frac{\partial T_n(x)}{\partial x} - T_n(x) + 2 \overline{T_w}(x) - T_w(x) + 0 = 0 \\ 0 + 0 - T_w(x) + \frac{1}{H} \frac{\partial T(x)}{\partial x} + T(x) = 0 \\ T_n(x) - 2 \overline{T_w}(x) + (1 - f_h) T_w(x) + f_h T(x) = 0 \\ T_n(x) - (2 + f_k) \overline{T_w}(x) + (1 + f_k) T_w(x) + 0 = 0 \end{array} \right.$$

Laplace transform from $x \rightarrow S$:

$$(9) \left\{ \begin{array}{l} \left(\frac{S}{H_n} - 1 \right) T_n(s) + 2 \overline{T_w}(s) - T_w(s) + 0 = \frac{T_{no}}{H_n} \\ 0 + 0 - T_w(s) + \left(\frac{S}{H} + 1 \right) T(s) = \frac{T_o}{H} \\ 1 T_n(s) - 2 \overline{T_w}(s) + (1 - f_h) T_w(s) + f_h T(s) = 0 \\ 1 T_n(s) - (2 + f_k) \overline{T_w}(s) + (1 + f_k) T_w(s) + 0 = 0 \quad ; \end{array} \right.$$

in matrix representation,

$$(9) \quad T_n(s) \overline{T}_w(s) : T_w(s) T(s) = \begin{pmatrix} \frac{S}{H_n} - 1 & 2 & -1 & 0 \\ 0 & 0 & -1 & \frac{S}{H} + 1 \\ 1 & -2 & (1 - f_h) & f_h \\ 1 & -(2 + f_k) & (1 + f_k) & 0 \end{pmatrix} = \begin{pmatrix} \frac{T_{no}}{H_n} \\ \frac{T_o}{H} \\ 0 \\ 0 \end{pmatrix}$$

The determinant of the matrix is

$$(10) \quad D_p = -\frac{S}{H_n} \left[d_o + d_l \frac{S}{H} \right]$$

The results (after conversion to X, and with the restraint that $0 < j < 4$) are:

$$(11) \quad \Delta T_{nj}(x) = b t_j (1 - e^{-x/L_j}) = T_{nj}(x) - T_{nj}(o)$$

$$(12) \quad \Delta T_{wj}(x) = \left(1 - \frac{1}{H_j L_j} \right) t_j (1 - e^{-x/L_j})$$

$$(13) \quad \Delta T_j(x) = t_j (1 - e^{-x/L_j})$$

1. Nonboiling Section

The preheat region may have a nonboiling section (as indicated by dotted line in Figure 47).

$$(14) \quad \Delta T_{no}(x) = b t_o (1 - e^{-x/L_o}) = T_{no}(x) - T_{no}$$

$$(15) \quad \Delta T_{w0}(x) = \left(1 - \frac{1}{H_0 L_0}\right) t_0 (1 - e^{-x/L_0})$$

$$(16) \quad \Delta T_0(x) = t_0 (1 - e^{-x/L_0})$$

From the solution for $T_w(x)$, setting $x = 0$, one obtains:

$$(17) \quad T_{w0} = T_0 + \frac{t_0}{H_0 L_0} .$$

(See Figure 47 for nomenclature.)

The nonboiling section exists if

$$(18) \quad T_{w0} < T_s$$

or

$$(19) \quad T_s - T_0 - \frac{t_0}{H_0 L_0} > 0 .$$

The nonboiling section extends to

$$(20) \quad T_{w1} = T_{w0} + \Delta T_{w0} = T_s$$

or

$$T_s - T_{w0} = T_s - T_0 - \frac{t_0}{H_0 L_0} = \left(1 - \frac{1}{H_0 L_0}\right) t_0 (1 - e^{-X_0/L_0})$$

since

$$\left(1 - \frac{1}{H_0 L_0}\right) t_0 \text{ is always } > 0 .$$

(See Section IX. g.)

$$T_s - T_0 - \frac{t_0}{H_0 L_0} > 0$$

if

$$\delta_0 = (1 - e^{-X_0/L_0}) > 0$$

therefore:

$$\delta_0 = (1 - e^{-X_0/L_0}) \geq 0 \quad .$$

This means if $T_{w0} = T_s$, Equations (18) to (21) result in:

$$\Delta T_{n0} = \Delta T_{w0} = \Delta T_0 = 0 \quad .$$

The end-temperatures of the nonboiling region become:

$$(22) \quad T_{n1} = T_{n0} + b t_0 \delta_0$$

$$(23) \quad T_{w1} = T_{w0} + \left(1 - \frac{1}{H_0 L_0}\right) t_0 \delta_0$$

$$(24) \quad T_1 = T_0 + t_0 \delta_0$$

with

$$(25) \quad \delta_0 = \frac{T_s - T_0 - \frac{t_0}{H_0 L_0}}{t_0 \left(1 - \frac{1}{H_0 L_0}\right)} \geq 0$$

as the criterion for the existence of this section.

2. Nucleate Boiling

By using an average boiling heat transfer coefficient and assuming no void fraction, the following equations similar to (14) to (16) are obtained.

$$(26) \quad \Delta T_{n1}(x) = t_1 b (1 - e^{-X/L_1})$$

$$(27) \quad \Delta T_{w1}(x) = \left(1 - \frac{1}{H_1 L_1}\right) t_1 \left(1 - e^{-X/L_1}\right)$$

$$(28) \quad \Delta T_1(x) = t_1 \left(1 - e^{-X/L_1}\right)$$

$$t_1 = \frac{T_{n1} - T_1}{1 - b} = \frac{T_{n0} - T_0}{1 - b} + \frac{\Delta T_{n0} - \Delta T_0}{1 - b} = t_0 + \frac{b - 1}{1 - b} t_0 \delta_0$$

thus (31)

$$t_1 = (1 - \delta_0) t_0$$

The end-temperatures of the nucleate boiling region are as follows.

$$T_{n2} = T_{n1} + \Delta T_{n1}$$

$$(29) \quad T_{n2} = T_{n0} + b t_0 \delta_0 + (1 - \delta_0) t_0 b (1 - e^{-X_1/L_1})$$

$$T_{w2} = T_{w1} + \Delta T_{w1}$$

$$(30) \quad T_{w2} = T_{w1} + \left(1 - \frac{1}{H_1 L_1}\right) t_0 (1 - \delta_0) \left(1 - e^{-X_1/L_1}\right)$$

$$(31) \quad T_2 = T_s = T_1 + \Delta T_1 = T_0 + t_0 \delta_0 + t_0 (1 - \delta_0) (1 - e^{-X_1/L_1})$$

or

$$(32) \quad t_0 (1 - \delta_0) (1 - e^{-X_1/L_1}) = T_s - T_0 - t_0 \delta_0$$

Equation (32) into (29) and (30) gives:

$$(33) \quad T_{n2} = T_{n0} + b t_0 \delta_0 + b (T_s - T_0) - b t_0 \delta_0$$

$$(34) \quad T_{w2} = T_{w1} + \left(1 - \frac{1}{H_1 L_1}\right) (T_s - T_0 - t_0 \delta_0)$$

for

$$\delta_0 > 0 : T_{w1} = T_s = T_{w0} + \left(1 - \frac{1}{H_0 L_0}\right) t_0 \delta_0 ,$$

and for

$$\delta_0 = 0 : T_{w1} = T_0 + \frac{t_0}{H_1 L_1} \equiv T_1 + \frac{t_1}{H_1 L_1} ,$$

from (17).

B. SOLUTION FOR BOILING REGION

Assume:

$$T = T_s = \text{constant},$$

$$\frac{\partial T}{\partial x} = 0 ,$$

inlet quality = 0, and

outlet quality = 100% ,

Then Equation (6) becomes:

$$(35) \left\{ \begin{array}{l} \alpha) \quad \frac{1}{H_n} \frac{\partial T_n(x)}{\partial x} - T_n(x) + 2 \bar{T}_w(x) - T_w(x) = 0 \\ \beta) \quad T_s - T_w(x) + \frac{1}{H} \frac{\partial \alpha_t(x)}{\partial x} \frac{\zeta_v}{\zeta_{av}} \frac{h_{fg}}{C} = 0 \\ \gamma) \quad T_n(x) - 2 \bar{T}_w(x) + (1 - f_h) T_w(x) + f_h T_s = 0 \\ \delta) \quad T_n(x) - (2 + f_k) \bar{T}_w(x) + (1 + f_k) T_w(x) = 0 \end{array} \right.$$

The constants H and f_h of Equation (35) contain an average heat transfer coefficient for the boiling region which will be called:

$$h_{av,b}(x,w) = \text{function of } t_w(x) \text{ and } W.$$

By defining:

$$(36) \quad \zeta_{av} = (1 - \alpha) \zeta + \alpha \zeta_v = \zeta - \alpha (\zeta - \zeta_v)$$

$$(37) \quad t_w(x) = T_w(x) - T_s$$

$$(38) \quad t_n(x) = T_n(x) - 2 \bar{T}_w(x) + T_w(x) \quad ,$$

and using the simplified symbols, Equation (35) reduces to:

$$(39) \left\{ \begin{array}{l} \alpha) \quad \frac{1}{H_n} \frac{\partial T_n(x)}{\partial x} - t_n(x) = 0 \\ \beta) \quad -\frac{C}{h_{fg}} H_c \frac{h_{av,b}(x)}{h_c} t_w(x) + \frac{\zeta_v}{\zeta_{av}} \frac{\partial \alpha_t(x)}{\partial x} = 0 \\ \gamma) \quad t_n(x) - f_h \frac{h_{av}(x)}{h_c} t_w(x) = 0 \\ \delta) \quad t_n(x) - f_k (\bar{T}_w(x) - T_w(x)) = 0 \end{array} \right.$$

One of the few approaches to boiling heat transfer, which are in common use and which have been more or less reliably confirmed in many works, is the superposition principle assumed by Rohsenow (Reference 2, at end of Appendix).

$$(40) \quad q = q_{nb} + q_b \quad ,$$

where q_{nb} is the nonboiling - and q_b the boiling heat flux. For q_{nb} , one can put $h_{nb} t_w(x)$; because of lack of more than one node for the boiling region, the following plausible assumption was made:

$$(41) \quad h_{nb} = (1 - X) h_c(w) + X h_v(w) \quad .$$

For q_b , Levy's (1) approach is assumed:

$$(42) \quad q_b = \frac{C_t}{B_l T_s} \frac{h_{fg}}{C} (1 - X) t_w^3(x) \quad .$$

Combining Equations (40) to (42) gives:

$$(43) \quad q = \left[H_{fg} t_w^2 (1 - X) + h_c(w) (1 - X) - (1 - X) h_v(w) + h_v(w) \right] t_w$$

$$(44) \quad \frac{q}{t_w(x)} = h_{av}(x) = (1 - X) \left[H_{fg} t_w^2(x) + h_c(w) - h_v(w) \right] + h_v(w)$$

$$(45) \quad X = \frac{\zeta_v A_v V_v}{\zeta_{av} A V_{av}} = \frac{\zeta_v}{\zeta_{av}} \alpha_t(x) \quad \text{for nonslip.}$$

With Equations (44) and (45), Equation (39 β) becomes:

$$\begin{aligned} & \left[\zeta_{av} - \zeta_v \alpha_t(x) \right] \left[\frac{H_{fg} t_w^2(x)}{h_c(w)} + 1 - \frac{h_v(w)}{h_c(w)} \right] + \zeta_{av} \frac{h_v(w)}{h_c(w)} \\ & = \frac{h_{fg}}{CH_c} \frac{\zeta_v}{t_w(x)} \frac{\partial \alpha_t(x)}{\partial x} \quad . \end{aligned}$$

Equation (36) introduced into (39 β) gives

$$(46) \quad \left[1 - \alpha_t(x) \right] \zeta \left[\frac{H_{fg} t_w^2(x)}{h_c(w)} + 1 - f_v \right] + \zeta f_v - (\zeta - \zeta_v) \alpha_t(x) f_v$$

$$= \frac{h_{fg}}{CH_c} \frac{\zeta_v}{t_w(x)} \frac{\partial \alpha_t(x)}{\partial x} \quad .$$

With f_v as defined in the nomenclature, or after rearranging

$$(47) \quad \left[\frac{H_{fg} t_w^2(x)}{h_c(w)} + 1 - t_w(x) - \alpha_t(x) t_w(x) \right] \left[\frac{H_{fg} t_w^2(x)}{h_c(w)} + 1 - \frac{\zeta_v}{\zeta} f_v \right]$$

$$= \frac{h_{fg}}{CH_c} \frac{\zeta_v}{\zeta} \frac{\partial \alpha_t(x)}{\partial x} \quad .$$

In order to obtain an analytical solution for Equations (43) to (46), further assumptions are necessary.

Case 1 - Assume $t_w(x) = t_{wav} = t_w = \text{constant}$ and substitute in Equation (46), to obtain

$$(48) \quad -\alpha_t(x) \frac{b_w - \frac{\zeta_v}{\zeta} f_v}{b_w} = \frac{h_{fg}}{CH_c} \frac{\zeta_v}{\zeta} \frac{1}{b_w t_w} \frac{\partial \alpha_t(x)}{\partial x} \quad .$$

Simplifying,

$$1 - \alpha_t(x) \frac{b_w - \frac{\zeta_v}{\zeta} f_v}{b_w} = U(x)$$

$$\frac{\partial \alpha_t(x)}{\partial x} = \frac{-b_w}{b_w - \frac{\zeta_v}{\zeta} f_v} \frac{\partial U(x)}{\partial x}$$

$$(48a) \quad -U(x) = \underbrace{\frac{h_{fg}}{C} \frac{\zeta_v}{t_w H_c} \frac{1}{b_w - \frac{\zeta_v}{\zeta} f_v}}_{L_2} \frac{\partial U(x)}{\partial x}$$

$$U(x) = e^{-X/L_2},$$

we finally obtain

$$(49) \quad \alpha_t(x) = \frac{b_w}{b_w - \frac{\zeta_v}{\zeta} f_v} \left(1 - e^{-X/L_2} \right).$$

The length of the boiling region is determined by the exit quality of 100%, or

$$(50) \quad \alpha_t(X_2) = 1.$$

Thus from (49) and (50),

$$(51) \quad X_2 = L_2 \left[\ln(b_w) - \ln\left(\frac{\zeta_v}{\zeta} f_v\right) \right].$$

Equation (51) is the analytical result of Case 1, based on Equations (39 β) and (47).

Case 2 - Assume: (52) $h_{av, b}(x) = h_{av} = \text{constant}$.

The use of Equations (39 α , β , γ), (37), (38) and (52) leads to:

$$(53) \quad \frac{1}{H_n} \frac{\partial T_n(x)}{\partial x} = f_h \frac{h_{av}}{h_c} t_w(x).$$

The integration of Equation (53) gives:

$$(54) \quad \Delta T_{n2} = H_n f_h \frac{h_{av}}{h_c} \int_0^{X_2} t_w(x) dx = f_h \frac{h_{av}}{h_c} H_n X_2 t_w \quad ,$$

since

$$(55) \quad t_w = \frac{1}{X_2} \int_0^{X_2} t_w(x) dx$$

with:

$$(56) \quad \Delta T_{n2} = b \frac{h_{fg}}{C} \quad \leftarrow \text{from the energy balance over the boiling region.}$$

$$(57) \quad X_2 = \frac{b h_{fg}}{C H_n t_w} \frac{h_c}{f_h h_{av}}$$

Equation (57) is the result of Case 2, based on Equations (39) and (52); it can also be obtained from the definition,

$$(58) \quad h_{av} = \frac{\int_0^{X_2} h_{av,b}(x) t_w(x) dx}{\int_0^{X_2} t_w(x) dx} = \frac{\int_0^{X_2} g(x) dx}{X_2 t_w} \quad .$$

The end temperatures of the boiling region are:

$$T_{n3} = T_{n2} + \Delta T_{n2} = T_{n2} + b \frac{h_{fg}}{C}$$

or:

$$(59) \quad T_{n3} = T_{n0} + b (t_s - T_0)$$

$$(60) \quad T_3 = T_2 = T_s \quad .$$

The boundary wall temperatures for the boiling region are obtained from the adjacent regions: at the beginning of the region

$$T_w(0) = T_{w2} \quad \text{from (34)} \quad ,$$

(61)

$$T_w(X_2) = T_{w3} \quad \text{from (67)} \quad .$$

C. SOLUTION FOR SUPERHEAT REGION

Since the superheat region is a one-phase region, the system of equations to be used is mathematically identical to the one used in the nonboiling section; the solutions are directly transcribed from Equations (14) to (17).

The numerical parameters are different (all physical properties on the water side have to be replaced by the corresponding vapor properties).

$$(62) \quad \Delta T_{n3}(x) = b_v t_3 (1 - e^{-X/L_3})$$

$$(63) \quad \Delta T_{w3}(x) = t_3 (1 - e^{-X/L_3})$$

$$(64) \quad \Delta T_3(x) = t_3 (1 - e^{-X/L_3})$$

$$(65) \quad T_{w3} = T_3 + \frac{t_3}{H_3 L_3} = T_s + \frac{t_3}{H_3 L_3}$$

$$t_3 = \frac{T_{n3} - T_3}{1 - b_v} = \frac{T_{n0} - T_s + b(t_s - T_0)}{1 - b_v}$$

$$(66) \quad t_3 = \frac{T_{n0} + T_s(b - b_v) + b\left(\frac{h_{fg}}{C} - T_0\right)}{1 - b_v} - T_s$$

Substituting (66) into (65),

$$(67) \quad T_{w3} = \left(1 - \frac{1}{H_3 L_3}\right) T_s + \frac{T_{no} + b \left(\frac{h_{fg}}{C} - T_0\right) + T_s (b - b_v)}{H_3 L_3 (1 - b_v)}$$

The end-temperatures of the superheat regions are:

$$T_{n4} = T_{n3} + \Delta T_{n3} \quad ,$$

or

$$(68) \quad T_{n4} = T_{n0} + b (t_s - T_0) + b_v t_3 \delta_3$$

$$(69) \quad T_{w4} = T_{w3} + \Delta T_{w3} = T_{w3} + \left(1 - \frac{1}{H_3 L_3}\right) t_3 \delta_3$$

$$(70) \quad T_4 = t_3 \delta_3$$

$$(71) \quad \delta_3 = 1 - e^{-X_3/L_3} \geq 0$$

$$X_3 = X_t - \sum_{i=0}^2 X_i \quad .$$

V. TRANSIENT SOLUTION

For short transients, the following averaging technique is used to get rid of the space variable.

$$(72) \quad F(t) = \frac{1}{X_j} \int_0^{X_j} F(x, t) dx$$

Applying Equation (72) on every term of Equations (1) and (3) gives the following set of equations for each one of the three main regions.

$$(73) \quad \left\{ \begin{array}{l} \alpha) \quad \frac{T_n(x_j, t) - T_n(0, t)}{X_j} = H_n \left[T_n(t) + T_w(t) - 2 \bar{T}_w(t) \right] + \frac{1}{V_n} \frac{\partial T_n(t)}{\partial t} \\ \beta) \quad \frac{T(x_j, t) - T(0, t)}{X_j} = \frac{H}{1 - \alpha_x(t)} \frac{\zeta_{av}}{\zeta} \left[T_w(t) - T(t) \right] - \frac{1}{V} \frac{\partial T(t)}{\partial t} \\ \quad - \frac{1}{1 - \alpha_x(t)} \frac{\alpha(x_j, t) - \alpha(0, t)}{X_j} \frac{\zeta_v}{\zeta} t_s + \frac{1}{1 - \alpha_x(t)} \frac{\zeta_v}{\zeta} \frac{1}{X_j} I_j \\ \gamma) \quad T_n(t) + T_w(t) - 2 \bar{T}_w(t) - f_h \left[T_w(t) - T(t) \right] = t_w \frac{\partial \bar{T}_w(t)}{\partial t} \end{array} \right.$$

with (74),

$$I_j = \int_0^{X_j} \frac{\partial \alpha(x, t)}{\partial x} T(x, t) dx$$

By partial integration,

$$(74) \quad \begin{aligned} I_j &= \alpha(x, t) T(x, t) \Big|_0^{X_j} - \int_0^{X_j} \alpha(x, t) \frac{\partial T(x, t)}{\partial x} dx \\ &= \alpha(x_j, t) T(x_j, t) - \alpha(0, t) T(0, t) - \int_0^{X_j} \alpha(x, t) \frac{\partial T(x, t)}{\partial x} dx \end{aligned}$$

Assume that the balances for each region are conserved during transients, thus.

$$(75) \quad T_n(x_j, t) - T_n(0, t) = \Delta T_{nj}$$

$$T(x_j, t) - T(0, t) = \Delta T_j$$

where X_j is also determined from steady state.

In fact it is therefore concluded that the average slopes (l. h. s. of 73 α, β) are approximately the same for steady state and transients.

A. SOLUTION FOR PREHEAT REGION WITH ASSUMPTION g) IN SECTION II

$$(76) \quad \left\{ \begin{array}{l} \alpha) \quad \frac{X_p}{V_n} \frac{\partial T_{np}(t)}{\partial t} = \Delta T_{np} - H_{np} X_p [T_{np}(t) + T_{wp}(t) - 2 \bar{T}_{wp}(t)] \\ \beta) \quad \frac{X_p}{V} \frac{\partial T_p(t)}{\partial t} = H_p X_p [T_{wp}(t) - T_p(t) - \Delta T_p] \\ \gamma) \quad t_w \frac{\partial \bar{T}_{wp}(t)}{\partial t} = T_{np}(t) + T_{wp}(t) - 2 \bar{T}_{wp}(t) - f_{hp} [T_{wp}(t) - T_p(t)] \end{array} \right.$$

ΔT_{wp} , ΔT_{np} and X_p are taken from the steady state solution.

Assume linearity in X for all transient temperatures if necessary. Thus:

$$(77) \quad T_{np}(t) = \frac{1}{2} [T_{n0}(t) + T_{n1}(t)]$$

$$(78) \quad T_{wp}(t) = \frac{1}{2} [T_{w0}(t) + T_{w1}(t)]$$

$$(79) \quad \bar{T}_{wp}(t) = \frac{1}{2} [\bar{T}_{w0}(t) + \bar{T}_{w1}(t)]$$

$$(80) \quad T_p(t) = \frac{1}{2} [T_0(t) + T_1(t)] = \frac{T_0(t) + T_s}{2}$$

From (33)

$$\Delta T_{np} = b \left[T_s - T_0(t) \right] ;$$

from (21), (32)

$$(81) \quad X_p = X_1 + X_2 = L_0 \ln \left(\frac{1}{1 - \delta_0} \right) + L_1 \ln \left(\frac{t_0 (1 - \delta_0)}{t_0 + T_0 - T_s} \right) ,$$

which gives:

$$(82) \quad X_p = (L_1 - L_0) \ln (1 - \delta_0) + L_1 \ln \left[\frac{t_0}{t_0 - (T_s - T_0)} \right] .$$

From (31)

$$\Delta T_p = T_s - T_0(t) ,$$

from (34)

$$\Delta T_{w1} = \left(1 - \frac{1}{H_1 L_1} \right) (T_s - T_0 - t_0 \delta_0) .$$

$$\left. \begin{array}{l} \text{For:} \quad T_0 = \text{constant} \\ \text{and:} \quad \delta_0 = 0 , \end{array} \right\} (83) ,$$

the following set of equations is obtained.

Summary for preheat region:

$$\begin{array}{ll} \text{a.} & t_n \left(\frac{X_p}{X_t} \right) \dot{T}_{np} = b (T_s - T_0) - (H_{np} X_t) \left(\frac{X_p}{X_t} \right) \left[T_{np} + T_{wp} - 2 \bar{T}_{wp} \right] \\ \text{b.} & t \left(\frac{X_p}{X_t} \right) \dot{T}_p = (T_s - T_0) + (H_p X_t) \left(\frac{X_p}{X_t} \right) \left[T_{wp} - T_p \right] \end{array}$$

$$c. \quad t_w \dot{\bar{T}}_{wp} = T_{np} + T_{wp} - 2 \bar{T}_{wp} - f_{hp} \left[T_{wp} - T_p \right]$$

$$d. \quad T_{wp} = T_0 + \frac{t_0}{H_p L_p} + \frac{1}{2} \left(1 - \frac{1}{H_p L_p} \right) (T_s - T_0)$$

$$e. \quad T_{n2} = 2 T_{np} - T_{n0}$$

$$f. \quad \left(\frac{X_p}{X_t} \right) = \left(\frac{L_p}{X_t} \right) \ln \left(\frac{t_0}{t_0 - T_s + T_0} \right)$$

$$g. \quad t_0 = \frac{T_{n0} - T_0}{1 - b}$$

$$h. \quad b = 0.754 \frac{\frac{W}{W_0}}{\frac{W_n}{W_{n0}}}$$

Further numerical constants are:

$$t_n, t, t_w, (H_{np} X_t), (H_p X_t), \left(\frac{L_p}{X_t} \right), \text{ and } f_{hp}$$

B. SOLUTION FOR BOILING REGION

$$(84) \quad \left\{ \begin{array}{l} \alpha) \quad \frac{X_b}{V_n} \frac{\partial T_{nb}(t)}{\partial t} = \Delta T_{nb} - H_{nb} X_b \left[T_{nb}(t) + T_{wb}(t) - 2 \bar{T}_{wb}(t) \right] \\ \beta) \quad \frac{X_b}{V} \frac{\partial T_b(t)}{\partial t} \left[1 - \alpha_x(t) \right] = - \frac{\zeta_v}{\zeta} t_s + H_b X_b \frac{\zeta_{av}}{\zeta} \left[T_{wb}(t) - T_s \right] + \frac{\zeta_v}{\zeta} I_b \\ \gamma) \quad t_w \frac{\partial \bar{T}_{wb}(t)}{\partial t} = T_{nb}(t) + T_{wb}(t) - 2 \bar{T}_{wb}(t) - f_{hb} \left[T_{wb}(t) - T_s \right] \end{array} \right.$$

Similar assumptions as in the preheat region were made as follows. Specifically, ΔT_{nb} , t_{wb} , $\Delta\alpha$ and X_b are taken from steady state, with

$$T_b = T_s, \text{ a constant at all times,}$$

and from (74):

$$I_b = 1 T_s - 0 - 0 = T_s$$

Substitution in (84) and simplifying give

$$1. \quad t_n \left(\frac{X_b}{X_t} \right) \dot{T}_{nb} = b \frac{h_{fg}}{C} - (H_{nb} X_t) \left(\frac{X_b}{X_t} \right) \left[T_{nb} + T_{wb} - 2 \bar{T}_{wb} \right]$$

$$2. \quad \frac{\zeta_v}{\zeta_{av(t)}} \frac{h_{fg}}{C} = (H_b X_t) \left(\frac{X_b}{X_t} \right) \left[T_{wb} - T_s \right]$$

$$3. \quad t_w \dot{\bar{T}}_{wb} = T_{nb} + T_{wb} - 2 \bar{T}_{wb} - f_{hb} \left[T_{wb} - T_s \right]$$

$$4. \quad t_{wb} = T_{wb} - T_s = b \frac{h_{fg}}{C} \frac{1}{f_{hb} (H_{nb} X_t)} \left(\frac{X_t}{X_b} \right)$$

$$5. \quad T_{n3} = 2 T_{nb} - T_{n2}$$

$$6. \quad \left(\frac{X_b}{X_t} \right) = \left(\frac{L_b}{X_t} \right) \ln \left(\frac{\zeta b_w}{\zeta_v f_v} \right)$$

$$7. \quad b_w = 1 + \frac{H_{fg}}{h_c(w)} t_{wb}^2$$

Numerical constants are as follows.

$$t_n, t, t_w, (H_{nb} X_t), (H_b X_t), \left(\frac{L_p}{X_t}\right), f_{hb} \text{ and } \zeta_{av}$$

Steady state average values may, however, take time-dependent arguments.

C. EQUATIONS OBTAINED FROM SOLUTION OF SUPERHEAT REGION

$$a) \quad t_n \left(\frac{X_s}{X_t}\right) \dot{T}_{ns} = b_v (T_{sh} - T_s) - (H_{ns} X_t) \left(\frac{X_s}{X_t}\right) \left[T_{ns} + T_{ws} - 2 \bar{T}_{ws}\right]$$

$$b) \quad t_v \left(\frac{X_s}{X_t}\right) \dot{T}_{su} = -(T_{sh} - T_s) + (H_s X_t) \left(\frac{X_s}{X_t}\right) \left[T_{ws} - T_{su}\right]$$

$$c) \quad t_w \dot{\bar{T}}_{ws} = T_{ns} + T_{ws} - 2 \bar{T}_{ws} - f_{hs} \left[T_{ws} - T_{su}\right]$$

$$d) \quad T_{ws} = T_s + \frac{t_3}{H_s L_s} + \frac{1}{2} \left(1 - \frac{1}{H_s L_s}\right) (T_{sh} - T_s)$$

$$e) \quad T_{ns} = \frac{1}{2} (T_{ni} + T_{n3})$$

$$f) \quad \left(\frac{X_s}{X_t}\right) = 1 - \left(\frac{X_p}{X_t}\right) - \left(\frac{X_b}{X_t}\right) \geq 0 !$$

$$g) \quad t_3 = \frac{T_{n3} - T_s}{1 - b_v}$$

$$h) \quad b_v = 0.0715 \frac{\frac{W}{W_0}}{\frac{W_n}{W_{n0}}}$$

Further numerical constants are

$$t_n, t_v, t_w, (H_{ns} X_t), (H_s X_t), \left(\frac{L_s}{X_t}\right) .$$

VI. AVERAGE VALUES
(All taken from steady state solutions)

A. SOLUTION FOR PREHEAT REGION

for

$$\delta_0 = 0$$

$$t_n = \frac{X_i}{V_n} = \frac{X_t A_n \zeta_n}{W_{no}} \frac{W_{no}}{W_n} = \frac{t_{no}}{\left(\frac{W_n}{W_{no}}\right)}$$

$$t = \frac{X_t}{V} = \frac{X_t A \zeta}{W_o} \frac{W_o}{W} = \frac{t_o}{\left(\frac{W}{W_o}\right)}$$

$$t_w = \frac{C_w A_w \zeta_w}{h_{no} P_{en}} \frac{W_o}{\left(\frac{W_n}{W_{no}}\right)^{0.6}} = \frac{t_{wo}}{\left(\frac{W_n}{W_{no}}\right)^{0.6}}$$

$$(H_{np} X_t) = \frac{h_{no} P_{en} X_t}{C_n W_{no}} \frac{\left(\frac{W_n}{W_{no}}\right)^{0.6}}{\left(\frac{W_n}{W_{no}}\right)} = (H_{npo} X_t) \frac{\left(\frac{W_n}{W_{no}}\right)^{0.6}}{\left(\frac{W_n}{W_{no}}\right)}$$

$$(H_p X_t) = \frac{h_{co} P_e X_t}{\left(\frac{h_{av}}{h_{co}}\right)_p} \frac{W_o}{W} = (H_{po} X_t) \frac{\left(\frac{h_{av}}{h_{co}}\right)_p}{\left(\frac{W}{W_o}\right)}$$

$$\frac{L_p}{X_t} = \frac{H_p L_p}{(H_p X_t)} = \frac{1 + f_{np}}{1 - b} \frac{1 + \left(\frac{2}{f_k}\right)}{(H_p X_t)}$$

$$f_{np} = \left(\frac{h_{co}}{h_{no}} \right) \left(\frac{P_e}{P_{en}} \right) \left(\frac{h_{av}}{h_{co}} \right)_p \frac{1}{\left(\frac{W_n}{W_{no}} \right)^{0.6}} = f_{npo} \frac{\left(\frac{h_{av}}{h_{co}} \right)_p}{\left(\frac{W_n}{W_{no}} \right)^{0.6}}$$

$$\frac{2}{f_k} = \frac{2}{2h_w} h_n (W_n) + \frac{h_{no}}{h_w} \left(\frac{W_n}{W_{no}} \right)^{0.6}$$

$$\left(\frac{h_{av}}{h_{co}} \right)_p = \frac{1}{h_{co}} \frac{C_t}{B_1 T_s} \left[\frac{h_{fg}}{C} + T_s - \frac{1}{2} T_o \right] \frac{(T_{wp} - T_s)^3}{T_{wp} - \frac{1}{2} (T_s + T_o)}$$

or:

$$\left(\frac{h_{av}}{h_{co}} \right)_p = \underbrace{\frac{C_t \left(\frac{T_s - T_o}{2} \right)^2}{B_1 h_{co}}}_{K_1} \underbrace{\left[\frac{h_{fy}}{C} + \frac{1}{2} (T_s - T_o) \right]}_{K_s} \frac{1}{T_s} \left(\frac{2 (T_{wp} - T_s)}{T_s - T_o} \right)^3$$

$$1 + 2 \frac{T_{sp} - T_s}{T_s - T_o}$$

B. SOLUTION FOR BOILING REGION

$$t_n, tw, (H_{nb} X_t)$$

same as in preheat region

$$(H_b X_t) = (H_{po} X_t) \frac{\left(\frac{h_{av}}{h_{co}} \right)_b}{\left(\frac{W}{W_o} \right)}$$

$$\left(\frac{L_b}{X_t}\right) = \underbrace{\frac{h_{fg}}{C(H_{po} X_t)}}_{K_b} \frac{\zeta_v}{\zeta} \frac{\left(\frac{W}{W_o}\right)}{\left(\frac{W}{W_o}\right)^{0.8}} \frac{1}{t_{wb} \left(b_w - \frac{\zeta_v}{\zeta} f_v\right)}$$

$$\left(\frac{L_b}{X_t}\right) = K_b \frac{\left(\frac{W}{W_o}\right)}{\left(\frac{W}{W_o}\right)^{0.8}} \frac{1}{t_{wb} \left(b_w - \frac{\zeta_v}{\zeta}\right)}$$

$$f_{nb} = f_{npo} \frac{h_{avb}}{h_{co}} \frac{1}{\left(\frac{W_n}{W_{no}}\right)^{0.6}}$$

$$\zeta_{avx} = \zeta - \alpha_x (\zeta - \zeta_v)$$

from (36)

$$\alpha_x = \frac{1}{x_p} \int_0^{xp} \alpha(x) dx = \frac{b_w}{b_w - \frac{\zeta_v}{\zeta} f_r} \left[1 - \frac{L_b}{X_b} \left(1 - e^{-\frac{X_b}{L_b}} \right) \right]$$

$$\alpha_x = \frac{b_w}{b_w - \frac{\zeta_r}{\zeta} f_r} \left[1 - \frac{L_b}{X_b} \frac{b_w - \frac{\zeta_v}{\zeta} f_r}{b_w} \right]$$

or

$$\alpha_x = \frac{b_w}{b_w - \frac{\zeta_r}{\zeta} f_r} - \left(\frac{L_b}{X_t}\right) \left(\frac{X_t}{X_b}\right)$$

$$\left(\frac{h_{av}}{h_{co}}\right)_b = \left(\frac{W}{W_o}\right)^{0.8} \left[f_r + \left(\frac{\zeta - \alpha_x \zeta + \alpha_x \zeta_v - \alpha_x \zeta_v}{\zeta_{av}} \right) (b_w - f_r) \right]$$

$$= \left(\frac{W}{W_o}\right)^{0.8} \left[f_r + \frac{\zeta}{\zeta_{av}} (b_w - f_v) \left(\frac{b_w - \frac{\zeta_r}{\zeta} f_r - b_w}{b_w - \frac{\zeta_r}{\zeta} f_r} + \left(\frac{L_b}{X_t} \right) \left(\frac{X_t}{X_b} \right) \right) \right]$$

or

$$\left(\frac{h_{av}}{h_{co}}\right)_b = \left(\frac{W}{W_o}\right)^{0.8} \left[f_v + \left(\frac{b_w - f_r}{1 - \left(1 - \frac{\zeta_r}{\zeta} \right) \alpha_x} \right) \left(\frac{L_b}{X_t} \frac{X_t}{X_b} - \frac{\frac{\zeta_r}{\zeta} f_r}{b_w - \frac{\zeta_r}{\zeta} f_r} \right) \right]$$

$$b_w = \frac{H_{fy}}{H_{co}} t_{wb}^2 \frac{1}{\left(\frac{W}{W_o}\right)^{0.8} + 1}$$

C. SOLUTION FOR SUPERHEAT REGION

$$t_n = \frac{t_{no} \frac{\zeta_{ns}}{\zeta_n}}{\left(\frac{W_n}{W_{no}}\right)}$$

$$t_r = \frac{t_o \frac{\zeta_v}{\zeta}}{\left(\frac{W}{W_o}\right)}$$

$$H_{ns} X_t = H_{np} X_t \left(\frac{C_n}{C_{ns}} \right) \left(\frac{W_n}{W_{no}} \right)^{0.6}$$

$$(H_s X_t) = H_{po} X_t \left(\frac{C}{C_v} \right) \frac{\left(\frac{W}{W_o} \right)^{0.8}}{\left(\frac{W}{W_o} \right)}$$

$$\frac{1}{H_s L_s} = \frac{1 - bv}{1 + f_{ns} \left(1 + \frac{2}{f_k} \right)}$$

$$f_{hs} = f_{hpo} f_v \frac{\left(\frac{W}{W_o} \right)^{0.8}}{\left(\frac{W}{W_o} \right)^{0.6}}$$

VII. NUMERICAL VALUES

A. SOLUTION FOR PREHEAT REGION

$$t_{no} = \frac{x_t A_n \zeta_n}{W_{no}} = \frac{3.2 \text{ ft} \times 1.41 \text{ ft}^3 \times 5.27 \text{ lb} \times 3.6 \times 10^3 \text{ sec}}{\text{ft}^3 \times 8.55 \times 10^5 \text{ lb} \times 10^{-2}} = 10 \text{ sec}$$

$$t_o = \frac{X_t A \zeta}{W_o} = \frac{3.2 \times 2.163 \times 3.6 \times 39.9 \text{ sec}}{144 \times 1.16} = 5.95 \text{ sec}$$

$$t_{wo} = \frac{C_w A_w \zeta_w}{h_{no} P_{en}} = \frac{0.12 \text{ Btu} \times 37.27 \text{ ft}^2 \times 4.88 \text{ lb} \times 3.6 \times 10^5 \text{ sec } ^\circ\text{F ft}^2}{\text{lb } ^\circ\text{F} \times 144 \text{ ft}^3 \times 4000 \text{ Btu} \times 37.9 \text{ ft}}$$

$$= 0.36 \text{ sec}$$

$$\begin{aligned} (H_{npo} X_t) &= \frac{h_{no} P_{en} X_t}{C_n W_{no}} = \frac{4000 \text{ Btu} \times 37.9 \text{ ft} \times 32 \text{ ft lb } ^\circ\text{F hr}}{\text{hr ft}^2 ^\circ\text{F} \times 0.303 \text{ Btu} \times 8.55 \times 10^5 \text{ lb}} \\ &= \frac{128 \times 37.9}{8.55 \times 30.3} = 18.7 \end{aligned}$$

$$(H_{po} X_t) = \frac{96 \times 22.95}{1.16 \times 168} = 11.3$$

$$f_{hpo} = \frac{h_{co} P_e}{h_{no} P_{en}} = \frac{3 \times 22.95}{4 \times 37.9} = 0.454$$

$$h_w = \frac{k_w P_{ew}}{\zeta P_{en}} = 1170 \frac{\text{Btu}}{\text{hr ft}^2 \text{ } ^\circ\text{F}}; \frac{h_{no}}{h_w} = \frac{4}{1.17} = 3.42$$

$$K_s = \frac{\frac{h_{fg}}{C} + 1/2 (T_s - T_o)}{T_s} = \frac{\frac{424.4}{1.68} + 25}{650} = 0.428$$

$$K_\ell = \frac{C_t \left(\frac{T_s - T_o}{2} \right)^2}{B_1 h_{co}}$$

with

$$C_t = 3.64 \times 10^5 \frac{\text{Btu}}{\text{hr ft}^2 \text{ } ^\circ\text{F}} \frac{1}{(\text{ } ^\circ\text{F})^2}$$

and

$$\frac{1}{B_1} = 1.7 \times 10^{-3}$$

$$K_\ell = \frac{3.64 \times 10^2 \times 625 (\text{ } ^\circ\text{F})^2 \times 1.7}{(\text{ } ^\circ\text{F})^2 3000} = 129$$

thus

$$\left(\frac{h_{av}}{h_{co}} \right)_p = 55.2 \frac{(t_w^*)^3}{1 + t_w^*}$$

with

$$t_w^* = \left(\frac{T_{wp} - T_s}{25^\circ\text{F}} \right)$$

B. SOLUTION FOR BOILING REGION

$$(H_b X_t) = \frac{1.68}{1.82} \times 11.3 \frac{\left(\frac{h_{av}}{h_{co}} \right)_b}{\left(\frac{W}{W_o} \right)} = 10.43 \frac{\left(\frac{h_{av}}{h_{co}} \right)_b}{\left(\frac{W}{W_o} \right)}$$

$$\left(\frac{L_b}{X_t} \right) = \frac{424.4 \text{ Btu lb } ^\circ\text{F}}{1\text{b} \times 1.68 \times 11.3 \text{ Btu}} \times 0.164 \frac{\left(\frac{W}{W_o} \right)}{\left(\frac{W}{W_o} \right)^{0.8}} \frac{1}{t_{wb} (b_w - 0.0356)} =$$

$$= \frac{\frac{\left(\frac{W}{W_o} \right)}{\left(\frac{W}{W_o} \right)^{0.8}}}{t_{wb} b_w - (0.0365)}$$

$$\frac{\zeta_v}{\zeta} = \frac{6.15}{37.5} \quad ; \quad \frac{\zeta_v f_v}{\zeta} = 0.0356 \quad ; \quad f_r = \frac{h_v}{h_{co}} = \frac{652}{3000} = 0.217$$

$$f_{hb} = 0.454 \frac{\left(\frac{h_{av}}{h_{co}} \right)_b}{\left(\frac{W_n}{W_{no}} \right)^{0.6}}$$

$$\alpha_x = \frac{b_w}{b_w - 0.0356} - \left(\frac{L_b}{X_t} \right) \left(\frac{X_t}{X_b} \right)$$

$$\left(\frac{h_{av}}{h_{co}} \right)_b = \left(\frac{W}{W_o} \right)^{0.8} \left[0.217 + \left(\frac{b_w - 0.217}{1 - 0.836 \alpha_x} \right) \left(\frac{L_b}{X_t} \frac{X_t}{X_b} - \frac{0.0356}{b_w - 0.0356} \right) \right]$$

$$H_{fg} = \frac{C_t h_{fy}}{B_1 T_s C} = \frac{3.64 \times 1.7 \times 10^2 \text{ Btu lb } ^\circ\text{F} \times 424.4 \text{ Btu}}{\text{hr ft}^2 \times ^\circ\text{F} (^\circ\text{F})^\pm \times 650 ^\circ\text{F} \times 1.68 \text{ Btu lb}} =$$

$$= \frac{1.7 \times 424.4}{1.68 \times 650} 364 \frac{[h]}{(^{\circ}\text{F})^2} = \frac{240}{(^{\circ}\text{F})^2} [h]$$

$$b_w = 1 + \frac{0.08}{(^{\circ}\text{F})^2} t_{wb}^2 \frac{1}{\left(\frac{W}{W_o} \right)^{0.8}}$$

$$t_{wb} = 0.754 \frac{\left(\frac{W}{W_o} \right)}{\left(\frac{W_n}{W_{no}} \right)} \frac{424.4 ^\circ\text{F}}{1.68} \frac{1 \left(\frac{W_n}{W_n} \right)}{0.454 \times 18.7 \left(\frac{W_n}{W_{no}} \right)^{0.6} \left(\frac{X_t}{X_b} \right) \left(\frac{h_{co}}{h_{av}} \right)_b}$$

$$= \frac{22.4 ^\circ\text{F}}{\left(\frac{W_n}{W_{no}} \right)^{0.6} \left(\frac{X_t}{X_b} \right) \left(\frac{h_{co}}{h_{av}} \right)_6}$$

C. SOLUTION FOR SUPERHEAT REGION

$$t_n = \frac{9.8 \text{ sec}}{\left(\frac{W_n}{W_{no}} \right)}$$

$$t_v = 0.56 \text{ sec } \frac{1}{\left(\frac{W}{W_o}\right)}$$

$$(H_{ns} X_t) = 18.7 \frac{\left(\frac{W_n}{W_{no}}\right)^{0.6}}{\left(\frac{W_n}{W_{no}}\right)}$$

$$(H_s X_t) = 4.33 \frac{\left(\frac{W}{W_o}\right)^{0.8}}{\left(\frac{W}{W_o}\right)}$$

$$\frac{1}{H_s L_s} = \frac{1 - b_v}{1 + f_{hs} \left(1 + \frac{2}{f_k}\right)}$$

$$\frac{2}{f_k} = 3.42 \left(\frac{W_n}{W_{no}}\right)^{0.6}$$

$$f_{hs} = 0.0985 \frac{\left(\frac{W}{W_o}\right)^{0.8}}{\left(\frac{W_n}{W_{no}}\right)^{0.6}}$$

$$b_v = 0.0715 \frac{\left(\frac{W}{W_o}\right)}{\left(\frac{W_n}{W_{no}}\right)}$$

VIII. SUMMARY OF EQUATIONS FOR $\delta_0 = 0$

A. SOLUTION FOR PREHEAT REGION

$$1) \quad 10 \sec \left(\frac{X_p}{X_t} \right) \dot{T}_{np} = 37.7^\circ F \left(\frac{W}{W_0} \right) - 18.7 \left(\frac{W_n}{W_{no}} \right)^{0.6} \left(\frac{X_p}{X_t} \right) \left[T_{np} + T_{wp} - 2 \bar{T}_{wp} \right]$$

$$2) \quad 5.95 \sec \left(\frac{X_p}{X_t} \right) \dot{T}_p = -50^\circ F \left(\frac{W}{W_0} \right) + 11.3 \left(\frac{h_{av}}{h_{co}} \right)_p \left(\frac{X_p}{X_t} \right) \left[T_{wp} - T_p \right]$$

$$3) \quad 0.36 \sec \dot{\bar{T}}_{wp} = \left(\frac{W_n}{W_{no}} \right)^{0.6} \left[T_{np} + T_{wp} - 2 \bar{T}_{wp} \right] - 0.454 \left(\frac{h_{av}}{h_{co}} \right) \left[T_{wp} - T_p \right]$$

$$4) \quad T_{wp} = 625^\circ F + \frac{t_o - 25^\circ F}{H_p L_p}$$

$$5) \quad T_{n2} = 2 T_{np} - T_{no}$$

$$6) \quad \left(\frac{X_p}{X_t} \right) = \frac{\left(\frac{W}{W_o} \right)}{11.3 \left(\frac{h_{av}}{h_{co}} \right)_p} (H_p L_p) \ln \left(\frac{t_o}{t_o - 50^\circ F} \right)$$

$$7) \quad b = 0.754 \frac{\left(\frac{W}{W_o} \right)}{\left(\frac{W_n}{W_{no}} \right)}$$

$$8) \quad t_o = \frac{T_{n0} - T_0}{1 - b}$$

$$9) \quad t_w^* = \frac{T_{wp} - 650^\circ F}{25^\circ F}$$

$$10) \quad (H_p L_p) = \frac{1}{1-b} \left\{ 1 + 0.454 \frac{\left(\frac{h_{av}}{h_{co}}\right)_p}{\left(\frac{W_n}{W_{no}}\right)^{0.6}} \left[1 + 3.42 \left(\frac{W_n}{W_{no}}\right)^{0.6} \right] \right\}$$

$$11) \quad \left(\frac{h_{av}}{h_{co}}\right)_p = 55.2 \frac{(t_w^*)^3}{1+t_w^*}$$

B. SOLUTION FOR BOILING REGION

$$12) \quad 10 \sec \left(\frac{X_b}{X_t}\right) \dot{T}_{nb} = 190^\circ F \left(\frac{W}{W_o}\right) - 18.7 \left(\frac{X_b}{X_t}\right) [T_{nb} + T_{wb} - 2 \bar{T}_{wb}]$$

$$13) \quad 41.4^\circ F \left(\frac{W}{W_o}\right) = 11.3 \left(\frac{h_{av}}{h_{co}}\right)_b \left(\frac{X_b}{X_t}\right) (1 - 0.836 \alpha_x) (T_{wb} - 650^\circ F)$$

$$14) \quad 0.36 \sec \dot{\bar{T}}_{wb} = \left(\frac{W_n}{W_{no}}\right)^{0.6} [T_{nb} + T_{wb} - 2 \bar{T}_{wb}] - 0.454 \left(\frac{h_{av}}{h_{co}}\right)_b (T_{wb} - 650^\circ F)$$

$$15) \quad T_{wb} = t_{wb} + 650^\circ F$$

$$16) \quad T_{n3} = 2 T_{nb} - T_{n2} \quad \frac{3.36^\circ F \left(\frac{W}{W_o}\right)^{0.8}}{\left(\frac{W}{W_o}\right)}$$

$$17) \quad \left(\frac{X_b}{X_t}\right) = \left(\frac{L_b}{X_t}\right) [3.34 + \ln(bw)] ; \left(\frac{L_b}{X_t}\right) = \frac{\left(\frac{W}{W_o}\right)}{t_{wb} (b_w - 0.0356)}$$

$$18) \quad b_w = 1 + 50 \frac{\left(\frac{t_{wb}}{25^\circ F}\right)^2}{\left(\frac{W}{W_o}\right)^{0.8}}$$

$$19) \quad t_{wb} = \frac{22.4^\circ F \left(\frac{W}{W_o}\right)}{\left(\frac{X_b}{X_t}\right) \left(\frac{h_{av}}{h_{co}}\right)_b \left(\frac{W_n}{W_{no}}\right)^{0.6}}$$

$$20) \quad \alpha_x = \frac{b_w}{b_w - 0.0356} - \frac{1}{3.34 + \ln(bw)}$$

$$21) \quad \left(\frac{h_{av}}{h_{co}} \right)_b = \left(\frac{W}{W_o} \right)^{0.8} \left[0.217 + \left(\frac{b_w - 0.217}{1 - 0.836 \alpha_x} \right) (1 - \alpha_x) \right]$$

0.0356 << b_w and may be neglected !

C. SOLUTION FOR SUPERHEAT REGION

$$22) \quad 9.8 \text{ sec} \left(\frac{X_s}{X_t} \right) \dot{T}_{ns} = 0.0715 \left(\frac{W}{W_o} \right) (T_{sh} - 650^\circ\text{F}) - 18.7 \left(\frac{W_n}{W_{no}} \right)^{0.6}$$

$$\left(\frac{X_s}{X_t} \right) \left[T_{ns} + T_{ws} - 2 \bar{T}_{ws} \right]$$

$$23) \quad 0.56 \text{ sec} \left(\frac{X_s}{X_t} \right) \dot{T}_{sh} = - \left(\frac{W}{W_o} \right) (T_{sh} - 650^\circ\text{F}) + 4.33 \left(\frac{W}{W_o} \right)^{0.8} \left(\frac{X_s}{X_t} \right) \left[T_{ws} - T_{su} \right]$$

$$24) \quad 0.36 \text{ sec} \dot{\bar{T}}_{ws} = \left(\frac{W_n}{W_{no}} \right)^{0.6} \left[T_{ns} + T_{ws} - 2 \bar{T}_{ws} \right] - 0.0985 \left(\frac{W}{W_o} \right)^{0.8} \left[T_{ws} - T_{sh} \right]$$

$$25) \quad T_{ws} = \frac{1}{2} \left[T_{sh} + 650^\circ\text{F} \right] + \frac{t_3 - \frac{1}{2} (T_{sh} - 650)}{H_s - L_s}$$

$$26) \quad T_{ni} = 2 T_{ns} - T_{n3}$$

$$27) \quad \left(\frac{X_s}{X_t} \right) = 1 - \left(\frac{X_p}{X_t} \right) - \left(\frac{X_b}{X_t} \right) \geq 0 !$$

$$28) \quad b_v = 0.0715 \frac{\left(\frac{W}{W_o}\right)}{\left(\frac{W_n}{W_{no}}\right)}$$

$$29) \quad t_3 = \frac{T_{n3} - 650^\circ\text{F}}{1 - b_v}$$

$$30) \quad (H_s \ L_s) = \frac{1}{1 - b_v} \left\{ 1 + 0.0985 \left(\frac{\left(\frac{W}{W_o}\right)^{0.8}}{\left(\frac{W_n}{W_{no}}\right)^{0.6}} \right) \left(1 + 3.42 \left(\frac{W_n}{W_{no}} \right)^{0.6} \right) \right\}$$

if necessary also (instead of (5), (16) and (26):

$$\text{e. g. } 31) \quad T_{ni} = T_{ns} + \frac{1}{2} \Delta T_{n3} ; \Delta T_{n3} \text{ from (62)}$$

and from an "all over" - energy balance:

$$32) \quad T_{ni} - T_{no} = b \left[303^\circ\text{F} + 0.567 (T_{sh} - 650^\circ\text{F}) \right]$$

for steady state and slow transients !

IX. NOMENCLATURE

a) Flows

W_n = sodium flow (lb/hr)

W = water or steam flow (lb/hr)

b) Temperatures (°F)

T = sodium, wall, water or steam temperatures

See Figure 47 for subscripts

T_s = saturation temperature of water

c) Areas (ft²)

A_n = total flow area for sodium

A = total flow area for water

A_w = total cross section of wall

d) Lengths (ft)

P_{en} = total heated perimeter on sodium side

P_e = total heated perimeter on water side

P_{ew} = average perimeter of wall

δ = thickness of wall

X_t = total heated length of steam generator

X = regional lengths, see Figure 47 for subscripts

a = void volume per unit heated area (refers to water vapor)

e) Times (sec)

$t_n = \frac{X_t}{V_n}$, total flow time of sodium through steam generator

$t = \frac{X_t}{V}$, total flowtime of water through steam generator

$t_v = \frac{X_t}{V_v}$, total flowtime of vapor through steam generator.

t_w = time constant of wall

t_t = transient period

f) Velocities (ft/sec)

V_n = average sodium velocity

V = average water velocity

V_v = average velocity of superheated vapor

g) Simplified Symbols

$$H_n = \frac{h_n (W_n) P_{en}}{C_n W_n}$$

$$H = \frac{h (w, \Delta T) P_e}{C W}$$

$$f_h = \frac{h (w, \Delta T)}{h_n (W_n)} \frac{P_e}{P_{en}}$$

$$f_k = \frac{2 h_w}{h_n (W_n)}$$

$$h_w = \frac{k_w}{\zeta} \frac{P_{ew}}{P_{en}}$$

$$V_n = \frac{W_n}{A_n \zeta_n}$$

$$V = \frac{W}{A \zeta}$$

$$\alpha (x) = \frac{a (x) P_e}{A} = \frac{A_v}{A}$$

$$t_s = \frac{h_{fg}}{C} + T_s$$

$$t_w = \frac{C_w \zeta_w A_w}{h_n (W_n) P_{en}}$$

$$b = \frac{H_n}{H} f_h = \frac{C W}{C_n W_n}$$

$$d_o = f_k (1 - b)$$

$$t_w (x) = T_w (x) - T_s$$

$$t_n (x) = T_n (x) - 2 \bar{T}_w (x) + T_w (x)$$

$$H_{fg} = \frac{C_t}{B_l T_s} \frac{h_{fg}}{C}$$

$$C_t = \frac{K C^2 \zeta^2}{\partial (\zeta - \zeta_v) h_{fg}}$$

$$f_v = \frac{h_v (w)}{h_c (w)}$$

$$b_w = \frac{H_{fg}}{h_c (w)} t_{ws}^2 + 1$$

$$t_{wb} = T_{wb} - T_s$$

$$L_6 = \frac{h_{fy} \zeta_v}{t_{wb} H \zeta b_w - \frac{\zeta_v}{\zeta} f_v}$$

Notice:

∂ = surface tension

B_l = boiling N_o (Levy)

$$d_1 = f_k \left[1 + f_h \left(1 + \frac{2}{f_k} \right) \right]$$

$$L_j^1 H_j = \left(\frac{d_o}{d_1} \right)_j$$

$$T_j = \frac{T_{nj} - T_j}{1 - b(v)} \quad , \quad j = 0 \rightarrow 4$$

h) Heat Transfer Coefficients (Btu/hr ft²°F)

h_{co} = convectonal heat transfer coefficient at full water flow

$h(w, \Delta T)$ = heat transfer coefficient (local) at any water flow and ΔT

h_{av} = regional average heat transfer coefficient for water (see Figure 47 for subscripts)

h_{no} = convectonal heat transfer coefficient at full sodium flow

$h_n(W_n)$ = convectonal heat transfer coefficient, local and average, at any sodium flow

h_w = equivalent heat transfer coefficient, representing conduction in wall. (See abbreviations.)

i) Physical Properties

ζ = density (lb/ft³)

C = specific heat (Btu/lb°F)

K = thermal conductivity (Btu/hr ft°F)

h_{fg} = heat of vaporization for water (Btu/lb)

ζ_{av} = average density of mixture of water and water vapor.

j) Others

X = steam quality (%)

α = vapor void volume fraction (%)

k) Variables

x = space variable (ft)

t = time variable (sec)

l) Subscripts

Group 1: medium

n = sodium

none = water

w = wall

v = vapor (of water)

Group 2: regions

p = preheat

b = boiling

s = superheat

(c = convection)

j = 0, 1, 2, 3, 4 (See Figure 47)

i = inlet

Group 3: averages

av = any average

t = time average

x = space average

Group 4: others

0 = initial steady state value for full sodium and water flow

S = saturation (of water)

nb = nonboiling

TABLE 15

STEAM GENERATOR PROPERTIES AND DIMENSIONS

A. Physical Properties Used in Calculations, Heat Transfer Coefficients and Flow Rates

WATER

$T_s = 650^\circ\text{F}$	$P_r = 0.853$ Prandtl Number
$T_o = 600^\circ\text{F}$	$h_{co} = 3000 \text{ Btu/hr ft}^2\text{F}$
$C = 1.68 \text{ Btu/lb}^\circ\text{F}$	$W_o = 116,000 \text{ lb/hr}$
$\zeta = 39.9 \text{ lb/ft}^3$	$V_{av} = 5.37 \text{ ft/sec}$
(at saturation condition, $P_s = 2200 \text{ psia}$, $T_s = 650^\circ\text{F}$)	

$h_{fg} = 424.4 \text{ Btu/lb}$	$\zeta = 37.5 \text{ lb/ft}^3$
$C = 1.82 \text{ Btu/lb}^\circ\text{F}$	$\zeta_v = 6.15 \text{ lb/ft}^3$
$C_v = 2.86 \text{ Btu/lb}^\circ\text{F}$	

VAPOR(superheated) $T = 650 \rightarrow 1050^\circ\text{F}$

$C_{vav} = 0.952 \text{ Btu/lb}^\circ\text{F}$	$V_{vav} = 57.2 \text{ ft/sec}$
$\zeta_{vav} = 3.75 \text{ lb/ft}^3$	$h_{vav} = 652 \text{ Btu/hr ft}^2\text{F}$

SODIUM

(for all regions)

$C_n = 0.303 \text{ Btu/lb}^\circ\text{F}$	$h_{no} = 4000 \text{ Btu/hr ft}^2\text{F}$
$\zeta_n = 52.7 \text{ Lb/ft}^3$	$W_n = 855,000 \text{ lb/hr}$
$\zeta_{nv} = 0.981 \zeta_n$	$V_n = 3.19 \text{ ft/sec}$
(for superheat region)	$P_{rn} = 0.0047$

WALL

$\zeta_w = 488 \text{ lb/ft}^3$	$K_w = 12 \text{ Btu/hr ft}^\circ\text{F}$
$C_w = 0.12 \text{ Btu/lb}^\circ\text{F}$	

TABLE 15 (continued)

STEAM GENERATOR PROPERTIES AND DIMENSIONS

B. Dimensions of Steam Generator

$$A_n = 1.41 \text{ ft}^2$$

$$A = 21.63 \text{ in.}^2$$

$$P_{en} = 37.9 \text{ ft}$$

$$P_e = 22.95 \text{ ft}$$

$$X_t = 32 \text{ ft}$$

$$D_o = 0.5 \text{ in.}$$

$$D_i = 0.303 \text{ in.}$$

No. of tubes: 300

$$A_w = 37.27 \text{ in.}^2$$

REFERENCES

1. S. Levy, "Generalized Correlation of Boiling Heat Transfer," ASME paper 58-HT-8.
2. Griffith, Clark and Rohsenow, "Void Volumes in Subcooled Boiling Systems," Technical Report No. 12, MIT, 1958.

APPENDIX D
NOMENCLATURE

I. FOR PART II AND APPENDIX B OF REPORT

A	Area (ft^2)
C_p	Specific heat (Btu/lb-°F)
C_v	Butterfly valve coefficient (gpm/ $\sqrt{\text{psi}}$)
K	Controller constants
P	Pressure (psi)
ΔP	Pressure drop (psi)
Q	Volumetric flow rate (gpm)
q	Heat flux (Btu/hr-ft ²)
R	Air-to-gas ratio
t	Temperature (°F)
U	Overall heat transfer coefficient (Btu/hr-ft ² -°F)
W	Weight rate of flow (lb/hr)
X	Distance (ft)
α	Constant in IHX equations ($\text{ft}^2/\text{°F-Btu}$)
β	Constant in IHX equations (hr^{-1})
γ	Controller corrective signal for furnace fuel
ϵ	Controller corrective signal for air-to-gas ratio
η	Controller corrective signal for furnace air
θ	Time (hr)
ρ	Weight density (lb/ft ³)
τ	Time delay (hr)
ϕ	Valve position (fraction of 90°)

SUBSCRIPTS

1	Sodium flow-into high temperature mixing tee
2	Sodium flow-hot bypass
3	Sodium flow through cooler
4	Sodium flow-cold bypass
5	Sodium flow-secondary side of IHX
14	Sodium flow-sum of flows 1 and 4
123	Sodium flow-sum of flows 1, 2, and 3
1234	Sodium flow-sum of flows, 1, 2, 3, and 4
fs0	Furnace sodium-outlet
fs1	Furnace sodium-entering radiant region
fs2	Furnace sodium-leaving economizer or entering connection region
fs3	Furnace sodium-entering economizer
fsr	Furnace sodium-radiant region
fgf	Furnace gas-flame
fg1	Furnace gas-radiant region
fg2	Furnace gas-entering convection region
fg3	Furnace gas-leaving convection region or entering economizer
fg4	Furnace gas-leaving economizer
fr	Furnace radiant region
fa	Furnace air
ff	Furnace fuel
fc	Furnace convection region
fe	Furnace economizer region
fgc	Furnace gas convection region
fsc	Furnace sodium convection region
fwc	Furnace tube wall in convection region
fg3	Furnace gas economizer region
fse	Furnace sodium economizer region
fwe	Furnace tube wall in economizer region
fwr	Furnace wall in radiant region
x	Intermediate heat exchanger abbreviated to IHX
xc	IHX-shell wall
xp	IHX-primary side

SUBSCRIPTS (continued)

xsp	IHX-sodium primary side
xspi	IHX-sodium primary side inlet
xspo	IHX-sodium primary side outlet
xspip	IHX-sodium primary side inlet piping
xspit	IHX-sodium primary side inlet thermocouple
xs	IHX-secondary side
xss	IHX-sodium secondary side
xssi	IHX-sodium secondary side inlet
xssso	IHX-sodium secondary side outlet
xw	IHX-tube wall
c	Sodium-to-air cooler abbreviated to cooler
cs	Cooler sodium
csi	Cooler-sodium inlet
cso	Cooler-sodium outlet
ca	Cooler-air
cai	Cooler-air inlet
cao	Cooler-air outlet
cw	Cooler-tube wall
ca(o)	Reference value of cooler air
fa(o)	Reference value of furnace air
ff(o)	Reference value of furnace fuel
(o)	Reference value
iv2	Reset rate constant used in controller equation for valves 2, 3, 4, 6, 7, 10, and 11
iv3	
iv4	
iv5	
iv6	
iv7	
iv10	
iv11	

SUBSCRIPTS (continued)

2v2	}	Rate constant used in controller equations for valves 2, 3, 4, 6, 7, 10, and 11
2v3		
2v4		
2v6		
2v7		
2v10		
2v11		
3v2	}	Proportional band constant used in controller equations for valves 2, 3, 4, 6, 7, 10, and 11
3v3		
3v4		
3v6		
3v7		
3v10		
3v11		
v1		Valve No. 1
v2		Valve No. 2
v3		Valve No. 3
v4		Valve No. 4
v6		Valve No. 6
v7		Valve No. 7
v10		Valve No. 10
v11		Valve No. 11
ln		Logarithmic
m		Mean
s		Sodium
sp		Sodium piping
spw		Sodium piping wall
*		Delayed in time
**		Delayed in time
l		Control valves

II. FOR PART III OF REPORT

A. TEMPERATURES

1. Sodium

T_{ns} = superheat region inlet sodium temperature

T_{ns} = superheat region average sodium temperature

T_{nb} = boiling region inlet sodium temperature

T_{nb} = boiling region average sodium temperature

T_{np} = preheat region inlet sodium temperature

T_{np} = preheat region average sodium temperature

T_{no} = preheat region outlet sodium temperature

2. Tube Metal

T_{wns} = superheat region sodium side average temperature

T_{wnb} = boiling region sodium side average temperature

T_{wnp} = preheat region sodium side average temperature

T_{ws} = superheat region average tube wall temperature

T_{wb} = boiling region average tube wall temperature

T_{wp} = preheat region average tube wall temperature

T_{wss} = superheat region steam side average temperature

T_{wsb} = boiling region steam side average temperature

T_{wsp} = preheat region water side average temperature

3. Steam/Feedwater

T_{sh} = superheat region steam outlet temperature

T_s = superheat region steam average temperature

T_s = boiling region steam/water saturation temperature

T_w = preheat region water average temperature

T_w = preheat region water inlet temperature

B. PRESSURES

1. Sodium

P_n = pump outlet pressure

ΔP_n = pump pressure rise

ΔP_{v106} = valve pressure drop

ΔP_{ihx} = intermediate heat exchanger pressure drop

ΔP_{bn} = boiler pressure drop

ΔP_{cn} = piping pressure drop

2. Steam/Feedwater

P_{sh} = steam pressure at steam generator outlet

P_p = pump outlet pressure

ΔP_p = pump pressure rise

ΔP_{v201} = feedwater valve pressure drop

ΔP_{v200} = pressure control valve pressure drop

ΔP_b = boiler pressure drop

ΔP_l = loop components and piping pressure drops

C. FLOWS

W_s = steam/feedwater flow

W_n = secondary loop sodium flow

D. DIMENSIONS

X_s = superheat region length

X_b = boiling region length

X_p = preheat region length

X_t = total length of steam generator heat transfer region

E. DISPLACEMENTS

X_{v200} = pressure control valve position

X_{v201} = feedwater flow control valve position

X_{v106} = sodium flow control valve position

X_{st} = pump speed control scoop tube position

F. FEEDWATER PUMP

N = pump rotational speed

η = pump impeller efficiency