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ACCELERATOR DEPARTMENT
Informal Report

INTERSECTING STORAGE ACCELERATOR NOTES

On the Value of Measuring Single Particle Inclusive Cross Sections
at 90° in the ISABELLE CM Frame

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The potential importance to the study of strong interaction dynamics of a systematic program to measure single particle inclusive spectra over the wide range of s values unique to ISABELLE (3600 GeV^2 to $160,000 \text{ GeV}^2$) is presumably well-appreciated. (Such a program is referred to in section 3.3 of the ISABELLE preliminary design study.) This report assumes that such a series of measurements, spanning a complete range of $x = 2P_L/\sqrt{s}$ and P_T^2 values, and for a variety of secondary particles ($\pi^\pm, K^\pm, p, \bar{p}$), will be carried out using ISABELLE as soon as experimentally feasible. Its purpose is to point out, however, that a reasonable fraction of the new physical insight which can be expected to be gained from the complete program could more easily be obtained from an experimentally simpler series of measurements made only at 90° in the ISABELLE CM frame. These measurements have the significant advantage, in light of the limited number of interaction regions planned for the facility, of being compatible with the simultaneous operation of other aspects of the proposed ISABELLE experimental program, and consequently could, and should, be carried out very early in the operation of the ISABELLE accelerating storage rings.

We will base our argument on the scaling hypothesis of Feynman.¹⁾

This states that for the single particle inclusive reaction

$$p + p \rightarrow q + \dots$$

where particle q may be $\pi^\pm, K^\pm, p, \bar{p}, \dots$ we have

$$E \frac{d^3\sigma(q)}{dP^3} = \sqrt{x^2 + \frac{4(P_T^2 + m^2)}{s}} \frac{d^3\sigma}{\pi dP_T^2 dx} = F(x, P_T^2, s) \xrightarrow{s \rightarrow \infty} f(x, P_T^2)$$

where

$$x = \frac{2P_L}{\sqrt{s}} \approx \frac{2P_L}{\sqrt{s - 4M^2}} = \frac{P_L}{P_{ISA}} \quad (M \text{ being the proton mass})$$

and m , P_T , and P_L are the mass, transverse CM momentum, and longitudinal CM momentum, respectively, of particle q .

By symmetry, $f(x, P_T^2)$ must be an even function of x for the pp collisions characterizing ISABELLE. Consequently, we can expand $f(x, P_T^2)$ about $x = 0$

$$f(x, P_T^2) = \sum_{n=0}^{\infty} f_{2n}(P_T^2) \frac{x^{2n}}{(2n)!}$$

where

$$f_{2n}(P_T^2) = \left. \frac{d^{2n}}{dx^{2n}} (f(x, P_T^2)) \right|_{x=0}$$

and obtain

$$\frac{d^2\sigma(q)}{dP_T^2} \xrightarrow{s \rightarrow \infty} 2\pi f(0, P_T^2) \log s + g(P_T^2)$$

where

$$g(P_T^2) = \pi \sum_{n=1}^{\infty} \frac{1}{n} \frac{1}{(2n)!} f_{2n}(P_T^2) - 2\pi f(0, P_T^2) \log[2(P_T^2 + m^2)]$$

and is finite as $s \rightarrow \infty$.

Integrating over transverse momentum yields

$$\sigma^{(q)} = \langle n_q \rangle \sigma_{inel} \xrightarrow{s \rightarrow \infty} \left\{ 2\pi \int_0^{s/4 \approx \infty} f(0, P_T^2) dP_T^2 \right\} \log s + \int_0^{s/4 \approx \infty} g(P_T^2) dP_T^2$$

where $\langle n_q \rangle$ is the average multiplicity of particle q in an inelastic pp collision. (The rapid transverse damping characteristic of strong interaction processes implies that as $s \rightarrow \infty$ the only significant s dependence of $\sigma^{(q)}$ arises from the $\log s$ factor.)

The preceding is, of course, a well known result.²⁾ The significant factor which we wish to emphasize here is that the coefficient of the $\log s$ term depends only on the single particle inclusive cross section at 90° in the CM frame. Explicitly, we have that if $d\Omega$ represents the angular acceptance of the 90° spectrometer, then since, if ψ represents the angular deviation of the particle q from 90° scattering (so that $d\Omega \approx d\psi d\phi$, where ϕ is the azimuthal angle),

$$x = \frac{2P_L}{\sqrt{s}} = \frac{2P_T}{\sqrt{s}} \tan\psi \approx \frac{2P_T}{\sqrt{s}} \psi$$

we have that $dx = \frac{2P_T}{\sqrt{s}} d\psi$ and

$$f(0, P_T^2) = 2\sqrt{1 + (m/P_T)^2} \left. \frac{d^3\sigma}{dP_T^2 d\Omega} \right|_{90^\circ}$$

Consequently, the asymptotic rate of increase of the multiplicity of any particle species is completely determined from just a measurement of its production cross section at 90° in the collision CM frame.

The preceding assumes perfect scaling. The way in which the scaling limit is approached, however, is perhaps the most critical point as far as the study of strong interaction dynamics is concerned. For example, if

one adopts the Mueller-Regge point of view, and assumes the dominance at high energies of processes involving pomeron exchange, then one obtains the prediction that:³⁾

$$F(0, P_T^2, s) \xrightarrow{s \rightarrow \infty} f(0, P_T^2) + h(P_T^2) s^{-1/2}$$

Verification of this prediction at ISABELLE energies would be an important result.⁴⁾ Moreover, since under the same assumptions a more rapid approach to scaling is expected for $x \neq 0$:

$$F(x, P_T^2, s) \xrightarrow{s \rightarrow \infty} f(x, P_T^2) + k(x, P_T^2) s^{-1/2}$$

the region in the vicinity of $x = 0$ may turn out to be the only experimentally interesting region for the study of single particle inclusive processes at ISABELLE (at least for the pion-producing reactions $p + p \rightarrow \pi^\pm + \dots$).

In addition to the preceeding, the 90° inclusive cross section measurements are, of course, of considerable interest in themselves since not much is currently known experimentally about the behavior of $d\sigma/dP_T^2$ for large P_T^2 values, and measurements made at 90° are a natural way to investigate this behavior. Presently available data do not, for example, reliably distinguish between fall-offs proportional to $e^{-\alpha P_T^2}$ or to $e^{-\beta P_T}$. However, while for $\alpha = 3(\text{GeV}/c)^{-2}$ and $\beta = 6(\text{GeV}/c)^{-1}$ (values suggested by presently available data) the exponential terms are equal at $P_T = 2 \text{ GeV}/c$, they differ by a factor of $e^{24} \approx 3 \times 10^9$ at $P_T = 4 \text{ GeV}/c$. The use of ISABELLE to investigate hadron production at large values of transverse momentum is mentioned in section 3.7 of the preliminary design study; it is perhaps worth emphasizing here that it is the high luminosity of ISABELLE, rather than simply its increased value of s , which makes it

a particularly attractive instrument for the investigation of the large P_T^2 region (relative to the ISR, for example).

Finally, from an experimental point of view single particle inclusive measurements made just at 90° in the CM are considerably simpler than a more complete survey. Particle momenta are low enough that a spectrometer of moderate size can yield $\sim 1\%$ momentum resolution as well as separation of pions, kaons, and protons. (A 20ft. long spectrometer with approximately these characteristics is referred to in section 3.7 of the preliminary design study.) Moreover, measurements utilizing a 90° spectrometer should be spatially compatible with the simultaneous execution of the large angle elastic scattering experiment referred to in section 3.2 of the preliminary design study. Hence, the program outlined in this report can be carried out very early in the operation of the accelerating storage rings using the high luminosity interaction region.

1. R. P. Feynman, Phys. Rev. Letters 23, 1415 (1969).
2. See, for example, N. F. Bali et al., Phys. Rev. Letters 25, 557 (1970).
3. See, for example, Chan Hong-Mo et al., Phys. Rev. Letters 26, 672 (1971). In the central region, the appropriate value of s from the Mueller-Regge point of view involves a projectile and a particle at rest in the collision CM, and therefore increases only in proportion to the overall CM energy, not its square.
4. For a summary of presently available evidence concerning the rate of approach to limiting behavior of $F(0, P_T^2, s)$, see University of Rochester Report, UR-381 (1972); a review paper presented by T. Ferbel at the International Colloquium on Multiparticle Reactions, Zakopane, Poland (June 1972).