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ANNUAL PROGRESS REPORT

"Plasma Instabilities and Waves Excited
by an Anisotropic Energy Distribution
Produced by Electron Cyclotron Resonance"

U.S. Atomic Energy Commission
Contract No. AT(11-1) 1695

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December 15, 1969

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Plasma Instabilities and Waves Excited by an Anisotropic Energy
Distribution Produced by Electron Cyclotron Resonance

Contract No. AT(11-1) 1695

U.S. Atomic Energy Commission

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I. Introduction

The research performed under this contract involves the study of plasmas produced by electron cyclotron resonance heating in a magnetic mirror device. Specifically, the work has centered on three major problems:

- I. The measurement of free-free bremsstrahlung from the plasma and the theoretical problem of relating the results to the electron energy distribution.
- II. The measurement of parallel and perpendicular electron energies by means of an $\vec{E} \times \vec{B}$ analyzer and diamagnetic loop along with microwave diagnostics, and the comparison of the results with theoretical predictions and with the results of computer simulation.
- III. The investigation and identification of an electromagnetic instability.

This report summarizes the progress in the above areas during the period of December 15, 1968 to December 14, 1969 and outlines a program for further theoretical and experimental effort.

The free-free bremsstrahlung measurements have been refined by a more accurate calculation of the scintillator-photomultiplier sensitivity, so that the theoretical efficiency of the detector is known to a higher degree of accuracy. The observed anisotropy in the x-ray flux is in qualitative agreement with the radiation expected for an anisotropic electron distribution. A quantitative measurement of the electron distribution requires a theoretical calculation of the radiation spectrum. The Sommerfeld, quantum-mechanical, non-relativistic theory has been used to interpret the results, and the theory has been extended to relativistic energies.

The $\vec{E} \times \vec{B}$ analyzer measurements have been corrected for the non-uniform magnetic field and for the threshold of the detector. Calibration with an

electron beam is being planned. Perpendicular energy measurements have been improved by the construction of a better, shielded, diamagnetic loop. Since density measurements made by microwave techniques in a hot plasma are often subject to question, an independent measurement of the density by probes or optical methods will be attempted. Currently, the parallel and perpendicular energies are being measured vs time for various experimental conditions. Since the $\vec{E} \times \vec{B}$ analyzer measures the energy distribution of particles that escape from the mirror along the axis, it was necessary to relate the measured distribution to the distribution of parallel energies at the center of the mirror. A computer program has been written to simulate rf heating of electrons in an arbitrary, non-uniform, time independent, magnetic field. The plasma simulation program will first be applied to simple geometries for which analytic solutions are available, and then to the experimental apparatus under study. A long pulse rf source is being developed to permit more flexibility in the heating conditions.

An electromagnetic instability has been observed and identified. The instability is characterized by microwave emission below the electron cyclotron frequency, bursts of high energy x-rays, particle losses along the magnetic axis, and variation in stored energy density. The dispersion relation for transverse electromagnetic waves propagating in a plasma with an anisotropic Maxwellian electron velocity distribution with loss cones was solved numerically using parameters characteristic of the experiment. This was found to yield instability frequencies and growth rates which agreed well with those observed.

Section II describes the experimental apparatus and diagnostics. Sections III through VII discuss the status of the work in the areas previously mentioned. Section VIII outlines the direction of future research.

II. Apparatus and Diagnostics

The experimental device is shown schematically in Fig. 1. The double ended vacuum system has a base pressure of about 1×10^{-7} torr. Helium is continuously admitted through a variable leak so that the pressure during operation is between 2 and 10×10^{-4} torr. The dc magnetic mirror field is formed by a set of 24 pancake coils which allow variation of the mirror ratio from 1.02:1 to 2.38:1. A digital voltmeter across a calibrated series resistor permits accurate measurement of the magnet current, and the shape and strength of the field has been calculated to a high precision from the geometry of the coils by computer.

Microwave pulses of 1.5 μ sec duration for plasma heating are produced by a 100 kW, 8.5 GHz magnetron. The magnetron operates at a repetition rate of typically 360 Hz, and a special two-pulse system allows the magnetron to be pulsed twice in rapid succession during each period. A second microwave system capable of producing 20 kW at pulse lengths up to 1 msec is being developed. The microwaves pass through a hybrid junction and enter the vacuum chamber through two ports located 180° apart in the midplane of the device. The microwave cavity is formed by the cylindrical vacuum chamber (15.4 cm in diameter) and by two perforated copper end plugs (axial separation of 42 cm). Forward and reflected power in the waveguide feeding the cavity is measured with directional couplers.

Plasma diamagnetism is measured by a shielded diamagnetic loop¹ located near the midplane of the magnetic field against the inside wall of the microwave cavity. A passive RC integrator with a long time constant ($\tau = RC = 1.5$ msec) is used to process the signal. Assuming that the magnetic field over the cross section of the loop is relatively constant, the measured voltage $V(t)$ is related to the stored energy density (nkT_1) by

Figure 1

DIAGRAM OF EXPERIMENTAL APPARATUS

$$nkT_{\perp} = \frac{B_{av}}{\mu_0} \frac{RC}{NA} V^*(t)$$

where $V^*(t) = \int_0^t V(t)dt$, and B_{av} is the average dc magnetic field passing through the loop.

The electron density is determined by using two different microwave techniques. The first makes use of the resonant frequency shift of the TE_{111} cylindrical cavity mode (at 1204 MHz) during the plasma lifetime,² while the second relies on the resonant frequency shift of very high order cylindrical cavity modes³ at 35.2 GHz during plasma buildup and decay.

The energy of the energetic electrons can be obtained from the absorption of thick target x-ray bremsstrahlung in aluminum foil. The x-rays, originating either from the microwave cavity walls and end plugs, or from an aluminum target situated outside the microwave cavity centered about the magnetic axis, yield identical results. The x-ray detection is made by an NaI (Tl) scintillator, a 6810A photomultiplier, and a 256 channel pulse height analyzer. When the x-rays are detected with no absorbers, data relating to the intensity of the x-rays vs energy is obtained. It can be shown⁴ that a given distribution of electron energies, $f(W)$, is related to the intensity, I , of thick target x-ray bremsstrahlung produced by these electrons by the following relation:

$$f(W) \approx Ad^2 I/dW^2 .$$

Measurement of thin target, free-free bremsstrahlung is made using a collimated NaI (Tl) scintillator that views the plasma either along the axis or across the midplane. The collimation is such that the walls of the vacuum chamber are outside the view of the detector. The output of the photomultiplier is shaped by a short time constant integrator, amplified, and processed

by a 256 channel pulse height analyzer. The analyzer is operated in a coincidence mode so as to accept pulses only when a gating signal is applied. The gating allows time resolved measurements to be made. The interpretation of the x-ray spectrum so obtained will be discussed in detail in section III.

Measurement of the energy distribution of electrons which escape from the magnetic mirror along the magnetic axis is made using an $\vec{E} \times \vec{B}$ energy analyzer.⁵ The output of the analyzer is detected with an aluminized plastic scintillator viewed by a photomultiplier through a lucite light guide. Electrons entering the input collimating slit of the analyzer experience an $\vec{E} \times \vec{B}$ drift. Since the input and output slits are laterally displaced, varying the electric field within the analyzer allows the energy distribution of electrons entering the slit to be measured.

If we assume that electrons reach the analyzer by small angle coulomb scattering into the loss cone, and if we neglect potential gradients parallel to the magnetic field, the output signal from the analyzer $S(W_M)$ is related to the distribution of energies $f(W_M)$ on the loss cone by⁵

$$S(W_M) \propto f(W_M) [W_M - W_{TH} + eV_{bias}] W_{HA} W_M^{-3/2} \ln(W_M/A),$$

where W_{TH} is the threshold energy of the detector and W_{HA} is the parallel energy at the analyzer:

$$W_{HA} = W_M \left(1 - \frac{B_A}{B(0)}\right) \approx W_M.$$

Although the analyzer measures the total energy of particles on the loss cone, it is possible to estimate the parallel temperature at the center of the mirror by the following argument. Assume a distribution function of the form

$$f(W_{\perp}, W_{\parallel}) = \begin{cases} e^{-W_{\perp}/W_1} e^{-W_{\parallel}/W_2} & (W_{\parallel} < R-1) \\ 0 & (W_{\parallel} > R-1) \end{cases}$$

where R is the mirror ratio. Since

$$W_{\perp} = \frac{1}{R} W_M \quad \text{and} \quad W_{\parallel} = (1 - \frac{1}{R}) W_M,$$

the measured distribution function can be written as

$$f(W_M) = e^{-W_M/RW_1} e^{-(1 - \frac{1}{R})W_M/W_2} = e^{-W_M/kT_M}.$$

The measured temperature T_M is then given by

$$kT_M = \frac{RW_1W_2}{W_2 + (R-1)W_1}.$$

The parallel temperature $T_{\parallel}$ is defined by

$$f(W_{\parallel}) = \int_{W_{\perp 0}}^{\infty} f(W_{\perp}, W_{\parallel}) dW_{\perp} = e^{-W_{\parallel}/kT_{\parallel}},$$

where

$$W_{\perp 0} = \frac{1}{R-1} W_{\parallel}.$$

Evaluating the integral gives

$$kT_{\parallel} = \frac{(R-1)W_1W_2}{W_2 + (R-1)W_1},$$

or

$$T_{\parallel} = \frac{R-1}{R} T_M.$$

The measured temperature is therefore larger than the parallel temperature by a factor that depends only on the mirror ratio.

The non-uniform magnetic field in the analyzer allow only particles with small magnetic moment to pass through the analyzer for a given voltage setting.⁴ Given a particle at the input slit of the analyzer with any value

of parallel energy ($W_{\parallel}(z)$), the gain in parallel energy within the analyzer given by

$$\Delta W_{\parallel} = \mu \Delta B$$

will be too great to allow the particle sufficient deflection into the output slit, if its magnetic moment is above the cutoff value. The greater the particle parallel energy, the smaller will be its deflection. In the case of small μ values the particle gains little parallel energy within the analyzer so the effect is not much different from the uniform field case, i.e. only a narrow band of parallel energies is observed to pass through the analyzer. It should be noted here that this band consists of particle magnetic moments from zero up to the maximum allowed value for the given voltage setting on the analyzer plates. This maximum value of allowed magnetic moment varies as the square of the applied analyzer plate voltage.

Another effect, however, will limit the maximum value of magnetic moment to a value that is independent of the analyzer voltage. Since the particles are traveling in a monotonically decreasing magnetic field, the gyroradius, ρ , will change with axial location according to the formula:

$$\rho(z) = m_e v_{\perp}(z) / eB(z) = \sqrt{2m_e u / B(z)} / e$$

This equation shows that ρ varies inversely as the square root of the magnetic field and hence increases as $B(z)$ decreases. The input and output slits of the analyzer are each approximately 1.55 mm wide. Then one would expect that for $\rho > (1.55/2)$ mm, the reliability of the analyzer would be degraded. If this relation is assumed as an upper limit of the allowed values of particle magnetic moments which will be adequately passed through the analyzer, then the restriction sets

$$\nu_{\max} = \frac{1}{2m_e} e^2 p_{\max}^2 B(z^*) = 5.27 \times 10^4 B(z^*) \text{ eV/Tesla}$$

at the input slit of the $\vec{E} \times \vec{B}$ analyzer where $p_{\max} = 0.78 \text{ nm}$. This limitation depends only on the width of the input slit to the analyzer and the value of magnetic field at that point. The larger the value of $\nu_{\max}$, then the greater the range of parallel energies which will be reliably measured by the $\vec{E} \times \vec{B}$ energy analyzer.

The particles leaving the exit slit of the analyzer were detected using an aluminized plastic scintillator. The scintillator was known to exhibit a detection threshold, i.e., only electrons having energies $\geq W_{TH}$ would be detected. Biasing the scintillator accelerates electrons and thus lowers the detection threshold. By appropriately choosing the scintillator bias voltage and analyzer voltage, a narrow energy window can still be obtained.

An attempt will be made to calibrate the analyzer using an electron gun. An electron beam of known energy and current will be produced inside the mirror field and directed along the axis to the entrance slit of the analyzer. The sensitivity of the detector will then be measured vs electron energy in order to check the validity of the equation used to determine $S(E)$.

III. Theoretical Calculations of the Free-Free Bremsstrahlung from an Anisotropic Plasma

Calculations of free-free bremsstrahlung are being extended from non-relativistic calculations to include all relativistic electrons which contribute significantly to the bremsstrahlung. The theoretical problem may be separated into two parts: (1) write an energy distribution function for electrons in a plasma when the energy distribution of electrons is anisotropic and some of the electrons have relativistic velocities; (2) using this energy distribution function, calculate the free-free bremsstrahlung emission pattern from the plasma using the techniques described in the last annual progress report. The bremsstrahlung intensity is given in terms of the differential cross section ds by

$$dI = I_0 dN v_0 ds. \quad (1)$$

dN is the density of incident electrons having velocity v_0 :

$$dN = N_0 f(p, \theta) p^2 \sin \theta dp d\theta \quad (2)$$

Plasmas produced by electron cyclotron resonance have electrons with high perpendicular velocity and low parallel velocity relative to the dc magnetic field. To describe this anisotropic electron velocity distribution, E. G. Harris⁶ assumed that it could be represented as a Maxwellian in any given direction, but that the characteristic electron temperature depends on the direction:

$$f(v_{\parallel}, v_{\perp}) = \exp\left(\frac{-mv_{\parallel}^2}{2kT_{\parallel}} - \frac{mv_{\perp}^2}{2kT_{\perp}}\right). \quad (3)$$

In the derivation of equation (3) a non-relativistic expression for the energy, $E = mv^2/2$, is used, and so the resulting distribution function is

limited to nonrelativistic electrons. A relativistic expression for the energy is given by

$$E = mc^2 \sqrt{\frac{v_x^2 + v_y^2 + v_z^2}{1 - v^2/c^2}} + c^2 \quad (4)$$

In equation (4), a characteristic temperature cannot be conveniently assigned to each component of the velocity. Furthermore, in relativistic mechanics even the case of isotropic electron velocities (thermal equilibrium with only one characteristic temperature) requires a complicated distribution function. F. Jüttner⁷ derived this distribution function for the isotropic case with the result that

$$dN = \frac{\exp(-E/kT) p E dE}{\frac{\pi}{4NC} (U_0)^2 kT [-iH_2^{(1)}(iU_0/kT)]} \quad (5)$$

where $U_0 = m_0 c^2$ is the electron rest mass energy, E is the total energy, p is the momentum ($pc = \sqrt{E^2 - U_0^2}$), and $H_2^{(1)}$ is the Bessel function of the second kind with imaginary argument. The denominator of equation (5) is a normalization chosen so that there are N particles with $U_0 < E < \infty$.

Thus, to satisfy part (1) of the problem, the electron energy distribution function has to:

- (1) be anisotropic, but reducible to equation (5) if the anisotropy is taken as one,
- (2) include relativistic electrons, and
- (3) be normalized.

A variety of distribution functions meeting all or some of these requirements have been investigated. A spherical coordinate geometry is used. The plasma is assumed to be in a dc magnetic field in the z direction with a symmetrical distribution of electron momenta in the ϕ direction. Normali-

zation is accomplished by numerical integration of equation (6).

$$2\pi N_0 \int_0^\infty \int_0^\pi f(p, \theta) p^2 \sin \theta d\theta dp = 1. \quad (6)$$

The following distribution functions have been considered:

$$f_1 = \exp(-Y)$$

$$f_2 = \exp(-X)$$

$$f_3 = \exp\left(-\left[\frac{kT_\perp}{kT_\parallel} \cos^2 \theta + \sin^2 \theta\right] p^2 c^2 + u_0^2\right)^{1/2} / kT_\perp$$

$$f_4 = 1/X^4$$

$$f_5 = 1/X^6$$

$$f_6 = 1/X^8$$

$$f_7 = \exp(-E/kT_\parallel)$$

$$f_8 = \exp(-E/kT_\perp)$$

$$f_9 = B \exp(-E/27.9) + \sin^2 \theta \exp(E/kT_\perp)$$

$$f_{10} = \sin^2 \theta \exp(-E/kT_\perp)$$

$$f_{11} = \cos^2 \theta \exp(-E/kT_\parallel) + \sin^2 \theta \exp(-E/kT_\perp)$$

$$f_{12} = \exp(-X^2/1600)$$

$$f_{13} = \exp(-X/2)$$

$$f_{14} = \exp(-X(15.5))$$

$$f_{15} = \exp(-Y/2)$$

$$f_{16} = \exp(-X(1.55))$$

$$f_{17} = \exp(-Y(15.5))$$

$$f_{18} = \exp(-\sqrt{Y})$$

$$f_{19} = \exp(-Y^2/1600)$$

$$f_{20} = \exp(-2X)$$

$$f_{21} = \exp(-Y(21.7))$$

$$f_{22} = \exp(-X(21.7))$$

$$f_{23} = \exp(-X(21.7)^2)$$

$$f_{24} = \exp(-Y(21.7)^2)$$

$$f_{25} = \exp(-\sqrt{X})$$

where $X(kT_{||}) = E/(kT_{||}\cos^2\theta + kT_{\perp}\sin^2\theta)$,

$$X = X(3.1 \text{ keV})$$

$$Y(kT_{||}) = E\left(\frac{\cos^2\theta}{kT_{||}} + \frac{\sin^2\theta}{kT_{\perp}}\right), \text{ and}$$

$$Y = Y(3.1 \text{ keV}).$$

f_1 is the Harris distribution with the non-relativistic expression for energy, $mv^2/2$, replaced by the relativistic expression given in equation (4).

f_2 is derived by considering the Maxwell-Boltzmann distribution function for relativistic electrons given by

$$f d^3p = \exp(-\sqrt{p^2 c^2 + U_0^2}/kT) p^2 \sin\theta dp d\theta \quad (7)$$

By partial integration kT is found to be proportional to the average value of $v^2/\sqrt{1-\beta^2}$ (β is the electron velocity divided by c). v^2 may be separated into components as follows:

$$kT = D(\overline{v^2/\sqrt{1-\beta^2}}), \quad (8)$$

where D is the proportionality constant ($D=2m/3$)

$$\begin{aligned} kT &= D(\overline{v_N^2/\sqrt{1-\beta^2}}) + D(\overline{v_{\perp}^2/\sqrt{1-\beta^2}}) \\ &= D(\overline{v^2 \cos^2\theta/\sqrt{1-\beta^2}}) + D(\overline{v^2 \sin^2\theta/\sqrt{1-\beta^2}}) \\ &= D(\overline{v^2/\sqrt{1-\beta^2}}) \cos^2\theta + D(\overline{v^2/\sqrt{1-\beta^2}}) \sin^2\theta \end{aligned} \quad (9)$$

The last step is taken holding θ constant (its veracity may be seen immediately from equation (8)). Now T is replaced by an anisotropic temperature function

with the property that at $\theta=0$, $T=T_{\parallel}$ and at $\theta=90^\circ$, $T=T_{\perp}$. Based on the form of equation (9), we replace T with $T_{\parallel} \cos^2 \theta + T_{\perp} \sin^2 \theta$,

where: $kT_{\parallel} = D(v^2/\sqrt{1-\beta^2})$ for $\theta=0$ and

$$kT_{\perp} = D(v^2/\sqrt{1-\beta^2}) \text{ for } \theta=90^\circ.$$

f_2 is then $\exp(-E/(kT_{\parallel} \cos^2 \theta + kT_{\perp} \sin^2 \theta))$.

f_3 is derived by a Taylor expansion of the argument of the exponent of the function

$$f_3 = \exp(-c\sqrt{a p_{\parallel}^2 + p_{\perp}^2 + U_0^2/c^2} / kT)$$

which is of a form similar to the distribution function developed by Noerdlinger and Yui⁸. Assuming $a p_{\parallel}^2$ is small relative to $p_{\perp}^2 + U_0^2/c^2$,

$$\begin{aligned} f_3 &= \exp[(-c\sqrt{p_{\perp}^2 + U_0^2/c^2} / kT) (\sqrt{1 + a p_{\parallel}^2 / (p_{\perp}^2 + U_0^2/c^2)})] \\ &= \exp[(-c\sqrt{p_{\perp}^2 + U_0^2/c^2} / kT) (1 + a p_{\parallel}^2 / 2(p_{\perp}^2 + U_0^2/c^2) + \dots)]. \end{aligned}$$

At $\theta=0$, $f_3 = \exp[(-U_0/kT) (1 + a p_{\parallel}^2 / 2m^2 c^2 + \dots)]$

$$= c \exp(-a p_{\parallel}^2 / 2mkT),$$

which corresponds to the classical distribution (with $p_{\parallel}$ non-relativistic):

$$c \exp(-p_{\parallel}^2 / 2m kT_{\parallel}).$$

Therefore $a = T/T_{\parallel}$.

At $\theta=90^\circ$, $f_3 = \exp(-c\sqrt{p_{\perp}^2 + U_0^2/c^2} / kT)$

which corresponds to the relativistic distribution:

$$c_2 \exp(-c\sqrt{p_{\perp}^2 + U_0^2/c^2} / kT_{\perp}),$$

so $T = T_{\perp}$ and $a = T/T_{\parallel} = T_{\perp}/T_{\parallel}$. Also $p_{\parallel} = p \cos \theta$, and $p_{\perp} = p \sin \theta$.

Thus $f_3 = \exp[-c(\frac{T_{\perp}}{T_{\parallel}} p^2 \cos^2 \theta + p^2 \sin^2 \theta + U_0^2/c^2)^{1/2} / kT_{\perp}]$.

f_4 , f_5 , and f_6 are simple power functions of an anisotropic expression for kT divided by the total energy. Since the volume element in momentum space is proportional to p^2 , the lowest power considered for E was the fourth power to insure rapid convergence.

f_7 and f_8 are isotropic energy distributions.

f_9 is primarily an isotropic distribution function with characteristic temperature $T_{||}$ combined with a high energy tail of characteristic temperature $T_{\perp}$ weighted by $\sin^2\theta$. Thus parallel velocities remain characterized by $T_{||}$ while perpendicular velocities are affected by $T_{\perp}$.

f_{10} is the same as f_9 without the background isotropic distribution.

f_{11} adds a $\cos^2\theta$ term so that when $T_{||} = T_{\perp}$, the distribution function will reduce to the isotropic case.

f_{18} and f_{25} have the energy dependence reported for Wisconsin's toroidal octupole.⁹

f_{12} , f_{19} , f_{23} , and f_{24} are like the distributions reported for the magnetic mirror device at Berkeley.¹⁰ It has been suggested that factors of X^n be included in future computations.

In the remaining distribution functions of the f_1 and f_2 types, the characteristic temperatures of both the parallel and perpendicular components are varied.

$kT_{||}$ is taken as 3.1 keV unless otherwise noted in $X(T_{||})$ and $Y(T_{||})$, and $kT_{\perp}$ is 31.0 keV (equal to 10 times $kT_{||}$). This makes f_{10} and f_{11} indistinguishable since $\exp(-E/kT_{||})$ is negligible when compared to $\exp(-E/kT_{\perp})$.

A function g_1 is defined as $\int_0^\infty f_1 p^2 \sin\theta d\theta$ which yields the density of electrons at particular electron energies.

Examples of g_1 vs electron energy are shown in Figures 2 and 3. The values of the normalization constants for all the above distribution functions, f_1 , are given in Table 1.

Figure 2

Figure 3

Table I

Normalization constants, $N_i = 1/2\pi \int_0^\pi \int_0^\pi f_i p^2 \sin\theta d\theta dp$

$N_1 = .286571 \times 10^{24}$	$N_{14} = .702108 \times 10^{23}$
$N_2 = .941977 \times 10^{23}$	$N_{15} = .177503 \times 10^{20}$
$N_3 = .630532 \times 10^{23}$	$N_{16} = .967783 \times 10^{23}$
$N_4 = .167239 \times 10^{20}$	$N_{17} = .955237 \times 10^{23}$
$N_5 = .229606 \times 10^{23}$	$N_{18} = .487117 \times 10^{16}$
$N_6 = .146450 \times 10^{26}$	$N_{19} = .312440 \times 10^{14}$
$N_7 = .187235 \times 10^{89}$	$N_{20} = .551067 \times 10^{31}$
$N_8 = .199383 \times 10^{23}$	$N_{21} = .625411 \times 10^{23}$
$N_9 = .987264 \times 10^{22}$	$N_{22} = .543883 \times 10^{23}$
$N_{10} = .299089 \times 10^{23}$	$N_{23} = .42877 \times 10^{137}$
$N_{11} = .299089 \times 10^{23}$	$N_{24} = .55178 \times 10^{137}$
$N_{12} = .120465 \times 10^{14}$	$N_{25} = .187136 \times 10^{16}$
$N_{13} = .600637 \times 10^{19}$	

The second part of the problem of the theoretical calculations of the free-free bremsstrahlung is the integration of the Heitler differential cross-section¹¹ for relativistic, quantum mechanical free-free bremsstrahlung weighted by the energy distribution function formulated in the first part of the problem to yield the x-ray intensity at various angles as a function of x-ray energy. The Heitler cross section, using the Born approximation, is given by

$$\begin{aligned}
 ds(k, p, p_o, E, E_o, \theta, \theta_o, \phi) = & \frac{Z^2 e^4}{2\pi^{1/2}} \frac{dk}{k} \frac{p}{p_o} \frac{\sin\theta d\theta \sin\theta_o d\theta_o d\phi}{q^4} \left\{ \frac{p^2 \sin^2\theta}{(E-p \cos\theta)^2} (4E_o^2 - q^2) \right. \\
 & + \frac{p_o^2 \sin^2\theta_o}{(E_o - p_o \cos\theta_o)^2} (4E^2 - q^2) - \frac{2 p p_o \sin\theta \sin\theta_o \cos\phi}{(E-p \cos\theta)(E_o - p_o \cos\theta_o)} (4EE_o - q^2 + 2k^2) \\
 & \left. + \frac{2k^2 (p^2 \sin^2\theta + p_o^2 \sin^2\theta_o)}{(E-p \cos\theta)(E_o - p_o \cos\theta_o)} \right\} \quad (10)
 \end{aligned}$$

where: $\vec{k}$ = photon momentum times the velocity of light (omission of the vector sign indicates magnitude of the quantity),

$\vec{p}$ = electron momentum times the velocity of light

(zero subscripts refer to the incident electron and no subscript refers to the scattered electron),

θ and θ_o are the angles between $\vec{k}$ and $\vec{p}$ and between $\vec{k}$ and $\vec{p}_o$ respectively,

ϕ is the angle between the planes $(\vec{p}\vec{k})$ and $(\vec{p}_o\vec{k})$, and

$\vec{q}$ is the recoil momentum of the nucleus times the velocity of light.

From conservation of momentum, q^2 is given by

$$q^2 = p_o^2 + p^2 + k^2 - 2p_o k \cos\theta_o + 2pk \cos\theta - 2p_o p (\cos\theta \cos\theta_o + \sin\theta \sin\theta_o \cos\phi) \quad (11)$$

Equation (10) is derived using Dirac's relativistic wave equation for an electron interacting with an electromagnetic field. The interaction Hamiltonian is the sum of two parts: (1) the interaction, H_{rad} , of the electron with the radiation field giving rise to the emission of the photon and (2) the interaction H_{nuc} , of the electron with the field of the nucleus. Both interactions are needed to allow conservation of both momentum and energy. H_{nuc} and H_{rad} are treated as perturbations of the total Hamiltonian of the system resulting in an expansion in powers of $(\frac{2\pi Ze^2}{\hbar v})^2$ for H_{nuc} and $e^2/\hbar c = 1/137$ for H_{rad} . In the expansion only the first term that produces radiation is retained in the Born approximation. Thus the Born approximation is good only for incident and scattered electron energies much greater than 2.2 keV. (2.2 keV is the energy of an electron with a velocity of $\frac{2\pi Ze^2}{\hbar}$.)

The following discussion of H_{rad} will also illustrate Born's approximation. At time zero a fast incoming electron has not interacted with the nucleus, and no photon is present. The probability that, at a later time, a given lower electron energy state and associated photon is populated depends on the population of other states and the rate of their population change with time. The Born approximation considers only transitions for which the number of photons differ by one from the initial state. This means that there is only one photon in the final state. This first order transition matrix element has a magnitude of order $e^2/\hbar c = 1/137$. The next higher order term in calculating H_{rad} produces two photons for the final state. This is accomplished by assuming a transition from the initial state to an intermediate state with one photon, followed by emission of a second photon or the reabsorption of the first photon. Since the case of reabsorption is a nonradiative transition, it will not be further considered. The final population probability for the two photons is the product of the transition

probability from the initial state to the intermediate state times the transition probability from the intermediate state to the final state, and is thus of order $(1/137)^2$.

Likewise the series expansion for H_{nuc} yields a first order transition matrix element of magnitude $\frac{2\pi Ze^2}{\hbar v}$ and hence a transition probability of order $(\frac{2\pi Ze^2}{\hbar v})^2$. Since the total perturbation is H_{nuc} plus H_{rad} , the total first order transition probability is of order $(1/137) (\frac{2\pi Ze^2}{\hbar v})^2$ which may be recognized in the coefficient of equation (10). The next largest terms which are neglected in the Born approximation are of order $(1/137)^2 (\frac{2\pi Ze^2}{\hbar v})^2$ and $(\frac{2\pi Ze^2}{\hbar v})^4$. Evaluation of these higher order terms will indicate the accuracy of the Heitler cross-section at lower energies.

Multiple integration of the Heitler differential cross section for bremsstrahlung weighted by the electron energy distribution functions described in the previous section has been carried out. Some of the results (bremsstrahlung intensity versus photon energy) are shown in Figures 4, 5, and 6.

General observations from the data producing these graphs include:

- 1) for low energy incident electrons, the photon intensity perpendicular to the magnetic field is less than the photon intensity parallel to the field;
- 2) however, on including electrons of relativistic energies, the perpendicular x-ray intensity exceeds the parallel (non-relativistic electrons gyrating around magnetic field lines emit bremsstrahlung x-rays perpendicular to their direction of motion which means parallel x-rays, whereas relativistic gyrating electrons radiate in their direction of motion which means perpendicular x-rays); and 3) also the characteristic "temperature" obtained from the slopes of x-ray intensity versus photon energy graphs, is greater for the x-rays in the perpendicular direction than for those in the parallel

Figure 5

Normalized X-Ray Spectra for Various Electron Energy Distributions

Electron Energy Distributions.

$$A \sim \text{Exp}[-X(1.55)]$$

$$B \sim \text{Exp}[-X(3.1)]$$

$$C \sim \text{Exp}[-X(15.5)]$$

$$D \sim \text{Exp}[-X(21.7)]$$

$$E \sim \text{Exp}[-X(31.0)]$$

$$X(T_{11}) = E_0 / (T_{11} \cos^2 \theta + 31.0 \sin^2 \theta)$$

Figure 6

direction. This is seen by the fact that the curves for perpendicular x-ray intensity and for parallel x-ray intensity diverge toward high values of photon energy. Since the semilog graphs are not straight lines, a true "temperature" is not defined for the x-rays. For comparison of slopes we may take the instantaneous slope at a photon energy of 40 keV for example.

In conclusion, these theoretical studies show that the x-ray spectrum is not strongly dependent on the form of the electron distribution function, and experimental techniques are not sufficiently accurate to distinguish various distributions. For a given form of distribution, however, bremsstrahlung measurements can be used to determine the average energy of the higher energy perpendicular component, but not the lower energy parallel component.

IV. Observations of Free-Free Bremsstrahlung from an ECR Plasma

X-rays, on passing through an absorber, are attenuated by the factor $\exp(-\mu x)$, where μ is the mass absorption coefficient of the absorbing material and x is its thickness. After the x-rays pass from the plasma through various absorbers and are detected using a detector with energy dependent counting efficiency, $\eta(E)$, the x-ray intensity is then given by

$$I_m(E) = \eta(E) \exp(-\mu_A x_A) \exp(-\mu_B x_B) I(E). \quad (14)$$

μ_A and μ_B are the mass absorption coefficients for materials A and B, and x_A and x_B are their respective thicknesses. Let the efficiency $\eta(E)$ of the sodium iodide scintillator-photomultiplier combination be defined as the ratio of light energy detected by the photomultiplier (which views the scintillator) to the incident x-ray energy.

When an x-ray photon impinges on a piece of scintillation material, its energy is converted into light energy in the following way: 1) all or part of the x-ray energy is converted into kinetic energy of one or more electrons, and 2) the electron kinetic energy is then converted into light energy which is emitted by the scintillator.

Since x-ray photons of energies below 100 keV are being considered here, the main process by which x-ray photons are absorbed in the sodium iodide is photoabsorption.¹² This means that the energy of the x-ray photon is completely converted into kinetic energy of an inner shell electron and into the atomic binding energy necessary to free this electron. In the sodium iodide material, the iodine, which is the heavier of the two elemental constituents, will dominate the x-ray absorption processes.¹³ The binding energy for a K shell electron in iodine is 33.2 keV.

Consider an incident beam of x-ray photons of energies $E = h\nu < 33.2$ keV. These photons will be able to release electrons from shells other than the

K shell. The efficiency of the crystal will be determined from the mass absorption coefficient of sodium iodide, $\mu(h\nu)$, and the crystal thickness, x , by:

$$\eta(h\nu) = \eta_0 (1 - \exp(-\mu(h\nu)x)). \quad (15)$$

The exponential factor indicates how many beam x-rays are not absorbed. The constant η_0 refers to an absolute scintillator efficiency determined at some low x-ray energy at which all the x-ray photons are absorbed.

For x-ray photons incident upon the crystal with energies $h\nu > 33.2$ keV, it is possible that a K electron will absorb some of this energy. In fact, in 84% of the photoabsorption events taking place in sodium iodide for energies above 33.2 keV, a K electron will be excited. In the remaining 16% of the photoabsorption events an inner shell electron (from L, M, N... shells) will absorb the photon energy. There is a high probability (87.5% in NaI) that an electron will fill the vacancy in the K shell and release a photon of energy $h\nu_k = 33.2$ keV. This probability is known as the fluorescence yield of the material.

The probability that the absorption of an incident x-ray photon gives rise to a K photon, which in fact escapes from the crystal, is called the escape probability, P . If a K photon does escape, then the energy deposited in the crystal is $h\nu - h\nu_k$. The possibility of the escape of a K photon will modify the efficiency of a NaI scintillator by the factor:

$$(1 - P \frac{h\nu_k}{h\nu}) \eta_0 (1 - \exp(-\mu(h\nu)x)). \quad (16)$$

Adlam and Taylor¹³ present a theoretical escape probability formula as:

$$P = (\beta \omega_k / 2) \{1 - r \ln(\frac{r+1}{r})\}, \quad (17)$$

where β^* is the probability that a K-electron is involved in a photo absorption event (0.84 for NaI), ω_k is the fluorescence yield (0.875 for NaI), and $r = \mu(\nu_k)/\mu(\nu)$. $\mu(h\nu_k)$ is the maximum falus of $\mu(h\nu)$ at the absorption edge.

To summarize, the sodium iodide crystal efficiency depends on energy both through the energy dependence of the mass absorption coefficient μ and through the probability that a K-photon will be lost from the crystal. For energies $h\nu < 33.2$ keV the efficiency is given by (15), and for $h\nu \geq 33.2$ keV by (16).

A correction to the measured intensities of the free-free bremsstrahlung due to the geometry of the experimental setup of plasma, pinholes, and detectors has been calculated. The geometry of the experimental setup is shown in Figure 7. Note that the same geometry describes both the parallel and the perpendicular direction, so that one computation with subscripts deleted will solve the problem for both directions.

The detector located at pinhole #1 receives radiation from the plasma. The exposed detector area equals the pinhole area, $A_1 = \pi r_1^2$. Each point on A_1 "sees" a volume of plasma equal approximately to a frustum defined in Figure 8. The approximation is very good because the angle of the frustum is very small. The error in the approximation would result from the bases of the frustum being not quite planes (because the plasma boundaries may be curved surfaces) and the bases being not quite perpendicular to the axis of frustum. From Figure 8 the volume of the frustum is seen to be V , which is related to other dimensions in the figure by

$$V = \frac{1}{3} A_4 (X+Y) - \frac{1}{3} A_3 (Y). \quad (18)$$

The light intensity, F , arriving on the detector is thus proportional to the product of the frustum volume, V , times the pinhole area, A_1 :

where:

γ is the frustum angle.

Figure 8

$$F = C V A_1 \quad (19)$$

The frustum angle, γ , is given in equation (20). The fact that γ is small allows the angle to be set equal to the tangent of the angle:

$$\gamma = \tan^{-1} r_2/z = r_2/z = r_3/y = r_4/(x+y). \quad (20)$$

Simple algebraic steps reduce F to

$$F = C\pi \frac{r_2^2}{2} A_1 \left[\frac{x^3}{3} + x^2 y + xy^2 \right]. \quad (21)$$

The ratio of parallel intensity to perpendicular intensity is $F_{\parallel}/F_{\perp}$, given by equation (22). All pinhole areas are the same.

$$\begin{aligned} F_{\parallel}/F_{\perp} &= \frac{C\pi r_{2\parallel}^2 A_{1\parallel} z_{\perp}^2 [x_{\parallel} y_{\parallel}^2 + x_{\parallel}^2 y_{\parallel} + x_{\parallel}^3/3]}{C\pi r_{2\perp}^2 A_{1\perp} z_{\parallel}^2 [x_{\perp} y_{\perp}^2 + x_{\perp}^2 y_{\perp} + x_{\perp}^3/3]} \\ &= \frac{z_{\perp}^2 x_{\parallel} [y_{\parallel}^2 + x_{\parallel} y_{\parallel} + x_{\parallel}^2/3]}{z_{\parallel}^2 x_{\perp} [y_{\perp}^2 + x_{\perp} y_{\perp} + x_{\perp}^2/3]} \quad (22) \end{aligned}$$

The physical dimensions of the experimental setup are given in Figure 7.

The dimensions of the plasma are taken in the perpendicular direction as the inside of the tungsten limiter and in the parallel direction as the distance between the copper endplugs which define the microwave cavity.

$\gamma_{\parallel}$ and $\gamma_{\perp}$ are 8.02×10^{-4} and 3.32×10^{-3} radians respectively, so that the small angle approximations are justified. $F_{\parallel}/F_{\perp}$ is calculated to be 4.12.

This correction factor is independent of the x-ray energy so that on a logarithmic graph of x-ray intensity versus x-ray energy, the parallel intensity will be lowered relative to the perpendicular intensity, but the slope will be unaffected. These calculations neglect inhomogeneities in the plasma that may well require additional corrections.

The detector sensitivity corrections and the geometric corrections are made to the experimental data as follows. A graph of the multichannel analyzer output for a 40 minute count of x-ray flux, both parallel and perpendicular to the magnetic field axis, is shown in Figure 9A. Since the measured data yield the number of photons per unit energy versus energy, $N(E)$ must be multiplied by E to obtain the x-ray intensity $I_m(E)$ per unit energy. Figure 9B shows $I_m(E)_\parallel$ and $I_m(E)_\perp$ versus E . For reference, equation (14) is rewritten here with the absorbing materials specified:

$$I_m(E) = n(E) \exp(-\mu_A X_A) \exp(-\mu_B X_B) \exp(-\mu_C X_C) I_s(E) = s(E) I_s(E) \quad (23)$$

where:

$I_s(E)$ is the x-ray intensity from the plasma source (to be found by correcting for absorbers and scintillator efficiency),

$n(E)$ is the scintillator efficiency,

μ_A is the absorption coefficient of the .003" Mylar window,

μ_B is the absorption coefficient of air,

μ_C is the absorption coefficient of the .005" beryllium window, and

X_A , X_B , and X_C are the thicknesses of Mylar, air, and beryllium, respectively. (Note: $X_{B\parallel} \neq X_{B\perp}$.)

From Table II it may be seen that the corrections due to the mylar window and the beryllium window are negligible for all energies. The scintillator efficiency corrections increase the slopes of the spectra while the corrections due to air path absorption decrease the slopes. The correction for the unequal air path lengths decreases the slope for the parallel spectrum more than for the perpendicular spectrum.

The experimental measurements are not yet sufficiently accurate, but it appears from Figure 10 that at low energies, the slope of the parallel

Figure 9A

Figure 9B

Table II

Correction Factors for Experimental Measurements of Bremsstrahlung

E	$\eta(E)$	$\exp(-\mu_A X_A)$ mylar	$\exp(-\mu_B X_B)_{//}$ air	$\exp(-\mu_B X_B)_{\perp}$ air	$\exp(-\mu_C X_C)$ beryllium	$S(E)_{//}$	$S(E)_{\perp}$
18	.99983	.9920	.626	.8943	.9930	.6165	.8808
22	.9932	.9946	.735	.9292	.9945	.7221	.9128
26	.9592	.9961	.817	.9521	.99525	.7769	.9054
30	.8835	.9968	.854	.9629	.9957	.7489	.8444
34	.8962	.9971	.868	.9665	.9958	.7724	.8600
38	.9235	.9973	.881	.9699	.99586	.8080	.8896
42	.9407	.9975	.894	.9733	.99595	.8355	.9096
46	.9474	.9978	.907	.9767	.99605	.8540	.9196
50	.9429	.99795	.913	.9783	.99613	.8558	.9170
54	.9257	.99804	.917	.9790	.99618	.8440	.9010
58	.8913	.99812	.920	.9797	.99622	.8153	.8682
62	.8536	.9982	.922	.9805	.9963	.7827	.8324

Figure 10

spectrum is less than the slope of the perpendicular spectrum, whereas at higher energies, where relativistic effects are more important, this difference in slopes is reversed.

V. Measurements of the Electron Energy Distribution in an Electron Cyclotron Resonance Plasma

An experimental analysis is being made to determine the time dependence of the anisotropy of the electron energy distribution. The measurements are being made in the afterglow of the plasma, during and immediately following the second pulse of our variable spacing two-pulse code system.

The primary device used to measure the perpendicular energy was a shielded diamagnetic loop. The output from the loop is connected to a differential preamplifier to cancel out any electrostatic signal picked up by the coil. The unintegrated diamagnetic loop output is the change in energy density in time, that is, the heating rate.

Copper end plugs placed in the machine allow the electron density to be measured using cavity resonant frequency shifts. The dimensions of the cavity are determined by the separation of the plugs and the cylinder radius. The TE_{111} cylindrical cavity mode is the lowest frequency mode present. Since this mode exhibits a polarization degeneracy, mechanical imperfections cause this mode to split. The presence of plasma in the cavity causes additional shifting of the resonant frequencies.¹⁴

A Tektronix 1L30 Spectrum Analyzer plug-in was used as a fixed frequency microwave receiver to determine the resonant frequency as a function of time and thus the density.

The parallel energies of particles escaping axially from the mirror are determined with an $\vec{E} \times \vec{B}$ analyzer similar to that developed for measurement of electron energies by Barr and Perkins.⁵ Experimental verification was made of the previously determined theoretical value for detector threshold energy--6000 eV. Figure 11 shows that for two cases, the sum of the incident electron

Figure 11

energy and the extrapolated post acceleration voltage add to 6000 eV.

As was discussed in section II, we may determine what the parallel temperature is from the distribution of energies measured at the analyzer. One of the critical assumptions was that $f(W_M)$ - where W_M is the measured energy - can be approximated by a simple, separable exponential. In Figures 12 and 13 the distribution of energies 1 and 4 μ sec after the end of the second pulse are shown. (The units on the vertical scales are arbitrary but the same in both.) The straight line shows how good this assumption is. The dashed line is a maxwellian. Since the parallel temperature measurements assume that particles are lost by coulomb scattering into the loss cone, the measurements are suspect when instabilities such as described in section VII are present. For the present experiments, no such instabilities were noted.

Three parameters have been varied in the experiments. First, in Figure 14, showing the mod B surfaces in our machine for a mirror ratio of 1.19:1, it can be seen that the heating may take place in the end zone, midplane, or off axis. These three cases are under study.

Second, the background gas pressure can be varied from 6×10^{-4} Torr to 10^{-2} Torr with the plasma on. Third, the spacing between the two power pulses can be varied. However, there has been no noticeable variation with pulse spacing in the range between 30 μ s - 175 μ s. This can be explained by the fact that the density decay after the first pulse is slow enough so that the second pulse always encounters essentially the same plasma.

Figures 15 and 16 show one of the two classes of results. This type of characteristic resulted when the pressure was low for all heating zones when the mirror ratio was 1.02:1 and for endzone heating and midplane heating when $R=1.19:1$. In both cases shown, the heating was at the midplane. The perpendicular temperature decay constants are 4 μ s for Figure 15 and 2 μ s for Figure 16.

Figure 12

Figure 13

CONTOURS OF CONSTANT MAGNETIC FIELD AMPLITUDE

Figure 14

Figure 15

Figure 16

Future work on this problem includes increasing the mirror ratio to 1.43:1, 1.78:1, and 2.37:1.

As mentioned elsewhere, the $\vec{E} \times \vec{B}$ analyzer will be calibrated to check the parallel temperature measurements. More quantitative measurements of the pressure dependence of the anisotropy will be made. Finally, we will use the data obtained to test the predictions of various theoretical models for the heating mechanism.

VI. Plasma Simulation

A program that provides a numerical evaluation of the equations of motion of large numbers of particles under the influence of their self-consistent electric field, an arbitrary dc magnetic field, and an applied external rf electric field has been written to complement our study of the anisotropy of the electron energy as produced by ECH devices. The procedure is similar to the Particle-In-Cell methods developed at the Los Alamos Scientific Laboratory.¹⁵

The basic method is the following. The particles are located on, in this case, a 2 dimensional region, consisting of a rectangular grid. A rectangular grid is used rather than the circular one, which might represent our cylindrical cavity, since it is much easier to work with on a computer. When results have been obtained for the rectangular case, they may be transformed to circular to achieve the results pertinent to our apparatus. We do not follow the trajectory of a single isolated particle due to the lack of sufficient computational facilities. Rather, though, a collection of particles is lumped together to form a super particle. The super particle, if it is composed of identical elements, has the same charge-to-mass ratio as that of a single particle, and hence obeys the same equation of motion, namely

$$\frac{d\vec{v}}{dt} = (q/m)[\vec{E}_{EXT} + \vec{E}_{S.C.} + \vec{v} \times \vec{B}_0],$$

where $\vec{B}_0$ is the external dc magnetic field, q/m is the charge to mass ratio of the super particle, $\vec{E}_{EXT}$ is the external impressed electric field, and $\vec{E}_{S.C.}$ is the self consistent field due to the other super particles.

Initial potentials, if any, are set on the boundaries of the grid. Each super particle is assumed to have an area of one rectangular mesh cell, as shown in Figure 17. Note that a super-particle is free to move and may

Figure 17

occupy, in general, portions of four mesh cells. The charge, but not the position, of each super particle is placed at the four nearest mesh cell centers by dividing the charge of each superparticle in proportion to the area it covers in each of the four mesh cells. Note that this assumes that the super particle charge is distributed uniformly over its area. If the charge were concentrated at the particle center, the charge should be divided inversely as the distance to the mesh cell centers. Since this is a plasma, the positive ions must also be included to insure charge neutrality. They may either be treated as the electrons or as is more often the case in our present work, they are assumed fixed and located at the mesh cell centers; that is, the positive charge is always assumed to be distributed evenly over the entire grid.

The potential from this arrangement is then found by using an algorithm developed by O. Buneman¹⁶ for solving Poisson's equation numerically. The gradient of the potential is then taken, giving the values of electric field at each mesh cell center. The field is then bilinearly interpolated to obtain its value at the super particle coordinates. The external field is added, and the equation of motion is solved in a finite difference form. The velocity change of each super particle will be approximated by

$$\vec{v}_{j+1} = \vec{v}_{j-1} + \vec{F}_j(2\Delta T), \quad (24)$$

where ΔT is the time step between units j and $j+1$. $\vec{F}_j$ is the force evaluated at time j . A symmetric finite difference approximation to the derivative of the position change may then be obtained by the use of the following expression:

$$\vec{x}_{j+2} = \vec{x}_j + \vec{v}_{j+1}(2\Delta T).$$

Note that the velocity used in this calculation is that evaluated at the time half-way between successive position changes. The system can now be allowed to progress in time from some initial set of conditions, on the position and velocity of the super-particles. The external electric field may vary in time and the time step is adjusted so that no instability in the numerical solution will occur. This condition does not eliminate the possibility of instabilities due to the equation of motion, etc., but tends to eliminate those solutions which blow up due to accumulated round-off error, etc.

Note that the force F_j must be evaluated at time "j". If no magnetic field force is present, then the force is only due to the electric fields and may be evaluated from the positions of the super-particles which are known at time j, plus the external field.

However, the presence of the magnetic field requires the knowledge of the super-particle velocities at time j. These are not explicitly known. An implicit formulation is required therefore to obtain the required velocity. The following expression is used:

$$v_j = \frac{v_{j+1} + v_{j-1}}{2} \quad (25)$$

This may be placed in the expression for F_j , i.e.,

$$\vec{F}_j = \frac{q}{m} [\vec{E}_{EXT} + E_{S.C.} + (\frac{\vec{v}_{j+1} + \vec{v}_{j-1}}{2}) \times \vec{B}_0]. \quad (26)$$

This results in equation (24) becoming

$$\vec{v}_{j+1} - \frac{q}{2m} (\vec{v}_{j+1} \times \vec{B}_0) = \frac{q}{m} [\vec{E}_{EXT} + E_{S.C.} + \frac{\vec{v}_{j-1}}{2} \times \vec{B}_0]. \quad (27)$$

Equation (27) may be solved in component form to obtain the required change in velocity. It should be noted here that particles are presently "reflected" when they strike the potential boundary, although this condition may be modified to show absorption and/or periodic boundary conditions.

The initial positions of the super-particles may be specified by distributing them uniformly over the grid. The initial velocities are obtained from a weighted distribution of random numbers to produce a Maxwellian (gaussian) distribution. If two different uniform series of random numbers between 0 and 1 are obtained, then two weighted distributions are given by

$$V_x = \sigma_x (-2 \log_e X_1)^{1/2} \cos (2\pi X_2), \quad (28)$$

and

$$V_y = \sigma_y (-2 \log_e X_1)^{1/2} \sin (2\pi X_2). \quad (29)$$

σ_x and σ_y are the standard deviations, which for a Maxwellian are $(kT/m)^{1/2}$. X_1 and X_2 are the two random number series.

After each time step calculation, both the kinetic energy and the electrostatic energy of the system are calculated so that growth and decay may be monitored. The information obtained about particle position may then be used for graphical output displays, and particle motion can be observed. The effects of rf heating may be observed during the heating, but in order to study the relaxation after heating, long computer runs are required.

The program at present has not been used for spatially varying $\vec{B}$ -fields. This will soon be done, and simple heating problems will be considered, including $\vec{B}$ -fields with constant gradients and simple mirror configurations. After we have compared the results obtained with problems that are at least partially analytically tractable, the more complicated situation of the realistic dc mirror field and rf field combination of our machine can be studied.

VII. Electromagnetic Instability in an Electron

Cyclotron Resonance Plasma

Observations

In the course of these experiments, a plasma instability was observed. The instability was characterized by bursts of microwave emission, loss of electrons along the magnetic axis, bursts of high energy x-rays, and correlated variations in the plasma diamagnetic signal. The time of occurrence of the instability was generally within a few microseconds after the end of the single microwave heating pulse. In addition, it was found that the location of the heating zone in the median plane of the experimental device was a necessary condition for the occurrence of the unstable behavior. The background pressure, likewise, needed to be below about 10×10^{-4} torr for the instability to occur.

The microwave emission signals were detected by using (1) a microwave crystal diode in the reflected power arm of the input waveguide and (2) a spectrum analyzer. These detection arrangements allowed determination of the frequencies and times of occurrence of the microwave signals. Representative signals obtained by detecting the microwaves in these ways are shown in Figure 18. The frequency band in which the microwave emission occurred was determined to be between about 7.94 and 8.14 GHz for a mirror ratio of 1.02 and between about 7.79 and 7.99 GHz for a mirror ratio of 1.19. It should be noted that these bands of microwave emission are below 8.54 GHz, the frequency of the microwave heating pulse.

The oscillographs in Figure 19 show the plasma diagnostic signals when an instability is present. Plasma diamagnetism, x-ray output, and axial particle flux are shown. When the instability was absent, the variations in

REFLECTED
POWER
SIGNAL

THE ARROWS DENOTE
THE DURATION OF
THE HEATING PULSE

SPECTRUM
ANALYZER
DISPLAY

$R = 1.02$

$I_{MAG} = 295$ amps

PRESSURE =

9.7×10^{-4} TORR

Figure 18

Figure 19

these signals, which are seen in Figure 19 to appear within a few microseconds after the end of the microwave heating pulse, were absent.

Instability Model

An attempt was made to find a plasma instability theory which would predict some of the experimentally observed instability characteristics. Such a theory must yield a complex instability frequency $\omega = \omega_R + i\omega_I$ with $\omega_R < \omega_{ce}$, and ω_I , the growth rate, corresponding to the observed values. Furthermore, the ratio $(\omega_{pe}/\omega_{ce})^2$ must be of the order of 10^{-2} , where ω_{pe} is the radian electron plasma frequency and ω_{ce} is the radian electron cyclotron frequency. The theory properties should reflect, in a reasonable way, a plasma having properties similar to those found experimentally.

A number of theories were examined¹⁷⁻²³ and that of Scharer¹⁷ was found to satisfy the requirements stated above. This theory considers the stability of right hand circularly polarized electromagnetic waves propagating along $\vec{B}_0$ (the dc magnetic field) in a plasma characterized by an anisotropic Maxwellian electron velocity distribution with loss cones. The distribution function f_0 takes the following form:

$$f_0(v_{\perp}, v_{\parallel}) = \begin{cases} (N_0 \pi^{3/2} a_{\perp}^2 a_{\parallel})^{-1} \exp[-(v_{\perp}/a_{\perp})^2 + (v_{\parallel}/a_{\parallel})^2] & \text{for } v_{\perp} > |v_{\parallel}| (R-1)^{-1/2} \\ 0 & \text{for } v_{\perp} < |v_{\parallel}| (R-1)^{-1/2} \end{cases} \quad (30)$$

where $a_{\perp}^2 = 2 k T_{\perp}/m_e$ and $a_{\parallel}^2 = 2 k T_{\parallel}/m_e$; R is the magnetic mirror ratio; the symbols $\perp$ and $\parallel$ refer to directions perpendicular and parallel to the magnetic axis; and N_0 is a normalization factor which is given by $N_0 = \{(R-1) \theta / [(R-1) \theta + 1]\}^{1/2}$. The quantity θ is the ratio of $T_{\perp}$ to $T_{\parallel}$, the transverse and longitudinal electron temperatures. The dispersion

relation, derived by Scharer, for this distribution function is given by:

$$\omega^2 = X^2 c^2 - \omega_{pe}^2 \{ Z(\phi) [(R-1)^{-1} \phi^3 + \theta \phi - \omega_{ce} / K B_0 a_H] + (R-1)^{-1} [1/2 + \phi^2] + (\theta - 1) \} \quad (31)$$

where $Z(\phi)$ is the plasma dispersion function²⁴ for the argument ϕ , and ϕ is defined by $\phi = (\omega + \omega_{ce}) / K N_0 a_H$. Solutions are now sought to equation (31) with ω complex and K real. Instability corresponds to $\text{Im}(\omega) > 0$ since the dispersion relation was derived for perturbations of the form $\exp -i(\omega t + Kz)$.

Numerical solutions of equation (31) were first found for $\text{Im}(\omega) = 0$, corresponding to marginal stability. The parameters used in the computations were R , $T_{||}$, θ , and K (mirror ratio, parallel temperature, anisotropy and wave number, respectively). Typical results are shown in Figure 20 where the marginal stability boundary is plotted for the axes $(\omega_{pe} / \omega_{ce})^2$ and K . The region above each curve is unstable. These results indicate that instability is possible at a given density for sufficiently small wave numbers, while, for a disturbance with a given wave number, instability is possible for a plasma density larger than some minimum density. The computations also showed⁴ that $\text{Re}(\omega)$ was always less than ω_{ce} for marginal stability. Note that the dependence of ω on $T_{||}$ in Figure 15 is fairly weak, so that the difficulty in measuring $T_{||}$ accurately in the presence of the instability is not a serious source of error.

Equation (31) was then solved numerically using parameters which were typical of the experimental conditions at the time of the plasma instability. The calculation procedure was designed to search for the maximum value of ω_I corresponding to the maximum growth rate of the instability. In solving equation (31), the parameters R , n_e , $T_{||}$, and θ were fixed. Solutions for the complex frequency ($\omega = \omega_R + i\omega_I$) were found as a function of the independent variable K . One such solution is shown plotted in Figure 21. The maximum

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Figure 20

K (WAVENUMBER) (IN UNITS OF 10^2 m^{-1})

REAL PART OF FREQUENCY
 $\text{Re} [\omega]$ (Radians/sec)

IMAGINARY PART OF FREQUENCY
 $\text{Im} [\omega]$ (Radians / sec)

growth rate corresponds to the complex frequency solution $\omega = (5.12 \times 10^{10} + i 6.10 \times 10^8)$ radians/second for a wave number K of 179 m^{-1} .

Due to the finite length ($L=42 \text{ cm}$) of the resonant cavity, the allowed values for the axial wave number should be given by $K=l \pi/L$ where l is an integer. The solutions for maximum growth rate will then need to be found for a wave number K which satisfies this requirement.

Table III gives results found for the complex frequency with the discreteness condition taken into account. For a given magnetic mirror ratio (R), the computations were carried out for the parameters of electron density (n_e), anisotropy (Φ), and parallel temperature ($T_{\parallel}$) which were chosen to be approximately representative of the plasma conditions at the time of the instability.

If we identify the growth rate of the instability as the shortest measurable time for the diode detected microwave signal to reach its maximum value ($\sim 0.05 \text{ } \mu\text{sec.}$), then the corresponding maximum growth rate is 1.3×10^8 radians/second. It is expected that the conditions which correspond to the theoretical maximum growth rate will be most likely to lead to instability. The bands of frequency in which microwave emission was experimentally observed were between 4.99×10^{10} and 5.12×10^{10} radians/second (for $R = 1.02$) and between 4.89×10^{10} and 5.02×10^{10} radians/second (for $R = 1.19$). These experimental results compare well with the theoretical predictions shown in Table III. The wave number, K , was not measured in these experiments. However, the theoretical result is well within the lower and upper bounds on K set by the length of the resonant cavity ($K_{\text{lower}} = \pi/L = 7.5 \text{ m}^{-1}$) and by the requirement that K be smaller than $2\pi/\lambda_D$, where λ_D is the hot electron Debye length ($K_{\text{upper}} = 597 \text{ m}^{-1}$).

Table III

R	ω_R	ω_I	K	L	θ	T_H	n_e
1.021	5.15×10^{10}	5.87×10^8	187	25	1.0	6000	6.5×10^9
1.021	5.15×10^{10}	5.87×10^8	187	25	2.0	3000	6.5×10^9
1.021	5.16×10^{10}	5.87×10^8	187	25	4.0	1500	6.5×10^9
1.021	5.16×10^{10}	5.86×10^8	187	25	6.0	1000	6.5×10^9
1.021	5.12×10^{10}	6.80×10^8	187	25	1.0	6000	1.0×10^{10}
1.021	5.12×10^{10}	6.76×10^8	187	25	2.0	3000	1.0×10^{10}
1.021	5.12×10^{10}	6.67×10^8	187	25	4.0	1500	1.0×10^{10}
1.021	5.12×10^{10}	6.59×10^8	187	25	6.0	1000	1.0×10^{10}
1.190	4.78×10^{10}	1.46×10^8	165	22	2.0	5000	5.0×10^9
1.190	4.80×10^{10}	1.71×10^8	165	22	4.0	2500	5.0×10^9
1.190	4.80×10^{10}	1.61×10^8	165	22	6.0	1666	5.0×10^9
1.190	4.96×10^{10}	1.93×10^8	172	23	10.0	1000	5.0×10^9
1.190	4.69×10^{10}	2.60×10^8	165	22	1.0	10000	1.0×10^{10}
1.190	4.70×10^{10}	2.63×10^8	165	22	2.0	5000	1.0×10^{10}
1.190	4.84×10^{10}	2.35×10^8	172	23	4.0	2500	1.0×10^{10}
1.190	4.85×10^{10}	2.78×10^8	172	23	6.0	1666	1.0×10^{10}
1.190	4.87×10^{11}	2.91×10^8	172	23	10.0	1000	1.0×10^{10}

Solutions of the dispersion relation for the maximum instability growth rates with the condition $K = L/L$ taken into account. The units of the parameters are:
 ω_R, ω_I = radians/sec; $K = m^{-1}$; T_H = electron volts;
 n_e = electrons/cm³.

VIII. Future Research

Future research will concentrate on understanding the dynamics of the rf heating process by theoretical calculations, experimental measurements, and computer simulation.

The theoretical work will include an attempt to calculate the heating rate expected for resonance heating in a magnetic mirror. A number of attempts at solving this problem have appeared in the literature,^{10, 25-32} The predictions will be compared with the observed heating rate. Calculations of the free-free bremsstrahlung spectrum for an anisotropic electron distribution will be continued in an attempt to estimate the importance of this as an energy loss mechanism as well as to provide an independent check of electron energies as measured by other techniques.

The experimental program will be enhanced by the operation of a long pulse magnetron which should permit operation at lower background pressures and produce a more gentle heating, making possible detailed measurements of the electron heating rate as a function of experimental parameters such as mirror ratio, location of the resonance zones, microwave power, and background gas pressure. The primary diagnostics will continue to be the diamagnetic loop, the $\vec{E} \times \vec{B}$ analyzer and x-ray detectors. Attempts will be made to measure density and potential in the plasma with Langmuir probes. Calculations are being made that will hopefully allow us to determine the anisotropy from measurements of the density profile along the magnetic axis. Optical techniques may also be attempted.

The computer simulation program will be used to determine the heating and cooling rate for some simple geometries for which theoretical solutions are available. It will then be applied to a magnetic mirror field similar

to that presently under study. Ultimately, we would like to use the program to study rf heating in even more complicated geometries, such as stellarators and multipoles.

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Figure Captions

1. Diagram of experimental apparatus
2. Spectra of electron energies for various energy distributions (integrated over all angles)
3. More spectra of electron energies
4. X-ray intensity vs photon energy
5. X-ray intensity vs photon energy
6. X-ray intensity vs photon energy
7. Geometry of experimental setup for measuring x-rays
8. View of plasma through pinholes
9. Measured bremsstrahlung flux vs photon energy and corrections
10. Corrected bremsstrahlung intensity vs photon energy
11. Experimental verification of theoretically determined threshold energy of detector for $\vec{E} \times \vec{B}$ analyzer
12. Distribution of electron energies measured by the $\vec{E} \times \vec{B}$ analyzer
13. Distribution of electron energies measured by the $\vec{E} \times \vec{B}$ analyzer
14. Constant B surfaces for a mirror ratio of 1.19
15. Perpendicular and parallel electron energy vs time after the beginning of the second microwave pulse
16. Perpendicular and parallel electron energy vs time after the beginning of the second microwave pulse
17. Super particle in mesh
18. Plasma microwave emission
19. Plasma diamagnetism variation, x-ray bursts, and particle losses correlated with occurrence of the instability
20. Marginal stability boundary for $T_{\parallel} = 10^1, 10^2, 10^3, 10^4$ electron volts
21. Dispersion relation for right hand circularly polarized waves in an anisotropic Maxwellian electron plasma with loss cones

Visits of Laboratory Staff, Publications and Conference Papers

Presented During the Period of This Report

1. S. J. Gitomer, Electromagnetic Instability in an Electron Cyclotron Resonance Plasma, Ph.D. Thesis, January 1969.
2. J. L. Shohet, K. A. Connor and B. D. Cutting, A Long Pulse, 3 cm Microwave Source, University of Wisconsin PLP Report No. 253.
3. J. L. Shohet and A. J. Hatch, Eigenvalue of a Microwave Cavity Filled with a Plasma of Variable Radial Density, J. Appl. Phys., Submitted for publication.
4. S. J. Gitomer and J. L. Shohet, Electromagnetic Instability in an Electron Cyclotron Resonance Plasma, Phys. Fluids, to be published.
5. J. L. Shohet, D. G. S. Greene and K. A. Connor, visited the Thermonuclear Division, Oak Ridge National Laboratory, in January, 1969.
6. J. L. Shohet, Comments on "Resonant Absorption in Zero-Temperature Non-uniform Plasma," Phys. Fluids, submitted for publication.
7. S. J. Gitomer and J. L. Shohet, Electromagnetic Instability in an Electron Cyclotron Resonance Plasma, University of Wisconsin, PLP Report No. 274.
8. J. L. Shohet, Comments on "Resonant Absorption in Zero-Temperature Non-uniform Plasma," University of Wisconsin PLP Report No. 281.
9. J. L. Shohet and A. J. Hatch, A Computer Simulation of Single Particle R.F. Confinement, presented at the 11th Annual Meeting, Div. Plasma Physics, American Physical Society (1969).
10. J. L. Shohet and D. G. S. Greene, The Dependence of the Free-Free Bremsstrahlung Intensity on the Nature of the Anisotropic Energy Distribution of Relativistic Electrons, presented at the 11th Annual Meeting, Div. Plasma Physics, American Physical Society (1969).
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12. J. L. Shohet, K. A. Connor and T. A. Casper, Time Resolved Anisotropy of the Electron Energy Distribution in an Electron Cyclotron Resonance Plasma, presented at the 11th Annual Meeting, Div. Plasma Physics, American Physical Society (1969).