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ACCELERATOR DEPARTMENT
Informal Report

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THE HARMONIC CONTENT OF THE ENDS IN THE
COSINE THETA ISABELLE DIPOLES

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Abstract

Utilizing the theory developed by Mills and Morgan,¹ the harmonic content of an ISABELLE model dipole has been calculated and a modification arrived at which eliminates the sextupole and minimizes higher harmonics.

It was shown by Mills and Morgan¹ that the harmonic content q_n , the Taylor expansion coefficients of the integral value of the axial or z component of the vector potential, are given by

$$q_n = \frac{\mu_0 I}{2\pi n} \int_{\lambda} N(\lambda) d\lambda \int_{c_\lambda} \cos n\theta \left(\frac{r^n}{R^{2n}} + \frac{1}{r^n} \right) dz \quad (1)$$

where λ is a parameter specifying a turn or group of turns, I is the current per turn, θ and r are polar coordinates and c_λ is the space curve followed by the λ element; c_λ is in the form $G_1(\lambda, r, \theta, z) = 0$ and $G_2(\lambda, r, \theta, z) = 0$. R is the radius of the infinite- μ iron shield.

If c_λ specifies a 2-m pole magnet, by the use of symmetry arguments it can be shown that (1) becomes $q_n = 0$ for K even, and for K odd

$$q_n = \frac{2\mu_0 I}{K\pi} \int_{\theta_0} N(\theta_0) d\theta_0 \int_0^{Z_{\max}} \cos n\theta \left(\frac{r^n}{R^{2n}} + \frac{1}{r^n} \right) dz \quad (2)$$

where $K = n/m$ and λ is now the angle $\theta = \theta_0$ at which the linear elements of the two-dimensional part of the magnet are located at $z = 0$ where the end begins. For a dipole, $m = 1$ and $K = n$.

In the present analysis, $N(\theta_0)$ is the number of turns per block, and all the current is assumed to lie in a filament at the center of the block, of which there are six. Thus $N(\theta_0)$ is 14, 13, 12, 9, 6, 3 for $\theta_0 = 7.06, 20.56, 35.32, 49.39, 63.68$, and 77.18 degrees, respectively.

In the linear section of the magnets, the blocks have an inner radius of 4 cm and the block centers lie on a radius of 5.0418 cm. To improve the accuracy of the computation, an analytic form given by Mills and Morgan¹ was modified to the following:

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1. "A Flux Theorem for the Design of Magnet Coil Ends," F. Mills and G. Morgan to be submitted to Particle Accelerators.

$$G_1: \frac{x^2}{g^2(z)} + \frac{y^2}{R_c^2} = 1$$

$$G_2: \frac{z^2}{T^2(\theta_0)} + \left(\frac{x - K_1 R_c}{K_1 - \sin \theta_0} \right)^2 = R_c^2$$

where $x = r \sin \theta$, $y = r \cos \theta$, R_c is the block center radius, and K_1 is a constant. $g(z)$ is R_c at $z = 0$ and decreases with increasing z , so that G_1 is a circle in the $z = 0$ plane, and an ellipse of increasing eccentricity for $z > 0$. The function $T(\theta_0)$ has four constants chosen to make the block centers agree with measurements at $y = 0$, the top of the coil. Thus

$$g(z) = \{a + [a^2 + 4b(c + z)]^{1/2}\} / 2a + 4$$

and

$$T(\theta_0) = a_1 + a_2 \theta_0 + a_3 \sin \theta_0 + a_4 \sin 2\theta_0.$$

The constant $K_1 = 4.0800$ is chosen using G_2 and dx/dz from G_2 with T^2 eliminated, to make dx/dz at $\theta_0 = 7.06$ deg agree with observation, and the remaining constants a , b , c and a_1 are fitted by least squares, with the results 65.845, 52.493, 11.623 for a , b , and c , and 3.037, -0.1741, 12.915, -2.200 for the a_1 , respectively, if θ_0 is in degrees. The iron inner radius R is 6.35 cm. The numbers $z_{\max}$ in (2) are the intercepts of G_1 and G_2 at $y = 0$, rather than the observed block center positions; the difference is slight. The integral over θ_0 in (2) is replaced by a summation.

For convenience, the quantities $Q_n = q_n \pi / (2\mu_0 I)$ are computed in place of the q_n , and only the ratios of the Q_n to Q_1 computed for the straight section are reported. The straight section Q_1 is $L(R_c/R^2 + 1/R_c) \sum N(\theta_0) \cos \theta_0 = 934.16$ for a straight section length $L = 64.135$ cm ($25\frac{1}{2}$ inch). The Q_1 computed for one end is 82.67, which is 0.0885 of $Q(ss)$. Hence the ends contribute 15.0% of the BL product, and the effective length of an end is 2.23 inch. The Q_n computed are then divided by $Q_1(ss) + 2Q_1(end) = 1099.5$. To convert this ratio to the coefficients b_i in the median plane expression $B = B_0(1 + b_2 x^2 + b_4 x^4 + \dots)$, note that¹

$$q_n = \frac{1}{n!} \int \frac{\partial^{n-1} B}{\partial x^{n-1}} \bigg|_{r=0} dz,$$

i.e., $\bar{B}_n = nq_n x^{n-1}/L_n$, so that $b_i = 2(i+1)Q_{i+1}(2.54)^i/Q_1(\text{total})$, giving the b_i in inch^{-i} . The results are given in Table I.

TABLE I

i	2	4	6	8	10	12	14
$10^3 b_i$	-4.77	-1.76	-0.019	-0.038	-0.008	-0.0011	-0.0004

Note that all b_i are negative with respect to b_1 . This has implications for correcting the ends, as will be seen.

The correction procedure is simply to extend the length of a block in the straight section, preserving the shape of the end, i.e., to put spacers between blocks at the ends. By removing inert turns in the ends only, a limited amount of negative displacement of some of the blocks can be obtained, though this changes the end shape to a certain extent.

A computer program was written to compute the matrix of block displacements and coefficient change, invert it, and multiply the inverted matrix times the end harmonic content, with from 1 to 5 blocks and coefficients. The result was that only one coefficient could be nulled. Study of the matrix and block structure showed that using the maximum amount of inert turn removal combined with moving the outermost block ($\theta_o = 7.06$ deg) minimized the b_4 term without greatly increasing the remaining terms. The block displacements decided upon are 2.676, 0, -0.935, -0.472, 0, 0 from outer block inwards, and the resulting b_i 's are given in Table II, along with the maximum values proposed by Parzen.² The b_i 's for a magnet having the same ends but an effective length of 2.9 m are also shown.

TABLE II

i	2	4	6	8	10
$10^3 b_i$	0	-0.65	0.132	-0.026	-0.0003
max b_i (Parzen) $\times 10^3$	0.13	0.04	0.02	0.007	not given
$10^3 b_i$ (2.9 m magnet)	0	-0.165	0.033	-0.0066	-0.0001

2. G. Parzen, "Tolerances for Bending Magnets in a Superconducting Storage Accelerator," Int. Rept. AADD-194 (12/5/72).

The b_4 term is unacceptably large, even when averaged over a 2.9 meter length, but the remaining terms are reasonable.

It should be kept in mind that actual displacements will be based upon measurements as yet unmade. A larger bore magnet will have lower errors.

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