PCTR MEASUREMENT OF k_{∞}
FOR URANIUM LITHIUM SUPERCELL

D. F. Newman

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by

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December 20, 1967

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I. INTRODUCTION

Experimental measurements have been made on a test lattice as a benchmark to which theoretical models can be compared for the development of calculation methods.

This report describes the development of techniques and the experimental measurements utilizing the Physical Constants Testing Reactor (PCTR) for the measurement of the infinite medium neutron multiplication factor of a supercell consisting of separate graphite moderated cells of enriched uranium fuel and lithium-in-aluminum target in a three-to-one fuel-to-target ratio. The results of the experiment are to be used primarily to improve various theoretical models for calculating such mixed lattices.

II. SUMMARY

Experimental values for infinite medium neutron multiplication factor, k_{∞} , have been determined for a graphite moderated supercell utilizing 2.1% enriched uranium fuel and 2.93 wt% lithium-in-aluminum target in separate process tubes, in a three-to-one ratio. Determinations of k_{∞} were made with the coolant channels wet and dry to obtain an estimate of the reactivity coefficients due to water loss. Both a 6x6 and a 5x5 array of test cells were inserted in the PCTR test cavity for these measurements. Results of experimental measurements are listed in Table I.

TABLE I
Lattice Parameters Inferred from PCTR Measurements

<u>Test Array</u>	<u>Lattice Condition</u>	<u>M_O^{Cu}</u>	<u>Thermal Utilization</u>	<u>k_∞ (Air Atmos.)</u>	<u>k_∞ (vacuum Atmos.)</u>
6x6	Wet fuel - Wet target (wet buffers)	65 _{±15}	0.590 ± .002	1.0036 _{±.0008}	1.0064 _{±.001}
6x6	Dry fuel - Dry target (dry buffers)	-26 _{±28}	0.583 ± .002	0.9983 _{±.0018}	1.0008 _{±.002}
5x5	Wet fuel - Wet target (wet buffers)	2 ₋₃₇ ⁺⁶⁰	0.590 ± .002	1.0001 _{-.0020} ^{+.0033}	1.0029 _{-.0022} ^{+.0035}
5x5	Dry fuel - Dry target (dry buffers)	-237 _{±13}	0.583 ± .002	0.9847 _{±.0008}	0.9872 _{±.001}

Several perturbation measurements were taken to determine the reactivity changes due to limited water loss in the target and fuel, respectively. These results indicate that water loss in the target will decrease reactivity but that water loss in the fuel will slightly increase reactivity. Absolute neutron multiplication factors were not inferred from perturbation data since the mass of copper which is required to poison the lattice to unit multiplication is sensitive to the mismatch of the neutron spectrum incident on the perturbed cell. Non-linear relative differences are readily apparent in measuring the relative masses of copper necessary to compensate for the change in reactivity, ΔM. Results of the perturbation measurements are listed in Table II. Dry perturbations in the matched spectrum of the wet lattice appear to be more linear than the wet perturbations in the matched spectrum of the dry lattice.

TABLE II

Mass of Copper Required to Compensate for Coolant Perturbations

Measurements in Matched Spectrum of Wet Lattice

<u>Lattice Condition</u>	<u>ΔM, 5x5 Test Array</u>	<u>ΔM, 6x6 Test Array</u>
Dry target, Wet U_1 Wet U_2 (wet buffers)	-252 grams	-164 grams
Dry U_1 , Wet U_2 , Wet target (wet buffers)	-4 grams	59 grams
Dry U_2 , Wet U_1 , Wet target (wet buffers)	18 grams	85 grams

Measurements in Matched Spectrum of Dry Lattice

Wet target, Dry U_1 Dry U_2 (dry buffers)	201 grams	192 grams
Wet U_1 , Dry U_2 , Dry target (dry buffers)	34 grams	62 grams
Wet U_2 , Dry U_1 , Dry target (dry buffers)	-6 grams	20 grams

A blocking effect due to neutrons incident on the target coolant water being reflected away from the target material is indicated by the above results. The change in reactivity, Δk , due to loss of coolant water in the target cell only is estimated to be -0.012 ± 0.003 .

III. DESCRIPTION OF EXPERIMENT

A. Test Lattices Inserted into PCTR Cavity

The PCTR has a 41.25" x 41.25" x 37.5" cavity into which test lattices are placed. The test lattices in this experiment consisted of a 5x5 array of 7.5" graphite cells with 18 fuel columns and 7 target columns as shown in Figure 1, and a 6x6 array of 7.5" graphite cells with 27 fuel columns and 9 target columns, as shown in Figure 2. In each array, measurements

were made with four center cells (the combination of three fuel cells and one target cell is called a supercell). As seen in Figure 1, the central supercell of the 5x5 array is "T" shaped having ten surfaces exposed to the radial buffer cells. When the cell was removed, nine surfaces of fuel cells and one surface of a target cell were exposed to the central void. However, as seen in Figure 2, the central supercell of the 6x6 array is square shaped, having eight surfaces exposed to the radial buffer cells. The void formed by removal of the center supercell in the 6x6 array exposes six surfaces of fuel cells and two surfaces of target cells (precisely a 3-to-1 ratio). The stacking pattern of the two test lattice arrays were similar, so the difference in the k_{∞} measurement may be in part due to the difference in the uranium-to-target surface ratio of a single supercell. The supercell used in the 6x6 array is considered to be more typical of an infinite array of three-to-one supercells.

B. Fuel and Target Dimensions

The fuel was 2.099 at% enriched uranium and the target material was 2.93 wt% lithium (42.8 at% Li^6) in aluminum. A 21.875" long supercell was used as a typical sample of the test lattice to permit spectrum buffering in the axial direction as well as the radial direction. Details of the center fuel cells are listed in Table III; center target cell details are listed in Table IV.

C. Supercell Construction and Composition

The cells comprising the supercell can be further distinguished by their physical relation to the target cells. Two of the fuel columns per supercell are adjacent to one target column and will be called U_1 cells. The remaining fuel cell per supercell is between two target columns and will

be called a U_2 cell. The characteristic neutron spectrum and the neutron absorption rates of these cell types were different so that in an experiment involving such a supercell, relative absorption rates in each cell type were required simultaneously in order to determine the effect of adjacent cell interactions. Results of isotopic analyses on the 2.1% enriched uranium cores and the 2.93 wt% lithium target cores are listed in Table V and Table VI, respectively. Extensive measurements were made with the coolant channels filled with water and with the coolant channels dry for both the 6x6 array of test cells and for the 5x5 array of test cells, previously described.

FIGURE 1. PCTR Core Fuel Loading Diagram
(5 x 5 Test Cell Array)

U = 2.1% Enriched Uranium

T = Lithium in Aluminum Target

7.50" Lattice Pitch

FIGURE 2. PCTR Core Fuel Loading Diagram
(6 x 6 Test Cell Array)

TABLE III

Composition and Geometry of the Wet Fuel Central Cell

Region		Radii		Density	Center Cell	
Name	No.	Isotope	Outside cm	gm/cm ³	Mass, gms	Volume, cm ³
Water	1		0.660	0.9955	75.763	76.1
		H		0.1113	8.474	
		O		0.8842	67.289	
Clad	2	Al	0.804	2.699	99.042	36.7
Fuel	3		1.746	17.071(a)(b)	7161.325	419.5
		²³⁵ U		0.3590	150.60	
		²³⁸ U		16.532	6935.27	
		Al		0.17987	75.455	
Clad	4	Al	1.873	2.699	216.17	80.1
Water	5		2.187	0.9955	221.60	222.6
		H		0.1113	24.78	
		O		0.8842	196.82	
Process Tube	6	Zr	2.287	6.400	499.20	78.0
Void	7		2.338	1.2090x10 ⁻³	0.04981	41.2
		N		9.2554x10 ⁻⁴	0.03813	
		O		2.8361x10 ⁻⁴	0.01168	
Graphite	8		10.747(c)	1.6082	30888.1	19206.6
		C		1.6079	30881.4	
		N		2.6543x10 ⁻⁴	5.0984	
		O		8.1336x10 ⁻⁵	1.5623	

(a) Reduced density to account for fuel element end caps

(b) Density of 2.1% enriched uranium core is 18.93 gm/cm³

(c) Cylindricized radius

TABLE IV

Composition and Geometry of the Wet Target Central Cell

Region		Isotope	Radii	Density	Center Cell	
Name	No.		Outside cm	gm/cm ³	Mass, gms	Volume, cm ³
Target	1		1.907	2.46996	1567.987(a) (b)	634.9
		Li ⁶		0.02697	17.1235	
		Li ⁷		0.04205	26.6947	
		Al		2.40064	1524.1688(a)	
Clad	2	Al	2.032	2.699	231.844	85.9
Water	3		2.187	0.9956	113.6957	114.2
		H		0.1114	12.7227	
		O		0.8842	100.973	
Process Tube	4	Zr	2.287	6.400	499.20	78.0
Void	5		2.338	1.2090x10 ⁻³	0.04981	41.2
		N		9.2554x10 ⁻⁴	0.03813	
		O		2.8361x10 ⁻⁴	0.01168	
Graphite	6		10.747(c)	1.6082	30888.1	19206.6
		C		1.6079	30881.4	
		N		2.6543x10 ⁻⁴	5.0984	
		O		8.1336x10 ⁻⁵	1.5623	

(a) Includes aluminum end caps on target elements = 72.487 gm

(b) Mass of target core material is 1495.5 gm

(c) Cylindricized radius

TABLE V

FUEL ISOTOPIC ANALYSIS (a)

2.1% Enriched Uranium

<u>Element</u>	<u>Isotopic Content at%</u>
^{235}U	2.0990
^{236}U	.0193
^{238}U	97.8120
Cu	.0005
Al	.0033
H	.00015
Si	.0188
Ni	.0002
Mg	.0005
Fe	.0123
N	.0015
C	.0311
B	.000014
Cr	.0004
Mn	.0003
Be	.0001
Ca	.000019
Zr	.0005

(a) Personal communication, C. A. Strand

TABLE VI

Target Isotopic Analysis (a)

<u>Element</u>	<u>Isotopic Content at%</u>
Li ⁶	4.7092
Li ⁷	6.2935
Al	88.6188
B	.0001
Cd	.0005
Co	.0005
Cr	.0010
Cu	.0005
Fe	.2000
Ga	.0020
Mg	.0100
Mn	.0020
Mo	.0005
Na	.1000
Ni	.0002
Si	.0500
Sn	.0002
Ti	.0010
Zn	.0100

Conc. Li⁶ = $42.8 \pm .0$ at%

Total Li = 2.93 wt%

(a) Private communication, Hurshal G. Powers

IV. EXPERIMENTAL TECHNIQUES AND RESULTS

A. The PCTR Method for Determining k_{∞}

Since the Physical Constants Testing Reactor first went critical in October of 1955, many different types of experiments on various lattices have been performed. A frequent application of the reactor is the measurement of the infinite medium neutron multiplication factor (k_{∞}) for test lattices. The general techniques are well documented^(1,2) for measurements performed on single cells. A summary of the method for measuring k_{∞} of a reactor lattice will be given to allow a clearer understanding of the specific experiments presented in this report.

There are three related parts in a k_{∞} measurement in the PCTR:

1. The neutron energy distribution in the central test cell is adjusted so that it is identical to that which would be present in an infinite array of test cells.
2. The mass of neutron absorber (copper) required to absorb all the excess neutrons of the central test cell is accurately measured in the central cell having the proper neutron energy distribution, obtained in Part 1.
3. The relative reaction rates of the test cell components are measured in both the lattice poisoned to unit multiplication and the unpoisoned lattice when the proper neutron energy distribution exists in the test cell.

B. Neutron Energy Distributions in the Lattices

The neutron energy distribution is usually inferred by measuring cadmium ratios of resonance detectors, and the proper cadmium ratio is determined by comparisons of ratios between the central test cell and

surrounding cells at equivalent positions within each cell. One of the most useful materials for such measurements is gold which, when covered with cadmium, is activated almost entirely by neutrons in a narrow region around 4.9 ev. The use of this technique at intermediate energies is not at all complicated, but the use of a large number of foils having slightly different thicknesses made it necessary to correct the measured gold cadmium ratios to a standard foil thickness gold cadmium ratio using experimental data published by R. A. Bennett.⁽³⁾

Gold foils approximately 1/2" in diameter and 0.005" thickness were irradiated bare and in 0.040" thickness cadmium covers to obtain cadmium ratios from which neutron energy distributions could be inferred. The foils were placed on a radius approximately 1.75" away from the center of both fuel and target cells so that reasonable changes in the spectrum could be measured. Generally, the objective was to match the spectrum of all three cell types in the radial buffer region of the test lattice (i.e., U_1 , U_2 , and Target) so that each type would have the same cadmium ratio as the center cell of the same type. For the 5x5 test array (see Figure 1) the following comparisons were made:

<u>Cell Type</u>	<u>Center Cell</u>	<u>Equivalent Radial Buffer Cells</u>
U_1	12 and 14	2, 4, 7, 9, 17, 19, 22, 24
U_2	18	1, 5, 8, 11, 15, 21, 25
Target	13	3, 6, 10, 16, 20, 23

The 6x6 array of test cells (see Figure 2) used the following comparisons in the determination of a flux match:

<u>Cell Type</u>	<u>Center Cell</u>	<u>Equivalent Radial Buffer Cells</u>
U_1	16 and 22	2, 4, 6, 8, 10, 12, 14, 18, 20, 24, 26, 28, 30, 32, 34, 36
U_2	21	1, 5, 9, 13, 17, 25, 29, 33
Target	15	3, 7, 11, 19, 23, 27, 31, 35

Specifically, a weighted average of the cadmium ratio of the equivalent radial buffer cells was calculated for comparison with the cadmium ratio of the respective center cell. The weighting factor used is proportional to the inverse of the linear distance, center to center, from a radial buffer cell to the nearest center cell of the same type. Thus, the average cadmium ratio of a type of radial buffer is defined as:

$$\overline{CDR} = \frac{\sum W_i CDR_i}{\sum W_i} \quad (1)$$

where CDR_i is the cadmium ratio of the i^{th} radial buffer cell of a particular type, and W_i is its weighting factor. Supercell average cadmium ratios are obtained by linearly weighting the average cadmium ratios of each cell in the supercell. For the central supercell the average cadmium ratio is defined as:

$$\overline{CTR} = \frac{2 \overline{U}_1 + \overline{U}_2 + \overline{T}}{4} \quad (2)$$

where, $\overline{U}_1$ = average cadmium ratio of central U_1 type cells;

$\overline{U}_2$ = average cadmium ratio of central U_2 type cell;

$\overline{T}$ = average cadmium ratio of central target cell.

The radial buffer supercell average cadmium ratio is similarly defined, from quantities obtained with Equation (1);

$$\overline{RD} = \frac{2 \overline{CDR}_{U_1} + \overline{CDR}_{U_2} + \overline{CDR}_{Target}}{4} \quad (3)$$

The specific criteria for obtaining matched spectral conditions, therefore, is defined by the equivalence of Equations (2) and (3):

$$\overline{CTR} = \overline{RD} \quad (4)$$

Variation of the cadmium ratio was accomplished by changing the positions of the PCTR driver fuel columns outside the test lattice. The proper matched spectral conditions were obtained in the central supercell for each lattice condition by a judicious choice of driver fuel positions. The details of the cadmium ratio matching are presented in Table VII and Figures 3 and 4.

TABLE VII
Matched Supercell Average Gold Cadmium Ratios

<u>Coolant Channels</u>	<u>Test Lattice</u>	<u>Matched 5 mil Gold Cadmium Ratio</u>
Wet	5x5	4.47 $\pm$ 0.05
Wet	6x6	4.44 $\pm$ 0.05
Dry	5x5	3.30 $\pm$ 0.03
Dry	6x6	3.31 $\pm$ 0.03

The degree of mismatch in the fuel cells ($\Delta CDR_{Uranium}$) and in the target cells (ΔCDR_{Target}), which existed between the central supercell and the surrounding radial buffer cells, is plotted in Figure 3A for the

FIGURE 3. Wet Supercell Average Gold Cadmium Ratios

FIGURE 3A. Gold Cadmium Ratio Matching in the Wet Lattice

FIGURE 4. Dry Supercell Average Gold Cadmium Ratios

wet lattice. The central supercell would have been properly matched when both ΔCdR Uranium and ΔCdR Target were zero. When the 6x6 array of test cells was loaded into the central cavity of the reactor, the number of PCTR driver fuel columns necessary to achieve critical prevented measurement of a spectral mismatch in the positive region of Figure 3A. However, when the 5x5 array of test cells was loaded into the central cavity of the reactor, it was possible to load PCTR driver fuel far enough away from the test cells so that a positive cadmium ratio mismatch in the uranium cells was obtained. The mismatch in the target cell remained negative. It was not possible to obtain a positive cadmium ratio mismatch in the target cell because of the close proximity of PCTR driver fuel columns necessary to achieve critical to the radial buffer target cells. Water loss perturbation measurements in the target cell indicate that the target cell is much less sensitive to the spectral mismatch than is a fuel cell. However, the difference in the target cell mismatch in the two test cell arrays may account for some of the difference in the measured mass of copper required to absorb the excess neutrons of the central supercell.

C. Null Reactivity Measurements

The mass of neutron absorber required to absorb the excess neutrons produced by the test cell is determined by a null reactivity technique. This technique forms the basis for the PCTR measurement and depends upon the observation that a properly poisoned test cell will have the same reactivity effect on the reactor as an empty cavity. In a properly poisoned cell the neutron absorptions balance the neutron productions. The central supercell which was the test cell for the null reactivity measurements is shown being removed from the 5x5 test array in Figure 5. The same central

FIGURE 5
Central Supercell Loading in PCTR Core, 5 x 5 Test Cell Array

supercell is shown being removed from the 6x6 test array in Figure 6. Short end buffer cells enclose the central supercell along the long axis. Slots are provided in the lattice and on the central supercell for insertion of copper poison strips at the cell boundaries.

A reactivity series was done for each reactor loading for which gold cadmium ratios were measured. A reactivity series consisted of measuring the reactivity of the lattice in four configurations: ρ cell in, ρ cell out, ρ cell out with copper in the cavity, and ρ cell in with copper on the boundary. From these measurements the reactivity worth of three quantities are obtained:

1. The worth of the central supercell, $\Delta\rho_{\text{cell}} = \rho_{\text{cell in}} - \rho_{\text{cell out}}$.

2. The worth of a small increment of copper on the cell boundary,

$$\Delta\rho_{\text{Cu}}^{\text{boundary}} = \rho_{\text{cell in}} - \rho_{\text{cell in with copper on the boundary}}$$

3. The worth of a small increment of copper in the cavity,

$$\Delta\rho_{\text{Cu}}^{\text{cavity}} = \rho_{\text{cell out}} - \rho_{\text{cell out with copper in the cavity}}, \text{ which}$$

is vacated by the central supercell when it is out of the reactor.

These three quantities were combined to calculate the amount of copper that must be added to or subtracted from the test cell so that the null reactivity condition could be satisfied.

An estimate of the mass of copper required to absorb the excess neutrons of the central supercell (M_o) was obtained for each gold cadmium ratio measurement from the equation:

$$M_o = \frac{\Delta\rho_{\text{cell}} - \Delta\rho_{\text{N2}}}{\Delta\rho_{\text{Cu}}^{\text{boundary}}} M_{\text{Cu}}^{\text{boundary}} \quad (5)$$

FIGURE 6
Central Supercell Loading in PCTR Core, 6 x 6 Test Cell Array

where $M_{\text{Cu}}^{\text{boundary}}$ = the mass of the small increment of copper inserted at the boundary of the central supercell in the $\rho_{\text{cell in with copper on the boundary}}$ measurement

$\Delta\rho_{\text{N}_2}$ = the reactivity worth of the nitrogen in the air which is present in the cavity vacated by the central supercell when it is out of the reactor (See V-A, "Nitrogen Corrections").

It is readily apparent that the reactivity worth of the central supercell, and hence the mass of copper required to absorb the excess neutrons of the test cell, will change as a function of the neutron energy distribution in the supercell. Because it is impractical to achieve an exact matched spectrum loading, several measurements are made which bracket the desired match and converge toward it. The correct mass of copper required to achieve unit multiplication is then interpolated as a function of the central supercell average gold cadmium ratio ($\overline{\text{CTR}}$) in Figure 7 for the wet lattice, and in Figure 8 for the dry lattice.

FIGURE 7. Correct Mass of Copper to Achieve Unit Multiplication

FIGURE 8. Correct Mass of Copper to Achieve Unit Multiplication

TABLE VIII

Relative Activities of Copper Foils in Central Supercell Graphite

Traverse Position	Radius Inches	Wet Lattice		Dry Lattice	
		Activity		Activity	
		Bare	Cd Covered	Bare	Cd Covered
Ref.*		1.000	1.000	1.000	1.000
r ₁	3.750	5.654	.349	5.308	.390
r ₂	2.420	5.486	.358	5.082	.389
r ₃	1.000	4.540	.338	3.702	.371
r ₄	1.000	4.404	.337	3.667	.368
r ₅	2.420	5.354	.356	4.998	.387
r ₆	3.750	5.400	.347	5.184	.388
r ₇	2.420	4.974	.340	4.699	.377
r ₈	1.000	3.386	.298	2.774	.332
r ₉	1.170	3.570	.306	3.279	.342
r ₁₀	1.670	4.250		4.031	
r ₁₁	2.170	4.684	.329	4.519	.370
r ₁₂	2.670	5.018		4.963	
r ₁₃	3.170	5.231	.340	5.074	.385
r ₁₄	3.670	5.381		5.267	
r ₁₅	4.230	5.491	.358	5.383	.397
r ₁₆	3.670	5.598		5.357	
r ₁₇	3.170	5.543	.350	5.407	.396
r ₁₈	2.670	5.529		5.154	
r ₁₉	2.170	5.364	.347	4.925	.388
r ₂₀	1.670	5.133		4.561	
r ₂₁	1.170	4.784	.338	3.972	.372
r ₂₂	1.170	4.485	.347	3.942	.371
r ₂₃	1.670	4.983		4.606	
r ₂₄	2.170	5.261	.340	4.947	.383
r ₂₅	2.670	5.431		5.175	
r ₂₆	3.170	5.555	.347	5.330	.391
r ₂₇	3.670	5.619		5.431	
r ₂₈	4.230	5.647	.350	5.486	.394
r ₂₉	1.170	3.488	.304	3.184	.341
r ₃₀	1.670	4.236		3.983	
r ₃₁	2.170	4.702		4.445	
r ₃₂	2.670	5.017	.335	4.828	.376
r ₃₃	3.170	5.269		5.075	
r ₃₄	3.670	5.442		5.262	
r ₃₅	4.170	5.577	.358	5.410	.387
r ₃₆	4.170	5.707	.359	5.753	.399
r ₃₇	3.670	5.673		5.666	
r ₃₈	3.170	5.605		5.525	
r ₃₉	2.670	5.507	.366	5.270	.390
r ₄₀	2.170	5.331		5.065	

*External thermal column (bare foils)

TABLE VIII (Continued)

Traverse Position	Radius Inches	Wet Lattice		Dry Lattice	
		Activity		Activity	
		Bare	Cd Covered	Bare	Cd Covered
r ₄₁	1.670	5.030		4.697	
r ₄₂	1.170	4.589	.338	4.057	.367
r ₄₃	1.170	4.778	.339	4.063	.371
r ₄₄	1.670	5.164		4.686	
r ₄₅	2.170	5.404	.349	5.057	.383
r ₄₆	2.670	5.562		5.297	
r ₄₇	3.170	5.677	.352	5.407	.394
r ₄₈	3.670	5.740		5.558	
r ₄₉	4.230	5.815	.370	5.633	.397
r ₅₀	2.000	4.493	.328	4.177	.362
r ₅₁	2.000	5.332	.347	4.760	.380
r ₅₂	2.000	5.205	.353	4.745	.382

D. Relative Reaction Rate Measurements

Relative reaction rates of the test cell components are determined from foil activation measurements. Energy dependences of the reaction rates are determined from cadmium ratios. Bare and cadmium covered foils were activated in separate irradiations, and normalization between irradiations was accomplished by placing monitor foils in an external thermal column.⁽⁴⁾

The relative reaction rates of the supercell components were determined in loadings where the gold cadmium ratio data indicated that the neutron spectrum had the proper energy distribution. Copper foils (0.010" thickness) were used for traverses in the graphite, in the fuel, in the target, and in the coolant channels. Foil locations in the central supercell graphite, shown in Figure 9, are sufficiently provided so that two dimensional plots of relative reaction rates can be made.

The specific activities of the copper foils placed in the graphite relative to the foil in the external thermal column are presented in Table VIII. The activities were calculated using a revised version of the computer code APDAC-I,⁽⁵⁾ which corrects the counting data for activity decay during counting, room background, foil mass variations, and residual activity. Representative curves of the total, sub-cadmium, and epi-cadmium relative copper-63 activity variation along the angular traverse direction of Figure 9 and in the target and fuel rods are shown in Figure 10 for the wet lattice, and Figure 11 for the dry lattice.

FIGURE 9. Foil Locations in the Central Supercell Graphite

FIGURE 10. Cylindricized Radial Traverse Between Target and Fuel in the Wet Lattice Relative Reaction Rates in Copper-63

FIGURE 11. Cylindricized Radial Traverse Between Target and Fuel in the Dry Lattice Relative Reaction Rates in Copper-63

The measured spatial distribution of the relative reaction rates in copper-63, $SA^{63}(r)$, was corrected for non-1/v absorption rates. Non-1/v corrections were inferred from copper-63 cadmium ratio measurements and resonance integral data.⁽³⁾

$$SA_{1/v}^{63}(r) = SA_{total}^{63}(r) - SA_{epi\ Cd}^{63}(r) \left(\frac{RI_{ex}}{RI_{epi\ Cd}} \right) \quad (6)$$

where, RI_{ex} = effective excess resonance integral for copper-63 in a 0.010" thick foil, 1.63073 barns.

$RI_{epi\ Cd}$ = effective total epi-cadmium resonance integral for copper-63 in a 0.010" thick foil, 3.42011 barns.

Axial distribution of 1/v copper-63 reaction rates in the wet lattice are shown in Figure 12 for the target cell, and in Figure 13 for the U_1 fuel cell. Axial distributions in the dry lattice target and U_1 fuel cells are shown in Figures 14 and 15, respectively.

Figure 16 shows a target cell and a fuel cell from the central supercell with the special separable elements and other provisions for foil traverses. Special sector and quadrant foils were made to integrate the neutron reaction rate over the area of the fuel and target, respectively. In order to obtain spatial detail of the reaction rate integrals, a copper sector foil in the fuel and a copper quadrant in the target were dissected into several annular regions after irradiation. These foil pieces were counted separately and yielded a histogram of specific activities relative to the monitor foil in the external thermal column. Cadmium dishes in the separable fuel element were designed so that they displaced the aluminum cladding in order to allow the inside dimensions of the dish to be identical to the fuel core dimensions. Foils placed inside the cadmium dish, having one mil aluminum fission product

FIGURE 12. Axial Distributions of $1/v$ Copper-63 Neutron Reaction Rate in the Wet Target Central Cell

FIGURE 13. Axial Distributions of $1/v$ Copper-63 Neutron Reaction Rate in the Wet U_1 Type Fuel Central Cell

FIGURE 14. Axial Distributions of $1/v$ Copper-63 Neutron Reaction Rate in the Dry Target Central Cell

FIGURE 15. Axial Distributions of $1/v$ Copper-63 Neutron Reaction Rate in the Dry U_1 Type Fuel Central Cell

FIGURE 16
Separable Elements and Provisions for Foil Measurements

catcher foils on their surfaces, were between two 0.045" wafers of the fuel core material, also inside the cadmium dish. Bare foils, having one mil aluminum fission product catcher foils on their surfaces, were placed in holders machined from the fuel core material.

E. Neutron Cross Section Generating Technique

Experimentally determined boundary conditions were utilized in transport theory calculations for generation of spectrum averaged cross sections. Two-dimensional plots of the sub-cadmium neutron reaction rate in copper-63 were obtained from experimental data presented in Table VIII. Radial activation traverses in all center cells at various angles were used to determine the sub-cadmium zero net neutron current boundary between adjacent cells of the supercell. Equivalent cell radii were determined by integration of the area inside the sub-cadmium zero net neutron current boundaries, and calculation of the radii having the same cylindricized cross sectional area. Topographical plots of the sub-cadmium copper-63 relative activation and the adjacent cell zero net neutron current boundaries are shown in Figure 17 for the wet lattice and in Figure 18 for the dry lattice, with equivalent cell radii tabulated for each cell type.

Thermal neutron spectrum averaged microscopic cross section values, calculated by transport theory, were obtained from computer code THERMOS.⁽⁶⁾ Single cell calculations were made for each type of cell both wet and dry in cylindricized geometry, and a reflecting boundary condition was used at the outer boundary. The location of the outer boundary was obtained from the equivalent cell radius determined experimentally from sub-cadmium zero net neutron current boundaries.

FIGURE 17. Sub-Cadmium Iso-Flux Lines in Wet Uranium-Lithium Supercell

FIGURE 18. Sub-Cadmium Iso-Flux Lines in Dry Uranium-Lithium Supercell

This technique is considered to be a distinct improvement over the direct use of the Wescott approximation⁽⁷⁾ for calculating effective thermal cross sections, $\hat{\sigma} = \sigma_0(g + rs)$, wherein one must, in practice, assume both the energy distribution of neutrons and its characteristic spectral indices, e.g., a Maxwellian with an estimated neutron temperature. In general, the cross sections calculated by THERMOS are defined as:

$$\hat{\sigma}(r) = \frac{\int_0^{E_{ec}} \sigma^c(E) \phi(E, r) dE}{\int_0^{E_{ec}} \sqrt{E_0/E} \phi(E, r) dE}, \quad (7)$$

where, E_{ec} = upper limit of the thermal neutron spectrum, taken as 0.683 ev.

$$E_0 = 0.0253 \text{ ev}$$

$\phi(E, r)$ = space-energy distribution of neutrons.

Region averaged THERMOS cross sections calculated using the previously described technique are presented in Table IX for the wet lattice. One may note that as defined in Equation 7, the values will be 2200 m/sec values unless the cross section is non-1/v for $0 \leq E \leq 0.683$ ev, e.g., see ²³⁵U in Table IX.

TABLE IX

Thermal Average Cross Sections from THERMOS

<u>U₁ Type Fuel Cell, Wet Lattice</u>				
Isotope	Region	$\bar{\sigma}_a$	$\bar{\sigma}_a 1/v$ (a)	$\hat{\sigma}_i = \frac{(\bar{\sigma}_a)_i}{(\bar{\sigma}_a 1/v)_i}$
Al	1	0.15631	0.67959	0.23000
O	1	1.4270x10 ⁻⁴		0.00021
H	1	0.22290		0.32800
²³⁵ U	2	404.724	0.62997	642.45
²³⁸ U	2	1.7445		2.7692
Al	2	0.14489		0.23000
Al	3	0.16844	0.73235	0.23000
H	3	0.24021		0.32800
O	3	1.5378x10 ⁻⁴		0.00021
Zr	4	0.13483	0.74904	0.18000
N	5	1.3133	0.75042	1.7500
O	5	1.5758x10 ⁻⁴		0.00021
C	6	2.5409x10 ⁻³	0.74732	0.00340
N	6	1.3079		1.7500
O	6	1.5693x10 ⁻⁴		0.00021
Cu	6	2.8324		3.7900

<u>U₂ Type Fuel Cell, Wet Lattice</u>				
Al	1	0.15510	0.67434	
O	1	1.4160x10 ⁻⁴		
H	1	0.22118		
²³⁵ U	2	400.382	0.62389	641.75
²³⁸ U	2	1.7283		
Al	2	0.14349		
Al	3	0.16687	0.72550	
H	3	0.23787		
O	3	1.5234x10 ⁻⁴		
Zr	4	0.13350	0.74169	
N	5	1.3003	0.74301	
O	5	1.5602x10 ⁻⁴		
C	6	2.5152x10 ⁻³	0.73976	
N	6	1.2946		
O	6	1.5534x10 ⁻⁴		
Cu	6	2.8037		3.7900

(a) $\sigma_{a_o} = 1.000$

TABLE IX (cont.)

Target Cell, Wet Lattice (cont.)

Isotope	Region	$\bar{\sigma}_a$	$\bar{\sigma}_a$ (a) 1/v	$\hat{\sigma}_1 = \frac{(\bar{\sigma}_a)_1}{\bar{\sigma}_a \text{ 1/v }_1}$
Al	1	0.12449	0.54127	0.23000
Li ₆	1	510.97		944.02
Li ₇	1	1.7862x10 ⁺²		0.03300
Al ₇	2	0.15867	0.68987	0.23000
H	3	0.23606	0.71968	0.32800
O	3	1.5112x10 ⁻⁴		0.00021
Zr	4	0.13255	0.73639	0.18000
N	5	1.2919	0.73799	1.7506
O	5	1.5501x10 ⁻⁴		0.00021
C	6	2.4723x10 ⁻³	0.72715	0.00340
N	6	1.2726		1.7501
O	6	1.5269x10 ⁻⁴		0.00021
Cu	6	2.7559		3.7900

F. Determination of 1/v Utilizations

The 1/v utilization of a cell component is defined as the ratio of the 1/v absorption rate in that component, to the sum of 1/v absorption rates in all cell components. Experimental values are inferred from measured spatial distributions of the 1/v neutron reaction rate in ^{63}Cu . At space point r the 1/v absorption rate in ^{63}Cu is given by:

$$\overline{SA}^{63}(r) = c \frac{N_0}{A^{63}} \hat{\sigma}^{63}(r) \int_0^\infty \sqrt{E_0/E} \phi(E, r) dE \quad (8)$$

where c is a space independent normalizing constant.

A similar value of $\overline{SA}^x$ for the material, x , in any given region, may be obtained from the simple ratio,

$$\overline{SA}^x(r) = \left(\frac{\hat{\sigma}^x(r) A^{63}}{\hat{\sigma}^{63}(r) A^x} \right) \overline{SA}^{63}(r) \quad (9)$$

where A is the atomic weight.

The total 1/v absorption rate in material x , $(\text{Abs})^x$, of a region may be obtained from the volume weighted average of $\overline{SA}^x(r)$:

$$(\text{Abs})^x = M^x \frac{\int_{\text{all of region } x} \overline{SA}^x(r) dV}{\int_{\text{all of region } x} dV} \quad (10)$$

where M^x is the mass of material x in the region.

Finally, then, the 1/v utilization of material x is:

$$f^x = (\text{Abs})^x / \sum_i (\text{Abs})^i \quad (11)$$

Absorption rates for all supercell components and 1/v utilizations of these components are presented in Table X.

TABLE X

Experimental $1/v$ Absorption Rates and k_{∞} Measurements

I. Wet Lattice:

#	Region	Isotope	Abs. $1/v$	Cell $1/v$ Util.	Unpoisoned Super-cell $1/v$ Util.
1	Target	Li ⁶	31244.245	.958240	.334612
		Li ⁷	1.459	.000045	.000016
		Al	143.987	.004416	.001542
2	Target Clad	Al	49.310	.001512	.000528
3	Water	H	188.862	.005792	.002023
		O	.062	.000002	.000001
4	Process Tube	Zr	63.954	.001961	.000685
5	Air	N	.241	.000007	.000002
		O	.000	.000000	.000000
6	Graphite	C	850.180	.026075	.009105
		N	62.545	.001918	.000670
		O	.000	.000000	.000000
TOTAL TARGET CELL:			32605.849		
1	Water	H	130.028	.006463	.001393
		O	.037	.000002	.000000
2	Fuel Inner Clad	Al	35.400	.001759	.000379
3	Fuel	²³⁵ U	15241.188	.757507	.163226
		²³⁸ U	2982.439	.148231	.031941
	End caps	Al	23.811	.001183	.000255
4	Fuel Outer Clad	Al	124.085	.006167	.001329
5	Water	H	540.690	.026873	.005791
		O	.172	.000009	.000002
6	Process Tube	Zr	76.958	.003825	.000824
7	Air	N	.339	.000017	.000004
		O	.000	.000000	.000000
8	Graphite	C	904.824	.044971	.009690
		N	66.065	.003284	.000708
		O	.000	.000000	.000000
TOTAL U ₁ FUEL CELL:			20120.186		

$$f_{1/v} = .905738$$

TABLE X (Cont.)

Wet Lattice

#	Region	Isotope	Abs. l/v	Cell l/v Util.	Unpoisoned Supercell - l/v Util.
1	Water	H	124.171	.006049	.001330
		O	.040	.000002	.000000
2	Fuel inner clad	Al	42.139	.002053	.000451
3	Fuel	²³⁵ U	15566.925	.758318	.166715
		²³⁸ U	3050.473	.148599	.032669
	End caps	Al	24.354	.001186	.000261
4	Fuel outer clad	Al	124.820	.006080	.001337
5	Water	H	546.335	.026614	.005851
		O	.173	.000008	.000002
6	Process tube	Zr	78.021	.003801	.000836
7	Air	N	.449	.000022	.000005
		O	.000	.000000	.000000
8	Graphite	C	904.314	.044052	.009685
		N	66.020	.003216	.000707
		O	.000	.000000	.000000
TOTAL U ₂ FUEL CELL:			20528.234		

$$f_{l/v} = .906917$$

TOTAL SUPERCELL: 93374.455

$$f_{l/v} = .589718 \text{ (air atmosphere)}$$

$$f_{l/v} = .591378 \text{ (vacuum atmosphere)}$$

Copper Poison Requirements

Wet Lattice:	M _O	M _{l/v}	Abs l/v	k _∞ (air atm)	k _∞ (vacuum atm)
Square shaped supercell (6x6 array)	65±15	67.44	333.299	1.0036±.0008	1.0064±.001
"T" shaped supercell (5x5 array)	2 ⁺⁶⁰ ₋₃₇	2.07	10.230	1.0001 ^{+.0033} _{-.0020}	1.0029 ^{+.0035} _{-.0022}

II. Dry Lattice

1	Target	Li ⁶ ₇	31242.920	.969768	.370918
		Li ⁷	1.459	.00045	.000017
		Al	149.515	.004641	.001775
2	Target clad	Al	42.222	.001311	.000501
3	Air	N	.214	.000007	.000003
		O	.000	.000000	.000000
4	Process tube	Zr	42.412	.001316	.000503
5	Air	N	.182	.000006	.000002
		O	.000	.000000	.000000
6	Graphite	C	687.743	.021347	.008165
		N	50.229	.001559	.000596
		O	.000	.000000	.000000
TOTAL TARGET CELL:			32216.896		

TABLE X (cont.)

Dry Lattice:					
#	Region	Isotope	Abs. l/v	Cell l/v Util.	Unpoisoned Super-cell - l/v Util.
1	Air	N	.251	.000014	.000003
		O	.000	.000000	.000000
2	Fuel inner clad	Al	24.109	.001401	.000286
3	Fuel	²³⁵ U	13501.498	.784818	.160291
		²³⁸ U	2752.350	.159989	.032676
	End caps	Al	21.918	.001274	.000260
4	Fuel outer clad	Al	88.740	.005158	.001053
5	Air	N	1.240	.000072	.000015
		O	.000	.000000	.000000
6	Process tube	Zr	49.231	.002862	.000585
7	Air	N	.261	.000015	.000003
		O	.000	.000000	.000000
8	Graphite	C	712.925	.041441	.008464
		N	52.062	.003026	.000618
		O	.000	.000000	.000000
TOTAL U ₁ FUEL CELL:			17203.345	$f_{1/v} = .944807$	
1	Air	N	.251	.000014	.000003
		O	.000	.000000	.000000
2	Fuel inner clad	Al	24.110	.001369	.000286
3	Fuel	²³⁵ U	13806.174	.784099	.163908
		²³⁸ U	2806.445	.159387	.033318
	End caps	Al	22.348	.001269	.000265
4	Fuel outer clad	Al	106.329	.006039	.001262
5	Air	N	1.486	.000084	.000018
		O	.000	.000000	.000000
6	Process tube	Zr	65.395	.003714	.000776
7	Air	N	.322	.000018	.000004
		O	.000	.000000	.000000
8	Graphite	C	722.123	.041012	.008573
		N	52.709	.002994	.000626
		O	.000	.000000	.000000
TOTAL U ₂ FUEL CELL:			17607.692	$f_{1/v} = .943486$	
TOTAL SUPERCELL:			84231.278	$f_{1/v} = .583160$ (air atmosphere)	
				$f_{1/v} = .584639$ (vacuum atmosphere)	

Copper Poison Requirements

Dry Lattice:	M _O	M _{1/v}	Abs l/v	k _∞ (air atm)	k _∞ (vacuum atm)
Square shaped supercell (6x6 array)	-26+28	-27.32	-141.627	.9983+.0018	1.0008+.002
"T" shaped supercell (5x5 array)	-237+13	-249.01	-1290.865	.9847+.0008	.9872+.001

G. Determination of k_{∞} from These Experiments

The equations for calculating k_{∞} are derived from the four factor formula⁽⁸⁾:

$$k_{\infty} = \eta \epsilon p f \quad (12)$$

It is assumed that the addition of copper to the boundary of the supercell will change only the thermal utilization. Since the k_{∞}' of the supercell with the correct mass of copper of the boundary (Mo) is unity,

$$k_{\infty}' = \eta \epsilon p f' = 1 \quad (13)$$

Thus, k_{∞} has been derived^(1,2) as the ratio of the unpoisoned to poisoned thermal utilizations of the supercell,

$$k_{\infty} = \frac{f}{f'}, \text{ where} \quad (14)$$

f = the $1/v$ utilization of the unpoisoned supercell;

f' = the $1/v$ utilization of the poisoned supercell.

The derivation of the $f_{1/v}$ (in equation 11) is based on the assumption that the absorption rates in all the cell components, except for the resonance absorptions in ^{238}U , can be calculated by using the flux measured by a detector whose absorption cross section varies as $1/\sqrt{E}$ multiplied by the effective 2200 m/sec cross sections calculated by THERMOS. The material used as the absorber in the determination of $k_{\infty} 1/v$ must also have an effective cross section that varies as $1/\sqrt{E}$. The use of the four factor formula the derivation of $k_{\infty} 1/v$ implies that:

$$k_{\infty} 1/v = \eta_{1/v} \epsilon p_{\text{non-1/v}} f_{1/v} \quad (15)$$

All non- $1/v$ epithermal absorption is accounted for in the resonance escape probability, $p_{\text{non-1/v}}$.

Equation 14 can be rewritten in terms of absorption rates by use of equation 11,

$$k_{\infty} - 1 = \frac{\sum_i (Abs)_p'}{\sum_i (Abs)_i} + \frac{\sum_i [(Abs)_i' - (Abs)_i]}{\sum_i (Abs)_i} \quad (16)$$

$(Abs)'_p$ = relative $1/v$ absorption rate in the copper poison.

$(Abs)'_i$ = relative $1/v$ absorption rate in the i^{th} component of the poisoned supercell.

$(Abs)_i$ = relative $1/v$ absorption rate in the i^{th} component of the unpoisoned supercell.

A condition on Equation 16 is that the absorption rates in the fuel for the poisoned and unpoisoned conditions are normalized to be equal. Since the correct mass of copper required to achieve a multiplication factor of unity in this supercell is very small (for the wet lattice) or negative (for the dry lattice), the copper addition would not measurably have changed the relative absorption rate, $(Abs)_i$, in any of the cell components of the unpoisoned lattice. With this assumption, equation 16 reduces to the expression, $k_{\infty} - 1 = \frac{\sum_i (Abs)'_p}{\sum_i (Abs)_i}$ (17)

Since the absorption rate in the copper poison is small with respect to the total supercell absorption rate, the measurement of k_{∞} in this experiment should be very precise.

Values for k_{∞} determined from these PCTR experiments are presented in Table X for both the 5X5 array and the 6X6 array of test cells. As seen in Figures 19 and 20 the only differences between the two lattices, assuming a matched spectrum in each, is the geometry of the cavity formed by removal of the central supercell. Neutron energy distributions inferred by gold cadmium, ratio measurements (See Table VII) in the 5X5 and the 6x6 array were the same.

FIGURE 19
Test Lattice Loaded into PCTR Cavity, 5 x 5 Array

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FIGURE 20
Test Lattice Loaded into PCTR Cavity, 6 x 6 Array

Close inspection of the lattice stacking shows that there are three possible supercell building blocks which can be used to reproduce the same three-to-one supercell lattice:

Building block (A) was used as the central supercell of the 5x5 array. This "T" shaped central supercell has ten surfaces exposed to the radial buffer cells. Nine graphite surfaces of fuel cells and one graphite surface of a target cell lie on the boundary between the central supercell and the radial buffer cells. In contrast, the "T" shaped building block (B) has seven graphite surfaces of fuel cells and three graphite surfaces of a target cell on the boundary between the central supercell and the radial buffer cells. Building block (C) was used as the central supercell of the 6x6 array. This square shaped central supercell has eight surfaces exposed to the radial buffer cells. Six graphite surfaces of fuel cells and two graphite surfaces of a target cell (precisely a three-to-one ratio) lie on the boundary between the central supercell and the radial buffer cells.

If one imagines a semi-infinite slab reactor, two cells high, made up of building blocks (A), the ratio of the number of graphite surfaces of fuel cells to the number of graphite surfaces of target cells in the slab is given by:

$$S.R. = \frac{3N+6}{N} , \quad (18)$$

where N = number of supercell building blocks. For building block (B), this surface ratio is given by:

$$S.R. = \frac{3N+4}{N+2} . \quad (19)$$

Similarly, the surface ratio for building block (C) is given by:

$$S.R. = \frac{3N+3}{N+1} = 3 . \quad (20)$$

It can be readily seen that in the limit as $N \rightarrow \infty$ in Equations (18) and (19), the surface ratio converges to a value of 3.0, as given by Equation (20). Since the lattice geometry is reproduced by stacking the two-cell high semi-infinite slab reactors on top of one another, the square shaped building block (C) is considered to be most typical of an infinite array of the three-to-one supercell lattice. However, the "T" shaped central supercell k_{∞} measurements yielded inferred values for the magnitude of this surface effect on reactivity for a single building block of type (A): -3.4 mk in the wet lattice, and -13.4 mk in the dry lattice. This information should be valuable in estimating the reactivity of a block loading of these cells into a reactor in a ratio slightly different from three-to-one. Since the reactivity effect due to coolant loss in the 6x6 array is -5.1 mk, the effect of increasing the uranium-to-target surface ratio will be a further loss of reactivity due to coolant loss.

Relative reaction rate measurements on the central supercell boundary were made both with the central cell in, and with the central cell removed. Determination of the infinite medium neutron multiplication constants, k_{∞} , utilizing the unpoisoned technique, and the determination of the adjoint weighted excess neutron production cross sections, $\langle \sum_p \rangle$, will be reported in a future paper.

V. CORRECTIONS AND SYSTEMATIC ERRORS

A. Nitrogen Corrections

Nitrogen affects the null reactivity measurement of the supercell in two ways: first, air occupies the void spaces in the graphite and acts as a poison; second, upon removal of the central supercell ambient air enters and fills the cavity. Measured values of k_{∞} for the supercell in the PCTR are for such a lattice stacked in an air environment. To obtain the values for k_{∞} of these lattices stacked in a helium or vacuum environment, the $1/v$ utilization of nitrogen in the supercell (see Table X) should be removed. This will increase the k_{∞} values by 2.6 mk.

Determination of the reactivity worth of the nitrogen, which fills the cavity formed by removal of the central supercell, $\Delta\rho_N$, is inferred from a copper sensitivity, S_{Cu}^{cavity} , measurement using the expression:

$$\Delta\rho_N = D C \frac{P}{T} (M_N)(1-c) S_{Cu}^{cavity} \quad (21)$$

where D = thermal disadvantage factor for the thickness of copper
used in the determination of $S_{Cu}^{cavity} = 1.0110$;

$S_{Cu}^{cavity} = \Delta\rho_{Cu}^{cavity} / M_{Cu}^{cavity}$, ϕ /gram of copper;

$\Delta\rho_{Cu}^{cavity}$ = worth of mass of copper, M_{Cu}^{cavity} , in the supercell cavity, ϕ ;

P = atmosphere pressure during reactivity measurement, mb;

T = ambient temperature in cavity during reactivity measurement, $^{\circ}K$;

M_N = mass of nitrogen (at standard temperature and pressure)
in the cavity = 79.1577 grams.

$$C = \frac{\sigma_o^N}{\sigma_o^{Cu}} \frac{A^{Cu}}{A^N} \frac{T_o}{P_o}$$

σ_o^N = 2200 m/sec microscopic absorption cross section of nitrogen = 1.88 b.

σ_o^{Cu} = 2200 m/sec microscopic absorption cross section of natural copper = 3.79 b

A^{Cu} , A^N = copper and nitrogen atomic weights.

T_o , P_o = standard temperature and pressure.

The quantity (1-c) is a correction for the non-1/v nature of the epi-thermal energy dependance of the absorption cross section of natural copper, approximated by:

$$1 - c = 1 - \left(\frac{m_1}{m_2} \right) \frac{RI_{ex}^{Cu}}{RI_{epi-Cd}^{Cu} (CdR-1)} \quad (22)$$

where $\frac{m_1}{m_2}$ = ratio of the fast-to-thermal adjoint fluxes and is approximately equal to the resonance escape probability of the test lattice = 0.9081, calculated by MOFDA. (9)

RI_{ex}^{Cu} = effective excess resonance integral of natural copper for the thickness of copper used in measuring $S_{Cu}^{cavity} = 1.3705$ b.

RI_{epi-Cd}^{Cu} = effective epi-cadmium resonance integral of natural copper for the thickness of copper used in measuring $S_{Cu}^{cavity} = 2.8805$ b. (3)

CdR = cadmium ratio of copper in the cavity, for the thickness of copper used in measuring $S_{Cu}^{cavity} = 13.7 \pm .2$.

B. 1/v Mass Equivalent of a Mass of Copper

The requirement that the material used as the absorber in the determination of $k_{\infty 1/v}$ (see IV-G) have an effective cross section that varies as $1/\sqrt{E}$, makes it necessary to define a 1/v equivalent mass of copper, $M_{1/v}$, so that its

reactivity coefficient is equivalent to the correct mass of copper, M_o .

$$M_{1/v} SA_{1/v} M_2 = M_o (SA_{1/v} M_2 + SA_{res} M_1) \quad (23)$$

Account is taken of the worth of epi-thermal resonance absorptions in copper relative to the worth of $1/v$ absorptions. Using notation similar to that in the preceeding section, it has been shown⁽¹⁰⁾ that

$$M_{1/v} = M_o \left[1 + \left(\frac{m_1}{m_2} \right) \left(\frac{RI_{ex}^{Cu}}{CdR \cdot RI_{epi-Cd}^{Cu} - RI_{ex}} \right) \right] \quad (24)$$

Copper strips 0.003" thick were inserted at the boundary of the supercell, and the cadmium ratio of the copper at the boundary was measured with 0.003" thick copper foils inserted in these strips. Effective resonance integrals for 0.003" thick natural copper were taken from measurements by Bennett.⁽³⁾ Values for $M_{1/v}$ are only a few percent greater than M_o , and are included in Table X.

C. Counter Efficiency Correction for Dissected Foils

The irradiated copper foils used to measure the relative reaction rates in the target material and the fuel material had a variable specific activity as a function of radial symmetry due to large disadvantage factors in the target and fuel materials. The distribution of the activity induced in a detector foil affects the overall efficiency of the scintillation counting system, since the counter efficiency varies with distance from the center of the crystal. Copper foils irradiated in the external thermal column of the PCTR had a constant specific activity as a function of radial symmetry. Thus, the counter efficiency function must be removed from the measured data in order to obtain relative reaction rates in the target and fuel materials.

The observed specific activity for a given geometry shaped detector foil at a given height above the crystal can be expressed as the specific activity

of the detector foil as a function of its radial symmetry convoluted with the counter efficiency as a function of radial position.

$$\overline{SA}_{\text{observed}} = \{SA(R)\} * \{\epsilon(r)\} \quad (25)$$

Counter efficiency measurements with a ^{137}Cs pin point source were made by R. A. Bennett⁽¹¹⁾ to determine the relative efficiency of the counter as a function of distance between the pin point source and the center of the surface of the scintillation crystal. The quadrant and sector foils used in the target and fuel, respectively, were dissected into small annular regions of approximately constant specific activity. By superposition of the dissected annular regions on the counter efficiency topographical plot, an effective counter efficiency for each region was obtained,

$$\epsilon_R = \frac{\int_R \epsilon(r) dA}{\int_R dA}, \quad (26)$$

where $\epsilon(r)$ = counter efficiency at a distance r from the center of the surface of the crystal;

dA = element of area on the foil of a distance r from the center of the surface of the crystal.

The dissected annular region was counted in the position that it would normally assume had the whole foil been placed on the crystal. The counter efficiency corrected specific activity for that region is given by:

$$SA_R(\text{corrected}) = \frac{SA_R(\text{observed})}{\epsilon_R} \quad (27)$$

Thus, the overall counter efficiency for a foil having regional specific activities obtained from Equation 24 is determined by:

$$\bar{\epsilon} = \frac{\sum_{l=1}^n SA_R(\text{corrected}) A_R \epsilon_R}{\sum_{l=1}^n SA_R(\text{corrected}) A_R} \quad (28)$$

where, A_R = total area of the region R.

n = number of annular regions in a whole foil.

D. Flux Enhancement by Detectors

Three dimensional Monte Carlo calculations made by F. E. Dunn⁽¹²⁾ explicitly including foils have indicated that the flux in a foil is often considerably different from the flux in the medium in which it is placed, even for optically thin foils in which no such effect would be expected. A correction must be applied to the measured foil activities to obtain the proper flux averages to be used in determining the thermal disadvantage factors.

The derivation for the fraction of the measured foil activity which is due to flux peaking in a flat foil of thickness t , and diameter or width L , was reported in reference 12 and is summarized as follows:

If a foil of macroscopic absorption cross section Σ_{foil} is placed in a medium of macroscopic absorption cross section Σ_{medium} and a group of neutrons have a path length ℓ_o through the foil, then their contribution to the activation of the foil, A_{foil} , is $\ell_o \Sigma_{\text{foil}}$. The fractional flux peaking in the foil is $\ell_o (\Sigma_{\text{medium}} - \Sigma_{\text{foil}})$ if $\ell_o \Sigma_{\text{foil}}$ and $\ell_o \Sigma_{\text{medium}}$ are small compared to 1. Therefore, if A_o is the activation that would occur if the flux in the foil were the same as the flux in the medium,

$$\frac{A_{\text{foil}} - A_o}{A_{\text{foil}}} = \frac{\int \ell_o (\Sigma_{\text{medium}} - \Sigma_{\text{foil}}) \ell_o \Sigma_{\text{foil}} f(\ell_o) d\ell_o}{\int \ell_o \Sigma_{\text{foil}} f(\ell_o) d\ell_o} \quad (29)$$

where, $f(\ell_o) =$ probability of the occurrence of ℓ_o .

If θ is the angle of incidence between the neutron path and the foil surface, then

$$\ell_o = \frac{t}{\sin \theta} \quad , \quad \text{for } \theta > \frac{t}{L} \quad . \quad (30)$$

For an isotropic flux, neglecting edge effects,

$$f(\ell_o) d\ell_o = \sin \theta \cos \theta d\theta \quad (31)$$

Substitution of equations (30) and (31) into equation (29) gives:

$$\frac{A_{\text{foil}} - A_o}{A_{\text{foil}}} = t \left(\sum_{\text{medium}} - \sum_{\text{foil}} \right) \ln(L/t) \quad . \quad (32)$$

Sub-cadmium activities of 0.010" thick copper foils placed in the target and fuel materials were corrected by use of equation (32) in conjunction with average cross sections calculated from transport theory⁽⁶⁾ for copper placed in the neutron spectrum of the target and fuel materials, as well as the cross sections for lithium and uranium.

E. Central Cell End Effects

When the central supercell is removed from the test lattice in the course of performing null reactivity measurements, the front and rear end buffer cells each have one end of their process tubes exposed to the central supercell cavity. This exposure could affect the $\rho_{\text{cell out}}$ reactivity measurement if neutron streaming from the cavity to these ends is appreciable. An attempt was made to measure these end effects. Correlations of three dimensional calculations with computer code WHIRLAWAY^(a) with these experimental measurements will be discussed in a future report.

If the length of the central supercell removed is decreased, the solid angle of the central cavity seen by the exposed portion of an end buffer cell

(a) G. L. Bennett, private communication

is increased. The front and rear end buffers (7.072 and 7.254 inches long, respectively) were used as central supercells in null reactivity measurements in the matched spectrum of each lattice condition. The correct mass of copper required to absorb the excess neutrons of the central supercell per unit central cell length was used to infer values for k_{∞} of each supercell. Reactivity worths of copper (0.8416 gm/in = 1 mk for the wet lattice, and 0.7223 gm/in = 1 mk for the dry lattice) were obtained from measurements with the 21.875 inch long central supercell. Results of these measurements are shown in Figure 21. Correlation of these results with the three dimensional calculations from WHIRLAWAY are necessary to determine the magnitude of the end effects on the 21.875 inch long center cell in relation to a center cell of infinite length, where the end effect is zero.

FIGURE 21. Infinite Medium Neutron Multiplication Constant as a Function of the Central Supercell Length

VI. APPENDICES

A. Correlation of THERMOS Calculations with Sub-Cadmium Copper Activation Measurements

Experimentally determined boundary conditions were utilized in transport theory calculations with computer code THERMOS⁽⁶⁾ as discussed in IV-E. Since thermal activation in copper is proportional to the neutron density, a direct comparison of relative activity of a $1/v$ material calculated by THERMOS and measured by copper is possible. Cylindricized radial traverses in the target, U_1 fuel, and U_2 fuel cells of the wet lattice are compared in Figures 22, 23, and 24, respectively. In each cell the average activation in the core material calculated by THERMOS is normalized to the experimental data. Dry lattice cylindricized radial traverses in the target, U_1 fuel, and U_2 fuel, are compared in Figures 25, 26, and 27, respectively.

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FIGURE 22. Lithium in Aluminum Target Cell Radial Traverse

FIGURE 23. U_1 Type Fuel Cell Radial Traverse

FIGURE 24. U_2 Type Fuel Cell Radial Traverse

FIGURE 25. Lithium in Aluminum Target Cell Radial Traverse

FIGURE 26. U_1 Type Fuel Cell Radial Traverse

FIGURE 27. U_2 Type Fuel Cell Radial Traverse

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