FEB. 21 1963

Internal Report
Brookhaven National Laboratory
Upton, New York

BNL 6717

PD-54 ✓

February 7, 1963

Facsimile Price \$ 1.20

Microfilm Price \$.80

Available from the
Office of Technical Services
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MASTER

Trajectories for a Rectangular Magnet

with Uniform Field*

R. M. Sternheimer

In connection with the design of a hodoscope system for the momentum analysis of secondary particles from a target bombarded by monoenergetic particles, it has been considered to use a large-aperture rectangular magnet with uniform field, and a hodoscope set of counters behind the magnet after a suitably long flight path of the particles. For accurate calibration of the momentum of the particles, it will probably be necessary to map very completely the field inside the magnet and the fringing fields at the entrance and exit boundaries, in order to calculate typical trajectories as a function of the position of the particle path at the entrance to the magnet, and the entrance angle and particle momentum. Nevertheless, for the sake of orientation, it may be useful to obtain the projections of the trajectory in the horizontal

* Work performed under the auspices of the U.S. Atomic Energy Commission.

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and vertical planes in the approximation that the effect of the fringing field can be represented by adding a correction to the length of the magnet and assuming that the field is uniform over the distance of the total effective length, and is zero outside the effective field boundaries. For the treatment of the vertical motion, we will also assume that the object (source) lies in the median plane of the magnet. This condition will be approximately satisfied in the important practical case that the source is a target which is small in the vertical direction and from which the secondary particles are emitted.

Fig. 1

As discussed above, we assume that the magnetic field can be replaced by a uniform field of length l . The radius of curvature (for particles of a given momentum p) will be denoted by R . The entrance and exit angles are denoted by θ_1 and θ_2 , respectively. θ_1 and θ_2 are taken as positive if the trajectory is upward at the respective boundaries. Thus in Fig. 1, $\theta_1 > 0$, and $\theta_2 < 0$. The x and y axes are as shown in Fig. 1; the origin is

at 0 (lower left corner of the effective magnetic field). The y coördinates at the entrance and exit are denoted by y_1 and y_2 , respectively.

The coordinates (x_c, y_c) of the center of curvature of the orbit are given by:

$$x_c = R \sin \theta_1, \quad (1)$$

$$y_1 - y_c = R \cos \theta_1. \quad (2)$$

We have:

$$(x_c - l)^2 + (y_c - y_2)^2 = R^2, \quad (3)$$

which can be rewritten as follows:

$$(R \sin \theta_1 - l)^2 + (Y + R \cos \theta_1)^2 = R^2, \quad (4)$$

where: $Y \equiv y_2 - y_1$.

Upon solving Eq. (4) for Y, one obtains

$$Y = -R \cos \theta_1 + [R^2 \cos^2 \theta_1 + 2lR \sin \theta_1 - l^2]^{\frac{1}{2}}. \quad (5)$$

The angle θ_2 of the outgoing trajectory can be obtained from the relation:

$$\sin \theta_2 = \frac{x_c - l}{R}, \quad (6)$$

whence:

$$\sin \theta_2 = \sin \theta_1 - \frac{l}{R}. \quad (7)$$

The angle of deflection ϕ in the magnet is given by: $\theta_1 - \theta_2$.

In order to obtain the projection of the trajectories in the vertical direction, we will assume that the source lies in the median plane of the magnet, at a distance S in front of the magnet. Since the field is assumed to be uniform inside the region of the effective magnetic field, there will be a force in the z direction (and hence a deflection of the orbit) only at the fringing fields at the entrance and exit of the magnet (see Fig. 2). The change in ^{the} momentum p_z at the entrance of the magnet is given by:

$$\Delta p_z^{(s)} = -p \left(\frac{\tan s}{R} \right) z_s, \quad (8)$$

where $z_s = S \alpha_0$ is the z displacement at the magnet entrance (α_0 = vertical angle at the source), p = momentum, $s = \theta_1$ = entrance angle with respect to the normal to the entrance face of the magnet. Thus the change of the angle α is given by:

$$\Delta \alpha_s = \frac{\Delta p_z^{(s)}}{p} = - \left(\frac{\tan s}{R} \right) z_s = - \left(\frac{\tan s}{R} \right) S \alpha_0. \quad (9)$$

The vertical angle inside the magnet is:

$$\alpha_1 = \alpha_0 + \Delta\alpha_s = \alpha_0 \left(1 - \frac{S}{R} \tan \delta\right) \equiv \alpha_0 U_s. \quad (10)$$

At the exit of the magnet, the change of α is given by:

$$\Delta\alpha_t = - \left(\frac{\tan t}{R} \right) z_t, \quad (11)$$

where $t = \text{exit angle} = -\theta_2$, with our present convention for the sign of θ_2 .

The displacement z_t at the exit of the magnet is given by:

$$z_t = S\alpha_0 + R\varphi\alpha_1 = \alpha_0 (S + R\varphi U_s). \quad (12)$$

Hence:

$$\Delta\alpha_t = - \left(\frac{\tan t}{R} \right) \alpha_0 (S + R\varphi U_s). \quad (13)$$

After passage through the magnet, α is given by:

$$\alpha_2 = \alpha_1 + \Delta\alpha_t = \alpha_0 U_s - \alpha_0 \left(\frac{\tan t}{R} \right) (S + R\varphi U_s). \quad (14)$$

If the hodoscope is placed at a distance T from the exit

face of the magnet, the z displacement at the detector is obtained from:

$$z_T = z_t + \alpha_2 T = \alpha_0 (S + R\varphi U_s + T U_s) - \alpha_0 \left(\frac{T \tan t}{R} \right) (S + R\varphi U_s). \quad (15)$$

If we let:

$$U_t \equiv 1 - \frac{T \tan t}{R}, \quad (16)$$

Eq. (15) can be rewritten as follows:

$$z_T = \alpha_0 (S + R\varphi U_s) U_t + T \alpha_0 U_s = \alpha_0 (S U_t + T U_s + R\varphi U_s U_t). \quad (17)$$

As expected, if we set $z_T = 0$ in Eq. (15), and solve for T , we obtain the usual expression for the distance to the vertical focus formed by a magnet with uniform field:

$$T_v = \frac{R\varphi + S(1 - \varphi \tan s)}{\left(\frac{S}{R} \right) [\tan s + (1 - \varphi \tan s) \tan t] - (1 - \varphi \tan t)} \quad (18)$$

Fig. 2 (case with $s > 0$, $t < 0$).

I wish to thank Dr. G. B. Collins for suggesting the present work and for helpful discussions.

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