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REPORT

THEORY-EXPERIMENT COMPARISON OF
 Pu^{240} RESONANCE ABSORPTION

May, 1965

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THEORY-EXPERIMENT COMPARISON OF Pu^{240} RESONANCE ABSORPTION

by

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PHYSICS AND INSTRUMENTS DEPARTMENT
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INTRODUCTION

The calculation of neutron absorption rates in reactor systems containing Pu^{240} is an important and, in at least one respect, a unique problem. This plutonium isotope is present in all plutonium-burning reactors and in uranium burners which contain appreciable amounts of U^{238} . Furthermore, most of the neutron absorption occurs in one gigantic resonance at 1 eV incident neutron energy.

An experiment to measure the Pu^{240} absorption was performed at Hanford by P. F. Nichols. The results of this experiment have been published in Nuclear Science and Engineering, Vol. 17, pages 144-148 (1963). Since the publication of these experimental results, improved methods of applying the theory of resonance absorption to specific problems have become available, hence it is both interesting and useful to compare the calculational and experimental results. It is particularly important to do so because many of the original resonance absorption calculation routines are primarily concerned with U^{238} and Th^{232} and not with Pu (see for instance references 9, 10). Agreement between theory and experiment would, therefore, to some extent, "justify" the use of these methods in the calculation of resonance absorption in Pu^{240} .

SUMMARY AND CONCLUSIONS

It is concluded that the INT and NRIA methods do not conflict with the experimental results, and that these methods can be used to calculate the resonance absorption in Pu^{240} containing systems until newer, more definitive experiments prove differently. It is also rather clear that the values of the Pu^{240} resonance integrals at high Pu^{240} concentration quoted in the

original experiment may be too low, and hence that these results tend to over-estimate the resonance self-shielding effect. For a comparison of the new values with the calculated values and with the original values turn to Table IV.

SYNOPSIS OF THEORY

We set forth the principal equations with which we shall be concerned. The derivations are contained in references 8, 9, and 11.

The slowing down equation in lethargy expressed in terms of the collision density $F(u) = \Sigma_t(u) \phi(u)$ is:

$$F(u) = (1 - P_o^*) \left[\frac{1}{\alpha_o} \int_{u-\Delta u_o}^u F(u') \frac{\sigma_{so}(u')}{\sigma_{to}(u') + \sigma_m} du' + \frac{1}{\alpha_m} \int_{u-\Delta u_m}^u F(u') \frac{\sigma_m}{\sigma_{to}(u') + \sigma_m} du' \right] + P_o^* (\sigma_{to}(u) + \sigma_m) \frac{e^u}{E_1} \quad (1)$$

The problem for which this equation is the proposed solution is that of a lump of arbitrary shape containing a resonant scattering species (zero subscript) mixed with a non-resonant scattering species (subscript m), and embedded in an external moderator from which neutrons are scattered into the lump. The main assumption made in the derivation is that of a spatially uniform flat source of fast neutrons. The parameters of the equation are defined as follows:

P_o^* = the corrected lump escape probability. This function of lump dimensions in mean free path units differs from those derived in Reference 1 in that it is modified to account for the probability that a neutron may re-enter an adjacent lump without making a collision with the intervening external moderator. For homogeneous geometry $P_o^* = 0$.

$$\alpha_i = \frac{4A_i}{(A_i + 1)^2} , \text{ for an isotope with mass in neutron units of } A_i$$

$$\Delta u_i = \ln \frac{1}{1 - \alpha_i} .$$

$\sigma_m = N_1 \sigma_{sl} / N_0$, where σ_{sl} is the scattering cross section of the admixed non-resonant scatterer, and N_1 is the usual nuclei density.

E_1 = the energy chosen for the base of lethargy, $u = \ln \frac{E_1}{E}$.

Three methods are available to solve Equation 1:

NR: The narrow resonance approximation consists of assuming that the resonance is so narrow that no resonance scattering or absorption occurs in the determination of $F(u)$. In this case the cross section substitution is $\sigma_{so} = \sigma_{to} = \sigma_{po}$, the potential scattering cross section of the absorber, and the collision density ($F = \Sigma_t \phi$) assumes its asymptotic value $F_{as} = \sigma_{po} / E = \sigma_{po} e^{u/E_1}$. With these substitutions in equation 1), we get

$$F(u) = e^{u/E_1} \left[\sigma_{po} + P_0 (\sigma_{to} - \sigma_{po}) \right] . \quad (2)$$

NR1A: In the narrow resonance, infinite absorber approximation one assumes that no slowing down occurs upon collision with the absorber (absorber of infinite A), while collisions with the admixed moderator involve such large energy changes with respect to the width of the resonance that one can use the NR approximation for the moderator source integral.

With the assumption of infinite absorber mass:

$$\lim_{A_0 \rightarrow \infty} \alpha_0 = \lim_{A_0 \rightarrow \infty} \frac{4A_0}{(A_0 + 1)^2} = 0, \text{ and similarly}$$

$$\lim_{A_0 \rightarrow \infty} \Delta u_0 = \lim_{A \rightarrow \infty} \ln \frac{1}{(1 - \alpha_0)} = 0 .$$

Thus the absorber integral in Equation 1) becomes;

$$\begin{aligned} \lim_{\substack{\Delta u_0 \rightarrow 0 \\ \alpha_0 \rightarrow 0}} \frac{1}{\alpha_0} \int_{u-\Delta u_0}^u F(u') \frac{\sigma_{so}(u')}{\sigma_{to}(u') + \sigma_m} du' &= \int_0^\infty \delta(u-u') \frac{F(u') \sigma_{so}(u')}{\sigma_{to}(u') + \sigma_m} du' \\ &= \frac{\sigma_{so}(u) F(u)}{\sigma_{to}(u) + \sigma_m} , \end{aligned}$$

and the moderator integral in Equation 1) becomes;

$$\frac{1}{\alpha_m} \int_{u-\Delta u_m}^u F(u') \frac{\sigma_m}{\sigma_{to}(u') + \sigma_m} du = \frac{1}{\alpha_m} \int_{u-\Delta u_m}^u \sigma_m \frac{e^u}{E_1} du = \frac{\sigma_m}{E_1} e^u .$$

With these substitutions, Equation 1) changes to

$$F(u) = (1-P_0^*) \left[\frac{\sigma_{so}(u) F(u)}{\sigma_{to}(u) + \sigma_m} + \frac{\sigma_m}{E_1} e^u \right] + P_0^* (\sigma_{to}(u) + \sigma_m) \frac{e^u}{E_1} .$$

Solving for $F(u)$:

$$F(u) = \frac{(\sigma_{to}(u) + \sigma_m)(P_0^* \sigma_{to} + \sigma_m)}{\sigma_{to}(u) + \sigma_m - (1 - P_0^*) \sigma_{so}(u)} \frac{e^u}{E_1} . \quad (3)$$

INT: The integral method consists of solving Equation 1) directly by numerical methods. This is accomplished in the G.A. Code ZUT and its progeny NIT and GANIT. (References 9 and 10.)

The integral method can be made equivalent to the NR and NRIA approximation by merely making the appropriate approximations for the absorber and moderator integrals.

Having obtained an expression (or set of numbers) for $F(u)$, we then proceed to evaluate the resonance integral:

$$I = E_1 \int_0^{\infty} \sigma_{a_o}(u) \frac{F(u)}{\sigma_{to}(u) + \sigma_m} e^{-u} du \quad (4)$$

For each approximation we have:

$$\text{NR: } I = \int_0^{\infty} \frac{\sigma_{a_o}(u)}{\sigma_{to}(u) + \sigma_m} \left[\sigma_{po} + \sigma_m + P_o^* (\sigma_{to} - \sigma_{po}) \right] du \quad (5)$$

$$\text{NRIA: } I = \int_0^{\infty} \frac{\sigma_{a_o}(u) (P_o^* \sigma_{to}(u) + \sigma_m)}{\sigma_{to}(u) + \sigma_m - (1 - P_o^*) \sigma_{so}(u)} du \quad (6)$$

$$\text{INT: } I = E_1 \sum_{j=1}^N \frac{(\sigma_{a_o})_j F_j \Delta u}{(\sigma_{to})_j + \sigma_m} \quad (7)$$

where

$F_j = F(u_j)$, $u_j = j \Delta u$, Δu being the lethargy mesh interval.

The corrected escape probability, P_o^* can be obtained from the single lump probabilities, P_o , by using the relation (Reference 8, pg. 76):

$$P_O^* = P_O \frac{1 - C}{1 - C \left[1 - \bar{\ell} N_O (\sigma_{to}(u) + \sigma_m) P_O \right]}$$

C is the Dancoff factor defined as the probability that a neutron which has escaped from the lump reaches another, neighboring lump without colliding with the intervening moderator (Reference 5). $\bar{\ell}$ is just the mean chord length in the lump ($\bar{\ell} = 4 V/S$, V, the volume, S the surface of the lump). The uncorrected escape probabilities are obtained from References 1 and 6.

A useful approximation for the uncorrected escape probability is Wigner's rational approximation (References 3 and 11):

$$P_O = \frac{1}{1 + \bar{\ell}/\lambda}, \text{ where } \lambda \text{ is the mean free path within the lump.}$$

Although the errors introduced by using this approximation can be as high as 10%, (Reference 8, pg. 71) it brings about a great simplification of the heterogeneous expressions, especially where non-Doppler-broadened cross sections are used.

The resonance cross section shapes are given by the usual Breit-Wigner single level formula without Doppler broadening:

$$\sigma_{to} = \sigma_o (1 - x^2)^{-1} + (\sigma_o \sigma_{po} g_j \frac{\Gamma_n}{\Gamma})^{\frac{1}{2}} 2x (1 + x^2)^{-1} + \sigma_{po},$$

$$\sigma_{ao} = \sigma_o \frac{\Gamma_y}{\Gamma} (1 + x^2)^{-1}, \quad x = \frac{E - E_o}{\Gamma/2}, \text{ where } E_o \text{ is the resonance energy.}$$

$$\sigma_o \cong \frac{2.6 \times 10^6}{E_o} \left(\frac{\Gamma_n}{\Gamma} \right) g \text{ (the maximum cross section at resonance),}$$

where g is the statistical factor. Using these expressions for the cross

sections in Equation 5) and 6), it can be readily shown that (References 3 and 4):

$$\text{NR: } I = \frac{I_{\infty}}{\sqrt{1 + \sigma_o/\sigma_{po}}}, \text{ Homogeneous} \quad (9)$$

$$\text{NR: } I = \frac{I_{\infty}}{\sqrt{1 + \frac{\sigma_o}{(\sigma_{po} + \sigma_m) \left[1 + \frac{1}{\bar{\ell} N_o (\sigma_{po} + \sigma_m)} \right]}}}, \text{ Heterogeneous} \quad (10)$$

$$\text{NR1A: } I = \frac{I_{\infty}}{\sqrt{1 + \frac{(\Gamma_Y/\Gamma) \sigma_o}{\sigma_m}}}, \text{ Homogeneous} \quad (11)$$

$$\text{NR1A: } I = \frac{I_{\infty}}{\sqrt{1 + \frac{\sigma_o (\Gamma/\Gamma)}{\sigma_m \left[1 + \frac{1}{N_o \sigma_m \bar{\ell}} \right]}}}, \text{ Heterogeneous} \quad (12)$$

Wigner's rational approximation has been made for the escape probability in the heterogeneous cases.

With Doppler-broadened cross sections, these relatively simple formulas are replaced by expressions involving the J, K, and L functions, the details of which are contained in References 2 and 3.

DESCRIPTION OF THE EXPERIMENT

To determine the value of the resonance integral for Pu^{240} , the reactivity difference between pairs of 0.65 inch diameter Pu-Al alloy

rods differing mainly only in the Pu^{240} concentration were measured in the PCTR. An equation of the form

$$\Delta\rho_i = C \left[(N_B^{240} I_B^{240} V_B - N_A I_A^{240} V_A) \right]_i \quad (13)$$

was assumed.

$\Delta\rho_i$ = the observed reactivity difference between rods B and A of the i^{th} pair. (ϕ)

I^{240} = the effective resonance integral for the rod (barns)

V = the rod volume (cm^3)

N^{240} = nuclei density of Pu^{240} in the rod (nuclei/barn cm).

To minimize reactivity differences due to differences in thermal absorption in the rods, the rods were provided with cadmium cans making them essentially completely black to neutrons below approximately 0.600 eV. The rods were also contained in zirconium cans which will be ignored here.

The experimental data is reproduced in Table I. The measured value of the Pu^{240} resonance integral was obtained by calculating the resonance integral for the first four rods using the NR1A approximation, Doppler-broadened cross sections, the rational approximation to the cylindrical geometry, and the J function tables (References 2 and 3). These values were used to obtain the constant C in equation 13. The observed reactivity change $\Delta\rho_4$ was then used to evaluate I_B^{240} , the calculated value of 7865 barns being used for I_A^{240} . The value of 6730 barns for I_B^{240} in the fourth set was taken to be I_A^{240} for the fifth set and so on until all five of the more concentrated sets (4 through 8) were evaluated. The addition of 314 barns (200 barns for all resonances above 1.056 eV and 114 barns for the epicalcium $1/v$ absorption) was made to all J-table values in order to arrive at the calculated value for the first three sets.

TABLE I

<u>ORIGINAL EXPERIMENTAL RESULT</u>						
<u>Set</u>	<u>Rod</u>	N^{240} <u>(n/barn cm)</u>	<u>V</u> <u>cm³</u>	$\Delta\rho$ <u>ϕ</u>	I^{240} <u>Barns</u>	<u>Source</u>
1	A	5.0×10^{-8}	160.0		8599	Calculated
	B	1.42×10^{-7}	160.0	0.033 ± 0.02	8522	Calculated
2	A	1.42×10^{-7}	160.0		8522	Calculated
	B	4.08×10^{-7}	160.0	0.058	8323	Calculated
3	A	4.08×10^{-7}	160.0		8323	Calculated
	B	1.17×10^{-6}	160.0	1.160	7865	Calculated
4	A	1.17×10^{-6}	160.0		7865	Calculated
	B	3.35×10^{-6}	160.0	0.369	6730 ± 430	Measured
5	A	3.35×10^{-6}	160.2		6730 ± 430	Measured
	B	9.59×10^{-6}	160.2	0.725	5080 ± 450	Measured
6	A	9.59×10^{-6}	160.5		5080 ± 450	Measured
	B	2.74×10^{-5}	160.5	1.160	3300 ± 370	Measured
7	A	2.74×10^{-5}	161.0		3300 ± 370	Measured
	B	7.80×10^{-5}	161.0	1.800	1990 ± 300	Measured
8	A	7.80×10^{-5}	164.0		1990 ± 300	Measured
	B	2.20×10^{-4}	164.0	2.910	1180 ± 270	Measured

THE ZUT AND TUZ CODES

Resonance integrals can be evaluated by direct numerical solution of Equation 1) for $F(u)$, with subsequent insertion of these values into Equation (4) followed by numerical integration. The G.A. code ZUT affords the following flexibility in performing these computations.

1. Cross sections: Doppler-broadened, single level Breit Wigner shape including the term for interference between potential and resonance scattering.
2. Geometry: Homogeneous infinite medium, cylindrical, spherical, slab.
3. Moderators: Up to two admixed, non-resonant moderators.
4. Method of solution: Equation (1) can be solved by NR, NR1A, or INT methods for the resonant absorber integral in the first term. The moderator integrals can be done either numerically as written, or the $1/E$, asymptotic value for the flux may be used.
5. Lump interaction: Dancoff factors can be used in Equation 8) for closely-packed lump ensembles.

The code TUZ (Reference 10) can be used to estimate the resonance integrals from unresolved resonances. The rational and $F(u') = \sigma_m C^{u'}/E_1$ approximations are made in the code. The Porter-Thomas distribution of resonance width (Reference 7) is assumed above a cutoff energy, E_c , specified in the input. Only the additional specification of an average reduced width ($\langle \Gamma_n^0 \rangle$) and an average level spacing (D) is required.

The codes ZUT and TUZ are very suitable for refining the analysis of the original Nichols experiment. When the nuclear data given in Appendix A is used as input along with the experimental conditions given in Table I the output listed in Table II results, using the INT method of solution.

TABLE II

ZUT, INT METHOD VALUES FOR THE Pu-Al RODS

	A	B	C	D	E	F	G
	N^{240}		$\sum I$ above 1 eV				
Rod #	n/barn-mc	I at 1 eV barns	barns	Unresolved barns	Epi Cd 1/V barns	Total barns	Nichols' Data barns
1 A	5.0×10^{-7}	8,259	113	32	114	8,518	8,699
2 A	1.42×10^{-7}	8,210	113	32	114	8,469	8,522
3 A	4.08×10^{-7}	8,074	113	32	114	8,332	8,323
4 A	1.17×10^{-6}	7,712	113	32	114	7,970	7,865
5 A	3.35×10^{-6}	6,865	112	31	114	7,122	$6,730 \pm 430$
6 A	9.59×10^{-6}	5,312	110	31	114	5,567	$5,080 \pm 450$
7 A	2.74×10^{-5}	3,453	106	31	114	3,704	$3,300 \pm 370$
8 A	7.80×10^{-5}	2,045	95	29	114	2,283	$1,990 \pm 300$
8 B	2.20×10^{-4}	1,185	76	26	114	1,401	1180 ± 270

NOTES: 1) Column F is the sum of Columns B, C, D, and E.

2) TUZ has been used to calculate the contribution from the unresolved resonance listed in Column D.

The difference in the first four rods are purely calculational. They result from different choices for the resonance parameters. Nichols' data would give an infinitely dilute value of 8,293 barns, whereas the corresponding value using the nuclear data from Appendix A is 8,306 barns. The principal difference, however, is Nichols' assumption of 200 barns for the resonances above 1 eV versus the calculated values in Columns C and D, which vary from 113 to 76 barns. The remaining discrepancies are due to the effect of using the rational approximation, and perhaps to interpolation inaccuracies in using J-function tables. The differences between the values at higher concentrations are due to three sources:

- 1) The different values of $\underline{C}$ in $\Delta\rho = C \left[(VIN)_B - (VIN)_A \right]$ resulting from different values of the first four rods will cause one to arrive at different values for I_B for an observed value of $\Delta\rho$.
- 2) Discrepancies, if any, between theory and experiment.
- 3) Experimental error.

Since we are primarily interested in item (2) we will proceed to analyze the experiment using the calculated values from Table II, Column F.

ANALYSIS OF THE EXPERIMENT

Old Method, New Numbers

First we will apply the results of Table II to the method used in the original experiment, i.e., using the first three dilute points to arrive at a value of $\underline{C}$, and then using this value to carry out a pairwise evaluation of the five remaining pairs.

Define $x = \frac{I_B V_B}{I_A V_A} - \frac{I_N V_A}{I_A V_A}$, $\Delta\rho = y$. The experimental relationship is $y = Cx$. The original value of $\underline{C}$ given by Nichols is $C = 0.173 \pm .014$.

The first four values in Column F give $C = 0.170 \pm .008$ from a least squares fit of the model $y = Cx$. Thus the difference created by the newer values of the low concentration integrals is very small. When the evaluation by pairs of each rod set is carried out using the slightly different value of C , values very close to Nichols' original value result as can be seen in Table III.

TABLE III
COMPARISON OF NEW OBSERVED VALUES
WITH CALCULATED VALUES

<u>Rod</u>	N^{240} <u>n/barn-cm</u>	<u>I_{OBS}</u> <u>barns</u>	<u>I_{NICHOLS}</u> <u>barns</u>	<u>I_{CALC}</u> <u>barns</u>
4 B	3.35×10^{-6}	6,839	$6,730 \pm 430$	7,122
5 B	9.59×10^{-6}	5,173	$5,080 \pm 450$	5,567
6 B	2.74×10^{-5}	3,289	$3,300 \pm 370$	3,704
7 B	7.80×10^{-5}	2,000	$1,990 \pm 300$	2,283
8 B	2.20×10^{-4}	1,171	$1,180 \pm 270$	1,401

I_{OBS} ~ Resonance Integrals from the new calibration line

$I_{NICHOLS}$ ~ Resonance Integrals reported by Nichols.

I_{CALC} ~ Calculated values from Column F, Table II

Test of the Experimental Model

One method of attempting to check the validity of the mathematical model would be to fit the five pair-points at high absorber concentration and then to test the value of the slope, C so obtained with the value at low concentration. If the two values are statistically different, then the validity of the linear model is in doubt. The five upper pair values, when fitted to the observed y 's yield the value $C = 0.141 \pm .002$

as compared to $C = 0.170 \pm .008$ for the low concentration line. The plotted lines are shown in Figure 1. A test of the hypothesis that the two observed values of C may be independent estimates of the same true value (see Appendix B) shows that they are not estimates of different true C 's at the 95% level. Thus the difference in the estimates of the slope is not large enough to invalidate the model, however this difference does illustrate an inherent danger in an analysis of this type. In this procedure, the indirectly "observed" values are deduced from what amounts to a very large extrapolation. The rationale of such a procedure is that the computations are more accurate at low nuclei density, and so a line obtained from low concentration data benefits from more accurate theory. Unfortunately, the experimental error is higher when small changes in reactivity must be measured, hence there is an appreciable risk of going off on the wrong slope. This danger could be minimized in a more carefully designed experiment involving much repeated measurement. Lacking such data, we will adopt the classic procedure of treating all points as equivalent and proceed to fit all of the data points to the linear model and then test the resulting line to see how well the model fits the data.

New Method, New Numbers

When all eight experimental points are used, the value of the slope becomes $C = 0.142 \pm .0016$, not much of a change from the high concentration line slope of $C = 0.141 \pm .002$. Figure 2 shows the complete line along with 95% confidence level bands for determining the range within which we can feel fairly certain (with probability .95) that the true value of x must be located for any single observation of y (the PCSTR reactivity).

Figure 2
ALL POINTS LINE
Reactivity vs. Absorption Difference,
95% CL Band Indicated

All three of the lines (high concentration, low concentration lines, all points) are good statistical fits. However the best fit, as judged by an F-ratio test (see Appendix B) is given by the line which includes all of the experimental points. The agreement obtained from the experiment using the calculated resonance integrals is very good, as can be seen from an examination of Figure 2.

One can use the confidence level bands to indicate the precision of the experimental technique as a function of absorber nuclei density. For a given value of y , the best experimental value obtainable is just the x -value on the regression line, while the x values of the upper and lower bands define the 95% CL range. One can then evaluate each experimental integral by rearranging the definition of x :

$$I_B = \frac{1}{N_B V_B} (x + N_A V_A I_A) .$$

In Table IV the calculated values of each previous I_A has been used to give the experimental values of I_B shown in the table. The limits shown correspond to those values of I found from the 95% CL bands. The error range narrows sharply with increasing absorber density until it becomes quite narrow at high absorber concentration. This is due directly to multiplying any error in the experimentally inferred quantity, x , by $\frac{1}{N_B V_B}$. It is interesting to note that the calculated values from Table IV differ most from the experimental values precisely in the range of greatest experimental uncertainty, and that the observed discrepancies narrow almost as sharply with increasing absorber concentration as do the estimates of experimental uncertainty.

TABLE IV

COMPARISON OF EXPERIMENTAL AND
CALCULATED RESONANCE INTEGRAL VALUES

<u>Rod</u>	<u>N_0 n/barn-cm</u>	<u>$I_{\text{experimental}}$ barns</u>	<u>$I_{\text{calculated}}$ barns</u>	<u>I_{Nichols} barns</u>
4	3.35×10^{-6}	$7,647 \pm 1,286$	7,122	$6,730 \pm 430$
5	9.59×10^{-6}	$5,826 \pm 456$	5,567	$5,080 \pm 450$
6	2.74×10^{-5}	$3,724 \pm 163$	3,704	$3,300 \pm 370$
7	7.80×10^{-5}	$2,314 \pm 61$	2,283	$1,990 \pm 300$
8	2.20×10^{-4}	$1,389 \pm 26$	1,401	$1,180 \pm 270$

$I_{\text{experimental}} \sim$ the value corresponding to $x = (N_B V_B I_B - N_A V_A I_A)$
 where $x = y/c$ and $N_A V_A I_A$ contains the calculated
 value of the resonance integral. This is the best
 estimate of the value of the resonance integral
 that would result from a great many repetitions
 of the experiment.

$I_{\text{calculated}} \sim$ the value calculated with ZUT and TUZ code
 previously listed in Table II, Column F.

$I_{\text{Nichols}} \sim$ the original resonance integral values as quoted
 by Nichols.

COMPARISON OF METHODS OF CALCULATION

The most commonly used method of calculating the resonance integral for the 1 eV resonance is the NRIA approximation. NRIA calculations were accomplished with the ZUT code by using Equation (3) with Doppler-broadened cross sections. Equation (12) was also used to calculate the same integrals. The results are shown in Table V. The effect of Doppler broadening is very small at 20°C, the experimental temperature, so the differences between the hand calculation method using Equation (12) and the ZUT NRIA output can be ascribed to the use of the rational approximation in the derivation of Equation (12). The rational approximation uniformly underestimates the escape probability, P_0 when compared to the correct expression for cylinders (Reference 8, page 71) which would overestimate the absorption and hence result in high resonance integral values.

The sums of the ten resolved resonance integrals above 1.056 eV are given in the last two columns of Table V, one column for the integral method and one for the NRIA applied to all of the resonances. The growing discrepancies with increasing absorber concentration give a rough feel for the size of the error resulting from misapplication of the NRIA approximation to some of these resonances.

TABLE V

	1.056 eV	1.056 eV	1.056 eV	>1.056 eV	>1.056 eV
N_0 n/barn-cm	I_{INT} barns	I_{NRIA} barns	$I_{Eqn. 12}$ barns	$\sum I_{INT}$ barns	$\sum I_{NRIA}$ barns
5.00×10^{-8}	8,259.3	8,259.4	8,279.8	112.7	112.7
1.42×10^{-7}	8,210.3	8,210.5	8,230.0	112.7	112.7
4.08×10^{-7}	8,073.7	8,074.0	8,093.7	112.6	112.7
1.17×10^{-6}	7,712.1	7,713.0	7,737.0	112.4	112.5
3.35×10^{-6}	6,865.3	6,866.8	6,930.3	111.8	112.1
9.59×10^{-6}	5,312.4	5,313.5	5,356.6	110.1	111.0
2.74×10^{-5}	3,453.4	3,453.1	3,886.2	105.7	107.8
7.80×10^{-5}	2,043.0	2,042.1	2,458.5	95.3	100.1
2.20×10^{-4}	1,184.6	1,183.9	1,503.3	76.1	83.7

$I_{INT} \sim$ the 1.056 eV resonance integral value calculated using ZUT with the INT method for the absorber and the asymptotic value of $F(u')$ for the admixed aluminum scatterer integral.

$I_{NRIA} \sim$ the 1.056 eV resonance integral value calculated using ZUT with the infinite absorber mass method of solution for the absorber integral and the asymptotic value of $F(u')$ for the admixed aluminum scatterer integral.

$I_{Eqn 12} \sim$ the 1.056 eV resonance integral value calculated using equation 12.

$\sum I_{INT} \sim$ sum of all resolved resonance integrals above 1.056 eV using the INT method.

$\sum I_{NRIA} \sim$ sum of all resolved resonance integrals above 1.056 eV using the NRIA approximation method as calculated by the ZUT code.

APPENDIX A

NUCLEAR DATA FOR Pu²⁴⁰ AND Al²⁷

Pu²⁴⁰

$$A = 240.0, \sigma_{po} = 10.4 \text{ barns}, E_c = 120.0 \text{ eV}, D = 10.722 \text{ eV},$$

$$\langle \Gamma_m^0 \rangle = 2.03 \times 10^{-3} \text{ eV}, \Gamma_\gamma = 0.0259 \text{ eV}, g = 1.0$$

Resonance Data

$E_o(\text{eV})$	$\Gamma_n(\text{eV})$	$\Gamma_\gamma(\text{eV})$
1.056	2.45×10^{-3}	3.05×10^{-2}
20.4	2.35×10^{-3}	3.40×10^{-2}
38.2	1.52×10^{-2}	3.40×10^{-2}
41.7	1.90×10^{-3}	3.40×10^{-2}
66.0	4.50×10^{-2}	3.40×10^{-2}
72.0	2.90×10^{-2}	3.40×10^{-2}
90.0	1.70×10^{-2}	3.40×10^{-2}
92.0	4.00×10^{-3}	3.40×10^{-2}
104.0	6.00×10^{-2}	3.40×10^{-2}
116.0	4.00×10^{-2}	3.40×10^{-2}
119.0	5.00×10^{-2}	3.40×10^{-2}

Al²⁷

$$A = 27.0 \quad \sigma_s = 1.5 \text{ barns}$$

APPENDIX B

STATISTICS

Three linear regressions through the origin were used as estimates of the true model $\eta = \beta x$. η is the true PCTR reactivity, β is the true slope and x is $N_{B B}^{I V} - N_{A A}^{I V}$. The analysis of variance for the three is as follows:

1. Low Concentration Line

<u>Source</u>	<u>Sum of Squares</u>	<u>Degrees of Freedom</u>	<u>Mean Square</u>	<u>F-Ratio</u>
Due to line	2.9907×10^{-2}	1	2.9907×10^{-2}	
Residual	1.455×10^{-4}	2	0.7275×10^{-4}	411.0
Total	3.005×10^{-2}	3	$\hat{\beta}_1 = 0.16975$	

2. High Concentration Line

Due to line	13.603	1	13.603	
Residual	0.0102	4	0.02755	4,937.6
Total	13.6132	5	$\hat{\beta}_2 = 0.14148$	

3. All points

Due to line	13.63332	1	13.63332	
Residual	0.01192	7	0.00170	8,020
Total	13.64524	8	$\hat{\beta}_3 = 0.014152$	

The F-ratios for each of the three estimating lines are highly significant. For Line 1, the F-ratio is 411 as compared to the tabular value of $F_{99.5}(1,2) = 198.5$. Hence the line 1 regression is significant at greater than the 99.5% level. The line 2 ratio of 4,937.6 is even more significant: $F_{99.9}(1,4) = 74.14$. But line 3 is best yet with an F-ratio of 8020 as compared to $F_{99.9}(1,7) = 29.5$. Certainly the inclusion of all the data points only increases the validity of the model and hence line 3 serves as the best estimate of the true model: $\eta = \beta x$.

The slopes of lines 1 and 2 are $\hat{\beta}_1 = 0.16975$ and $\hat{\beta}_1 = 0.14148$ respectively. In order to test the significance of these apparently different slopes we must test to see first if they have the same variance. If so, we can pool the variance estimates to test the significance of the difference in the estimates of the slope. $V[\hat{\beta}_1] = \frac{s^2}{\Sigma X^2}$

$$V[\hat{\beta}_1] = 70.1 \times 10^{-6} \quad V[\hat{\beta}_2] = 4.0537 \times 10^{-6}$$

$$F_p(1, 1) = 17.3$$

$$F_{95}(1, 1) = 161.4$$

The estimates of the variance do not therefore differ at the 95% level hence the estimates may be pooled according to the equation

$$s^2 = \frac{(n_1 - 1) s_1^2 + (n_2 - 1) s_2^2}{n_1 + n_2 - 2} \quad (14)$$

and the test for equality of b_1 and b_2 is

$$\frac{b_1 - b_2}{s \sqrt{\frac{1}{\Sigma X_1^2} + \frac{1}{\Sigma X_2^2}}} \sim t(n_1 + n_2 - 2)$$

The test yields $t(6) = 0.66572$ whereas $t_{.95}(6) = 1.943$, therefore, the two estimates may both be of the same true value β and hence do not differ at the 95% level.

The confidence level bands shown in Figure 2 are derived from:

$$X \pm = \frac{y}{\hat{\beta}} \pm \frac{tS}{\hat{\beta}^2} \sqrt{y^2/\Sigma X^2 + \hat{\beta}^2} \quad . \quad S \text{ is the pooled estimate from}$$

Equation 14. For a two-tailed test at the 95% level $t = 2.365$.

The statistical formula and conclusions derive from the model (Reference 12, page 298) wherein the experimentally observed reactivities, y , are distributed normally about the true value $\eta = \beta X$ and with variance σ^2 . The quantity S^2 is the estimator of σ^2 in the various treatments. We also assume that the value of X is known without error. Of course, the compositions of the test rods are subject to error in fact, and this will increase the variance associated with the observed reactivity; however, these errors are not truly independent of one another since the same rod appears in two different determinations of y . The experimental observations are independent, however, and the model used will give an upward estimate of the true experimental variance. Since the model is approximate and hence may depart somewhat from a normal distribution, the robust analysis of variance has been used instead of the more delicate goodness of fit tests. More exact models would not be necessary in this application since the results are judged to be very highly significant with even an approximate model.

APPENDIX C

NOTATION

1. Subscripts: "0" refer to the absorber nuclei

"1" refer to non resonant admixed moderator

"m" refer to macroscopic non-resonant admixed moderator
per absorber nucleons

2. Resonance Parameters:

Γ = total level width

Γ_n = neutron width

Γ_γ = radiation width

$\langle \Gamma_n^0 \rangle$ = average value of the reduced $(\Gamma_n/\sqrt{E})$ neutron width

E_0 = energy at the maximum of the resonance cross section

E_c = cut-off energy above which the resonances are unresolved

σ_{p0} = potential scattering cross section of the absorber
nucleous

I_∞ = infinitely dilute resonance integral

g = the statistical factor $g = (2J + 1)/2(2I + 1)$

J = nuclear spin of the target plus a neutron

I = nuclear spin of the target nucleous

3. Statistical Parameters

$V[b]$ = variance of b

$\hat{b}$ = estimator of b

t = ordinate of Student's t -distribution

$s^2 = \hat{\sigma}^2$

σ^2 = the true variance

$F_p(n_1, n_2)$ = the F distribution with n_1 and n_2 degrees of freedom.

p is the probability that a given value of F will be
equal of less than $F_p(n_1, n_2)$.

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