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REPORT TITLE **Stability Analysis of a SNAP Core Equipped with a Mechanical Temperature Coefficient Augmentor**

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NAA-SR-MEMO - 10687

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ATOMICS INTERNATIONAL A Division of North American Aviation, Inc.		MAA-SR-MDSO- TDR NO. 10687	APPROVALS <i>Lohm</i>
TECHNICAL DATA RECORD		PAGE 1 OF 22	
AUTHOR R. L. Johnson, D. J. Arnold, 727-76 M. W. Halseth		DEPT. & GROUP NO.	DATE 11-12-64
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STATEMENT OF PROBLEM To investigate the stability of a SNAP Core equipped with a Mechanical Temperature Coefficient Augmentor.			
ABSTRACT: A transfer function is developed for a somewhat simplified SNAP system equipped with a proposed device for increasing the temperature coefficient. The system stability is then examined by constructing Nyquist and Bode Plots by means of the digital program TRAFFIC, for several different power levels and coolant flow rates. Indications are that the system is quite stable for a wide range of these parameters. The response of the system to a sunshade orbit is also calculated. Oscillations in the reactor power are less than 0.5% of the steady-state value.			

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I. System Description

The system under study here consists of a SNAP 10-A core and reflector assembly. The core is considered here to consist of fuel rods, coolant, container vessel, and grid plates. The reflector assembly consists of a molybdenum strip attached to the container vessel by a mechanical device so designed that changes in the vessel or strip temperature produce a significant relative radial motion between strip and vessel. The Beryllium reflector itself is attached to the molybdenum strip. Figure I shows the system and its components.

For simplicity the analysis is restricted to a lumped parameter model, that is, all spatial variation of quantities is ignored. Thus, the fuel is taken as a point mass, as is the coolant, the molybdenum strip, and the reflector. No variations in coolant flow rate were considered and the coolant was assumed to have constant temperature at the core inlet.

The 7094 Program TRAFFIC, (Ref. 1), as presently written allows only feedback functions in the form of a rational function, or the sum of rational functions. The polynomials in the numerator or the denominator of these functions must not be of degree greater than 4. To keep the transfer function within these bounds, it is necessary to lump the upper grid plate with the fuel. Further, since the container vessel is quite thin, it will be assumed to be at the same temperature as the coolant.

For this system we can therefore write down the following lumped parameter equations which represent merely energy balances for the various components defined above.

Fuel Temperature, T_f :

$$(1.1) \quad W_f C_f \frac{dT_f}{dt} = P(t) - UA_f (T_f - T_c)$$

The symbols are defined as:

- W_f = Total fuel weight, lbs. (including clad)
- C_f = Fuel specific heat, Btu/lb°F
- $P(t)$ = Total reactor power, Btu/sec
- U = Overall heat transfer Coefficient, Fuel to coolant $\frac{\text{Btu}}{\text{ft}^2 \cdot \text{°F} \cdot \text{sec}}$
- A_f = Total fuel surface area, ft^2
- T_c = Coolant (NaK) temperature, °F

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Coolant Temperature, T_c :

$$(1.2) \quad C_c \rho_c V_c \frac{dT_c}{dt} = W_c C_c (T_{ci} - T_c) + UA_f (T_f - T_c) - \sigma f_m A_i (T_c^4 - T_m^4)$$

where:

C_c = NaK specific heat Btu/lb°F

ρ_c = NaK density lbs/ft³

V_c = Coolant volume in core, ft³

W_c = Coolant flow rate lbs/sec

T_{ci} = Coolant inlet temperature - °F

= Boltzmann constant - $\frac{\text{Btu}}{\text{ft}^2} (^{\circ}\text{R})^4 \text{ sec}$

f_m = Shape factor for radiation from coolant vessel to molybdenum, dimensionless

T_m = Molybdenum temperature, °R

Molybdenum Temperature, T_m :

$$(1.3) \quad C_m \rho_m V_m \frac{dT_m}{dt} = \sigma f_m A_m (T_c^4 - T_m^4) - h_{mr} A_m (T_m - T_R)$$

where:

C_m = Molybdenum specific heat Btu/lbs°F

ρ_m = Molybdenum density lbs/ft³

V_m = Molybdenum volume ft³

A_m = Area of molybdenum receiving heat by radiation from the reactor vessel, ft²

h_{mr} = A coefficient for the conduction of heat from molybdenum to beryllium reflector Btu/ft²°F sec

Reflector Temperature, T_R

$$(1.4) \quad C_R \rho_R V_R \frac{dT_R}{dt} = h_{mr} A_m (T_m - T_R) - \sigma f_R A_R (T_R^4 - T_s^4)$$

C_R = Reflector specific heat, Btu/lb°F

ρ_R = Reflector density lbs/ft³

V_R = Reflector volume, ft³

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f_n = Shape factor for radiation from reflector to space
dimensionless

T_s = Effective space temperature, $^{\circ}R$

The molybdenum is assumed to have the same surface area both for receiving heat from the vessel and in contact with the reflector.

In addition, the standard power equations are used,

$$(1.5) \quad \frac{dP}{dt} = \frac{\rho}{\lambda} (P-1)P + \frac{\rho}{\lambda} \sum \alpha_i W_i$$

$$(1.6) \quad \frac{dW_i}{dt} = \lambda_i (P - W_i)$$

The symbols have the usual meaning. W_i the normalized precursor concentration has the units of power. (For further details on the equations in this form see reference 2). The reactivity, ρ , is given as the input reactivity δ , and the feedback reactivity. Therefore, we can write

$$(1.7) \quad \rho = \delta + \sum_i \gamma_i (T_i - T_{i0})$$

where: The γ_i are the temperature coefficients.

For the present problem the significant reactivity changes are due to fuel temperature changes, and reflector position changes. Any change in the reflector position depends entirely on the container vessel temperature (equal to the coolant temperature by assumption) and on the temperature of the molybdenum plate to which the reflector is mounted. The reflector assembly is so designed that an increase in container vessel temperature at constant molybdenum temperature moves the reflector away from the core thus decreasing the reactivity. An increase in the molybdenum temperature at constant vessel wall temperature would have the opposite effect. Therefore a negative temperature coefficient is assigned to the container vessel temperature, a positive one to the molybdenum. The reactivity can then be expressed as

$$(1.8) \quad \rho = \delta + \gamma_f (T_f - T_{f0}) + \gamma_c (T_c - T_{c0}) + \gamma_m (T_m - T_{m0})$$

where: γ_f , γ_c are negative and γ_m is a positive number. A discussion of the various reactivity coefficients will be given later.

II. Formulation of the System Transfer Function

The stability of the system will be investigated for the case of an arbitrary reactivity input δ . Alternatively one could use other forcing functions, for example, a varying coolant inlet temperature, or a varying

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effective space temperature. The latter case is also investigated in this report. To formulate the transfer function the various reactor parameters are assumed to vary about their equilibrium values, that is

$$T_f = T_{fo} + T_f' \quad , \quad P(t) = P_o + p(t)$$

$$T_c = T_{co} + T_c' \quad \text{etc.}$$

are substituted into equations (1.1)-(1.4) giving the following equations in the primed quantities.

$$(2.1) \quad W_f C_f \frac{dT_f'}{dt} = p(t) - UA_f (T_f' - T_c')$$

$$(2.2) \quad C_c \rho_c V_c \frac{dT_c'}{dt} = -W_c C_c T_c' + UA_f (T_f' - T_c') - 4\sigma f_m A_1 T_{co}^3 T_c' + 4\sigma f_m A_1 T_{mo}^3 T_m'$$

where we have used the assumption that $T_{ci}' = 0$ and

$$W_c C_c (T_{cio} - T_{co}) + UA_f (T_{fo} - T_{co}) - \sigma f_m A_1 (T_{co}^4 - T_{mo}^4) = 0$$

at steady state. Further the expansion

$$(T_{co} + T_c')^4 = T_{co}^4 \left(1 + 4 \frac{T_c'}{T_{co}} + \frac{6 T_c'^2}{T_{co}^2} + \frac{4 T_c'^3}{T_{co}^3} + \frac{T_c'^4}{T_{co}^4} \right)$$

has been approximated by

$$T_{co}^4 \left(1 + 4 \frac{T_c'}{T_{co}} \right)$$

by neglecting terms of higher order in T_c' . This restricts the analysis to linear effects, that is small changes in the temperatures and power. In a similar manner

$$(2.3) \quad C_m \rho_m V_m \frac{dT_m'}{dt} = 4\sigma f_m A_m T_{co}^3 T_c' - 4\sigma f_m A_m T_{mo}^3 T_m' - h_{mr} A_m (T_m' - T_r')$$

and

$$(2.4) \quad C_r \rho_r V_r \frac{dT_r'}{dt} = h_{mr} A_m (T_m' - T_r') - 4\sigma f_r A_r T_{ro}^3 T_r'$$

where it has been assumed, that for the purposes of calculating core stability, the effective space sink temperature may be considered constant. This is permissible, of course, because of the wide difference in the time scales between nuclear changes in the core and orbital position changes.

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From (1.5) the change in reactor power, $p(t)$ is given by

$$\frac{dp}{dt} = \frac{\beta}{\lambda} (P-1) P_0 + \frac{\beta}{\lambda} (P-1) p(t) + \frac{\beta}{\lambda} \sum \alpha_i (W_{i0} + W_i')$$

or

$$\frac{dp}{dt} = -\frac{\beta}{\lambda} P_0 + \frac{\beta}{\lambda} p P_0 + \frac{\beta}{\lambda} (P-1) p(t) + \frac{\beta}{\lambda} \sum \alpha_i W_{i0} + \frac{\beta}{\lambda} \sum \alpha_i W_i'$$

Since $-\frac{\beta}{\lambda} P_0 + \frac{\beta}{\lambda} \sum \alpha_i W_{i0} = 0$, (Steady state values)

then $\frac{dp}{dt} = \frac{\beta}{\lambda} P_0 p(t) + \frac{\beta}{\lambda} (P-1) p(t) + \frac{\beta}{\lambda} \sum \alpha_i W_i'$

And neglecting the product of two, by hypothesis, small quantities $P(t)$ and $p(t)$ eq. (1.5) becomes

$$(2.5) \quad \frac{dp}{dt} + \frac{\beta}{\lambda} p(t) = \frac{\beta}{\lambda} P_0 p(t) + \frac{\beta}{\lambda} \sum \alpha_i W_i'$$

From equation (1.6), the precursor equation, W_i' is given by

$$(2.6) \quad \frac{dW_i'}{dt} = \lambda_i (p(t) - W_i')$$

Finally the reactivity is from equation (1.8)

$$(2.7) \quad \rho(t) = \delta(t) + \gamma_f T_f' + \gamma_c T_c' + \gamma_m T_m' = \delta(t) + \rho_f(t)$$

where $\rho_f(t)$ is the feedback reactivity.

Taking the Laplace Transform of (2.6) and solving for $\bar{W}_i(s)$, the transform of the change in precursor concentration gives,

$$(2.8) \quad \bar{W}_i(s) = \frac{\lambda_i \bar{p}(s)}{s + \lambda_i}$$

Taking the transform of (2.5) and using (2.8) the transformed power equation is found to be:

$$(2.9) \quad s \bar{p}(s) + \frac{\beta}{\lambda} \bar{p}(s) = \frac{\beta}{\lambda} P_0 \bar{p}(s) + \frac{\beta}{\lambda} \sum_i \frac{\alpha_i \lambda_i}{s + \lambda_i} \bar{p}(s)$$

where the primes have been dropped for convenience. Defining now the reactivity feedback transfer function

$$\bar{\rho}_f(s) = -H(s) \bar{p}(s)$$

and using this in (2.9) gives after multiplying by $\frac{\lambda}{\beta}$

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$$(2.10) \quad s \frac{1}{\beta} \overline{p}(s) + \overline{p}(s) + P_0 H(s) \overline{p}(s) - \sum_i \frac{\alpha_i \lambda_i}{s + \lambda_i} \overline{p}(s) = P_0 \overline{\delta}(s)$$

Since $\sum \alpha_i = 1$ (2.10) may be written as

$$(2.11) \quad s \frac{1}{\beta} \overline{p}(s) + \overline{p}(s) \left(\sum_i \alpha_i - \sum_i \frac{\alpha_i \lambda_i}{s + \lambda_i} \right) + P_0 H(s) \overline{p}(s) \\ = s \frac{1}{\beta} \overline{p}(s) + P_0 H(s) \overline{p}(s) + \overline{p}(s) \left(s \sum_i \frac{\alpha_i}{s + \lambda_i} \right) = P_0 \overline{\delta}(s)$$

Identifying $s \left(\frac{1}{\beta} + \sum_i \frac{\alpha_i}{s + \lambda_i} \right)$ as the reciprocal of the zero power transfer function $G(s)$ (2.11) becomes

$$G^{-1}(s) \overline{p}(s) + P_0 H(s) \overline{p}(s) = P_0 \overline{\delta}(s)$$

And solving for $\overline{p}(s)$ one gets the standard form

$$(2.12) \quad \overline{p}(s) = \frac{P_0 \overline{\delta}(s)}{G^{-1}(s) + P_0 H(s)} = \frac{P_0 \overline{\delta}(s) G(s)}{1 + P_0 G(s) H(s)}$$

The transfer function is now in the correct form for the program TRAFFIC. It is only necessary to find $H(s)$ in the correct form, that is the quotient of two polynomials.

III. Feedback Transfer Function

From equation (2.7)

$$p_f(t) = \gamma_f T_f' + \gamma_c T_c' + \gamma_m T_m'$$

Taking the Laplace Transform of $p_f(t)$ and dropping the primes again on the T's, get

$$(3.1) \quad \overline{p}_f(s) = \gamma_f \overline{T}_f(s) + \gamma_c \overline{T}_c(s) + \gamma_m \overline{T}_m(s)$$

which by the definition of $H(s)$ is equal to $-H(s) \overline{p}(s)$. Thus

$$(3.2) \quad \gamma_f \overline{T}_f(s) + \gamma_c \overline{T}_c(s) + \gamma_m \overline{T}_m(s) = -H(s) \overline{p}(s)$$

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From equations (2.1) through (2.4) each of the functions $\overline{T_f(s)}$, $\overline{T_c(s)}$ and $\overline{T_m(s)}$ can be found in terms of $\overline{p(s)}$. Putting

$$\overline{T_f(s)} = \overline{g_f(s)} \overline{p(s)}$$

$$\overline{T_c(s)} = \overline{g_c(s)} \overline{p(s)}$$

$$\overline{T_m(s)} = \overline{g_m(s)} \overline{p(s)}$$

into eq. (3.2) gives for $H(s)$,

$$(3.3) \quad H(s) = -\gamma_f \overline{g_f(s)} - \gamma_c \overline{g_c(s)} - \gamma_m \overline{g_m(s)}$$

The determination of the various $\overline{g}$'s from equations (2.1) through (2.4) is somewhat tedious. One starts by taking the Laplace Transform of equations (2.1)-(2.4) and eliminating the variables one by one. Thus the transformed equations are starting with (2.4)

$$(3.4) \quad (s + a_1 + a_2) \overline{T_R(s)} = a_1 \overline{T_m(s)}$$

or

$$\overline{T_R(s)} = \frac{a_1}{(s + a_1 + a_2)} \overline{T_m(s)}$$

where

$$a_1 = \frac{h m_R A_m}{C_R \rho_R V_R} \quad a_2 = \frac{4 \sigma f_R A_R T_{R0}^3}{C_R \rho_R V_R}$$

$$(3.5) \quad (s + a_3 + a_4) \overline{T_m(s)} = a_3 \overline{T_R(s)} + a_4 \overline{T_c(s)}$$

where

$$a_3 = \frac{h m_R A_m}{C_m \rho_m V_m} \quad a_4 = \frac{4 \sigma f_m A_m T_{m0}^3}{C_m \rho_m V_m}$$

$$a_5 = \frac{4 \sigma f_m A_m T_{c0}^3}{C_m \rho_m V_m}$$

$$(3.6) \quad (s + a_6 + a_7 + a_8) \overline{T_c(s)} = a_7 \overline{T_f(s)} + a_8 \overline{T_m(s)}$$

$$a_6 = \frac{W_c C_c}{C_c \rho_c V_c} \quad a_7 = \frac{U A_f}{C_c \rho_c V_c} \quad a_8 = \frac{4 \sigma f_m A_1 T_{c0}^3}{C_c \rho_c V_c}$$

$$a_9 = \frac{4 \sigma f_m A_1 T_{m0}^3}{C_c \rho_c V_c}$$

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$$(3.7) \quad (s + a_{10}) \bar{T}_f(s) = a_{11} \bar{p}(s) + a_{11} \bar{T}_c(s)$$

$$a_{10} = \frac{UA_f}{W_f C_f}$$

$$a_{11} = \frac{1}{W_f C_f}$$

These equations may now be solved for $\bar{T}_f$, $\bar{T}_c$, and $\bar{T}_m$ in terms of $\bar{p}(s)$. From these expressions we can write down the g 's of eq. (3.3). The results are:

$$g_f(s) = \frac{a_{11} \{ [(s+a_1+a_2)(s+a_3+a_4) - a_1 a_2] [s+a_6+a_7+a_8] - a_5 a_9 [s+a_1+a_9] \}}{g_1(s)}$$

$$g_c(s) = \frac{a_7 a_{11} [(s+a_1+a_2)(s+a_3+a_4) - a_1 a_2]}{g_1(s)}$$

$$g_m(s) = \frac{a_5 a_7 a_{11} (s+a_1+a_2)}{g_1(s)}$$

where

$$g_1(s) = [s+a_{10}] [(s+a_1+a_2)(s+a_3+a_4) - a_1 a_2] [s+a_6+a_7+a_8] - a_5 a_9 [s+a_{10}] [s+a_1+a_2] - a_7 a_{10} [(s+a_1+a_2)(s+a_3+a_4) - a_1 a_2]$$

After expanding out the numerators and denominators of $g_f(s)$, $g_c(s)$ and $g_m(s)$ equation (3.3) takes the desired form, that is the quotient of two polynomials, or rather the sum of three such terms.

Since the constants a_i , depend on the reactor initial temperatures which depend on the initial power, and on the coolant flow rate W_c , they must be calculated anew for each different case. Therefore, in order to make a stability survey of a large number of power levels and flow rates it is necessary to machine calculate the input. A program SPEEDY was written for this purpose by D. J. Arnold which starting with only the initial power level P_0 and flow rate W_0 , as well as the physical parameters of the system, prepares the input for TRAFFIC in punched card form. This enabled the survey of a wide range of parameters quickly without hand labor.

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The Nyquist and Bode plots for a representative power and flow rate are shown by Figures 2 through 4. Other cases were calculated for various steady state power levels and flow rates. The power selected ranged from 11 kw to 50 kw while the flow was varied from 0.40 to 2.50 lbs/sec. The results showed the system stability to be essentially independent of flow rate and power within these ranges.

To interpret the Nyquist plots one constructs the reflection in the real axis of the curves shown, starting from the origin. From this point the curve is closed in a clockwise manner. If the resulting curve encircles the $-1+j0$ point, marked by X on the plots, it indicates a root of $1+RCH$ with $Re s > 0$, that is, an instability. The system response will then be of the form $e^{\lambda t}$, $Re \lambda > 0$, where λ is the corresponding root. Alternatively the Bode plots can be used. Here an instability corresponds to a magnitude > 1 and phase angle of 180° for some frequency. All of the curves shown in this report indicate stable behavior.

IV. Effect of Space Environment

The same equations are used here as in the foregoing except that the space temperature T_s is allowed to vary. Thus:

Reflector temperature,

$$(4.1) \quad C_R P_R V_R \frac{dT_R}{dt} = h_{mR} A_m (T_m - T_R) - \sigma f_R A_R (T_R^4 - T_s^4)$$

Molybdenum temperature,

$$(4.2) \quad C_m P_m V_m \frac{dT_m}{dt} = \sigma f_m A_m (T_c^4 - T_m^4) - h_{mR} A_m (T_m - T_R)$$

Coolant Temperature,

$$(4.3) \quad C_c P_c V_c \frac{dT_c}{dt} = W_c C_c (T_{c1} - T_c) + U A_f (T_f - T_c) - \sigma f_m A_1 (T_c^4 - T_m^4)$$

Fuel Temperature,

$$(4.4) \quad W_f C_{pf} \frac{dT_f}{dt} = P(t) - U A_f (T_f - T_c)$$

The symbols have the same meaning as before.

An exact analysis for the true effective space temperature would be tedious and would, of course, depend on the surface conditions of the reflector. To determine order of magnitude changes in the reactor power a rather crude approximation should be adequate. It was therefore assumed that the reactor was in a 2.5 hour orbit, half sun, half shade and that the change in the effective space temperature T_s has the form of square wave. A reasonable estimate of the change in T_s is from -200°F to 35°F , shade to sun.

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The initial conditions for the calculations were found by setting all time derivatives equal to zero in equations (4.1)-(4.4). For simplicity it was assumed that $T_m = T_R$, which is very nearly true. Then given an initial power level and flow rate, the initial temperatures of all the elements can be calculated. The calculations, both steady state, and time dependent were carried out by means of CREKIN, a digital kinetics code which can solve several simultaneous first order equations of the type under consideration here.

The results of these calculations are shown in Figures 5 through 7. Figure 5 shows the assumed effective space temperature. Figure 6 shows the change in the reactor power. It is seen to oscillate in a sawtooth fashion, the maximum deviation being about 1% of the mean power. Thus fluctuations in the reactor power are too small to be of any significance. The drift of the power profile from the initial point is due to the approximate nature of the equilibrium calculations. That is the values of T_c , T_R , T_m and T_m calculated for time $t = 0$ are not exact with the result that the initial calculation of the derivatives (at the first time step) does not give zero. This is also the case for the initial calculation of the reactivity. In other words the initial conditions calculated were not quite exact and as a consequence the reactor is not initially in equilibrium.

The reactivity versus time curve, Figure 7, shows the rapid change in reactivity that occurs when the system passes from sun to shade and conversely. The rapid change in reactivity, that is the spike, is due to the molybdenum. This is assumed to have a positive coefficient, thus the reactivity change is in the same direction as the environmental temperature change. This is quickly followed by a change of opposite sign due to the coolant. These transients die quickly and are followed by a period in which the reactor seeks equilibrium. (reactivity tends towards zero) This is then interrupted by a new change in reactivity. Ultimately the power and reactivity oscillate about the equilibrium points. The magnitude of the power oscillations can be read direct from Figure 6.

V. Temperature Coefficients

As has been stated the reflector assembly is assumed to have two temperature feedback paths, one negative and following closely the coolant temperature, the other positive and due to changes in the molybdenum temperature. The magnitude of these coefficients is evaluated as follows: The coefficient γ_c is, for small changes in T_c , equal to

$$\gamma_c = \left(\frac{\partial \rho}{\partial T_c} \right)_{T_m, T_f}$$

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If the reactivity worth of the reflector is known as a function of the change in its distance X from the container vessel, then

$$\gamma_c = \left(\frac{d\rho}{dX} \right) \left(\frac{\partial X}{\partial T_c} \right)_{T_m, T_f}$$

The quantity $\frac{d\rho}{dX} = \frac{\Delta\rho}{\Delta X}$, the reactivity change associated with an incremental change in reflector position, has been calculated by the usual nuclear codes. The quantity $\left(\frac{\partial X}{\partial T_c} \right)_{T_m, T_f}$ can be calculated from the geometry.

A simplified sketch of the system is shown below. For simplicity, in the following analysis, we assume the connections at A, A', B, B' are pins. This will give an adequate approximation. For a more detailed drawing of the system under study see Figure 1.

From the sketch above

$$l_s = l_R + 2l_B \cos \theta$$

$$\Delta l_s = \Delta l_R - 2l_B \sin \theta \Delta \theta = -2l_B \sin \theta \Delta \theta$$

since the molybdenum and the brackets (length l_B) are not stressed. Thus:

$$\Delta \theta = - \frac{\Delta l_s}{2l_B \sin \theta}$$

If β_s is the thermal expansion coefficient of the steel then

$$\Delta l_s = \beta_s l_s \Delta T_c$$

Further

$$\Delta X = -l_B \Delta \theta \cos \theta$$

Thus

$$\Delta X = -l_B \cos \theta \left(\frac{-\Delta l_s}{2l_B \sin \theta} \right) = \frac{1}{2} \cot \theta \Delta l_s$$

$$\Delta X = \frac{1}{2} \cot \theta \beta_s l_s \Delta T_c$$

$$\frac{\Delta X}{\Delta T_c} = \frac{1}{2} \cot \theta l_s \beta_s$$

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For the motion of the molybdenum.

$$\Delta l_R = k_R l_R \Delta T_R$$

$$\Delta l_S = 0$$

$$\Delta l_R = 2 l_B \sin \theta \Delta \theta$$

$$\Delta \theta = \frac{\Delta l_R}{2 l_B \sin \theta}$$

$$\Delta x = -l_B \cos \theta \Delta \theta$$

$$\Delta x = -l_B \cos \theta \frac{\Delta l_R}{2 l_B \sin \theta} = -\frac{1}{2} \cot \theta \Delta l_R$$

$$\Delta x = -\frac{1}{2} \cot \theta k_R l_R \Delta T_R$$

$$\left(\frac{\Delta x}{\Delta T_R} \right) = -\frac{1}{2} \cot \theta k_R l_R$$

This is of opposite sign to $\frac{\Delta x}{\Delta T_C}$. The ratio of the two reactivity temperature coefficients is

$$\frac{Y_C}{Y_R} = \frac{(\Delta x / \Delta T_C)}{(\Delta x / \Delta T_R)} = \frac{\frac{1}{2} \cot \theta l_S k_S}{-\frac{1}{2} \cot \theta k_R l_R} = - \left(\frac{l_S k_S}{l_R k_R} \right)$$

The values of k_S and k_R are about

$$k_S = 10.0 \times 10^{-6} \frac{\text{in}}{\text{in}^\circ \text{F}} \quad (\text{Steel})$$

$$k_R = 3.0 \times 10^{-6} \frac{\text{in}}{\text{in}^\circ \text{F}} \quad (\text{Molybdenum})$$

The reactivity changes associated with a change in reflector position has been calculated as $\left(\frac{\Delta \rho}{\Delta x} \right) = -20.0 \text{ } \$/\text{in}$ (Ref. 5)

Therefore

$$Y_C = -(20.0 \text{ } \$/\text{in}) \frac{1}{2} \cot \theta l_S k_S \quad (\cot \theta \sim 10.0)$$

$$Y_C = -(200.0 \text{ } \$/\text{in}) (12 \text{ in}) (10.0 \times 10^{-6} \frac{\text{in}}{\text{in}^\circ \text{F}}) = -0.024 \text{ } \$/^\circ \text{F}$$

$$Y_R = +0.008 \text{ } \$/^\circ \text{F} \quad \text{if } l_S \sim l_R$$

For the fuel the negative temperature coefficient was given the value

$$Y_f = -0.00065 \text{ } \$/^\circ \text{F}$$

This value is taken from Ref. 3.

Values used for other physical parameters used are in the appendix.

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- (1) W. A. Rhoades, M. A. Boling; "TRAFFIC - A Computer Program for Reactor Transfer Functions Calculations in the Complex Domain". NAA-SR-8607
- (2) W. A. Rhoades, W. B. Green; "SNAP Reactor Handbook - Transient Analysis". NAA-SR-9368
- (3) I. Miller, R. L. Brehm, W. J. Roberts; "Temperature Coefficients and Spectra in the Hydride Moderated SNAP Reactors". NAA-SR-7140
- (4) R. L. Johnson, "Core Transfer Function Models for SNAP System Kinetics". NAA-SR-8440
- (5) Based on preliminary estimates made from reviewing SCA-4A data. More recent calculations based on later SCA-4A data correlated with FS-1 data (see NAA-SR-9754 "SNAP 10A Nuclear Analysis", A. R. Dayes, W. A. Flynn, J. P. Hawley (CRD), 12/31/64, give an average value for the reflector gap of \$13.00/inch. The basic conclusions of this report are not affected by this change in value.

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Appendix I - Numerical Data

W_f	=	Total fuel weight, including moderator and clad	115.3 lbs
C_f	=	Fuel mean heat capacity	0.12 BTU/lb°F
U	=	Overall heat transfer coefficient	0.013 BTU/sec ft ² °F
A_f	=	Fuel surface area	17.1 ft ²
C_c	=	NaK specific heat	0.21 Btu/lb°F
P_c	=	NaK density	44 lbs/ft ³
V_c	=	Coolant volume in core	0.083 ft ³
W_c	=	Boltzmann Constant	$0.481 \times 10^{-12} \frac{\text{Btu}}{\text{sec ft}^2(^{\circ}\text{R})^4}$
W_c	=	Coolant flow rate	various lbs/sec
	=	Molybdenum Emissivity	0.10
	=	Stainless emissivity	0.75
f_m	=	$1 / (\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1)$	shape factor
C_m	=	Molybdenum Specific heat	0.061 $\frac{\text{Btu}}{\text{lb}^{\circ}\text{F}}$
P_m	=	Molybdenum density	636 $\frac{\text{lbs}}{\text{ft}^3}$
V_m	=	Molybdenum volume	0.011 ft ³ (band on 0.1" thick strip)
A_m	=	Molybdenum heat transfer area	1.29 ft ²
h_{ma}	=	Coefficient for conduction of heat from Molybdenum to Beryllium. (Based on assumed linear temperature distribution; $= 2k/l$ where k is the Beryllium conductivity and l is the Beryllium thickness)	
k	=	Beryllium conductivity	0.0235
l	=	Reflector thickness	3 inches
C_R	=	Reflector specific heat	0.5 $\frac{\text{Btu}}{\text{lb}^{\circ}\text{F}}$
P_R	=	Reflector density (Beryllium)	115 $\frac{\text{lbs}}{\text{ft}^3}$
V_R	=	Reflector volume	0.322 ft ³
f_n	=	Shape factor for radiation from reflector to outer space, equal to the surface emissivity of the Beryllium, about 0.5.	
A_R	=	Outer surface area of reflector	2.32 ft ²

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Figure 1. Schematic Reactor Mechanical Temperature Coefficient Augmentor

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Figure 2. Closed Loop Phase Angle vs Frequency
P-NOT = Thermal Power, Kilowatts
WC = Coolant Flow Rate, lb/sec

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Figure 3. Closed Loop Magnitude vs Frequency
P-NOT = Thermal Power, Kilowatts
WC = Coolant Flow Rate, lb/sec

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CASE 3 P-NOT = 11.38, WC = 1.22

POWER = 0.113800 6-1.2

OPEN LOOP PGH -- IMAGINARY PART (DECIBELS)

OPEN LOOP PGH -- REAL PART (DECIBELS)

Figure 4. System Nyquist Plot

P-NOT = Thermal Power, Kilowatts

WC = Coolant Flow Rate, lb/sec

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Figure 5. Effective Space Temperature for a SNAP Reactor in a 150-Minute Sun-Shade Orbit

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Figure 6. Total System Reactivity as a Function of Time for a SNAP Reactor in a Sun-Shade Orbit

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Figure 7. Reactor Power as Function of Time in a Sun-Shade Orbit

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