THE USE OF SCATTERING PARAMETERS IN THE DESIGN
OF AMPLIFIERS OF SUBNANOSECOND RISETIME

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SUMMARY

This paper discusses techniques useful in the design of wide-band linear amplifiers intended for pulse operation. A suitable model for high-frequency transistors is given, and use is made of the scattering characterization and of a special computer program to deduce element values in the model.

The design of a fast risetime series-shunt pair and series-series triple is carried out, and computer simulations are used to predict performance of the actual designs inclusive of all parasitic reactances consequent upon physical realization of the circuits.

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CHAPTER 1. INTRODUCTION

The need often arises in experiments in high-energy physics for an amplifier possessing moderate gain and having rise-times in the subnanosecond region. Good d-c stability and zero offset between input and output, as well as favourable noise performance are additional characteristics desirable in such an amplifier; however, since in this paper the primary concern is with techniques of realizing the necessary transient response, suffice it to say that the d-c requirements can be met using the operational-amplifier stabilizing technique described elsewhere. (1) The topic of noise minimization is an extensive one in its own right, and could provide the basis for further research.

The minimal specification to be satisfied by the circuit in respect of transient performance may be stated concisely thus:

- (i) Rise-time ≤ 500 psec.
- (ii) Overshoot on step-function input $\leq 10\%$.
- (iii) Gain of 20 dB ($\times 10$) in a 50 Ω system.

It is evident that the amplifier must be capable of passing frequencies approaching one gigahertz. At the other end of the spectrum, a passband extending down to d-c is desirable from the aspect of reducing tilt on wide input pulses, and of simplifying the application of operational-amplifier d-c stabilizing methods.

The low-frequency requirement is easily satisfied by the use of direct coupling within the amplifier; however, the provision of the requisite high-frequency response is a considerably more difficult problem, and requires in addition to the use of transistors having suitably high f_T figures the application of stripline and thin-film construction techniques in order to reduce to a minimum the effects upon circuit performance of parasitic reactances and transversal delay around the feedback loop.

Two amplifier designs are considered in this paper, and in both series feedback is used at the input in order to raise the input impedance of the circuit. In this way an approximation can be attained to a $50\ \Omega$ input impedance through use, at the base of the input transistor, of a $50\ \Omega$ resistor to ground. Moreover, earlier work⁽²⁾ has shown that the two configurations chosen for treatment here, namely the series-shunt pair and series-series triple, are optimal from the standpoint of maximum gain-bandwidth product in comparison with alternate two- and three-stage arrangements.

A preliminary to the design of any amplifier is the determination of suitable models of the active devices intended for use therein. Such models may be deduced directly from physical reasoning provided accurate data are available regarding device construction. Frequently this information is proprietary and hence is not readily available; even if it were, manufacturing processes are often sufficiently ill-defined as to make rather inaccurate a device model determined solely on the basis of a device processing schedule.

The approach to transistor characterization adopted in this paper relies upon physical reasoning only to deduce the topology of the equivalent circuit, and in this way is relieved of dependence upon details of transistor fabrication. Direct measurement of the two-port

parameters of the transistor, in conjunction with the use of a digital computer program devised especially for the task, then enables values to be assigned to the elements of the model. This method has the virtue of reflecting the actual characteristics of the device, and an arbitrary increase in model accuracy is possible simply by incorporating into the model those additional elements necessary to deal with effects not accounted for in more elementary versions.

The two-port measurements necessary for the characterization of the transistors used in the amplifier were done using scattering parameters in order to circumvent the need for accurate short- or open-circuits of device ports necessary for the determination of the more common Y- or Z-parameters. The difficulty at high frequencies of attaining these states can lead to significant inaccuracies in measurement.

Included in this paper is a complete description of the various design and modelling techniques already described in brief above. Chapter 2 is devoted to a brief introduction to the scattering matrix and to the development of formulae useful in the remainder of the report. In Chapter 3, the theoretical aspects of automatic modelling are presented, along with a short description of a computer program written to realise the theory. The results of automated modelling of a high-frequency chip transistor are contained in Chapter 4. Two amplifiers were designed using the transistor characterized in Chapter 4; the design of a series-shunt feedback pair forms the subject of Chapter 5. Chapter 6 discusses the design of a series-series feedback triple; this particular design was accomplished using a variant of the optimization program

dealt with in Chapter 3. Chapter 7 concludes the report with a discussion of important results from the preceding chapters and with suggestions for further work. Appendices give details of matters considered too specialized for more than preliminary mention in other parts of the paper.

CHAPTER 2. ELEMENTARY THEORY OF THE SCATTERING MATRIX

The scattering parameter method of two-port characterization was devised originally in connection with microwave network theory, where the description of two-port behaviour in terms of incident and reflected wave quantities is a physically realistic approach lending itself to easy visualization.

In measurements upon active devices at high frequencies, the use of scattering parameters is further commended by the fact that at these frequencies, the open- and short-circuits requisite to measurement of the Y- or Z-parameters are exceedingly difficult to attain due to the presence of spurious reactances. Such parasitics can also render an active device unstable and susceptible to free oscillation when the more conventional two-port characterizations are employed. These and attendant problems are greatly reduced in the scattering formulation, for in this, the two ports of the device under test are always terminated in a constant and precisely known reference impedance.

The scattering formulation commonly used, and that to be presented in this chapter, is carried out in terms of a resistive reference immittance. The scattering formalism is completely general, however, and can accomodate complex reference immittances. (3)

Shown in Fig. (2-1) are the basic definitions to be employed in the ensuing discussion which, although it will be given on the basis of a two-port configuration, is easily extensible to the general n-port case.

(a) Calculation of $S_{11}(j\omega)$, $S_{21}(j\omega)$
 $a_2(j\omega) = 0$

(b) Calculation of $S_{12}(j\omega)$, $S_{22}(j\omega)$
 $a_1(j\omega) = 0$

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Fig. 2-1

The customary definition of the basis-free normalized scattering matrix is contingent upon the introduction of an hypothetical pair of vector waves comprising the wave $\underline{a}(j\omega)$ incident upon the two-port, and its reflected counterpart $\underline{b}(j\omega)$. The scattering matrix $\underline{S}(j\omega)$ relates the incident and reflected waves according to the formula: (4)

$$\underline{b}(j\omega) = \underline{S}(j\omega) \underline{a}(j\omega), \quad (2-1)$$

where,

$$\underline{a}(j\omega) = \frac{1}{2} \underline{R}^{-\frac{1}{2}} (\underline{V}(j\omega) + \underline{R}\underline{I}(j\omega)), \quad (2-2)$$

$$\text{and,} \quad \underline{b}(j\omega) = \frac{1}{2} \underline{R}^{-\frac{1}{2}} (\underline{V}(j\omega) - \underline{R}\underline{I}(j\omega)). \quad (2-3)$$

By $\underline{R}$ is denoted the diagonal matrix of reference normalizing resistances, namely,

$$\underline{R} = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix},$$

and $\underline{V}(j\omega)$ and $\underline{I}(j\omega)$ are the vectors of port voltages and currents, respectively.

It is apparent from these definitions that a scattering-matrix specification of a network must necessarily include mention of the reference impedances under which the characterization was done.

The evaluation of the individual entries in the scattering matrix is easily undertaken through the expansion of Eq. (2-1) according to Eq. (2-4),

$$\begin{bmatrix} b_1(j\omega) \\ b_2(j\omega) \end{bmatrix} = \begin{bmatrix} s_{11}(j\omega) & s_{12}(j\omega) \\ s_{21}(j\omega) & s_{22}(j\omega) \end{bmatrix} \begin{bmatrix} a_1(j\omega) \\ a_2(j\omega) \end{bmatrix}, \quad (2-4)$$

whence it is evident that the following relations are obeyed:

$$s_{11}(j\omega) = \frac{b_1(j\omega)}{a_1(j\omega)} \Big|_{a_2(j\omega) = 0} \quad (2-5a)$$

$$s_{12}(j\omega) = \frac{b_1(j\omega)}{a_2(j\omega)} \Big|_{a_1(j\omega) = 0} \quad (2-5b)$$

$$s_{21}(j\omega) = \frac{b_2(j\omega)}{a_1(j\omega)} \Big|_{a_2(j\omega) = 0} \quad (2-5c)$$

$$s_{22}(j\omega) = \frac{b_2(j\omega)}{a_2(j\omega)} \Big|_{a_1(j\omega) = 0} \quad (2-5d)$$

Achievement of the condition $a_k(j\omega) = 0$ is readily attained by removing any excitation from the k -th port, whereupon $V_k(j\omega) = r_k I_k(j\omega)$ and so, according to Eq. (2-2), $a_k(j\omega)$ will vanish. Figure (2-1) gives a summary of the actual use of Eqs. (2-5a) to (2-5d).

In practice, scattering parameters are measured using the instrumentation arrangement shown in schematic form in Fig. (2-2). The dual-directional couplers of that figure are passive devices giving at their two output terminals a sample of the wave traversing them in the direction of the associated arrow.

The $\underline{a}(j\omega)$ and $\underline{b}(j\omega)$ vectors are defined as the wave vectors existing at the terminals of the network under consideration. In a practical situation, where the points of evaluation of the incident and reflected waves are located some distance from the network ports, the following relation exists between $\hat{\underline{a}}(j\omega)$, the measured quantity corresponding to $\underline{a}(j\omega)$, and $\underline{a}(j\omega)$ itself,

$$\begin{bmatrix} \hat{a}_1(j\omega) \\ \hat{a}_2(j\omega) \end{bmatrix} = \begin{bmatrix} K_1 e^{-j\omega\tau_1} & 0 \\ 0 & K_2 e^{-j\omega\tau_2} \end{bmatrix} \begin{bmatrix} a_1(j\omega) \\ a_2(j\omega) \end{bmatrix}. \quad (2-6)$$

A similar formula holds true in respect of $\underline{b}(j\omega)$ and its measured counterpart $\hat{\underline{b}}(j\omega)$,

$$\begin{bmatrix} \hat{b}_1(j\omega) \\ \hat{b}_2(j\omega) \end{bmatrix} = \begin{bmatrix} K_1 e^{-j\omega\tau_1} & 0 \\ 0 & K_2 e^{-j\omega\tau_2} \end{bmatrix} \begin{bmatrix} b_1(j\omega) \\ b_2(j\omega) \end{bmatrix}. \quad (2-7)$$

The constants K_1 and K_2 in these two equations account for the attenuation between the waves appearing at the network terminals, and the corresponding samples thereof provided at the directional couplers.

Fig. 2-2

The linear phase (delay) operator exponentials simulate the effects of propagation delay through the directional couplers and transmission lines, and for simplicity the transmission lines are assumed loss-less.

It is usual in practice to arrange that the channels associated with the two network ports are symmetric, whereupon $K_1 = K_2$ and $\tau_1 = \tau_2$ since then attenuation and delay in the paths are equal. For this commonly encountered case, Eqs. (2-5a) to (2-5d) can be rewritten as:

$$s_{11}(j\omega) = \left. \frac{\hat{b}_1(j\omega)}{\hat{a}_1(j\omega)} \right|_{\hat{a}_2(j\omega) = a_2(j\omega) = 0} \quad (2-8a)$$

$$s_{12}(j\omega) = \left. \frac{\hat{b}_1(j\omega)}{\hat{a}_2(j\omega)} \right|_{\hat{a}_1(j\omega) = a_1(j\omega) = 0} \quad (2-8b)$$

$$s_{21}(j\omega) = \left. \frac{\hat{b}_2(j\omega)}{\hat{a}_1(j\omega)} \right|_{\hat{a}_2(j\omega) = a_2(j\omega) = 0} \quad (2-8c)$$

$$s_{22}(j\omega) = \left. \frac{\hat{b}_2(j\omega)}{\hat{a}_2(j\omega)} \right|_{\hat{a}_1(j\omega) = a_1(j\omega) = 0} \quad (2-8d)$$

Equations (2-8a) to (2-8d) provide the necessary relations for direct experimental determination of the scattering matrix of an arbitrary network. They were obtained by substitution into Eq. (2-4) of Eqs. (2-6) and (2-7).

For the case of non-symmetrical channels, it is easy to show that,

$$s_{11}(j\omega) = \left. \frac{\hat{b}_1(j\omega)}{\hat{a}_1(j\omega)} \right|_{\hat{a}_2(j\omega) = a_2(j\omega) = 0} \quad (2-9a)$$

$$s_{12}(j\omega) = \frac{K_2}{K_1} e^{j\omega(\tau_1 - \tau_2)} \left. \frac{\hat{b}_1(j\omega)}{\hat{a}_2(j\omega)} \right|_{\hat{a}_1(j\omega) = a_1(j\omega) = 0} \quad (2-9b)$$

$$s_{21}(j\omega) = \frac{K_1}{K_2} e^{j\omega(\tau_2 - \tau_1)} \left. \frac{\hat{b}_2(j\omega)}{\hat{a}_1(j\omega)} \right|_{\hat{a}_2(j\omega) = a_2(j\omega) = 0} \quad (2-9c)$$

$$s_{22}(j\omega) = \left. \frac{\hat{b}_2(j\omega)}{\hat{a}_2(j\omega)} \right|_{\hat{a}_1(j\omega) = a_1(j\omega) = 0} \quad (2-9d)$$

In many applications of the scattering parameters to network analysis and synthesis it is necessary, by using standard techniques of analysis, to be able to compute the scattering matrix for a given network whose topology and element values are specified. Towards this end, it is useful to deduce formulae relating the scattering parameters to network port voltages and currents.

Presuming the desirability of employing nodal analysis to accomplish the task of analytic determination of the scattering matrix, it is clear that the preferred excitation and response bases are currents and node-pair potential differences, respectively.

Treating first the case of $s_{11}(j\omega)$ and with reference to Fig. (2-1a), substitution into Eq. (2-5a) of the appropriate scalar version of Eqs. (2-2) and (2-3) yields,

$$s_{11}(j\omega) = \frac{V_1(j\omega) - r_1 I_1(j\omega)}{V_1(j\omega) + r_1 I_1(j\omega)} \quad (2-10)$$

The variable $I_1(j\omega)$ can be eliminated from this expression to give a dependence upon the excitation current source $J_1(j\omega)$ and port voltage $V_1(j\omega_1)$ by noting that,

$$I_1(j\omega) = J_1(j\omega) - \frac{V_1(j\omega)}{r_1} \quad (2-11)$$

whence substitution of this into Eq. (2-10) gives the required formula, namely,

$$s_{11}(j\omega) = \left(\frac{2}{r_1} \left(\frac{V_1(j\omega)}{J_1(j\omega)} \right) - 1 \right) \quad (2-12)$$

In the case of $s_{21}(j\omega)$, a similar development commences with a substitution into Eq. (2-5b) from Eqs. (2-2) and (2-3) to give,

$$s_{21}(j\omega) = \frac{V_2(j\omega) - r_2 I_2(j\omega)}{V_1(j\omega) + r_1 I_1(j\omega)} \quad (2-13)$$

Since in addition to Eq. (2-11), the following relation is also valid,

$$I_2(j\omega) = \frac{V_2(j\omega)}{r_2}, \quad (2-14)$$

these can be inserted into Eq. (2-13) to yield,

$$s_{21}(j\omega) = \left(\frac{2}{r_1} \right) \left(\frac{V_2(j\omega)}{J_1(j\omega)} \right) \quad (2-15)$$

In similar fashion, the two remaining scattering parameters can be determined as follows:

$$s_{12}(j\omega) = \left(\frac{2}{r_1} \right) \left(\frac{V_1(j\omega)}{J_2(j\omega)} \right), \quad (2-16)$$

and,

$$s_{22}(j\omega) = \left(\frac{2}{r_2} \right) \left(\frac{V_2(j\omega)}{J_2(j\omega)} \right) - 1, \quad (2-17)$$

where reference is now to Fig. (2-1b).

Through the applications of these four expressions, the scattering matrix may readily be obtained using a nodal analysis.

The description of the scattering matrix given in this chapter has of necessity been brief and has been limited to the derivation of results useful in subsequent sections of the paper. For a more detailed exposition of the subject, the reader is referred to the references.

CHAPTER 3. THE THEORY OF AUTOMATED MODELLING USING THE SCATTERING MATRIX

The objective of the automatic modelling process is, in most general terms, the determination of element values in a pre-specified network (model) N such that the scattering parameters of this circuit agree optimally in some sense with a given set of scattering matrices taken at a sequence of l frequencies $\omega_1, \omega_2, \dots, \omega_1, \dots, \omega_l$ in the band wherein optimal agreement is sought. The problem can be stated mathematically as one of finding the minimum of an error function whose magnitude is a function of the discrepancy existing between a given set of scattering matrices $\underline{S}^G(j\omega_1), \underline{S}^G(j\omega_2), \dots, \underline{S}^G(j\omega_1), \dots, \underline{S}^G(j\omega_l)$, and those calculated from N , $\underline{S}^N(j\omega_1, \underline{x}), \underline{S}^N(j\omega_2, \underline{x}), \dots, \underline{S}^N(j\omega_1, \underline{x}), \dots, \underline{S}^N(j\omega_l, \underline{x})$. The functional dependence upon the vector $\underline{x} = (x_1, x_2, \dots, x_m)^T$ of variable elements in N of the $\underline{S}^N$ accounts for the fact that variation of the $\underline{S}^N$ can be accomplished through changing the values of prescribed elements in N .

Although alternative choices are certainly possible, the error function adopted in this instance is of the weighted sum-of-squares variety, and is expressible in the space of elements subject to variation as,

$$\begin{aligned} \epsilon(\underline{x}) = & \sum_i \sum_{p,q} W_{pq}^2 \left[\left(\operatorname{Re} \left\{ s_{pq}^N(j\omega_i, \underline{x}) \right\} - \operatorname{Re} \left\{ s_{pq}^G(j\omega_i) \right\} \right)^2 \right. \\ & \left. + \left(\operatorname{Im} \left\{ s_{pq}^N(j\omega_i, \underline{x}) \right\} - \operatorname{Im} \left\{ s_{pq}^G(j\omega_i) \right\} \right)^2 \right]. \quad (3-1) \end{aligned}$$

The index of summation i in Eq. (3-1) is taken over all frequencies at which $\underline{S}^G$ are given, the indices p and q being used to signify summation over all four entries in the 2×2 scattering matrices. A weight vector $\underline{W}$ is also included in the formulation of the error function in order to enable preference to be given in the fitting process to one or more entries in the scattering matrix. Note in passing that $\epsilon(\underline{x})$ of Eq. (3-1) is a non-negative scalar possessing the property of vanishing when the $\underline{S}^N$ and $\underline{S}^G$ agree for all ω_i .

It transpires that the most efficient schemes of functional minimization require both the function and its gradient. Differentiation of Eq. (3-1) yields for the gradient of $\epsilon(\underline{x})$ with respect to $\underline{x}$,

$$\begin{aligned} \frac{\partial \epsilon(\underline{x})}{\partial \underline{x}} = & \sum_i \sum_{p,q} 2^{-W_{pq}} \left[\left(\operatorname{Re} \left\{ \frac{\partial s_{pq}^N(j\omega_i, \underline{x})}{\partial \underline{x}} \right\} - \operatorname{Re} \left\{ s_{pq}^G(j\omega_i) \right\} \right) \right. \\ & \left. + \left(\operatorname{Im} \left\{ \frac{\partial s_{pq}^N(j\omega_i, \underline{x})}{\partial \underline{x}} \right\} - \operatorname{Im} \left\{ s_{pq}^G(j\omega_i) \right\} \right) \right]. \quad (3-2) \end{aligned}$$

The notation used here for the gradient is rather informal, and is expandible as, $\partial \epsilon(\underline{x}) / \partial \underline{x} = \left(\frac{\partial \epsilon(\underline{x})}{\partial x_1}, \frac{\partial \epsilon(\underline{x})}{\partial x_2}, \dots, \frac{\partial \epsilon(\underline{x})}{\partial x_m} \right)^T$. Apparently, the

determination of $\epsilon(\underline{x})$ and $\partial \epsilon(\underline{x}) / \partial \underline{x}$ reduces to the problem of calculating for N its scattering parameters at all ω_i , and of finding at these same frequencies the gradients of all four entries in the calculated scattering matrices $\underline{S}^N$.

The former task is easily accomplished via nodal analysis in the complex frequency domain and use of Eqs. (2-12) and (2-15) to (2-17); at the i -th frequency ω_i , nodal analysis is done upon network N augmented by the normalizing resistances for which the $\underline{S}^G$ are specified, and excited at its input port with a current source of intensity $J_1(j\omega_i)$ according to Fig. (2-1a). Solution is made for $V_1(j\omega_i)$ and $V_2(j\omega_i)$ of that figure, whereupon application of Eqs. (2-12) and (2-15) permits direct computation of $\underline{S}_{11}^N(j\omega_i, \underline{x})$ and $\underline{S}_{21}^N(j\omega_i, \underline{x})$. A similar development using the excitation current source in the position of Fig. (2-1b) in conjunction with Eqs. (2-16) and (2-17) enables $\underline{S}_{12}^N(j\omega_i, \underline{x})$ and $\underline{S}_{22}^N(j\omega_i, \underline{x})$ to be found.

The problem of calculating the gradients of the two-port scattering parameters can be recast into one of finding the gradients of the terminal voltages of network N to which appropriate excitation has been applied. Differentiation of Eqs. (2-12) and (2-15) then yields,

$$\frac{\partial \underline{S}_{11}^N(j\omega_i, \underline{x})}{\partial \underline{x}} = \left(\frac{2}{J_1(j\omega_i) r_1} \right) \frac{\partial V_1(j\omega_i, \underline{x})}{\partial \underline{x}}, \quad (3-3)$$

$$\frac{\partial \underline{S}_{21}^N(j\omega_i, \underline{x})}{\partial \underline{x}} = \left(\frac{2}{J_1(j\omega_i) r_1} \right) \frac{\partial V_2(j\omega_i, \underline{x})}{\partial \underline{x}}. \quad (3-4)$$

For these two equations, the exciting source for N is a current source of value $J(j\omega_i)$ connected at the input port of N as in Fig. (2-1a).

For Eqs. (3-5) and (3-6), the source is in the location of Fig. (2-1b), and the differentiation is done upon Eqs. (2-16) and (2-17).

$$\frac{\partial S_{12}^N(j\omega_1, \underline{x})}{\partial \underline{x}} = \frac{2}{J_2(j\omega_1) r_2} \frac{\partial V_1(j\omega_1, \underline{x})}{\partial \underline{x}} \quad (3-5)$$

$$\frac{\partial S_{22}^N(j\omega_1, \underline{x})}{\partial \underline{x}} = \frac{2}{J_2(j\omega_1) r_2} \frac{\partial V_2(j\omega_1, \underline{x})}{\partial \underline{x}} \quad (3-6)$$

The voltages $V_1(j\omega_1, \underline{x})$ and $V_2(j\omega_1, \underline{x})$ are dependent upon the variable elements $x_1, x_2, \dots, x_n$ in N , hence their functional dependence upon $\underline{x}$.

Calculation of the derivatives on the right-hand side of Eqs. (3-3) to (3-6) in the numerically precise form required by the functional minimization algorithm is greatly facilitated through application of the concept of the adjoint network. (5,6,7)

The adjoint $\hat{N}$ of N possesses a topology identical to that of N . All conductance, capacitance, and reciprocal inductance branches in N are associated in $\hat{N}$ with identical branches; voltage-controlled current sources in N are associated with similar elements in the adjoint, however the roles of controlled and controlling branches are reversed in $\hat{N}$.

These definitions are illustrated in Fig. (3-1), along with other useful terminology. Essential to the simplification of the subsequent exposition is the introduction of the augmented networks $\hat{N}^*$ and $\hat{N}^*$ connected with N and $\hat{N}$; the augmented network is simply the original network with the scattering normalizing conductances (reciprocal normalizing resistances) connected across its two ports. It is essential to note that the port voltages of $\hat{N}^*$ and N , and of $\hat{N}^*$ and $\hat{N}$ are identical.

Note $g_i = 1/r_i$

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Fig. 3-1

Given the augmented networks $\hat{N}$ and $\hat{N}^*$, the calculation of the voltage derivatives in Eqs. (3-3) to (3-6) becomes an exercise in the application of Tellegen's theorem.⁽⁸⁾ Using the general notation $I_B(j\omega_i, \underline{x})$, $V_B(j\omega_i, \underline{x})$ for branch currents and voltages in $\hat{N}$, and $\phi_B(j\omega_i, \underline{x})$, $\psi_B(j\omega_i, \underline{x})$ for their analogues in $\hat{N}^*$, Tellegen's theorem states that since $\hat{N}$ and $\hat{N}^*$ are topologically equivalent,

$$\left[\sum_B V_B(j\omega_i, \underline{x}) \phi_B(j\omega_i, \underline{x}) \right] - V_1(j\omega_i, \underline{x}) \hat{J}_1(j\omega_i) = 0, \quad (3-7)$$

and,
$$\left[\sum_B I_B(j\omega_i, \underline{x}) \psi_B(j\omega_i, \underline{x}) \right] - J_1(j\omega_i) \Psi_1(j\omega_i, \underline{x}) = 0. \quad (3-8)$$

Summation is done over all branches in $\hat{N}$ and $\hat{N}^*$, and both networks are assumed to be driven by constant current sources according to Fig. (3-2a).

Now, an infinitesimal perturbation in one of the variable elements of N leads to changes in the branch voltages and currents in $\hat{N}$; the perturbation is symbolically accomplished by altering the vector $\underline{x}$ by an infinitesimal amount $\delta \underline{x}$ in the case of $\hat{N}$ so that,

$$\left[\sum_B V_B(j\omega_i, \underline{x} + \delta \underline{x}) \phi_B(j\omega_i, \underline{x}) \right] - V_1(j\omega_i, \underline{x} + \delta \underline{x}) \hat{J}_1(j\omega_i) = 0, \quad (3-9)$$

and,
$$\left[\sum_B I_B(j\omega_i, \underline{x} + \delta \underline{x}) \psi_B(j\omega_i, \underline{x}) \right] - J_1(j\omega_i) \Psi_1(j\omega_i, \underline{x}) = 0. \quad (3-10)$$

Fig. 3-2

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Subtraction of Eq. (3-7) from Eq. (3-9), followed by subtraction from the resulting expression of the difference between Eqs. (3-8) and (3-10) yields,

$$\left[\sum_B \left\{ \phi_B(j\omega_1, \underline{x}) \left(V_B(j\omega_1, \underline{x} + \delta \underline{x}) - V_B(j\omega_1, \underline{x}) \right) - \psi_B(j\omega_1, \underline{x}) \left(I_B(j\omega_1, \underline{x} + \delta \underline{x}) - I_B(j\omega_1, \underline{x}) \right) \right\} \right] - \left(V_1(j\omega_1, \underline{x} + \delta \underline{x}) - V_1(j\omega_1, \underline{x}) \right) \hat{J}_1(j\omega_1) = 0. \quad (3-11)$$

Assume now that the perturbed element is a conductance in the K-th position of $\underline{x}$ so that in $\hat{N}^*$, $\underline{x} + \delta \underline{x}^G$ where, $\delta \underline{x}^G$ denotes the vector $(0, 0, \dots, \delta x_K^G, \dots, 0)^T$, and the superscript G signifies treatment of a conductance element. The branch relations for the element x_K^G are given by Eqs. (3-12) and (3-13) for the case of unperturbed and perturbed $\hat{N}^*$, respectively.

$$I_{x_K^G}(j\omega_1, \underline{x}) = x_K^G V_{x_K^G}(j\omega_1, \underline{x}) \quad (3-12)$$

$$I_{x_K^G}(j\omega_1, \underline{x} + \delta \underline{x}^G) = (x_K^G + \delta x_K^G) V_{x_K^G}(j\omega_1, \underline{x} + \delta \underline{x}^G) \quad (3-13)$$

In $\hat{N}^*$, the branch equation,

$$\phi_{x_K^G}(j\omega_1, \underline{x}) = x_K^G \psi_{x_K^G}(j\omega_1, \underline{x}) \quad (3-14)$$

will be satisfied. The subscript scheme on voltage and current

variables denotes quantities appropriate to the branch in $\hat{N}$ and $\hat{N}^*$ containing x_K^G . Substitution of Eqs. (3-12) to (3-14) into Eq. (3-11) leads directly to,

$$\begin{aligned} & - \psi_{x_K^G(j\omega_1, \underline{x})} V_{x_K^G(j\omega_1, \underline{x} + \delta \underline{x}^G)} \delta x_K^G \\ & = J_1(j\omega_1) V_1(j\omega_1, \underline{x} + \delta \underline{x}^G) - V_1(j\omega_1, \underline{x}) . \end{aligned} \quad (3-15)$$

By considering the branch relations for the other unperturbed elements in $\hat{N}$ and their correspondents in $\hat{N}^*$ it can readily be verified that all terms on the left-hand side of Eq. (3-11) vanish, leaving only the term due to the branches of $\hat{N}$ and $\hat{N}^*$ containing x_K^G . Taking the limit as $\delta x_K^G \rightarrow 0$ and $\delta \underline{x}^G \rightarrow 0$ yields upon slight rearrangement of Eq. (3-15),

$$\frac{\partial V_1(j\omega_1, \underline{x})}{\partial x_K^G} = \frac{\psi_{x_K^G(j\omega_1, \underline{x})} V_{x_K^G(j\omega_1, \underline{x})}}{\hat{J}_1(j\omega_1)} , \quad (3-16)$$

which enables direct computation of those elements of vector $\partial V_1(j\omega_1, \underline{x}) / \partial \underline{x}$ arising from conductances.

The gradient terms arising from voltage-controlled current sources are derived using the two-branch representation for that element shown in Fig. (3-1). The canonical form chosen for this element is one providing both a transconductance term gm and a linear phase (delay) operator τ .

Considering now that position k of the vector $\underline{x}$ contains a delay term x_k^τ associated with a voltage-controlled current source, and employing notation similar to that used previously in deriving the gradient

terms arising from conductances, the branch relations of the element in question can be written as,

$$I_{VDI_{x_K^T}}(j\omega_i, \underline{x}) = g_m e^{-j\omega_i x_K^T} V_{VCI_{x_K^T}}(j\omega_i, \underline{x}), \quad (3-17)$$

and

$$I_{VCI_{x_K^T}}(j\omega_i, \underline{x}) = 0 \quad (3-18)$$

in the unperturbed N . A perturbation in N of $\underline{x} + \delta \underline{x}^T$, with $\delta \underline{x}^T = (0, 0, \dots, \delta x_K^T, \dots, 0)^T$ causes $g_m e^{-j\omega_i x_K^T} + \delta(g_m e^{-j\omega_i x_K^T})$

and therefore yields,

$$I_{VDI_{x_K^T}}(j\omega_i, \underline{x} + \delta \underline{x}^T) = \left[g_m e^{-j\omega_i x_K^T} + \delta(g_m e^{-j\omega_i x_K^T}) \right] \times V_{VCI_{x_K^T}}(j\omega_i, \underline{x} + \delta \underline{x}^T), \quad (3-19)$$

and,

$$I_{VCI_{x_K^T}}(j\omega_i, \underline{x} + \delta \underline{x}^T) = 0. \quad (3-20)$$

Since in the adjoint $\hat{N}$, no perturbation is done,

$$\phi_{VCI, x_K^T}(j\omega_i, \underline{x}) = g_m e^{-j\omega_i x_K^T} \psi_{VDI, x_K^T}(j\omega_i, \underline{x}), \quad (3-21)$$

and,

$$\phi_{VDI, x_K^T}(j\omega_i, \underline{x}) = 0. \quad (3-22)$$

The foregoing six equations, Eqs. (3-17) to (3-22), can be inserted directly into Eq. (3-11) with the consequence that,

$$\begin{aligned} & -\psi_{VDI, x_K^T}(j\omega_i, \underline{x}) V_{VCI, x_K^T}(j\omega_i, \underline{x} + \delta \underline{x}^T) g_m e^{-j\omega_i x_K^T} \\ & = \hat{J}_1(j\omega_i) [V_1(j\omega_i, \underline{x} + \delta \underline{x}^T) - V_1(j\omega_i, \underline{x})]. \end{aligned} \quad (3-23)$$

By taking the limit as $\delta x_K^T \rightarrow 0$ and $\delta \underline{x}^T \rightarrow 0$, using the identity, $\delta(g_m e^{-j\omega_i x_K^T}) = \delta x_K^T \delta(g_m e^{-j\omega_i x_K^T}) / \delta x_K^T$, and expanding the complex exponential according to Euler's theorem, it can readily be shown that,

$$\begin{aligned} \frac{\partial V_1(j\omega_i, \underline{x})}{\partial x_K^T} & = g_m \omega_i (\sin \omega_i x_K^T + j \cos \omega_i x_K^T) \psi_{VDI, x_K^T}(j\omega_i, \underline{x}) \\ & \times V_{VCI, x_K^T}(j\omega_i, \underline{x}). \end{aligned} \quad (3-24)$$

It is now possible to deduce the contributions to the gradient of the scattering parameters arising from each type of element in N . Substituting Eq. (3-16) into Eq. (3-3) yields, for conductances,

$$\frac{\partial s_{11}^N(j\omega_i, \underline{x})}{\partial x_K^G} = \left(\frac{2}{r_1 J_1(j\omega_i) \hat{J}_1(j\omega_i)} \right) \Psi_{x_K^G(j\omega_i, \underline{x})} V_{x_K^G(j\omega_i, \underline{x})}. \quad (3-25)$$

For the remaining types of passive elements, it is easy also to show that, for capacitors,

$$\frac{\partial s_{11}^N(j\omega_i, \underline{x})}{\partial x_K^C} = \left(\frac{j2\omega_i}{r_1 J_1(j\omega_i) \hat{J}_1(j\omega_i)} \right) \Psi_{x_K^C(j\omega_i, \underline{x})} V_{x_K^C(j\omega_i, \underline{x})}, \quad (3-26)$$

and for reciprocal inductances,

$$\frac{\partial s_{11}^N(j\omega_i, \underline{x})}{\partial x_K^I} = \left(\frac{2}{j\omega_i r_1 J_1(j\omega_i) \hat{J}_1(j\omega_i)} \right) \Psi_{x_K^I(j\omega_i, \underline{x})} V_{x_K^I(j\omega_i, \underline{x})}. \quad (3-27)$$

For the two parameters characterizing the voltage-controlled current source, the gradient terms taken in respect of the delay τ are given by substitution of Eq. (3-24) into Eq. (3-3) to yield,

$$\begin{aligned} \frac{\partial s_{11}^N(j\omega_i, \underline{x})}{\partial x_K^\tau} &= \left(\frac{2 \sin \omega_i}{r_1 J_1(j\omega_i) \hat{J}_1(j\omega_i)} \right) \left(\sin \omega_i x_K^\tau + j \cos \omega_i x_K^\tau \right) \\ &\quad \times \Psi_{VDI_{x_K^\tau}(j\omega_i, \underline{x})} V_{VCI_{x_K^\tau}(j\omega_i, \underline{x})}, \end{aligned} \quad (3-28)$$

and for the gradient with respect to the transconductance g_m , it can be shown that,

$$\frac{\partial s_{11}^N(j\omega_1, x)}{\partial x_K g_m} = \frac{-2}{r_1 J_1(j\omega_1) \hat{J}_1(j\omega_1)} (\cos \omega_1 \tau - j \sin \omega_1 \tau) \times \psi_{VDI, x_K g_m}(j\omega_1, x) \psi_{VCI, x_K g_m}(j\omega_1, x). \quad (3-29)$$

The formulae given in Eqs. (3-25) to (3-29) are applicable in their stated form for calculation of the gradients of the additional three entries in the scattering matrix of N ; only the subscripts on the normalizing resistor and scattering variables need be changed, and the excitation modes in $\hat{N}$ and $\hat{N}^*$ for the three remaining cases are illustrated in Fig. (3-2b) to (3-2d).

All gradient-based multidimensional minimization algorithms endeavour to reduce the problem of finding the local minimum of a function to one of successive linear searches. The means employed in determining the direction of search, as well as the actual realization of the search, account in large measure for the proliferation of computation techniques available today to solve the problem of functional minimization.

The method adopted in this work is due to R. Fletcher,⁽⁹⁾ and represents a significant improvement over earlier versions of the variable metric algorithm.^(10,11) Although in this new algorithm, a linear search is done at every iteration, it is not necessary as in

the Fletcher-Powell procedure that it locate a minimum along the line of search; rather, only at least an arbitrary prespecified decrease in the error function undergoing minimization is sought.

A defining property of all variable-metric algorithms is the retention of an approximation $\underline{H}^{-1}(\underline{x})$ to the inverse Hessian matrix of second derivatives of the error function; the approximation is improved as the minimum is approached, and converges at the minimum to the true inverse Hessian $\underline{H}^{-1}(\underline{x})$. To commence the theoretical exposition of the method, let the error function $\epsilon(\underline{x})$ of Eq. (3-1) be expanded into a multivariate Taylor's series about a point $\underline{x}_p$, and let only three terms of the series be retained so that, (12)

$$\epsilon(\underline{x}) = \epsilon(\underline{x}_p) + (\underline{x} - \underline{x}_p)^T \frac{\partial \epsilon(\underline{x}_p)}{\partial \underline{x}} + \frac{1}{2} (\underline{x} - \underline{x}_p)^T \underline{H}(\underline{x}_p) (\underline{x} - \underline{x}_p). \quad (3-30)$$

In order that Eq. (3-30) be valid, it is necessary that $\epsilon(\underline{x})$ be twice differentiable and that its Hessian matrix $\underline{H}(\underline{x})$ be non-singular for all $\underline{x}$ in a sufficiently large neighborhood of $\underline{x}_p$. Each entry h_{ij} in $\underline{H}(\underline{x})$ is defined according to convention as $h_{ij} = \partial^2 \epsilon(\underline{x}) / \partial x_i \partial x_j$. Setting the gradient of the right-hand side of Eq. (3-30) equal to zero at the minimum $\underline{x}_0$ of $\epsilon(\underline{x})$, since there the gradient vanishes, enables definition of the Newton-Raphson formula, (13) Eq. (3-31).

$$\underline{x}_0 = \underline{x} + \underline{H}^{-1}(\underline{x}) \frac{\partial \epsilon(\underline{x})}{\partial \underline{x}} \quad (3-31)$$

STUDY OF NUCLEAR STRUCTURE FROM THE ANALYSIS OF γ RAYS
FOLLOWING μ^- CAPTURE IN Al, Si, Ca, AND Co.

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ABSTRACT

An analysis of γ rays emitted following μ^- capture in Al, Si, Ca, and Co has been carried out with the intention of studying the structure of the nuclear states excited in the muon capture reaction. The results of this experiment are correlated and compared with the results of single nucleon pick-up reactions, photoexcitation and electroexcitation studies, and thermal neutron capture experiments as well as with theoretical nuclear structure calculations. These correlations reveal the similarities of the various excitation techniques and show how muon capture studies compliment and extend the information one can obtain about nuclear structure.

The close correlation of the states populated in ^{39}K following muon capture in ^{40}Ca with those populated in the single nucleon pickup reaction $^{40}\text{Ca}(t, \alpha)^{39}\text{K}$ indicates that single hole states are populated preferentially in the residual ^{39}K nucleus when a single neutron is emitted following the muon capture. An identification of the states formed in ^{28}Al following muon capture in ^{28}Si with levels thought to be configuration mixed particle-hole states is made.

The results of the photoexcitation reaction $^{40}\text{Ca}(\gamma, p\gamma')^{39}\text{K}$ are compared with the $^{40}\text{Ca}(\mu^-, \nu n)^{39}\text{K}$ results and indicate the presence of a giant resonance mechanism in the muon capture process. The relative frequency of populating $1d_{5/2}$ proton hole states to that of populating

where $\underline{\xi}$ is taken as a scalar multiple β of a direction of linear search chosen in analogy with Eq. (3-32) so that,

$$\underline{\xi} = -\beta \hat{H}^{-1}(\underline{x}_{K-1}) \frac{\partial \varepsilon(\underline{x}_{K-1})}{\partial \underline{x}} \quad (3-33)$$

In order to force a sufficient decrease in $\varepsilon(\underline{x})$ on an iteration, it suffices to require that,

$$0 < \left(\varepsilon(\underline{x}_{K-1}) - \varepsilon(\underline{x}_K) \right) / \underline{\xi}^T \frac{\partial \varepsilon(\underline{x}_K)}{\partial \underline{x}} \ll 1. \quad (3-34)$$

Inequality (3-34) can ultimately be satisfied by trying values of $\beta = 1, w, w^2, \dots$ with $0 < w < 1$, however, the empirical device was adopted of fitting a cubic interpolating polynomial⁽¹⁴⁾ to the error function if $\beta = 1$ does not satisfy the criterion of Eq. (3-34). By re-evaluating $\varepsilon(\underline{x})$ and its gradient at the minimum of the interpolating polynomial, an estimate of the minimum of $\varepsilon(\underline{x})$ and its gradient there along the direction of search, Eq. (3-33), may be obtained; this strategy injects an element of independence into the successive $\underline{\xi}$, so improving the approximation $\hat{H}^{-1}(\underline{x}_K)$ to $H^{-1}(\underline{x}_K)$. Experience has shown that usually the choice $\beta = 1$ will be taken, hence the efficiency of the algorithm in requiring an average of very nearly one function and gradient evaluation per iteration.

The formula used in updating the matrix $\hat{H}^{-1}(x)$ are two, namely,

$$\hat{H}_0^{-1}(x_K) = \hat{H}^{-1}(x_{K-1}) + \frac{\xi \xi^T}{\xi^T \xi} - \frac{\hat{H}^{-1}(x_{K-1}) \underline{\gamma} \underline{\gamma}^T \hat{H}^{-1}(x_{K-1})}{\underline{\gamma}^T \hat{H}^{-1}(x_{K-1}) \underline{\gamma}} \quad (3-35)$$

and,

$$\hat{H}_1^{-1}(x_K) = \left(\underline{I} - \frac{\xi \underline{\gamma}^T}{\xi^T \underline{\gamma}} \right) \hat{H}^{-1}(x_{K-1}) \left(\underline{I} - \frac{\underline{\gamma} \xi^T}{\xi^T \underline{\gamma}} \right) + \frac{\xi \xi^T}{\xi^T \underline{\gamma}} \quad (3-36)$$

Equation (3-35) is that deduced by Fletcher and Powell;⁽¹⁵⁾ Eq. (3-36) is derived from it by observing that Eq. (3-35) obeys the equality $\hat{H}^{-1}(x_K) \underline{\gamma} = \xi$. Then, solving Eq. (3-35) for an updating relation for $\hat{H}(x_K)$ which will hence force $\hat{H}(x_K) \xi = \underline{\gamma}$, and interchanging $\underline{\gamma}$ and ξ in the resulting expression leads directly to Eq. (3-36).

Both Eqs. (3-35) and (3-36) are the extreme members of a general class of updating formula generated by the scalar parameter ϕ according to the convex relation,

$$\hat{H}_\phi^{-1}(x_K) = (1-\phi) \hat{H}_0^{-1}(x_K) + \phi \hat{H}_1^{-1}(x_K); \quad (3-37)$$

Fletcher proves in his description of the updating method⁽¹⁶⁾ that members of the class given by Eq. (3-37) possess the property that through their use, convergence in a certain sense of $\hat{H}^{-1}$ to $\underline{H}^{-1}$ is assured through tendency of the eigenvalues of $\hat{H}^{-1}$ to those of $\underline{H}^{-1}$.

In order to deduce a criterion for the choice on a particular iteration of using Eq. (3-35) or (3-36) to accomplish the updating of $\hat{H}^{-1}$, it is useful to examine the relation, (17)

$$\hat{H}_\phi^{-1}(x_K) = \hat{H}^{-1}(x_{K-1}) + \frac{(\underline{\xi} - \hat{H}^{-1}(x_{K-1})\underline{Y})(\underline{\xi} - \hat{H}^{-1}(x_{K-1})\underline{Y})^T}{\underline{Y}^T(\underline{\xi} - \hat{H}^{-1}(x_{K-1})\underline{Y})}. \quad (3-38)$$

This formula is in fact a member of the class governed by Eq. (3-37), but with a parameter $\hat{\phi}$ corresponding to ϕ being given by,

$$\hat{\phi} = \frac{\underline{\xi}^T \underline{Y}}{(\underline{\xi}^T \underline{Y} - \underline{Y}^T \hat{H}^{-1}(x_{K-1})\underline{Y})}. \quad (3-39)$$

For $\underline{\xi}^T \underline{Y}$ strictly positive, a property guaranteed by Eq. (3-34), and with $\hat{H}^{-1}$ positive definite, it is demonstrable that $\hat{\phi}$ cannot lie in the closed interval [0,1]. Now, if $\underline{\xi}^T \underline{Y} > \underline{Y}^T \hat{H}^{-1}(x_{K-1})\underline{Y}$, then $\hat{\phi} > 1$ hence use of Eq. (3-36) is proper, whereas if $\underline{\xi}^T \underline{Y} < \underline{Y}^T \hat{H}^{-1}(x_{K-1})\underline{Y}$, then $\hat{\phi} < 0$ and so Eq. (3-35) should be used for updating $\hat{H}^{-1}$.

A means is needed of sensing when the minimum of $\varepsilon(x)$ has been determined, hence use is made of the fact that there the Euclidean norm of the gradient vector vanishes so that at the minimum x_0 , $\| \partial(x_0) / \partial x \| = 0$. In practice, convergence is assumed when $\| \partial(x_a) / \partial x \| \leq \epsilon$, where ϵ is a small number, and where x_a denotes the approximation to the minimum x_0 of $\varepsilon(x)$.

As is the case with most multidimensional minimization algorithms, convergence occurs most rapidly when the surface being traversed is free of elongated contours. In an effort to achieve this condition, the gradient vector is scaled to unity at commencement of minimization. By so doing, it was found that as much as a fivefold decrease in number of iterations required for convergence was attained in comparison to the unscaled case. Let $\underline{Z}$ be the square matrix having scale factors on the diagonal and zeroes elsewhere, and let $\underline{x}'$ represent the vector of variable parameters so scaled as to cause the initial gradient vector to be unity. Then, writing,

$$\underline{x}' = \underline{Z} \underline{x}, \quad (3-40)$$

differentiation of both sides of this equation followed by application of the chain-rule yields,

$$\frac{\partial \varepsilon(\underline{x}')}{\partial \underline{x}'} = \underline{Z}^{-1} \frac{\partial \varepsilon(\underline{x})}{\partial \underline{x}}. \quad (3-41)$$

The matrix $\underline{Z}$ is determined so that the initial gradient vector, $\partial \varepsilon(\underline{x}') / \partial \underline{x}'$ becomes unity, whence,

$$\underline{Z} = \underline{1} \frac{\partial \varepsilon(\underline{x})}{\partial \underline{x}}. \quad (3-42)$$

The theory described in the preceding portions of this chapter has been translated into Fortran IV code, and forms the basis of a digital computer program especially designed for automated modelling.

A full listing of the code forms the subject of Appendix B, and contained in Appendix A are instructions for use of the program.

In order to maximize the efficiency of the error-function and gradient computation sections of the program, use has been made of sparse-matrix techniques in the solution of nodal equations. The non-zero structure of the nodal admittance matrix is recorded by a pointer system, and the nodal equations are then reordered by an algorithm⁽¹⁸⁾ which seeks to minimize the number of additional non-zero entries introduced into the admittance matrix by the process of decomposing it into the product of a lower triangular matrix having ones on the diagonal and zeroes above, and an upper triangular matrix having zeroes above the diagonal.^(19,20) From these two matrices, a sequence of forward substitution followed by back substitution is employed to solve for nodal potentials. Further computational efficiency is realized by making use of the fact that the nodal admittance matrix of the adjoint network is the transpose of that for the original network, hence only one decomposition of the admittance matrix is required per frequency point.⁽²¹⁾

Storage space is economized in the minimization routine by storing only the upper portion of the inverse hessian matrix; this is possible owing to the numerical symmetry of the matrix.

Figure (3-3) gives a simple flow-chart of program operation. Input data cards are read in free-format by a routine⁽²²⁾ called by the main program, which then passes control to the sparse-matrix and

pointer set-up subprograms, whereupon the minimization routine is invoked and repeatedly calls a subroutine to evaluate the error function and its gradient. Final element values are printed upon conclusion of the minimization, and execution is terminated. Error traps are incorporated to catch elementary faults in input data, thereby avoiding possible wastes of program time arising from faulty or improperly specified parameters.

XBL 724 - 2812

Fig. 3-3

CHAPTER 4. MODELLING OF BIPOLAR JUNCTION TRANSISTORS IN THE SMALL-SIGNAL REGIME

The use of modelling techniques in the design of circuits employing bipolar junction transistors is dictated by the necessity of obtaining an analytically tractable representation of a device characterized by the solutions to rather complex partial differential equations. The actual choice of model configuration and degree of complexity is a compromise between several factors. In particular, the complexity of the model is heavily dependent upon the accuracy expected of it, whereas its configuration is governed by the network structure required to simulate the physical phenomena responsible for transistor operation. These two considerations are, of course, interdependent, so that the optimal equivalent circuit would be one so contrived as to yield maximal accuracy and yet require a minimum number of elements in its realization.

It is convenient in devising a model for the bipolar junction transistor or, for that matter, for any physically realized device, to partition the model into two components.

The primary or intrinsic portion of the model accounts for the physical processes directly responsible for device operation. Additional to this are those model elements necessary to describe spurious parasitic effects external to the active region of the device.

The model adopted to characterize the bipolar junction transistor for the purposes of the work described in this paper is the 'hybrid- π ' configuration to which a delay operator $e^{-j\omega\tau}$ has been added to the

voltage-controlled current source in order to account for the effects of phase-shift through the device caused by poles non-dominant relative to that causing the initial 20 dB/decade fall in forward current transfer ratio. Since the effective loads imposed upon the device by the type of circuits treated here are of the order of 100 Ω , and whereas the device output resistance is commonly of the order of thousands of ohms, no account has been taken of output resistance arising from base-width modulation effects. The important extrinsic elements of the model are the two inductances in the 'base' and 'emitter' leads of the hybrid-pi circuit; these reactances are included to simulate the inductive effects of the two bonding leads necessary to make connection between the transistor chip being characterized and the transmission lines of the test fixture. Figure (4-1) shows the complete model.

For the hybrid-pi model, certain of the elements can be estimated on the basis of commonly available information, as follows: (23)

$$g_m = q|I_C|/kT$$

$$r_\pi = \beta_F(0)/g_m$$

$$C_\pi = \frac{g_m}{2\pi f_T} - C_\mu$$

In the above equations, I_C represents the quiescent device collector current, $\beta_F(0)$ the static forward current transfer ratio, and f_T the beta transition frequency. The physical constants q , k , and T denote, respectively the electronic charge, Boltzmann's constant, and Kelvin temperature. The remaining elements, C_π , τ , and r_b are most readily estimated either by direct measurements upon the device or by physical reasoning based upon an intimate knowledge of the device construction, but in this work an initial guess was made of these values on the basis of previous experience with high-frequency devices.

HP 704-2700

Fig. 4-1

The transistor chip type HPA 35820-A, ⁽²⁴⁾ manufactured by the Hewlett-Packard Company Microwave Division, was chosen for the amplifier described here for the reasons to be outlined in Chapter 5, and the operating point was selected to be $I_C = 10$ mA and $V_{CE} \approx 5$ V. On this basis, estimates can be calculated for g_m , r_π , and C_π using the knowledge that $\beta_F(0) \approx 35$ and $f_T \approx 4$ GHz. Therefore, $g_m \approx 0.38 \Omega^{-1}$, $r_\pi \approx 100 \Omega$, and $C_\pi \approx 12$ pF. Guesses were taken of $C_\mu \approx 0.6$ pF, $\tau \approx 50$ psec, and $r_b' \approx 15 \Omega$. The modelling program refined these values to $g_m = 0.3458 \Omega$, $r_\pi = 96.45 \Omega$, $C_\pi = 14.87$ pF, $C_\mu = 0.5253$ pF, $\tau = 20.33$ psec, and $r_b' = 9.46 \Omega$.

A complete set of scattering data were taken on an actual 35820-A chip in the common-emitter configuration using a Hewlett-Packard Type 8745A scattering parameter test set, Type 8405A Vector voltmeter, and Type 11400B transistor test fixture. ⁽²⁵⁾ This equipment provides a convenient realization of the basic measurement arrangement of Fig. (2-2). Data were taken at an operating point of 10 mA and for V_{CE} of +3, +5, and +8 V at frequencies 50, 75, 100, 200, 400, 600, and 800 MHz; the data taken for $V_{CE} = +5$ V are plotted in polar form in Figs. (4-2) through (4-5).

In order to single out the two parasitic inductances of Fig. (4-1) from further consideration during the modelling process, a set of scattering matrices were measured at frequencies 200, 400, 600, 800, and 1000 MHz for the chip-to-test-fixture bonding leads. These data were provided to the automated modelling program together with estimates of 1.5 nH for each inductance, a topological description of the 'L' of inductances resulting after the transistor chip had been short-circuited, and a set of weights set uniformly to unity. The refined values returned by the program were $L_E = 1.345$ nH and $L_B = 1.763$ nH.

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Fig. 4-2

XSL724-2777

Fig. 4-3

XBL724-2776

Fig. 4-4

XBL724-2781

Fig. 4-5

The actual modelling of the chip was accomplished by entering the topology of the complete circuit of Fig. (4-1) into the modelling program. Also included were the estimates already given for the values of the elements in the hybrid-pi model and the values calculated previously by the program for L_E and L_B . The optimization was done simultaneously on the parameters r_b' , r_π , C_π , C_μ , g_m , and τ . Since s_{11} and s_{21} are most important in untuned wideband amplifier work, weight factors of 15, 10, 1, and 4 were chosen for s_{11} , s_{12} , s_{21} , and s_{22} , respectively. The parameter values in the hybrid-pi model resulting from use of the program are plotted in Figs. (4-6) through (4-11) for the device operating point $I_C = 10$ mA and $V_{CC} = 3, 5, \text{ and } 8$ V.

Two tests were done on the model determined in this way for the 35820-A transistor chip in an effort to check its validity. The first was to see if certain trends, predictable on the basis of qualitative physical reasoning, could be discerned in the model parameters. Of especial interest in this respect are the plots of r_b' and C_μ versus V_{CE} given in Figs. (4-6) and (4-9). Evident in the latter plot is the decrease of C_μ with increasing collector-base voltage, an effect predicted by the third-power law relating graded junction depletion-layer capacitance to junction reverse bias. Apparent also in the plot of r_b' versus V_{CE} is the rise of r_b' with increasing V_{CE} due to the fact that base-width narrows with increasing $|V_{CE}|$. Base-width shrinkage causes the rise in r_b' owing to the consequent lengthening of the path from the base contact pad to the active region of the base.

XBL724-2782

Fig. 4-6

XBL724-2783

Fig. 4-7

XBL724-2784

Fig. 4-8

XBL724-2785

Fig. 4-9

XBL724-2786

Fig. 4-10

Fig. 4-11

XBL724-2775

The second, and by far the more dramatic, verification ~~made~~ of the efficacy of the model, was to work backwards and calculate the scattering parameters of the equivalent circuit of Fig. (4-1) and compare these with those measured on the actual device. This was done for all three values of V_{CE} used in biasing the transistor, and the results in the three cases were roughly equivalent; those for the case of $V_{CE} = 5$ V are plotted in Figs. (4-2) through (4-5) along with the experimentally determined data, and it will be seen that the fit is excellent. The effect of the predominant weighting accorded to s_{11} and s_{21} is apparent in the virtually perfect fit of these two parameters at the expense of slight inaccuracies in the fits of s_{12} and s_{22} .

It seems likely that with improvements in experimental procedure the fit of the model could be improved, since under measurement techniques more refined than those used here, the effects of spurious reactances could be greatly reduced. In any event, all the benefits hoped for in automated modelling have been realized, and it should be obvious that the techniques used here can be readily extended to the characterization of other linear elements on the basis of scattering data.

CHAPTER 5. DESIGN OF A FAST-RISE SERIES-SHUNT PAIR

The series-shunt feedback pair to be dealt with in this chapter has been designed with a view towards satisfying the risetime (≤ 500 ps) and gain (20 dB) requirements set down in Chapter 1. Additionally, it is desirable that the amplifier possess the ability to operate within the context of a 50Ω system, and to this end a receiving-end termination of appropriate value has been incorporated into the design. The circuit of the amplifier is the subject of Fig. 5-1.

Hewlett-Packard Type HPA 35820-A NPN transistors were chosen for this design because they possess a unique combination of several highly desirable characteristics, namely, a large gain-bandwidth product f_T of 4 GHz, the capability of handling substantial collector current of up to 45 mA, and an extremely low base spreading resistance of approximately 10Ω . Another transistor, the Motorola MM-4049, was investigated as a possible candidate for a PNP complement to the HPA 35820-A, but it was found to suffer the severe disadvantage of having a base spreading resistance in excess of 80Ω . The gain-bandwidth product is optimum in the HPA 35820-A for collector currents of 10-20 mA, hence the biasing arrangement employed in the amplifier is designed to insure standing currents in both transistors of 10 mA. In the case of the output transistor, since the amplifier works into 50Ω cable, a maximum permissible output transition from 0 V to -0.5 V requires a 10 mA increase in collector current, and it is apparent that provided this limit on output swing is not exceeded, the second transistor operates in a

state guaranteeing maximum f_T . Small-signal fluctuations in collector current of the first transistor are considerably less than those in the output stage, so that the first stage is also optimally biased with a 10 mA collector current. A quiescent value of about 5 V was arbitrarily set for the V_{CE} of both transistors in order to effect a compromise between power dissipation on the one hand, and the desirability of minimizing the V_{CB} -dependent collector-base depletion layer capacitance, on the other.

Under these operating conditions, the small-signal model of the transistors is that found by the method of Chapter 4, and use of this equivalent circuit in the appropriate locations in Fig. (5-1) enables the entire amplifier to be characterized for small-signal operation as shown in Fig. (5-2). Analysis of the resulting amplifier model is best undertaken with the aid of a digital computer; use thereof enables many of the approximations essential to manual analysis to be dispensed with, giving a consequent increase of accuracy. Since the ultimate criteria of amplifier performance are to be assessed in the time-domain, the pole-zero approach was adopted for the analysis owing to the intimate relation existing between complex-plane singularities and transient behaviour. Initially, the pole-zero computation program, "Frank"⁽²⁶⁾ was used for these calculations, and no account was taken of parasitic elements or of the excess-phase of transistors. Therefore, these elements are not shown in Fig. (5-2), but they will be re-introduced later in the design process, so that the final circuit simulations are an accurate reflection of true circuit performance.

KDL 724-2804

Fig. 5-1

XBL 724-2802

Fig. 5-2

The resistor R_s of Fig. (5-2) replaces the transmission-line characteristic impedance and the line terminating resistor of Fig. (5-1) in accordance with the relation,

$$R_s = R_o R_T / (R_o + R_T). \quad (5-1)$$

Capacitor C_{E1} is incorporated into the circuit for reasons of frequency-response compensation, and a discussion will be given later regarding the determination of an appropriate value for it.

The series-shunt feedback configuration may be regarded as an interconnection of two two-port networks; the partition of the overall circuit into a forward gain path $a_v(s)$ and a feedback network $f(s)$ is shown symbolically in the Fig. (5-2). It should be apparent that the hybrid-basis two-port matrices are the appropriate ones to use in this situation. Identification of $a(s)$ and $f(s)$ is therefore easily made, and the circuits pertinent to the computation of these transfer functions are given as Fig. (5-3a) and (5-3b), respectively.

Using the pole-zero analysis program "Frank", the open-loop gain transfer function $a_v(s)$ of the amplifier was found to be,

$$a_v(s) = \frac{V_o(s)}{E_s(s)} = \frac{\prod_{k=1}^4 (1 - s/z_{ak})}{\prod_{i=1}^6 (1 - s/p_{ai})} \cdot a_v(0) \quad (5-2)$$

(a) Circuit for open-loop gain calculations

(b) Circuit for feedback network transfer function

Fig. 5-3

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where,

$$\begin{aligned}
 P_{a1} &= -43 \times 10^7, & Z_{a1} &= -82.2 \times 10^8, \\
 P_{a2} &= -73.76 \times 10^8 - j 62.2 \times 10^8, & Z_{a2} &= -85.88 \times 10^8, \\
 P_{a3} &= -73.76 \times 10^8 + j 62.2 \times 10^8, & Z_{a3} &= 22.0 \times 10^{10}, \\
 P_{a4} &= -80.9 \times 10^8, & Z_{a4} &= 65.83 \times 10^{10}, \\
 P_{a5} &= -73.95 \times 10^9, & \text{and,} & \\
 P_{a6} &= -25.24 \times 10^{10}, & a_v(0) &= 67.1.
 \end{aligned}$$

For practical purposes, since the response of the amplifier is of interest only for frequencies below about two gigahertz, or 12.56×10^9 rad/sec, a simplification of Eq. (5-2) can be made on the basis of retaining only those singularities dominant in their influence upon $a_v(s)$ in that band. Therefore, to good approximation, Eq. (5-2) can be recast as,

$$a_v(s) = \frac{\prod_{k=1}^2 (1 - s/Z_{ak})}{\prod_{i=1}^4 (1 - s/P_{ai})} \cdot a_v(0). \quad (5-3)$$

The poles and zeroes in this equation are identical to their correspondants in Eq. (5-2).

The transfer function of the feedback network is easily calculable analytically through consideration of the circuit of Fig. (5-3b), whence it is seen to be a single-pole function of form,

$$f(s) = \frac{V_f(s)}{V_o(s)} = \frac{f(0)}{1 - s/P_f}, \quad (5-4)$$

where $f(0) = -R_{EI}/(R_{EI} + R_F)$ and $P_f = -(R_{EI} + R_F)/(C_{EI} R_{EI} R_F)$. For the values of R_{EI} , R_F and C_{EI} used here, $P_f = 82.6 \times 10^8$. The standard feedback formula, namely

$$A_V(s) = \frac{a_V(s)}{1 - a_V(s) f(s)}, \quad (5-5)$$

can now be applied to relate closed-loop gain $A_V(s)$ to the amplifier and feedback network transfer functions.

Proceeding first in the low-frequency case in the limit as $s \rightarrow 0$, substitution into Eq. (5-5) of the low-frequency analogues of Eqs. (5-3) and (5-4) and solution of the resulting formula for R_F enables Eq. (5-6) to be deduced.

$$R_F = R_{EI} \cdot \frac{a_V(0) - A_V(0) - a_V(0) A_V(0)}{A_V(0) - a_V(0)} \quad (5-6)$$

Through the intermediary of this relation, a value of R_F can be calculated appropriate to give the 20 dB gain figure $A_V(0) = 10$. Note that this calculation must be done iteratively, for in fact $a_V(0)$ is dependent upon the value of R_F as is evident from consideration of Fig. (5-3a). The iterative cycle commenced with an estimate for R_F of 150 Ω , whereupon convergence of the process led to a value for R_F of 170 Ω .

Analytic determination of a suitable value for C_{EI} would commence with multiplication of Eq. (5-5) by the Laplace transform of the Heaviside unit step function, followed by expansion of the resultant product

into partial fractions and the application of the Laplace inversion integral to effect a transformation of the response into the time-domain. From the expression obtained in this way, formulae can be deduced relating C_{EI} to rise-time and overshoot. The ultimate objective is minimization of rise-time to a degree commensurate with the restriction that overshoot not exceed ten per cent.

The algebra involved in these computations is exceptionally lengthy and involves in several instances the solution of complicated transcendental equations, a task most efficiently done iteratively using a computer. For these reasons, it was decided to use the computer from the start and to adopt the rather empirical approach of variation of C_{EI} followed by pole-zero analysis of the closed-loop model of Fig. (5-2).

Direct substitution into Eq. (5-5) of Eqs. (5-3) and (5-4) and algebraic rearrangement yields,

$$A_V(s) = \frac{\frac{a_V(0) \prod_{k=1}^2 (1 - s/z_{a_k})}{\prod_{i=1}^4 (1 - s/p_{a_i})}}{1 - \frac{a_V(0) \prod_{k=1}^2 (1 - s/z_{a_k})}{\prod_{i=1}^4 (1 - s/p_{a_i})} \cdot \frac{f(0)}{(1 - s/p_f)}} \quad (5-7)$$

$$= \frac{a_V(0) \left(1 - \frac{s}{p_f}\right) \prod_{k=1}^2 \left(1 - \frac{s}{z_{a_k}}\right)}{\left(1 - \frac{s}{p_f}\right) \prod_{i=1}^4 \left(1 - \frac{s}{p_{a_i}}\right) - a_V(0) f(0) \prod_{k=1}^2 \left(1 - \frac{s}{z_{a_k}}\right)}$$

Equation (5-7) indicates the fact that by variation of C_{E1} , and hence of R_F , the position of the closed-loop poles of the amplifier can be altered. It is notable that besides appearing in both numerator and denominator of Eq. (5-7) through P_F , capacitor C_{E1} also gives rise to zero Z_{a1} in the open-loop amplifier response $a_v(s)$ because of its interaction with the equivalent emitter resistance $R_{E1}R_F/(R_{E1}+R_F)$ of the input transistor. This is an elementary consequence of the fact that a common-emitter stage with a simple parallel R-C network in its emitter lead has a zero lying at $1/RC$.

A plot of the trajectory of the roots of the denominator of Eq. (5-7) as $a_v(o)f(o)$ is varied⁽²⁷⁾ is the subject of Fig. (5-4). The circles and x's represent zeroes and poles, respectively, of the quantity $a_v(s)f(s)$, whereas the boxes denote the location of the closed-loop zeroes of $1+a_v(s)f(s)$, which are hence the poles of $A_v(s)$.

Under influence of the quantity $a_v(o)f(o)$, the complex pair of poles of $a_v(s)$, P_{a2} and P_{a3} , is displaced to the location $P_{A1} = -43.3 \times 10^8 - j 48.2 \times 10^8$, $P_{A2} = -43.3 \times 10^8 + j 48.2 \times 10^8$, whereas P_{a1} is forced out to the location $P_{A3} = -66.2 \times 10^8$. The remaining poles P_{a4} and P_F completely subsume zeroes Z_{a1} and Z_{a2} , and so cancel these same quantities in the numerator of Eq. (5-7). Thus, the closed-loop response of the circuit is governed by a constellation of three poles and one zero, as shown in Fig. (5-5), hence it is possible to write for the dominant effects in the closed-loop response the expression,

$$A_v(s) = \frac{A_v(o) \left(1 - \frac{s}{P_{A1}} \right)}{\prod_{k=1}^3 \left(1 - \frac{s}{P_{Ak}} \right)} \quad (5-8)$$

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Fig. 5-4

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Fig. 5-5

The value of C_{E1} adopted in the design of $C_{E1} = 8$ pF is that required to place the closed loop complex poles of $A_v(s)$ on nearly the 45° radials; this positioning yields optimal transient response. (28) Larger values of C_{E1} cause undesirable post- and pre-transition oscillation in the transient response, whereas smaller values leave opportunity for rise-time improvement through the introduction of a tolerable amount of overshoot in the response.

Shown also in Fig. (5-5) are the closed-loop singularity locations for values of $C_{E1} = 6$ pF and $C_{E1} = 10$ pF; the coordinates of these singularities are given explicitly in Table (5-1).

C_{E1}	$A_v(0)$	P_{A1}	P_{A2}	P_{A3}	Z_{A1}
6 pF	9.8	$-48.9 \times 10^8 - j39.4 \times 10^8$	$-48.9 \times 10^8 + j39.4 \times 10^8$	-92.5×10^8	-11.3×10^9
8 pF	9.8	$-43.3 \times 10^8 - j48.2 \times 10^8$	$-43.3 \times 10^8 + j48.2 \times 10^8$	-66.2×10^8	-84.2×10^8
10 pF	9.8	$-35.9 \times 10^8 - j51.0 \times 10^8$	$-35.9 \times 10^8 + j51.0 \times 10^8$	-57.7×10^8	-67.2×10^8

TABLE 5-1. Dominant Closed-Loop Series-Shunt Pair Singularities.

The onset of ringing in the transient response for $C_{E1} > 8$ pF is reflected in the entries of Table (5-1) by the increasing radial angle of the complex pole pair P_{A1} , P_{A2} for increasing C_{E1} ; the angle exceeds 45° for $C_{E1} > 8$ pF.

In order to predict in more refined fashion the actual transient response of the amplifier, account must be taken of the physical realization of the circuit. This requires estimates to be obtained for

parasitic effects, so that the final simulation of amplifier performance more closely conforms to physical reality than the idealized analyses just made.

The need for extreme miniaturization of a feedback amplifier possessing subnanosecond risetime is manifest from consideration of the fact that the velocity of propagation of electromagnetic waves in free space is 3×10^{10} cm/s; at this velocity, a path length of 36 cm would contribute a 360° phase shift at 1 GHz. This is a liberal figure, for in most media, the propagation velocity is somewhat less than that in free space. It should be apparent from these figures that at microwave frequencies, the propagation delay of signals around its feedback loop can render an amplifier unstable. For this reason, it is essential that the physical length of the loop be maintained at an absolute minimum.

Parasitic reactances also pose a problem at frequencies in the gigahertz region, and these can be dealt with in either of two ways. A common approach is the incorporation of the parasitics into transmission lines. An alternative to this technique, and one more suited to the demands of feedback circuitry, where size is of paramount importance, is the minimization of spurious reactances through reduction of circuit size.

Fortunately, both these problems can be abated simultaneously through application to circuit construction of hybrid thin-film technology. Therefore, the amplifier described herein was fabricated upon a

low-loss sapphire substrate 1.0 inch square by 0.025 inch thick. Resistors were formed from a nichrome film deposited to a thickness giving 50 Ω /square resistivity, and a pattern of pure gold was used to provide conductive interconnections between elements.

Microstrip transmission lines of 50 Ω characteristic impedance provide the input and output signal feeds, and in this way irregularities in the signal path are kept to an absolute minimum. The requisite conductor width required to realize this value of impedance was determined to be 0.025 inch using data published elsewhere, ⁽²⁹⁾ in conjunction with the fact that for the sapphire substrate, $\epsilon_r \approx 9.8$.

Shown in Fig. (5-6) is a diagram of the layout adopted for the amplifier. Connections to the microstrip lines were made using strip-line launchers which provide a smooth transition from coaxial cable to microstrip. All supply rails are bypassed to the extensive ground-plane using ceramic chip capacitors; in this way, low impedance levels are maintained on the supply rails from dc to well into the microwave region. A large-area contact is provided by the test fixture to the ground-plane, and fine wires were used to convey supply potentials to the pads provided on the substrate for that purpose.

In spite of the reduction of parasitics afforded by hybrid construction, spurious reactances still exist, and in an accurate simulation of the amplifier, they must be taken into account.

Connexion schedule

- A -0.86 V.
- B -5.7 V.
- C +20 V.
- D -20 V.
- E Ground (0 V.)
- F Input
- G Output

Not to scale

- Gold
- ▨ Nichrome

XBL 724-2607

Fig. 5-6

Parasitic capacitance is not, as might perhaps first be expected, solely a function of resistor and pad area and substrate thickness.

Rather, it is a complex function of resistor pattern and material properties of the metal out of which the resistances are fabricated. For the amplifier described here, estimation of these capacitances was made on the basis of earlier work. (30)

Device bonding wires contribute parasitic inductance, and appropriate estimates of values can be done in a straightforward fashion using the fact that the leads are an average 0.03 inch long and 0.0007 inch in diameter. These parasitics manifest themselves in actuality as distributed effects, and the lumped-element approximations used here are only adopted out of the necessity of maintaining a tractable model.

The level-shifting zener diode of Fig. (5-1) can be modelled by the parallel combination of its 18Ω dynamic resistance shunted by a 500 pF capacitor.

A difficulty in using existing transient analysis programs in the simulation of high-frequency circuits arises from their inability to accept delay operators directly. Consequently, it becomes necessary to formulate an adequate approximation to the delayed voltage-controlled current sources in the models of the transistors if account is to be taken of excess-phase effects.

A digression will therefore be made at this point to develop an adequate equivalent to the delayed voltage-controlled current source representing that element accurately over the spectrum of frequencies at which the overall amplifier is expected to operate.

Let the complex exponential operator be expanded into Mac Laurin's series, and retain only the first two terms of the expansion, whereupon

$$e^{-j\omega\tau} \approx 1 - j\omega\tau \quad (5-9)$$

The rational approximation of Eq. (5-9) possesses a phase characteristic falling in asymptotic fashion from zero to $-\pi/2$ at high frequencies, however, it possesses the disadvantage of having considerable magnitude error. For example, a single-pole R-C network having the phase response of the right-hand side of Eq. (5-9) for $\tau = 20$ psec causes nearly a 10% magnitude error at 1 GHz, and for accurate simulation, this is considered too high.

A superior candidate is a cascade of one or more lossless π -sections of the form of Fig. (5-7); the appropriate formulae for these sections can be deduced in terms of image parameters, and the characteristic impedance shown to be,

$$Z_0 = \frac{\sqrt{L/C}}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2}} \quad (5-10)$$

where the parameter ω_0 is given by,

$$\omega_0^2 = 2/\sqrt{LC} \quad (5-11)$$

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Fig. 5-1

It can be shown also, that provided the input and output ports of the π -section are terminated in an impedance Z_0 , then the delay through the section is given by, (31)

$$\tau_s = \frac{\sqrt{LC}}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2}} \quad (5-12)$$

provided $\omega \leq \omega_0$.

It can easily be verified that values of $C = 2$ pF, and $L = 0.2$ nH yield $f_0 = \omega_0/2\pi = 16$ GHz. In this case, for $f \leq 2$ GHz, $\frac{\omega}{\omega_0} \leq 0.13$,

whence $\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2} \approx 1$, and $\tau_s \approx 20$ psec and $Z_0 = 10 \Omega$.

Figure (5-7) shows the discrete approximate equivalent of the ideal delayed source, and plotted in Fig. (5-8) are the phase and magnitude curves of the approximating network along with those of the ideal element.

Note the excellence of the agreement, which attests to the validity of the approximation.

All parasitic elements can now be incorporated into the basic equivalent circuit of Fig. (5-2) to yield the circuit given in Fig. (5-9). Although not shown explicitly in that illustration, all voltage-controlled current sources are replaced by the equivalent network of Fig. (5-7) for purposes of computer transient-response simulation using the "Cancer" Program. (32)

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Fig. 5-9

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The results of that simulation, which was carried out with a step-function input to the amplifier equivalent circuit, are plotted in Fig. (5-10). From that graph, the 10% to 90% risetime on the output is seen to be 330 psec, and the overshoot is 9%. These figures are well within the design bounds stated at the outset of this chapter.

In order experimentally to assess the transient response of the actual amplifier a Hewlett-Packard Type 141A oscilloscope equipped with the Type 1425A sampling time base and Type 1411 sampling amplifier was employed. The sampling head used is specified for a risetime ≤ 28 psec, so that in the measurements to be described, the effects of oscilloscope response can essentially be neglected.

A step transition input to the amplifier was generated by a Hewlett-Packard Type 1105A pulser operating into a Type 1106A tunnel-diode mount.

Illustrated in Fig. (5-11a) is a photograph of the output of the tunnel-diode pulser after attenuation sufficient to reduce the pulse amplitude to a 10 mV transition. It is evident from that figure that the testing waveform in fact possesses a finite risetime of about 60 psec, hence a small correction must be made for this fact when evaluating the output of the amplifier in the calculation of amplifier risetime. Figs (5-11b,c) show the amplifier output corresponding to the input of Fig. (5-11a). The risetime of the output is about 400 psec, with an overshoot of 10%. These figures agree excellently with the values of risetime ≈ 370 psec and overshoot $\approx 10\%$ as computed using program "Cancer" for the model of Fig. (5-9) driven with a pulse possessing a linear front edge of 60 psec risetime.

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Fig. 5-10

FIG. 5-11a

INPUT WAVEFORM

50 ps/div.

2 mV/div.

FIG. 5-11b

OUTPUT WAVEFORM VIA

20 dB ATTEN.

400 ps/div.

2 mV/div.

FIG. 5-11c

OUTPUT WAVEFORM VIA

20 dB ATTEN.

200 ps/div.

2 mV/div.

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FIG. 5-11 SMALL SIGNAL STEP RESPONSE
OF THE SERIES-SHUNT PAIR

Although the amplifier was designed so as to pass negative-going pulses, the use of the positive-going pulse employed in these tests was dictated by the unavailability of a negative-going pulse generator of adequately fast risetime.

To ensure a proper transient response of the amplifier, it is necessary properly to terminate the driving coaxial cable at the amplifier input for minimum reflections in the cable. This is especially important when the sending end is unterminated, as would be the case in a cascade of amplifiers of this type described here.

The input impedance at low frequencies is virtually that of the 50Ω terminating resistor, since the series feedback at the input effectively increases the open-loop input impedance by a factor of $1 + a(o)f(o)$. The magnitude of this quantity decreases with increasing frequency, hence so also does the impedance looking into the first stage, and with it the input impedance of the entire circuit.

The scattering parameter approach provides the most convenient vehicle of input impedance measurement, and from the discussion of Chapter 2, it follows that an ideal input termination corresponds, for identical system and normalizing impedances, to a value for s_{11} of zero.

The magnitude and phase of s_{11} were calculated for the model of Fig. (5-9) using the A-C analysis capability of the "Cancer" program in conjunction with a small additional program designed to calculate the scattering parameter s_{11} from the impedance looking into the amplifier.

Measurements were made in the laboratory on the actual circuit, and the data obtained are plotted alongside the analytically determined values of s_{11} in Fig. (5-12).

The forward scattering parameter s_{21} is also of interest, insofar as it gives a measure of the frequency response of the amplifier, hence there are plotted in Fig. (5-13) both analytically determined and experimentally measured values of this parameter. Once again, agreement is excellent between theory and practice.

Hitherto, all measurements done on the amplifier have been of 'small-signal' nature. Even the theoretical analysis has been carried out on the small-signal basis, since linearization of the inherently non-linear phenomena responsible for transistor action demands that only small perturbations about a quiescent operating point be considered. Extension of the analysis to the large-signal case would require use of a full non-linear model for the transistors.

The situation frequently arises, however, that in nuclear pulse amplifiers high-amplitude pulses will be encountered that do not satisfy the small-signal assumption, and so it is instructive at least experimentally to examine the resultant response of the amplifier. The experimental arrangement employed for lack of better equipment a Hewlett-Packard pulse generator Type 215-A having a risetime approximating to 1 nanosecond to generate a large-scale negative-going excursion at the amplifier input. Figure (5-14) shows some of the results of this test. A more detailed interpretation of large-signal response results requires a pulse generator of faster risetime than that used to produce Fig. (5-14).

Any amplifier other than the hypothetical ideal amplifier will contribute noise to the signal passing through it in addition to the intrinsic noise of the signal source. For signals of moderate to large amplitudes, the effect of amplifier noise may be negligible. However, noise performance of the amplifier may in some instances be of importance, for example, if it is to be used to preamplify very low-level signals.

A common measure of the noise behaviour of an amplifier is given by its noise figure F , which is defined as the quotient of input signal-to-noise ratio to output signal-to-noise ratio thus, (33)

$$F = \frac{P_{si}/P_{ni}}{P_{so}/P_{no}}, \quad (5-13)$$

where P_{si} and P_{ni} are the input signal power and input signal noise power, respectively, and P_{so} and P_{no} are the corresponding quantities for the signals at the amplifier output. The noise figure is proportional to the decadic logarithm of according to the relation,

$$N = 10 \log_{10} F \text{ dB.} \quad (5-14)$$

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Fig. 5-12

XBL724-2787

Fig. 5-13

FIG. 5-14a AMPLIFIER INPUT WAVEFORM
 1 nsec/div.
 20 mV/div.

**FIG. 5-14b AMPLIFIER OUTPUT VIA
 20 dB ATTENUATOR**
 1 nsec/div.
 20 mV/div.

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**FIG. 5-24 LARGE-SIGNAL RESPONSE
 OF THE SERIES-SHUNT PAIR**

Spot-noise measurements were taken for N at selected points within the passband of the amplifier using a white noise generator and micro-wave receiver equipped with a 100 KHz notch filter, and the data obtained thereby are plotted in Fig. (5-15).

As often useful alternative index of amplifier noise behaviour is given by its input-referred noise, a quantity deducible from N if Eq. (5-13) is written in its equivalent form,

$$F = \frac{\overline{e_{si}^2} \overline{e_{no}^2}}{\overline{e_{ni}^2} \overline{e_{so}^2}}, \quad (5-15)$$

where F is now expressed in terms of mean-square noise voltages. If A_V denotes amplifier voltage gain, Eq. (5-15) becomes, since

$$\overline{e_{so}^2} = A_V \overline{e_{si}^2},$$

$$F = \frac{1}{A_V} \frac{\overline{e_{no}^2}}{\overline{e_{ni}^2}}. \quad (5-16)$$

Let a new variable $\overline{e_{ai}^2}$ be defined to represent the noise generated in the amplifier itself and referenced to the input, so that,

$$\overline{e_{no}^2} = \overline{e_{ni}^2} A_V + \overline{e_{ai}^2} A_V. \quad (5-17)$$

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Fig. 5-15

Since the amplifier input is assumed to be terminated properly by the resistance R_T of 50 Ω , thermal noise generated within R_T comprises $\overline{e_{ni}^2}$ according to the Nyquist formula,

$$\overline{e_{ni}^2} = 4kT R_T \Delta f, \quad (5-18)$$

where T is the temperature in degrees Kelvin, and k is the Boltzmann constant. The thermal noise is considered over a frequency interval Δf in width. Substitution into Eq. (5-16) of Eqs. (5-17) and (5-18) then yields after rearrangement,

$$\overline{e_{ai}^2} = (F-1) (4kT R_T \Delta f). \quad (5-19)$$

For an example of the use of Eq. (5-14), F at 400 MHz is, from Eq. (5-19) and Fig. (5-15), $F \approx 1.5$. Assuming room temperature operation of the amplifier input terminating resistor Eq. (5-19) works out to a value of $\overline{e_{ai}^2} \approx 40 \times 10^{-20} \text{ V}^2/\text{Hz}$.

The noise figures found for this amplifier at midband are about 3.5 dB. This value could be improved upon by selecting a lower collector current for the transistors, since under the assumption of constant β_{dc} versus I_c in the contemplated range of I_c , the high-frequency shot noise generated in the collector and base regions is proportional to I_c .⁽³⁴⁾ Noise figure will increase over the midband value at the low end of the amplifier passband due to the onset there of the effects of colored flicker noise.

CHAPTER 6. DESIGN OF A FAST RISE SERIES-SERIES TRIPLE

The series-series triple feedback configuration offers the very substantial advantage over the series-shunt pair just treated of a much higher potentially available loop gain. This in turn leads to a superior gain stability than that possible with the feedback pair. This advantage is not obtained without penalty, however, for owing to the more complex singularity pattern of the triple, it is correspondingly more difficult to compensate for a given transient behaviour than the series-shunt pair. Another difficulty with the feedback triple manifest at high frequencies, is that its increased circuit complexity leads to increased parasitic effects when compared with the simpler series-shunt feedback pair. These factors, in addition to the excess phase delay contributed by the additional transistor of the triple, tend to destabilize the circuit.

Shown in Fig. (6-1) is the full electrical diagram of a series-series triple designed with a view towards meeting the elementary specification set down in Chapter 1; it can easily be verified from the figure that each of the three transistors passes a standing current of 10 mA and sustains $V_{CE} \approx 5$ V. In order that the output transistor operate at all times at the 10 to 20 mA level of I_C necessary for optimal f_T , the output of the amplifier is constrained to be in the range 0 to -0.5 V.

XBL724-2803

Fig. 6-1

A drawing of the physical layout adopted for the circuit is illustrated in Fig. (6-2). Figure (6-3) shows the full small-signal equivalent circuit, using the hybrid pi model deduced in Chapter 4 for the HPA 35820-A, and including all the attendant parasitic effects. The resistor R_s in series with the input voltage source E_s is defined identically to its counterpart of the preceding chapter.

As in the analysis done for the series-shunt pair, the low-frequency amplifier characteristics are first determined. For the low-frequency case, the series-series triple can be partitioned into its forward gain path and feedback network by opening all capacitors and short-circuiting all inductors of Fig. (6-3), and by noting that the open-circuit impedance matrix is the appropriate two-port formulation for the case of series-series feedback. The result of this operation is shown in Fig. (6-4), whence it is apparent that provided the forward alpha of the output transistor is near unity, the feedback function at d-c can be written as,

$$f(o) = \frac{R_{E1} R_{E3}}{R_L (R_{E1} + R_{E3} + R_F)} \quad (6-1)$$

This formula relates the voltage fed back to the amplifier input, to the voltage developed at the amplifier output node. Now, the amplifier closed-loop d-c gain $A_V(o)$ is related to the d-c open-loop gain $a_V(o)$ and to $f(o)$ by the equation,

$$A_V(o) = \frac{a_V(o)}{1 - a_V(o) f(o)} \quad (6-2)$$

Connexion schedule

A	- 6 V.
B	+ 12 V.
C	- 20 V.
D	- 2.5 V.
E	Ground
F	Input
G	Output
H	-10.7+ V.

Not to scale

- ☐ Gold
☒ Nichrome

XSL 724 - 2801

Fig. 6-2

(b) d-c Feedback function calculation

(a) d-c open-loop gain calculation

Fig. 6-4

XBL724-2800

Substitution into Eq. (6-2) of Eq. (6-1) followed by algebraic rearrangement yields an expression for the value of feedback resistor R_F in terms of other circuit parameters, as follows,

$$R_F = \frac{R_L [a_V(o) - A_V(o)] [R_{E1} + R_{E3}] - A_V(o) a_V(o) R_{E1} R_{E3}}{R_L [A_V(o) - a_V(o)]} \quad (6-3)$$

Several principal factors enter into the choice of values for the emitter resistances R_{E1} and R_{E3} . In order to secure a large closed-loop bandwidth in the triple, it is desirable to locate the second and third dominant poles as far from the origin of the complex phase as possible. This requires broadbanding of the first and third stages of the amplifier, a condition achieved by using large values for R_{E1} and R_{E3} . On the other hand, high d-c loop gain in the triple, with the consequent advantage of increased desensitivity, demands low values for the two emitter resistances. An additional factor favouring choice of low values for R_{E1} and R_{E3} arises out of the practical consideration that any compensating capacitor used across R_F must have a reasonably large value in order to be physically realizable. Arguing on a first-order basis, it is therefore desirable that R_F , hence also R_{E1} and R_{E2} , be of low value.

These several conflicting requirements are resolved in the particular selection made for the emitter resistors of $R_{E1} = R_{E2} = 33 \Omega$. Once this choice has been taken, Eq. (6-3) must be solved iteratively for the requisite $A_V(o)$ of -10 since $a_V(o)$ depends upon the value of R_F as is apparent from consideration of Fig. (6-3a). Commencement of the iteration

with a value of $R_F = 150 \Omega$ led after two cycles to a value of $R_F = 175 \Omega$, at which $a_V(0)$ was calculated to be -158.

With the d-c characteristics of the amplifier thus determined in closed-loop, it remains to take up the more difficult topic of the arrangement of suitable compensation to provide the fastest risetime on the output waveform, for a step-function input to the amplifier, consistent with an overshoot not exceeding ten per cent.

Although in the design of the series-shunt pair considered in Chapter 5, a pole-zero analysis was used owing to the intimate connection existing between this representation and time-domain behaviour, it was decided here to use optimization techniques to compensate the circuit. This approach enables direct use to be made of scattering parameters, and provides a link between the methods used for transistor characterization, and the design of the final amplifier. The disadvantage of the method is, of course, that an immediate and readily apparent correlation does not exist between transient response and the scattering parameter representation. Consequently, it was sought only to ensure small peaking and maximum break frequency for $|s_{21}|$ in the hope that this would yield adequate step response.

The program discussed in Chapter 3 was used as the vehicle for optimization; a modification was incorporated into the program to enable the optimization to be carried out with respect only to the magnitude of the forward scattering parameter s_{21} .

In particular, the error function of Eq. (3-1) was reformulated as,

$$\epsilon(\underline{x}) = \sum_i \left\{ |s_{21}^A(j\omega_i, \underline{x})| - |s_{21}^G(j\omega_i)| \right\}^2, \quad (6-4)$$

where,

$$|s_{21}^A(j\omega_i, \underline{x})| = \sqrt{\left[\operatorname{Re} \left\{ s_{21}^A(j\omega_i, \underline{x}) \right\} \right]^2 + \left[\operatorname{Im} \left\{ s_{21}^A(j\omega_i, \underline{x}) \right\} \right]^2}.$$

The superscript G signifies a given parameter to which fit is sought, and superscript A denotes a quantity calculated from the amplifier circuit of Fig. (6-3). The gradient vector corresponding to the error function of Eq. (6-4) can be computed in terms of $\partial s_{21}^A(j\omega_i, \underline{x}) / \partial \underline{x}$ as,

$$\begin{aligned} \frac{\partial \epsilon(\underline{x})}{\partial \underline{x}} = \sum_i 2 \left[|s_{21}^A(j\omega_i, \underline{x})| - |s_{21}^G(j\omega_i)| \right] \cdot \frac{1}{|s_{21}^A(j\omega_i, \underline{x})|} \cdot \left[\operatorname{Re} \left\{ s_{21}^A(j\omega_i, \underline{x}) \right\} \cdot \right. \\ \left. \operatorname{Re} \left[\frac{\partial s_{21}^A(j\omega_i, \underline{x})}{\partial \underline{x}} \right] + \operatorname{Im} \left\{ s_{21}^A(j\omega_i, \underline{x}) \right\} \cdot \operatorname{Im} \left[\frac{\partial s_{21}^A(j\omega_i, \underline{x})}{\partial \underline{x}} \right] \right]. \quad (6-5) \end{aligned}$$

The capacitor C_p , located across the feedback resistor R_p , was selected as the sole compensating element, and the optimization routine was employed to locate optimally the phantom zero and the inevitable higher-order pole introduced by C_p . No application was made of 'pole-splitting' techniques⁽³⁵⁾ within the amplifier forward gain path, since these would require the use of a capacitor of impractically small value.

The form chosen for the $|s_{21}^G|$ was that of a constant 20 dB out to 1.2 GHz, after which a fixed 60 dB/decade rolloff was followed. The slope used for the rolloff of 60 dB/decade was chosen on the basis of a three pole transfer function for $a_V(s)$. Thus, with the mode of compensation used here, the closed loop gain $A_V(s)$ will also possess three poles, which in the limit yields an asymptotic approach of $|A_V(j\omega)|$ to the -60 dB/decade line. The optimization was done on the full model of Fig. (6-3), thus all parasitics are accounted for from the outset.

Figure (6-5) shows the form of $|s_{21}|$ calculated from the optimized circuit for the optimal value of $C_F = 1.6$ pF. Shown also in Fig. (6-5) is a plot of the phase characteristics of s_{21} of the optimized amplifier. Figure (6-6) shows the input reflection parameter s_{11} in both phase and magnitude, as computed from the optimized circuit. The absence of severe peaking in the plot of $|s_{21}|$ implies a transient response having tolerable overshoot, whilst the plot of s_{11} shows that the input impedance of the amplifier is maintained reasonably constant at 50 Ω up to nearly 1 GHz.

In order to assess the transient response of the amplifier, the model of Fig. (6-3) was entered into the "Cancer" program along with the approximation developed in the preceding chapter for the delayed voltage-controlled current sources. The results of this simulation are plotted in Fig. (6-7), whence the figures of risetime ≈ 270 psec and overshoot $\approx 8\%$ are calculable.

XBL 724-2790

Fig. 6-5

XBL724-2791

Fig. 6-6

XBL724-2792

Fig. 6-7

Actual test of the amplifier was made using the equipment employed for evaluation of the series-shunt pair described in Chapter 5. Figure (6-8a) shows the amplifier driving waveform used in the test, and shown in the Figs. (6-8b,c) is the resulting output. The output waveform possesses a risetime of approximately 320 psec with maximum overshoot of about 13%.

The risetime of the experimental device is about 16% slower than the predicted risetime, and the overshoot is about 5% higher. These discrepancies are certainly within the range of acceptability, since estimation of all circuit parasitics was made on a simple first-order basis, and since the passive components are not at their exact values due to fabrication tolerances.

The errors notwithstanding, the response of the amplifier meets for all practical purposes the specification laid down for it in Chapter 1. The overshoot can easily be brought down to less than 10% simply by increasing slightly the value of C_p . This procedure would, of course, increase the risetime, but an adequate safety margin of 180 psec exists before the limiting value quoted in the specification of 500 psec is reached.

Scattering parameter evaluations were also made on the amplifier for the input scattering quantity s_{11} , and the forward scattering parameter s_{21} . The results of these measurements are plotted in Fig. (6-6) and (6-5), respectively, alongside the analytically determined parameters. Agreement is good, and the slightly lower frequency rolloff of s_{21} in the experimental case may be taken as indication of the observed fact that the step response of the actual amplifier is a little slower than predicted.

FIG. 6-8a AMPLIFIER INPUT WAVEFORM
2mV/div.
50 psec/div.

**FIG. 6-8b AMPLIFIER OUTPUT WAVEFORM
VIA 20dB ATTENUATOR**
2mV/div.
1nsec/div.

**FIG. 6-8c AMPLIFIER OUTPUT WAVEFORM
VIA 20dB ATTENUATOR**
2mV/div.
400psec/div.

XBB 722-816

**FIG. 6-8 SMALL-SIGNAL RESPONSE OF THE
SERIES-SERIES TRIPLE.**

As was done in Chapter 5 for the series-shunt pair, it is of interest to make an experimental assessment of the large-signal behaviour of the series-series triple. To this end, the amplifier was driven with a variable amplitude version of the same pulse used to evaluate its small-signal response, and risetime measurements were made on the output. Figure (6-9) gives a pictorial example of the large-signal response of the amplifier; the upper photograph shows the input waveform, and shown in the lower photograph is the associated amplifier output. It is apparent that even with an 800 mV amplitude on the output waveform, the risetime and overshoot are impaired only slightly over the small-signal case. A graphical presentation of the large-signal transient-response measurements is the subject of Fig. (6-10), and it will be noted that in all cases, the risetime at the output is held appreciably below the design limit of 500 psec. This very high slew rate is a consequence of the fact that the design of the amplifier incorporates no large capacitors which have to be charged through high impedance sources, and which would hence limit the speed of the output. Slew rate can be calculated directly from Fig. (6-10) for any value of output level by taking the ratio of output risetime, to 80% of the total output voltage excursion. A typical value for slew rate is that for an output transition amplitude of 500 mV, at which an elementary calculation shows that $t_{slew} \approx 1 \text{ V/ns}$. The mechanism responsible for risetime degradation in the large signal case is most probably the effect of transistor nonlinearities, especially those in the output device, which in large signal cases is required to operate over a nearly 3:1 variation in I_C , with

a rather lesser change in V_{CE} . Quantitative evaluation of these effects requires analysis of the amplifier to be done using the full nonlinear model of the constituent transistors.

Spot measurements were taken of the noise figure N of the series-series triple, and the results plotted in Fig. (6-11) where it is seen that a midband noise-figure of about 4 dB is attained. Corresponding to this, from Eq. (5-19), is an input-referred noise voltage of $\overline{e_{a_1}^2} \approx 47 \times 10^{-20} \text{ V}^2/\text{Hz}$. As was the case with the series-shunt pair noise figure, N can be expected to increase for low frequencies due to the effects of flicker noise in the transistors. Similarly, noise figure increases at the high-frequency end of the amplifier passband due to rolloff of amplifier gain, and the increase of transistor $1/f^2$ noise.

FIG. 6-9a AMPLIFIER INPUT WAVEFORM
20 mV/div.
50 psec/div.

FIG. 6-9b AMPLIFIER OUTPUT WAVEFORM
VIA 20 dB ATTENUATOR
20 mV/div.
1 nsec/div.

FIG. 6-9c AMPLIFIER OUTPUT WAVEFORM
VIA 20 dB ATTENUATOR
20 mV/div.
400 psec/div.

XBB 722-815

FIG. 6-9 LARGE-SIGNAL RESPONSE OF THE
SERIES-SERIES TRIPLE.

KBL724-2793

Fig. 6-10

XM. 725-K52

Fig. 6-11

CHAPTER 7. CONCLUSION

The efficient design and accurate prediction of performance of linear pulse amplifiers of subnanosecond risetime requires, in addition to standard design and measurement practice, the use of scattering parameters and of the computer.

The use of scattering parameters is especially necessary in the small-signal characterization of high-frequency transistors owing to the difficulty of obtaining reliable data by the more usual approaches of Y- or Z-matrix measurement, or through the application to the device of a series of measurements designed to single out for evaluation a particular element of the small-signal model. The accurate interpretation of scattering data for a particular device is greatly aided by the use of computer programs designed to fit an arbitrary model to the measured data. That this operation can be done efficiently can be verified by noting that one complete characterization of the type discussed in Chapter 4 took 2.07 seconds of central processor time on a CDC 6600 machine.

In the actual design of amplifier circuits intended for high-frequency operation, the use of the computer is essential, for it enables full account to be taken of all spurious reactances consequent to the physical realization of any circuit. Computer aided analysis is further commended if hybrid or integrated circuit methods of circuit construction are contemplated due to the difficulties attendant upon prototype alteration.

Upon conclusion of the amplifier design and construction, scattering parameters provide a convenient vehicle of performance evaluation in the frequency domain, and are useful directly if it is desired that the amplifier be designed to meet a frequency-domain response specification.

The excellent agreement observed in this work between theoretical and experimental data confirms that the approaches described herein are valid, and gives confidence that they can be applied in future work in linear wideband circuit design and yield accurate results.

Of the two feedback configurations treated in this paper, the series-series triple appears theoretically to have a definite superiority over the series-shunt pair. This is a consequence of the improved risetime of the triple, a prediction which was also borne out experimentally. An ancillary benefit of the triple lies in its possession of a potentially higher loop gain than that of the pair; this leads to an increased desensitization of the triple to parameter variations in comparison with that of the feedback pair.

In respect of noise performance, both the series-shunt pair and series-series triple appear from an experimental standpoint to be about equal. The accuracy of the noise measurements made on the amplifiers was limited by that of the test equipment used; and a more detailed comparison of amplifier noise figure awaits measurements of higher accuracy and greater frequency range than those made here.

The performance of these amplifiers is to an extent limited by the capabilities of the apparatus and technology used in their fabrication, and also by the fact that the designs were carried out as an investigation into the feasibility of attaining subnanosecond risetime feedback amplifiers rather than with the intention of designing a circuit for general use. Suggestions for a number of improvements in, and additions to, the original designs can therefore be made.

In regard to device fabrication, there appears to be no bar to reducing by a factor of, say, two, the size of the active area of the circuit. Such a reduction would yield further decreases in magnitude of the parasitic elements. The effects of a large emitter inductance can be serious, for it leads to degenerative local series feedback which increases with frequency; this causes the stage gain to fall off at a faster rate than would be the case were the emitter inductance absent. It has been found, however, by means of repeated computer aided analysis, that only a small (≈ 20 psec in the series-series triple) theoretical improvement in circuit risetime is possible by reducing to zero the emitter inductance parasitics in the circuits described here. This degree of improvement is not significant for the order of magnitude of risetime possessed by these circuits, however, as risetimes are improved, the effects of lead inductance may increase in dominance.

As a safety precaution in a user-oriented design, it would seem advisable to incorporate into the output stage some sort of output current limiting in order to avoid catastrophic failure of the output stage due to excessive current drain. For the HPA 35820-A transistor used in the amplifiers described herein, 45 mA is quoted as the maximum safe collector current. Hence, output stage current limiting could reasonably be set to limit at somewhat above 30 mA, a value corresponding to more than one volt output into 50 ohms, assuming a quiescent emitter current of 10 mA.

As a final modification, it would be of utility to incorporate d-c baseline stabilization into the amplifier. Especially at low pulse duty-cycle rates, an efficient way to accomplish this is through the use of an ancillary circuit which integrates the input and output of the wideband amplifier and furnishes a d-c error voltage to the wideband amplifier of such a polarity as to cause the amplifier output to track the input d-c level. High loop gain in the correction circuit can be attained by using a standard integrated operational amplifier.

Future work along the lines of circuit realization to provide extremely high-speed amplification will most probably necessitate the abandonment of multi-stage feedback configurations due to the inevitable problems in them occasioned by loop length and the consequent destabilizing effect of the transversal delay introduced thereby. Moreover, the parasitics in the circuit tend to introduce additional lags over and above those due to dominant singularities; these additional phase shifts reduce the loop phase margin, thereby increasing the difficulty of applying overall feedback around several stages.

A more profitable approach for amplifiers intended to have risetime of the order of say 100 psec, would be the use of a cascade of single stages, perhaps with local feedback. The use of impedance level mismatching between adjacent stages has the additional virtue of permitting an overall amplifier pole-zero pattern to be synthesized by appealing to each stage in turn since, at least to first order, mismatching would ensure response independence of an intermediate stage from its predecessor and successor.

Transistors are now available with f_T figures of the order of 6 GHz, and although these devices presently suffer the limitation of a restricted range of operating conditions, this disadvantage may be remedied in future.⁽³⁶⁾ An alternate route to the direct use of high f_T devices might be the application of novel circuit techniques; for example, the ' f_T -doubler' circuit proposed recently⁽³⁷⁾ by workers at the Tektronix Company is capable of a theoretical doubling of the f_T of the constituent transistors.

The development of higher speed amplifiers will also necessitate the tighter control over parasitic elements, and the more universal application of transmission-line technique than has been used in the circuits described in this paper.

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APPENDICES

APPENDIX A. USE OF THE AUTOMATED MODELLING PROGRAM

Data input to the program is via punched cards; in order to facilitate the task of entering data, the cards are read by the program under a 'free' format wherein fields on a card are separated by a delimiter 'd' which may be a comma or one or more spaces. Numerical information may be given in fixed-point, floating, or exponential form, and any card may be continued an arbitrary number of times by punching as the first character on each continuation card an asterisk. Batch-processing is used, so that any number of data decks may be processed consecutively by the program.

What follows is a description of the various cards required by the program.

[A] TITLE CARD

The title card must be the first card in each data deck, and it may contain arbitrary alphanumeric information descriptive of the problem being done, for example, MODEL OF HPA 35820-A TRANSISTOR AT IC = 10 MA, $V_{CE} = 5 \text{ V}$.

[B] SCATTERING PARAMETER CARD

Up to sixty scattering parameter cards may be used. These cards contain the scattering matrices to which the circuit undergoing optimization is to be fitted by the program. Their general form is:

S d FREQUENCY IN Hz d MAGNITUDE S_{11} d ANGLE S_{11} d
 MAGNITUDE S_{12} d ANGLE S_{12} d MAGNITUDE S_{21} d ANGLE S_{21} d
 MAGNITUDE S_{22} d ANGLE S_{22}

[C] WEIGHT-FACTOR CARD

This card contains the weighting factors to be used in the error function and also the normalizing resistance under which the scattering parameters of [B], above, are taken. This card is of general form:

WEIGHT d WEIGHT S_{11} d WEIGHT S_{12} d WEIGHT S_{21} d
 WEIGHT S_{22} d NORMALIZING RESISTANCE

One weight card must be included with each problem.

[D] ELEMENT CARDS

These cards describe the elements of the circuit in terms of their topological position and value. The nodes of the circuit are assumed to have been assigned integer numbers with the sole restrictions that the datum node must be numbered zero, and that no number must be used more than once. Element cards are of four classes, as follows:

(1) Resistors

NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH R d
 ESTIMATED OR TRUE VALUE d + NODE d - NODE.

(ii) Capacitors

NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH C d
ESTIMATED OR TRUE VALUE d + NODE d - NODE.

(iii) Inductors

NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH L d
ESTIMATED OR TRUE VALUE d + NODE d - NODE.

(iv) Voltage-Controlled Current Sources

NAME OF UP TO SEVEN CHARACTERS BEGINNING WITH GM d
ESTIMATED OR TRUE VALUE OF TRANSCONDUCTANCE d. NAME OF UP
TO SEVEN CHARACTERS BEGINNING WITH TO d ESTIMATED OR TRUE
VALUE OF DELAY d + NODE d - NODE d + CONTROL NODE d - CONTROL
NODE.

[E] PORT CARDS

Two port cards must accompany each problem; that for the input
port is as follows:

INPORT d + NODE d - NODE.

while that describing the output port has the form:

OUTPORT d + NODE d - NODE.

[F] OPTIMIZE CARD

This card describes by name the circuit parameters subject to
variation by the program in the fitting process. This card is of the
following form:

.OPT d MAXIMUM PERMISSIBLE NUMBER OF ITERATIONS IN THE
OPTIMIZATION d PRINT INTERVAL IN THE OPTIMIZATION d NAME
OF FIRST PARAMETER TO BE OPTIMIZED d NAME OF SECOND PARAMETER
TO BE OPTIMIZED d ----.

Up to 30 parameters may be optimized simultaneously.

[G] END CARD

This card must be the last card in each problem, and has the form:

.END

An example circuit and data deck are illustrated in Fig. A-1.

① Circuit

② ESTIMATED VALUES SCALED TO R IN OHMS, C IN PF, T IN PS, AND F IN $\times 10^{12}$ Hz

$R_B = 10 \Omega$
 $C_{PI} = 12 \text{ PF}$
 $R_{PI} = 100 \Omega$
 $C_{MU} = .5 \text{ PF}$
 $G_M = .38 \Omega^{-1}$
 $T_O = 30 \text{ psec}$

③ MEASURED SCATTERING MATRICES.

FREQUENCY | S_{11} | $\angle S_{11}$ | S_{12} | $\angle S_{12}$ | S_{21} | $\angle S_{21}$ | S_{22} | $\angle S_{22}$
 ($\times 10^{12}$ Hz)

0.0002	0.3	-95°	.034	67°	.112	123°	.69	-26°
0.0004	0.22	-145°	.052	68°	6.5	100°	.95	-25°
0.0006	0.22	-176°	.072	68°	4.4	87°	.50	-26°

50 Ω NORMALIZING RESISTANCE USED.

④ PROBLEM.

CALCULATE C_{MU} , G_M , AND TO SO AS TO GIVE
 BEST AGREEMENT OF CIRCUIT SCATTERING
 PARAMETERS AND MEASURED SCATTERING MATRICES.

⑤ PROGRAM INPUT DATA.

TEST EXAMPLE

WEIGHT, 1, 1, 1, 1, 50
 R_B , 10, 01, 02
 R_{PI} , 100, 02, 00
 C_{PI} , 12.0, 02, 00
 C_{MU} , 0.50, 02, 03
 $INPORT$, 01, 00
 G_M , .38, T_O , 30, 03, .70, 02, 00
 $OUTPORT$, 03, 00
 $.OPT$, 300, 10, G_M , C_{MU} , T_O
 S , 2E-4, .3, -.95, .034, .69, 11.2, 123, .69, -26
 S , 4E-4, .22, -145, .052, 6.5, 100, .95, -25
 S , .0006, .22, -176, .072, 6.8, 4.4, 87, .50, -26
 $.END$

NOTE THAT UP TO 300 ITERATIONS WILL BE MADE, THAT
 PRINT-OUTS OF PROGRAM STATUS WILL BE TAKEN EVERY 10
 ITERATIONS, AND THAT ALL SCATTERING PARAMETER WEIGHTS
 HAVE BEEN SET TO UNITY.

XBL725-3057

Fig. A-1

APPENDIX B.

Given in this appendix is a full Fortran listing of the program "MODEL". In its present form, it is suitable for use on any of the CDC 6400, 6600, or 7600 computers; minor alterations may be necessary to suit the program for use on other machines. In particular, it may be found necessary on computers having smaller word sizes than the CDC machines to use double-precision in the matrix solution and decomposition routines, and in the optimization subprogram in order to avoid numerical difficulties arising from rounding errors.

PROGRAM NMODEL(INPUT,OUTPUT,TAPES=INPUT)

R. WILSON. VERSION WITHOUT POST-OPTIMISATION SCATTERING
PARAMETER CALCULATION.

#MODEL* IS A COMPUTER PROGRAM TO DETERMINE THE VALUES OF PRESCRIBED
ELEMENTS OF AN R-L-C-GM-TO (DELAY) NETWORK REQUIRED TO REALISE
THE BEST FIT OF THE NETWORK TO A GIVEN SET OF TWO-PORT RESPONSES. THE
DESIRED RESPONSE IS FURNISHED TO THE PROGRAM IN FORM OF A SERIES
OF UP TO SIXTY SETS OF TWO-PORT SCATTERING PARAMETERS TAKEN AT
ARBITRARY FREQUENCIES OVER THE SPECTRUM IN WHICH BEST FIT OF THE
NETWORK TO THE GIVEN DATA IS SOUGHT. THE INCORPORATION INTO THIS
PROGRAM OF PROVISION FOR SIMULATING DELAY TIME AND FOR ACCEPTING
SCATTERING PARAMETERS MAKES IT PARTICULARLY USEFUL IN THE MODELLING
OF MICROWAVE DEVICES IN THE SMALL-SIGNAL REGION OF OPERATION.

PROGRAM CAPACITY IS AS FOLLOWS.

30 NODES, NOT INCLUDING DATUM.
120 ELEMENTS.
R, L, C, AND GM/TO ARE ACCEPTED AS ELEMENTS.
30 OPTIMISABLE PARAMETERS.
ZERO-VALUED ELEMENTS ARE NOT PERMITTED.

THIS IS THE CONTROLLING PROGRAM.

COMMON DECLARATIONS.

COMMON /KILL/ NOGO,JEOP
COMMON /OPTIMUM/ WEIGHT(4),JLOCK(30),IDOPT(30),NDPT,NPRINT,
1 NITER,X(30),SCALEX(30)
COMPLEX YDATH,YNL,VATH,VN
COMMON /MATRIX/ YDATH,YNL(300),VATH,VN(30),MATLOC(488)
COMPLEX PORT
COMMON /PORTDAT/ PORT(60,4),OMEGA(60),NPOINTS,RNORM,LOCPR1,
1 LOCPR2
COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
1 NOOPLC(484),NUMNOD
COMMON /CARD/ FIELD(32),NUMFLD,IO,IHALT,ICARD
DIMENSION TEMPOT(60,8)
EQUIVALENCE (TEMPOT(1,1),YNL(1))
DIMENSION ITITLE(8)
DIMENSION IDNO(120),JUNODE(30)
EQUIVALENCE (IDNO(1),MATLOC(1)),(JUNODE(1),MATLOC(121))
DATA ISHIFT/1000000000000000/
DATA ILETR/22B/
DATA ILETC/03B/
DATA ILETL/14B/
DATA ILETC/07B/
DATA ILETT/24B/

C
C CALCULATE CONSTANTS AND INITIALISE.

```

C      PROGRAM MMODEL(INPUT,OUTPUT,TAPES=INPUT)
C
C      R. WILSON. VERSION WITHOUT POST-OPTIMISATION SCATTERING
C      PARAMETER CALCULATION.
C
C      *****
C
C      MMODEL IS A COMPUTER PROGRAM TO DETERMINE THE VALUES OF PRESCRIBED
C      ELEMENTS OF AN R-L-C-GH-TO (DELAY) NETWORK REQUIRED TO REALISE
C      THE BEST FIT OF THE NETWORK TO A GIVEN SET OF TWO-PORT RESPONSES. THE
C      DESIRED RESPONSE IS FURNISHED TO THE PROGRAM IN FORM OF A SERIES
C      OF UP TO SIXTY SETS OF TWO-PORT SCATTERING PARAMETERS TAKEN AT
C      ARBITRARY FREQUENCIES OVER THE SPECTRUM IN WHICH BEST FIT OF THE
C      NETWORK TO THE GIVEN DATA IS SOUGHT. THE INCORPORATION INTO THIS
C      PROGRAM OF PROVISION FOR SIMULATING DELAY TIME AND FOR ACCEPTING
C      SCATTERING PARAMETERS MAKES IT PARTICULARLY USEFUL IN THE MODELLING
C      OF MICROWAVE DEVICES IN THE SMALL-SIGNAL REGION OF OPERATION.
C
C      PROGRAM CAPACITY IS AS FOLLOWS.
C
C      30 NODES, NOT INCLUDING DATUM.
C      120 ELEMENTS.
C      R, L, C, AND GH/TO ARE ACCEPTED AS ELEMENTS.
C      30 OPTIMISABLE PARAMETERS.
C      ZERO-VALUED ELEMENTS ARE NOT PERMITTED.
C
C      *****
C
C      THIS IS THE CONTROLLING PROGRAM.
C
C      COMMON DECLARATIONS.
C
C      COMMON /KILL/ NOGO,JEOP
C      COMMON /OPTIMUM/ WEIGHT(4),JLOCK(30),IDOPT(30),HOPT,NPRINT,
C      1 NITER,X(30),SCALEX(30)
C      COMPLEX YDATUM,YNL,VDATUM,VN
C      COMMON /MATRIX/ YDAIUM,YNL(300),YDATUM,VN(30),MATLOC(400)
C      COMPLEX PORT
C      COMMON /PORTDAT/ PORT(60,4),OMEGA(60),NPOINTS,RNORM,LOCPR1,
C      1 LOCPR2
C      COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(123),LOCATE(5),
C      1 NODPLC(494),NUMNOD
C      COMMON /CARD/ FIELD(32),NUMFLD,IO,INHALT,ICARD1
C      DIMENSION TEMPLT(60,8)
C      EQUIVALENCE (TEMPLT(1,1),YNL(1))
C      DIMENSION ITITLE(8)
C      DIMENSION IDNO(120),JUNODE(30)
C      EQUIVALENCE (IDNO(1),MATLOC(1)),(JUNODE(1),MATLOC(121))
C      DATA ISHIFT/1000000000000000/
C      DATA ILETR/220/
C      DATA ILEYC/030/
C      DATA ILETL/140/
C      DATA ILEYG/070/
C      DATA ILETT/240/
C
C      CALCULATE CONSTANTS AND INITIALISE.

```

```

C      TWOPI=8.0*ATAN2(1.0,1.0)
5      RAOIAN=TWOPI/360.0
C
C      BEGIN A JOB.
C
7      15 CALL SECOND(TIME1)
11      NPOINTS=0
12      NUMMOD=0
12      JEOF=0
13      NOSTOP=0
13      IPORT1=0
14      IPORT2=0
14      NUMEL=0
15      IWEIGHT=0
15      NUMOPT=0
16      NOGO=0
C
C      READ JOB TITLE CARD.
C
17      READ (5,20) (ITITLE(I),I=1,8)
20      FORMAT (8R10)
24      IF (EOF,5) 23,24
27      23 CALL EXIT
30      24 PRINT 25, (ITITLE(I),I=1,8)
25      FORMAT (1H6,1X,120(1H*))///1X,8R10,20X,0----- MMODEL -----
1//2X,120(1H*)/////5X,0--- INPUT DATA ---//)
C
C      BEGIN, GET A CARD.
C
36      ICARD1=1
37      30 CALL PARSE
40      IF (JEOF.EQ.1) GO TO 2000
42      IF (IHALT.EQ.0) GO TO 31
43      NOGO=1
44      GO TO 30
C
C      CHECK FOR CONTROL OR ELEMENT CARD.
C
45      31 IF (ID.GT.8) GO TO 135
C
C      ELEMENT CARD .
C
51      IF (ID.GT.4) GO TO 50
53      NUMEL=NUMEL+1
54      IF (NUMEL.LT.120) GO TO 40
56      PRINT 35
35      FORMAT (//5X,0----- MORE THAN 120 ELEMENTS///)
62      NOGO=1
63      GO TO 30
64      40 IF (ID.EQ.4) GO TO 45
C
C      PASSIVE ELEMENTS.
C
66      IPOINT=3
67      ISTOP=2
70      GO TO 65
C

```


C VOLTAGE CONTROLLED CURRENT SOURCES.

C

```

71 45 IPOINT=5
72 ISTOP=4
73 GO TO 65
74 50 IF (ID.GT.6) GO TO 105

```

C

C PORTS.

C

```

100 IPOINT=2
100 ISTOP=2
101 IF (ID.EQ.6) GO TO 52
104 IPORT1=IPORT1+1
105 IF (IPORT1.LE.1) GO TO 65
107 NOG0=1
110 PRINT 51
51 FORMAT (//5X,'----- MORE THAN 1 INPUT PORT*//')
113 GO TO 30
114 52 IPORT2=IPORT2+1
116 IF (IPORT2.LE.1) GO TO 65
117 NOG0=1
120 PRINT 53
53 FORMAT (//5X,'----- MORE THAN 1 OUTPUT PORT*//')
124 GO TO 30

```

C

C SET NODE NUMBERS AND RENUMBER NODES SEQUENTIALLY.

C

```

125 65 IF ((IPOINT+ISTOP-1).NE.NUMFLO) GO TO 900
131 LOC1=NOSTOP+1
132 DO 85 I=1,ISTOP
134 NODE=FIELD(IPOINT)
135 IF (NODE.EQ.0) GO TO 80
137 IF (NUMNOD.EQ.0) GO TO 75
140 DO 70 J=1,NUMNOD
141 IF (NODE.NE.JUNODE(J)) GO TO 70
143 NODE=J
144 GO TO 80
144 70 CONTINUE
147 75 NUMNOD=NUMNOD+1
151 JUNODE(NUMNOD)=NODE
153 NODE=NUMNOD
153 80 NOSTOP=NOSTOP+1
155 NODPLC(NOSTOP)=NODE
157 IPOINT=IPOINT+1
160 85 CONTINUE
162 GO TO (90,90,90,95,100,100), ID

```

C

C SET ELEMENT VALUES AND LOAD ARRAYS.

C

C

C PASSIVE ELEMENTS.

C

```

176 90 NAME(NUMEL,1)=FIELD(1)
177 LOCAL(NUMEL)=LOC1
200 VALUE(NUMEL,1)=FIELD(2)
202 IDNO(NUMEL)=ID
204 GO TO 30

```

C

C VOLTAGE CONTROLLED CURRENT SOURCES.

C

```

206 95 NAME(NUMEL.1)=FIELD(1)
207   VALUE(NUMEL.1)=FIELD(2)
211   NAME(NUMEL.2)=FIELD(3)
212   VALUE(NUMEL.2)=FIELD(4)
214   LOCAL(NUMEL)=LOC1
215   IDNO(NUMEL)=ID
217   GO TO 30

```

C

C PORTS.

C

```

220 100 IF (ID.EQ.5) LOCPR1=L3C1
224   IF (ID.EQ.6) LOCPR12=L3C1
230   GO TO 30
231 105 IF (ID.GT.7) GO TO 115

```

C

C WEIGHTING FACTORS AND NORMALISING RESISTANCE CARD.

C

```

235   IWEIGHT=1
235   IF (NUMFLD.NE.6) GO TO 900
240   RNORM=FIELD(6)
241   DO 110 I=1,4
246 110 WEIGHT(I)=FIELD(I+1)
250   GO TO 30

```

C

C PORT FREQUENCY RESPONSE DATA.

C

```

250 115 IF (NUMFLD.NE.10) GO TO 900
252   NPOINTS=NPOINTS+1
254   IF (NPOINTS.LE.60) GO TO 125
256   NOGO=1
257   PRINT 120
262 120 FORMAT (//5X, '----- MORE THAN 60 DATA POINTS---')
262   GO TO 30
263 125 OMEGA(NPOINTS)=FIELD(2)
265   DO 130 I=1,8
275 130 TEMPOR(I,NPOINTS)=FIELD(I+2)
277   GO TO 30

```

C

C CONTROL CARDS.

C

```

300 135 IF (ID.EQ.10) GO TO 155

```

C

C OPTIMISATION CARD.

C

```

302   IF (NUMFLD.LT.3) GO TO 900
305   NUMOPT=1
306 145 DO 150 I=3,NUMFLD
314 150 IDOPT(I-2)=FIELD(I)
316   NOPT=NUMFLD-2
320   NPRINT=FIELD(2)
321   NITER=FIELD(1)
323   GO TO 30
324 900 NOGO=1
325   PRINT 905
331 905 FORMAT (//5X, '----- ERROR IN ABOVE CARD---')
331   GO TO 30

```

```

C ENTRY INTO THE FOLLOWING PART OF THE PROGRAM IMPLIES THAT AN
C END CARD HAS BEEN ENCOUNTERED.
332 155 IF (N0G0.EQ.0) GO TO 170
C
C ERROR MESSAGES.
C
333 PRINT 165
165 FORMAT (///10X,--- ERRORS IN INPUT DATA. EXECUTION HALTED ---//)
337 GO TO 2000
340 170 IF (NUMEL.GT.0) GO TO 176
343 N0G0=1
344 PRINT 175
175 FORMAT (///10X,--- CIRCUIT HAS NO ELEMENTS. EXECUTION HALTED ---//
1//)
347 176 IF (NUMOPT.GT.0) GO TO 180
352 N0G0=1
353 PRINT 177
177 FORMAT (///10X,--- NO OPTIMISATION REQUESTS. EXECUTION HALTED ---
1//)
356 180 IF (IPOINT1.EQ.1) GO TO 186
360 N0G0=1
361 PRINT 185
185 FORMAT (///10X,--- CIRCUIT HAS NO INPUT PORT. EXECUTION HALTED ---
1//)
365 186 IF (NPOINTS.GT.0) GO TO 188
370 N0G0=1
371 PRINT 187
187 FORMAT (///10X,--- NO SCATTERING PARAMETER DATA. EXECUTION HALTED
1---//)
374 188 IF (IPOINT2.EQ.1) GO TO 190
376 N0G0=1
377 PRINT 189
189 FORMAT (///10X,--- CIRCUIT HAS NO OUTPUT PORT. EXECUTION HALTED --
1---//)
403 190 IF (IWEIGHT.EQ.1) GO TO 200
405 N0G0=1
406 PRINT 191
191 FORMAT (///10X,--- NO WEIGHT CARD. EXECUTION HALTED ---//)
412 200 IF (N0G0.EQ.1) GO TO 2000
C
C PROCESS TWO-PORT DATA.
C
C PRINT OUT TWO-PORT DATA.
C
414 PRINT 205
205 FORMAT (////////45X,--- SCATTERING PARAMETER DATA ---//7X,*FREQ*,7X
1,*MAG S11*,7X,*ANG S11*,7X,*MAG S12*,7X,*ANG S12*,7X,*MAG S21*,7X,
2,*ANG S21*,7X,*MAG S22*,7X,*ANG S22*//)
420 DO 215 I=1,NPOINTS
422 PRINT 210, OMEGA(I), (TEMPORT(I,J),J=1,8)
210 FORMAT (5X,E10.3,8(4X,E10.3))
441 215 CONTINUE
C
C CONVERT PORT DATA FROM POLAR TO RECTANGULAR COMPLEX FORM.
C

```

```

444 DO 220 I=1,NPOINTS
445 OMEGA(I)=THOPI*OMEGA(1)
447 DO 216 J=1,4
451 ANG=RADIANT*TEMPORT(I,2*J)
455 XMAG=TEMPORT(I,2*J-1)
460 216 PORT(I,J)=CMPLX(XMAG*COS(ANG),XMAG*SIN(ANG))
477 220 CONTINUE

```

C PRINT PORTS AND WEIGHTING FACTORS.

```

502 PRINT 225
225 FORMAT (//////40X,--- TWO PORT DATA ---//20X,*,NODE*,7X,*,NODE*)
506 KPNODE=NODPLC(LOCPT1)
507 KNNODE=NODPLC(LOCPT1+1)
511 IF (KPNODE.EQ.0) GO TO 230
512 KPNODE=JUNODE(KPNODE)
513 IF (KNNODE.EQ.0) GO TO 235
514 KNNODE=JUNODE(KNNODE)
515 235 PRINT 240,KPNODE,KNNODE
240 FORMAT (//5X,*INPUT PORT*,5X,15,7X,15)
526 KPNODE=NODPLC(LOCPT2)
527 KNNODE=NODPLC(LOCPT2+1)
531 IF (KPNODE.EQ.0) GO TO 245
532 KPNODE=JUNODE(KPNODE)
533 245 IF (KNNODE.EQ.0) GO TO 250
534 KNNODE=JUNODE(KNNODE)
535 250 PRINT 251, KPNODE,KNNODE
251 FORMAT (//5X,*OUTPUT PORT*,4X,15,7X,15)
545 PRINT 255, (4EIGHT(I),I=1,4)
255 FORMAT (//5X,*S11 WEIGHT*,14X,*S12 WEIGHT*,14X,*
1522 *WEIGHT*,5X,E10.3,3(14X,E10.3))
553 PRINT 256, RNORM
256 FORMAT (//5X,*NORMALISING RESISTANCE = 0.E10.3)

```

C PROCESS WEIGHTING FACTORS.

```

561 DO 257 I=1,4
566 257 WEIGHT(I)=WEIGHT(I)**2

```

C PROCESS CIRCUIT ELEMENT DATA.

C SORT ELEMENTS INTO THE ORDER R, C, L, GN/TO, AND ESTABLISH
C THE LOCATE ARRAY.

```

570 LOCATE(1)=1
570 NPLACE=1
571 DO 266 I=1,4
573 IF (NPLACE.GT.NUMEL) GO TO 265
576 DO 260 J=NPLACE,NUMEL
577 IF (IDNO(J).NE.1) GO TO 260
601 IF (J.EQ.NPLACE) GO TO 259

```

C EXCHANGE ELEMENTS.

```

607 NTEMP=NAME(J,1)
610 NAME(J,1)=NAME(NPLACE,1)
611 NAME(NPLACE,1)=NTEMP

```

```

613 NTEMP=NAME(J,2)
614 NAME(J,2)=NAME(NPLACE,2)
615 NAME(NPLACE,2)=NTEMP
616 NTEMP=LOCAL(J)
617 LOCAL(J)=LOCAL(NPLACE)
620 LOCAL(NPLACE)=NTEMP
621 NTEMP=IDNO(J)
622 IDNO(J)=IDNO(NPLACE)
623 IDNO(NPLACE)=NTEMP
624 TEMP=VALUE(J,1)
625 VALUE(J,1)=VALUE(NPLACE,1)
627 VALUE(NPLACE,1)=TEMP
630 TEMP=VALUE(J,2)
631 VALUE(J,2)=VALUE(NPLACE,2)
632 VALUE(NPLACE,2)=TEMP
633 259 NPLACE=NPLACE+1
634 260 CONTINUE
637 265 LOCATE(I,1)=NPLACE
641 266 CONTINUE

C
C PRINT ELEMENT TABULATION.
643 PRINT 270
270 FORMAT (//////40X,*-- CIRCUIT SUMMARY --*////)

C
C PASSIVE ELEMENTS.
647 ISTART=1
650 ISTOP=LOCATE(4)-1
652 IF (ISTOP.LT.ISTART) GO TO 300
654 PRINT 275
275 FORMAT (//5X,*-- PASSIVE ELEMENTS --*//5X,*NAME*,15X,*VALUE*,15X,*
INODE*,15X,*-NODE*//)
657 DO 295 I=ISTART,ISTOP
661 LOC=LOCAL(I)
663 KPNODE=NODPLC(LOC)
664 KNNODE=NODPLC(LOC+1)
666 IF (KPNODE.EQ.0) GO TO 280
667 KPNODE=JUNODE(KPNODE)
670 280 IF (KNNODE.EQ.0) GO TO 285
671 KNNODE=JUNODE(KNNODE)

672 285 PRINT 290, NAME(I,1),VALUE(I,1),KPNODE,KNNODE
290 FORMAT (//4X,R7,11X,E10.3,12X,15,15X,15)
710 295 CONTINUE

C
C VOLTAGE-CONTROLLED CURRENT SOURCES.
713 300 ISTART=LOCATE(4)
714 ISTOP=LOCATE(5)-1
716 IF (ISTOP.LT.ISTART) GO TO 338
721 PRINT 305
305 FORMAT (//////5X,*-- VOLTAGE CONTROLLED CURRENT SOURCES --*//5X,*B
14X,*GM*,10X,*DELAY*,5X,*DELAY*,6X,*-NODE*,3X,*-NODE*,3X,*-CON*,
24X,*-CON*/5X,*NAME*,6X,*VALUE*,8X,*NAME*,6X,*VALUE*,22X,*NODE*,6X,
3X,*NODE*//)
724 DO 335 I=ISTART,ISTOP
726 LOC=LOCAL(I)

```

```

730 KPNODE=NODPLC(LOC1)
731 KNNODE=NODPLC(LOC1+1)
733 KPCNODE=NODPLC(LOC1+2)
734 KNCNODE=NODPLC(LOC1+3)
736 IF (KPNODE.F2.0) GO TO 310
737 KPNODE=JUNODE(KPNODE)
740 310 IF (KNNODE.E0.0) GO TO 315
741 KNNODE=JUNODE(KNNODE)
742 315 IF (KPCNODE.E0.0) GO TO 320
743 KPCNODE=JUNODE(KPCNODE)
744 320 IF (KNCNODE.E0.0) GO TO 325
745 KNCNODE=JUNODE(KNCNODE)
746 325 PRINT 330, NAME(I,1),VALUE(I,1),NAME(I,2),VALUE(I,2),KPNODE,
      1 KNNODE,KPCNODE,KNCNODE
330 FORMAT (2(3X,R7.3X,E10.3),4(3X,I5))
776 335 CONTINUE

```

C INCORPORATE SCATTERING-PARAMETERS NORMALISING RESISTANCE INTO STACKS.

```

1001 338 ISTART=LOCATE(2)
1002 ISTOP=LOCATE(5)-1
1004 IF (ISTOP.LT.ISTART) GO TO 3005
1007 K=ISTOP
1007 J=ISTOP+2
1011 DO 3000 I=ISTART,ISTOP

```

C MOVE ELEMENTS DOWN BY TWO LOCATIONS IN STACK.

```

1023 NAME(J,1)=NAME(K,1)
1026 NAME(J,2)=NAME(K,2)
1027 LOCAL(J)=LOCAL(K)
1031 VALUE(J,1)=VALUE(K,1)
1035 VALUE(J,2)=VALUE(K,2)
1036 K=K-1
1037 J=J-1
1040 3000 CONTINUE

```

C AMEND LOCATE ARRAY TO REFLECT NEW STATUS OF STACK.

```

1045 3005 DO 3010 I=2,5
1052 3010 LOCATE(I)=LOCATE(I)+2

```

C INSERT RNORM ACROSS INPUT AND OUTPUT PORTS.

```

1056 I=LOCATE(2)-1
1060 VALUE(I,1)=RNORM
1061 LOCAL(I)=LOCPRT1
1062 I=I-1
1063 VALUE(I,1)=RNORM
1064 LOCAL(I)=LOCPRT2

```

C INVERT RESISTANCES AND INDUCTANCES.

```

1067 336 ISTART=1
1070 ISTOP=LOCATE(2)-1
1072 DO 337 I=ISTART,ISTOP
1100 337 VALUE(I,1)=1.0/VALUE(I,1)
1102 I=ISTART+LOCATE(3)

```

```

1103      ISTOP=LOCATE(4)-1
1105      IF (ISTOP.LT.ISTART) GO TO 340
1110      DO 339 I=ISTART,ISTOP
1115      339 VALUE(I,1)=1.0/VALUE(I,1)
C
C ESTABLISH ADMITTANCE MATRIX SPARSE POINTER SYSTEM.
C
1117      340 CALL SETUP
1120      IF (NUGO.EQ.1) GO TO 2000
C
C PROCESS OPTIMISATION REQUESTS.
C
1122      PRINT 344
1122      344 FORMAT (//////40X,*** OPTIMISATION SUMMARY ***//)
C
C ESTABLISH OPTIMISATION POINTER SYSTEM.
C
1126      PRINT 345, NITER,NPRINT
1126      345 FORMAT (//////5X,'A MAXIMUM OF *.14.* ITERATIONS WILL BE MADE'//5X,
1*PRINT-OUTS WILL BE MADE EVERY *.14.* ITERATIONS'//5X,*** ELEMENT
25 TO BE OPTIMISED ---//)
1136      DO 396 I=1,NOPY
1141      NAME=IDOPT(I)
1142      NAMELET=NAME/ISHIFT
1145      PRINT 346, NAME
1145      346 FORMAT (/5X,R7)
1153      IF (NAMELET.NE.ILETRY) GO TO 350
1155      NPLACE=1
1156      ISTART=1
1157      ISTOP=LOCATE(2)-3
1161      IDOPT(I)=1
1163      GO TO 370
1166      350 IF (NAMELET.NE.ILETC) GO TO 355
1166      NPLACE=1
1167      ISTART=LOCATE(2)
1170      ISTOP=LOCATE(3)-1
1172      IDOPT(I)=2
1175      GO TO 370
1175      355 IF (NAMELET.NE.ILETL) GO TO 360
1177      NPLACE=1
1180      ISTART=LOCATE(3)
1201      ISTOP=LOCATE(4)-1
1203      IDOPT(I)=3
1206      GO TO 370
1206      360 IF (NAMELET.NE.ILETG) GO TO 365
1210      NPLACE=1
1211      ISTART=LOCATE(4)
1212      ISTOP=LOCATE(5)-1
1214      IDOPT(I)=4
1217      GO TO 370
1217      365 IF (NAMELET.NE.ILETT) GO TO 380
1221      NPLACE=2
1222      ISTART=LOCATE(4)
1223      ISTOP=LOCATE(5)-1
1225      IDOPT(I)=5
1230      370 IF (ISTART.GT.ISTOP) GO TO 380
1234      DO 375 J=ISTART,ISTOP
1235      IF (NAME.NE.NAME(J,NPLACE)) GO TO 375

```

```

1241      JLOC(I)=J
1243      GO TO 390
1244      375 CONTINUE
1246      380 PRINT 385
1247      385 FORMAT (1/5X,***** ERROR IN ABOVE ELEMENT SPECIFICATION*)
1252      NOGO=1
1253      390 CONTINUE
1256      IF (NOGO.EQ.0) GO TO 393
1257      392 FORMAT (1/5X,*** ERRORS IN OPTIMISATION REQUESTS, EXECUTION HALT
1262      10 ***)
      GO TO 2000
C
C LOAD INITIAL PARAMETER VALUES INTO X.
C
1263      393 DO 394 J=1,NOPT
1271      JLOC=JLOC(J)
1272      NPLACE=1
1273      IF (1DOPT(J).EQ.5) NPLACE=2
1276      394 X(J)=VALUE(JLOC,NPLACE)
C
1303      PRINT 395
1304      395 FORMAT (1/5X,*** OPTIMISATION RESULTS ***)
C
C CALL THE OPTIMISATION ROUTINE.
C
1306      CALL MINIMUM
C
C PRINT OUT FINAL CIRCUIT ELEMENT VALUES.
C
1307      PRINT 405
1308      405 FORMAT (1/5X,*** FINAL CIRCUIT ELEMENT VALUES ***)
C
C PASSIVE ELEMENTS.
C
1313      DO 410 I=1,3
1315      ISTART=LOCATE(I)
1316      ISTOP=LOCATE(I+1)-1
1320      IF (I.EQ.1) ISTOP=ISTOP-2
1323      IF (ISTOP.LT.ISTART) GO TO 410
1326      DO 415 J=ISTART,ISTOP
1330      VAL=VALUE(J,I)
1333      IF ((I.EQ.1).OR.(I.EQ.3)) VAL=1.0/VAL
1336      PRINT 406, NAME(J,I),VAL
1337      406 FORMAT (1/5X,R7,* = *,E10.3)
1353      407 CONTINUE
1356      410 CONTINUE
C
C VOLTAGE CONTROLLED CURRENT SOURCES.
C
1360      ISTART=LOCATE(4)
1361      ISTOP=LOCATE(5)-1
1363      IF (ISTOP.LT.ISTART) GO TO 2000
1366      DO 415 I=ISTART,ISTOP
1367      415 PRINT 420, NAME(I,1),VALUE(I,1),NAME(I,2),VALUE(I,2)
1368      420 FORMAT (1/5X,R7,* = *,E10.3,10X,R7,* = *,E10.3)
C
C EXIT.

```


1411 C 2000 CALL SECOND(TIME2)
 1413 ET=TIME2-TIME1
 1415 PRINT 2005, ET
 2005 FORMAT (////5X, *----- TOTAL JOB TIME = *, F10.5, * SECONDS*)
 1423 GO TO 15
 1424 END

PROGRAM LENGTH INCLUDING I/O BUFFERS
 003230

FUNCTION ASSIGNMENTS

STATEMENT	ASSIGNMENTS
15	- 000010 20 - 001442 23 - 000030 24 - 000031
25	- 001445 30 - 000040 31 - 000046 35 - 001461
40	- 000065 45 - 000072 50 - 000075 51 - 001467
52	- 000115 53 - 001475 65 - 000126 70 - 000145
75	- 000150 80 - 000154 90 - 000175 95 - 000205
100	- 000221 105 - 000232 115 - 000251 120 - 001503
125	- 000264 135 - 000301 145 - 000307 155 - 000333
165	- 001517 170 - 000341 175 - 001526 176 - 000350
177	- 001536 180 - 000357 185 - 001546 186 - 000366
187	- 001556 188 - 000375 189 - 001566 190 - 000404
191	- 001576 200 - 000413 205 - 001605 210 - 001627
225	- 001633 230 - 000514 235 - 000516 240 - 001643
245	- 000534 250 - 000536 251 - 001650 255 - 001655
256	- 001670 259 - 000633 260 - 000635 265 - 000640
270	- 001676 275 - 001703 280 - 000671 285 - 000673
290	- 001714 300 - 000714 305 - 001721 310 - 000741
315	- 000743 320 - 000745 325 - 000747 330 - 001747
336	- 001070 338 - 001002 340 - 001120 344 - 001755
345	- 001763 346 - 002003 350 - 001165 355 - 001176
360	- 001207 365 - 001220 370 - 001231 375 - 001244
380	- 001247 385 - 002005 390 - 001254 392 - 002014
393	- 001264 394 - 001277 395 - 002024 405 - 002032
406	- 002042 410 - 001357 420 - 002046 900 - 000325
905	- 001511 2000 - 001412 2005 - 002054 3005 - 001046

BLOCK NAMES AND LENGTHS

KILL	- 000002	OPTIMUM-	000177	MATRIX -	002200	PORTDAT-	001040
NETDAT	- 002114	CARD	- 000044				

VARIABLE ASSIGNMENTS

ANG	- 002141	ET	- 002161	FIELD	- 000000506	I	- 002133
ICARD1	- 000043506	ID	- 000041506	IDNU	- 001230503	IDOPT	- 000042502
INALT	- 000042506	IETC	- 002116	IETG	- 002120	IETL	- 002117
IETR	- 002115	IETT	- 002121	IPONT	- 002134	IPART1	- 002126
IEXPORT	- 002127	IShift	- 002114	ISTART	- 002150	ISTOP	- 002135
ITITLE	- 002104	IWEIGHT	- 002131	J	- 002140	JEOF	- 000001501
JLOC	- 002156	JLOCX	- 000040502	JUNODE	- 001420503	K	- 002153
KNCNODE	- 002152	KNNODE	- 002144	KPCNODE	- 002151	KPNODE	- 002143
LOCAL	- 000750505	LOCATE	- 001142505	LOCPT1	- 001036504	LOCPT2	- 001637504
LOC1	- 002136	MATLOC	- 001230503	NAM	- 002154	NAME	- 000000505
NAMLET	- 002155	NITER	- 000102502	NODE	- 002137	NODPLC	- 001147505
NOGO	- 000000501	NOPT	- 000100502	NOSTOP	- 002125	NPLACE	- 002145

SUBROUTINE PARSE

ROUTINE TO INTERPRET A CARD READ UNDER ALPHANUMERIC FORMAT.

ELEMENT CARD

ID NUMBER	ELEMENT TYPE	CARD CODE
1	RESISTOR	R (22B)
2	CAPACITOR	C (03B)
3	INDUCTOR	L (14B)
4	V.C.C.S. (G4 AND TQ)	G (07B)
5	INPUT PORT	I (11B)
6	OUTPUT PORT	O (17B)
7	WEIGHTS	W (27B)
8	S-PARAMETERS	S (23B)

CONTROL CARD

ID NUMBER	OPERATION	CARD CODE
9	OPTIMISE	.OP (17209)
10	END	.EN (05168)

COMMON DECLARATIONS

```
COMMON /KILL/ NOGS,JEOP,  
COMMON /CARD/ FIELD(32),NUMFLD,ID,INWLT,ICARD1  
DIMENSION IOEL(8),ILEL(10)  
DATA IOEL /228,038,148,078,118,178,278,238/  
DATA ISHIFT/1008/  
DATA JSHIFT/10000000000000008/  
DATA IBLNK/5555555555555555B/  
DATA ISPACE,IASTK,IPEM,ICOMMA,IZERO,IPLUS,IMINUS,ILETE/558,478,  
578,568,338,458,468,058/  
DATA JOPT/17208/  
DATA JEND/05168/
```

INITIALISE.

```
NUMFLD=0
ID=0
IHALT=0
IF (ICARD1.EQ.0) GO TO 20
```

FEAD A CARD.

```

400 READ (5,401) (IFIELD(I),I=1,10)
401 FORMAT (10R8)
    IF (EOF,5) 405,410
405 PRINT 406
406 FORMAT (///5X,--- INCOMPLETE INPUT DATA. PROGRAM EXECUTION TERMINAT

```

```

21      1ED --*//)
22      JEOF=1
      RETURN
C
C REMOVE BLANK FIELDS.
C
23      *10 ISTOP=10
24      *15 IF (IFIELD(ISTOP).NE.13LNK) GO TO 420
27      ISTOP=ISTOP-1
30      IF (ISTOP.LT.1) GO TO 400
31      GO TO 415
32      *20 IF (ICARD1.EQ.0) GO TO 430
33      ICARD1=0
34      GO TO 20
34      *30 NOFLD=0
35      JUMP=15
36      GO TO 301
C
C CHECK FOR PRESENCE OF CONTINUE CARD.
C
37      *35 IF (ICHAR.EQ.ISPACE) GO TO 300
41      IF (ICHAR.NE.IASTK) RETURN
43      PRINT 500, (IFIELD(I),I=1,ISTOP)
52      IF (IHALT.EQ.1) GO TO 400
54      GO TO 55
C
C FIRST PASS.
C
55      20 PRINT 500, (IFIELD(I),I=1,ISTOP)
64      NOFLD=0
65      JUMP=1
66      GO TO 301
67      25 IF (ICHAR.EQ.ISPACE) GO TO 300
71      IF (ICHAR.EQ.IPER) GO TO 31
C
C ELEMENT CARD, GET ID.
C
73      DO 30 I=1,8
74      IF (ICHAR.NE.IDEL(I)) GO TO 30
76      ID=I
77      GO TO 95
77      30 CONTINUE
101     GO TO 50
C
C CONTROL CARD, PACK LEADING PERIOD AND FIND ID.
C
102     31 JUMP=14
103     GO TO 300
104     32 JCHAR=ICHAR
105     JUMP=2
107     GO TO 300
107     35 ICHAR=JCHAR+ISHIFT+ICHAR
112     IF (ICHAR.NE.JOPT) GO TO 36
114     ID=9
114     JUMP=3
116     GO TO 300
116     40 IF (ICHAR.EQ.ISPACE) GO TO 55
120     IF (ICHAR.EQ.ICOMMA) GO TO 61

```

```

122      GO TO 300
122      36 IF (ICAR.NE.JEND) GO TO 50
124      3D=10
125      RETURN

```

C UNIDENTIFIABLE CARD ERROR MESSAGE.

```

126      50 INALT=1
127      PRINT 51
127      51 FORMAT (//5X,***** ABOVE CARD IS UNIDENTIFIABLE**//)
133      GO TO 400

```

C THIS LOOP IS USED FOR ALL PASSES OTHER THAN THE FIRST.

C IF LAST FIELD WAS ENDED BY A SPACE, IGNORE SPACES AND FIRST COMMA.

```

134      55 JUMP=4
135      GO TO 300
136      60 IF (NOFLD.GT.ISTOP) GO TO 400
142      IF (ICAR.EQ.ISPACE) GO TO 300
144      IF (ICAR.NE.ICOMMA) GO TO 65

```

C IF LAST FIELD WAS ENDED BY A COMMA, IGNORE ONLY SPACES.

```

145      61 IF (NUMFLD.EQ.32) GO TO 400
147      JUMP=5
150      GO TO 300
151      62 IF (NOFLD.GT.ISTOP) GO TO 400
155      IF (ICAR.EQ.ISPACE) GO TO 300
157      IF (ICAR.NE.ICOMMA) GO TO 65

```

C TWO COMMAS SEPARATED BY SPACES DENOTES A ZERO-VALUED FIELD.

```

160      NUMFLD=NUMFLD+1
162      FIELD(NUMFLD)=0.0
163      GO TO 61

```

C CHECK WHETHER NUMBER OR NAME.

```

164      65 IF (ICAR.GE.IZERO) GO TO 125

```

C NAME FIELD.

```

167      95 IF (NUMFLD.EQ.32) GO TO 212
171      NUMFLD=NUMFLD+1
172      NAM=0
173      ICHK=0
173      ICOUNT=1
175      100 NAM=NAM*ISHIFT+ICAR
200      ICOUNT=ICOUNT+1
202      IF (ICOUNT.GT.7) GO TO 110
205      JUMP=6
206      GO TO 300
206      105 IF (ICAR.EQ.ISPACE) GO TO 118
210      IF (ICAR.EQ.ICOMMA) GO TO 117
212      GO TO 100

```

C

C FINISH THE FIELD.

```

212 110 FIELD(NUMFLD)=NAM
215     JUMP=7
216     GO TO 300
216 115 IF (ICAR.EQ.ISPACE) GO TO 55
220     IF (ICAR.EQ.ICOMMA) GO TO 61
222     GO TO 300

```

C PACK THE FIELD WITH BLANKS.

```

222 117 ICHK=1
223 118 DO 120 I=ICOUNT,7
231 120 NAM=NAM+IShift*ISPACE
234     FIELD(NUMFLD)=NAM
236     IF (ICHECK) 55,55,61

```

C NUMBER FIELD.

```

240 125 IF (NUMFLD.EQ.32) GO TO 212
242     ICHK=0
242     NUMFLD=NUMFLD+1
244     NUM=0
245     IEXP=0
245     ISET=0

```

C CHECK FOR A SIGN.

```

246     ISIN=0.
246     IF (ICAR.EQ.IPLUS) ISIN=1
252     IF (ICAR.EQ.IMINUS) ISIN=-1
255     IF (ISIN.NE.0) GO TO 130
256     ISIN=1
257     JUMP=9
260     GO TO 138
261 130 JUMP=8
262     GO TO 300
263 135 IF (ICAR.EQ.ISPACE) GO TO 300
265     JUMP=9
266 138 IF (ICAR.EQ.ICOMMA) GO TO 205
270     IDIGIT=ICAR-IZERO
271     IF (IDIGIT.LT.0) GO TO 140
273     IF (IDIGIT.GT.9) GO TO 140
276     NUM=NUM*10+IDIGIT
277     IF (ISET.EQ.0) GO TO 300
301     IEXP=IEXP+1
302     GO TO 300

```

C CHARACTER IS NOT A DIGIT.

C CHECK FOR SPACE, DECIMAL POINT, E-NOTATION.

```

303 140 IF (ICAR.EQ.ISPACE) GO TO 206
305     IF (ICAR.EQ.IPER) GO TO 155
307     IF (ICAR.EQ.ILETE) GO TO 160
310     GO TO 210

```

C DECIMAL POINT.

C
311 155 IF (ISET.EQ.1) GO TO 210
313 ISET=1
314 GO TO 300

C
C E-FORMAT.

C
315 160 ITEMP=0
316 JUMP=11
317 GO TO 300
320 165 IF (ICCHAR.EQ.1SPACE) GO TO 300
322 IEXSIN=0
322 IF (ICCHAR.EQ.IPLUS) IEXSIN=1
325 IF (ICCHAR.EQ.IMINUS) IEXSIN=-1
330 IF (IEXSIN.NE.0) GO TO 170
331 IEXSIN=1
332 JUMP=13
333 GO TO 178
334 170 JUMP=12
335 GO TO 300
336 175 IF (ICCHAR.EQ.1SPACE) GO TO 300
340 JUMP=13
341 178 IF (ICCHAR.EQ.ICOMMA) GO TO 179
343 IF (ICCHAR.EQ.1SPACE) GO TO 180
345 IDIGIT=ICCHAR-IZERO
346 IF (IDIGIT.GT.9) GO TO 210
351 ITEMP=ITEMP*10+IDIGIT
353 GO TO 300
353 179 ICHK=1
354 180 IEXP=IEXP+IEXSIN*ITEMP
357 GO TO 206

C
C ASSEMBLE THE NUMBER.

C
360 205 ICHK=1
361 206 IF (NUM) 207,209,207
362 207 IF (IEXP.GT.40) GO TO 210
366 IF (IEXP.LT.-40) GO TO 209
367 IF (IEXP.EQ.0) GO TO 208
370 VAL=NUM*10.0**IEXP
374 IF (VAL.GT.1.0E20) GO TO 210
400 IF (VAL.LT.1.0E-40) GO TO 209
402 FIELD(NUMFLD)=ISIN*VAL
404 IF (ICHECK) 55,55,61
406 208 FIELD(NUMFLD)=ISIN*NUM
412 IF (ICHECK) 55,55,61
414 209 FIELD(NUMFLD)=0.0
416 IF (ICHECK) 55,55,61

C
C ERROR MESSAGES.

C
420 210 IHALT=1
421 PRINT 211, NUMFLD
211 FORMAT (//5X,----- ABOVE CARD HAS AN ERROR IN FIELD #.15//)
427 GO TO 400
430 212 IHALT=1
431 PRINT 213

```

435 213 FORMAT (/5X,*----- ABOVE CARD HAS MORE THAN 32 FIELDS*/)
      GO TO 400
C
C LOCAL SUBROUTINE TO GET NEXT CHARACTER. THE POINT OF RETURN IS
C DETERMINED BY #JUMP#.
C
436 300 IFDPOS=IFDPOS+1
440 IF (IFDPOS.LT.9) GO TO 305
442 301 NOFLD=NOFLD+1
444 IF (NOFLD.GT.1STOP) GO TO 310
447 IFLD=IFIELD(NOFLD)
450 IFDPOS=1
451 305 ICHAR=IFLD/JSHIFT
454 IFLD=(IFLD-ICHAR*JSHIFT)*JSHIFT
460 GO TO 315
461 310 ICHAR=ISPACE
463 315 GO TO (25,35,40,60,62,103,115,135,138,210,165,175,178,32,
      1 435), JUMP
506 500 FORMAT (5X,10R8)
      END

```

SUBPROGRAM LENGTH
000653

FUNCTION ASSIGNMENTS

STATEMENT	ASSIGNMENTS
20	- 000056 25 - 000070 30 - 000100 31 - 000103
32	- 000105 35 - 000110 36 - 000123 40 - 000117
50	- 000127 51 - 000526 55 - 000135 60 - 000137
61	- 000146 62 - 000152 65 - 000165 95 - 000170
100	- 000176 105 - 000207 110 - 000213 115 - 000217
117	- 000223 118 - 000224 125 - 000241 130 - 000262
135	- 000264 138 - 000267 140 - 000304 155 - 000312
160	- 000316 165 - 000321 170 - 000335 175 - 000337
178	- 000342 179 - 000354 180 - 000355 205 - 000361
206	- 000362 207 - 000363 208 - 000407 209 - 000415
210	- 000421 211 - 000540 212 - 000431 213 - 000547
300	- 000437 301 - 000443 305 - 000452 310 - 000462
315	- 000464 400 - 000005 401 - 000513 405 - 000016
406	- 000516 410 - 000024 415 - 000025 420 - 000033
430	- 000035 435 - 000040 500 - 000556

BLOCK NAMES AND LENGTHS

KILL - 000002 CARD - 000044

VARIABLE ASSIGNMENTS

FIELD	- 000000S02	I	- 000630	IATK	- 000617	IBLNK	- 000615
ICARD1	- 000043S02	ICHAR	- 000634	ICBK	- 000637	ICOMMA	- 000621
ICOUNT	- 000640	ID	- 00041S02	IDEL	- 000571	IDIGIT	- 000645
IEXP	- 000642	IEXSIN	- 000647	IFDPOS	- 000651	IFIELD	- 000601
IFLD	- 000652	IHALT	- 000042S02	ILETE	- 000625	IMINUS	- 000624
IPER	- 000620	IPLUS	- 000623	ISET	- 000643	ISHIFT	- 000613
ISIN	- 000644	ISPACE	- 000616	ISTOP	- 000631	ITEMP	- 000646
IZERO	- 000622	JCHAR	- 000635	JEND	- 000627	JEOF	- 00001S01
JOPT	- 000626	JSHIFT	- 000614	JUMP	- 000633	NAM	- 000636

SUBROUTINE SETUP

C ROUTINE TO ESTABLISH THE SPARSE-MATRIX POINTER SYSTEM.

C
C
C COMMON DECLARATIONS.

```
COMMON /KILL/ NOGO,JEOP
COMMON /COMPLEX/ YDATTUM,YNL,VODATTUM,VN
COMMON /MATRIX/ YDATTUM,YNL(300),VODATTUM,VN(30),MATLOC(486)
DIMENSION KNODES(4)
COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5)
1 NODPLC(484),NUMNOD
COMMON /POINTS/ IORDER(30),JMNODE(30),IUC(300),IUR(31),MEMO(300)
1 IUS,ILS,LASTUT,LASTLT
DIMENSION ITABLE(30,5)
EQUIVALENCE (ITABLE(1,1),MATLOC(1))
DATA IUS/30/
DATA ILS/165/
DATA MIRROR/135/
```

C
C SET UP NODE TABLE.

```
6 DO 5 I=1,NUMNOD
10 5 ITABLE(I,1)=0
11 DO 50 ID=1,4
12 ISTART=LOCATE(ID)
13 ISTOP=LOCATE(ID,1)-1
14 IF (ISTART.GT.ISTOP) GO TO 50
15 NODEND=2
16 IF (ID.EQ.4) NODEND=4
17 DO 40 I=ISTART,ISTOP
18 LOC=LOCAL(I)
19 JSTOP=LOC+NODEND-1
20 DO 40 J=LOC,JSTOP
21 NODE=NODPLC(J)
22 IF (NODE.EQ.0) GO TO 40
23 ISPOT=ITABLE(NODE,1)+2
24 ITABLE(NODE,ISPOT)=1
25 ITABLE(NODE,1)=ISPOT-1
26 40 CONTINUE
27 50 CONTINUE
```

C
C SET UP IUC AND IUR POINTERS.

C THE POINTERS ARE SET UP TO PRESERVE MAXIMUM SPARSENESS OF THE MATRIX.

```
57 IUCEND=0
60 DO 2000 I=1,NUMNOD
63 IUR(I)=IUCEND+1
65 ISTOP=ITABLE(I,1)+1
67 DO 1800 J=2,ISTOP
72 IELNUM=ITABLE(I,J)
73 LOC=LOCAL(IELNUM)
74 KNOSTP=2
75 IF (IELNUM.GE.LOCATE(4)) KNOSTP=4
76 DO 1025 L=1,KNOSTP
102 LOC=LOC-L-1
112 NODE=NODPLC(LOC)
113
```



```

114      KNODES(L)=NODE
115 1025 CONTINUE
C
C NOW SET THE POINTER. IUR POINTS TO THE LOCATION OF THE FIRST
C CROSS-TERM FOR THAT NODE. IUR IS THE CROSS-TERM NODE.
C
117      DO 1400 K=1,KNOSTP
120      NODEC=KNODES(K)
121      IF (NODEC.EQ.0) GO TO 1400
123      IF (NODEC.EQ.1) GO TO 1400
124      ISTART=IUR(I)
125      IF (IUCEND.LT.ISTART) GO TO 1320
130      DO 1310 L=ISTART,IUCEND
135 1310 IF (NODEC.EQ.IUC(L)) GO TO 1400
140 1320 IUCEND=IUCEND+1
142      IF (IUCEND.LT.271) GO TO 1330
144      NOGO=1
145      PRINT 1321
1321 FORMAT (///10X,*** MATRIX OVERFLOW TYPE 1. EXECUTION HALTED ---//)
150      RETURN
151 1330 IUC(IUCEND)=NODEC
153 1400 CONTINUE
156 1000 CONTINUE
160 2000 CONTINUE
163      IUR(NUMNOD+1)=IUCEND+1
C
C MAKE OPTIMAL REORDERING OF NODAL ADMITTANCE MATRIX.
C
165      CALL OPTORD
166      IF (NOGO.EQ.1) RETURN
C
C ESTABLISH MATRIX LOCATIONS.
C
C
C TWO-TERMINAL ELEMENTS.
C
171      IFIND=0
172      ISTOP=LOCATE(4)-1
174      IF (ISTOP.LT.1) GO TO 130
176      DO 120 I=1,ISTOP
177      LOC=LOCAL(I)
201      NODE1=NODPLC(LOC)
202      NODE2=NODPLC(LOC+1)
204      IF (NODE1.EQ.NODE2) GO TO 106
206      MATLOC(IFIND+1)=NODE1
207      MATLOC(IFIND+2)=NODE2
211      IF (NODE1.EQ.0) GO TO 110
211      IF (NODE2.EQ.0) GO TO 110
212      ISPOT=INDEX(NODE1,NODE2)
214      MATLOC(IFIND+3)=ISPOT
217      IF (ISPOT.GT.1LS) GO TO 106
221      MATLOC(IFIND+4)=ISPOT+4*ERROR
223      IFIND=IFIND+4
224      GO TO 120
224 106 MATLOC(IFIND+4)=ISPOT+4*ERROR
227      IFIND=IFIND+4
230      GO TO 120
231 103 MATLOC(IFIND+1)=0

```

```

232 MATLOC(IFIND+2)=0
233 110 MATLOC(IFIND+3)=0
235 MATLOC(IFIND+4)=0
236 IFIND=IFIND+4
237 120 CONTINUE

C VOLTAGE CONTROLLED CURRENT SOURCES.
C
242 130 ISTART=LOCATE(4)
243 ISTOP=LOCATE(5)-1
245 IF (ISTART.GT.ISTOP) GO TO 160
251 DO 150 I=ISTART,ISTOP
253 LOC=LOCAL(I)
254 NODE1=NODPLC(LOC)
255 NODE2=NODPLC(LOC+1)
257 NODE3=NODPLC(LOC+2)
260 NODE4=NODPLC(LOC+3)
262 MATLOC(IFIND+1)=INDEX(NODE1,NODE3)
266 MATLOC(IFIND+2)=INDEX(NODE1,NODE4)
272 MATLOC(IFIND+3)=INDEX(NODE2,NODE3)
276 MATLOC(IFIND+4)=INDEX(NODE2,NODE4)
302 IFIND=IFIND+4
303 150 CONTINUE

C SET UP THE ARRAY #MEMO#.
C
306 160 LE=NUMNOD-1
310 IF (LE.EQ.0) GO TO 175
311 ISQ=0
312 DO 171 L=1,LE
313 IS=IUR(L)
314 IE=IUR(L+1)-1
316 IF (IE.LT.IS) GO TO 171
321 DO 170 IL=IS,IE
323 IO=IUC(IL)
324 I=IORDER(IO)
326 DO 165 IU=IS,IE
331 JO=IUC(IU)
332 J=IORDER(JO)
334 J=INDEX(I,J)
336 ISQ=ISQ+1
340 IF (ISQ.GT.300) GO TO 180
343 MEMO(ISQ)=J
344 165 CONTINUE
346 170 CONTINUE
350 171 CONTINUE
353 175 LASTUT=IUS+IUR(NUMNOD+1)-1
356 LASTLT=ILS-IUS+LASTUT
360 RETURN
361 180 NOGO=1
362 PRINT 185
185 FORMAT (///10X,!-- MATRIX OVERFLOW TYPE 2. EXECUTION HALTED ---//)
366 RETURN
367 END

```

SUBROUTINE OPTORD

RENUMBERS THE ROWS OF A STRUCTURALLY SYMMETRIC SPARSE MATRIX IN ORDER TO MINIMISE THE NUMBER OF MAJOR ARITHMETIC OPERATIONS REQUIRED TO DECOMPOSE IT INTO LU FORM.

COMMON DECLARATIONS.

COMMON /KILL/ NOGO,JEOP
COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
1 NODPLC(484),NUMNOD
COMMON /POINTS/ IORDER(30),JMNODE(30),IJC(300),IUR(31),MEMO(300),
1 IUS,ILS,LASTUT,LASTLT
DIMENSION ILC(30),ILR(30),NUMOFF(30),ITA(30),IFILL(30)
EQUIVALENCE (NUMOFF(1),ILC(1),MEMO(1))
EQUIVALENCE (ITA(1),ILR(1),MEMO(31))
EQUIVALENCE (IFILL(1),MEMO(61))

NN=NUMNOD+1
LE=NUMNOD-1

ASCERTAIN THE NUMBER OF OFF-DIAGONAL ELEMENTS IN EACH ROW.

DO 1 J=1,NUMNOD
IORDER(J)=J
JMNODE(J)=J
1 NUMOFF(J)=IUR(J,1)-IUR(J)
IF (LE.EQ.0) GO TO 110

FIND ROWS WITH ONLY ONE OFF-DIAGONAL ELEMENT.

LOAD=1
IRO=1
16 IF (IRO.LT.LOAD) IRO=LOAD
IRO=IORDER(IRO)
NUM=1
15 IF (NUMOFF(IRO)-1) 18,20,17
17 IRO=IRO+1
IF (IRO-NUMNOD) 16,16,40
18 NUM=0

LOAD THE NEWLY SELECTED ROW.

20 ITEMP=IORDER(LOAD)
IROT=JMNODE(IRO)
IF (IROT.GT.NUMNOD) GO TO 21
IF (IROT.LT.LOAD) GO TO 21
IORDER(IROT)=ITEMP
JMNODE(ITEMP)=IROT
IORDER(LOAD)=IRO
JMNODE(IRO)=LOAD
LOAD=LOAD+1
IF (LOAD-NUMNOD) 21,110,110

SUBTRACT THE COLUMN ELEMENTS OF THE ROW JUST LOADED FROM THE RESPECTIVE ROWS IN WHICH THE TERM ALSO APPEARS.

```

57 21 IF (NUM) 350,30,23
58 23 ICS=IUR(IR+1)-1
59 ICT=IUR(IR)
60 24 IR=IJC(ICT)
61 IF (JMNODE(IR)-LOAD) 29,25,25
62 25 NUMOFF(IR)=NUMOFF(IR)-1
63 28 ICT=ICT+1
64 IF (ICT-ICS) 24,24,30
65 30 IRO=LOAD
66 GO TO 16

```

C CALCULATE THE FILL-IN OF EACH REMAINING ROW IF IT WERE TO
 C BE LOADED NEXT. A FILL-IN OCCURS AT THE INTERSECTION OF EACH OFF-
 C DIAGONAL ELEMENT ROW AND COLUMN. FOR LOADING A ROW, THE POINTER IS
 C SET IN FOR THE FILL-INS.

```

104 40 IPUTIN=0
105 41 IROF=LOAD
106 42 IR=IORDER(IROF)
107 IFILL(IROF)=0
108 ICS=IUR(IR+1)-1
109 NUM=0
110 ICT=IUR(IR)
111 43 IC=IJC(ICT)
112 IF (JMNODE(IC)-LOAD) 45,44,44
113 44 NUM=NUM+1
114 ITA(NUM)=IC
115 45 ICT=ICT+1
116 IF (ICT-ICS) 43,43,46

```

C DETERMINE FILL-INS OF REMAINING ROWS.

```

134 46 NE=NUM-1
135 I=1
136 47 IF (I.GT.NE) GO TO 59
137 IRT=ITA(I)
138 J=I+1
139 48 IF (J.GT.NUM) GO TO 56
140 IC=ITA(J)
141 ICS=IUR(IRT+1)-1
142 ICT=IUR(IRT)
143 51 IF (IUC(ICT).EQ.IC) GO TO 55
144 ICT=ICT+1
145 IF (ICT-ICS) 51,51,53
146 53 IFILL(IROF)=IFILL(IROF)+1
147 IF (IPUTIN.EQ.0) GO TO 55

```

C ADJUST IUC AND IUR.

```

172 IDOWN=IUR(NUM)-IUR(IRT+1)
173 DO 155 L=1, IDOWN
174 K=IUR(NUM)-L
175 155 IUC(K+1)=IUC(K)
176 IUC(K)=IC
177 NUMOFF(IRT)=NUMOFF(IRT)+1
178 JS=IRT+1
179 DO 156 L=JS, NUM
180 156 IUR(L)=IUR(L)+1

```

```

225 IF (IUR(NN).GT.300) GO TO 350
231 IDOWN=IUR(NN)-IUR(IC+1)
233 DO 157 L=1,100*NN
245 K=IUR(NN)-L
246 157 IUC(K+1)=IUC(K)
252 IUC(K)=IRT
254 NUMOFF(IC)=NUMOFF(IC)+1
255 JS=IC+1
256 DO 158 L=JS,NN
262 158 IUR(L)=IUR(L)+1
264 55 J=J+1
266 GO TO 48
266 56 I=I+1
270 GO TO 47
270 59 IF (IPUTIN.EQ.0) GO TO 60
271 IPUTIN=0
272 GO TO 20
272 60 IF ((IFILL(IROF).GT.0) GO TO 61
275 GO TO 20
275 61 IROF=IROF+1
277 IF (IROF-NUMNOD) 42,42,78

```

C SEARCH FOR THE ROW THAT WILL CAUSE THE MINIMUM NUMBER OF
C FILL-INS. IF TWO OR MORE ROWS SATISFY THIS CONDITION, SELECT
C THE ROW WITH THE GREATEST NUMBER OF OFF-DIAGONAL ELEMENTS.

```

301 78 DO 75 J=LOAD,LE
304 ITEST=IORDER(J)
305 KS=J+1
307 DO 74 K=KS,NUMNOD
320 IF ((IFILL(K).GE.IFILL(J)) GO TO 74
323 IR=IORDER(K)
324 IORDER(K)=ITEST
324 JMNODE(ITEST)=K
325 IORDER(J)=IR
327 JMNODE(IR)=J
327 ITEST=IR
331 IR=IFILL(K)
331 IFILL(K)=IFILL(J)
332 IFILL(J)=IR
333 74 CONTINUE
336 IF ((IFILL(LOAD).NE.IFILL(J)) GO TO 76
341 75 CONTINUE
343 76 KE=J-1
345 ITEST=IORDER(LOAD)
347 KS=LOAD+1
352 IF (KE.LT.KS) GO TO 80
352 DO 78 K=KS,KE
352 IR=IORDER(K)
363 IF (NUMOFF(IR).LE.NUMOFF(ITEST)) GO TO 78
366 IORUEH(K)=ITEST
367 JMNODE(ITEST)=K
370 IORDER(LOAD)=IR
371 JMNODE(IR)=LOAD
372 ITEST=IR
373 78 CONTINUE
376 80 IPUTIN=1
377 GO TO 41

```

141

END

SUBPROGRAM LENGTH

000165

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

1	-	000041	2	-	000044	4	-	000020	5	-	000064
6	-	000113	7	-	000116	10	-	000137			

BLOCK NAMES AND LENGTHS

NETDAT - 002114 POINTS - 001267 MATRIX - 002200

VARIABLE ASSIGNMENTS

I	-	000160	IC	-	000163	IORDER	-	000000302	IJ	-	000161
IUC	-	000074S02	IUE	-	000162	IUR	-	00050502	JB	-	000164
JMNODE	-	000036S02	L	-	000157	LE	-	000146	LOCAL	-	000750S01
LOCATE	-	001142S01	MATLOC	-	001230S03	MEMO	-	000607S02	NAME	-	000000S01
MODPLC	-	001147S01	NUMNOD	-	002113S01	U	-	000076S03	UL	-	000514S03
VALUE	-	000364S01	VDATUM	-	001132S03	VN	-	001134S03	VDATUM	-	000000S03
YNL	-	000002S03									

START OF CONSTANTS

000144

START OF TEMPORARIES

000145

START OF INDIRECTS

000147

UNUSED COMPILER SPACE

024500

SUBROUTINE SOLVEA

C ROUTINE TO PERFORM ADJOINT FORWARD AND BACKWARD SUBSTITUTION
C ON THE DECOMPOSED NODAL ADMITTANCE EQUATIONS.

C
C
C COMMON DECLARATIONS.

COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
1 NODPLC(488),NUMNOD
COMMON /POINTS/ IORDER(30),JMNODE(30),IJC(300),IUR(31),MEMO(300),
1 IUS,ILS,LASTUT,LASTLT
COMPLEX YDATUM,YNL,VDATUM,VN
COMMON /MATRIX/ YDATUM,YNL(300),VDATUM,VN(30),NATLOC(488)
COMPLEX U,UL
DIMENSION U(135),UL(135)
EQUIVALENCE (U(1),YNL(31)),(UL(1),YNL(166))

C
LE=NUMNOD-1
3 IF (LE.GT.0) GO TO 4

C
C ACCOUNT FOR CIRCUITS HAVING ONLY ONE NODE.

5 L=IORDER(1)
5 VN(L)=VN(L)/YNL(L)
17 RETURN

C
C FORWARD SUBSTITUTION.

17 4 DO 5 I=1,LE
21 L=IORDER(I)
24 VN(L)=VN(L)/YNL(L)
35 IU=IUR(I)
36 IUE=IUR(I+1)-1
40 1 IF (IU-IUE) 2,2,5
43 2 IC=IJC(IU)
45 IC=IORDER(IC)
46 VN(IC)=VN(IC)-U(IU)*VN(L)
61 IU=IU+1
62 GO TO 1
63 5 CONTINUE

C
C BACKWARD SUBSTITUTION.

66 L=IORDER(NUMNOD)
67 VN(L)=VN(L)/YNL(L)
101 DO 10 I=1,LE
102 L=IORDER(NUMNOD-I)
105 IU=IUR(NUMNOD-I)
106 IUE=IUR(NUMNOD-I+1)-1
112 6 IF (IU-IUE) 7,7,10
117 7 JB=IJC(IU)
120 JB=IORDER(JB)
122 VN(L)=VN(L)-UL(IU)*VN(JB)
134 IU=IU+1
136 GO TO 6
136 10 CONTINUE
141 RETURN

SUBROUTINE SOLVE

ROUTINE TO PERFORM FORWARD AND BACKWARD SUBSTITUTION ON THE DECOMPOSED
NODAL ADMITTANCE EQUATIONS.

COMMON DECLARATIONS.

COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
I MODPLC(484),NUMNOD
COMMON /POINTS/ IORDER(30),JHNODE(30),IUC(300),IUR(31),MEMO(300),
I IUS,ILS,LASTUT,LASTLT
COMPLEX YDATUM,YNL,VDATUM,VN
COMMON /MATRIX/ YDATUM,YNL(300),VDATUM,VN(30),MATLOC(488)
COMPLEX U,UL
DIMENSION U(135),UL(135)
EQUIVALENCE (U(1),YNL(31)),(UL(1),YNL(166))

LE=NUMNOD-1
IF (LE.GT.0) GO TO 5

ACCOUNT FOR CIRCUITS HAVING ONLY ONE NODE.

L=IORDER(1)
VN(L)=VN(L)/YNL(L)
RETURN

FORWARD SUBSTITUTION.

5 DO 20 J=1,LE
21 JS=IUR(J)
22 JE=IUR(J+1)-1
24 IF (JE.LT.JS) GO TO 20
27 L=IORDER(J)
30 DO 15 IL=JS,JE
40 IO=IUC(IL)
41 I=IORDER(IO)
43 15 VN(I)=VN(I)-UL(IL)*VN(L)
61 20 CONTINUE

BACKWARD SUBSTITUTION.

L=IORDER(NUMNOD)
VN(L)=VN(L)/YNL(L)
DO 40 I=1,LE
100 L=IORDER(NUMNOD-I)
103 JS=IUR(NUMNOD-I)
104 JE=IUR(NUMNOD-I+1)-1
107 IF (JE.LT.JS) GO TO 40
112 DO 25 IU=JS,JE
122 JO=IUC(IU)
123 J=IORDER(JO)
125 25 VN(I)=VN(L)-U(IU)*VN(J)
142 40 VN(L)=VN(L)/YNL(L)
155 RETURN
156 END


```

104 26 PRINT 27
27 FORMAT (//5X, 'UNDERFLOW DURING LU DECOMPOSITION OF ADMITTANCE MATR
11X, EXECUTION TERMINATED')
110 NOGO=1
111 RETURN
112 END

```

SUBPROGRAM LENGTH
000162

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

```

21 - 000020 22 - 000041 25 - 000102 26 - 000165
27 - 000117

```

BLOCK NAMES AND LENGTHS

```

KILL - 000002 NETOAT - 002114 POINTS - 001267 MATRIX - 002200

```

VARIABLE ASSIGNMENTS

```

EPS - 000150 IE - 000156 IL - 000157 IORDER - 000000S03
IS - 000155 ISO - 000151 IU - 000160 IUC - 000074S03
IUR - 000550S03 J - 000161 JMNODE - 000036S03 KK - 000153
L - 000154 LE - 000152 LOCAL - 000750S02 LOCATE - 001142S02
MATLOC - 001230S04 MEMO - 000687S03 NAME - 000000S02 NDOPLC - 001147S02
NOGO - 000000S01 NUMNOD - 002113S02 U - 000076S04 UL - 000514S04
VALUE - 000300S02 VDATUM - 001132S04 VN - 001134S04 VDATUM - 000000S04
YNL - 000002S04 YPARTS - 000002S04

```

START OF CONSTANTS

000115

START OF TEMPORARIES

000131

START OF INDIRECTS

000140

UNUSED COMPILER SPACE

024600

SUBROUTINE DECOMP

C PERFORMS LU DECOMPOSITION ON THE COMPLEX ADMITTANCE MATRIX.
C
C YNL COMPLEX SPARSE MATRIX OF SYMMETRIC STRUCTURE.
C IUR(L) LOCATION IN Y 1ST NON-ZERO ELEMENT IN ROW L.
C IUC(IU) REORDERED COLUMN NUMBER FOR ELEMENT IU IN U.
C IORDER(I) STORED IN THIS ARRAY ARE THE USER-SPECIFIED NODE NUMBERS
C APPEARING IN THE ORDER IN WHICH DECOMPOSITION
C WILL PROCEED. AN LU DECOMPOSITION OF THE REARRANGED
C MATRIX IS THEN DONE TAKING ADVANTAGE OF SPARSITY.
C IO REORDERED MATRIX ROW NUMBER.
C JO REORDERED MATRIX COLUMN NUMBER.
C IL UL ARRAY INDEX NUMBER.
C IU U ARRAY INDEX NUMBER.
C J ORIGINAL ADMITTANCE MATRIX COLUMN NUMBER.
C I ORIGINAL ADMITTANCE MATRIX ROW NUMBER.

C COMMON DECLARATIONS.

C
COMMON /KILL/ NOGO,JEOP
COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
1 NOOPLC(484),NUMNOD
COMMON /PRINTS/ IORDER(30),JMNODE(30),IUC(300),IUR(31),MEMO(300),
1 IUS,ILS,LASTUT,LASTLT
COMMON YDATUM,YNL,YDATUM,VN
COMMON /MATRIX/ YDATUM,YNL(300),YDATUM,VN(30),YATLOC(488)
DIMENSION YPARTS(2,300)
EQUIVALENCE (YPARTS(1,1),YNL(1))
COMMON U,UL
DIMENSION U(135),UL(135)
EQUIVALENCE (U(7),YNL(3)), (UL(1),YNL(166))
DATA EPS/1.0E-20/

C BEGIN LU DECOMPOSITION:

C
1 ISQ=0
2 LE=NUMNOD-1
4 KK=IORDER(NUMNOD)
5 IF (ABS(YPARTS(1,KK))+ABS(YPARTS(2,KK)),LE,EPS) GO TO 26
16 IF (LE.EQ.0) RETURN
C
17 21 DO 25 L=1,LE
21 IS=IUR(L)
22 IE=IUR(L+1)-1
24 IF (IE.LT.IS) GO TO 25
27 KK=IORDER(L)
30 IF (ABS(YPARTS(1,KK))+ABS(YPARTS(2,KK)),LE,EPS) GO TO 26
40 22 DO 24 IL=IS,IE
42 UL(IL)=UL(IL)/YNL(KK)
53 DO 23 IU=IS,IE
62 ISQ=ISQ+1
63 J=MEMO(ISQ)
65 YNL(J)=YNL(J)-UL(IL)*U(IU)
75 23 CONTINUE
76 24 CONTINUE
101 25 CONTINUE
104 RETURN

```

162 LOC4=MATLOC(IFIND+4)
164 IFIND=IFIND+4
165 YPARTS(2,LOC1)=YPARTS(2,LOC1)+VAL
171 YPARTS(2,LOC2)=YPARTS(2,LOC2)+VAL
174 YPARTS(2,LOC3)=YPARTS(2,LOC3)-VAL
176 YPARTS(2,LOC4)=YPARTS(2,LOC4)-VAL
200 30 CONTINUE

```

```

C
C LOAD RECIPROCAL INDUCTANCES.
C

```

```

206 40 ISTART=LOCATE(3)
207 ISTOP=LOCATE(4)-1
211 IF (ISTART.GT.ISTOP) GO TO 60
215 DO 50 I=ISTART,ISTOP
230 VAL=-VALUE(I,1)/FHED
233 LOC1=MATLOC(IFIND+1)
234 LOC2=MATLOC(IFIND+2)
236 LOC3=MATLOC(IFIND+3)
240 LOC4=MATLOC(IFIND+4)
242 IFIND=IFIND+4
243 YPARTS(2,LOC1)=YPARTS(2,LOC1)+VAL
247 YPARTS(2,LOC2)=YPARTS(2,LOC2)+VAL
252 YPARTS(2,LOC3)=YPARTS(2,LOC3)-VAL
254 YPARTS(2,LOC4)=YPARTS(2,LOC4)-VAL
256 50 CONTINUE

```

```

C
C LOAD VOLTAGE-CONTROLLED CURRENT SOURCES.
C

```

```

264 60 ISTART=LOCATE(4)
265 ISTOP=LOCATE(5)-1
267 IF (ISTART.GT.ISTOP) RETURN
272 DO 70 I=ISTART,ISTOP
274 Z=FREQ*VALUE(I,2)
275 VALREAL=VALUE(I,1)*COS(Z)
301 VALIMAG=-VALUE(I,1)*SIN(Z)
317 LOC1=MATLOC(IFIND+1)
321 LOC2=MATLOC(IFIND+2)
323 LOC3=MATLOC(IFIND+3)
326 LOC4=MATLOC(IFIND+4)
330 IFIND=IFIND+4
332 YPARTS(1,LOC1)=YPARTS(1,LOC1)+VALREAL
336 YPARTS(2,LOC1)=YPARTS(2,LOC1)+VALIMAG
342 YPARTS(1,LOC2)=YPARTS(1,LOC2)-VALREAL
346 YPARTS(2,LOC2)=YPARTS(2,LOC2)-VALIMAG
351 YPARTS(1,LOC3)=YPARTS(1,LOC3)+VALREAL
354 YPARTS(2,LOC3)=YPARTS(2,LOC3)+VALIMAG
357 YPARTS(1,LOC4)=YPARTS(1,LOC4)-VALREAL
362 YPARTS(2,LOC4)=YPARTS(2,LOC4)-VALIMAG
365 70 CONTINUE
370 RETURN
370 END

```

SURPROGRAM LENGTH
060434

FUNCTION ASSIGNMENTS

SUBROUTINE LOADY(FREQ)

LOADS THE COMPLEX ADMITTANCE MATRIX.

COMMON DECLARATIONS.

```
COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
1 NODPLC(484),NUMNOD
COMMON /POINTS/ IORDER(30),JMNODE(30),IUC(300),IUR(31),MEMO(300),
1 IUS,ILS,LASTUT,LASTLT
COMPLEX YDATUM,YNL,VDATUM,VN
COMMON /MATRIX/ YDATUM,YNL(300),VDATUM,VN(30),MATLOC(488)
DIMENSION YPARTS(2,300)
EQUIVALENCE (YPARTS(1,1),YNL(1))
```

IFIND=0

INITIALISE ADMITTANCE MATRIX TO ZERO.

```
YDATUM=CHPLX(0.0,0.0)
DO 99 I=1,NUMNOD
  YPARTS(1,I)=0.000
  99 YPARTS(2,I)=0.000
DO 999 I=IUS,LASTUT
  YPARTS(1,I)=0.000
  999 YPARTS(2,I)=0.000
DO 9999 I=ILS,LASTLT
  YPARTS(1,I)=0.000
  9999 YPARTS(2,I)=0.000
```

LOAD CONDUCTANCES.

```
ISTOP=LOCATE(2)-1
IF (ISTOP.LT.1) GO TO 20
DO 10 I=1,ISTOP
  VAL=VALUE(I,1)
  LOC1=MATLOC(IFIND+1)
  LOC2=MATLOC(IFIND+2)
  LOC3=MATLOC(IFIND+3)
  LOC4=MATLOC(IFIND+4)
  IFIND=IFIND+4
  YPARTS(1,LOC1)=YPARTS(1,LOC1)+VAL
  YPARTS(1,LOC2)=YPARTS(1,LOC2)+VAL
  YPARTS(1,LOC3)=YPARTS(1,LOC3)-VAL
  YPARTS(1,LOC4)=YPARTS(1,LOC4)-VAL
  10 CONTINUE
```

LOAD CAPACITANCES.

```
20 ISTART=LOCATE(2)
ISTOP=LOCATE(3)-1
IF (ISTART.GT.ISTOP) GO TO 40
DO 30 I=ISTART,ISTOP
  VAL=VALUE(I,1)*FREQ
  LOC1=MATLOC(IFIND+1)
  LOC2=MATLOC(IFIND+2)
  LOC3=MATLOC(IFIND+3)
```

SUBROUTINE SCALE(GRAD)

C SUBROUTINE SCALES PARAMETERS BEING OPTIMISED SO AS TO
 C IMPROVE CONVERGENCE OF THE MINIMISATION ROUTINE. SCALING IS DONE
 C TO NORMALISE TO UNITY THE GRADIENT VECTOR AT THE COMMENCEMENT
 C OF OPTIMISATION.

COMMON DECLARATIONS.

COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122),LOCATE(5),
 1 NODPLC(484),NUMNOD
 COMMON /OPT/MUM/ WEIGHT(4),JLOCX(30),IDOPT(30),NOPT,NPRIN,
 1 NITER,X(30),SCALEX(30)
 DIMENSION GRAD(30)
 DATA EPS/1.0E-10/

C VERIFY THAT SCALING IS POSSIBLE. IF NOT, SET SCALE TO UNITY.

DO 5 I=1,NOPT
 IF (ABS(GRAD(I)).GT.EPS) GO TO 5
 PRINT 2
 11 2 FORMAT (//5X,*.AT LEAST ONE COMPONENT OF THE INITIAL UNSCALED GRADIENT VECTOR IS TOO SMALL TO PERMIT SCALING*//5X,*.SCALES WILL THEREFORE BE SET TO UNITY*)
 DO 3 J=1,NOPT
 15 3 SCALEX(J)=1.0
 23 4 RETURN
 24 5 CONTINUE
 25

C COMPUTE VECTOR OF SCALE FACTORS.

DO 10 I=1,NOPT
 Z=GRAD(I)
 X(I)=X(I)*Z
 30 SCALEX(I)=1.0/Z
 36 10 GRAD(I)=1.0
 37 RETURN
 40
 41
 42

C UNSCALE X-VECTOR AND UPDATE ELEMENT STACKS.

ENTRY DESCALE
 DO 20 I=1,NOPT
 43 LOC=JLOCX(I)
 51 J=1
 60 IF (IDOPT(I).EQ.5) J=2
 61 20 VALUE(LOC,J)=X(I)*SCALEX(I)
 62 RETURN
 65
 73

C RESCALE THE GRADIENT VECTOR.

ENTRY RESCALE
 DO 30 I=1,NOPT
 74 30 GRAD(I)=GRAD(I)*SCALEX(I)
 102 RETURN
 107
 111 END
 112

```

563      DGDG=DGDG+DGDG
565      DGDG=DGDG
566      145  IJ=0
567      DO 150 I=1,NOPT
572      W=DELX(I)/DGDG
573      Z=WORK(I)/DGDG
576      DO 150 J=1,NOPT
605      IJ=IJ+1
606      150  HESS(IJ)=HESS(IJ)+W*DELX(J)-Z*WORK(J)
C
C INCREMENT THE ITERATION COUNTERS.
C
616      ITN=ITN+1
620      NFIRST=NFIRST+1
621      GO TO 20
621      END

```

SUBPROGRAM LENGTH
002243

FUNCTION ASSIGNMENTS

STATEMENT	ASSIGNMENTS
10	000626 11 - 000022 15 - 000040 20 - 000045
30	000072 45 - 000177 50 - 000677 51 - 000214
52	000715 55 - 000227 60 - 000730 65 - 000241
70	000257 71 - 000750 72 - 000274 84 - 000401
85	000771 90 - 000411 91 - 000440 115 - 000444
119	000466 125 - 000522 145 - 000567 500 - 000635
505	000650 510 - 000673

BLOCK NAMES AND LENGTHS

KILL - 000002 NETDAT - 002114 OPTIMUM- 000177

VARIABLE ASSIGNMENTS

ALFA	001101	BOUND	002234	CONVTOL	002212	DELB	001175
DELX	001137	DGDG	002241	DGDG	002242	ERANEN	002236
ERROR	002213	GDX	002225	GO	002224	GDX	002237
GPDG	002240	GRAD	001043	HESS	001271	I	002222
IDOPT	000042503	IFIRST	002232	II	002227	IJ	002221
ITN	002216	J	002223	JLOX	000004503	K	002231
KPRINT	002215	LIMIT	002233	LOCAL	000750502	LOCATE	001142502
NAME	000000502	NFCALL	002214	NFIRST	002217	NITER	000102503
NOOPLC	001147502	NOGO	000000501	NOPT	000100503	NPRINT	000101503
SCALEX	000141503	STEP	002220	VALUE	000364502	W	002235
WEIGHT	000000503	WORK	001233	X	000103503	Z	002226
ZZ	002230						

START OF CONSTANTS
000624

START OF TEMPORARIES
001017

START OF INDIRECTS
001035

```

353 IF ((IFIRST.GT.NOPT).AND.(GPDZ.LT.0.0)) GO TO 84
363 IF ((IFIRST.EQ.0).AND.((GPDZ.GT.0.0).OR.(ERRNEW.GT.ERROR)))
1 GO TO 90
374 ZZ=BOUND
377 GO TO 91
400 84 ITN=ITN+1
402 PRINT 85, ITN
85 FORMAT (//5X,*ULTRA-QUADRATIC BEHAVIOUR DETECTED IN THE ERROR FUNC
TION ON THE *I4,*-TH ITERATION*/5X,*REVERSION TO STEEPEST DESCEN
2T NECESSARY*)
407 GO TO 11
C
C INTERPOLATE CUBICALLY TO ESTIMATE A STEP-SIZE IF A STEP-SIZE
C OF ONE DID NOT LEAD TO A SUFFICIENT DECREASE IN THE ERROR FUNCTION,
C AND IF THE INTERVAL BOUNDARY CONDITIONS ARE CORRECT.
C
410 90 ZZ=3.0*(ERROR-ERRNEW)+3PDX*GDDX
415 W=SQRT(1.0-GDDX/ZZ*GPDZ/ZZ)*ABS(ZZ)
424 ZZ=1.0-(GPDZ+W-ZZ)/(GDDX+2.0*W)
432 IF (ZZ.LT.BOUND) ZZ=BOUND
436 IFIRST=1
437 91 BOUND=BOUND+0.1
441 LIMIT=LIMIT+1
442 GO TO 70
443 115 ERROR=ERRNEW
445 DO 116 I=1,NOPT
452 116 GRAD(I)=WORK(I)
454 IF (GDDX.GT.0.0) GO TO 119
456 GDX=GPDZ
456 ZZ=4.0
460 BOUND=0.1
461 LIMIT=LIMIT+1
463 IFIRST=0
464 GO TO 70
465 119 STEP=ZZ
466 IF (GPDZ.LT.(0.5*GDDX)) STEP=2.0*STEP
C
C UPDATE THE INVERSE HESSIAN MATRIX.
C
474 DGHG3=0.0
475 DO 135 I=1,NOPT
477 Z=0.0
477 IJ=I
501 IF (I.EQ.1) GO TO 125
503 II=I-1
504 DO 120 J=1,II
513 Z=Z+HESS(IJ)*DELG(J)
515 120 IJ=IJ+NOPT-J
521 125 DO 130 J=I,NOPT
530 Z=Z+HESS(IJ)*DELG(J)
532 130 IJ=IJ+1
535 DGHG3=DGHG3+Z*DELG(I)
540 135 WORK(I)=Z
543 IF (DGHG3.LT.0.0) DGHG3=GDDX*0.01
546 IF (DGHG3.LT.DGHG3) GO TO 145
551 W=1.0+DGHG3/DGHG3
553 DO 140 I=1,NOPT
560 140 DELX(I)=W*DELX(I)-WORK(I)

```

```

C
C CHECK FOR ROUND-OFF ERRORS OR SINGULAR HESSIAN.
C SHOULD THESE CONDITIONS OCCUR, THE SEARCH WILL BE IN AN ASCENT
C DIRECTION.
C
226 55 IF (GDX.LT.0.0) GO TO 65
230 ITN=ITN+1
231 PRINT 65, ITN
60 FORMAT (//5X,'ROUND-OFF ERRORS OR SINGULAR HESSIAN ON *I4,*-TH IT
237 1ERATION. REVERSION TO STEEPEST DESCENT! NECESSARY*)
GO TO 11

C
C LINEAR SEARCH.
C
C
C ESTIMATE A SUITABLE STEP SIZE.
C
240 65 ZZ=1.0
241 IFIRST=0
242 LIMIT=0
243 BOUND=0.1
244 IF (NFIEST.LT.NOPT) ZZ=STEP
250 W=-2.0*ERROR/GDX
252 IF (d.LT.ZZ) ZZ=W
256 70 IF (LIMIT.LT.10) GO TO 72
261 ITN=ITN+1
262 PRINT 71, ITN,ERRNEW
71 FORMAT (//5X,'INTERPOLATION LIMIT IN LINEAR SEARCH ENCOUNTERED ON
ITHE *I4,*-TH ITERATION. EXECUTION TERMINATED*//5X,'FINAL VALUE OF
2 THE ERROR = *E10.2)
272 RETURN
273 72 GDX=GDX*ZZ
275 DO 75 I=1,NOPT
284 Z=ALFA(I)*ZZ
285 DELX(I)=Z
286 75 X(I)=X(I)+Z
287 CALL DESCALE(WORK)
288 CALL ERRFCN(WORK,ERRNEW)
289 IF (NOGO.EQ.1) RETURN
290 CALL HESCAL(WORK)
291 NFCALL=NFCALL+1
292 GPOX=0.0
293 DO 80 I=1,NOPT
294 DELG(I)=WORK(I)-GRAD(I)
295 80 GPOX=GPOX+WORK(I)*DELX(I)
DGOX=DPOX-GDX

C
C CHECK FOR SUFFICIENT DECREASE IN THE ERROR FUNCTION DUE TO THE LAST
C STEP TAKEN.
C
336 IF (ERROR.GT.(ERRNEW-1.0E-04*GDX)) GO TO 115
344 DO 81 I=1,NOPT
351 81 X(I)=X(I)-DELX(I)

C
C CHECK FOR ULTRA-QUADRATIC BEHAVIOUR IN THE ERROR FUNCTION PROVIDED .
C AT LEAST NOPT ITERATIONS HAVE BEEN EXECUTED SINCE THE LAST
C REVERSION TO STEEPEST DESCENT.
C

```



```

35     IF (I.EQ.J) HESS(I)=1.0
37     15 IJ=IJ+1
C
C COMPUTE SOME USEFUL QUANTITIES.
C
44     20 GG=0.0
45     GD=0.0
46     DO 40 I=1,NOPT
47     Z=0.0
48     IJ=1
51     IF (I.EQ.1) GO TO 30
53     II=I-1
54     DO 25 J=1,II
63     Z=Z-HESS(IJ)*GRAD(J)
65     25 IJ=IJ+NOPT-J
71     30 DO 35 J=1,NOPT
100    Z=Z-HESS(IJ)*GRAD(J)
102    35 IJ=IJ+1
106    GG=GG+GRAD(I)**2
110    ALFA(I)=Z
111    40 GD=GD+GRAD(I)*Z
115    GG=SQRT(GG)
C
C MAKE A PRINT-OUT IF APPROPRIATE.
C
116    IF (KPRINT.LT.NPRINT) 30 TO 45
121    KPRINT=K
122    PRINT 500, ITN
500    FORMAT (//////15X,***** STATUS OF THE MINIMISATION ROUTINE AFTER
176    176 ITH=*,I4,*,TH ITERATION -----*)
127    PRINT 505, ERROR,ZZ,NFCALL
505    FORMAT (//5X,*ERROR = *,E10.3,5X,*STEP SIZE = *,E10.3,5X,*NUMBER OF
IF CALLS TO ERRFCN = *,I2,///5X,*PARAMETER NAME*,10X,*X-VECTOR ENTRY
Z*,10X,*GRADIENT VECTOR ENTRY*,10X,*SCALE FACTOR*//)
141    DO 515 I=1,NOPT
144    J=JLOCK(I)
145    K=I
146    IF (IDOPT(I).EQ.5) K=2
151    PRINT 510, NAME(J,K),X(I),GRAD(I),SCALEX(I)
510    FORMAT (//10X,R7.3(17X,E10.3))
173    515 CONTINUE
176    45 KPRINT=KPRINT+1
C
C CHECK CONVERGENCE.
C
200    IF (GG.GT.CONVTOL) GO TO 51
203    PRINT 50, ITN,ERROR
50    FORMAT (//5X,*CONVERGENCE ATTAINED IN *,I4,* ITERATIONS OF THE MIN
176    176 IMISATION ROUTINE*///5X,*FINAL VALUE OF THE ERROR = *,E10.3)
212    RETURN
C
C CHECK IF NUMBER OF ITERATIONS EXCEEDS NITER.
C
213    51 IF (ITN.LE.NITER) GO TO 55
216    PRINT 52, NITER,ERROR
52    FORMAT (//5X,*NO CONVERGENCE IN *,I4,* ITERATIONS. EXECUTION HALTE
176    176 D. FINAL VALUE OF THE ERROR = *,E10.3)
225    RETURN

```

SUBROUTINE MINIMUM

C SUBROUTINE TO FIND THE MINIMUM OF THE ERROR FUNCTION USING THE
C JUAL-UPDATING VARIABLE METRIC ALGORITHM OF R. FLETCHER.
C THE METHOD IS FULLY DESCRIBED BY FLETCHER IN: A NEW APPROACH
C TO VARIABLE METRIC ALGORITHMS, THE COMPUTER JOURNAL, VOLUME 13,
C NUMBER 3, AUGUST, 1970.

C ARRAY IDENTITIES ARE AS FOLLOWS.

C X VECTOR OF PARAMETERS BEING OPTIMISED
C DELX STORES THE CHANGE IN THE X VECTOR BETWEEN LAST AND
C PRESENT ITERATIONS
C GRAD VECTOR OF GRADIENT OF PARAMETERS BEING OPTIMISED
C DELG STORES THE CHANGE IN THE GRADIENT VECTOR BETWEEN LAST AND
C PRESENT ITERATIONS
C HESS STORES THE UPPER TRIANGULAR PORTION OF THE INVERSE HESSIAN
C MATRIX BY ROWS
C ALFA STORES THE TRIAL STEP-SIZE FOR THE LINEAR SEARCH
C WORK WORKING SPACE

C COMMON DECLARATIONS.

C COMMON /KILL/ NOGO,JEOP
C COMMON /NETDAT/ NAME(122,2),VALUE(122,2),LOCAL(122), LOCATE(5),
C 1 NODPLC(484),NUMNOD
C COMMON /OPTIMUM/ WEIGHT(4),JLOCK(30),IDOPT(30),NOPT,NPRINT,
C 1 NITER,X(30),SCALEX(30)
C DIMENSION GRAD(30),ALFA(30),DELX(30),DELG(30),WORK(30),HESS(465)

C SET CONVERGENCE CRITERION.

C DATA CONVTOI/.00002/

C INITIAL FUNCTION EVALUATION.

C CALL ERRFCN(GRAD,ERROR)
C IF (NOGO,EQ.1) RETURN
C CALL SCALE(GRAD)
C PRINT 10, ERROR
C 10 FORMAT (/5X,*INITIAL VALUE OF THE ERROR = *,E10.3)

C INITIALISE.

C NFCALL=1
C KPRINT=0
C ITN=0
C 11 NFIRST=0
C STEP=1.0

C SET INVERSE HESSIAN TO THE UNIT MATRIX.

C 1J=1
C DO 15 I=1,NOPT
C DO 15 J=1,NOPT
C HESS(IJ)=0.0

```

C
305 25 PSI=PSI*CRFREQ
313 GO TO 40
C
C CONTRIBUTIONS OF TRANSCONDUCTANCE OF VOLTAGE CONTROLLED
C CURRENT SOURCES.
C
314 30 ARG=FREQ*VALUE(LOC1,2)
316 PSI=PSI*CMPLX(COS(ARG),-SIN(ARG))
333 GO TO 40
C
C CONTRIBUTIONS OF DELAY OF VOLTAGE CONTROLLED CURRENT SOURCES.
C
335 35 ARG=FREQ*VALUE(LOC1,2)
337 GOMEG=-FREQ*VALUE(LOC1,1)
341 PSI=PSI*CMPLX(GOMEG*SIN(ARG),GOMEG*COS(ARG))
366 40 SDEL(II,ICOL)=SDEL(II,ICOL)*PSI
374 SDEL(II,JCOL)=SDEL(II,JCOL)*PSI
402 45 CONTINUE
404 ICOL=3
405 JCOL=4
406 1000 CONTINUE
410 S(1)=S(1)-1.0
414 S(4)=S(4)-1.0
C
C ITERATE OVER TWO-PORT PARAMETERS.
C
417 DO 500 K=1,4
C
C FORM THE ERROR FUNCTION.
C
426 WGT=ONEIGHT(K)
426 DIFF=S(K)-PORT(1,K)
433 DIFFRE=REAL(DIFF)
434 DIFFIM=AIMAG(DIFF)
436 ERROR=ERROR+JGT*(DIFFRE**2+DIFFIM**2)
C
C FORM THE GRADIENT OF THE ERROR FUNCTION.
C
442 DO 400 J=1,NOPT
452 GRAD(J)=GRAD(J)+WGT*(DIFFRE*REAL(SDEL(J,K))+
1 DIFFIM*AIMAG(SDEL(J,K)))
457 400 CONTINUE
462 500 CONTINUE
464 5000 CONTINUE
466 RINV=2.0*RINV
470 DO 400 I=1,NOPT
474 300 GRAD(I)=GRAD(I)*RINV
476 RETURN
476 END

```

SUBPROGRAM LENGTH
001550

FUNCTION ASSIGNMENTS

```

62      JNODE2=NODE2
63      IF (IK,EO,2).OR.(K,EO,4)) JNODE2=NODE4
C
C SET NODE VOLTAGE VECTOR TO ZERO.
C
74      DO 5 II=1,NUMNOD
106     VNPART(1,II)=0.0
107     5 VNPART(2,II)=0.0
C
C ESTABLISH EXCITATION.
C
115     VNPART(1,JNODE1)=-1.0
116     VNPART(1,JNODE2)=1.0
120     VDATJN=CMPLX(0.0,0.0)
122     IF (K,GT,2) GO TO 15
C
C PROCESS THE ORIGINAL NETWORK.
C
127     CALL SOLVE
C
C SOLVE FOR SCATTERING PARAMETERS.
C
130     S(NCOL)=RINV*(VN(NODE2)-VN(NODE1))
130     S(NCOL)=RINV*(VN(NODE4)-VN(NODE3))
C
C ACCOUNT FOR CONTRIBUTION TO THE GRADIENT OF THE NETWORK.
C
170     DO 10 II=1,NOPT
204     LOC=JLOCX(II)
205     LOC=LOCAL(LOC)
207     IF (IDOPT(II),GT,3) LOC=LOC+2
213     NODEX1=NODPLC(LOC)
215     NODEX2=NODPLC(LOC+1)
220     V=VN(NODEX2)-VN(NODEX1)
231     SOEL(II,NCOL)=V
233     10 SOEL(II,NCOL)=V
237     NCOL=2
240     NCOL=4
241     GO TO 1000
C
C ACCOUNT FOR CONTRIBUTIONS TO THE GRADIENT OF THE ADJOINT NETWORK.
C
243     15 CALL SOLVEA
244     DO 45 II=1,NOPT
250     LOC1=JLOCX(II)
251     LOC1=LOCAL(LOC1)
252     NODEX1=NODPLC(LOC1)
254     NODEX2=NODPLC(LOC1+1)
256     PSI=VN(NODEX1)-VN(NODEX2)
264     IO=IDOPT(II)
266     GO TO (40,20,25,30,35), IO
C
C CONTRIBUTIONS OF CAPACITORS.
C
276     20 PSI=PSI*CFREQ
304     GO TO 40
C
C CONTRIBUTIONS OF RECIPROCAL INDUCTANCES.

```

SUBROUTINE ERRFCN (GRAD, ERROR)

C SUBROUTINE TO COMPUTE THE ERROR FUNCTION AND ITS GRADIENT
C IN THE SPACE OF PARAMETERS BEING OPTIMISED. THE ERROR FUNCTION
C IS OF THE WEIGHTED SUM-OF-SQUARES TYPE.

COMMON DECLARATIONS.

COMMON /KILL/ N080, JEOP
COMMON /NETDAT/ NAME(122,2), VALUE(122,2), LOCAL(122), LOCATE(5),
1 NODPLC(484), NUMNOD
COMMON /OPTIMUM/ WEIGHT(4), JLOCX(30), IDOPT(30), NOPT, NPRINT,
1 NITER, X(30), SCALEX(30)
COMPLEX PORT
COMMON /PORTDAT/ PORT(60,4), OMEGA(60), NPOINTS, RNORM, LOCPRT1,
1 LOCPRT2
COMPLEX YDATUM, YNL, VDATUM, VN
COMMON /MATRIX/ YDATUM, YNL(300), VDATUM, VN(30), MATLOC(480)
DIMENSION VNPART(2,30)
EQUIVALENCE (VNPART(1,1), VN(1))
COMPLEX CFREQ, CRFREQ, V, PSI, DIFF
COMPLEX S, SDEL
DIMENSION S(4), SDEL(60,4), GRAD(30)

INITIALISE:

ERROR=0.0
DO 2 I=1, NOPT
2 GRAD(I)=0.0
NODE1=NODPLC(LOCPRT1)
NODE2=NODPLC(LOCPRT1+1)
NODE3=NODPLC(LOCPRT2)
NODE4=NODPLC(LOCPRT2+1)
RINV=2.0/RNORM

ITERATE OVER FREQUENCY:

DO 5000 I=1, NPOINTS
FREQ=OMEGA(I)
CFREQ=CMPLX(0.0, FREQ)
CRFREQ=CMPLX(0.0, -1.0/FREQ)

LOAD THE ADMITTANCE MATRIX AND DECOMPOSE IT INTO LU FORM.

CALL LOADY(FREQ)
CALL DECOMP
IF (N080, EQ, 1) RETURN

CALCULATE THE TWO-PORT SCATTERING PARAMETERS AND THEIR GRADIENTS.

NCOL=1
MCOL=3
ICOL=1
JCOL=2
DO 1000 K=1, 4
JNODE1=NODE1
IF ((K, EQ, 2), OR, (K, EQ, 4)) JNODE1=NODE3

FUNCTION INDEX(NODE1,NODE2)

C ROUTINE TO FIND MATRIX LOCATION GIVEN NODE SPECIFICATION.

C COMMON DECLARATIONS.

COMMON /NETDAY/ NAME(122*2),VALUE(122*2),LOCAL(122),LOCATE(5),
1 NOOPLC(484),NUMNOD
COMMON /POINTS/ IORDER(30),JMNODE(36),IUC(300),IUR(31),MEMO(300),
1 IUS,ILS,LASTUT,LASTLT

C

```

5      IF (NODE1.EQ.0) GO TO 15
6      IF (NODE2.EQ.0) GO TO 15
7      IF (NODE1.NE.NODE2) GO TO 1
8      INDEX=NODE1
9      RETURN
10     1 IO=JMNODE(NODE1)
11     JO=JMNODE(NODE2)
12     ISTART=IUS
13     IF (JO.GT.IO) GO TO 4
14     ISTART=ILS
15     ITEMP=JO
16     JO=IO
17     IO=ITEMP
18     4 IF (JO.LE.NUMNOD) GO TO 8
19     ITEMP=IO
20     IO=JO
21     JO=ITEMP
22     8 JS=IUR(IO)
23     JE=IUR(IO+1)-1
24     IF (JE.LT.JS) GO TO 15
25     DO 10 L=JS,JE
26     IF (IUC(L).NE.JO) GO TO 10
27     INDEX=ISTART+L
28     RETURN
29     10 CONTINUE
30     15 INDEX=0
31     RETURN
32     END

```

SUBPROGRAM LENGTH

000076

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

1	=	000011	4	=	000024	8	=	000032	10	=	000046
15	=	000051									

BLOCK NAMES AND LENGTHS

NETDAY - 002114 POINTS - 001267

VARIABLE ASSIGNMENTS

ILS	=	001264502	INDEX	=	000066	IO	=	000067	IORDER	=	000000502
ISTART	=	000071	ITEMP	=	000072	IUC	=	000074502	IUR	=	000550502

C
C NODE RENUMBERING IS COMPLETE. ILC AND ILR ARE NOW SET UP
C ACCORDING TO THE RE-ORDERED NODE NUMBERS.

```

400 110 J=0
401 DO 90 L=1,NUMNOD
402 ILC(L)=J+1
403 IN=IORDER(L)
404 KS=IUR(IN)
405 KE=IUR(IN+1)
406 IF (KS.GT.KE) GO TO 90
407 DO 89 K=KS,KE
408 IN=IJC(K)
409 IRN=JMNODE(IN)
410 IF (IRN.LT.L) GO TO 89
411 J=J+1
412 ILR(J)=IRN
413 89 CONTINUE
414 90 CONTINUE
415 IUTOT=J
416 ILC(NUMNOD+1)=J+1
417 DO 96 J=1,NN
418 IUR(J)=ILC(J)
419 IF (IUTOT.EQ.0) RETURN
420 DO 97 J=1,IUTOT
421 IJC(J)=ILR(J)
422 RETURN

```

C
C STOP REORDERING IN EVENT OF ERROR AND PRINT APPROPRIATE MESSAGE.

```

465 350 NOGO=1
466 PRINT 351
472 351 FORMAT (///10X,--- MATRIX OVERFLOW TYPE 3. EXECUTION HALTED ---//)
473 RETURN
474 END

```

SUBPROGRAM LENGTH
000566

FUNCTION ASSIGNMENTS

STATEMENT ASSIGNMENTS

15	-	000027	16	-	000021	17	-	000032	18	-	000036
20	-	000037	21	-	000060	23	-	000062	24	-	000067
25	-	000074	28	-	000077	30	-	000103	40	-	000105
41	-	000106	42	-	000110	43	-	000120	44	-	000125
45	-	000131	46	-	000135	47	-	000140	48	-	000147
51	-	000161	53	-	000167	55	-	000245	56	-	000267
59	-	000271	60	-	000273	61	-	000276	70	-	000302
74	-	000334	76	-	000344	78	-	000374	80	-	000377
89	-	000435	90	-	000440	110	-	000461	350	-	000466
351	-	000500									

BLOCK NAMES AND LENGTHS

KILL = 000002 NETDAT = 002114 POINTS = 001267