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AN ESTIMATION TECHNIQUE FOR MONTE CARLO CALCULATIONS OF DOSE FROM AN INTERNAL SOURCE OF PHOTONS

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MATHEMATICS DIVISION

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ACKNOWLEDGEMENT

This work was sponsored by the Health Physics Division and was accomplished with the direct assistance of W. S. Snyder of that Division.

AN ESTIMATION TECHNIQUE FOR MONTE CARLO CALCULATIONS
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ABSTRACT

Buildup factors and Monte Carlo techniques have been used to calculate specific absorbed fractions of photon energy in a man's internality. Difficulties are encountered when the distance between the selected source point and the selected target point is greater than 40 mean free path lengths or is quite small. Estimation techniques have been devised to handle these difficulties.

INTRODUCTION

Contained in this presentation are remarks about specific absorbed fractions, that is, fractions of photon energy absorbed per gram of absorbing material, and remarks on how a Monte Carlo estimation technique helps to produce better estimates of these fractions.

Martin J. Berger¹ has communicated to me a tabulation of buildup factors for point sources of monoenergetic photons in an infinite medium of water up to 40 mean free paths. The specific absorbed fraction Φ and the buildup factor B are related by

$$\Phi(x) = \frac{\mu_{ab} e^{-\mu x}}{4\pi x^2} B(\mu x), \quad (1)$$

where x is the distance in centimeters from the source point to a target

¹Martin J. Berger, "Energy Deposition in Water by Photons from Point Isotropic Sources," MIRD Pamphlet No. 2, Supplement No. 1, J. Nucl. Med., 15, (1968).

point, μ_{ab} is the mass absorption coefficient, μ is the attenuation coefficient, and Φ is the fraction of energy emitted by the point photon source which is absorbed per gram of target medium at a distance x from the source point. Snyder, et. al.,² have used B to express $\Phi(T \leftarrow S)$ for extended source and target organs as a six-dimensional integral over the source organ S and the target organ T :

$$\Phi(T \leftarrow S) = \frac{1}{|S||T|} \int_S dx \int_T dy \frac{\mu_{ab} e^{-\mu |X-Y|}}{4\pi |X-Y|^2} B(\mu |X-Y|). \quad (2)$$

A density of 1.0 is assumed. They have suggested that values of $\Phi(T \leftarrow S)$ obtained using B as defined for the infinite space are reasonable approximations to the values of $\Phi(T \leftarrow S)$ in the human body. $\Phi(T \leftarrow S)$ may be used with appropriate radionuclide decay scheme data and residence data to calculate dose to a target organ from sources in other organs.

The integral in (2) is evaluated by a Monte Carlo procedure on a computer. Several thousand X 's and Y 's are chosen uniformly distributed over the source organ and the target organ respectively and then a mean $\Phi(T \leftarrow S)$ is obtained.

Two difficulties are inherent in the use of (2). First of all, Berger's data extend only to 40 mean free paths and these are not adequate for some organ pairs--such as skeleton to other organs. This difficulty has been met by substituting $B(40)$ for $B(\mu |X-Y|)$ when $\mu |X-Y|$ is greater than 40. These substitutions have not affected the first 4 significant digits of the Φ obtained and thus this difficulty is dismissed. The second

²W. S. Snyder, et. al., "Absorbed Fractions for Internal Emitters," Health Physics Div. Ann. Progr. Rept., July 31, 1971, ORNL-4720, pp. 121-124.

difficulty is this: if X and Y are sufficiently close, the estimate of $\Phi(T-S)$ is unbounded. This may happen when one organ includes the other--the source organ and the target organ are the same. This difficulty is met by replacing the estimator

$$\frac{e^{-\mu|X-Y|}}{|X-Y|^2} \text{ by } \frac{3(1-e^{-\mu k})}{\mu k^3}.$$

This is the average of the estimator over a small sphere of radius k whenever $|X-Y| < k$.

For the purpose of demonstration, a unit sphere (one whose radius = 1) was chosen to represent both the source region and the target region. Figure 1 shows a band that indicates the range of all the values of Φ estimated for 7 different computer runs at each of 12 different energies extending over the energy range .015 MeV to 4 MeV. A computer run consisted of 30000 pairs of X's and Y's handled by the Monte Carlo method at each energy. Seven other runs at the same 12 energies but using the new estimator when $|X-Y| < .1$ gave data which was included in this band. As a matter of comment, data for runs where k was chosen to be .02, .05, and .2 also fell within the band. Coefficients of variation for the runs without the new estimator ranged from 1.6% to 11%. Coefficients of variation for the runs with the new estimator ranged from 2.0% to 4.2%.

The advantage of the use of the new estimator comes in the form of a dampening effect on the coefficients of variation of the estimates of Φ . The Monte Carlo evaluation of the integral requires the averaging of many estimates and requires statistics for establishing the reliability of the estimates. Figure 2 shows a mal-formed open band which includes all the

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Fig. 1

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Fig. 2

coefficients of variations for the 7 different runs without the new estimator mentioned above. No functional relationship is implied in this figure. It is only a pictorial representation of what may be described succinctly as, "The statistics vary all over the place." The coefficients of variation for the other runs using the new estimator when $|X-Y| < .1$ are in the solid band. There has been a great smoothing of the statistics which enhances the reliability of the estimates.

Putting this new technique to use and estimating $\Phi(T-S)$ for an organ to itself, estimates were made for the source to target combination of ovaries to ovaries. The mathematical description of these organs is found in MIRD 5.³ Results for one computer run per energy are shown in Figure 3. Again there is not much difference between the estimates based on the two methods. Figure 4 shows the dampening of the errant statistics.

The choice of $k = .1$ was arbitrary but seems to have been a wise choice. There's probably an optimum small sphere radius k for each organ and it may be determined to be related to the volume of the organ with some kind of shape factor thrown in. One needs to look at non-spherical shapes such as ellipsoids or cylinders or various polygons with different cross sections.

Data tables follow.

³W. S. Snyder, et. al., "Estimates of Absorbed Fractions for Monoenergetic Photon Sources Uniformly Distributed in Various Organs of a Heterogeneous Phantom," MIRD Pamphlet No. 5, Supplement No. 3, J. Nucl. Med., 10 (1969).

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Fig. 3

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Fig. 4

UNIT SPHERE DATA FOR $k=.1$

Energy (MeV)	Coefficients of Variation Without New Estimator						
	Run No.						
	1	2	3	4	5	6	7
.015	.07014	.04732	.05003	.05921	.07014	.03650	.04991
.02	.03328	.03467	.02448	.04659	.03328	.03244	.02994
.03	.04452	.03000	.02315	.04569	.04452	.07601	.03041
.05	.02374	.01563	.07555	.01655	.02374	.02115	.02076
.1	.02394	.03640	.04068	.05902	.02394	.02667	.02051
.2	.02695	.05195	.02515	.1073	.02695	.03464	.09892
.5	.05612	.01883	.02117	.03399	.05612	.02121	.02407
1.	.02319	.01744	.02185	.03127	.02319	.1589	.01903
1.5	.03755	.03030	.03387	.06713	.03755	.04131	.1087
2.	.02938	.02966	.03883	.02585	.02938	.09943	.02751
3.	.02422	.02779	.06668	.02315	.02422	.02372	.02891
4.	.03100	.03608	.01932	.05906	.03100	.03167	.03267

Coefficients of Variation With New Estimator ($k = .1$)							
.015	.04133	.04282	.03943	.04231	.04133	.03902	.04035
.02	.02973	.03059	.02867	.02967	.02973	.03246	.03251
.03	.02415	.02423	.02356	.02629	.02415	.02327	.02507
.05	.02291	.02172	.02261	.02018	.02291	.02265	.02138
.1	.02568	.02410	.02356	.02340	.02568	.02308	.02351
.2	.02720	.02510	.02256	.02454	.02720	.02371	.02634
.5	.02582	.02353	.02311	.02514	.02580	.02597	.02555
1.	.02590	.02352	.02496	.02514	.02590	.02834	.02600
1.5	.02670	.02659	.02685	.02548	.02670	.02821	.02781
2.	.02496	.02613	.02785	.02586	.02496	.02548	.02703
3.	.02267	.02509	.02631	.02434	.02267	.02641	.02560
4.	.02597	.02411	.02446	.02660	.02597	.02319	.02601

TABLE OF DATA

(Average of Runs)

Unit Sphere (Mass = $4\pi/3$)

Energy (MeV)	Without Estimator		k=.05		k=.1		k=.2	
	Φ	Coef. of Variation	Φ	Coef. of Variation	Φ	Coef. of Variation	Φ	Coef. of Variation
.015	.1295	.05595	.1289	.05356	.1324	.04096	.1262	.02569
.02	.07510	.03409	.07948	.04905	.07704	.03051	.07460	.01919
.03	.02795	.04502	.02922	.04043	.02765	.02441	.02740	.01599
.05	.008330	.03429	.008003	.02240	.008438	.02207	.008269	.01498
.1	.004840	.03534	.005067	.04182	.004883	.02417	.004769	.01574
.2	.005554	.06243	.005434	.04168	.005423	.02530	.005352	.01684
.5	.005723	.03642	.005845	.03973	.005797	.02560	.005390	.01717
1.	.005402	.06375	.005626	.04588	.005465	.02570	.005673	.01782
1.5	.005084	.05721	.005003	.03697	.005063	.02692	.005061	.01761
2.	.004594	.04694	.004677	.03572	.004678	.02606	.004635	.01729
3.	.003991	.03449	.004008	.03575	.004044	.02477	.004105	.01744
4.	.003631	.03617	.003738	.03582	.003697	.02533	.003772	.01738

TABLE OF DATA
(Average of Runs)

Ovaries (Mass = 8.2675)

Energy (MeV)	Without Estimator		k=.1	
	Φ	Coef. of Variation	Φ	Coef. of Variation
.015	.06629	.1067	.06505	.06602
.02	.03387	.06859	.03415	.04895
.03	.01278	.09575	.01177	.03741
.05	.003429	.02703	.003592	.03568
.1	.002165	.03982	.002212	.03685
.2	.002427	.09964	.002250	.03893
.5	.002484	.03883	.002492	.03509
1.	.002378	.03847	.002428	.03593
1.5	.002061	.03390	.002142	.04043
2.	.002034	.06052	.002059	.04106
3.	ND		ND	
4.	.001602	.05417	.001635	.04138

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