MODEL FOR TWO-PHASE FLOW IN WATER

F. H. Clark

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JUNE 1973

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ABSTRACT

A model for two-phase flow was developed to provide detailed dynamics of water flow in the core of a light-water reactor during blowdown. Further model development has been necessary to meet the enlarged problem scope resulting when nonequilibrium conditions are allowed between the two phases. Moreover, because of apparent disagreement in the textbook methods of representing some effects, a return to prior principles seems mandatory.

Nine thermodynamic regions are defined according to whether there is one or two phases present and also to the relation that the enthalpy of each phase bears to its equilibrium value. Three of these regions are discarded because they have no bearing on problems of interest.

A complete set of equations appropriate to each region is developed, different, in general, for each region. Appropriate region boundary conditions are devised (although there still remains a question about the correct enthalpy conditions at a boundary between a two-phase and a one-phase region).

Two kinds of interphase mass exchange are considered: that which is driven by heat or work input or pressure change, and that which arises from crossings of a phase boundary due to a statistical resolution of differences on the two sides. These are quite different effects, requiring different kinds of coefficients for their description.

The formal structure of the model is one dimensional. Transverse flows and interphase transfers are handled through the conventional strategy of inhomogeneous terms. There is a wide difference of opinion in the literature concerning the treatment of pressure derivatives, the space derivative in the momentum equation, and the temporal derivative which appears when the energy equation is written in terms of enthalpy. One can find the temporal derivative in the same context given as a partial, a total, or a substantial derivative. This study indicates that in the formulation chosen the derivative should be partial.

In the separate phase formulation, a factor α or $(1 - \alpha)$ appears in each pressure derivative term, α being the local volume fraction of gas phase. This investigation indicates that the α and $(1 - \alpha)$ terms should fall within the differential operator, although a number of workers treat it as a coefficient to the operator.

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1. INTRODUCTION

The model of two-phase flow of water presented here was developed to apply to the study of the possible loss of coolant accident in light-water reactors. Its application is by no means limited to such studies, however. The severe and rapid variations in state variables which might be expected from a loss of coolant accident have projected investigations into domains only imperfectly studied in the past. While major efforts must be made in the laboratory to develop a body of observational information relating to these problems, a companion effort of some magnitude must be undertaken to provide appropriate analytical tools.

It is always preferable to operate from information well based in measurement. It must, however, be obvious that to ask for experimental observation of one reactor accident of major proportions may be unreasonable and to ask for experimental observations covering all the ranges of possible reactor accidents is utterly outlandish.

Hence, a computational model tailored to the realities of a reactor core in such a failure situation is a necessity. Only with such a model available is it possible to contemplate extrapolating effects which can within reason be measured to accidents which within reason may be imagined. It has not been found possible to adapt very readily to this severe situation calculation models which were developed for other problems. Hence, we have found it necessary to produce a model to fit the problem.

Three major conditions guided the development:

1. The two phases must be described in a consistent fashion which permits them to be out of equilibrium with each other.

2. The model must adapt readily to the transverse flows and circulatory patterns which could be encountered in a real reactor core.
3. It must be adaptable to an order of computations which has some reasonable chance of producing stable, converged solutions consistently.

There were, of course, other motivations along the way: the attempt to resolve some differences appearing in the open literature, and the desire to do one or two things a little better. But the three items listed we consider essential to analysis of the loss of coolant accident in the core.

2. DEFINITIONS

Subscripts: Subscript s denotes saturation value. Subscripts i and j denote flow channels: i is always the current channel, and when no channel subscript is present i is understood; and j is a running subscript which always excludes i . Subscript k denotes an external heat transfer surface.

Superscripts: Superscript g denotes water in the vapor state or a quantity associated therewith. The superscript ℓ denotes the same as g for the liquid state. Superscript $g\ell$ indicates flow from gas to liquid; and ℓg , liquid to gas.

t time variable

Z space variable (one dimensional)

V volume variable [$V(Z_1, Z_2)$ is the total volume between Z_1 and Z_2] = $V^g + V^\ell$

ρ density

α volume fraction of vapor = $\lim_{V \rightarrow 0} V^g/V$

x	mass fraction of vapor = $\lim_{V \rightarrow 0} \rho^g V^g / (\rho^g V^g + \rho^l V^l)$
v	fluid velocity
S_m	external mass source
S_f	external momentum source
S_h	external enthalpy source
P	pressure
h	specific enthalpy
A_{ki}	k^{th} heat transfer surface per unit volume in i^{th} channel
H_{ki}	heat transfer coefficient belonging to A_{ki}
θ_k	temperature of k^{th} heat transfer surface
T	water temperature
B	effective cross-section area, so that if $\tilde{V}$ = volume per unit length , $B^g = \alpha \tilde{V}$ and $B^l = (1 - \alpha) \tilde{V}$
ψ	angle between flow direction and vertical
g	acceleration of gravity
Gr	friction between fluid and externals (force per unit area per unit length)
Γ	total interphase mass flow, mass per unit total volume (liquid plus gas) per unit time
μ	driven interphase mass flow, mass per unit total volume (liquid plus gas) per unit time
ν	spontaneous interphase mass flow, mass per unit phase separation surface area per unit time
C	interphase separation surface area per unit of gas volume

F	function which specifies slip velocity or α
I_r	interphase friction (units of Gr)
F_r	total friction = $Gr + I_r$
n_{ij}	mass flow from channel i to channel j , mass per unit volume per unit time

3. ASSUMPTIONS FOR A TWO-PHASE FLOW MODEL

1. Only one-dimensional flow is considered.
2. At every point measured along the single space dimension every state variable has a unique value. This eliminates the possibility of computing conditions within a film which are, in general, quite different from those in the neighboring bulk. The assumption does not permit any variations in values transverse to the direction of translation. Effects arising from the special conditions within films must be handled through appropriate correlations.
3. This study is concerned with the thermodynamic behavior of pure water only.* Specifically, the influence of external surfaces on the thermodynamic behavior of the water is not considered. Such externals can make their presence felt through heat transfer correlations and friction factors only. On account of this assumption, even more so than the geometric limit above, films cannot be dealt with explicitly in the model.
4. No solid phases and no supersaturated conditions are considered.
5. At every point the liquid phase is in equilibrium with itself, and the vapor phase with itself, but, in general, not with each other.

* However, things like bubble growth can be permitted to correspond to water with impurities present.

6. Mass can move from one phase to another non-adiabatically (or as a result of pressure change or of work) only if the phase from which it moves is at saturation. This assumption is more or less consistent with the requirement that each phase be in equilibrium with itself. It is not an absolute limitation but could be removed following the methods of ref. 1 with some increase in complexity.

7. The pressure has the same value in each phase at any point. (However, within small bubbles a pressure increment may be included.)

8. The system has constant geometric cross section.

9. The energy conservation equation is written in terms of enthalpy, with friction heating and translational kinetics energy neglected.

4. FUNDAMENTAL EQUATIONS

Let α be the fraction of volume occupied by vapor at some point and x the fraction of mass. For any system quantity f , let us define mass and volume means as follows:

$$f_x = xf^g + (1 - x)f^l ; \quad (1)$$

$$f_\alpha = \alpha f^g + (1 - \alpha)f^l . \quad (2)$$

Then we may, from first principles, write^{2,3}

$$\frac{\partial \rho}{\partial t} \alpha + \frac{\partial}{\partial Z} (\rho v)_\alpha = S_m ; \quad (3)$$

$$\frac{\partial (\rho v)_\alpha}{\partial t} + \frac{\partial}{\partial Z} (\rho v^2)_\alpha = -\frac{dP}{dZ} + \rho_\alpha g \cos \psi - Fr + S_f ; \quad (4)$$

$$\frac{\partial(\rho h)}{\partial t} \alpha + \frac{\partial}{\partial Z} (\rho h v) \alpha = \sum_k \frac{H_{ki} A_{ki}}{B} (\theta_k - T) + \frac{\partial P}{\partial t} + S_h . \quad (5)$$

Further, we have for the equation of state

$$\rho^g = \rho(P, h^g) , \quad (6)$$

$$\rho^l = \rho(P, h^l) , \quad (7)$$

$$T^g = T(P, h^g) , \quad (8)$$

$$T^l = T(P, h^l) , \quad (9)$$

and, under saturation conditions,

$$\rho_s^g = \rho_s^g(P) , \quad (10)$$

$$\rho_s^l = \rho_s^l(P) , \quad (11)$$

$$T_s = T_s(P) , \quad (12)$$

$$h_s^g = h_s^g(P) , \quad (13)$$

$$h_s^l = h_s^l(P) . \quad (14)$$

The three source terms, S_m , S_f , and S_h , appearing in Eqs. (3), (4), and (5) are discussed in Sect. 7.

5. SEPARATION INTO TWO PHASES

Treatment of the vapor and liquid phases separately and explicitly increases the number of variables which must be solved for and, therefore, increases the requisite number of equations. We must then search for a complete set of equations.

It is easy to get two useful equations from each of Eqs. (3) and (5) as follows:

$$\frac{\partial \rho^g \alpha}{\partial t} + \frac{\partial}{\partial Z} \rho^g v^g \alpha = S_m^g + \Gamma^{lg} - \Gamma^{gl} ; \quad (15)$$

$$\frac{\partial \rho^l (1 - \alpha)}{\partial t} + \frac{\partial}{\partial Z} \rho^l v^l (1 - \alpha) = S_m^l + \Gamma^{gl} - \Gamma^{lg} ; \quad (16)$$

$$\begin{aligned} \frac{\partial \rho^g h^g \alpha}{\partial t} + \frac{\partial}{\partial Z} \rho^g h^g v^g \alpha = S_h^g + \sum_k \frac{H_{ki}^g A_{ki}^g}{B^g} \left(\theta_k^g - T^g \right) + \frac{\partial P}{\partial t} + \mu^g h_s^g \\ - \mu^g h_s^g + v^{lg} C \alpha h_s^g - v^{gl} \alpha C h^g ; \end{aligned} \quad (17)$$

$$\begin{aligned} \frac{\partial \rho^l h^l (1 - \alpha)}{\partial t} + \frac{\partial}{\partial Z} \rho^l h^l v^l (1 - \alpha) = S_h^l + \sum_k \frac{H_{ki}^l A_{ki}^l}{B^l} \left(\theta_k^l - T^l \right) + \frac{\partial P (1 - \alpha)}{\partial t} \\ + \mu^g h_s^l - \mu^l h_s^g + v^{gl} h^g \alpha C - v^{lg} h_s^g \alpha C ; \end{aligned} \quad (18)$$

$$\Gamma^{gl} = \mu^g + \alpha C v^{gl} ; \quad (19)$$

$$\Gamma^{lg} = \mu^l + \alpha C v^{lg} . \quad (20)$$

The quantity $\Gamma^{g\ell}$ is the mass transfer (mass per unit total volume per unit time) from gas to liquid phase; $\mu^{g\ell}$ is that portion which is driven by heat source, work, or pressure change. The term $\nu^{g\ell} C$ is the "spontaneous" portion that arises from contact between phases not in equilibrium with each other. It flows across an interphase surface in amounts determined statistically.

$$C = C(\xi_i) , \quad (21)$$

ξ_i are the various system parameters. C is an empirical function which gives the interphase surface area per unit volume of vapor. Hence, $\nu^{g\ell}$ is the spontaneous mass transfer from gas to liquid (mass per unit interphase area per unit time). (The same symbols with g and ℓ reversed designate liquid to gas transfer.) In principle, C could be computed from the rather well developed equations for bubble dynamics and from the various hydraulic parameters of the system. In practice any such computation would be fantastically difficult to incorporate, and we assumed that there is an appropriate empirical function. The empirical rules that enable one to distinguish flow regimes (bubbly, slug, separated, water droplet) come very close to defining the function C . The quantities $\nu^{\ell g}$ and $\nu^{g\ell}$ can be precomputed as functions of pressure and enthalpy only. For pure water an elementary computation from kinetic theory is sufficient.

Consider a closed volume of liquid water and steam in equilibrium in a gravitational field. The two phases are separated with the steam above the horizontal separation surface. There is a continuous exchange of mass between the two phases. Since they are in equilibrium, equal amounts move in each direction. Apart from the negli-

gible pressure difference due to differences in height, both phases are at the same temperature and pressure. One can, from elementary kinetic considerations, compute the mass of vapor striking a unit surface of liquid in unit time. Hence, the mass movement across unit area of interphase surface is a function only of the temperature and pressure of the phase from which it is moving. We denote this quantity by v^{lg} or v^{gl} .

In our model water droplets in steam or steam bubbles in water are not necessarily in temperature equilibrium with their surroundings. Moreover, the spherical phase, if the sphere is small enough, may have a decidedly higher pressure on account of surface tension. At any point, therefore, v^{gl} may be quite different from v^{lg} ; both are computable from pressure and temperature alone, given enough information about bubble or droplet size-- $C(\xi_i)$ --to determine excess pressures. Neither bears any relation to the quantities μ^{lg} or μ^{gl} which are the interphase mass transfers due to heat, work, or pressure change. There exists empirical information to modify such results to account for the presence of impurities.⁴

To these equations let us add another empirical relation:

$$F(\alpha, v^g, v^l) = 0 . \quad (22)$$

This equation can specify slip velocity from some observed relations, i.e., place a condition upon v^g and v^l , or it can specify α from some reasonable constraint. In principle, one could compute slip, given sufficient information about the hydraulic parameters of the system by dealing carefully with the shear forces between liquid and vapor. We shall not attempt this. In regions where one of the phases would be

supersaturated, we have eliminated that phase by choosing α to be 0 or 1. We can consider such choices to be alternative statements of F , i.e.,

$$\alpha = 0 ; \quad (22a)$$

$$\alpha - 1 = 0 . \quad (22b)$$

In a very loose manner of speaking we might consider that Eq. (3) provides us with an equation for velocity and Eq. (5) one for enthalpy. By expansion of Eqs. (3) and (5) into Eqs. (15), (16), (17), and (18), and by addition of Eqs. (19), (20), (21), and (22), Eqs. (3) and (5) are replaced with eight equations. In the expanded formulation the new unknown variables include ρ^g , ρ^l , h^g , h^l , C , α , Γ^{lg} , Γ^{gl} , μ^{lg} , and μ^{gl} . From first appearances, two equations dealing with two unknowns ρ and h were replaced by eight equations dealing with ten unknowns. Closer examination shows, however, that with assumption 6 we can set

$$\mu^{lg} = 0 \quad (23)$$

if the liquid is not at saturation, and

$$\mu^{gl} = 0 \quad (24)$$

if the vapor is not at saturation. Further, if either or both phases are at saturation, there are additional relations from the group Eqs. (10) through (14). It follows that more variables than equations have not been introduced, at least not in this section of the problem.

6. THE MOMENTUM RELATION IN TWO PHASES

In Sect. 5, conservation Eqs. (3) and (5) were replaced with separate equations for each phase and some auxiliary relations were introduced. This operation enabled expansion of the number of available equations to meet the expanded number of variables. In particular, the objective was to go from ρ and h to ρ^ℓ , ρ^g and h^ℓ , h^g , so that however many auxiliary quantities were introduced a net increase of two equations to the set was required. This net increase was achieved.

Under some circumstances, as in separated flow of the two phases, it is desirable to separate Eq. (4), as well as Eqs. (3) and (5), into two parts. It is shown (Appendix, Sect. 12) that this is reasonably done as follows:

$$\frac{\partial}{\partial t} \rho^g v_{\alpha}^g + \frac{\partial}{\partial Z} \rho^g v_{\alpha}^{g^2} = - \frac{d\alpha P}{dZ} + \rho^g g \alpha \cos \psi - Fr^g + S_f^g + \Gamma^{\ell g} v_{\alpha}^{\ell} - \Gamma^{g \ell} v_{\alpha}^g ; \quad (25)$$

$$\begin{aligned} \frac{\partial \rho^{\ell}}{\partial t} v^{\ell} (1 - \alpha) + \frac{\partial}{\partial Z} \rho^{\ell} v^{\ell^2} (1 - \alpha) = & - \frac{d}{dZ} (1 - \alpha) P + \rho^{\ell} g (1 - \alpha) \cos \psi - Fr^{\ell} \\ & + S_f^{\ell} + \Gamma^{g \ell} v_{\alpha}^g - \Gamma^{\ell g} v_{\alpha}^{\ell} . \end{aligned} \quad (26)$$

With this explicit separation of the momentum equation, it is not necessary to provide another relation for slip or for α [as in Eqs. (22), (22a), or (22b)]. In regimes other than separated flow of the phases, it is probably more reasonable and certainly simpler to retain Eq. (4) instead of Eqs. (25) and (26) and add to it Eq. (22), (22a), or (22b).

The friction effects in Eqs. (25) and (26) may sometimes be conveniently broken down as follows:

$$Fr^{\ell} = Gr^{\ell} + Ir ; \quad (27)$$

$$Fr^g = Gr^g - Ir ; \quad (28)$$

$$Fr = Fr^{\ell} + Fr^g . \quad (29)$$

Gr represents frictional force between the fluid and the containing walls, and Ir represents shearing forces between the liquid and the vapor phases. If it could be assumed that measurements of two-phase flow could be analyzed carefully enough to abstract the quantities Gr^{ℓ} , Gr^g , and Ir , one might expect to generalize a fairly limited group of such experiments over a very broad range of two-phase flow. Having data available in such form or having a model available to assist in analyzing measurements to yield such data would be more than adequate justification for use of a separated momentum relation. With friction data available only for combined effects of the two phases, one would not expect to be able to generalize it very much beyond the conditions for which it was measured.

7. THE SOURCE TERMS

The terms S_m , S_f , and S_h appearing in Eqs. (3), (4), and (5), respectively, and the subsequently appearing derived terms S_m^g , S_m^{ℓ} , S_f^g , S_f^{ℓ} , S_h^g , and S_h^{ℓ} are inhomogeneous source terms. They are included in this model to provide a convenient method for dealing with cross flow. An inhomogeneous term is known or is computable

from relations other than those that comprise the model. (In this case, the other relations will be those that relate cross flow to pressure difference in neighboring channels or to channel velocities.) For specialization to cross flow, the following relationships are set:

$$S_m = \sum_i (n_{ji} - n_{ij}), \quad (30)$$

$$S_f = \sum_i (n_{ji} v_i - n_{ij} v_i), \quad (31)$$

$$S_h = \sum_i (n_{ji} h_i - n_{ij} h_i), \quad (32)$$

$$S_m^\ell = \sum_i (n_{ji}^\ell - n_{ij}^\ell), \quad (33)$$

$$S_m^g = \sum_i (n_{ji}^g - n_{ij}^g), \quad (34)$$

$$S_f^\ell = \sum_i (n_{ji}^\ell v_i^\ell - n_{ij}^\ell v_i^\ell), \quad (35)$$

$$S_f^g = \sum_i (n_{ji}^g v_i^g - n_{ij}^g v_i^g), \quad (36)$$

$$S_h^\ell = \sum_i (n_{ji}^\ell h_i^\ell - n_{ij}^\ell h_i^\ell), \quad (37)$$

$$S_h^g = \sum_i (n_{ji}^g h_i^g - n_{ij}^g h_i^g). \quad (38)$$

The symbol n_{ji} represents mass flow from channel j to channel i in mass per unit volume per unit time. Superscripts ℓ and g retain their previous meanings. All n 's are non-negative. Net flow, $n_{ji} - n_{ij}$, may be negative.

It is customary to effect a further decomposition of the space and time derivatives such as those appearing in Eqs. (4)-(5), (17)-(18), and (25)-(26):

$$\frac{\partial \rho^g h^g \alpha}{\partial t} + \frac{\partial}{\partial Z} \rho^g h^g v^g \alpha = \rho^g \alpha \frac{\partial h^g}{\partial t} + \rho^g v^g \alpha \frac{\partial h^g}{\partial Z} + h^g \left[\frac{\partial \rho^g \alpha}{\partial t} + \frac{\partial}{\partial Z} \rho^g v^g \alpha \right]; \quad (39)$$

$$\begin{aligned} \frac{\partial \rho^l h^l (1 - \alpha)}{\partial t} + \frac{\partial}{\partial Z} \rho^l h^l v^l (1 - \alpha) &= \rho^l (1 - \alpha) \frac{\partial h^l}{\partial t} + \rho^l v^l (1 - \alpha) \frac{\partial h^l}{\partial Z} \\ &+ h^l \left[\frac{\partial \rho^l (1 - \alpha)}{\partial t} + \frac{\partial}{\partial Z} \rho^l v^l (1 - \alpha) \right]; \quad (40) \end{aligned}$$

$$\frac{\partial \rho^g v^g \alpha}{\partial t} + \frac{\partial}{\partial Z} \rho^g v^g \alpha = \rho^g \alpha \frac{\partial v^g}{\partial t} + \rho^g v^g \alpha \frac{\partial v^g}{\partial Z} + v^g \left[\frac{\partial \rho^g \alpha}{\partial t} + \frac{\partial}{\partial Z} \rho^g v^g \alpha \right]; \quad (41)$$

$$\begin{aligned} \frac{\partial \rho^l v^l (1 - \alpha)}{\partial t} + \frac{\partial}{\partial Z} \rho^l v^l (1 - \alpha) &= \rho^l (1 - \alpha) \frac{\partial v^l}{\partial t} + \rho^l v^l (1 - \alpha) \frac{\partial v^l}{\partial Z} \\ &+ v^l \left[\frac{\partial \rho^l (1 - \alpha)}{\partial t} + \frac{\partial}{\partial Z} \rho^l v^l (1 - \alpha) \right]. \quad (42) \end{aligned}$$

The square bracket on the right side of each of the last four equations is the right-hand side of Eqs. (15) or (16). In most thermohydraulic applications those terms are zero. Not so here, however. When these expansions with the explicit forms of inhomogeneous terms are placed in the system equations, the inhomogeneous terms and the interphase transfer terms enter the enthalpy and momentum equations in two ways. Their effects on Eqs. (17) and (25) will be examined in some detail. All other equations affected will be affected in virtually the same way as one of these.

Eqs. (15), (17), (19), (20), (30), (38), and (39) lead to

$$\begin{aligned}
& \rho^g_\alpha \frac{\partial h^g}{\partial t} + \rho^g_\alpha v^g_\alpha \frac{\partial h^g}{\partial Z} + h^g \left[\sum_i (n^g_{ii} - n^g_{ii}) + (\mu^{lg} - \mu^{gl}) + C_\alpha (v^{lg} - v^{gl}) \right] \\
& = \sum_k \frac{H^g_{ki} A^g_{ki}}{B^g} (\theta^g_k - T^g) + \frac{\partial \alpha P}{\partial t} + \sum_i (n^g_{ii} h^g_i - n^g_{ii} h^g_i) + \mu^{lg} h^g_s \\
& \quad - \mu^{gl} h^g_s + C_\alpha (v^{lg} h^g_s - v^{gl} h^g_s) . \quad (43)
\end{aligned}$$

Eqs. (15), (19), (20), (25), (31), (36), and (41) lead to

$$\begin{aligned}
& \rho^g_\alpha \frac{\partial v^g}{\partial t} + \rho^g_\alpha v^g_\alpha \frac{\partial v^g}{\partial Z} + v^g \left[\sum_i (n^g_{ii} - n^g_{ii}) + (\mu^{lg} - \mu^{gl}) + C_\alpha (v^{lg} - v^{gl}) \right] \\
& = - \frac{d\alpha P}{dZ} + \rho^g_\alpha g \cos \psi - Fr^g + \sum_i (n^g_{ii} v^g_i - n^g_{ii} v^g_i) \\
& \quad + (\mu^{lg} v^g - \mu^{gl} v^g) + C_\alpha (v^{lg} v^g - v^{gl} v^g) . \quad (44)
\end{aligned}$$

The square-bracketed terms on the left side of both Eqs. (43) and (44) can be considered deficiency terms in the following sense: if there is a net mass source of $\sum_i (n^g_{ii} - n^g_{ii})$, it must be accompanied by a net enthalpy change of $h^g \sum_i (n^g_{ii} - n^g_{ii})$ to prevent change in the specific enthalpy at this point and by a net momentum change of $v^g \sum_i (n^g_{ii} - n^g_{ii})$ to prevent change in the specific momentum at this point. In fact, the change in mass due to this source is accompanied by a change in enthalpy of $\sum_i (n^g_{ii} h^g_i - n^g_{ii} h^g_i)$ and in momentum of $\sum_i (n^g_{ii} v^g_i - n^g_{ii} v^g_i)$, source terms appearing on the right-hand sides of Eqs. (43) and (44), respectively. When

$$h^g = h^g_i \quad (45a)$$

and

$$v^g = v_i^g, \quad (45b)$$

the deficiency terms are just balanced by their corresponding sources. In like manner, the bracketed v terms on left and right represent deficiencies and sources due to the thermodynamic interphase mass exchanges and the μ terms, to the heat or work driven interphase mass exchanges. It appears from Eq. (43) that the deficiency balances the source in the thermodynamic exchange case as long as the steam is at saturation, independent of the enthalpy of the liquid.

In Eq. (43) the driven source term is $\mu^{lg} h_s^g - \mu^{gl} h_s^l$. In $\mu^{lg} h_s^g$ there is an apparent surplus of

$$\mu^{lg} (h_s^g - h_s^l), \quad (46a)$$

and in $\mu^{gl} h_s^l$, an apparent deficit of

$$\mu^{gl} (h_s^g - h_s^l). \quad (46b)$$

These two terms [Eqs. (46a) and (46b)] represent, in the non-adiabatic cases, for example, the heat flow between the fluid and its surroundings, and this heat flow causes the phase change. Assumption 6 in Sect. 4 requires that this effect be zero unless the fluid to which Eq. (43) refers is saturated.

With the modifications indicated by this section, Eq. (16) becomes

$$\rho^l (1 - \alpha) \frac{\partial h^l}{\partial t} + \rho^l v^l (1 - \alpha) \frac{\partial h^l}{\partial Z} + h^l \left[\sum_i (n_{ii}^l - n_{ii}^g) \right] + (\mu^{lg} - \mu^{gl})$$

$$\begin{aligned}
& + C\alpha(v^{\ell g} - v^{g\ell}) \Big] = \sum_k \frac{H_{ki}^{\ell} A_{ki}^{\ell}}{B^{\ell}} (\theta_k^{\ell} - T^{\ell}) + \frac{\partial P(1 - \alpha)}{\partial t} \\
& + \sum_i (n_{ii}^{\ell} h_i^{\ell} - n_{ii}^{\ell} h_i^{\ell}) + \mu^{\ell g} h_s^{\ell} - \mu^{\ell g} h_s^{\ell} + C\alpha(v^{\ell g} h_s^{\ell} - v^{\ell g} h_s^{\ell}) ; \quad (47)
\end{aligned}$$

Eq. (26) becomes

$$\begin{aligned}
& \rho^{\ell}(1 - \alpha) \frac{\partial v^{\ell}}{\partial t} + \rho^{\ell} v^{\ell}(1 - \alpha) \frac{\partial v^{\ell}}{\partial Z} + v^{\ell} \left[\sum_i (n_{ii}^{\ell} - n_{ii}^{\ell}) + (\mu^{\ell g} - \mu^{\ell g}) \right. \\
& \left. + C\alpha(v^{\ell g} - v^{\ell g}) \right] = - \frac{d(1 - \alpha)P}{dZ} + \rho^{\ell}(1 - \alpha)g \cos \psi - Fr^{\ell} \\
& + \sum_i (n_{ii}^{\ell} v_i^{\ell} - n_{ii}^{\ell} v_i^{\ell}) + (\mu^{\ell g} v^{\ell} - \mu^{\ell g} v^{\ell}) + C\alpha(v^{\ell g} v^{\ell} - v^{\ell g} v^{\ell}) ; \quad (48)
\end{aligned}$$

Eq. (4) becomes

$$\begin{aligned}
& \rho^g \alpha \frac{\partial v^g}{\partial t} + \rho^{\ell}(1 - \alpha) \frac{\partial v^{\ell}}{\partial t} + \rho^g v^g \alpha \frac{\partial v^g}{\partial Z} + \rho^{\ell} v^{\ell}(1 - \alpha) \frac{\partial v^{\ell}}{\partial Z} \\
& + v^g \left[\sum_i (n_{ii}^g - n_{ii}^g) + (\mu^{\ell g} - \mu^{\ell g}) + C\alpha(v^{\ell g} - v^{\ell g}) \right] \\
& + v^{\ell} \left[\sum_i (n_{ii}^{\ell} - n_{ii}^{\ell}) + (\mu^{\ell g} - \mu^{\ell g}) + C\alpha(v^{\ell g} - v^{\ell g}) \right] \\
& = - \frac{dP}{dZ} + [\rho^g \alpha + \rho^{\ell}(1 - \alpha)]g \cos \psi + Fr^{\ell} + Fr^g \\
& + \sum_i (n_{ii}^g v_i^g - n_{ii}^g v_i^g + n_{ii}^{\ell} v_i^{\ell} - n_{ii}^{\ell} v_i^{\ell}) ; \quad (49)
\end{aligned}$$

Eq. (15) becomes

$$\frac{\partial \rho^g \alpha}{\partial t} + \frac{\partial}{\partial Z} \rho^g v^g \alpha = \sum_i (n_{ii}^g - n_{ii}^g) + \mu^{lg} - \mu^{gl} + C\alpha(v^{lg} - v^{gl}) ; \quad (50)$$

and Eq. (16) becomes

$$\frac{\partial}{\partial t} \rho^l (1 - \alpha) + \frac{\partial}{\partial Z} \rho^l v^l (1 - \alpha) = \sum_i (n_{ii}^l - n_{ii}^l) + (\mu^{gl} - \mu^{lg}) + C\alpha(v^{gl} - v^{lg}) . \quad (51)$$

8. THERMODYNAMIC REGIONS

Nine thermodynamic regions defined by the relations of h^l to h_s^l and h^g to h_s^g (Table 1) will be considered.

Table 1. Thermodynamic regions

Region No.	Liquid Relation	Vapor Relation	Description
1	$h^l < h_s^l$	$h^g < h_s^g$	Liquid subcooled, vapor supersaturated; set $\alpha = 0$ to make pure subcooled.
2	$h^l > h_s^l$	$h^g > h_s^g$	Liquid supersaturated, vapor superheated; set $\alpha = 1$ to make pure superheated.
3	$h^l = h_s^l$	$h^g = h_s^g$	Mixed region at saturation; require F definition.
4	$h^l < h_s^l$	$h^g = h_s^g$	Liquid subcooled, vapor saturated; require F.
5	$h^l = h_s^l$	$h^g > h_s^g$	Liquid saturated, vapor superheated; require F.

Table 1. Thermodynamic Regions (Continued)

Region No.	Liquid Relation	Vapor Relation	Description
6	$h^l < h_s^l$	$h^g > h_s^g$	Liquid subcooled, vapor superheated; require F .
7	$h^l = h_s^l$	$h^g < h_s^g$	Liquid saturated, vapor supersaturated; set $\alpha = 0$ to make saturated liquid; this region has probability zero of occurrence and is of little interest.
8	$h^l > h_s^l$	$h^g = h_s^g$	Liquid supersaturated, vapor saturated; set $\alpha = 1$ to make saturated vapor; this region has probability zero of occurrence and is of little interest.
9	$h^l > h_s^l$	$h^g < h_s^g$	Supersaturated vapor, supersaturated liquid; exclude such a possibility from consideration.

From the model developed the set of equations that is appropriate to each region must be selected. Further, system variables must be defined at region boundaries. The only variables that present any region boundary problems are h^g , h^l , v^g , and v^l . At a region boundary where α does not move from zero to non-zero or from one to non-one, all system variables are taken to be continuous. When α becomes non-zero or non-one, a phase not formerly present is appearing, and the velocity and enthalpy of that phase must be initialized. In every such case the region boundary velocity of the appearing phase is taken equal to the velocity of the existing phase.

$$v^g = v^l \quad (52)$$

This is based on the assumption that the phase present in small quantity will be swept along, initially, by the dominant phase.

There is no equivalent simple argument for choosing initial region values for h^g or h^l . For these one must rely on empirical information. In the present state of information, it is more reasonable to call the choices arbitrary. References 5 and 6 describe work that leads to models which can be used to select such h^g and h^l . The calculations are laborious and perhaps not yet definitive. (We mention them to indicate that the way to determine these values does exist, and that in calling upon empirical selection we are not therefore placed in the position of not knowing the equations that complete the system.)

With the model as developed to this point, a set of equations can be chosen for each region. This is done for regions 1-6 in Table 2. Regions 7 and 8 were omitted, since they are zero probability (though thermodynamically possible) regions, and region 9 was excluded from consideration. In setting up the table we have associated equations with state variables. This is a matter of convenience only. In fact, the whole set of variables corresponds to the whole set of equations.

Table 2. Equations by Thermodynamic Regions
(Tabulated Entries Are Equation Nos.)

Momentum [*] Choice	Variable	Region					
		1	2	3	4	5	6
1	P	49	49	49	49	49	49
1	α	22a	22b	22	22	22	22
2	P	49	49	44	44	44	44
2	α	22a	22b	48	48	48	48
Either 1 or 2	h^g	x ^{**}	43	13	13	43	43
	h^l	47	x	14	47	14	47
	v^g	x	50	50	50	50	50
	v^l	51	x	51	51	51	51
	ρ^g	x	6	10	10	6	6
	ρ^l	7	x	11	7	11	7
	μ^{gl}	24	24	43	43	24	24
	μ^{lg}	23	23	47	23	47	23
	C	21	21	21	21	21	21

*Momentum choice: (1) no phase separation; (2) phase separation.

**Not needed.

9. EQUILIBRIUM

The phase-separated model with non-equilibrium conditions between the phases, as developed to this point, is substantially more complex than a standard equilibrium model. The concept of velocity does not enter elementary thermodynamic considerations. However, if equilibrium in the thermodynamic sense is to prevail over the system for other than a few degenerate cases, it is clear that an essential condition is

$$v^g = v^l = v . \quad (53)$$

Given Eq. (53), i.e., the conditions for thermodynamic equilibrium for purposes of this model, there is the additional condition that

$$T^g = T^l = T . \quad (54)$$

With h_x defined by the notation of Eq. (1), the following can be stated:

$$x = \frac{h_x - h_s^l}{h_s^g - h_s^l} , \quad (55)$$

and

$$\alpha = \frac{\rho^l x}{\rho^l x + \rho^g (1 - x)} . \quad (56)$$

From Eqs. (1), (2), (55), and (56) it readily follows that

$$(\rho f)_\alpha = \rho_\alpha f_x . \quad (57)$$

The left-hand sides of Eqs. (3), (4), and (5) become, respectively,

$$\frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial Z} \rho_{\alpha} v ; \quad (58)$$

$$\rho_{\alpha} \frac{\partial v}{\partial t} + \rho_{\alpha} v \frac{\partial v}{\partial Z} + v \left[\frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial Z} \rho_{\alpha} v \right] ; \quad (59)$$

and

$$\rho_{\alpha} \frac{\partial h_x}{\partial t} + \rho_{\alpha} v \frac{\partial h_x}{\partial Z} + h_x \left[\frac{\partial \rho_{\alpha}}{\partial t} + \frac{\partial}{\partial Z} \rho_{\alpha} v \right] . \quad (60)$$

It follows that equilibrium can be adequately described with Eqs. (3), (4), and (5) [as modified by expressions (58), (59), and (60)], (6) through (14), and (53) through (56). Moreover, only regions 1, 2, and 3 (and the generally ignorable 7 and 8) need be considered. The problem is, therefore, very much simpler than the non-equilibrium phase-separated problem which was discussed earlier.

10. EQUILIBRIUM MODEL APPROXIMATION TO TWO-PHASE FLOW WITH SLIP

On account of the simplicity of the equilibrium system, it is customary to attempt to solve two-phase problems in the same model format, even when the essential condition of Eq. (53) is not met. In any kind of realistic system where conditions vary with Z , the failure of Eq. (53) will imply the failure of Eq. (54).

Failure of Eq. (53) also causes the left-hand sides of Eqs. (3), (4), and (5) to be equivalent not to Eqs. (58), (59), and (60), but to the following:

$$\frac{\partial \rho_\alpha}{\partial t} + \frac{\partial}{\partial Z} \rho_\alpha v_x ; \quad (61)$$

$$\rho_\alpha \frac{\partial v_x}{\partial t} + \rho_\alpha v_x \frac{\partial}{\partial Z} v^{(1)} + v_x \left[\frac{\partial \rho_\alpha}{\partial t} + \frac{v^{(1)}}{v_x} \frac{\partial \rho_\alpha v_x}{\partial Z} \right] ; \quad (62)$$

$$v^{(1)} = (v^2)_x / v_x ; \quad (63)$$

$$\rho_\alpha \frac{\partial h_x}{\partial t} + \rho_\alpha v_x \frac{\partial h^{(1)}}{\partial Z} + h_x \left[\frac{\partial \rho_\alpha}{\partial t} + \frac{h^{(1)}}{h_x} \frac{\partial \rho_\alpha v_x}{\partial Z} \right] ; \quad (64)$$

and

$$h^{(1)} = (vh)_x / v_x . \quad (65)$$

When this form is used to describe two-phase flow with slip, it is customary to make the approximations

$$v_x = v^{(1)} \quad (66)$$

and

$$h_x = h^{(1)} , \quad (67)$$

thereby causing Eqs. (61), (62), and (64) to assume the simple form of Eqs. (58), (59), and (60). These assumptions cause many of the effects of slip to be ignored; however, an attempt is made to preserve some effects of slip through the use of a slip correlation to calculate average quantities and in the use of heat transfer correlations developed under slip conditions.

One can assess the error of this approach in the following example. One of the empirically based correlations of slip velocity, whose application is, at best, limited, is

$$\rho^g v^g = \rho^l v^l \quad (68)$$

If one considers a region where Eq. (68) applies, that assumptions (66) and (67) are used, and the following relationship is set,

$$v^g = \beta v^l, \quad (69)$$

then it follows that

$$\beta = \sqrt{\rho^l / \rho^g}, \quad (70)$$

$$h^{(1)} / h_x = [h^g \beta x + h^l (1 - x)] / \{(\beta x + 1 - x)[h^g x + h^l (1 - x)]\}, \quad (71)$$

and

$$v^{(1)} / v_x = (\beta^2 x + 1 - x) / (\beta x + 1 - x)^2. \quad (72)$$

Eqs. (71) and (72) can be evaluated from the steam tables as a function of x and P . Tables 3, 4, and 5 illustrate, at certain values of P and x , the errors in the quantities $h^{(1)} / h_x$ and $v^{(1)} / v_x$. Departure from unity of the tabulated quantities in Tables 4 and 5 is the measure of the error. It is evidently significant.

Table 3. Values of some thermodynamic quantities

P (psi)	$1/\rho^{\ell}$ (ft ³ /lb)	$1/\rho^g$ (ft ³ /lb)	h^{ℓ} (Btu/lb)	h^g (Btu/lb)	β
2500	0.02859	0.1307	731.7	1093.3	2.14
2000	0.02565	0.1883	672.1	1138.3	2.71
1500	0.02346	0.2772	611.7	1170.1	3.44

Table 4. $h^{(1)}/h_x$

P\ x	0.1	0.5	0.9
2500	1.043	1.072	1.017
2000	1.0853	1.119	1.026
1500	1.148	1.172	1.034

Table 5. $v^{(1)}/v_x$

P\ x	0.1	0.5	0.9
2500	1.094	1.132	1.028
2000	1.192	1.212	1.041
1500	1.346	1.303	1.052

11. FURTHER DEVELOPMENTS

To apply this model to definite problems, it is necessary to develop specifications of a number of quantities that heretofore have been carried as coefficients in the model. Since different applications would require different treatments, these coefficients have been left unspecified.

To this point the model has dealt with two-phase flow in a one-dimensional channel. The n coefficients associated with external sources make possible the matrixization of the model to a multichannel (with cross flow) problem. The θ coefficients which appear in the equations for enthalpy make possible a great variety of external heating arrangements, including heating by several fuel rod sections per channel.

It is evident that the coefficients that go into the computation of n must come from the measurement. So, also, must the friction factors and heat transfer coefficients called for in this model. The geometry in which we seek to make its principal application is that of clustered rods. Hence, the channels, often called subchannels, are only interstitial gaps between rods. In general, they are not bounded by confining walls on all sides but have slots, running in the direction of principal flow, connecting with neighboring channels. It is difficult to see how existing correlated data can have much application to such geometries. There is, accordingly, a clear need for an analytical and measurement program, mutually supporting.

Certainly, a major objective of the program of which this model is a part is the determination of maximum values of θ . The principal thrust of the model as developed so far is the determination of the field of h^g , h^l , v^g , T^g , and T^l . These

are, of course, essential to a good determination of maximum θ . The interplay between θ and T is close. The adjunct model which will be next produced to provide θ for these calculations will in most cases probably provide sufficiently good values of θ . Where further refinement is required, output from this model can be used to drive a somewhat modified version of the THETA program.⁷

As the model is constructed, however, it is equally applicable to any two-phase flow situation that is consistent with the assumptions. That could include a steam generator as well as core flow in a blowdown situation. To specify the problem, one must do the following:

1. Define n 's to conform with our understanding of cross flow;
2. Define θ 's to provide a reasonable representation of diffusion heat sources in fuel rods;
3. Choose boundary conditions consistent with the physical problem;
4. Choose computation algorithms with regard to the following considerations:
 - (a) storage and management of data made necessary by replication of model in many channels, (b) double convergence criteria, i.e., convergence to boundary conditions in each channel, convergence in mass flow across channels, and (c) stability of solutions including the following cases, (1) steam and water flowing in the same direction, (2) steam and water flowing in opposite directions in some channels, (3) flow in both directions away from high pressure point in some channels, and (4) any degree of blockage of some channels.

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13. APPENDIX: THE DERIVATIVES OF THE PRESSURE

There does not appear to be full agreement in the literature concerning the treatment to be accorded the pressure derivatives that occur in the equations for two phase flow.^{2,3,8} Let us first consider the energy equation. In ref. 2, page 323, Eq. 10.1-9 (0)* is as follows:

$$\frac{\partial}{\partial t} \rho(\mu + k) = -\nabla \cdot \rho(\mu + k)\vec{v} - \nabla \cdot \vec{q} - \nabla \cdot P\vec{v} - \nabla \cdot [\bar{\bar{\tau}} \cdot \vec{v}] + \rho\vec{v} \cdot \vec{g}, \quad (73)$$

where μ and k are specific internal and kinetic energies of the fluid, respectively, $\vec{q}$ is the heat flux out of the fluid, and $\bar{\bar{\tau}}$ is the stress tensor.

The thermodynamic definition of specific enthalpy is

$$h = \mu + P/\rho. \quad (74)$$

When Eq. (74) is inserted into Eq. (73), the term $\nabla \cdot P\vec{v}$ (work done on fluid by pressure) is cancelled by a part of the first term on the right-hand side of Eq. (73), leaving

$$\frac{\partial}{\partial t} \rho(h + k) - \frac{\partial P}{\partial t} = -\nabla \cdot \rho(h + k)\vec{v} - \nabla \cdot \vec{q} - \nabla \cdot [\bar{\bar{\tau}} \cdot \vec{v}] + \rho\vec{v} \cdot \vec{g}. \quad (75)$$

In our model we neglected kinetic energy (k), friction energy ($\nabla \cdot [\bar{\bar{\tau}} \cdot \vec{v}]$), and gravitational energy $\rho\vec{v} \cdot \vec{g}$. Since $\nabla \cdot \vec{q}$ is simply the conduction of heat over

*A good heuristic justification of this equation term by term is provided on pp. 310-13 of the same reference.

external surfaces, one can readily perceive the correspondence between Eqs. (5) and (73).

The first point to be emphasized is that the time derivative of P is a partial derivative. [Reference 2, cited as the source of Eq. (73), carries it as a substantial derivative; p. 323, Eq. 10.1-9 (R).]

Next, if one writes Eq. (73) for separated phases in the one-dimensional assumption, the term on the left side, for the gas phase, say, becomes

$$\frac{\partial}{\partial t} \rho^g \alpha (v^g + k^g) ,$$

from which it would follow directly that in Eq. (75) it would be $\partial \alpha P / \partial t$ and not $\alpha \partial P / \partial t$.

Considering the pressure gradient that appears in the momentum equation, i.e., the term dP/dZ that appears in Eq. (4), and integrating this term over all the region composed of gas in some small volume V (V contains both liquid and gas) leads to

$$\begin{aligned} \iiint_{V^g} \frac{dP}{dZ} dV &= \iint_{A^g} P(Z) dA = P(Z_2) A^g(Z_2) - P(Z_1) A^g(Z_1) , \\ &= A [P(Z_2) \alpha(Z_2) - P(Z_1) \alpha(Z_1)] , \\ &= A \left(P(Z_1) \alpha(Z_1) + \frac{d}{dZ} [P(Z) \alpha(Z)] dZ - P(Z_1) \alpha(Z_1) \right) , \\ &= \frac{d}{dZ} [P(Z) \alpha(Z)] A dZ , \end{aligned}$$

$$= \iint_A \frac{d}{dZ} (\alpha P) dV . \quad (76)$$

In the above derivation we have assumed that the volume is essentially one-dimensional, i.e., $dV = AdZ$, the variables are functions of Z only, and (αP) is analytic. With A^g a bounding area touching only volumes that contain gas and A the total bounding area, the following expression is derived:

$$\iiint_{V^g} \frac{dP}{dZ} dV = \iiint_V \frac{d\alpha P}{dZ} dZ . \quad (77)$$

That is, the integral of the pressure gradient over the gas volume only is equivalent to the integral of the gradient of αP over a mixed volume. Since our integration schemes will involve taking meshes of mixed volumes, it is clear that the correct representation of the pressure gradient term in Eqs. (44) and (48) is

$$\frac{d\alpha P}{dZ} \text{ and } \frac{d(1 - \alpha)P}{dZ} . \quad (78)$$

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