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For a Na-Fe System**

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Sensitivity Analysis: A General Approach Illustrated
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Abstract

A general approach to sensitivity studies is introduced using a quantitative definition of cross-section sensitivity. The definition offered is intended to be both practical and physically meaningful. Its advantages in comparison with other approaches is discussed and its use in analysis is illustrated for a sodium-iron transport problem. Results of the analysis include a discussion of the role of important total cross-section minima in deep penetration in sodium and iron and the importance of certain energy ranges and partial cross sections for the sodium-iron system. The predictive aspect of the approach is demonstrated in comparisons of rigorous and predicted results for small cross-section perturbations and in conclusions about the order of Legendre expansion of the scattering kernel.

I. Introduction

Sensitivity analysis, the procedure by which one determines how sensitive a calculated result is to microscopic cross-section data, is finding ever-increasing applicability in shielding research. This trend is a direct result of the successful application of transport methods to shield analysis and the demand they create for more accurate cross-section data. With growing acceptance of the accuracy of these techniques, sensitivity studies have begun to play a major role in the task of uncovering cross-section inadequacies through analysis of integral experiments and determining their impact on overall shield design.

In practice, sensitivity studies fall into two broad categories: (1) those characterized by calculations made with differing data sets,¹⁻⁴ and (2) methods based on the use of adjoint fluxes and perturbation theory.^{5,6} Most work to date has been concentrated in the first area, but the methods employed in both of these groupings have strengths and weaknesses that govern their applicability in any specific instance. Assuming the positive aspects are well understood, two inherent limitations should be pointed out. In studies characterized by the use of differing data sets, there is a practical limit to the number of data sets or cross-section changes which can be analyzed. Additionally, computer arithmetic limits the size of changes which can be made while still retaining significance in computed results. The perturbation approach, as presently used, suffers from the constraints of first order perturbation theory, which usually limits analysis to the effects of small cross-section changes. Also, there has been a tendency to leave the practical goal of the sensitivity study vaguely defined when using this method, with the result that applications have chiefly been concerned with the effects of energy-dependent cross-section changes on the energy dependence of the scalar flux. In this instance, for practical applications, an enormous amount of information is available for analysis and it can be a question of "not seeing the forest for the trees."

Our approach to the problem of sensitivity analysis has been to recognize the inherent advantages of the perturbation model (e.g., for studying large amounts of data), and simply modify the approach so as to make it more practically oriented and to increase the physical relevance of the results produced. In this light, the method has been restricted to an analysis of the sensitivity of a flux-integrated quantity (such as dose, fluence, etc.) to basic cross-section data. The results are then of practical interest to shield designers and, in addition, make sensitivity a more problem-dependent concept. From the large number of choices of functional form for the sensitivity profiles, trial and error leads to a form which seems to convey maximum physical insight. The following definition of the "sensitivity profile," $P_{\Sigma}(E)$, is thus offered:

$$P_{\Sigma}(E) \equiv \frac{\int \int_{\bar{\Omega} \bar{r}} N^*(\bar{r}, E, \bar{\Omega}) T_{\Sigma} N(\bar{r}, E, \bar{\Omega}) d\bar{r} d\bar{\Omega}}{R} \quad (1)$$

where,

$$R = \int_{\bar{r}, E, \bar{\Omega}} \Sigma_R(\bar{r}, E, \bar{\Omega}) N(\bar{r}, E, \bar{\Omega}) d\bar{r} dE d\bar{\Omega} \quad (2)$$

is the flux-integrated response for response function Σ_R , N^* is the adjoint flux, N is the forward flux, and T_Σ is an energy-dependent reaction operator specific for each partial cross section whose sensitivity profile is being calculated. The domain in which T_Σ is defined determines the spatial and energy regions in which the cross sections are to be studied.

The physical meaning of the function $P_\Sigma(E)$ is almost evident from the way it is defined. $T_\Sigma N$ represents a reaction rate for partial cross-section Σ at energy E , and $N^* T_\Sigma N$ is then an importance weighting of the reaction rate (importance being represented by the adjoint flux N^*) as it relates to the response represented by R . Thus, the physical content of the sensitivity profile, as an importance-weighted energy-dependent reaction rate, can serve as a powerful tool for analyzing any calculation for the importance of each energy-dependent reaction process as it affects the final results.

In terms of the sensitivity of the response R to cross-section changes, the sensitivity profile rigorously represents the local logarithmic rate of change of R with respect to changes in reaction operator T_Σ at energy E , that is,

$$P(E) = \frac{d(\ln R)}{d(\ln T)} = \frac{dR/R}{d\Sigma/\Sigma} = \frac{\int \int_{\bar{r} \bar{\Omega}} N^* T_\Sigma N d\bar{r} d\bar{\Omega}}{R} \quad (3)$$

Here, use is made of the fact that T_Σ is simply an operator notation for the reaction cross section Σ which in general includes double differential data as well as partial cross sections. It is this derivative feature of the definition that allows one, within the framework of perturbation theory, to predict changes in the response function resulting from finite changes in the cross-section data. Thus, one integrates the profile multiplied by an energy-dependent cross-section change to get the change in the response as:

$$\frac{\delta R}{R} \approx \int_E \frac{\delta \Sigma}{\Sigma} P(E) dE = \frac{\int \int \int_{\bar{E} \bar{r} \bar{\Omega}} N^* \delta T_\Sigma N d\bar{r} d\bar{\Omega} dE}{R} \quad (4)$$

In this context, the use of N^* , the unperturbed adjoint, implies that linear perturbation theory is adequate for predicting δR . Equation 4, however, is more general in that it is also possible to obtain rigorous results (which include non-linear effects) if the perturbed adjoint N_p^* is used in the original definition, in place of N^* .

Finally, the definition offered in Equation 3 is clearly quite practical in that large amounts of cross-section data can be processed for a sensitivity analysis with the results of only one forward and one adjoint calculation. Yet each cross-section process will have its own profile and each profile is unique for the particular problem being studied. Sensitivity is thus unambiguously defined in a problem-dependent manner, taking into account the effects of the specific source, detector, and material configuration used, as well as cross-section considerations.

II. Analysis of a Na-Fe System

To illustrate both the analytic aspect of using a physically meaningful definition of sensitivity and the predictive aspect associated with perturbation theory, a particularly interesting problem of transport in a sodium-iron shield was investigated. The problem consists of a neutron spectrum incident on a slab made up of 15 ft of sodium followed by 3 ft of iron. The calculations were performed with the one-dimensional discrete ordinates transport code, ANISN,⁷ and the objective was the determination of the neutron tissue dose, as defined by a Snyder-Neufeld⁸ response function, at the exit face of the iron zone. Specifically, the source used was the neutron leakage spectrum from the FTR radial shield as calculated using DOT⁹ (a source peaked in the 300-keV to thermal energy range). The forward and adjoint ANISN calculations required were made using the 50-group energy structure employed at ORNL for FFTF design support calculations. Table 1 gives the details of the energy group structure, source spectrum, and tissue dose response function utilized in this analysis. The sodium cross sections used were those given in ENDF/B-III as Mat. 1156, and the iron cross sections were ENDF/B-III, Mat. 1180, which was re-evaluated using the recent total cross-section measurements of J. A. Harvey¹⁰ for minima below 400 keV. The sensitivity of this problem to a specific cross section, as defined by Equation 3, was determined for each cross section used in the transport calculation. In addition, sensitivity profiles were obtained for each of the partial reaction cross sections listed in the ENDF/B-III files for sodium and iron.

Figure 1 presents the sensitivity of the tissue dose to the total collision cross section in the sodium. Each bar of the histogram represents the summed sensitivities of all the cross sections for a particular group divided by the lethargy width of that group. The peaks generally correspond to minima in the total cross section, for example those minima at 0.3, 1.9, and 3.0 MeV. It is most interesting to note here that the sensitivity above 1 MeV is extraordinarily high considering the small number of source neutrons in this energy range (see Table 1), and that the sensitivity drops off sharply below 24 keV. The lower cutoff is due to the fact that the deepest minimum in iron is the big window at 24.5 keV below which the iron greatly attenuates neutrons. The high-energy importance is due largely to the moderating characteristics and thickness of the sodium slab (15 ft), which result in the high-energy source particles having a greater probability of entering the iron slab with energies above 24.5 keV. Additionally, as shown in Table 1, the dose response function decreases quite rapidly for lower energy neutrons.

This problem is therefore very sensitive to the high-energy sodium cross sections and to the total cross-section minima above 24.5 keV which serve as windows for the sodium transport. Analysis of the sensitivity to individual reaction cross sections for sodium indicates that the elastic cross section is predominant below 1 MeV, while the inelastic cross section has increasing importance at higher energies.

Figure 2 gives the sensitivity profile for the total collision cross section in the iron. The spectrum entering the iron slab is quite soft, due to the effects of slowing down in sodium and very few neutrons enter the iron with energies above 24.5 keV. The overall sensitivity of the problem to the iron cross sections therefore is much lower than the sensitivity to sodium cross sections. As could be expected, the iron sensitivity profile is dominated by the 24.5-keV minimum which is the widest and deepest window in the total cross section. Other prominent features correspond to the minima at 170 and 300 keV. Sensitivity studies of the individual reaction cross sections for iron indicate that the elastic cross section is dominant for this problem, accounting for approximately 95% of the total sensitivity. The sensitivity to the iron inelastic cross section is low because of the extremely low number of neutrons with energies above 1 MeV.

Table 1. Group Structure, Source Spectrum, and Detector Response
Function for Na-Fe Problem

Group	Tissue Dose Factor			Group	Tissue Dose Factor		
	Upper Energy (eV)	(mrem/hr/n/ cm ² /sec)	Source (n/sec)		Upper Energy (eV)	(mrem/hr/n/ cm ² /sec)	Source (n/sec)
1	14.92(+6)	0.150	4.3(-8)	26	8.652(+4)	0.0245	6.72(-2)
2	12.214(+6)	0.150	1.9(-7)	27	5.2475(+4)	0.0194	4.3(-2)
3	10.000(+6)	0.150	6.7(-7)	28	4.087(+4)	0.0162	3.5(-2)
4	8.187(+6)	0.149	1.8(-6)	29	3.183(+4)	0.0126	3.6(-2)
5	6.7032(+6)	0.143	3.9(-6)	30	2.479(+4)	0.0095	4.5(-2)
6	5.4881(+6)	0.136	7.2(-6)	31	1.930(+4)	0.0080	3.2(-2)
7	4.4933(+6)	0.132	1.2(-5)	32	1.503(+4)	0.0064	7.97(-2)
8	3.6788(+6)	0.129	2.5(-5)	33	7.102(+3)	0.0048	3.05(-2)
9	3.0119(+6)	0.126	3.52(-5)	34	4.307(+3)	0.0044	4.3(-3)
10	2.466(+6)	0.126	4.4(-5)	35	3.355(+3)	0.0045	8.1(-4)
11	2.019(+6)	0.128	8.1(-5)	36	2.613(+3)	0.0045	9.8(-3)
12	1.653(+6)	0.131	1.2(-4)	37	2.035(+3)	0.0046	2.6(-2)
13	1.353(+6)	0.134	1.6(-4)	38	1.585(+3)	0.0046	3.5(-2)
14	1.108(+6)	0.135	2.23(-4)	39	1.234(+3)	0.0047	3.3(-2)
15	9.072(+5)	0.122	3.2(-4)	40	9.611(+2)	0.0048	8.02(-2)
16	7.427(+5)	0.106	1.3(-3)	41	4.540(+2)	0.0049	5.91(-2)
17	6.081(+5)	0.091	3.8(-3)	42	2.145(+2)	0.0050	4.93(-2)
18	4.979(+5)	0.080	4.3(-3)	43	1.013(+2)	0.0050	4.23(-2)
19	4.076(+5)	0.073	9.0(-3)	44	4.785(+1)	0.0050	3.4(-2)
20	3.337(+5)	0.067	1.656(-2)	45	2.260(+1)	0.0050	2.8(-2)
21	2.732(+5)	0.060	1.7(-2)	46	1.068(+1)	0.0050	2.2(-2)
22	2.237(+5)	0.053	1.7(-2)	47	5.043(+0)	0.0050	1.5(-2)
23	1.832(+5)	0.046	2.24(-2)	48	2.382(+0)	0.0050	9.8(-3)
24	1.500(+5)	0.040	2.7(-2)	49	1.125(+0)	0.0050	6.09(-3)
25	1.228(+5)	0.031	5.6(-2)	50	0.414(+0)	0.00375	1.5(-3)
Thermal (~1 x 10 ⁻⁴)							

SENSITIVITY PER UNIT LETHARGY
 $[\Delta R/R / (\Delta \Sigma/\Sigma) / \Delta u]$

SENSITIVITY PER UNIT LETHARGY
 $[(dR/R)/(d\Sigma/\Sigma)/\Delta u]$

A close inspection of the sensitivity profiles given here reveals the reduced importance or absence of peaks corresponding to certain important cross-section minima, such as those at 0.5 MeV in sodium or at 80 keV in iron. Examination of the point cross-section data, however, reveals that the group structure employed tended to mask these minima, and perhaps therefore incorrectly reduce their importance in the sensitivity profile. Future sensitivity studies will employ calculations using point cross sections or tailored group structures which adequately represent the cross-section detail.

The predictive aspect of this sensitivity technique was investigated using Equation 4 with the unperturbed adjoint flux. Table 2 presents some of the results of this study. The change in the tissue dose caused by increasing all sodium or all iron cross sections for all energy groups by a given percentage (equivalent to an increase in the density of these materials) was predicted, and the prediction was then compared with the dose change as determined by a recalculation of the problem using ANISN with the perturbed cross sections. The change in dose with respect to change in the iron cross sections appears quite linear out to perturbations of the order of 5%, but non-linear effects are apparent in the 5% perturbations for sodium. Perturbations of the total collision cross section for only one important group in the sodium clearly indicates the non-linearity effect for this problem. This result may at least partially be explained by the importance of small angle scattering at high energies in the sodium which gives rise to a process approximating continuous slowing-down, thereby strongly coupling the fluxes in each group. In contrast, the iron one-group perturbation remains fairly linear, despite the fact that the group perturbed contains the 24.5-keV iron minimum. This effect is probably due simply to the overwhelming importance of sodium cross sections rather than considerations of linearity. Perturbations which involve a decrease in the sodium total collision cross section were included to illustrate that the linear approximation tends to underpredict the effect of a cross-section decrease, but tends to overpredict the effect of a cross-section increase for this problem.

In addition to predictions based on linear perturbation theory, an attempt has been made to predict the effects of truncating by group the number of moments used in the Legendre expansion of the scattering kernel. In this case, δT_T in Equation 4 represents the moments of the scattering cross sections eliminated from the original P_3 calculation. The truncation by group was then adjusted to give approximately equal errors in each group with a total error of less than 0.1%. Other cases involving more radical truncations were also considered.

The predictions are compared with recalculated results in Table 3. In the first case, the number of Legendre moments was varied by group for both sodium and iron, and, as shown, the tissue dose obtained from an ANISN recalculation was within 0.1% of the original calculation. Although most transport codes are not currently equipped to handle a variation in the number of Legendre moments by group, it can easily be done, and the results here indicate that such a calculation might result in a significant reduction in running time with little loss in accuracy, especially for two-dimensional codes. The next three cases show that since prediction and recalculation are in good agreement for this problem, iron can be run with a P_0 expansion and sodium can be run with a P_1 expansion without major changes in the dose. Using a P_0 expansion for sodium, however, causes a large change in the tissue dose. All the calculations used an S_{16} Gaussian Legendre quadrature so that these conclusions do not imply that diffusion theory is adequate for this system.

Table 2. A Comparison of Predicted Changes in Dose Due to Cross-Section Perturbations Versus Calculated Results Obtained With the Perturbed Cross Sections

Sodium			Iron		
Cross-Section Perturbation in Σ Collision (%)	Predicted Change in Tissue Dose (%)	Recalculated Change in Tissue Dose (%)	Cross-Section Perturbation in Σ Collision (%)	Predicted Change in Tissue Dose (%)	Recalculated Change in Tissue Dose (%)
+ 0.1, all groups	- 1.797	- 1.712	+ 0.1, all groups	- 0.391	- 0.376
+ 0.5, all groups	- 8.99	- 8.26	+ 0.5, all groups	- 1.955	- 1.87
+ 1.0, all groups	- 17.97	- 15.84	+ 1.0, all groups	- 3.91	- 3.72
+ 5.0, all groups	- 89.85	- 57.45	+ 5.0, all groups	- 19.55	- 17.25
+ 1.0, group 25	- 1.53	- 1.42	+ 1.0, group 30	- 1.58	- 1.48
+ 10.0, group 25	- 15.3	- 10.1	+ 10.0, group 30	- 15.8	- 13.1
- 1.0, group 25	+ 1.53	+ 1.55	- 1.0, group 30	+ 1.58	+ 1.52
- 10.0, group 25	+ 15.3	+ 25.0	- 10.0, group 30	+ 15.8	+ 17.4

Table 3. Predicted Versus Recalculated Variation of the
Tissue Dose With Number of Legendre Moments Used in
The Angular Expansion

Number of Moments		Predicted Change in Tissue Dose From Initial P_3 (%)	Recalculated Change in Tissue Dose From Initial P_3 (%)
Sodium	Iron		
P_3 , groups 1-12	P_1 , groups 1-10		
P_2 , groups 13-26	P_2 , groups 11-20	- 0.07	- 0.03
P_1 , groups 27-32	P_1 , groups 21-50		

P_3 , groups 1-12			
P_2 , groups 13-25	P_0 , all groups	- 20.8	- 16.8
P_1 , groups 27-32			
P_0 , groups 33-50			

P_1 , all groups	P_1 , all groups	- 18.0	- 13.8

P_0 , all groups	P_0 , all groups	- 278.0	- 87.6

III. Conclusions

In conclusion, the results presented provide confirmation of the usefulness of a quantitative definition of sensitivity. The physical meaning implicit in the definition made it possible to determine directly the important reaction cross sections, in an energy-dependent fashion, for the problem studied. Furthermore, the predictive aspect of the definition is consistent with a perturbation theory approach and the potential for evaluating cross-section uncertainties and approximate representation of basic cross-section data.

For the specific sodium-iron problem studied, the sensitivity analysis provided substantial confirmation of the role of total cross-section minima in deep penetration. The dominant effect of the 24.5-keV window in iron was demonstrated quantitatively in a manner that was expected, but the substantial importance of the sodium minima in the 1- to 3-MeV range was also revealed. Predictions concerning the number of Legendre expansion terms needed to represent both the sodium and iron scattering kernels were quite enlightening in that a P_1 representation for both sodium and iron were concluded to be adequate for this problem. Corroboration of this conclusion with recalculations pointed out one of the stronger predictive applications possible with this kind of sensitivity technique.

Finally, we expect that future extensions of this method will reveal the real promise of the method as a tool for both analysis and prediction. Development plans include the coupling of the technique to extremely fine energy-dependent point cross-section calculational methods (using up to 1000 energy point grids) for analyzing energy-dependent reaction rates in great detail and also for use in an importance-weighted group-averaging scheme. In more practical application, the method should prove to be extremely helpful in designing and analyzing integral experiments which will be more relevant, in the sense of having similar sensitivity to cross sections, to real design problems. The predictive aspect of the method should also be able to provide quantitative estimates of design result uncertainties based on the inadequacies in any reaction cross-section set. These estimates, to be generally applied, however, will have to be the result of further development work on practical methods for calculating perturbed adjoints.

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FIGURE CAPTIONS

Fig. 1. Sensitivity Profile for Sodium Total Collision Cross Sections.

Fig. 2. Sensitivity Profile for Iron Total Collision Cross Sections.