Transformation Properties of the Effective Transition Operator*

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Abstract:

A multipole decomposition of the matrix elements of the effective transition operator is made in both particle-particle (pp) and hole-hole (hh) nuclei. It is found that the many-body part of this operator may be represented by an effective two-body multipole coefficient. This coefficient transforms by a phase between the pp and hh nuclei which is different from the one-body part of the operator. This formalism is applied to the ^{42}Sc (pp) and ^{54}Co (hh) cases, where it is seen that the general structure of ^{42}Sc can be predicted from that of ^{54}Co .

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1. Introduction

Some recent works^{1,2} on spectral relations have stressed the calculation of differences in energy levels between particle-particle (pp) and particle-hole (ph) pairs of nuclei as well as between pp and hole-hole (hh) pairs rather than the calculation of individual spectra.

Among the advantages of such calculations is the result that the same complex configuration mixing, e.g. core polarization, which appear in both members of a pair of nuclei cancel in a theory of differences, and therefore need not be calculated. However, these configurations can be essential in the calculation of the spectrum for either of the individual nuclei. This view also allows immediate insight into the many-body structure of contributions to the spectra which do not behave as would be expected, based on the spectral relations.

The success of this viewpoint, i.e., the theory of differences, leads one to consider a similar approach to the analysis of electromagnetic transitions in these related nuclei, i.e., the pp, ph and hh nuclei. It is specifically the purpose of this paper to investigate the many-body structure of the electromagnetic operator through an analysis of the theoretical electromagnetic transitions and moments of ⁴²Sc and ⁵⁴Co using the theory of differences. We begin by using the simplest model states description of ⁴²Sc and ⁵⁴Co which are two-particles and two-holes in the of _{7/2} shell.

We may define the effective transition operator in general as,

$$\langle \phi_f | O^L(1) | \phi_i \rangle \equiv \langle \phi_f | O_{eff}^L | \phi_i \rangle \quad (1)$$

where $|\psi\rangle$ is the correlated many-particle wavefunction for the system in the initial state ($|\psi_i\rangle$) and final state ($|\psi_f\rangle$) and is the eigenfunction of the nuclear Hamiltonian. The quantity $O^L(1)$ is the ordinary one-body electromagnetic operator of rank L in the neutron-proton (np) formalism. The state $|\phi\rangle$ is the model state selected for the system, e.g. $|\phi\rangle = |(f_{7/2})_n (f_{u/2})_p\rangle$ for ^{42}Sc and $|\phi\rangle = (f_{7/2})_n^{-1} (f_{7/2})_p^{-1}$ for ^{54}Co , and O_{eff}^L is the effective transition operator. It should be noted that the many-body effects present in $|\psi\rangle$ on the left hand side of eqn.(1), have been absorbed by O_{eff}^L on the right hand side, so that the effective transition operator carries all the many-body information about the system.

The relation between the matrix elements of the one-body transition operator $[O^L(1)]$ in the model states of ^{42}Sc and ^{54}Co is,

$$\begin{aligned} \langle (f_{7/2})_n (f_{7/2})_p | O^L(1) | (f_{7/2})_n (f_{7/2})_p \rangle = \\ (-1)^{L+1} \langle (f_{7/2})_n^{-1} (f_{7/2})_p^{-1} | O^L(1) | (f_{7/2})_n^{-1} (f_{7/2})_p^{-1} \rangle \end{aligned} \quad (2)$$

Therefore our notion of differences will involve the quantity,

$$\begin{aligned} \langle \Delta O_{\text{eff}}^L \rangle = \langle (f_{7/2})_n (f_{7/2})_p | O_{\text{eff}}^L | (f_{7/2})_n (f_{7/2})_p \rangle - \\ (-1)^{L+1} \langle (f_{7/2})_n^{-1} (f_{7/2})_p^{-1} | O_{\text{eff}}^L | (f_{7/2})_n^{-1} (f_{7/2})_p^{-1} \rangle \end{aligned} \quad (3)$$

which is identically zero when there are no many-body effects present, i.e., $O_{\text{eff}}^L \equiv O^L(1)$, which implies that the model state description $\{|\phi\rangle\}$ is a true representation of the system $\{|\psi\rangle\}$. The difference defined by eqn. (3) is also zero when there are many-body forces present which obey

the appropriate transform equation between the pp and hh cases.

It is perhaps worthwhile to contrast this notion of differences, as defined by eqn. (3), with the concept of the effective charge. The effective charge³ is defined by,

$$\langle \psi_f | O^L(1) | \psi_i \rangle \equiv e_{\text{eff}}^L(i \rightarrow f) \langle \phi_f | O^L(1) | \phi_i \rangle \quad (4)$$

where $e_{\text{eff}}^L(i \rightarrow f)$, which is the effective charge, is a parameterization of the many-body effects present in O_{eff}^L . If one were to calculate an $e_{\text{eff}}^L(i \rightarrow f)$ for ^{42}Sc and ^{54}Co , then the difference as defined by eqn.(3) would yield a parameterization of the many-body effects which do not obey eqn.(2), which is numerically equal to the result obtained from eqn.(3) for the calculation of $\langle \Delta O_{\text{eff}}^L \rangle$. It should be noted, however, that in the effective charge case one must then calculate all the terms in each nucleus, a great many of which cancel when one takes the difference. Also, one still does not see the separate many-body characteristics of these discrepancies.

2. Theory

The general structure of the transition operator in terms of its many-body components is

$$O_{\text{eff}}^L = O^L(1) + O^L(2) + \dots + O^L(n) \quad (5)$$

which is of the same form as the valence effective interaction.⁴ The numbers in eqn. (5) refer to the actual number rank of the operator, that is, the number of particles involved in the interaction. By analogy we make the same approximation that is made in the valence effective

interaction, that is to say, we truncate O_{eff}^L , so that it becomes a two-body operator in the valence space. This truncation refers to the number of valence particles involved in the interaction and not the actual number of particles, e.g., there could be a large number of core particles interacting outside the valence space, but this would not affect the number rank of the operator within the valence space. To clarify this point we write

$$O_{\text{eff}}^L = \sum_{\mu} \langle \mu | O^L(1) | \mu \rangle U_{\mu\mu}^L + \sum_{\substack{\mu \leq \beta \\ \lambda_1, \lambda_2}} \alpha_{\lambda_1 \lambda_2}^L(\mu\beta) [U_{\mu\mu}^{\lambda_1} \times U_{\beta\beta}^{\lambda_2}]^L \quad (6)$$

where the U^{λ} 's are the Racah unit tensors of rank λ , and the λ_1 and λ_2 are the ranks of the electromagnetic operator for the proton and neutron, respectively, [$\vec{L} = \vec{\lambda}_1 + \vec{\lambda}_2$].

The coefficients of the two-body part of eqn.(6), $\alpha_{\lambda_1 \lambda_2}^L(\mu\beta)$, determine the strength of the interactions which occur outside the valence space and are many-body in character. This is analogous to, but more extensive than,¹ the distinction between valence and extra-valence excitations in the Brueckner ladder. That is, in a typical G-matrix, many-body ladder-type terms are included in the effective interaction which, however, is a two-body operator in the valence space.

The matrix elements of eqn. (6) are the natural generalization of those in eqn. (2). That is, the matrix elements of the effective operator defined by eqn.(6) in the model states obey a transformation relation between the pp and hh cases. To see this transform between the pp and hh matrix elements, we take the matrix elements of eqn.(6) in the state

$|jk:J\rangle$ and invertin we obtain,

$$\alpha_{\lambda_1 \lambda_2}^L(jk) = \sum_{J < J'} \frac{1+(-1)^{\lambda_1+\lambda_2-L}}{1+\delta_{JJ'}} [\lambda_1 \lambda_2] [JJ']^{\frac{1}{2}} \begin{Bmatrix} j & j & \lambda_1 \\ k & k & \lambda_2 \\ J' & J & L \end{Bmatrix}$$

$$\langle jk:J' || O_{eff}^L || jk:J \rangle$$

$$- [Lk]^{\frac{1}{2}} \delta_{L\lambda_1} \delta_{0\lambda_2} \langle j || O^L(1) || j \rangle \quad (7)$$

$$- [Lj]^{\frac{1}{2}} \delta_{0\lambda_1} \delta_{L\lambda_2} \langle k || O^L(1) || k \rangle$$

and

$$[\lambda_1] = 2\lambda_1 + 1, [\lambda_1 \lambda_2] = (2\lambda_1 + 1)(2\lambda_2 + 1)$$

Where j is a single proton label, k is a single neutron label, and $J(J')$ is the total angular momentum of the proton-neutron model system $[|jk:J\rangle]$ and $\vec{j} + \vec{k} = \vec{J}[\vec{J}']$.

It is clear from eqn. (7) that $\alpha_{\lambda_1 \lambda_2}^L$ has no one-body component. Thus, any contribution to $\alpha_{\lambda_1 \lambda_2}^L$ is produced by many-body effects, which are outside the model space $|jk:J\rangle$.

It is relatively easy to show that the matrix elements of eqn.(6) in the hh-model state $|j^{-1}k^{-1}:J\rangle$ may be related to the pp matrix elements, in terms of the multipole coefficients, as follows,

$$\alpha_{\lambda_1 \lambda_2}^L(j^{-1}k^{-1}) = \{(-1)^{\lambda_1+\lambda_2} + (-1)^{\lambda_1+1} \delta_{\lambda_2 0}[k] + (-1)^{\lambda_2+1} \delta_{\lambda_1 0}[j]\} \alpha_{\lambda_1 \lambda_2}^L(jk) \quad (8)$$

where the $\delta_{\lambda_2 0}[k]$ and $\delta_{\lambda_1 0}[j]$ terms are the contributions due to the difference in cores between the (jk) and $(j^{-1}k^{-1})$ nuclei, that is, both the j and k -shells are filled in the latter core. Because of the vector coupling $\vec{L} = \vec{\lambda}_1 + \vec{\lambda}_2$, it is clear that the effect of the core in eqn.(8) is reflected only in the monopole, $(\lambda_1, \lambda_2) = (0, L; L, 0)$, term; the higher order terms are related by the phase $(-1)^{\lambda_1 + \lambda_2}$ between the pp and hh cases.

We can see that eqn.(8) refers to a hierarchy of relations:

- (1) $L = 0$ implies $\lambda_1 = \lambda_2$ and, therefore, $\alpha_{\lambda\lambda}^{L=0}(jk) = \alpha_{\lambda\lambda}^{L=0}(j^{-1}k^{-1})$, which is the spectral relation of Talmi, Goldstein and Pandya.⁵
- (2) $L = 1$ implies $\lambda_1 = \lambda_2 \pm 1$ for non-zero $\alpha_{\lambda_1\lambda_2}^L$ (see eqn.7) and

$$[\alpha_{\lambda_1\lambda_2}^L(jk) \pm \alpha_{\lambda_2\lambda_1}^L(jk)] = (-1)^{\lambda_1 + \lambda_2} [\alpha_{\lambda_1\lambda_2}^L(j^{-1}k^{-1}) \pm \alpha_{\lambda_2\lambda_1}^L(j^{-1}k^{-1})] \quad (9)$$

where the monopole term has been omitted for clarity;

$$\alpha_{\lambda_1\lambda_2}^L(jk) \pm \alpha_{\lambda_2\lambda_1}^L(jk) = \sum_{J \leq J'} 2(1 \pm (-1)^{J+J'}) \{\lambda_1\lambda_2\} \{JJ'\}^{\frac{1}{2}} \left\{ \begin{matrix} j & j & \lambda_1 \\ k & k & \lambda_2 \\ J' & J & L \end{matrix} \right\} \langle jk:J' || O_{eff}^L || jk:J \rangle \quad (10)$$

Therefore, the moments ($J = J'$) and transition rates ($J = J' \pm 1$) for the pp and hh nuclei transform separately [see phase in right hand side of eqn. (10)].

It should be noted that the $(j^{-1}k^{-1})$ monopole terms α_{0L}^L and α_{LO}^L should be reduced by a factor $([k]-1)$ and $([j]-1)$ respectively, because

of the shift in cores [see eqn.(8)]. This reduction is made in predicting the (jk) monopole from the $(j^{-1}k^{-1})$ monopole. From a knowledge of the transition rates [minus sign in eqn.(9)] in the pp case, we can predict those in the hh case, but not the moments in the hh case. Similarly, a knowledge of the moments [plus sign in eqn.(9)] in the pp case will allow us to predict the moments but not the transition rates in the hh case.

This separation in the transformation properties between moments and rates does not occur for the comparison of particle-particle (jk) to particle-hole (jk^{-1}) or $(j^{-1}k)$ nuclei or between hole-hole $(j^{-1}k^{-1})$ to particle-hole (jk^{-1}) or $(j^{-1}k)$ nuclei. For example, one must know both the rates and moments in ^{42}Sc (jk) to make predictions of either the rates or moments in ^{48}Sc (jk^{-1}) .

(3) $L = 2$ implies $\lambda_1 = \lambda_2$, $\lambda_2 \pm 2$ for non-zero $\alpha_{\lambda_1 \lambda_2}^L$ [see eqn.(7)]. In this case, J and J' in eqn.(10) may differ by 0,1 or 2 units of angular momentums; therefore, the transitions in which $J = J' \pm 2$ transform with the moments ($J = J'$) and the transitions with $J = J' \pm 1$ transform separately. This separation is analogous to the $L = 1$ case above.

We do not consider operators of rank higher than $L = 2$ here, but the general trend of things is apparent.

3. Results

The multipole coefficients for the electromagnetic transition rates defined by eqn. (8) are obtained by calculating the transition rates and moments using the wavefunctions for ^{42}Sc and ^{54}Co calculated in ref. (2). These wavefunctions were obtained by diagonalizing the Hamiltonian in the

respective cases in the valence space of the f-p shell. Only single-particle excitations were allowed, that is either a neutron or proton was excited from the $(0f_{7/2})^{+2}$ ground state to the $\alpha = 0f_{5/2}$, $1p_{3/2}$ or $1p_{1/2}$ valence shells. The states included in the calculation are;

$$\begin{aligned}
 {}^{42}\text{Sc}(\text{pp}): & (f_{7/2})_n (f_{7/2})_p ; (f_{7/2})_n (\alpha)_p ; (\alpha)_n (f_{7/2})_p \\
 {}^{54}\text{Co}(\text{hh}): & (f_{7/2})_n^{-1} (f_{7/2})_p^{-1} ; (f_{7/2})_n^{-2} (f_{7/2})_p^{-1} (\alpha)_n ; \\
 & (f_{7/2})_n^{-1} (f_{7/2})_p^{-2} (\alpha)_p
 \end{aligned}$$

where the subscripts n and p refer to neutrons and protons respectively, and the coupling has not been indicated.

The calculation of the BML's and BE2's for both ${}^{42}\text{Sc}$ and ${}^{54}\text{Co}$ have been graphed in figures (1) and (2) respectively. To aid comparison we have plotted the ratio of the theoretical to single-particle values for all the calculations. In Fig. (1) we can see that the BML's are on the average suppressed from their single-particle values for both cases. This occurs because the valence nucleon favors the excitation of like nucleons in a singlet state and unlike nucleons in a triplet state and both of these effects result in a reduction of the effective moment of a single nucleon.⁶ There is, however, an oscillation about this average value which is 180° out of phase between ${}^{42}\text{Sc}$ and ${}^{54}\text{Co}$. This out of phase behavior also occurs for the BE2's in Fig. (2) which are transitions between states which differ by one unit of angular momentum. The reason that the pp and hh terms are out of phase between the pp and hh cases will become clear when we examine the multipole structure of the transitions.

In Table (1) we record the multipole structure for the BML's depicted

in Fig. (1). We record the multipole coefficients as sums and differences because these yield the multipoles for moments and transition rates, respectively [see eqn.(10)]. We note in Table (1), that the differences in the coefficients, which refer to transition rates, also display the periodicity noted in Fig.(1). This structure is, however, absent from the sums of coefficients since these, which reflect the structure of the magnetic moments, are almost equal to their single-particle value and would therefore have a value of unity in Fig. (1). We also see in Table (1) that there is a change in sign that occurs between the ^{42}Sc values and the ^{54}Co values for each multipole coefficient, (except the 0,1 term). The 0,1 term in Table (1) is analogous to the monopole term in the multipole decomposition of the effective interaction. For the effective transition operator this term is the average contribution of the many-body part of the interaction to the one-body part of the transition. The average suppression of BML's (which was discussed earlier) in both ^{42}Sc and ^{54}Co is reflected in the negative values of the 0,1 terms. Also, the larger value for the ^{54}Co case indicates its greater suppression.

Each of the higher multipole changes in sign between ^{42}Sc and ^{54}Co because of the phase $(-1)^{\lambda_1 + \lambda_2}$ in eqn.(9) which is (-1) for $L = 1$ and $L = 2$ for the cases in which $|J_i - J_f| = 1$. Thus for a particular transition when $|J_i - J_f| = 1$ the many-body terms contribute in the opposite direction for ^{42}Sc than for ^{54}Co . Therefore, if ^{42}Sc has a sawtooth-like structure, ^{54}Co would have an anti-correlated sawtooth-like structure.

Thus a sawtooth-like structure, referred to above, is present in a plot of M1 and E2 reduced transition rates ($J_i \rightarrow J_f$) against " $J_i + J_f$ " for transitions in which $|J_i - J_f| = 1$ (see figures 1 and 2). These

transitions are the M1's and E2's which transform together. That is to say, the E2's in which $|J_i - J_f| = 2$ transform with the quadrupole moments and they do not exhibit this sawtooth structure. The origin of the sawtooth structure can be understood by analyzing the wavefunctions. In ^{42}Sc , the dominant components of the $J = 0, 1, 2$ and 3 states are,

$$\begin{aligned} |^{42}\text{Sc}, J = 0, T = 1\rangle &= |f_{7/2}^2\rangle \\ |^{42}\text{Sc}, J = 1, T = 0\rangle &= .90|f_{7/2}^2\rangle + .40|f_{7/2}f_{5/2}\rangle, \\ |^{42}\text{Sc}, J = 2, T = 1\rangle &= .97|f_{7/2}^2\rangle, \text{ and} \\ |^{42}\text{Sc}, J = 3, T = 0\rangle &= .85|f_{7/2}^2\rangle + .18|f_{7/2}f_{5/2}\rangle. \end{aligned}$$

The relative purity of the $T = 1$ wavefunctions compared to the $T = 0$ wavefunctions is due to the fact that the particle-particle interaction is stronger for $T = 0$ than $T = 1$. Thus, there is more configuration mixing for the $T = 0$ states. Since every other spin state is relatively pure the transition moments take a particular form. Namely,

$$\begin{aligned} M(1^+ \rightarrow 0^+) &= .90|M_{f_{7/2}}(1^+ \rightarrow 0^+)| + .80|a|(f_{7/2}||M1||f_{5/2}) \\ M(2^+ \rightarrow 1^+) &= .87|M_{f_{7/2}}(2^+ \rightarrow 1^+)| + .80|a'| (f_{5/2}||M1||f_{7/2}), \\ \text{and} \\ M(3^+ \rightarrow 2^+) &= .82|M_{f_{7/2}}(3^+ \rightarrow 2^+)| + .34|a''|(f_{7/2}||M1||f_{5/2}), \end{aligned}$$

where

$$M_{f_{7/2}}(1^+ \rightarrow 0^+), M_{f_{7/2}}(2^+ \rightarrow 1^+), \text{ and } M_{f_{7/2}}(3^+ \rightarrow 2^+)$$

are the model space terms and $|a|$, $|a'|$, and $|a''|$ represent the modulus of the recoupling terms between the model space and the excitations outside of the $f_{7/2}$ -shell. Since

$$(f_{7/2} || M1 || f_{5/2}) = -(f_{5/2} || M1 || f_{7/2}),$$

we see that if the two terms which contribute to $M(1^+ \rightarrow 0^+)$ are in phase, then the two terms which contribute to $M(2^+ \rightarrow 1^+)$ must be out of phase and those which contribute to $M(3^+ \rightarrow 2^+)$ must be in phase. Thus, when we plot

$$\frac{M(J_i \rightarrow J_f)}{f_{7/2}(J_i \rightarrow J_f)}$$

against " $J_i \rightarrow J_f$ ", the points on the graph resemble a sawtooth because of the alternating contribution of terms outside the $f_{7/2}$ -shell.

Since E2 transitions in which $|J_i - J_f| = 2$ are between states of the same isospin, we do not see the alternating structure of the wavefunction that is present when $|J_i - J_f| = 1$. Thus, we will not see, for $|J_i - J_f| = 2$, the sawtooth structure or the anticorrelation between ^{54}Co and ^{42}Sc M1's and E2's that is present when $|J_i - J_f| = 1$.

4. Conclusion

Examining the transition operator in terms of its multipole components has enabled us to understand contributions of different particle rank operators to transition rates. In particular we have seen that the one and two-body parts of the effective transition operator transform differently between pp and hh nuclei. Further, that this two-body component is able to predict the change in the detailed structure of the pp and hh transitions and moments. A consequence of this change in the transformation is that one may transform the multipole coefficient for

the moments and transition rates, (for $L = 1$ and $L = 2$, $|J_i - J_f| = 1$) separately. This allows one to predict electromagnetic properties of the hh-nucleus with half the information previously needed about the pp-nucleus.

It is clear from the above analysis that this is a far better way to make predictions than by assuming an equality between the pp and hh moments and transition rates. It is clear from Table (1) that the general structure of the moments and transition rates in ^{42}Sc are accurately reflected in those of ^{54}Co . It is, however, unclear as to the exact quality of the prediction, as indicated by the discrepancies $[\Delta_+$ and $\Delta_-]$, since there is little or no experimental information with which to compare.

The monopole term for the moments in Table (1) given by ^{54}Co is almost exactly that calculated for ^{42}Sc , that is, less than a 4% enhancement of the moments above the single-particle value (on the average). The monopole term for the transition rates, however, do not agree. That from ^{54}Co predicts a 3% enhancement above ^{42}Sc single-particle value, while that calculation in ^{42}Sc yields a 13% suppression below its single-particle value. The implications of this latter results cannot be assessed without at least some experimental results to point the way. It does, however, seem clear that the transition rates are much more sensitive to the many-body effects reflected in the multipole decomposition of the effective transition operator than are the moments.

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Table (1)

We list here the multipole moments for both ^{42}Sc and ^{54}Co in terms of the quantity $C_{\pm}^{L=1} = [\alpha_{\lambda_1 \lambda_2}^{L=1} \pm \alpha_{\lambda_1 \lambda_2}^{L=1}]$, which is the multipole decomposition of the moments and transitions rates, respectively. The quantities Δ_+ and Δ_- are the size of the discrepancy for the moments and transitions, respectively.

$\lambda_1 \lambda_2$	$^{42}\text{Sc}: C_+^{L=1}$	$^{54}\text{Co}: (-)^{\lambda_1 + \lambda_2} C_+^{L=1}$	Δ_+	$^{42}\text{Sc}: C_-^{L=1}$	$^{54}\text{Co}: (-)^{\lambda_1 + \lambda_2} C_-^{L=1}$	Δ_-
0 1	0.20	0.23*	-0.03	-16.09	4.04*	-20.13
1 2	1.26	0.62	0.64	- 5.23	- 6.19	0.96
2 3	1.82	0.31	1.50	37.17	2.72	34.45
3 4	0.65	0.44	0.21	-28.49	-13.93	-14.56
4 5	4.10	1.13	2.97	52.62	19.87	32.75
5 6	-1.80	-0.36	-1.44	-26.81	- 7.12	-19.68
6 7	8.70	2.73	5.97	89.16	30.56	58.60

*The monopole has been reduced by a factor of seven [see eqn. (8)].

Figure Captions

Figure (1): The graph depicts the ratio of the magnetic transitions in both ^{42}Sc and ^{54}Co relative to the $(f_{7/2}^{\pm 1})_n (f_{7/2}^{\pm 1})_p$ single-particle transitions. The transitions are between the initial state (J_i) and the final state (J_f).

Figure (2): The graph depicts the ratio of the E2 transitions in both ^{42}Sc and ^{54}Co relative to the $(f_{7/2}^{\pm 1})_n (f_{7/2}^{\pm 1})_p$ single-particle transitions. The transitions are between the initial state (J_i) and final state (J_f) which differ by one unit of angular momentum.

Fig 1

$J_i \rightarrow J_f$

Fig 2

Fig 1