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**FINAL REPORT ON THE DEVELOPMENT
OF THE GEOGRAPHIC POSITION LOCATOR (GPL)
Volume 1
Summary**

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NOTICE

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MASTER

Preface

Navigation without external reference during a mission requires inertial navigation technology. If the mission length is more than a day or so, the position errors of the inertial navigator can become unacceptably large. However, if the mission includes stationary periods, the navigator's errors can be dramatically reduced by the effective use of these periods.

The idea of using the stationary periods to minimize inertial navigator errors has resulted in a concept called the Geographic Position Locator (GPL). A GPL is a completely automatic system with the following key functions:

- Sensing whether it is moving or stationary.
- Minimizing accumulated position errors during stationary periods.
- Comparing its computed position with stored position data and, as a result of this comparison, executing a prescribed action.

The GPL concept was investigated under the PASSPORT program. PASSPORT was conducted by the Lawrence Livermore Laboratory under the joint sponsorship of the U. S. Atomic Energy Commission's Division of Military Application and the Department of Defense's Advanced Research Projects Agency.

The objective of PASSPORT was to determine the feasibility of the GPL concept and to evaluate the capabilities of a GPL system using commercial-quality inertial components.

This objective was achieved. The excellent performance capabilities of the GPL error reduction techniques that resulted should make them attractive for use in any inertial navigation system that is occasionally stationary during a mission.

The GPL final report, UCRL-51130, consists of the following volumes:

1. Summary
2. Executive Summary
3. System Theory
4. Error Reduction
5. Motion Sensing
6. Performance
7. Field Considerations
8. Technical Support
9. WISPIN—An Inertial Navigation System Simulation Program
10. DINSII—A Navigation Data Reduction Program
11. Data Reduction Kalman Filter—Subroutine
12. Data Reduction A3FIX—Subroutine
13. Vulnerability (SDI)

The GPL development project was primarily staffed by an average of about 10 persons at LLL. In alphabetical order, those persons who were on, or directly associated with, the Livermore staff engaged in this project and who made significant and original contributions to the success of the GPL program were:

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VOLUME I - SUMMARY

Abstract

The Geographic Position Locator (GPL) is a small, self-contained unaided system for accurate, long-term, land-based inertial navigation. It uses the normal rest periods of land-based systems to reduce system errors and improve the subsequent navigational accuracy.

Operation can be fully automatic, with a motion-state sensor activating the error-reduction process during stationary periods. Stored map information for

automatic comparison with computed position, and selective response based on the comparison, also could be part of a fully-automated GPL system.

Possible use of the GPL with nuclear weapons was the primary motivation of our program. The techniques developed, however, apply to other navigation systems where the mission includes occasional stops and external references are unavailable.

Introduction

The Geographic Position Locator (GPL) was designed to provide long-term geographic positional information without access to external references. An inertial navigation system (INS) is a necessary but not sufficient requirement for such a mission. Even high quality, state-of-the-art inertial systems normally need external navigation aids to provide high accuracy for missions of extended duration. Without such aids a mission involving several days of continuous inertial navigation results in a position error of many nautical miles (nm). If the mission includes stationary periods, however, we can reduce the position error considerably even without external data. The information that it is not moving with respect to the earth can be used to great advantage by the system.

The GPL achieves improved accuracy by using the INS outputs as measurements

for reducing navigation errors whenever some other indicator establishes that the system is at rest. The GPL mission calls for occasional periods of motion interspersed with periods of rest over a time interval of many weeks. It is possible to detect these rest periods with a motion sensor that uses only data from the inertial platform. The motion sensor provides a velocity reference to the GPL by indicating that the true velocity is zero. Under these circumstances the INS-indicated velocities are, by definition, errors; they can then be used to estimate other system errors.

The development program's objective was to determine the feasibility of the GPL concepts, using a commercial-quality INS coupled to an additional computer. The major program effort aimed to develop a sensitive motion/rest discriminator, to explore the theoretical

means for position error minimization during rest periods, to simulate the error reduction techniques, and finally to evaluate them in the experimental system (GPL-A) under various assumed field conditions.

Design requirements for the GPL also include ruggedness, small size, low power and ability to deal with hostile environments. A GPL with these characteristics could have application to nuclear weapons and many other potential uses.

GPL System Description

The Geographic Position Locator (GPL) has three fundamental requirements:

- It must be capable of exercising a positive area control.
- It must be highly insensitive to accidentally or intentionally imposed environments.
- Its physical and logistic characteristics must be consistent with its intended operational uses.

Functionally, GPL is composed of three major subsystems—Navigator, System Condition Evaluator (SYSCON), and Responder—plus an integral standby power supply. See Fig. 1 for a block diagram of the system. Function of the subsystems are briefly as follows:

Navigator: computes position, compares it with stored information (MAP), and provides an appropriate signal to the Responder. Position is computed by an inertial navigation system comprising an inertial measuring unit (IMU) and the computer (NAV./E. RED.). In addition to navigation, the computer also carries out error minimizing procedures when the

The GPL system is described briefly in the next section. Subsequent sections describe the performance improvement concepts, system errors, error reduction, motion sensing and the navigation performance of GPL-A. Each of these is discussed in depth in the other volumes of this report.

The concluding section summarizes the results of the GPL program and our recommendations for future use of the GPL techniques.

system is motionless as determined by the motion sensor (MOTION).

System Condition Evaluator (SYSCON): identifies when the GPL is malfunctioning or when environments outside the design range are encountered.

Responder: receives signals from the Navigator and SYSCON, and provides an appropriate response.

The primary power source (EXTERNAL POWER) normally remains connected throughout a mission. The internal source (STANDBY POWER) provides power during short interruptions of the main power. The power is conditioned and distributed as necessary.

Although shown as separate blocks, the MAP and MOTION functions can probably best be integrated into the computer along with the navigation and error-reduction functions.

The system of Fig. 1 illustrates the basic elements needed for a GPL in its use with nuclear weapons. As stated previously, the GPL error-reduction techniques could be very effectively used

Fig. 1. Geographic Position Locator (GPL) system.

in normal land INS applications or any INS use involving occasional motionless periods. In such uses, moreover, most of the elements of Fig. 1 would probably be unnecessary. The only requirements would be an INS with a computer of sufficient additional memory capacity for the error-reduction and motion-sensing functions.

The motion sensor is not essential in the ordinary land navigation application, but it is recommended as a simple means of achieving automatic operation and of discriminating against very noisy environments; i.e., environments which might be detrimental to the error-reduction process even if the vehicle were not moving.

HARDWARE CONSIDERATIONS

Some consideration was given to predicting the characteristics that might be the most important in future applications of a GPL system. Approximate specifications applying to the high-quality future GPL system model shown in Fig. 2 include:

Weight	20-50 pounds
Volume	1/5-1/2 cubic foot
Power	100-200 watts
Reliability	1000-5000 hours MTBF

A GPL of the size shown in Fig. 2 could be made now using commercial quality inertial and computer hardware together with suitable interface electronics. Its performance would be similar to what we have achieved in the GPL-A system. More accurate INS units, when they become available, could lead to even better performance.

A trade-off between size and performance accuracy can probably be effected in the near future.

Several commercial-quality inertial navigation systems of very small size have been under development by various industrial firms. Primary emphasis in their development was to reduce cost, size, power and mechanical complexity and increase reliability. All of this is expected to be achieved without greatly reducing position accuracy performance from what is now available in commercial INS's.

MAP

In the GPL system a means (MAP) must be provided for determining if the

Fig. 2. Model of a high-quality, small-size GPL. Scale is in inches.

computed geographic position is within a prescribed geographic area. The problem of storing the area boundary with a given accuracy was studied with the objective of minimizing the storage requirement.

The MAP can best be stored in digital form in the memory of the GPL computer. In our study the MAP was undefined. It could range in complexity from a simple geometric shape to the irregular area of a large country with its boundary defined by natural features such as rivers, lakes and mountains. We covered the spectrum of possible area shapes by studying the worst case, an irregular area.

"Field Considerations," Volume 7 of this report, contains our recommenda-

tions and a discussion of several methods we studied for MAP storage. Computation speed is not a problem. The problem amounts to approximating an irregular boundary with straight-line segments.

To give the reader some idea of computer storage requirements imposed by MAP, India's boundaries can be stored to a useful accuracy for less than 1000 computer memory words. Probably most applications could be satisfied with only a fraction of this storage capacity.

COMPUTER

The INS used in GPL-A had a computer with 4000 memory words. Our studies indicate that approximately 6000 words

(24 bits each) of storage will be needed for the GPL with a suboptimal Kalman Filter and 7000 words if the optimal Kalman Filter is used. This is the total

memory requirement including 1000 words for MAP storage. See Volume 7 for a more complete discussion of the computer requirements.

Performance Improvement Concepts

Making use of the stationary periods to minimize position errors was a key element of the GPL system. Many systems such as tanks, aircraft, and portable missiles, associated with mobility, actually are at rest with respect to the earth's surface most of the time. Relative to our problem, then, the question is how best to take advantage of these rest periods. What information is available when a system is stationary? How can this rest condition be sensed? Can the rest period be used without degrading performance if motion is unexpectedly resumed?

The first and most obvious step is to freeze the computed position when the system is stationary; properly done this action causes the accumulated error to be more nearly proportional only to the time the system is in motion rather than total elapsed time. This is only a step in the right direction, however.

It also seems obvious that the north and east indicated (computed) velocities

could be set to zero. But these velocity errors contain useful information, and there is a preferred way to reset them. For example, information can be extracted from the east velocity to give a partial estimate of longitude error. Simultaneously, we can reduce or eliminate other errors that would contribute to the future buildup of more position error. Thus, considerably more can be done than just to freeze the position and zero the velocity.

We have demonstrated in an INS system that these performance improvement means can lead to greatly improved navigation. Their utility is limited by the characteristics of the system to which they are applied and the detailed interactions involved in their use. These factors are explored in the sections that follow. An estimate of the improvement to be expected in a particular application also requires, of course, a time-motion analysis of probable operational profiles.

System Errors

Many sources of error must be considered in the detailed analysis of an inertial navigation system. Most of these cause the INS to oscillate at its characteristic frequencies, with corresponding oscillations

in the errors of position and velocity. In general, the amplitudes of the oscillations grow with time, and they may become the dominant contribution to the total error in a short-term mission.

Fig. 3. Latitude error data from a continuous 72-hr test with unusually large 24-hr oscillation amplitude. The mean error is low, due to a low effective equatorial drift rate (0.0043 deg/hr).

However, oscillatory errors can be substantially reduced or removed when the system is at rest.

Figures 3, 4, and 5 show the nature of the errors in latitude, longitude, and velocity, respectively. The data were taken from a 72-hr continuous run of the experimental GPL—an aircraft inertial navigation system, modified for GPL ex-

perimental use, with a nominal position error growth rate of 1 nm/hr. The system was in motion for the first 7-1/2 hr and at rest for the remaining 64-1/2 hr. This run was typical of those made to get long-term base performance data (without error reduction).

Unusually large 24-hr oscillations were generated, as shown in Fig. 3. The

Fig. 4. Longitude error data from a continuous test of 72 hr. Note the Schuler (84-min) and 24-hr oscillations. The polar drift rate is derived from the test data and integrated to give the smooth curve shown here, which closely matches the mean longitude error.

higher frequency modulation is the Schuler (84-min) characteristic. The longitude error plot in Fig. 4 shows the oscillations as they appear in this error state. In the velocity channels, the Schuler oscillations usually dominate the 24-hr oscillations; this is apparent in Fig. 5 despite the large-amplitude 24-hr oscillations of this run. A simplified block diagram of the error dynamics of an inertial navigation system is shown in Fig. 6. This shows

the three loops that produce the system characteristic oscillations.

Certain other errors, though nonoscillatory, are also of little concern because they are bounded. The latitude bias error is an example of this type. (The 82-min and 24-hr components of error oscillate about the "bias value.") Note that the latitude bias error in Fig. 3 is only 1 arc-min (1 nm) at 72 hr and that this would be the final error if the oscillations were

correctly removed. This error corresponds to an equatorial drift rate of

$$U_e = \delta\lambda_0 = 0.0043 \text{ deg/hr} \quad (1)$$

where $\delta\lambda_0$ is the latitude bias error. (Equatorial drift rate is defined as the sum of the components of the north and vertical gyro drift rates resolved into a plane parallel to the equator.) Nomenclature for these and other terms in the GPL system is given in Table 1.

There is another error source, in longitude, which is of prime concern

because it is unbounded and increases faster than a linear function of time between rest periods. (By comparison, other errors are easier to control or make far less contribution.) This source is the drift rate, U_p , of the "polar gyro," a hypothetical gyro whose drift is the sum of the components of the north and vertical gyro drifts resolved into an axis parallel to earth's spin axis.

As an illustration, the polar drift rate was extracted from the test data and integrated to give a polar error angle plot (Fig. 4) which closely corresponds to the

Fig. 5. North and east velocity errors, showing the dominance of the Schuler (84-min) oscillations in these error states.

Fig. 6. Simplified diagram for the error dynamics of an inertial navigation system. These dynamics are dominant for velocities small compared to the earth spin rate.

longitude bias error ($\delta \lambda_0$). Polar gyro drift error remains after removal of the oscillatory error components. Most of the polar error angle curve can be modeled as a polar drift rate, U_p , which is ramping with a slope of 2.45×10^{-4} deg/hr²; that is,

$$\delta \lambda_0 \approx \int_0^t U_p dt$$

$$\approx \delta \lambda_0(t=0) + 1.22 \times 10^{-4} t^2 \text{ deg.} \quad (2)$$

In summary, if the oscillatory components of system error are properly handled, the only remaining errors of significance are those due to the gyros. The latitude bias error, normally small, is due to the unknown drift rate of the equatorial gyro and the longitude error is due to the unknown drift angle which the polar gyro develops. Finally, a direction

Table 1. GPL system nomenclature.

State	Description
α	x-tilt angle (about north axis)
β	y-tilt angle (about east axis)
γ	Azimuth error angle (about the vertical)
V_N	North velocity error
$\delta \omega_n$	North velocity error (angular) (V_N/R)
V_E	East velocity error
$\delta \omega_e$	East velocity error (angular) (V_E/R)
$\delta \lambda$	Latitude error
$\delta \ell$	Longitude error
U_x	x-gyro drift rate
U_y	y-gyro drift rate
U_z	z-gyro drift rate
n_x	x-gyro model white process noise
n_y	y-gyro model white process noise
n_z	z-gyro model white process noise
Ω	Earth spin rate
U_e	Equatorial drift rate
U_p	Polar drift rate
R	Earth radius
g	Gravity

bias error, normally of small value, is produced by the unknown drift rate of the east gyro.

Computer simulations verify that the polar gyro drift rate is the primary error

source. Analysis of experimental data also confirms this, as shown in Fig. 4. The following section on error reduction will explain how to reduce the effect of this error source.

Error Reduction

We have examined the nature of navigational errors and their relative importance to GPL. Some of the errors were oscillatory, usually with growing amplitude. Some were bounded. Most important to a long-term mission was the unbounded longitude error. We found that all these errors can be reduced and the oscillatory errors essentially removed during a rest period. However, the effectiveness of the rest period for error reduction depends in varying degrees on the length of time available, the technique used, and the time since the last rest period. Shorter navigation periods not only build up smaller errors, they also help the error-reduction technique make a greater percentage reduction of these errors.

Error reduction can be viewed, first, as a deterministic, directly calculable problem and second, as a purely statistical problem. Damping is an example of deterministic error reduction. It will be discussed briefly since it was used as part of an error-reduction technique. The statistical approach, epitomized by Kalman filtering, will be described in more detail since it was the most important aspect of the error-reduction methods.

Of the many error-reduction methods we studied, the optimal Kalman filter,

the suboptimal Kalman filter and the Kalman filter with platform rotation were judged to be the most important and are discussed in the following sections. Of these we implemented and tested, in the experimental system (GPL-A), the optimal Kalman filter and the suboptimal Kalman filter.

OPTIMAL KALMAN FILTER

In the past few years, several methods for reducing navigation system error have been introduced which treat the problem as purely statistical. This approach applies the theory of statistical filtering, pioneered by N. Wiener and further developed by R. E. Kalman.

Kalman filtering is the most powerful of the various techniques available for estimating and reducing errors. The Kalman filter requires more computer capability in the GPL than simple damping, for example. The small size of today's computers, however, makes this a reasonable penalty in exchange for improved performance, rapid error estimation, the ability to handle unobservable errors, and real-time statistical processing.

A Kalman filter incorporates statistical information on system disturbances and measurement noise, e.g. random gyro drift rates, in the design. This

additional information is the basis of the filter's improved error reduction performance.

The Kalman filter is optimal in the sense that it will minimize the difference between the INS errors and the error estimates at a maximum rate. This feature makes it possible to reduce errors substantially during very short rest periods.

It is possible with a Kalman filter to reduce some unobservable errors that deterministic methods cannot calculate. The filter algorithm computes the covariances between all the system errors. After the available measurements determine the observable errors, the filter uses the covariance to estimate statistically the nonobservable errors. This is very important when applied to the error in longitude. The longitude error is the larger component of position error and cannot be directly calculated from velocity error measurements alone. A significant portion of the longitude error is correlated with the polar drift rate (sum of those components of the gyro drift rates parallel to Earth's spin axis) and the oscillations in the stable platform azimuth angle, however. Both of these errors can be determined from the velocity errors and can be used with the covariances to make a "best guess" about the value of the longitude error.

The Kalman filter provides real time reduction of its input data or measurements to give a least squares (minimum variances) estimate of the system errors. Typically, the GPL error states to be modeled are given by the transposed state vector

$$\underline{x}^T = (\alpha \ \beta \ \gamma \ \delta\omega_n \ \delta\omega_e \ \delta\lambda \ \delta\lambda \ U_x \ U_y \ U_z). \quad (3)$$

Table 2. Kalman filter nomenclature.

State	Description
$\underline{x}$	State vector
$\dot{\underline{x}}$	Time derivative of state vector
F	Matrix of system dynamics
G, Q, R	Matrices describing random system disturbances and measurement errors
H	Measurement matrix
$\underline{z}$	Measurement vector
$\underline{v}$	Measurement noise vector
K	Kalman filter gain matrix
P	Error covariance matrix
$\dot{P}$	Time derivative of error covariance

Nomenclature for terms relating to the Kalman filter are given in Table 2.

The system error dynamics can be expressed as n simultaneous, first-order, linear differential equations of the form

$$\dot{\underline{x}} = F\underline{x} + G\underline{u} \quad (4)$$

where $\dot{\underline{x}}$ (n states) is the time derivative of the state vector, F is an n · n matrix of system dynamics, and G is an n · r matrix which relates the driving vector $\underline{u}$ (r elements) to the system dynamics. Figure 7 is a block diagram of Eq. (4), which is shorthand notation for Eqs. (A1) through (A7) of the appendix. In this case, G is the identity matrix, and the system disturbance vector is

$$\underline{u}^T = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ n_x \ n_y \ n_z). \quad (5)$$

The terms in $\underline{u}$ are due to the gyro drift model. In this case, a random walk process is assumed to be causing the system disturbance.

Fig. 7. Block diagram of first-order linear differential equations in state space notation. G is the $n \cdot r$ matrix relating the driving vector u (r elements) to the system dynamics, and F is the $n \cdot n$ matrix of system dynamics.

In the optimal filter all significant error states are modeled, although not all of the modeled errors converge to the values of the real system errors unless adequate measurements are provided. Considerable improvement in system performance results, however, even though the filter cannot estimate all the errors.

Figure 8 illustrates the essential parts of the Kalman filter. From a measurement

$$\underline{z} = H\underline{x} + \underline{v}, \quad (6)$$

the estimated error state vector can be updated with measurements according to the differential equation

$$\dot{\hat{\underline{x}}} = F\hat{\underline{x}} + K(\underline{z} - H\hat{\underline{x}}), \quad (7)$$

where the circumflex ($\hat{}$) indicates an estimate.

As shown in Eq. (7) the two elements which cause a change in the estimated state (error) variable, $\hat{\underline{x}}$, are the Kalman gain, K , and the difference between the

measurement and the estimate, $(\underline{z} - H\hat{\underline{x}})$. If both the measurement $\underline{z}$ and the estimate $H\hat{\underline{x}}$ were perfect, the expression would be zero and no change in the estimate would occur. Normally, however, there is a difference and the effect it has on $\hat{\underline{x}}$ depends upon K .

When at rest, the GPL gets its measurements from its own indications of north and east velocity and azimuth rate

Fig. 8. Block diagram of the continuous estimation equations. F is the matrix of system dynamics, G a system disturbance matrix, H a measurement matrix, and K the Kalman filter gain matrix.

(changes in indicated heading). The difference between the measurement and the estimate is weighted by K which is statistically optimum when derived by the equation

$$K = PH^T R^{-1}. \quad (8)$$

Equation (8) shows that K depends upon P and R^{-1} since H simply defines the relationship of the measurement to each error estimate. R is the measurement uncertainty. If R is small, R^{-1} is large and has a big effect on K . P is the system error covariance matrix. It describes the statistical uncertainty of the system error estimates; it grows when no measurements are present and decreases when measurements are provided. P propagates according to

$$\dot{P} = FP + PF^T + GQG^T - PH^T R^{-1} HP. \quad (9)$$

When measurements are available $PH^T R^{-1} HP$, being negative, causes P to decrease. This is logical since measurements should reduce the uncertainties of the system error estimates. P will continue to decrease until

$$GQG^T - PH^T R^{-1} HP = 0$$

that is, until the effect of additional measurements exactly balances the growth term, GQG^T . During navigation, when measurements are not available,

$$PH^T R^{-1} HP = 0$$

and P grows according to GQG^T .

This relationship reflects what one intuitively expects, namely that low dis-

turbance values to the system and low noise in the measurements permit the Kalman filter to converge rapidly to minimum uncertainty (variance) in estimating the errors. Conversely, high disturbance values and measurement noise slow down the convergence and produce a larger uncertainty in estimates of the errors.

As a control, we propagated the error covariance equations for the GPL navigator for 10 days without performing any error minimization, i.e., open loop. Equation (9) without the last term describes this operation. The resulting curves for RMS latitude and longitude errors appear in Fig. 8. The points were plotted at 24-hr intervals, to exclude the 24-hr oscillations. Note the complete dominance of longitude error over latitude error. This is typical of all the GPL covariance simulations. The longitude error therefore may be taken as the measure of the GPL total position error.

Covariance simulations of optimal Kalman filter error reduction were conducted over a 10-day interval with only one calibration period each day. The position error results are shown in Fig. 10. Comparison with the curves in Fig. 9 demonstrates the dramatic performance improvement possible with an optimal error reduction scheme; i.e., the 10-day error without calibration is about 60 nm, and the 10-day error with calibration once daily is less than 6 nm.

Figure 10 exhibits the capability of the filter to reduce the RMS longitude error each time calibration is performed. The partial error recovery is due to correlation of the longitude error (built up during the previous navigate mode) with the present value of polar gyro drift rate.

Fig. 9. Long-term open loop position error performance.

Fig. 10. Long-term optimal Kalman filter baseline performance. The scallops are due to longitude error recovery.

Figures 11a and b and 12 show actual performance results of two test runs of the experimental system, GPL-A, using the full Kalman filter for error reduction. The curves labeled "INS" are the uncorrected errors that developed during the test runs. The curves labeled "Filter"

are the errors remaining after applying the Kalman filter estimates; i.e., the corrected system errors. More detail on the performance results of this error reduction technique can be found in the "Performance" section of this report.

Fig. 11. Test results of the experimental system using the optimal (16-state) Kalman filter error reduction technique. (a) Longitude Error (b) Latitude Error.

Fig. 12. Example of total position (radial) error performance of GPL-A. Note the benefit of decreased time between measurements even though the measurement period is only half as long as in Fig. 11.

KALMAN FILTER WITH PLATFORM ROTATION

The optimal Kalman filter mechanization is north-seeking; i.e., the stable platform seeks to maintain a constant orientation with respect to true north. With constant orientation, the dominant source of error is minimized, and most of the potential for removing error is realized. Even so, many indeterminate error states remain. It is possible essentially to remove the remaining error states (except longitude bias) by rotating the stable platform. This is done normally in 90-deg steps, and at each orientation the filter is given time to achieve convergence to the affected states.

The Kalman filter with rotation is naturally more complex than the filter without rotation and, in general, provides

little additional performance improvement. This is because the azimuth and latitude bias errors are normally well-behaved. For a very long mission or for some other reason, however, occasional removal of all the latitude and azimuth errors might be desirable. For example, this scheme could be used between missions, during the periodic system reset activity, to perform a complete calibration. The GPL can do this only by means of the rotating filter. Further evaluation of the relative usefulness of this scheme versus the nonrotating one requires details on specific application requirements.

Neither the rotating nor the optimal Kalman filter uses damping. Instead these filters rapidly estimate the oscillatory error states, then use the data to correct the corresponding states in the inertial navigation system. Further, these filters do not require continuous measurements during the error reduction period. Their effectiveness is not significantly reduced even with measurements spaced by as much as a few minutes.

Needing neither conventional damping nor continuous measurements while in the error reduction state, the GPL can be highly isolated from any sudden motion. The INS remains in the navigation mode and the filter accepts no measurement until the motion sensor verifies the motion-free state. From the motion sensor standpoint this eliminates the need for rapid detection of motion, a significant point in some envisioned GPL applications.

DAMPING

In operation an INS gradually builds up platform oscillations which manifest

themselves as oscillatory errors in velocity and position. Since the peak amplitudes of the oscillations can rapidly grow intolerable they are removed at fairly frequent intervals. Usually this is done by damping.

Rapid oscillatory error removal requires heavy damping to increase the system's natural frequencies. For example, the Schuler (84-min) oscillations can be settled in 1 or 2 min by increasing the loop gain by a factor of several thousand. The net effect, so far as the rest of the system is concerned, is to remove the states in the Schuler loops. Similarly, the application of azimuth damping (gyro-compassing) effectively removes the 24-hr dynamics from the problem.

The penalty for heavy damping is that the system becomes hypersensitive to sudden motion and, if it occurs, subject to large dynamic errors. For the general case, therefore, damping design requires special attention to minimizing or eliminating system sensitivity to the onset of motion. For specific applications this refinement may not be needed. This is clearly true if the system can be alerted to imminent motion in a simple way.

The damping actions are commonly called platform leveling (vertical damping) and gyrocompassing (azimuth damping). These actions are performed serially; i.e., vertical damping is applied for a few minutes and then azimuth damping is added for a longer period.

Ordinarily damping is applied at a known location, making the oscillatory component of position error unimportant. In a GPL-type mission, damping is done in such a way that the indicated position

is corrected to its bias or most likely value. If this were not done the oscillatory position error would accumulate as position bias error.

SUBOPTIMAL KALMAN FILTER

The suboptimal filter can operate on the data only after damping removes the oscillatory errors. Transfer function analysis reveals that the velocity signals in the damped navigation system are effectively equivalent to direct measurements of the gyro drift states. Thus, if heavy damping is applied to the navigation system, the original velocity measurements may be used as inputs to the reduced-state Kalman filter.

In practice there are constraints which limit the attainable damping gains. Heavy damping makes the system hypersensitive to transient accelerations. Detailed study revealed, however, that the digital method of handling the accelerometer signals provided a means to detect the onset of motion and change from calibrate to navigate mode before using these signals. This made the damped GPL-A system immune to sudden motion regardless of the amount of damping used.

The suboptimal Kalman filter is a simplified approximation of the optimal filter. Normally, 10 to 16 error states are modeled in the GPL optimal Kalman filter. For the GPL-A suboptimal filter tests we chose a 3-state model; viz, longitude error, polar gyro random walk drift rate and polar gyro Markov drift rate. Both of the drift rate processes are modeled in Fig. 13. The Markov-type drift rate is dominant when the intervals between calibrations are of the order of the

$$E[n_{rw}(t)n_{rw}(\tau)] = q_{rw}\delta(t-\tau)$$

$$E[n_s(t)n_s(\tau)] = q_s\delta(t-\tau)$$

Fig. 13. Gyro drift error model elements.

correlation time. For longer intervals between calibration modes, the random walk drift-rate dominates.

Because of the matrix simplicity of the 3-state filter and to illustrate its makeup, the actual matrices that were used in the filter are presented as follows:

$$\underline{x} = \begin{bmatrix} \delta \ell \\ U_{p_{rw}} \\ U_{p_s} \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{\tau_s} \end{bmatrix}$$

$$Q = \begin{bmatrix} 0 & 0 & 0 \\ 0 & q_{rw} & 0 \\ 0 & 0 & q_s \end{bmatrix}$$

$$H = [0 \quad 1 \quad 1]$$

$$R = [r] \quad (10)$$

where r is the measurement noise power spectral density (PSD), q is the system disturbance noise PSD and τ_s is the Markov process/correlation time.

The measurement z used by the filter is obtained from the indicated velocities

Fig. 14. Three-state Kalman filter.

and heading according to

$$\begin{aligned}
 z &= -V_E \frac{K_V}{R} \cos \lambda - V_N \frac{K_Z}{R} \sin \lambda \\
 &\quad - \dot{\gamma} \sin \lambda \\
 &= U_x \cos \lambda - U_z \sin \lambda - \dot{U}_z \frac{K_V \sin \lambda}{K_Z} \\
 &= U_p - \dot{U}_z \frac{K_V \sin \lambda}{K_Z} .
 \end{aligned} \tag{11}$$

This shows that in a damped condition, V_N , V_E and $\dot{\gamma}$ provide a measurement of polar drift rate, U_p , corrupted by $\dot{U}_z$ (vertical gyro white noise).

Figure 14 shows how the filter uses the measurements. The filtering starts after damping settles the level and azimuth errors. The polar gyro drift estimate U_p is fed back as a correction to both the x and z gyro drift rates of the system in such a way that the equatorial component is unaffected.

The initial condition error covariance matrix must be specified for the filter. This matrix should include not only the expected mean squared values of the longitude error and polar gyro drift rate, but also the cross covariance terms. The cross terms represent the degree of correlation between U_p and $\delta \lambda$ built up over previous navigate mode. These terms allow the filter to estimate longitude error in addition to polar gyro drift rate. The longitude error recovery effect is demonstrated in Fig. 10. To generate the necessary terms, during the navigation mode the F-matrix is changed to

Fig. 15. Example of a test run using the suboptimal (3-state) filter.

$$F = \begin{bmatrix} 0 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\tau_s} \end{bmatrix} . \tag{11a}$$

This causes the covariance matrix to propagate during navigation to provide the initial condition covariance values for the next calibration period.

An example of the radial error performance of GPL-A with the suboptimal filter is given in Fig. 15. This is one of the plots of test data as it was recorded during the run. Because this error-reduction scheme uses damping and resets, a "before and after" picture, with some qualifications, is available only by partial reconstruction of the errors. The computer does this during the data reduction process by putting back the corrections that were made by the error-reduction filter. This process is not a

complete restoration, but it gives an indication of what the system performance

would have been if damping had been used alone.

Motion Sensing

GPL techniques are based on the condition that the mission will include one or more periods of rest. Before these periods may be used to reduce the accumulated navigation errors, the system must first verify that it is at rest.

In the GPL, sensing of the motion state is complicated by ground rules that include deliberate attempts to deceive the motion-sensing feature. Simply stated, if the system can be moved without detection by the motion sensor, the primary mission of the GPL might be subverted.

The GPL design requires sensing the motion state with enough sensitivity to detect any significant motion. Our study of the characteristics of motion has revealed that a simple acceleration threshold detector (normal accelerometer threshold $\sim 10^{-6}$ g) could be used to detect motion. Even relatively high thresholds of acceleration (10^{-2} g) are exceeded by any but the most painstaking attempts to move an object. If necessary, these thresholds can be made considerably more sensitive than 10^{-2} g because of the very high performance characteristics of the inertial components. In a noisy environment, however, an acceleration threshold detector would falsely indicate motion without significant displacement. By integrating the acceleration signal into a velocity signal and setting a velocity threshold we established a less ambiguous motion criterion.

If the threat of deceiving the motion sensor does not exist; for example, in normal land navigation, motion sensing signals could be supplied externally. This procedure is not recommended for three reasons, however: 1) we found it very simple to provide highly effective internal motion sensing. 2) The error-reduction process might be degraded by a rest period in a bad environment; i.e., an environment involving large amounts of vibration and/or shock. Correctly adjusted, an internal motion sensor would interpret this kind of rest period as motion and keep the GPL in the navigation mode, whereas the external means might not have this discrimination. 3) External connections invite potential problems. Furthermore, a navigation system that requires only a power source during its mission offers a maximum of convenience for installation and use.

An inertial navigation system provides a wealth of information about its own motion state. The GPL-A accelerometers, mounted on a stable platform, were sensitive to a few micro-g's. Pitch, roll and azimuth (heading) information could be resolved to less than 1 arc minute nominally, but the azimuth sensitivity was increased so that heading changes of as little as 4.5 arc seconds could be detected.

MOTION CHARACTERISTICS

Two types of motion can be ascribed to the GPL problem. "Gross motion" is

characteristic of normal activity. We define "careful motion" as that intended to delude the GPL by moving it gently enough to avoid triggering the motion sensor. Gross motion, although simple to detect, must be detected rapidly to prevent degradation of the error-reduction activity. This is especially important to the suboptimal error-reduction technique since, without it, gross motion during damping would induce large dynamic errors.

Interaction between the motion sensor and the optimal error-reduction technique is much less critical. This is because the INS is in navigate mode at all times. The navigation function remains unchanged during error-reduction. The motion-sensor function, in this case, is to determine whether or not to send to the error-reduction filter the INS measurements of heading and velocity. Furthermore, the filter does not require continuous measurements. In the GPL-A experiments, measurements were used once per minute for error-reduction. Thus, it was necessary only to ascertain that the velocity and heading indications were uncorrupted by motion during the previous one-minute period in order to use them as measurements—a less demanding requirement than that of the suboptimal filter.

For either filter, the more difficult task of the motion sensor is to detect Careful motion. Study effort was therefore concentrated in this part of the motion spectrum.

MOTION SENSORS

Three types of motion sensors were developed in the GPL program as follows:

1. TRANSIENT—fast response, medium sensitivity
2. AZIMUTH—slower response, more sensitivity
3. CAREFUL—very slow response, high sensitivity

The nominal characteristics of these motion sensors are given in Fig. 16. For uniformity and ease of comparison the sensitivity of all motion sensor types is given in acceleration units (g) even though the azimuth type responds only to heading (angular) changes. This is a useful way to characterize the azimuth type and it is based on our observation that motion, even with great care, is practically impossible without a consequent change of heading. Azimuth was used as this was the most sensitive axis of the GPL-A system.

Transient

When motion starts, velocity changes. This is the basis of the Transient motion sensor operation. Velocity is continually

Fig. 16. Motion sensor characteristics.*

* Note that, for ease of comparison, sensitivity is given in acceleration (g) units, even though the azimuth type responds only to change of heading. This is permissible because motion invariably causes heading changes.

monitored for changes. If the velocity change exceeds a given amount during a given period of time, this is interpreted as motion. Conversely, if the change of velocity is less than the preset amount during the time-period "window," this is interpreted as rest.

The values of velocity-change and time-period that were used in GPL-A were adjustable. Details on this and other features of motion sensing are contained in Volume 5, "Motion Sensing."

When going from motion to rest it is desirable to test the rest condition for several time periods. If no violation of the rest condition occurs during this longer time period, the GPL changes to the error-reduction mode. This procedure prevents frequent and possibly undesirable changes to and from the rest mode during short stops or from anomalous transient behavior of the system velocity indications.

Azimuth

The Azimuth motion sensor works on the same basis as the transient type except that it uses change of heading instead of change of velocity.

Careful

A somewhat more complex formulation was used in the Careful motion sensor design, based upon two hypotheses: (1) there are spectral differences between the background (ambient) vibration and that due to vehicle motion, (2) these differences can be detected by filtering the accelerometer outputs over long enough times to distinguish them and make a decision about the motion state.

This motion sensor was not tested in the GPL-A system because of its greater implementation requirements and because of the excellent performance of the Transient and Azimuth motion sensors. On the basis of our tests, these sensors adequately covered the entire motion spectrum. The Careful motion sensor is described here for completeness, in case an application should arise that could not be satisfied by the Transient and Azimuth motion sensors. It should be noted, however, that the Careful motion sensor's performance capability is based on theoretical work and not actual tests.

PERFORMANCE

The Transient and Azimuth motion sensors were integrated into the GPL-A system along with the suboptimal error reduction technique, Reset-3. Reset-3 is very sensitive to motion; its interaction with the motion sensor is much more complex and demanding than the optimal error-reduction technique, Reset-16. Successful motion sensor performance with Reset-3, therefore, would more than prove its ability to perform adequately with Reset-16. Time was not available to test motion sensors with both filters.

Tests were performed:

1. With the gentlest starts we could provide.
2. With simulated normal field operation.
3. With the most severe starts and stops the test vehicle could do.

The GPL-A system was completely automatic—making all decisions about its motion state and controlling whether

the GPL-A was in the navigate or error-reduction modes. All three parts of the tests were successfully accomplished with many starts and stops. Sudden motion, no matter how severe, was adequately handled and caused no degradation in Reset-3 performance.

Navigation Performance of GPL-A

RESET-16 RESULTS

We ran a wide variety of tests on the GPL-A experimental system to evaluate the 16-state optimal Kalman filter error-reduction technique, RESET-16. A brief summary of those tests is presented here. For more complete and detailed analyses, see Volume 6, "Performance."

The data discussed here are from eight experiments, representing about 350 hr of testing. These runs exhibited a broad spectrum of performance because of the variety of test conditions and the random nature of the navigation system error sources. Certain observations can be made, however. The implementation of a full Kalman filter as an on-line error-reduction scheme is definitely feasible and has been achieved in the GPL-A. Once the system computer program was debugged, RESET-16 operated reliably and with few surprises. A few initially strange results occurred because of inertial instrument imperfections that had been unobserved because of their subtle nature.

The performance of Reset-16 generally exceeded our expectations. On the average, for these data which include

Despite their tolerance to a reasonable operating level of background vibration, the motion sensors detected our best efforts to produce careful motion. The tests consisted of carefully pushing the test vehicle on a smooth surface. In all cases the sensors detected motion in less than 6 inches of travel.

test durations up to 80 hr, RESET-16 brought about a six-fold position error reduction relative to the errors of the INS alone despite the generally excellent performance of the INS. For example, our base performance tests would have indicated a navigator position error buildup to at least 25 nm in 70 hr, but Run 738, as shown in Fig. 17a, produced only about 6 nm. Of course, better-than-normal performance of the navigator makes the error reduction appear to be

Fig. 17a. Run 738 radial errors.

less effective. In all tests, however, filter performance exceeded that of the INS, reducing errors by factors of 3 to 20 for individual tests.

Test Description

The tests spanned a wide variety of navigate times, calibrate times, total duration, and INS initial conditions. They were all conducted at the LLL test facility, using the GPL-A experimental system.

Each test started with an initial 3-hr calibration period. This allowed the Reset-16 filter to reach fairly good gyro drift estimates. It also permitted the filter's initial covariance matrix to be propagated in time, thus generating off-diagonal terms (initially absent) which provide useful cross-correlation data to the filter.

Upon completion of the initial calibration, the GPL-A filter was put through repeated "navigate" and "calibrate" cycles. The status of the INS itself remained unchanged; it was only the filter's status which changed. During the calibrate periods, measurements of the INS velocity and azimuth errors were made available to the filter at 1-min intervals. Measurements were withheld during the navigate periods.

As mentioned above, RESET-16 and the INS operated open-loop during the test; i.e., without INS resets. This arrangement allowed us to evaluate unambiguously the effectiveness of RESET-16 in reducing INS position error; we could directly compare the INS position error with the filter's estimate of that error. This comparison was made throughout the test.

Fig. 17b. Run 745 radial errors.

Analysis of Results

The position error histories of the eight evaluation tests are presented in Figs. 17a-h. They are given in terms of the root-sum-square of the latitude and longitude errors (radial error). The upper curve is the INS radial error. The lower curve is the RESET-16 radial error, i.e. the error that would remain if the INS position errors were corrected by the RESET-16 estimates. The times in the navigate mode (NAV) and the calibrate mode (CAL) are indicated on each figure.

For ease of comparison the first four figures (17a-d) have very similar vertical scales and they are arranged by increasing time in the navigate mode (NAV). Figures 17e and 17f not only have similar scales, but also similar NAV times. In these figures the most significant difference is the calibrate time (CAL). Although Figs. 17g and 17h have different

Fig. 17c. Run 737 radial errors.

Fig. 17e. Run 739 radial errors.

Fig. 17d. Run 736 radial errors.

Fig. 17f. Run 746 radial errors.

Fig. 17g. Run 743 radial errors.

Fig. 17h. Run 748 radial errors.

vertical scales and NAV times, the filter errors are not greatly different.

These tests might be compared on several bases. Each has its particular drawback because the tests differed in

duration, mode profile, and, not the least, INS performance. Thus we have used several measures to characterize the tests, always keeping radial error as our primary measure of performance. For example, note the increase in RESET-16 errors as the NAV period (in Figs. 17a-d) increases. This is an expected result. In Figs. 17e and 17f, the different CAL times apparently dominate in the filter performance. The most significant comment that we can make in comparing Figs. 17g and 17h is the unusual way in which the INS errors developed. INS errors generally develop more like the plot of Fig. 17g than that shown in Fig. 17h.

In an attempt to quantify the performance of the runs analyzed here, we averaged the errors for both INS and filter for each run and called this average the "mean radial error." The averaged performance for each test, as well as for all tests combined, is given in Fig. 18. The RESET-16 estimation errors covered a wide range: 0.2 nm to 2 nm. The errors correlated (correlation coefficient = $r \approx 0.65$) with test duration, but they correlated even better ($r = 0.81$) with the cumulative time spent in the navigate mode during the test (Fig. 19). The mean-radial-error growth-rate estimated for Reset-16 is about 0.36 nm/day; this is about 1/6 the mean-radial-error growth-rate in the tests for the uncorrected INS.

Performance with Frequent Stops

The preceding data demonstrates acceptable performance under adverse conditions; e.g. operation in the NAV mode for 22 hr and in the CAL mode for 2 hr

Fig. 18. Mean radial errors for the INS and RESET-16.

each day. As a matter of interest we investigated the GPL's navigation capability under more favorable conditions, involving navigate periods of only 30 min and without the antideception feature. The NAV/CAL ratio is estimated at between 60 and 100, allowing less than 1/2 min for calibrations.

Figure 20 shows the precision navigation capability of the GPL using RESET-16 under these conditions. Actual performance of the filter was even better than indicated, since most of the variations were due to experimental limitations.

Oscillatory Error Removal

The previous analyses include the effects of the INS oscillatory error com-

ponents and the ability of the filter to estimate and compensate for them. Because of the unusual requirements for GPL-type error removal, however, the treatment by the filter of the oscillatory part of the errors seems worthy of separate comment.

In the usual case, removal of the INS oscillations is a trivial operation. This is because it is done at a known location and damping can be used without regard to its effects on position. When position error is unknown, however, removing oscillations is no longer trivial. Failure to determine both magnitude and phase of the oscillations brings in an unknown amount of position error bias from the damping action. The determination is

complicated by the long periods of the superposed Schuler and 24-hr oscillations. Despite this complication a full filter such as RESET-16 can, in a few minutes of measurement, form a very accurate estimate of the oscillatory content of the errors and remove or compensate for them. In fact, RESET-16 performs so efficiently that it makes this operation appear trivial. It is only when we con-

sider how we would do it without a Kalman filter that we appreciate the magnitude of the problem.

In other words, the Kalman filter is a superior way to achieve "damping." The oscillations may be removed by direct feedback correction to the INS or by propagating these error estimates and using them to correct the INS output. The tests of RESET-16 all use the latter procedure.

Fig. 19. RESET-16 mean radial error vs time in (a) navigate mode only, (b) navigate and calibrate modes (total). The slope of (a) is 0.5 nm/day and for (b) it is 0.38 nm/day.

Fig. 20. GPL-A performance using frequent brief stops.

RESET-3 RESULTS

Performance of the suboptimal error reduction technique RESET-3 in the GPL-A system was similar to that of RESET-16, based on the ratios of performance improvement (INS errors versus RESET-3 errors). For example, the RESET-3 mean radial error improvement ratio, averaged for all runs used in the evaluation, was 4.3/1 and the same ratio for RESET-16 was 6.5/1. Similarly the ratios of reduction in error growth rate were 5.1/1 for RESET-3 and 5.6/1 for RESET-16.

For brevity, the test data used to evaluate RESET-3 performance are not presented here. They may be found in Volume 6, "Performance." Instead the following discussion considers some significant differences

between the two test series and their evaluations.

Position Reconstruction

As explained under RESET-16 Results, those tests were conducted open-loop; that is, without feedback corrections to the INS. To get RESET-16 errors, its error estimates were subtracted from the corresponding INS output data during the data reduction procedure after a run. The remaining error was the RESET-16 error. This procedure is shown in the block diagram of Fig. 21a.

The primary purpose of RESET-3 was to minimize the GPL-A computer requirements without seriously degrading the error-reduction process. In order to achieve this end, the oscillatory errors were removed by damping during error reduction, and the INS resets were made at the end of the error-reduction period (see Fig. 21b). Since the output data from the INS was periodically corrected,

Fig. 21. GPL-A error reduction diagrams. These connections apply only to the error-reduction (motionless) periods.

there was no uncorrected INS data after the first reset. In order to evaluate the effects of RESET-3 it was necessary to reconstruct, as much as possible, the INS errors that would have built up if there had been no resets. The reconstructed INS errors approximate what would have occurred if only damping had been used during the error-reduction mode.

INS Performance with Resets

Although the performance improvement ratios for RESET-16 and RESET-3 were comparable, the actual INS mean errors during RESET-16 tests were only half as much as those observed in the RESET-3 tests. Consequently the difference in GPL-A mean radial errors using the two filters was even greater.

Part of this INS performance difference can be explained as normal variation. We would have been surprised if the INS performance had been the same in both test series. Some of the performance difference appears to be caused by the action of resetting, however. When RESET-16 experiments were performed using periodic resets, similar to RESET-3 normal action, the INS uncorrected performance also seemed to worsen.

Intuitively we expected frequent resets to improve the performance of the INS part of GPL-A. The indications are just the opposite. There is evidence to suggest that part of the advantage of using a suboptimal error-reduction technique may be lost because of its use of resets and damping; the well-settled system appears to have a greater rate of error-growth than the system with Schuler errors already established.

Damping Versus Estimation

In RESET-3 the oscillatory errors must be removed before the estimation process (Kalman filter action). The damping process uses the first 12 to 20 min of the error-reduction mode. During this time no data is available to the 3-state filter for estimating polar gyro drift and longitude error. Thus short stops are relatively ineffective for RESET-3 error reduction. An optimal filter such as RESET-16, on the other hand, can use measurements for error estimation from the start of the error-reduction mode. Just a few measurements, spaced at 1-min intervals, can provide a dramatic decrease in the estimation errors.

SUMMARY OF NAVIGATION PERFORMANCE

Our experience with GPL-A shows that RESET-16 is a very effective error-reduction technique. It can use motionless periods of only a few minutes duration. It appears to be most effective in the open-loop mode with infrequent resets depending upon the mission time duration or the magnitude of its estimation errors. A slight drawback is its greater computer memory requirement, but this is only a 10-20% increase over RESET-3.

Under somewhat restrictive conditions, involving relatively long and infrequent rest periods, RESET-3 can achieve performance comparable to that of RESET-16. RESET-3's main drawbacks are its inability to use short rest periods or to operate open-loop, and its more complex interaction with the motion sensor.

Conclusions

The GPL concept of using the stationary periods of otherwise mobile equipment for self calibration offers a major opportunity for navigator performance improvement. Compared to continuous navigation, computer simulations predicted that the RMS position errors could be reduced tenfold in a 10-day mission even if suitable stationary periods occurred as infrequently as once a day. Performance evaluation of GPL-A tests tended to confirm this prediction, although for experimental reasons we made no test runs that long.

Our studies and experiments revealed a number of attributes which may have broad utility. For example, we attained a significant amount of longitude error recovery. Even under conditions of unknown position error and navigate periods of several hours, the optimal Kalman filter can achieve oscillatory-error recovery using calibration periods as short as 5 min.

As predicted, the suboptimal Kalman filter scheme was able to approach the performance of a fully optimal Kalman filter. For the general GPL application, however, the suboptimal filter may not prove to be significantly less demanding of computer needs (only 10-20% less) than the optimal filter. Furthermore, the suboptimal filter's "simplifying" aspects actually introduce additional complexity in system operation and in the motion sensor interaction. The damping sequence discards Schuler error information and prevents the use of short calibration

periods. Also, there are fairly definite indications that, for the GPL-A at least, the INS does not perform as well (by about a factor of two in average position error) under the closed-loop or reset mode of operation required in this scheme, as it did open-loop.

While we cannot generalize that all of these comparisons would apply equally to GPL's using other INS's, our observation is that a GPL with the optimal Kalman filter more than justifies its requirement for additional computer memory. We recommend, therefore, that the optimal filter be used in future GPL systems or for improving performance in any INS applications involving stationary periods.

We developed three motion sensors and implemented and tested two of them in GPL-A. We determined from the tests that the motion sensors could be set for adequate sensitivity to careful motion and still tolerate normal levels of background vibration. Also, we verified that they could detect our most careful attempts to move the test vehicle and still change modes from calibrate to navigate under severe acceleration without detriment to the navigation accuracy.

Finally, the GPL concepts have been validated in theory and test. They offer markedly improved performance from any inertial navigator which comes to rest occasionally, without expensive instrument changes or external aids. The techniques should be useful for a wide range of applications.

Appendix A

The following equations describe the error dynamics in a terrestrial inertial navigation system using the notation of Table 1 in the section on System Errors. In a system which can occasionally remove or reduce the oscillatory errors, the only significant sources of errors are those due to the gyro drift rates, U_x , U_y , and U_z . These forcing terms are contained in Eqs. (A1), (A2), and (A3):

$$\dot{\alpha} = -\beta(\Omega \sin \lambda + \omega_e \tan \lambda) + \gamma \omega_n + \delta \omega_e - \Omega \delta \lambda \sin \lambda + U_x \quad (A1)$$

$$\dot{\beta} = \alpha(\Omega \sin \lambda + \omega_e \tan \lambda) + \gamma(\Omega \cos \lambda + \omega_e) - \delta \omega_n + U_y \quad (A2)$$

$$\dot{\gamma} = \alpha \omega_n - \beta(\Omega \cos \lambda + \omega_e) - \delta \omega_e \tan \lambda - \delta \lambda (\Omega \cos \lambda + \omega_e \sec^2 \lambda) + U_z \quad (A3)$$

$$\begin{aligned} \dot{\omega}_n = & -\beta(2\Omega \omega_e \cos \lambda + \omega_e^2 + \omega_n^2 - g/R) - \gamma(2\Omega \omega_n \sin \lambda + \omega_e \omega_n \tan \lambda) \\ & - \delta \omega_e (2\Omega \sin \lambda + 2\omega_e \tan \lambda) - \delta \lambda (2\Omega \omega_e \cos \lambda + \omega_e^2 \sec^2 \lambda) \end{aligned} \quad (A4)$$

$$\begin{aligned} \dot{\omega}_e = & \alpha(2\Omega \omega_e \cos \lambda + \omega_n^2 + \omega_e^2 - g/R) - \gamma(2\Omega \omega_e \sin \lambda + \omega_e^2 \tan \lambda) \\ & + \delta \omega_n (2\Omega \sin \lambda + \omega_e \tan \lambda) + \delta \omega_e \omega_n \tan \lambda \\ & + \delta \lambda (2\Omega \omega_n \cos \lambda + \omega_e \omega_n \sec^2 \lambda) \end{aligned} \quad (A5)$$

$$\delta \dot{\lambda} = \delta \omega_n \quad (A6)$$

$$\delta \dot{\lambda} = \delta \omega_e \sec \lambda. \quad (A7)$$

These equations can be greatly simplified by assuming, as an approximation, that the vehicle's velocity is negligible compared to the earth's velocity of rotation ($\omega_e = \omega_n = 0$). This assumption is reasonable for normal land vehicles, for example, the earth's velocity being 10-20 times that of the vehicle. In this case, and if the following combinations of equations are used in place of Eqs. (A1)-(A3), the simplified block diagram of Fig. 6 can be constructed:

$$\dot{\alpha} \sin \lambda + \dot{\gamma} \cos \lambda = -\Omega(\delta \lambda + \beta) + U_e \quad \text{[For (A1) and (A3)]}$$

$$\delta \dot{\lambda} + \dot{\beta} = \Omega(\alpha \sin \lambda + \gamma \cos \lambda) + U_y \quad \text{[For (A2) and (A6)]}$$

$$\dot{\alpha} \cos \lambda - \dot{\gamma} \sin \lambda = \delta \dot{\lambda} + U_p \quad \text{[For (A1) and (A3)]}$$

Equations (A1) and (A3) were multiplied by $\sin \lambda$ or $\cos \lambda$, as indicated on the left side, to get the desired form from the combined equations. U_e and U_p can be obtained from U_x and U_z by

$$U_p = U_x \cos \lambda - U_z \sin \lambda$$

$$U_e = U_x \sin \lambda + U_z \cos \lambda. \quad (A8)$$

The coordinate system for the above equations is shown in Fig. A-1.

Fig. A-1. Coordinate system.

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