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CEAL-1016

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AND POLARIZED ELECTRONS

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19 February 1965

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The research work described in
this report was performed under
Contract AT(30-1)-2076 between
the U.S. Atomic Energy Commission
and the President and Fellows of
Harvard College.

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Abstract

The use of a magnetic bottle in conjunction with the CEA for producing high energy positrons and polarized electrons is investigated. It appears to be possible to produce positrons with an intensity approaching 5 percent of the electron intensity, and polarized electrons with an intensity approaching the full electron intensity. The production of polarized electrons appears to be easier and more certain than the production of positrons.

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A. Magnetic Bottle for Producing Positrons

A magnetic bottle is a high field solenoid in which charged particles are trapped to travel with a spiral motion along tubes of constant magnetic flux. At each end of the solenoid there is a stopper to prevent particles from going out the end of the bottle. For the type of magnetic bottle used in plasma containment, this is an increase in the field strength of the solenoid, which reflects particles and prevents them going out to the end of the bottle if the pitch of the spiral motion is not too large. If one is only interested in one sign of charged particle, it is also possible to use an electrode with a potential to repel the particles, as a stopper at the ends of the bottle.

A magnetic bottle has two functions for use in producing positrons. If the high energy electron beam is used to produce the positrons, then the magnetic bottle serves to store them until the proper time to inject them into the linear accelerator and accelerate them in the synchrotron. Also the radiation loss in the field of the magnetic bottle reduces the transverse and longitudinal phase space occupied by the positrons, so that they can be accepted into the synchrotron.

For purposes of this analysis we assume that it will be possible to construct a solenoid of 2 inches bore and 10 feet long, with a superconducting coil that will produce a magnetic

field of 60 kilo-gauss.

If the positrons are trapped in the bottle with a small angle of pitch of their spiral motion, almost all their kinetic energy is due to the transverse motion. Under this condition, the electric potential required to reflect the particles at the end of the bottle is given by

$$V = \frac{\theta^2}{2} \frac{U^2 - (mc^2)^2}{U} \quad (1)$$

θ is the angle of pitch of the spiral motion, U is the total energy of the positrons. Thus for a particle of 20 Mev, and a pitch of .04 radian, the required potential is about 16 KV.

The power radiated by the positrons trapped in the bottle may be calculated by applying the relation

$$P_{\gamma} = \frac{2}{3} \frac{e^2 a^2}{c^3}$$

in the rest system of the particle. This results in a decreasing energy given by

$$T = \frac{2mc^2}{\left(\frac{U_0 + mc^2}{U_0 - mc^2} \right)^{t/\tau} - 1} \quad \tau = \frac{3}{4} \frac{c}{\omega_0^2 r_0} \quad (2)$$

T is the kinetic energy of the positron due to its transverse motion, ω_0 is the angular cyclotron frequency in the magnetic

field, r_o is the classical radius of the electron. Thus if the initial energy is 10 Mev, the kinetic energy will be reduced after $\frac{1}{30}$ second, to about 1.35 Mev. If the particle is observed in a system moving with the longitudinal velocity of the particle, the radiation is always emitted symmetrically. Therefore the radiation loss does not change the longitudinal velocity of the particle. The transverse velocity of the particle is determined by the energy of the particle as given by equation (2). Then as the energy decreases due to radiation loss, the angle of pitch of the spiral motion will increase. The angle of pitch will be given by

$$\theta = \tan^{-1} \theta_o \left[\frac{1 - \left(\frac{mc^2}{U_o}\right)^2}{1 - \left(\frac{mc^2}{U}\right)^2} \right]^{\frac{1}{2}} \quad (3)$$

Thus the angle of the spiral remains constant in the relativistic range, and increases in the non-relativistic range.

If the angle does not become too large, the variation of potential of the electrode required to stop the particle at the end of the bottle as the energy decreases is given by

$$V = \frac{\theta_o^2}{2} \left[1 - \left(\frac{mc^2}{U_o}\right)^2 \right] U = V_o \frac{U}{U_o} \quad (4)$$

Thus the required potential of the electrode decreases proportional to the total energy of the positron.

This reduction of required potential permits positrons to

be injected and stored in the magnetic bottle for many accelerator pulses.

A mode of operation in producing positrons would be to accelerate electrons in the reverse direction on alternate cycles of the Cambridge Electron Accelerator. The electrons would be extracted at high energy by a one turn extraction system and directed onto a converter to make the positrons.

The positrons would flow along the lines of magnetic flux into the bottle. The circulating beam in the accelerator will have a time length of about 0.5 microseconds, and will produce a pulse of positrons this long. At the beginning of this pulse, the potential on the stopper at the end of the bottle near the converter will be reduced to a fraction of its value so as to allow the positrons from the converter to be admitted into the bottle, while not allowing the trapped positrons to escape. If we want to trap particles up to an energy of 20 Mev, and an angle of pitch of .04, a potential of 16 KV is required. During the $\frac{1}{30}$ second interval between positron injection, the radiation loss will reduce the total energy of these positrons to about 2.0 Mev, and the required trapping potential to 1.6 KV. Thus the potential may be reduced to this value during the injection of positrons without allowing the trapped positrons to escape.

With a magnetic bottle 10 feet long, positrons with an angle of pitch of .04 radians will take 0.5 microseconds to traverse the length of the bottle, be reflected from the other end and

return to the injection end. Thus positrons with angles of spirals up to .04 radians may be trapped for the entire length of the 0.5 microsecond pulse.

The angular distribution of positrons from the converter will be somewhat peaked in the forward direction. The angular distribution may be transformed by having the magnetic field at the converter different from the magnetic field in the bottle. The relationship for transforming angles is given by

$$\frac{\cos^2 \theta_2}{B_2} = \frac{\cos^2 \theta_1}{B_1} \quad (5)$$

This transformation always results in an angular distribution which goes to zero as the angle of the spiral goes to zero. We assume that we can obtain a distribution in which the intensity per solid angle varies as $\sin \theta$. The total fraction of particles with angles between 0 and θ is then $\sin^2 \theta$, and the fraction between 0 and .04 radians is 1.6×10^{-3} . It may be possible to increase the number of positrons at small angles by proper arrangement of the shape of the converter and the angle of the electron beam, but it is not apparent how to do it.

If the electrons incident on the converter have an energy of 6Bev, there will be about 100 positrons per incident electron at the peak of the shower. Since the magnetic bottle will accept energies up to about 25 Mev, the fraction of positrons accepted should be determined essentially by the angular accept-

ance of 1.6×10^{-3} . For an electron intensity of 10^{11} per pulse, this gives an accepted positron intensity of 1.6×10^{10} per pulse.

In order to accept the positrons in the synchrotron it is necessary to leave the positrons in the magnetic bottle until the transverse momentum is damped to about 3×10^4 e.v./c. Since the diameter of the bottle is 5 cm, this results in a horizontal and vertical emittance of 1.5×10^{-3} cm radian, when accelerated to 100 Mev, which is expected to be smaller than the acceptance of the CEA. This corresponds to a transverse energy of about one kilovolt, and from equation (2) will require a damping time of about 0.5 second. Thus there will be 15 pulses of positrons stored in the bottle, and the average positron density will be 4×10^7 particles per cm^3 . It has been found that about 10^8 un-neutralized charged particles per cm^3 may be stored in a magnetic bottle without loss of particles due to collective effects (N. Milleron - private communication).

The energy corresponding to the maximum longitudinal velocity of $.04c$, is 400 e.v. This will have a negligible effect on the energy spread injected into the synchrotron.

In order to be able to selectively draw out the positrons which have been sufficiently damped by the radiation, it is useful to have a magnetic mirror at the end of the magnetic bottle opposite to the converter. Positrons with a transverse energy of 1000 e.v. and a longitudinal energy of 400 e.v. have

an angle of spiral of about 32° . To reflect them requires the mirror field to be about 40% higher or about 85 Kg. To draw positrons out of the bottle when it is desired to inject them into the synchrotron, the electrode at the position of the magnetic mirror would be pulsed somewhat negative, drawing out the positrons which have been sufficiently damped.

This magnetic mirror may also be useful to remove positive ions formed in the bottle. This would be done by operating the electrode at the mirror at a very small negative D.C. potential to continuously draw out the ions.

The gas pressure in the magnetic bottle must be sufficiently low that the positrons are not annihilated or scattered out of the bottle during the trapping time.

The annihilation may be determined from the annihilation rate for non-relativistic positrons given by

$$\frac{1}{N_p} \frac{dN_p}{dt} = -\pi r_o^2 N_e c$$

N_e is the electron density due to the gas. To make $\frac{1}{N_p} \frac{dN_p}{dt} = 10^{-2} \text{ sec}^{-1}$ requires $N_e = 1.3 \times 10^{12} \text{ per cm}^3$. This corresponds to a pressure of nitrogen of about 10^{-5} mm .

In the non-relativistic range where scattering is most important, the rms angle due to scattering, and the increasing angle of pitch of the spiral motion increase almost proportionately. The ratio between them is given by

$$\frac{(\theta_s^2)^{\frac{1}{2}}}{\theta_0} = \frac{1}{\theta_0} \left[\frac{1}{2} \frac{E_s^2}{(mc^2)^2} \frac{\rho}{X_0} \tau c \right]^{\frac{1}{2}}$$

Taking $X_0 = 40 \text{ gm/cm}$, $\theta_0 = .04$, $\tau = 7 \times 10^{-2} \text{ sec}$, a density of $1.5 \times 10^{-13} \text{ g/cm}^3$ is required to obtain a ratio of 0.5. This corresponds to a pressure of about $2 \times 10^{-7} \text{ mm}$ which should be easily achieved at the low operating temperature.

After drawing the positrons out of the magnetic field of the bottle by pulsing the electrode, an additional difficulty is presented in removing them from the very strong field of the bottle, while preserving the transverse phase space.

In principle this may be done by directing the positrons to flow along a decreasing magnetic field, thereby increasing the area of the beam inversely proportional to the field. When the positrons are in a sufficiently small field, they may be accelerated by an electric field to an energy such that their radius of curvature is comparable to the size of the beam, and they are effectively removed from the field. However this method requires quite a large diameter for the positron beam and may be practically undesirable.

Another method is to have the positrons flow into a region of sufficiently small field, that a ferromagnetic end plate may be used to terminate the field. This plate would have small slots in it to allow most of the positrons to pass through it into the field free region.

If the positrons flow into a region of 600 gauss field, the diameter of the beam will be increased by a factor of 10 to 20 inches, and the transverse momentum will be reduced to 3×10^3 e.v./c. An additional transverse momentum will be added by the fringing fields of the slots. The maximum transverse momentum added by a slot of height h is given by

$$p_t = \frac{Beh}{2c}$$

If the slots are 0.5mm high, the maximum transverse momentum produced is 4.5×10^3 e.v./c. This may be applied in the horizontal plane in which the acceptance of the synchrotron is considerably larger so as not to reduce the accepted intensity. If the electrons are accelerated to 200 kv before passing through the plate, the maximum angular deviation normal to the slots is 6×10^{-3} radian. It should then be possible to design an end plate such that most of the positrons will pass through it without being lost.

After passing through the end plate, the positrons may be focussed to a beam of small diameter by weak elements. The positron beam would probably be bunched at the synchrotron rf frequency by one or more klystron type cavities, and injected into the linear accelerator and accelerated by the synchrotron.

By the use of the magnetic bottle, it should then be possible to obtain positron intensities of about 10^{10} particles

per pulse, within the transverse and longitudinal phase space that can be accepted into the synchrotron.

B. Magnetic Bottle for Producing Polarized Electrons

The magnetic bottle may also be used to produce polarized electrons.

One method of doing this utilizes the fact that the energy difference of the two spin states of electrons trapped in the very high magnetic field of the bottle may be made larger than the energy corresponding to the low temperature of the walls of the bottle, such that in thermal equilibrium most of the electrons will be in the lower energy state.

For this purpose a shorter bottle should be adequate, as the electrons may be injected at low energies from a thermionic cathode. If the magnetic bottle is two feet long, with a two inch bore, and the electrons were injected with an energy of 5-10 volts, they would require about 0.5 microseconds to traverse the length of the bottle, be reflected from the other end, and return to the injection end. At that time the electrode would be pulsed to a negative potential to trap the electrons in the bottle. Magnetic mirrors could also be used as stoppers instead of electrodes, since it is not desired to store more than one injection pulse in the magnetic bottle. This would permit the space charge of the electrons to be neutralized, if this is desirable.

In order that the electrons become polarized there must be some medium to exchange energy between the trapped electrons and the cold walls of the bottle. The relaxation time of the electron spins due to radiation will be very long compared with the orbital damping time, as it is a magnetic dipole transition; and is not a practical means for polarizing the electrons.

A medium of energy exchange with the cold walls is the gas in the bottle. The electrons would then become polarized by collisions with the gas molecules.

In order to obtain a large cross section for a spin flip collision, it seems desirable to use a gas which can experience a spin exchange collision with the trapped electrons. A gas which can be used is atomic hydrogen. The cross section for a spin exchange collision between an electron of 5-10 volts energy and a hydrogen atom is estimated to be about 10^{-16} cm^2 .

A possible operating condition would be to have a superconducting solenoid which produces a magnetic field of 80kg in the 2 inch diameter, 2 foot long bottle. If the walls of the bottle were operated at a temperature of 4°K , in the state of thermal equilibrium the ratio of the populations of the two electron spin states would be about 7 to 1.

It does not appear necessary that the translational energy of the electrons should be damped to thermal energy in order that the electron spins may become polarized by spin exchange collisions with hydrogen atoms. If we take the electron energy

to be 5 e.v., and it is desired to obtain a relaxation time with the hydrogen atoms of 5 milliseconds, the required density of atomic hydrogen is 1.5×10^{10} per cm^3 , corresponding to a pressure of about 10^{-6} mm.

The atomic hydrogen would be produced in an external source and injected into the bottle. The electron spins of the atomic hydrogen would quickly become polarized as the atoms with the lower energy spin state would tend to be retained in the magnetic field of the bottle, while the higher energy spin states would go out through the apertures in the walls of the bottle or would be reflected by the magnetic field of the bottle when they are injected. In order for this to take place the tube conducting the atomic hydrogen from the source to the magnetic bottle, should be cooled to the temperature of the bottle. The hydrogen atoms may also be polarized by contact with the cold walls of the bottle.

In order that the atomic hydrogen should not adhere to the walls at the low temperature of operation, it would be necessary to coat the walls with a chemically inert substance such as argon.

With this method of operation the electrons trapped in the bottle would be polarized in the $\frac{1}{60}$ second between accelerator cycles by spin exchange collisions with the hydrogen atoms. If we inject 10^{11} electrons per pulse into the bottle, the ratio between the number of hydrogen atoms and trapped electrons

will be about 200. The lifetime of the atomic hydrogen will be determined by atoms going out the end of the bottle where the electrons are ejected. The lifetime for the atoms with electrons in the low energy spin state is estimated to be about 1/20 second, including the reflection of the low energy state at the end of the bottle. It would then be necessary to inject atomic hydrogen into the bottle at a rate of 4×10^{14} atoms per second, which appears to be feasible.

After the electrons become polarized, they will be drawn out by applying a positive pulse to the electrons at the end of the bottle. The polarized electrons will be removed from the magnetic field of the bottle by using a ferromagnetic endplate, similar to that used for positron production.

The other method of production of polarized electrons would be to inject hydrogen atoms with the electron spins polarized into the magnetic bottle, and to coat the walls with a substance which will not depolarize them, as is done in the hydrogen maser.

This method has the advantage that it does not require very high magnetic fields or very low operating temperatures.

The polarized hydrogen would be obtained by a sextupole magnet as is used in the hydrogen maser. This magnet focusses the $F=1, M=1$, and $F=1, M=0$, states into an aperture. If the magnetic field is sufficiently large that the energy difference of the electron spin states is large compared with the hyperfine energy difference, the electron spins of the hydrogen atoms are completely polarized. The intensity of polarized hydrogen atoms

which may be attained is perhaps 10^{15} atoms per second.

If the bottle has the same size of 2 inch bore and 2 feet long, the lifetime of the hydrogen atoms, due to going out one end of the bottle is estimated to be about 10^{-3} second at room temperature. The equilibrium density would be about 10^9 atoms per cm^3 , which would then require a time of about .1 seconds to polarize the electrons, so six pulses per second could be obtained,

This may be improved by putting a mechanical shutter at the end of the bottle, which is only opened when it is desired to draw out the electrons. The injected atoms will accumulate in the bottle for the time between electron ejection, thus achieving higher densities. It appears that this method would produce pulses of polarized electrons at a rate approaching 60 pulses per second, with an intensity approaching 10^{11} particles per pulse.

After being extracted from the magnetic bottle the spins of the electrons would be precessed to be transverse to the direction of motion, and injected into the linear accelerator and accelerated by the synchrotron.

During the acceleration cycle the electrons will be somewhat depolarized by a type of resonance which is encountered at points in the the acceleration cycle. The worst of these occur when the precession frequency due to the anomalous part of the magnetic moment of the electron is an integral multiple of the orbital frequency or is equal to the vertical betatron frequency. The latter resonance occurs at about 2.8 Gev.

It may be possible to reduce the depolarizing effect of these resonances by installing a special quadrupole lens in the synchrotron, whose strength can be programmed so that the resonances are crossed rapidly.

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