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FUNDAMENTALS IN THE OPERATION OF
NUCLEAR TEST REACTORS

VOL. 1 REACTOR SCIENCE AND TECHNOLOGY

MASTER

PHILLIPS
PETROLEUM
COMPANY

ATOMIC ENERGY DIVISION

NATIONAL REACTOR TESTING STATION
US ATOMIC ENERGY COMMISSION

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FUNDAMENTALS IN THE OPERATION OF NUCLEAR TEST REACTORS

Vol. 1	Reactor Science and Technology	IDO-16871-1
Vol. 2	Materials Testing Reactor Design and Operation	IDO-16871-2
Vol. 3	Engineering Test Reactor Design and Operation	IDO-16871-3
Vol. 4	Engineering and Design Considerations for In-Pile Tests	IDO-16871-4

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Printed in USA. Price \$2.50. Available from the Office of
Technical Services, U. S. Department of Commerce
Washington 25, D. C.

IDO-16871-1
AEC Research & Development Report
Reactor Technology
TID-4500 (20th Ed.)
June 1963

FUNDAMENTALS IN THE OPERATION OF NUCLEAR TEST REACTORS

VOL. 1 REACTOR SCIENCE AND TECHNOLOGY

PHILLIPS
PETROLEUM
COMPANY

Atomic Energy Division

Contract AT(10-1)-205

Idaho Operations Office

U. S. ATOMIC ENERGY COMMISSION

FUNDAMENTALS IN THE OPERATION OF NUCLEAR TEST REACTORS

FOREWORD

The safe and efficient operation of nuclear test reactors and their associated experimental programs requires a high degree of technical understanding of reactor behavior and careful attention to details on the part of the operators of such equipment. Regardless of how many protective devices are incorporated into the control circuitry of the reactor and the experimental loops, well-trained and intelligent operators are still the best assurance that the operations will be carried out safely and with a minimum of reactor down-time.

FUNDAMENTALS IN THE OPERATION OF NUCLEAR TEST REACTORS is intended to provide the basic informational background essential to a clear understanding of the problems associated with the conduct of irradiation test programs in the Materials Testing Reactor (MTR) and Engineering Test Reactor (ETR), and of the means of coping with them successfully. The authors of the various chapters are engineers and scientists on the staff of the Phillips Atomic Energy Division with several years of experience at the MTR and ETR.

This reactor operations training manual has been compiled under the general direction of a committee appointed by me and designated as the MTR-ETR Operator Training and Certification Committee. The membership of this committee is indicated on a preceding page. While the general scope of the manual and preliminary chapter headings were designated

by me, the selection of authors and final chapter coverage was left to the committee in consultation with interested responsible supervisors. In addition to its responsibilities in connection with the preparation of the training manual, the committee will have a continuing responsibility for organizing and directing the training program itself, as well as for the examination and certification of competence of all trainees graduating from the program. The training program will require approximately one year to complete and will comprise both classroom discussions based on the training manual and practical experience on the MTR and ETR. The general prerequisite for trainees is a college degree in engineering or one of the relevant sciences, although, in certain instances, talented individuals with less than a college degree will be admitted to the training program.

Rather than confining the manual to the bare minimum of what the reactor and experiment operator needs to know to carry out his immediate assignments, we have chosen a broader approach to the relevant subject matter for the benefit of those trainees who may not be well oriented in the rapidly expanding field of reactor development. It is our hope that by enlarging the informational background of the operator trainees we will stimulate a greater interest and, hence, help to develop reactor operators with a broader perspective and a better appreciation of the relationship of their efforts to the nation's over-all reactor development program.

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FUNDAMENTALS IN THE OPERATION OF NUCLEAR TEST REACTORS
VOL. 1 REACTOR SCIENCE AND TECHNOLOGY

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VIII	Reactor Safety	K. V. Moore

Errata Sheet for IDO-16871-1

(Corrected items are underlined)

Page 21, right column, line 18, should read:

"x direction, equation (2-27) becomes:"

Page 44, item H. (2), should read:

"(2) Mechanical Effects (meat and clad"

Page 67, line C. 2. a. (1), should read:

"(1) Careful pH control of p primary"

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CHAPTER I

NUCLEAR PHYSICS PERTINENT TO REACTOR OPERATION

A INTRODUCTION

This short account of nuclear physics is limited to those items most pertinent to the operation of MTR-ETR type reactors. An attempt is made to stress the physical concepts rather than the mathematical analyses of nuclear physics. Therefore, the treatment must be considered as subject to a number of limitations, qualifications and exceptions. In order to keep the text short and easily readable these qualifications are usually not mentioned.

A number of references for general supplementary reading are listed at the end of this chapter. It is not expected that any one student will read all of them. In general, the material has been taken from reactor engineering or physics textbooks rather than nuclear physics textbooks.

B UNITS AND BASIC CONCEPTS

1. Units

The units used in nuclear physics are usually taken from the metric system. Standard conversion tables can be used to convert to and from the usual engineering units.

The energy of a nuclear particle is usually expressed in "electron volts" (ev). By definition, one electron volt is the energy acquired by a particle carrying unit charge when it falls through a potential of one volt and is numerically equal to 1.6×10^{-12} ergs. For convenience, the "kiloelectron volt" (kev) and the "million electron volt" (Mev) are also used. The energy of electromagnetic radiation, such as gamma rays, is also usually given in ev.

The mass of nuclear particles is often expressed in "atomic mass units" (amu). The most abundant isotope of oxygen is taken as the standard for the physical scale of atomic weights and is arbitrarily assigned the value of 16.000000 amu. The atomic mass unit is then 1/16 of the mass of the most

abundant isotope of oxygen and is equal to 1.660×10^{-24} gm. The actual mass in grams of an atom is equal to its atomic weight in amu multiplied by 1.660×10^{-24} .

Other unusual units are introduced at the appropriate places in the text. An attempt is made to use standard terminology. This requires frequent use of Greek letters and use of the same symbol for more than one quantity. The context will usually make clear the meaning of the symbol.

2. Equivalence of Mass and Energy

In many processes the conservation of energy and the conservation of matter can be assumed to occur independently. However, in the operation of a reactor the transformation of matter into energy is an important factor. The relationship for this conversion was developed by Einstein and is given by

$$E = Mc^2 \quad (1-1)$$

where

E is the energy produced

M is the mass consumed or lost

c is the velocity of light

The reverse process also occurs and the equation (1-1) can be considered as expressing the equivalence of mass and energy. If **M** is in grams and **c** is in cm/sec, then **E** must be in ergs. Converting **E** to Mev and **M** to amu and substituting the measured velocity of light, equation (1-1) becomes

$$E(\text{Mev}) = 931 \times M(\text{amu}) \quad (1-2)$$

The energy of fission is produced by the transformation of mass into energy. The reverse process occurs to a limited extent in a reactor in some secondary nuclear reactions such as pair production.

3. Equivalence of Particles and Waves

A number of experiments have shown that

processes in which energy is usually considered as being transmitted by waves can exhibit the behavior of particles. Conversely, in many instances particles behave like waves. It has been shown that a particle of mass M moving with a velocity v has an equivalent wave length λ of:

$$\lambda = \frac{h}{Mv} \quad (1-3)$$

where h is Planck's constant.

An electromagnetic radiation of wave length λ has an equivalent mass of:

$$M = \frac{h}{\lambda c} \quad (1-4)$$

where c is the velocity of light.

The reactions of neutrons with matter are often most easily explained in terms of wave properties. The MTR crystal spectrometer utilizes the wave properties of neutrons.

4. Quantization of Electromagnetic Radiation

It is an experimental fact that the transfer of energy to a body absorbing or from a body emitting electromagnetic radiation occurs only in integral multiples of a fundamental unit known as a "quantum" or "photon." The magnitude of the energy of the photon depends upon the frequency ν of the radiation as given by the equation:

$$E = h\nu \quad (1-5)$$

In some situations the electromagnetic radiation acts like particles with an energy given by equation (1-5) and a mass given by equation (1-4). It should be noted that the amount of energy per unit time from a radiation source is determined by both the number of photons and the energy of an individual photon.

C THE ATOM AND THE NUCLEUS

1. Elementary Particles

The number of elementary particles recognized by modern physics has become quite large. However, only the few of these discussed below need be considered for the purposes of this text.

a. Electron and Positron

The electron carries a unit negative charge and has a mass of about 0.00055 amu. In nuclear reactions, the electron's mass number is considered to be zero. When an electron is emitted from the nucleus by radioactive decay it is often called a "negative beta particle." In some types of radioactivity a particle is emitted with unit positive charge and the same mass as the electron and is designated as a "positron" or "positive beta particle."

b. The Proton

The proton is a particle with a unit positive charge and a mass of 1.0076 amu. It is identical with the nucleus of a hydrogen atom. The proton has a mass number of one.

c. The Neutron

The neutron is an electrically neutral particle with a mass number of one and a mass of 1.0090 amu.

d. The Neutrino

The neutrino is a neutral particle of very small mass. It carries off a moderate amount of the energy produced by fission. If it is recognized that energy of the neutrino is not available, its existence can be neglected as far as reactor operation is concerned.

2. The Atom

a. Structure

The atom consists of two parts: a small positively charged core called the nucleus; and an electron or electrons revolving around the nucleus in definite orbits. For an atom in its normal state the number of electrons is the same as the number of positive charges in the nucleus so that the atom as a whole has no electrical charge. The atom has a diameter of about 10^{-8} cm, while that of the nucleus is in the order of 10^{-13} to 10^{-12} cm. The nucleus is a small dense particle at the center of the much larger electron orbit and is made up of neutrons and protons. It can be seen from the relative masses of a proton or neutron and from that of the electron that almost all of the mass of the atom is concentrated in the nucleus.

b. Nomenclature

The following definitions and symbols are used in discussing atomic or nuclear structures and reactions:

A = the "mass number," the sum of all the neutrons and protons in a nucleus.

Z = the "atomic number," the number of protons and thus, the number of positive charges in a nucleus. It is also equal to the number of orbital electrons in the non-ionized atom.

p = the symbol for the proton.

n = the symbol for the neutron.

e- or β^- = the symbol for the electron or negative beta particle.

e+ or β^+ = the symbol for the positron or positive beta particle.

Nuclide = a species of atom having a specified number of protons and neutrons. This term is often used instead of isotope.

Nucleon = a general term for either constituent of a nucleus, the neutron or the proton.

Isotopes = two or more nuclei having the same number of protons but different numbers of neutrons. Since isotopes have the same arrangement of orbital electrons their chemical properties are almost identical.

Isotope = a term sometimes used as having the same meaning as nuclide. The distinction between referring to one nucleus as an isotope and designating nuclei as isotopes of each other should be noted.

The following form is used for writing the formula for a nuclide:

$${}_Z(\text{chemical symbol of element})^A$$

Thus, uranium-235 is ${}_{92}\text{U}^{235}$, ordinary hydrogen is ${}_1\text{H}^1$; the electron is ${}_{-1}\text{e}^0$, the positron is ${}_1\text{e}^0$, the neutron is ${}_0\text{n}^1$; the proton is ${}_1\text{p}^1$ or ${}_1\text{H}^1$, etc. The subscript is fixed by the designation of the element and is not necessary. However, it is often convenient for balancing nuclear reactions.

Nuclear reactions are described by means of a nuclear equation. The atomic and mass numbers on each side of the equation must balance because in the nuclear reactions considered, the number of protons and of neutrons remains constant. For example, the absorption of neutrons by boron may be written:

It is not necessary, although often convenient, to give both the symbol and atomic number since one defines the other. Nuclear reactions are often written in shorter form so that equation (1-6) is expressed as:

3. The Nucleus

The nucleus consists of protons and neutrons. The electrostatic forces between the positively charged protons are balanced by nuclear forces that prevent immediate disintegration of the nucleus. The theory for these nuclear forces is not well developed, but their magnitude can be determined by a combination of theory and experiment. The masses for the constituents of atoms, electrons, protons and neutrons are well known. The mass of many atoms has been measured, and the measured mass is always less than that of the sum of the particles making up the atom. The difference between the total mass of the particles and the measured mass of the atoms is called the "mass defect." It represents the mass which would be transformed into energy if the atom were assembled from its constituent parts. The same amount of energy is required to break up the atom and thus the mass defect is a measure of the "binding energy" of the atom. The binding has been measured for a number of nuclides. The binding energy per nucleon is found by dividing the total binding energy of the nucleus by the total number of nucleons, i.e., the mass number A. Figure 1 shows the variation of binding energy per nucleon with mass number. It is seen that the binding energy per nucleon goes through a flat maximum at a mass number of about 60 and then drops slowly as the mass number decreases. Thus, nuclei of intermediate mass number are more stable than the heavy nuclei. If a nucleus of mass number 236

FIGURE 1
Binding energy per nucleon
for stable nuclides

is split into two equal nuclei, the binding energy per nucleon is increased since the number of nucleons does not change and therefore, the total binding energy is also increased. This increased binding energy is obtained by the conversion of mass into energy. It is this process which provides the energy of fission.

D. RADIOACTIVITY

1. The Radiations from Radioactive Elements

A number of isotopes undergo a spontaneous transformation into another type of nucleus. This process is called "radioactivity" and is accompanied by the emission of penetrating radiations. The nuclide which emits the particle is called the "parent" and the resulting nuclide, the "daughter." The three most common types of these radiations are described below.

a. Alpha Particles

The alpha (α) particle has a positive electric charge equal to twice that of the electron and a mass equal to that of the helium nucleus. It is identical to the helium nucleus. Alpha emission leads to a decrease of the mass number by 4 and the atomic number by 2. An example is described by the "nuclear reaction" equation:

This can be expressed in words as: the radium

nuclide with an atomic number of 88 and a mass number of 226 decays to form a radon isotope with an atomic number of 86 and a mass number of 222, plus an alpha particle with atomic number 2 and mass number 4; and the reaction is accompanied by the release of energy. In nuclear reaction equations, the orbital electrons are usually neglected and a symbol such as ${}_2\text{He}^4$ can be considered as referring to the nucleus rather than to the atom.

b. Beta Particles

Beta (β) particles are simply high speed electrons that result from nuclear disintegrations. While they are ejected from the nucleus, it is not true that electrons exist as such inside the nucleus. It can be considered that a neutron in the nucleus has undergone a transformation to a proton and an emitted electron according to the reaction:

An example is the reaction:

where Q is the energy released.

Negative beta emission increases the atomic number by one but leaves the mass number unchanged since the number of nucleons remains the same.

In some cases a positron is emitted and positive beta emission is said to occur. This decreases the atomic number by one and leaves the mass number unchanged as in the reaction:

c. Gamma Rays

Gamma (γ) rays are electromagnetic radiations of very short wavelengths. Emission of γ rays permits the nucleus to lose excess energy which it has acquired but no particle emission occurs and thus no change is produced in the type of nuclide. Frequently, after emission of an α or β particle, the resulting nucleus is left with an excess of energy and this appears as γ rays.

The term gamma ray is usually reserved for electromagnetic radiation resulting from reactions of the nucleus. Electromagnetic radiations arising from interactions with orbital electrons are designated as "X-rays." Usually the energy of an X-ray photon is lower than that of a gamma ray photon.

2. Decay Laws

a. One Radioactive Species

Consider the case where only one type of radioactive atom is present. The fundamental law of radioactive decay is that for a specified type of nucleus, the probability of a given nucleus decaying in a unit of time is a constant, i.e., independent of external conditions or the number of nuclei present. Stated another way, it is an experimental fact that the number of disintegrations occurring in unit time is proportional to the number of radioactive atoms present. From either approach the mathematical expression for the rate of decay is:

$$\frac{dN}{dt} = -\lambda N \quad (1-12)$$

where

N is the number of radioactive atoms at a given time

λ is the "decay constant," which is a measure of the decay probability.

If at any arbitrary zero time, N_0 radioactive

atoms are present it is found by integration of equation (1-12) that:

$$N = N_0 e^{-\lambda t} \quad (1-13)$$

The rate of decay is seen to be determined by the number of atoms present and the decay constant λ . The variation with time of the number of parent atoms is an exponential function.

b. Mean Life and Half Life

The "mean life" or "average life," designated t_m , of a radioactive species is related to the decay constant by the expression:

$$t_m = \frac{1}{\lambda} \quad (1-14)$$

It can be shown that the average life expectancy of the radioactive atoms present at any time is equal to the mean life.

It is customary to express the decay of a radioactive species in terms of the "half life," $T_{\frac{1}{2}}$. It is defined as the time required for the number of the radioactive atoms present to decay to half of the initial value. By substitution of N equals $1/2 N_0$ in equation (1-13), it is found that

$$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \frac{0.6931}{\lambda} \quad (1-15)$$

The half life is inversely proportional to the decay constant and directly proportional to the mean life as determined by the relation:

$$T_{\frac{1}{2}} = 0.6931 t_m \quad (1-16)$$

c. More Than One Radioactive Species

In a number of cases the daughter atom, produced by decay of the parent, is also radioactive and there may be a series of decay stages before a stable isotope is formed. For the case of the first two stages in a decay chain the following balance equation holds

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \quad (1-17)$$

where

N_A is the number of parent atoms with decay constant λ_A

N_B is the number of daughter atoms with decay constant λ_B .

The balance equation (1-17) can be expressed in words as follows:

The net number of daughter atoms formed equals the number of daughter atoms formed by decay of the parent minus the number of daughter atoms lost by radioactive decay.

If $N_B = 0$ at $t = 0$, that is if the initial number of daughter atoms is zero, the solution to equation (1-17) can be shown to be:

$$N_B = \frac{\lambda_A N_{A0}}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t}) \quad (1-18)$$

where

N_{A0} is the number of parent atoms present initially

Cases involving more than two steps in the decay chain are handled by solution of a balance equation similar to (1-17).

The general behavior of a parent-daughter decay chain depends on their relative half lives. If the parent A has a longer half life than the daughter B, then after some time $e^{-\lambda_B t}$ becomes negligible in comparison with $e^{-\lambda_A t}$ and from equation (1-18) the following equation can be derived:

$$\frac{N_B}{N_A} \cong \frac{\lambda_A}{\lambda_B - \lambda_A} \quad (1-19)$$

This equation states that the ratio of parent to daughter atoms becomes a constant and "transient equilibrium" results. The number of each nuclide decreases exponentially with time according to the half life of the parent

If the half life of the parent is very long and very much greater than that of the daughter then $\lambda_B - \lambda_A \cong \lambda_B$ and after some time

$$\frac{N_B}{N_A} \cong \frac{\lambda_A}{\lambda_B}$$

In this case, the amounts of parent and daughter are almost constant with time and "secular equilibrium" exists.

If daughter half life is longer than that of the parent, λ_A is greater than λ_B , no equilib-

rium is reached. Starting with a pure parent the amount of daughter increases to a maximum and then decreases while decaying according to the half life of the daughter as the supply of the parent is depleted.

In a reactor parents are formed by fission and neutron capture, and daughters are lost by neutron capture as well as by radioactive decay. Balance equations for this more complicated situation are discussed in paragraph G of this chapter.

3. Units of Radioactivity

The "curie" expresses radioactive intensity in terms of the number of atoms which disintegrate per unit time. The curie is defined as that quantity of any radioactive material which gives 3.70×10^{10} disintegrations per sec.

The "roentgen" is a unit which measures the amount of ionization and thus the amount of energy, produced by electromagnetic radiation, X- or gamma rays. The formal definition is: "The roentgen shall be the quantity of X- or gamma radiation such that the associated corpuscular emission per 0.001293 gm of air produces, in air, ions carrying one electrostatic unit of quantity of either sign." The value of the energy absorbed per roentgen is 0.108 ergs per cubic centimeter of air or 83.8 ergs per gram of air.

The "rad" is a unit of deposition of energy by ionization equal to 100 ergs per gram of absorbing material. The rad is preferred because its application is more straightforward than that of the roentgen in many situations

E CROSS SECTION AND MEAN FREE PATH

1. The Microscopic Cross Section

In order to develop quantitative expressions for the reactions of neutrons with nuclei, the concept of a cross section is introduced. Assume that a uniform parallel beam of neutrons is directed against a target of a one square centimeter area. The following terms are defined:

N = number of nuclei and thus atoms, per cubic centimeter of target.

I_0 = the initial beam intensity in neutrons per square centimeter per sec

I = the beam intensity after passing through the target

X = the thickness of the sample.

σ = the microscopic cross section of one target nucleus in square centimeters of effective area per nucleus. Its definition is developed in the following discussion.

In a target of thickness dX and unit area the number of atoms is NdX . Accepting σ as the effective area per nucleus, the total effective area in thickness dX is σNdX . Thus, the fraction of the total target area of 1 cm^2 that is effective in producing an interaction between a neutron and a nucleus is $\frac{\sigma NdX}{1}$ and this may be interpreted as the probability that an incident neutron will react with a target nucleus in a thickness dX . If it is considered that the neutron-nucleus interaction removes the neutron from the beam, the change in intensity of the beam in thickness dX is:

$$-dI = I \sigma N dX \quad (1-20)$$

The minus sign arises because the beam intensity decreases in traversing the sample. Rearranging equation (1-20) and integrating across the sample thickness X yields the following equations:

$$\int_{I_0}^I \frac{-dI}{I} = \sigma N \int_0^X dX \quad (1-21)$$

$$\ln \frac{I_0}{I} = \sigma NX \quad (1-22)$$

$$I = I_0 e^{-\sigma NX} \quad (1-23)$$

In the above equations σ and N occur only as a product which gives the cross section of all the nuclei in a cubic centimeter. This quantity is called the "macroscopic cross section" Σ . The microscopic and macroscopic cross sections are related by the equation:

$$\Sigma = N\sigma \quad (1-24)$$

For an element:

$$N = \frac{\rho}{A} N_0 \quad (1-25)$$

and

$$\Sigma = \frac{\rho \sigma N_0}{A} \quad (1-26)$$

where

ρ = the density in grams per cubic centimeter

A = the atomic weight

N_0 = the Avogadro number, 6.025×10^{23}

For a mixture of several types of nuclei:

$$\Sigma = N_1 \sigma_1 + N_2 \sigma_2 + \dots N_i \sigma_i \quad (1-27)$$

where

N_i = the number of nuclei per cm^3 of the i^{th} kind.

σ_i = the microscopic cross section of the i^{th} type of nucleus.

The definition of the microscopic cross section can be obtained in an equivalent but somewhat different manner. Consider that the uniform beam of $I \text{ n/cm}^2/\text{sec}$ impinges on a layer of target material one atom thick containing N_a atoms/ cm^2 . Then, let the number of nuclear processes occurring per unit time be C .

The constant C can represent the number of absorptions, fissions, scattering events or any other nuclear process of interest. It should be noted that a scattering event can be considered to remove the neutron from the beam although the neutron is not lost in the same sense as in an absorption. The microscopic cross section σ is then defined by the relation:

$$\sigma = \frac{C}{N_a I} \text{ cm}^2/\text{nucleus} \quad (1-28)$$

It is seen from equation (1-28) that σ has the dimensions of area and is the average number of events, of the type considered, per incident neutron per nucleus. The cross sectional area is thus defined as a reaction rate and need not have any correspondence to a geometrical area.

Nuclear cross sections are often in the range of 10^{-22} to $10^{-26} \text{ cm}^2/\text{nucleus}$. To obtain a convenient unit for the microscopic cross section, σ is customarily expressed in

terms of a unit of 10^{-24} cm²/nucleus, designated the "barn." The macroscopic cross section has units of cm² per cm³ or cm⁻¹.

2. Mean Free Path

For a medium in which only absorption occurs, it can be shown that the average distance λ , lambda, that a neutron travels before it is absorbed is given by

$$\lambda = \frac{1}{\Sigma} \quad (1-29)$$

This average distance λ is called the "mean free path" or, since it is the distance in which the beam is reduced to 1/e of initial value, the "relaxation length." Mean free paths for other processes such as scattering are defined in a similar manner

F NEUTRON FLUX AND CURRENT

1. Flux

Consider neutrons of velocity v cm/sec traveling in a medium in which λ cm is the average distance a neutron travels before undergoing a specified type of reaction. Then on the average the probability that a neutron will undergo a reaction in a second is v/λ . For a uniform beam of neutrons containing n neutrons per cm³ the number of neutron-nuclei reactions per sec is $\frac{nv}{\lambda}$ and the quantity nv , expressed as neutrons per cm²/sec, is an important quantity designated as the neutron flux and given the symbol ϕ , phi. In many cases of interest the neutrons are not in a beam but are traveling in all directions. In such cases nv is the sum of the distances traveled by all the neutrons in one cm³ in one sec. Therefore, the flux as defined in this manner is called the "track length flux"

As long as all of the neutrons are traveling with the same velocity there is no ambiguity in the definition of flux. However, in a reactor there is an important group of neutrons, the thermal neutrons, in which the neutrons have all velocities over a considerable range, as discussed in Chapter II. In this case two common definitions of flux exist. The first is the track length flux which is obtained by an integration to give the total length traveled by all neutrons in the thermal group regardless

of velocity. If the cross section varies with neutron velocity or energy, a properly averaged cross section must be used with the track length flux to obtain the correct reaction rate.

The second definition is that used in the flux measurements at the MTR-ETR Site and described in a report by Hogg.* This flux is referred to in this report as the "activation flux." It requires the use of the cross section value at 2200 meters per sec to give the correct reaction rate. The differences between the two definitions of flux will be explained more fully after the concept of thermal flux is developed. It is important to remember that the proper combination of flux and cross section must be used in order to obtain the correct reaction rate.

2. Current

In a parallel beam of neutrons having a velocity v cm/sec and a density of n neutrons/cm³, the number of neutrons crossing a plane perpendicular to the beam is n neutrons/cm². The "neutron current" is defined as the net number of neutrons crossing unit surface per unit time

and is given the symbol $\vec{J}$, where the arrow indicates a vector quantity. For the beam described above the magnitude of the current is the same as that of the flux. The current and flux both have dimensions of neutrons/cm²/sec. However, it must be recognized that the flux, as defined in the previous section, is a scalar quantity. That is, for the current, direction is important while for flux it is not.

In general the magnitude of the current and the flux is not the same. Consider a sample exposed to two equal but oppositely directed beams of neutrons. The net current in such a case is zero, but it is evident that the reaction rate and thus the flux is twice that of a single beam.

G INTERACTION OF RADIATION WITH MATTER

1. Neutron Interactions

Among the important reactions of neutrons with matter are (a) (n, γ) reaction or radiative capture, (b) the (n, α) reaction; (c) the (n, p) reaction, (d) fission, (e) elastic scattering; and (f) inelastic scattering. The fission reaction is discussed later in this chapter and the scattering reactions in Chapter II, Reactor Physics.

*Hogg, C. H., "Thermal Neutron Flux Measurement at the MTR-ETR Site," IDO-16538

a. Radiative Capture

In radiative capture, the target nucleus absorbs a neutron to produce a product nucleus with the same atomic number but with the mass number increased by one. Thus the product nucleus is an isotope of the target nucleus. The product nucleus is formed in an excited state and returns to the ground state by the emission of "capture gamma rays." Thus radiative capture is also referred to as an "(n,γ) reaction." Radiative capture reactions with slow neutrons are very common in a thermal reactor. Examples are:

The product is often radioactive, exhibiting negative-beta decay which is frequently accompanied by gamma rays. Thus radiative capture is an important source of gamma rays.

Most of the processes by which radioisotopes are made in reactors involve radiative capture. An example is the formation of Co-60 from Co-59:

The formation of a radioisotope in a reactor is described by the balance equation shown for the production of Co-60.

$$\frac{dN^{60}}{dt} = \phi \sigma^{59} N^{59} - \lambda N^{60} - \phi \sigma^{60} N^{60} \quad (1-33)$$

where

N^{60} is the number of Co-60 atoms present at any time

N^{59} is the number of Co-59 atoms present at any time.

σ^{59} and σ^{60} are the microscopic capture cross sections for Co-59 and Co-60 respectively

λ is the decay constant for Co-60.

Equation (1-33) states that cobalt-60 is formed by radiative capture in Co-59 and is lost by radioactive decay and neutron absorption by Co-60.

In a reactor, it is typical for the formation of radioisotopes, at any flux level, to approach a condition of equilibrium. At equilibrium, product atoms are lost as soon as formed. At the specified flux level, there exists a limiting concentration which cannot be exceeded, no matter how long the irradiation. The equilibrium condition can be found very simply from (1-33) by setting the derivative on the left equal to zero and rearranging to give:

$$N^{60} = \frac{\phi \sigma^{59} N^{59}}{\lambda + \phi \sigma^{60}} \quad (1-34)$$

If N^{59} is assumed constant, as is usually approximately true, then (1-34) gives the equilibrium concentration of N^{60} .

b. (n,α) and (n,p) Reactions

Slow neutron reactions resulting in the emission of an alpha particle (n,α) or a proton (n,p) occur only with a few light elements. This is because a positively charged particle must overcome an electrostatic potential in order to be expelled from the nucleus. The (n,α) reaction of thermal neutrons with boron-10 is of interest because of the use of boron in control rods and in the detection of neutrons. The equation is:

The α particle or helium nucleus is ejected with sufficient energy to produce considerable ionization in passing through a gas. This is the basis for use of the reaction in detection of slow neutrons. Only a few isotopes of low atomic number undergo (n,p) reactions with slow neutrons. An example is:

In general, reactions with fast neutrons, other than fission and scattering, are not important in reactor operation, although they

may be in some types of tests. However, an exception is the (n,p) reaction of fast neutrons with oxygen-16:

The nitrogen-16 formed is beta active with a half life of 7.3 sec and in addition emits a high energy gamma ray. It is this reaction that makes necessary the heavy shielding on the exit water lines of a water-cooled reactor.

2. Ionization

A fundamental process in the interaction of radiation with matter is ionization. A charged particle in passing through matter occasionally approaches sufficiently close to an atom or molecule that the electrical forces are sufficient to remove an external or orbital electron. The atom with one (or more) electron removed is a positively charged ion and together with the removed electron forms an ion-pair. It is thought that virtually all health effects are ultimately due to the ionization of atoms in the cells of the body. The majority of the positive ions and electrons eventually recombine and liberate heat in the process. Thus most of the energy of ionizing radiations eventually appears as heat. However, some chemical effects may be produced by the charged particles indicating that a portion of the radiation energy is transformed into chemical energy.

The intensity of ionization from a moving charged particle is expressed in terms of the number of ion-pairs formed per cm of path and is called the specific ionization. For particles of equal mass the specific ionization increases with the charge. The ability of a charged particle to produce ionization depends not only on the charge, but also on the time it spends in the vicinity of a given atom. Thus a slowly moving particle is a more effective ionizer and, for particles of the same energy, those of higher mass produce a higher specific ionization.

Alpha and beta particles, being charged, produce ionization directly and are referred to as ionizing radiation. Since gamma and X-rays are uncharged, they are not ionizing radiations. However, these rays eject electrons and thus indirectly produce ionization. Neutrons also produce ionization indirectly as they travel through matter.

3. Interaction of Alpha Particles with Matter

The double charge and high mass of an alpha particle make it an effective ionizer. Alpha particles produce about 50,000 to 100,000 ion-pairs per cm in passing through air at atmospheric pressure. Beta particles of similar energy produce about 30 to 300 ion-pairs per cm.

Alpha particles from any one type of source have nearly all the same energy or fall into a few definite energy groups. As an alpha particle proceeds along its path the production of ion-pairs causes a loss in energy and thus a decrease in velocity. Eventually the velocity becomes so low that electrons are captured and the alpha particle becomes a neutral helium atom which cannot cause ionization. For a given energy there is a rather definite distance from the source called the range, at which the particle is no longer ionizing. The range in air of some α -emitters, of interest in reactor operation, are shown in table 1.

Table 1

Energies and Ranges in Air
of Some Alpha Emitters

Isotope	Energy of Particle Mev	Range in Air	
		cm	in
Thorium-232	4.0	2.5	1.0
Uranium-238	4.2	2.7	1.06
Uranium-239	4.4 & 4.6	2.9 & 3.1	1.15 & 1.2
Uranium-233	4.8	3.3	1.3
Plutonium-239	5.1	3.6	1.4

In materials more dense than air the range is decreased because the greater concentration of atoms increases the specific ionization. The relative stopping power of a material is defined as:

$$\text{relative stopping power} = \frac{\text{range in air}}{\text{range in material}}$$

The relative stopping power of water paper or animal tissue is about 1000, of aluminum 1600 and of lead 5000. For alpha particles

of radioactive origin the range in air is less than 2 in. and in paper is thus less than 0.002 in. Ordinary paper is about 0.004 in. thick and so will stop almost all alpha particles of radioactive origin.

4. Interaction of Beta Particles with Matter

Beta particles are emitted with a continuous distribution of energies up to a limiting maximum energy determined by the source. Thus beta particles do not have a definite range as do alpha particles. There is however, a thickness of absorber beyond which the primary ionization from the beta particle is almost zero. Ranges for beta particles of several energies in air are listed in table 2.

Table 2

Approximate Ranges of Beta Particles in Air

Energy, Mev	Range in All	
	meters	ft
0.1	0.11	0.36
0.5	1.50	4.90
1.0	3.70	12.00
2.0	8.50	28.00
3.0	13.00	43.00

It is to be noted that the range of beta particles is much greater than that of alpha particles. In addition to ionization, beta particles or electrons also lose energy by the production of an electromagnetic radiation called "bremsstrahlung." The beta particles are so light that deflections produce sufficient deceleration to cause radiation of electromagnetic energy. The bremsstrahlung is quite penetrating, and ionization is produced beyond the range of the beta particle. The behavior of the bremsstrahlung is similar to that of gamma or X-rays as described in paragraph 5.

As a first approximation it may be assumed that the linear range in any medium is inversely proportional to the density. The rate of energy loss as bremsstrahlung per unit path length is approximately proportional to the energy of the particles and to the square of the atomic number of the absorber.

5. Interaction of Gamma Rays with Matter

The discussion on the interaction of gamma rays with matter also applies in general to other forms of high energy electromagnetic radiations such as X-rays and bremsstrahlung. The differences that occur are the result of the generally higher energy of gamma rays. The interaction of gamma rays with an absorber takes place by a number of processes. Three of the most important of these are the photoelectric effect, the Compton effect and the pair production.

a. The Photoelectric Effect

The photoelectric effect produces ionization by transfer of all of the energy of a gamma ray photon to an orbital electron. The energy of the photon must, of course, be greater than the binding energy of the electron to produce ionization, otherwise the atom is excited and the interaction usually results in production of an X-ray. The excess of the gamma photon energy over that of the binding energy is transferred as kinetic energy to the ejected electron which behaves exactly like a beta particle of the same energy. Replacement of the photoelectron, as the ejected electron is called, by another electron from an outer orbit produces a low energy X-ray. The photoelectric effect leads to essentially complete absorption of the gamma ray, although it may be replaced to some extent by low energy X-rays. In general few of these secondary X-rays escape from the absorbing material.

The photoelectric effect decreases with increasing energy of the gamma rays and decreasing atomic number of the absorber. In general the photoelectric effect is important only for energies less than about 1 Mev, and even then only for absorbers with high atomic number.

b. The Compton Effect

The Compton effect decreases the energy of a gamma ray photon by an elastic collision with an orbital electron of the absorbing material. This process is an example of an electromagnetic wave behaving as a particle. The Compton effect is proportional to the atomic number of the absorber and decreases with increasing energy. However, it is important for energies considerably higher than

those at which the photoelectric effect occurs. An interaction by the Compton effect does not produce a true absorption but merely a decrease in the gamma ray energy and a possible change in direction. The lowering of the energy increases the probability of absorption by the photoelectric effect. Thus, the gamma ray may suffer a number of collisions before it either escapes or is absorbed by photoelectric interaction.

c. Pair Production

The passage of a gamma ray photon with an energy greater than 1.02 Mev near a nucleus can result in annihilation of the photon and formation of an electron-positron pair, a process called "pair production." The minimum energy of 1.02 Mev for pair production is the energy equivalent, by the Einstein relation, of the total mass of the electron and the positron. The positron reacts with the electrons in the absorber to produce two 0.51 Mev photons of electromagnetic radiation. These secondary photons are distributed uniformly in all directions and are easily absorbed. The electrons resulting from pair production behave as beta particles. Thus, for shielding purposes pair production can be considered as a true absorption.

Pair production increases as the energy of the gamma ray increases above 1.02 Mev. Since the photoelectric effect and the Compton effect decrease with increasing energy, pair production becomes the dominant process for absorption of high energy gamma rays.

H. FISSION

1. The Fission Process

A nuclear reaction in which the nucleus is split into fragments is called "fission." The splitting is accompanied by the production of neutrons and the release of energy. In general the fragments are of unequal mass but each has a relatively high mass number. Thus, an alpha emission in which one of the products is a helium nucleus is not considered to be a fission. In a reactor the nucleus which fissions is nearly always split into two product nuclei.

The fissions of a chain reaction are produced by absorption of a neutron by a fissionable nucleus. The fissions in reactors of the MTR-ETR type are nearly all produced by thermal neutrons which have only a small amount of

kinetic energy. Therefore, this discussion is devoted largely to thermal fission.

2. Fission Neutrons

Since a neutron is required to produce a fission, it is only because each fission yields several neutrons that it is possible to obtain a chain reaction and utilize reactors as sources of power. The neutrons producing fission have velocities of the order of thousands of meters/sec and energies of the order of several hundredths of an ev. The neutrons resulting from fission have velocities of the order of tens of millions of meters/sec and an average energy of about 2 Mev. It will be seen later that this difference in energy between the neutrons causing fission and those produced by fission is very important in determining the characteristics of thermal reactors, such as the MTR and the ETR.

Over 99 percent of the fission neutrons are released simultaneously, within about 10^{-14} sec, with the fission process and are called "prompt neutrons." The prompt neutrons are produced with energies covering the range from 10 Mev down to thermal. The average energy is about 2 Mev with the most probable energy being around 0.7 Mev.

The remainder of the neutrons are emitted over a period of minutes after fission and are called "delayed neutrons." The intensity of emission decreases gradually with time according to the same exponential law as radioactive decay. The fraction which the delayed neutrons constitutes of the total number of prompt and delayed neutrons per fission is called the "delayed neutron fraction β (beta)." The delayed neutrons can be divided into six or more groups each having a characteristic half life. The fraction which a group constitutes of the total number of neutrons is called β_i . β is equal to the sum of all the β_i 's. The characteristics of the delayed neutrons for thermal fission of U-235 are given in table 3. The delayed neutrons are emitted at lower energies than the average prompt neutron and are somewhat more effective in producing fissions. For this reason a value of β equal to 0.75 percent is used in MTR-ETR control calculations instead of the absolute value as shown in table 3. In spite of their small numbers the delayed neutrons are very important in the

Table 3
Delayed Neutron Properties
for Thermal Fission
of Uranium-235

Group	Half Life, $T_{\frac{1}{2}}$	Delayed Neutron Fraction, β_1
1	55.7 sec	0.021 %
2	22.7 sec	0.140 %
3	6.2 sec	0.125 %
4	2.3 sec	0.253 %
5	0.61 sec	0.074 %
6	0.23 sec	0.027 %
		$\beta = \sum \beta_1 = 0.640 \%$

time dependent behavior and control of a reactor as discussed in Chapter VII. Plutonium-239 has a β of 0.210 percent and U-233 of 0.259 percent making control of reactors using these fuels more difficult in some respects than U-235 fueled reactors.

The average number of neutrons produced when a fission occurs is an important characteristic of a fissionable isotope and is designated as ν (nu). When a fissionable isotope absorbs a thermal neutron, one of two nuclear reactions takes place. With U-235 about 86 percent of the absorptions produce fission. In the other 14 percent of the captures a non-fission reaction, radiative capture, occurs and a U-236 atom which is not fissionable is formed. Thus the number of neutrons produced per capture in fuel is always less than the number of neutrons produced per fission. The average number of neutrons produced per capture in fuel is called eta (η). For thermal absorption in U-235, ν is 2.43 and η is 2.07. Of course, in any individual fission a fraction of a neutron cannot be emitted, but because there are a number of modes of fission the average need not be a whole number.

3. Energy of Fission

The amount of energy released by fission is quite large. The fissioning of one pound of material produces about the same amount of heat as the burning of 1400 tons of coal. Because of non-fission captures, a distinction

must be made between the amount of energy per unit mass of material fissioned and per unit mass of material consumed. In the MTR or ETR about 1-1/4 grams of U-235 is consumed/megawatt day (Mwd) and the amount fissioned/Mwd is about 85 percent of this value.

The energy of fission can be divided in a number of ways such as: (1) the type of radiation carrying off the energy, (2) the time of release of the energy, ie, at the time of fission or later, (3) the location at which the energy is transformed to heat, ie, near the place where the fission occurs or at some distance away. The distribution of fission energy by type of radiation is given in table 4.

Table 4
Distribution of Fission Energy
by Type of Radiation

Type of Radiation	Mev
Kinetic energy of fission fragments	168
Instantaneous gamma ray energy	5
Kinetic energy of fission neutrons	5
Beta particles from fission products	7
Gamma rays from fission products	6
Neutrinos	<u>10</u>
Total fission energy	201

The value of 201 Mev per fission is the primary fission energy of which about 191 Mev is available since the energy of the neutrinos cannot be recovered. In addition, radiative capture of neutrons by aluminum, uranium and water produces roughly 5 Mev of energy for each fission. Thus, the total available energy is about 195 to 200 Mev per fission.

Of the available fission energy about 92 percent is released instantaneously. About 87 percent of the total fission energy appears as heat at or near the point of fission. For the delayed energy, such as occurs from a fuel element after shutdown, about half of the heat is produced near the fuel.

4. Fission Products

More than 80 primary fission products, with mass number ranging from 72 to 160, are produced by the fission process. On the average each primary product goes through about three stages of radioactive decay before being transformed into a stable nucleus. After a short time there are more than 200 radioactive isotopes of over 30 elements present in the fission products. The distribution of the fission products by mass number is expressed by the "fission yields," which is defined as the fraction of the nuclear fissions that form products of a given mass number. Since two nuclei result from each fission the total yield for all mass numbers adds up to 200 percent. Most of the fission fragments undergo negative beta decay and the atomic numbers change with time. For this reason it is more convenient to express the fission yields in terms of mass number rather than atomic number.

The fission fragments are highly radioactive, most of them emitting negative beta particles and gamma rays. The half lives of the fission products range from a small fraction of a second up to millions of years. The complex mixture of isotopes of various half lives makes it impractical to express the rate of radioactive decay in terms of the individual fission yields and half lives. However, a simple empirical expression, that of Way and Wigner, is quite satisfactory for determining the rate of emission of beta particles and gamma rays. Expressions for these rates as a function of time after fission are as follows:

Rate of emission of gamma radiation equals

(1-38)

$$1.9 \times 10^{-6} t^{-1.2} \text{ photons/ (sec) (fission)}$$

* * * *

References

1. Stephenson, R , "Introduction to Nuclear Engineering," Chapters 1 and 2, McGraw-Hill, 1954.
2. Glasstone, S., "Principles of Nuclear Reactor Engineering," Chapters I and II, Van Nostrand, 1955.
3. Glasstone, S., and Edlund, M C , "The Elements of Nuclear Reactor Theory," Chapters I, II, III, and IV, Van Nostrand, 1952.
4. Littler, D J , and Raffle, J F , "An Introduction to Reactor Physics," Chapters 1, 2, 3, 4, and 5, McGraw-Hill, 1957.

Rate of emission of beta radiation equals

(1-39)

$$3.8 \times 10^{-6} t^{-1.2} \text{ particles/ (sec) (fission)}$$

where

t is the time after fission in "days" and the rates of emission are for one fission. Substituting average energies of 0.4 Mev for the beta particles and 0.7 Mev for the gamma photons it is found that

Rate of emission of beta and gamma energy equals

(1-40)

$$2.7 \times 10^{-6} t^{-1.2} \text{ Mev/ (sec) (fission)}$$

If a reactor has been operating for T days at a constant power of P watts and t is the time after startup, it is found by integration that

Rate of emission of beta and gamma energy at time t equals

(1-41)

$$5.9 \times 10^{-3} P \left[(t-T)^{-0.2} - t^{-0.2} \right] \text{ watts.}$$

The fission fragments are highly ionized carrying around 20 positive charges on the average. This high charge together with the large mass makes the specific ionization high and the range of the fission fragments is relatively short. In aluminum the range is about 1.4×10^{-3} cm. As a result of this short range, rather thin cladding is capable of preventing escape of the fission products from the fuel.

The fission products interfere with the chain reaction by introducing nuclei with high absorption cross section into the system. This effect is discussed in Chapter VII

CHAPTER II

REACTOR PHYSICS

A. INTRODUCTION

This chapter discusses some aspects of the reactor physics of the MTR and ETR. The treatment is approximate and limited to factors important for small, highly enriched, water-moderated reactors. Therefore, the results are subject to a number of limitations and qualifications which are usually not mentioned in order to keep the text short and easily readable. A number of references for general supplementary reading are listed on page 25. It is not expected that any one student will read all of them.

B THE CHAIN REACTION

1. The Neutron Cycle

The fission process is initiated by neutrons and also produces neutrons, and under the proper circumstances can produce a chain reaction. The operation of a reactor depends upon attaining a controlled self-sustaining chain reaction. In the reactors considered the following processes are important in achieving a chain reaction: (1) loss of neutrons from the core, that is "leakage," (2) capture of neutrons by any material other than U-235, designated as "parasitic capture," and (3) capture of neutrons by fuel, that is by U-235.

Figure 2 shows a typical neutron cycle for a highly enriched, well moderated reactor in a steady state. The values, while not exact, are representative of those for the MTR or ETR. The cycle starts with 100 neutrons being absorbed in fuel, U-235. Fourteen of the neutrons absorbed in fuel suffer radiative capture and result in the production of U-236, thus they are lost to the chain reaction. The 86 fissions yield 215 neutrons, if it is assumed each fission produces 2.5 neutrons. These neutrons from fission are emitted at high energies and high velocities. Collisions of the neutrons with the hydrogen atoms in the water, and to a lesser extent with other atoms, cause the neutrons to lose energy. While slowing down, 86 neutrons leak out of the core. When the neutrons have

lost sufficient energy, such that they are essentially in equilibrium with the surrounding atoms, they are called thermal neutrons. After the loss by fast leakage, 129 neutrons are left to become thermal neutrons. Of these, 5 leak from the core and 24 are absorbed parasitically in moderator and structural materials. Thus, 100 neutrons are left to be absorbed in fuel and start the next cycle, and the reactor is in a steady state.

2. The Two Factor Formula

The division of the neutron cycle into several factors in a somewhat arbitrary but natural way facilitates its use in determining the conditions for a chain reaction. It is necessary to introduce several concepts in order to discuss this division of the neutron cycle. The "effective multiplication factor," " k_{eff} ," is defined as the ratio of the average number of neutrons produced in a cycle to the total number absorbed or lost by leakage. If k_{eff} is equal to one, the number of neutrons remains constant from generation to generation, or from cycle to cycle, and the reactor is said to be "critical." If k_{eff} is less than one, the number of neutrons decreases from generation to generation and the reactor is "sub-critical." If k_{eff} is greater than one, the number of neutrons increases and the reactor is "super-critical."

The "infinite multiplication factor," " k_{∞} ," is defined as the ratio of the average number of neutrons produced in each generation to the number absorbed. It is the value that the effective multiplication factor would have if the reactor were infinite in size. The infinite multiplication factor is determined only by the composition and arrangement of materials in the reactor and is not dependent on the size.

The fraction of all the neutrons that do not escape by leakage is defined as the "nonleakage probability," " P ." The effective and infinite multiplication factors are then related by the equation

$$k_{\text{eff}} = kP \quad (2-1)$$

FIGURE 2
Neutron economy in MTR-ETR
type reactors

The nonleakage probability is broken down in two factors: the probability that a neutron escapes leakage while it is fast or being slowed down, P_f ; and the probability that it escapes leakage after becoming a thermal or slow neutron, P_s , so that:

$$P = P_f P_s \quad (2-2)$$

The "thermal utilization," "f," is the fraction of the thermal neutrons absorbed in fuel:

$$f = \frac{\text{thermal neutrons absorbed in fuel}}{\text{total thermal neutrons absorbed}} \quad (2-3)$$

For a system which can be treated as homogeneous:

$$f = \frac{\Sigma_u}{\Sigma_e + \Sigma_u} \quad (2-4)$$

where

Σ_u is the macroscopic thermal absorption for the U-235 fuel; Σ_e is the corresponding cross section for all other materials.

The number of neutrons produced per neutron absorbed in fuel is designated eta, η , as discussed in Chapter I

The ratio of the fission cross section, Σ_f , to the total cross section for absorption in fuel of thermal neutrons, Σ_u , is called "a":

$$a = \frac{\Sigma_f}{\Sigma_u} \quad (2-5)$$

Consider a system in which there is no leakage. Then if N thermal neutrons are absorbed in the reactor, fN neutrons are absorbed in fuel and ηfN fast neutrons are produced. In the highly enriched systems being considered the absorption of fast neutrons can be considered negligible, to a good approximation. Thus, in the next generation, ηfN neutrons are available for absorption in fuel and:

$$k = \frac{\eta f N}{N} = \eta f \quad (2-6)$$

Equation (2-6) is called the "two-factor" formula. It is reasonably valid only for well-moderated, highly enriched reactors. In reac-

tors using fuels of low enrichment, it is necessary to consider fissions produced by absorption of fast neutrons and loss of fast neutrons by absorption. In this case it is customary to use a four factor formula.

The neutron cycle for a finite reactor with leakage of neutrons using the concepts of the two-factor formula is shown in figure 3. It should be studied carefully and compared with figure 2 which uses a different starting point

C. NEUTRON SCATTERING

1. Moderation

The slowing down of neutrons to thermal energies is often referred to as "moderation" and the material which causes the slowing down as the "moderator." The neutrons lose energy by two types of collision.

An "elastic collision" is essentially the same as that between two billiard balls, and kinetic energy and momentum are conserved. In an "inelastic collision," part of the kinetic energy is lost to the production of gamma rays and only momentum is conserved. Inelastic scattering does not occur at all with hydrogen and in general occurs only for neutrons of rather high energy. For these reasons it can be neglected for the present discussion of the reactor physics of the MTR-ETR

The amount of energy lost by a neutron in an elastic scattering event can vary from zero up to a definite maximum depending on how near the collision is to being head-on. For a large number of collisions there is an average fractional energy loss that depends upon the ratio of the masses of the neutron and the nucleus with which it collides. The maximum average amount of energy is lost when the masses of the neutron and the target nucleus are equal, that is, for a collision with hydrogen. Thus, most of the neutron slowing down in MTR-ETR type reactors is done by collisions with the hydrogen atoms in the water which can be considered as the moderator.

The loss of energy from collisions of neutrons with a given type of nuclei is usually expressed in terms of the "average logarithmic energy decrement per collision" or the average value of the decrease in the natural logarithm of the neutron energy per collision. This

FIGURE 3
Neutron cycle for MTR-ETR
type reactors

quantity is given the symbol ξ . The value of ξ for hydrogen is one and for elements of higher mass it is ~

$$\xi \cong \frac{2}{A + 2/3} \quad (2-7)$$

where

A is the mass number. The average number of collisions required to slow neutrons from the average energy of fission neutrons of 2 Mev to the thermal energy of 0.025 ev is shown for a number of materials in table 5

Table 5
Neutron Thermalization

Element	ξ	Collisions to Thermalize
Hydrogen	1.000	18
Beryllium	0.206	86
Carbon	0.158	114
Aluminum	0.0723	252
Uranium	0.0084	2170

The ability of a moderator to thermalize neutrons depends not only on the average loss of energy, but also on the probability of a scattering event which is proportional to the macroscopic scattering cross section Σ_s . The "slowing down power" is used as a measure of the ability of a moderator to thermalize neutrons. It is defined as $\xi \Sigma_s$, the product of the average logarithmic energy decrement per collision and the macroscopic scattering cross section. It is important that a moderator have a low absorption cross section to minimize the loss of neutrons by parasitic absorption. The effectiveness of a moderator is often expressed in terms of the "moderating ratio," $\xi \Sigma_s / \Sigma_a$.

2. Thermal Neutrons

When the neutrons have been slowed to energies in the range of a few hundredths of an ev, they can gain energy as well as lose it in a collision. Thus, the slowing down process is stopped by the neutrons reaching thermal equilibrium with the surrounding atoms and acquiring a distribution of energies known as the Maxwell-Boltzmann distribution. These neutrons are then called "thermal neutrons"

The microscopic thermal absorption cross section of most materials is inversely proportional to the velocity or the square root of the

energy of the neutrons. It is, therefore, necessary to use the proper combination of flux and cross section to get the correct reaction rate as discussed in Chapter I. Many measurements of cross sections are normalized to give the value for neutrons with a velocity of 2200 meters/sec, or an energy of about 0.025 ev, which is the most probable energy for neutrons having a Maxwell-Boltzmann distribution corresponding to a temperature of about 20°C.

If the "track length flux" is used, an average cross section is required that is related to the cross section at 2200 meters by the relations:

$$\bar{\sigma}_{th} = 0.886 \sigma_{2200} \quad (2-8)$$

$$\bar{\Sigma}_{th} = 0.886 \Sigma_{2200} \quad (2-9)$$

The number of absorptions A/cm³/sec is then given by

$$A = \phi_{TL} \bar{\Sigma}_{th} = 0.886 \phi_{TL} \Sigma_{2200} \quad (2-10)$$

In flux measurements in the MTR and ETR the cross section at the most probable velocity is used and the flux is the "activation flux" giving the relation:

$$A = \phi_A \Sigma_{2200} \quad (2-11)$$

The value of A must be the same in equations (2-10) and (2-11) and

$$\phi_A = 0.886 \phi_{TL} \quad (2-12)$$

The "track length flux" is used in nearly all reactor physics texts, while the use of the "activation flux" is quite common in experimental determination of flux values.

The thermal absorption cross section is inversely proportional to the square root of the absolute temperature, that is

$$\sigma_{T_1} = \sigma_{T_0} \sqrt{\frac{T_0}{T_1}} \quad (2-13)$$

where

σ_{T_0} and σ_{T_1} are the absorption cross sections

at temperatures of T_0 and T_1 , in degrees Kelvin or Rankine, respectively. The equation (2-13) assumes that the cross section has a $1/v$ dependence and corrections are often required when it is used. It should be noted that the energy of the neutrons is largely determined by the moderator and the temperature to be used in determining the cross section is that of the moderator. For example in a fuel element the U-235 may be at a considerably higher temperature than the surrounding water but the cross section is determined by the water temperature

3. Reflection of Neutrons

The neutron economy of a reactor can be improved by surrounding the core with a "reflector" which returns neutrons to the core by scattering collisions. The same properties are important for a good reflector as for a moderator. The absorption cross section of a reflector is usually quite low and insertion of absorbing material, as in experiments, can markedly decrease the ability to reflect neutrons back to the core. The value of the reflector can be expressed in terms of the "reflector savings," which is defined as the increase in a dimension of the core required to keep the reactor just critical when the reflector is removed from the sides of the core perpendicular to the selected dimension. The reflector savings for water is about 5-1/2 in. and for beryllium about 8 in. Thus, to keep the MTR just critical if the reflector were removed, the height would have to be increased by about 5-1/2 in. and the length and width each increased by about 8 in.

The reflector also improves the thermal flux distribution in the core. In a bare or unreflected core, the thermal flux is very low at the edges as compared to the center. A reflector greatly reduces the ratio of the maximum to the average thermal flux. Since there is a limit to how much heat can be removed from any small volume of the core, this decrease in maximum-to-average flux ratio permits the reactor to operate at a higher power level.

The fast neutrons leaking from the core are slowed to thermal in a few inches in the reflector. Since the absorption in the reflector is low, the neutrons have a relatively long lifetime. This combination of the large source of thermal neutrons from the slowing down of fast neutrons and the long lifetime of the thermal neutrons often makes the maximum thermal

flux in the reflector several times as high as the maximum flux in the core. If a region consisting of a good moderator is surrounded by the core, this peaking effect is increased and the arrangement is called a flux trap.

D DIFFUSION THEORY

1. The Neutron Balance Equation

Previously, the conditions for obtaining a steady state chain reaction were developed by considering the whole reactor. The alternative approach developed in this section starts by considering any small volume of the reactor. In such a region the neutron density can be changed by absorption, by leakage into or out of the volume and by production of neutrons. The relation between these processes is the "neutron balance equation":

(2-14)

$$\text{production} - \text{absorption} - \text{leakage} = \frac{\partial n}{\partial t}$$

where

n is the neutron density and $\partial n / \partial t$ is its rate of change with time. When the system is in a steady state, the neutron density is not changing with time and the balance equation becomes

$$\text{production} = \text{absorption} + \text{leakage} \quad (2-15)$$

The production of neutrons is largely determined by the fission rate and presents no special difficulties in use of the balance equation. The absorption term is equal to $\Sigma_a \phi$ and its use is also straightforward. However, the calculation of the leakage is not as simple.

When a large number of neutrons are considered, the scattering collisions produce a net movement or "diffusion" of neutrons from regions of higher to those of lower neutron density. It can be shown that for a system in which the flux varies only in one direction, x , that the rate of neutron leakage at a point is

$$\text{neutron leakage/cm}^3 - \text{sec} \quad (2-16)$$

$$= \frac{-D \partial^2 \phi}{\partial x^2} = \frac{-D d^2 \phi}{d x^2}$$

where

D is the "diffusion coefficient." For a three dimensional flux variation the leakage is

leakage

$$= -D \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} \right) = -D \nabla^2 \phi \quad (2-17)$$

The diffusion coefficient is given by:

$$D = \frac{1}{3\Sigma_s(1-\bar{\mu}_0)} \quad (2-18)$$

where

$\bar{\mu}_0$ is the average cosine of the scattering angle and is related to the mass number A of the scattering nucleus by:

$$\bar{\mu}_0 = \frac{2}{3A} \quad (2-19)$$

2. The Diffusion Equation

Substituting the expressions for absorption and leakage into the balance equation (2-14) and letting S represent the source term, the "diffusion equation" is obtained by:

$$D \nabla^2 \phi - \Sigma_a \phi + S = \frac{\partial n}{\partial t} \quad (2-20)$$

For a steady state, the diffusion equation becomes:

$$D \nabla^2 \phi - \Sigma_a \phi + S = 0 \quad (2-21)$$

A hypothetical but interesting case is that of a uniform infinite reactor in the steady state. Then the leakage is zero and :

$$\Sigma_a \phi = S \quad (2-22)$$

The flux is then given by:

$$\phi = \frac{S}{\Sigma_a} \quad (2-23)$$

In this case for a given source of neutrons such as from thermalization of fast neutrons, the flux is inversely proportional to the absorption cross section. While equation (2-23) is derived for an infinite medium, it is often a good approximation for a finite region.

In an infinite medium the neutron lifetime (ℓ) is equal to the average distance traveled before absorption (λ_a) divided by the mean velocity of the neutrons (v); thus:

$$\ell = \frac{\lambda_a}{v} = \frac{1}{v \Sigma_a} \quad (2-24)$$

since

$$\lambda_a = \frac{1}{\Sigma_a} \quad (2-25)$$

as discussed in Chapter I. Equation (2-23) may be interpreted as stating that for a given source and velocity, the flux is proportional to the average time that the neutrons exist before being lost by absorption.

In a medium containing no fissionable material, the source term is zero except where a neutron source is placed and the steady-state diffusion equation becomes:

$$D \nabla^2 \phi - \Sigma_a \phi = 0 \quad (2-26)$$

or

$$\nabla^2 \phi - \kappa^2 \phi = 0 \quad (2-27)$$

where

$$\kappa^2 = \frac{\Sigma_a}{D} \quad (2-28)$$

For a case in which the flux varies only in the x direction, equation (2-13) becomes:

$$\frac{d^2 \phi}{dx^2} - \kappa^2 \phi = 0$$

The solution of which for this case of no fissionable material is:

$$\phi = A e^{-\kappa x} + C e^{\kappa x} \quad (2-29)$$

The problem becomes one of determining the arbitrary constants A and C from the boundary conditions. Consider a slab with a plane source emitting one n/cm²-sec at x = 0. The slab is infinite in the positive x direction and in the y and z directions. The boundary conditions for this case are:

(a) The flux is finite everywhere except x = 0.

(b) The current entering at x = 0 is S/2.

The first condition requires that C = 0 and using the second condition the solution is found to be:

$$\phi = \frac{S}{2} \frac{e^{-\kappa x}}{\kappa D} \quad (2-30)$$

The diffusion equation assumes that the scattering cross section Σ_s is much larger than the absorption cross section Σ_a . In addition the diffusion equation is not valid near boundaries. For these reasons the diffusion equation, while very useful, must be used only for situations to which it is applicable.

For a multiplying region, one in which there is fissionable material, a simple case is that in which all neutrons are considered as traveling at the same velocity, and thus there is no loss of neutrons by fast leakage. The source term or number of neutrons produced in a cm^3/sec is then:

$$S = v \Sigma_f \phi = f \eta \Sigma_a \phi = k \Sigma_a \phi \quad (2-31)$$

Then the diffusion equation becomes:

$$D \nabla^2 \phi - \Sigma_a \phi + k \Sigma_a \phi = 0 \quad (2-32)$$

The "diffusion length" L is a measure of the average distance thermal neutrons diffuse before being absorbed and is defined by the relation:

$$L^2 = \frac{D}{\Sigma_a} \quad (2-33)$$

Equation (2-32) can then be put in the form:

$$\nabla^2 \phi + B^2 \phi = 0 \quad (2-34)$$

where

$$B^2 = \frac{k-1}{L^2}$$

The "buckling B^2 ," is a measure of the curvature of the flux and for a given composition B^2 determines the size of the reactor that will give the amount of neutron leakage to make the reactor just critical.

The one velocity equation (2-32) underestimates the critical mass or size because of the neglect of fast leakage. The "Fermi age," " τ " is related to the distance neutrons produced by fission travel before being thermalized. An approximate expression for the fast non leakage probability P_f is:

$$P_f = e^{-B^2 \tau} \quad (2-35)$$

With the use of equation (2-35) for the fast leakage, the "critical condition" can be shown to be:

$$\frac{k e^{-B^2 \tau}}{1 + L^2 B^2} = 1 \quad (2-36)$$

It can also be shown that

$$P_s = \frac{1}{1 + L^2 B^2} \quad (2-37)$$

so that

$$P_f P_s k = 1 \quad (2-38)$$

or

$$P k = 1 \quad (2-39)$$

and

$$k_{\text{eff}} = 1 \quad (2-40)$$

Equation (2-40) gives the same critical condition as developed earlier but provides explicit relations, equations (2-35) and (2-36), for the nonleakage probabilities

E APPROXIMATE CALCULATIONS

1. By Use of the Two Factor Formula

For uniform changes of composition throughout the core simple methods of calculations can be used when the change in leakage P can be neglected. Then since:

$$k_{\text{eff}} = \eta f P = k P \quad (2-41)$$

and η can be considered as a constant, the change in k or k_{eff} is produced only by a change in the thermal utilization f . The change in infinite multiplication factor k is:

$$\Delta k = k_2 - k_1 = \eta P (f_2 - f_1) \quad (2-42)$$

and the fractional change is:

$$\frac{k_2 - k_1}{k_1} = \frac{\Delta k}{k} = \frac{f_2 - f_1}{f_1} \quad (2-43)$$

Consider the uniform addition of a thermal absorber with macroscopic cross section Σ_p to a core with macroscopic cross section Σ_c before addition of the poison, then.

$$f_1 = \frac{\Sigma_u}{\Sigma_u + \Sigma_e} = \frac{\Sigma_u}{\Sigma_c} \quad (2-44)$$

and

$$f_2 = \frac{\Sigma_u}{\Sigma_u + \Sigma_e + \Sigma_p} = \frac{\Sigma_u}{\Sigma_c + \Sigma_p} \quad (2-45)$$

where

Σ_u is cross section of the fuel, Σ_e is the cross section of everything else in the core. Substituting (2-44) and (2-45) into (2-43) the result is:

$$\frac{\Delta k}{k} = \frac{-\Sigma_p}{\Sigma_p + \Sigma_c} \quad (2-46)$$

similarly if the mass of fuel M in the core is changed by amount ΔM it can be shown that:

$$\frac{\Delta k}{k} = (1 - f) \frac{\Delta M}{M} \quad (2-47)$$

In using equations (2-46) and (2-47), the assumption of constant leakage must be considered. For changes which produce a large change in rod position, the equations are not directly applicable. For large changes in k , the fractional change depends on whether k_1 or k_2 is used in the denominator, the above equations being derived for k_1 . The use of this method assumes that the core can be considered as homogenized.

The effects of changes in the neutron cycle are often expressed in terms of the "reactivity" ρ , rho, defined as:

$$\rho = \frac{k_{eff} - 1}{k_{eff}} \quad (2-48)$$

2. Weighting Factors

The methods described above apply only to uniform changes throughout the core. The effect of a change in one part of a core can often be determined by establishing the weighting factor for that part of the core. A neutron produced in the center of the core has more chance of producing a fission than one near the edge and is said to be more important. The number of neutrons lost by an absorber or produced by a fissionable material is proportional to the flux. The effect of introducing absorber or fuel into the core depends on the importance and the flux. The experimental determination of the weighting factors

is made by introducing the same sample into a number of places in the core and from the changes in critical rod position determining the changes in reactivity. The central location of the core can arbitrarily be given a weighting factor of unity. Then the weighting factor for any other position is given by:

$$W_i = \frac{\Delta \rho_i}{\Delta \rho_c} \quad (2-49)$$

where

$\Delta \rho_i$ is the change in reactivity produced by insertion of the sample into the location i . $\Delta \rho_c$ is the change for insertion of the sample into the central location.

The effect of a local change in the core can be estimated by determining the effect of a uniform change throughout the core and then using the weighting factor of the location where the change is made.

F A REACTOR PHYSICS DESCRIPTION OF MTR-ETR TYPE REACTORS

The actual structure of a test reactor is so complicated that it is not feasible to describe it in the mathematical manner necessary for reactor calculations. In order to obtain a simplified representation or model, it is necessary to decide which features are important and which may be neglected. Consider calculations to determine the fast and thermal flux distributions and the effective multiplication factor. Then to a good approximation there may be no difference between curved and flat fuel plates, although as far as withstanding the forces produced by flow of coolant this difference may be very important. Among the features significant to the reactor physics of the MTR and ETR are the following: (1) they use highly enriched U-235 as fuel; (2) their cores are water moderated; (3) they have relatively small cores, that is, they have a large fraction of the neutrons escaping from the cores by leakage, and (4) the structural material of the cores is aluminum.

The fuel elements in the MTR and ETR have plates containing the fuel separated by channels filled with water. The water has a larger slowing down power and lower absorption cross section for thermal neutrons. As a result, the thermal flux is peaked in the water channels and depressed in the fuel plates.

However, this effect is sufficiently small because of the relatively thin fuel plates and water channels that for many purposes it can be neglected. In such cases the core can be considered to be homogenized. That is, the composition representing the core, or a region of it, has the same atom densities as if all of the materials in the core were thoroughly mixed.

The use of highly enriched fuel, together with the water moderator and aluminum structural material, make it a reasonably good approximation to assume that all absorptions and fissions are produced by thermal neutrons. This assumption results in the type of neutron cycle described earlier in this chapter. However, for the computation of threshold reactions such as the formation of N^{17} , this assumption cannot be made.

The cores of the MTR and ETR are small enough that a large fraction of the neutrons escape by leakage. Most of these leakage neutrons have relatively high energies and are thermalized after leaving the core. In the MTR most of the experiments are external to the core. In the ETR most of the major experiments are internal to the core. There is a basic difference in the behavior of the thermal flux in experiments which are essentially part of the core and those which are not. The thermal flux in the core increases at constant power with burnup of fuel. The number of fast neutrons produced remains unchanged with constant power operation, the usual manner of operation for the MTR and ETR. Thus experiments which receive their source of neutrons from fast leakage tend not to change during the cycle. Many experiments are intermediate between these two extremes. The characteristics of the flux in experiments in the interior of the core are strongly influenced by the amount of moderator around them.

To compensate for burnup of fuel and accumulation of fission products during the cycle a reactor must contain more fuel than required for criticality. However, except when the power is being changed, the reactor must be just critical while operating. Thus some means must be provided to vary the effective multiplication factor. In the MTR and ETR this variation is accomplished by the use of control rods. The changing position of the control rods produces changes in the flux

distribution which are often larger than the change from burnup of fuel.

The MTR has control rods that have sections in the core that are black to neutrons when the rods are fully inserted. That is, essentially all thermal neutrons arriving at the locations occupied by the rods are absorbed. When the rods are withdrawn, fuel is introduced at the bottom and poison removed at the top for the rods in the core. Additional rods in the reflector introduce beryllium. As the reflector rods are kept withdrawn in normal operation they are essentially a part of the reflector. When partially withdrawn the core rods suppress the flux in the top of the core and as they are withdrawn the flux increases in the top of the core and decreases in the bottom. The rods in the MTR are normally kept at the same height so that horizontal shifting of the flux pattern is minimized. The MTR and ETR are usually operated at constant power throughout the cycle and an increase in flux in one part of the core from rod movement must be accompanied by a decrease elsewhere.

Some of the rods in the ETR have gray poison sections; that is, the absorption is about the same as the surrounding fuel and they operate by changing the amount of fuel in the core. These rods produce less change in the vertical flux pattern, but the usual mode of operation in the ETR tends to produce horizontal flux shifts.

The control rods introduce local heterogeneities that make it difficult to assess their effect in terms of a core of uniform composition. However, the rods in the core can be considered to change the thermal utilization by varying the ratio of neutrons absorbed in fuel to those absorbed parasitically. Rods in the reflector change the non-leakage probability.

The effective multiplication factor is changed by anything which affects the neutron cycle. When the temperature is increased, important effects are: (1) the absorption cross section of the water decreases because of the decrease in density; (2) the age of the core increases because of the decreased density and this reduces the non-leakage probability; (3) all absorption cross sections are decreased because of the increased temperature; and (4)

the size of the core increases, thus increasing the non-leakage probability. However, the magnitude of this effect is such that it can usually be neglected. The change with temperature of the effective multiplication factor is then a balance between the decrease produced

by the larger leakage and the increase from the reduced absorption in water. For slow uniform changes the effective multiplication factor of the MTR and ETR decreases with increasing temperature and they have negative temperature coefficients of about $-1 \times 10^{-4} \Delta k/k/^{\circ}\text{F}$.

* * * *

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CHAPTER III

ELEMENTS OF REACTOR ENGINEERING

A. INTRODUCTION

Reactor Engineering, or Nuclear Reactor Engineering as it is frequently called, is a new and somewhat indistinct field of engineering involving the science and technology of atomic energy. The scope of reactor engineering includes the design, construction, testing and operation of reactors and reactor equipment and is largely concerned with the solution of problems involving fundamental scientific principles of conventional engineering but which must be given special interpretation to apply to the new physics and chemistry aspects of the nuclear processes. The engineering involved covers fluid flow, heat transfer, materials of construction, fuel materials, geometries and shielding, as well as the fundamentals of reactions underlying the release of nuclear energy. In a generalized approach, engineering, construction and operation costs may also be included. A single chapter on the subject of reactor engineering cannot hope to cover adequately so broad a field. Therefore, only those portions applicable to the testing reactors are described in this chapter. Additional details of reactor engineering may be found in texts such as Glasstone (ref 1), Murray (ref 2) and McLain (ref 3), which are listed as suggested reading at the end of this chapter. Also found at the end of this chapter is a list of subscripts and nomenclature used in this chapter.

Reactor engineering as covered in this chapter has been slanted towards operations and testing rather than design.

B. HYDRAULICS

1. Fundamentals

The basis of most hydraulic calculations dealing with liquid cooled reactors is the mechanical energy balance equation, which in integrated form is:

(3-1a)

$$\frac{-\Delta P}{\rho} = W + \frac{\Delta V^2}{2g_c} + \frac{g}{g_c} \Delta Z + \frac{V^2}{2g_c} \left[\frac{4fL}{d} + K_m \right]$$

and the continuity equation for a steady state system is:

$$\Delta (V \rho a) = 0 \quad (3-1b)$$

The equation (3-1a) simply states that the loss of pressure-volume energy is equal to the mechanical work done by the system, plus the increase in kinetic energy, plus the increase in potential energy, plus the loss of energy due to fluid friction in straight lengths of pipe and the loss of energy due to miscellaneous causes such as fittings, bends and sharp expansions, and contractions of the flow channel. The last term in equation (3-1a) is known as the frictional loss and represents the amount of energy which is irreversibly dissipated by the system. The remaining terms represent the reversible change of energy from one form to another. All of the terms of equation (3-1a), with the exception of the last term, may be calculated directly for a given system. The last term must be evaluated from experimental data or from empirical relationships based on experimental data. These empirical relationships for common systems are listed in most standard hydraulic texts and in many engineering handbooks (eg, ref 4, 5 and 6). It has been found that the Fanning Friction Factor, f^* , is dependent only on the Reynolds

number $\left(R_e = \frac{dV\rho}{\mu} \right)$ and upon the degree

of roughness of the flow channel. Plots of f^* vs Re and values of K_m for many fittings and for entrance and exit losses are given in the above references.

The terms in equation (3-1a) are represented graphically by vertical distances in figure 4. The vertical distances between the tube centerline and the pressure head line are proportional to the value of P/ρ . The vertical distances between the pressure head

*Some authors use an f which is 4 times the Fanning f , and thereby eliminate the 4 in the last term of equation (3-1a).

line and total head line indicate the velocity head, $V^2/2g_c$. The slope of the total head line is proportional to the friction loss per unit length of pipe, and discontinuities in the total head line are proportional to mechanical work or the minor losses.

For systems in which no mechanical work is done on or by the fluid and in which friction and change in elevation are negligible, equation (3-1a) reduces to Bernoulli's equation:

$$-\frac{\Delta P}{\rho} = \frac{\Delta V^2}{2g_c} \quad (3-2)$$

Bernoulli's equation is the basic equation which describes the action of most flow measuring devices such as venturies, orifices and Pitot tubes. Usually an empirical constant is required to correct Bernoulli's equation for practical application to allow for friction losses and flow disturbances.

2. Fluid Velocity

a. Average vs Distribution

In order to evaluate the heat transfer characteristics of an element, the fluid velocity must be known. Equation (3-1a) is useful for calculating the velocity when the pressure drop and physical properties of the system and fluid are known. However, if the flow path becomes quite involved and a large number of entrance and exit losses exist (as in an MTR element), then equation (3-1a) becomes difficult to evaluate and experimental measurements must be made to obtain accurate data. The minimum amount of data required from such experiments is the total flow rate through the test piece for several different over-all pressure drops. The velocity for each pressure drop is obtained from the equation:

$$V \left(\frac{\text{ft}}{\text{sec}} \right) = \frac{G \text{ (gal/min)}}{a \text{ (ft}^2\text{)} \times 7.48 \text{ (gal/ft}^3\text{)} \times 60 \text{ (sec/min)}} \quad (3-3)$$

The velocity, as determined from equation (3-3) is the average velocity for the entire test piece. Since most fuel elements have a number of flow channels in parallel and since differences in mechanical tolerances and the physical location of the channels affect the flow rate, it is necessary to determine the amount that the velocity in any given channel may vary from the average velocity. This may be done

directly by inserting Pitot tubes into each of the channels, or by placing at least two pressure taps in each of the channels and calculating the velocity from equation (3-1a). The effect of mechanical tolerance alone may be estimated directly from equation (3-1a).

b. Method of Measurement

(1) Hydraulic facility

A hydraulic test facility has been established and is in operation at the MTR-ETR for the purposes of obtaining the necessary experimental data on fuel assemblies, shim rods and experimental components. Figure 5 is a simplified schematic drawing of this facility.

The pump is required to circulate water through the experimental loop. The pressure differential measured across the orifice is combined with Bernoulli's equation and an empirical calibration constant to determine the rate of flow. The pressure drop through various portions of the element being tested is then measured by means of the pressure taps in the test fixture and a bank of manometers. The heat exchanger is used to control the temperature of the circulating water, and the bypass line is needed to control the flow through the test fixture without undue pressure build-up at the pump.

The flow through the test section may be accurately controlled and measured from 15 gpm to over 800 gpm. The low flows are limited by the accuracy of measurement and control, while the high flows are limited by the pump capacity and frictional pressure losses within the system.

Example: If the rate of flow through the test fixture containing an MTR type element is measured at 630 gpm when the pressure across the element is 40 psi, what is the average velocity of fluid in the fuel plate section?

Solution: From the drawings of the test fixture and the element, the total cross

FIGURE 4
Schematic diagram of flow system
showing terms in mechanical
energy balance equation

FIGURE 5
MTR-ETR hydraulic test facility

section area of flow in the test fixture fuel section is found to be 6.10 sq in. or 0.0424 sq ft.

$$V = \frac{630}{0.0424 \times 7.48 \times 60}$$

$$V = 33.2 \text{ ft/sec.}$$

This is the average velocity of flow through and around the fuel element in the test fixture.

Example: During the same test as described above, two static pressure taps were located in one of the flow channels. These taps were located 12 in. apart (6 in. above and below the centerline). The pressure difference between these two taps, as obtained from a manometer and converted to psi, was 3.95. How much does the velocity in this channel vary from the average? The temperature of the water was 110°F. The equivalent diameter of each channel is 0.222 in.

Solution: at $T = 110^\circ \text{ F}$,

$$\rho = 61.9 \text{ lb/ft}^3$$

$$\mu = 1.487 \text{ lb/hr-ft}$$

Equation (3-1a) may be used to calculate the velocity.

$$\frac{-\Delta P}{\rho} = W + \frac{\Delta V^2}{2g_c} + \frac{g}{g_c} \Delta Z + \frac{V^2}{2g_c} \left[\frac{4fL}{d} + K_m \right]$$

No work is done on or by the system between the two pressure taps, therefore $W = 0$.

Since the channel is uniform and the fluid is incompressible, the velocity is constant between the two pressure taps, therefore $\Delta V^2 = 0$.

Because the pressure loss was measured with a manometer (water over mercury) the potential energy term is not included in the ΔP measurement. Since the pressure taps are located in the center of the channel, there are no minor energy losses. Equation (3-1a) becomes:

$$\frac{-\Delta P}{\rho} = \frac{V^2}{2g_c} \frac{4fL}{d}$$

or

$$\frac{3.95 \times 144}{61.9} = \frac{V^2}{2 \times 32.2} \frac{4 \times 12}{0.222}$$

where

f is a function of the Reynolds number,

$$\frac{dV\rho}{\mu} = \frac{0.222 V \times 3600 \times 61.9}{12 \times 1.487}$$

as given in standard references. A trial and error solution shows the above equation, combined with a Reynolds number-friction factor plot, is solved by a velocity of 23 ft/sec.

Therefore, the velocity in this channel is 30 percent lower than the average velocity, or:

$$\frac{33 - 23}{33} \times 100 = 30\%$$

3. Pressure Losses

When evaluating a hydraulic system such as the primary coolant system of a reactor, it is necessary to analyze all of the components within the system, and thereby determine the over-all response of the system.

a. MTR

A simplified drawing of the primary coolant loop of the MTR is shown in figure 6. This loop is an excellent example of the manner in which differences in elevation may be used to obtain desired static pressures and flow rates. The working reservoir, the seal tank and the sump tank are all at atmospheric pressure. However, because of the differences in elevation, the working reservoir is able to supply water to reactor core at the rate of 24,200 gpm and maintain an absolute pressure of 62.5 psia at the top of the reflector. This 150,000 gal working reservoir not only pressurizes the

FIGURE 6
MTR primary coolant loop
simplified schematic

system, but also serves as an emergency source of water during short duration pumping failures. Because of the differences in elevation of the water levels in the seal tank and the evaporators, the pressure in the evaporators is about 1 psia. It is necessary to use ejectors to remove the uncondensed gases from the evaporators to maintain this pressure. The primary coolant of the MTR is also effectively degassed in the flash evaporators.

It is interesting to note that the primary pumps must impart a ΔP of approximately 97 psi to the coolant in order for a reactor tank ΔP of 42.5 psi to exist. Of the tank pressure loss, 40 psi is lost across the core.

b. ETR

The primary cooling system of the ETR, as shown in figure 7, is a closed loop in which elevation differences play a minor role. The primary pumps develop a pressure difference of about 78 psi to propel the coolant through the loop at the rate of 62,000 gpm.* Of this 78

psi, 2.6 psi are lost in flowing through the heat exchangers and 42.8 psi are dissipated in flowing through the reactor core. The remaining 32 psi are lost in the piping, fittings, controls and in entrance and exit effects. A surge tank filled to about 2/3 of its volume with air serves as a cushion on the system and as an emergency source of water in case of pump failure. Pressurizing pumps are required to establish the desired absolute pressure within the ETR primary coolant loop.

c. Fuel Element Pressure Losses

The pressure losses through the active lattice of the core are extremely important since these are the pressure drops which give the coolant the velocity necessary to remove the heat from the fuel elements. The velocities in the fuel elements also affect the saturation temperature of the coolant and generate differential pressures between adjacent channels which can limit the reactor operations. Within the fuel section of the element, the flow and pressure drop may be analyzed quite accurately by means of equation (3-1a).

*The coolant flow rate for the ETR may be changed in the near future.

Example: What is the frictional pressure loss in the MTR fuel assembly?

$$\text{Reynolds number} = \frac{DV\rho}{\mu}$$

Data: $V = 33 \text{ ft/sec}$

$d = 0.222 \text{ in.}$

$L = 24.625 \text{ in.}$

Total flow area immediately upstream of fuel plates = 7.7 in.^2

Total flow area between fuel plates = 5.47 in.^2

Total flow area immediately downstream of fuel plates = 7.7 in.^2

$T = 120^\circ\text{F}$

Solution: The frictional pressure drop is given by the last term of equation (3-1a).

$$\frac{-\Delta P}{\rho} = \frac{V^2}{2g_c} \frac{4fL}{d} + K_m \frac{V^2}{2g_c}$$

at $T = 120^\circ\text{F}$

$$\rho = 61.7 \text{ lb/ft}^3$$

$$\mu = 1.353 \text{ lb/ft-hr}$$

$$= \frac{0.222 \times 33 \times 3600 \times 61.7}{12 \times 1.353}$$

$$= 1.005 \times 10^5$$

The friction factor, f , associated with this Reynolds number as obtained from standard hydraulic reference books is 0.0047.

The minor friction losses are due to the entrance and exit effects, therefore:

$$K_m = K_{\text{entrance}} + K_{\text{exit}}$$

Standard hydrodynamics texts give K_{entrance} and K_{exit} as a function of the area ratios. The area ratio is:

$$5.47/7.70 = 0.711$$

Reference 5, p. 388, gives

$$K_{\text{entrance}} = 0.4 \left(1.25 - \frac{a_2}{a_1} \right)$$

for $a_2/a_1 < 0.715$,

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FIGURE 7
ETR primary coolant loop
simplified schematic

$$K_{\text{entrance}} = 0.75 \left(1 - \frac{a_2}{a_1}\right) \quad \text{for } a_2/a_1 > 0.715 \text{ and}$$

$$K_{\text{exit}} = \left(1 - \frac{a_2}{a_3}\right)^2$$

Therefore the minor loss coefficients are:

$$K_{\text{entrance}} = 0.4 (1.25 - 0.711) = 0.216$$

$$K_{\text{exit}} = (1 - 0.711)^2 = 0.084$$

$$K_m = 0.300$$

Then

$$-\Delta P = \frac{\rho V^2}{2g_c} \left[\frac{4fL}{d} + K_m \right]$$

$$= \frac{(61.7 \text{ lb/ft}^3)(33 \text{ ft/sec})^2}{2 \times 32.2 \text{ ft/sec}^2 \times 144 \text{ in}^2/\text{ft}^2}$$

$$\times \left[\frac{4 \times 0.0047 \times 24.625}{0.222} + 0.300 \right]$$

$$= 17.3 \text{ psi}$$

Thus, only 18 percent of the total pressure loss in the primary cooling system of the MTR occurs in the active lattice.

Although pressure losses in the end boxes may be calculated, experimental data is required for their accurate evaluation. This experimental data is obtained from the MTR-ETR hydraulic test facility. It has been determined that a core friction drop of ~40 psi is required to maintain an average coolant velocity of 33 ft/sec in the fuel element. Therefore, the pressure losses in the MTR end boxes are 40 minus 17.3 equals 22.7 psi. The larger portion of this loss occurs in the top end box.

4. Lateral Pressure Differentials

If manufacturing controls were such that all fuel plates were exactly evenly spaced in a fuel element and end boxes were not used, no lateral pressure differential would exist across the internal plates. Manufacturing tolerances being what they are, unequal channels do exist, and it is entirely possible that the widths of two adjacent channels in a fuel element may differ by as much as 0.020 in. In addition, differences in flow conditions inside and outside the fuel element will result in some pressure difference across

the outer plates. For a given flow rate through an element, the over-all pressure drop is the same for all channels, whether internal or external.

Figure 8 shows that even with equal over-all pressure drops, two adjacent channels which have unequal hydraulic diameters will generate a lateral pressure across the included fuel plate.

The smaller channel, because of its lower velocity, will have a smaller entrance loss and the conversion of pressure head to velocity head at the entrance will also be smaller, but the friction loss will be greater (because f/De in the Fanning equation increases more than V^2 decreases). At the outlet the narrow channel has a smaller exit loss, but the pressure recovery due to change in velocity head is also smaller than the larger channel. The result is a net recovery in both cases to the same final outlet pressure.

The above analogy, of course, has neglected the end box effects. These add to the ΔP across the core but do not affect individual channel flow except where the flow distribution is disturbed by poor end box design. Figure 8 shows that the maximum lateral pressure difference will be greatest at the top of the fuel plates.

The analogy discussed above assumed that the fuel plates were flat or, in the case of the MTR curved plates, that the curve was perfectly smooth and each plate parallel to the next. In actual practice, randomly sized and spaced ripples exist in nearly all plates. Since these variations either increase or decrease the width of the channel, they cause either a local decrease or increase in velocity with an associated change in velocity head and an increase or decrease in lateral pressure differential.

The effect of dimples in plates even where the adjacent channels are equal will be to cause a lateral pressure differential across the dimple because of the velocity differential. This pressure differential causes an elastic deformation in the direction tending to increase the size of the dimple. It is thus possible to develop accumulating pressures and depletions until the elastic limit is reached and the plate collapses.

Some of the effects of lateral pressure buildup may be illustrated mathematically using the flat plate ETR situation and assuming

the edges are simply supported and the plate uniformly loaded (due to the lateral pressure).*

The maximum allowable pressure difference, ΔP , is obtained from the equation (table 10, ref 17):

$$\text{Max stress} = \frac{0.75 \Delta P b^2}{t^2 \left[1 + 1.61 \left(\frac{b}{L} \right)^3 \right]} \quad (3-4)$$

where

t = plate thickness = 0.050 in.

b = plate width = 2.624 in.

L = plate length = 36 in.

For an ultimate strength of 5000 psi for 1100 aluminum, one obtains:

$$\Delta P = \frac{5000 \times (0.050)^2 \left[1 + 1.61 \left(\frac{2.624}{36} \right)^3 \right]}{0.75 (2.624)^2}$$

$$\Delta P = 2.4 \text{ psi}$$

If the maximum fuel plate surface temperature in the reactor is 280°F, the maximum allowable ΔP is 2.0 psi. For the 6061 alloy fully hardened (T6 condition), the corresponding value would be about 16 psi. It is generally accepted that the steady state lateral pressure differential does not cause failure, because it tends to widen the narrow channel (the high pressure) and narrow the wider channel, thus reducing the lateral pressures.

The amount of this movement of a fuel plate between two unequal channels may be calculated for the maximum elastic condition from the ΔP obtained in equation (3-4):

$$\begin{aligned} \text{Maximum deflection} &= \frac{0.1422 \Delta P b^4}{E t^3 \left[1 + 2.21 \left(\frac{b}{L} \right)^3 \right]} \\ &= \frac{0.1422 \times 2.0 (2.624)^4}{10^7 (0.050)^3 \left[1 + 2.21 \left(\frac{2.624}{36} \right)^3 \right]} \\ &= 0.0108 \text{ in. or 11 mils} \quad (3-5) \end{aligned}$$

*The deflection and stress developed in MTR type plates lie between those expected for simply supported and rigidly supported flat plates.

For the case in which dimples occur in a plate which is located between two unequal channels, the situation gets correspondingly worse. For example, with the addition of 20 mil dimples close to the top of a fuel plate (where the lateral pressure is still at a maximum) it is possible to obtain a ΔP of 7.5 psi which is in excess of the elastic limit for the fuel plate using annealed 1100 aluminum.

5. Methods of Measurement

An empirical measurement of strength of the outer fuel plates has been developed to determine the resistance to lateral pressures. A static pressure test is used to measure the deflection of the fuel element outer plates when a pressure is applied internally to the element. Figure 9 illustrates the test apparatus available at the MTR-ETR which has been used for this type of test. With the end boxes on the MTR element it is relatively simple to make the piece airtight, although it is sometimes necessary to seal the edges of the fuel plates where they join the side plates. This may be accomplished with sealing compounds such as Glyptol. To make the ETR element airtight, it is necessary to weld end-caps over both ends and over the ports at the lower end of the fuel plates.

With the element positioned as indicated, the dial gauges are adjusted to just touch the surface of the fuel plate. This becomes the zero reading. The pressurizing procedure consists of applying a small pressure increment, ie, 1 in. Hg, and noting the dial readings. The pressure is then released and the dials observed to determine if they return to zero. The pressure is increased again, this time to 2 in. Hg, and again released and this action continued until some permanent set is observed or until the maximum desired pressure is reached. From the data, curves of pressure vs deflection (stress-strain) may be plotted both in the elastic range and continued in the region of permanent set. Within the elastic range the curves are essentially the same for all alloys of aluminum, and thus there is no real advantage of one alloy over another for small fluctuations in operating conditions which cause some development of lateral pressure. The high strength alloy will have appreciable advantage where, because of manufacturing tolerances of wavy plates, relatively large lateral pressures are developed such that the elastic limit of annealed 1100 aluminum

FIGURE 8
Pressure losses in adjacent
uniform channels having
different hydraulic diameters

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FIGURE 9
Hydrostatic test facility

is exceeded while yet within the strength of 6061-T6 or some other hardened aluminum alloy.

C. HEAT REMOVAL

1. General

The nuclear reactor is essentially a device for transforming the nuclear energy of uranium, thorium or plutonium into heat with the simultaneous evolution of neutrons and conversion of elements to different isotopes or to new elements. Removal of this heat can be accomplished in a number of ways such as forced circulation, natural convection or boiling. In some respects the applications of heat transfer to reactor design and operation are simpler than in some conventional heating systems, eg, the furnace-boiler system. In the reactor the bulk of the heat is generated in fuel plates (rods or tubes, etc), and it is only necessary to consider the transfer (conduction) through these plates (rods or tubes, etc) and the film coefficient to the bulk of the coolant. Radiant energy need not be considered since normal operating temperatures are not high enough for this means of heat transfer to be significant. The only additional concept to be considered in reactor heat transfer is the calculation of heat transfer through solid materials in which uniform heat generation is taking place.

It is generally estimated that about 200 Mev of energy is released for each U-235 nucleus undergoing fission. This is equivalent to 3.2×10^{-11} watt-sec. Therefore, it requires 3.1×10^{10} fission events to produce 1 watt-sec of energy. When considering the heat removal problem in nuclear reactors, it is necessary to analyze these values more precisely. For example, when a reactor first starts up, the energy release is only about 185 Mev/fission since the delayed heat portion (β & γ energy, etc from fission products) is not yet available. Of the 185 Mev only about 90 percent is produced at or near the point of fission (ie, in the fuel elements), the remainder will be produced in the moderator, structure or reflector. During operation when the reactor system is at equilibrium, an additional 7 Mev/fission from the delayed heat fraction will appear in the fuel (for a total of 175 Mev/fission), the remaining 25 Mev/fission will appear elsewhere. During shutdown only the

delayed heat (~ 15 Mev/fission) need be considered, and only about half of this appears in the fuel.

Figure 10 illustrates schematically the generation of heat through the fission process in the fuel core of a plate type element and the conduction of heat through the core and cladding and across the coolant film to the bulk coolant. The corresponding temperature profile illustrates the situation encountered in a testing reactor of the MTR-ETR type. Heat generation and heat transfer occur simultaneously in reactor fuel elements. If we consider the case of the flat fuel plate where all the generated heat is transferred outward from the center core to both surfaces, we obtain the following:

$$\begin{aligned} \text{let } Q &= \text{total rate of heat generation} \\ &= \frac{qAz_c}{2} \end{aligned}$$

where

q = heat generation rate/unit volume. At equilibrium, the conduction equation for uniform heat generation is:

$$q Ax = -k_c A \frac{dT}{dx} \quad (3-6)$$

Integrating for the entire fuel portion of the plates

FIGURE 10
Heat generation and temperature
distribution in a reactor fuel plate

$$q \int_0^{z_c/2} x \, dx = k_c \int_{T_1}^{T_\xi} dT$$

$$q z_c^2 / 8 = k_c (T_\xi - T_1)$$

or the temperature drop through the fuel core
is

$$T_\xi - T_1 = \frac{q z_c^2}{8 k_c} = \frac{Q z_c}{4 A k_c} \quad (3-7)$$

For a solid rod the equivalent equation is:

$$T_\xi - T_1 = \frac{Q}{4 \pi k_c L} \quad (3-8)$$

If one continues with the flat plate analysis to consider the conduction through the cladding and film, it is sufficient to consider the cladding and coolant film as a series of slabs, each having a thermal resistance and transmitting the same quantity of heat under steady state conditions.

Thus for each clad and film one writes:

$$Q = k_{cl} A \frac{\Delta T_{cl}}{z_{cl}} = h A \Delta t_f \quad (3-9)$$

from which the individual temperature differences may be expressed as:

$$\Delta T_{cl} = T_1 - T_w = \frac{Q z_{cl}}{k_{cl} A} \quad (3-10)$$

$$\Delta T_f = T_w - T_b = \frac{Q}{h A} \quad (3-11)$$

Example The heat flux, $\frac{Q}{A}$, at a point in a reactor fuel plate is 4.76×10^5 Btu/hr-ft². The fuel plate is of the MTR type having 0.015 in. thick Al clad and 0.020 in. thick Al-U alloy core. The temperature at the outside surface of the cladding is 192°F. What is the temperature at the interface of the fuel and clad and at the fuel centerline?

Solution: Assume that no heat is generated within the clad. Then equation (3-10) applies.

$$T_i - T_w = \frac{Q}{A} \frac{z_{cl}}{k_{cl}}$$

k_{cl} for aluminum at 195°F is 119 Btu/hr-ft-°F.

$$T_i = 192^\circ\text{F} + \frac{4.76 \times 10^5 \text{ Btu/hr-ft}^2 \times 0.015 \text{ in}}{119 \text{ Btu/hr-ft-}^\circ\text{F} \times 12 \text{ in./ft}}$$

$$= 192 + 5.0 = 197.0^\circ\text{F}$$

Assume that the rate of heat generation in the fuel core is constant. Then equation (3-7) applies.

$$T_c = T_i + \frac{Q}{A} \frac{z_c}{4k_c}$$

The thermal conductivity for the U-Al core is found to be about 100 Btu/hr-ft-°F.

$$T_c = 197 + \frac{4.76 \times 10^5 \times 0.020}{4 \times 100 \times 12}$$

$$= 197 + 2 = 199^\circ\text{F}$$

The temperatures calculated above are well below the melting points of Al and U-Al alloy. However, in the case of thicker fuel elements (especially those containing ceramic fuels), it is possible for the centerline temperature to exceed the melting point of the fuel material.

The total temperature difference between the bulk coolant and the core centerline is:

(3-12)

$$\Delta T_{\text{Tot}} = \Delta T_c + \Delta T_{cl} + \Delta T_f = T_c - T_b$$

$$= \frac{Qz_c}{4Ak_c} + \frac{Qz_{cl}}{k_{cl}A} + \frac{Q}{hA} = \frac{Q}{A} \left(\frac{z_c}{4k_c} + \frac{z_{cl}}{k_{cl}} + \frac{1}{h} \right)$$

In the sections which follow, these basic relationships will be used to calculate the

maximum fuel element surface temperature and coolant bulk temperature with reference to the non-boiling and boiling conditions in the reactor and the safety factors incorporated in the design.

2. Calculation of T_b

All of the heat energy generated by the fission of reactor fuel must be carried away by the cooling medium of the reactor. This heat energy causes an increase in the temperature of the cooling medium as it passes through the reactor core. In order to determine the wall temperature at any point, it is necessary first to determine the bulk coolant temperature at that point. This may be done by means of a heat balance which yields:

$$T_b = T_i + \frac{\Delta Q}{V\rho C_p a} \quad (3-13)$$

where

ΔQ is the total rate at which heat is added to the coolant stream between the point the coolant enters the reactor and the point at which the calculation is being made.

Example: For Cycle 106B, element 37 produced 4.71 percent of the total MTR power of 40 Mw (ref 7, table 5). If the total effective cross sectional area of flow for this fuel element is 5.78 in.² and the average velocity through the fuel section of the element is 33 ft/sec, what is the temperature of the coolant leaving this element? The inlet temperature is 105°F and the coolant is light water.

Solution:

$$\Delta Q = \frac{4.71}{100} \times 40 \text{ megawatts}$$

$$\times 3.41 \times 10^6 \frac{\text{Btu}}{\text{hr-megawatt}}$$

$$= 6.42 \times 10^6 \text{ Btu/hr}$$

From equation (3-13):

$$\begin{aligned}
 T_b &= 105^\circ\text{F} + \frac{6.42 \times 10^6 \text{ Btu/hr} \times 144 \text{ in}^2/\text{ft}^2}{(33 \times 3600) \text{ ft/hr} \times 5.78 \text{ in}^2 \times 61.8 \text{ lb/ft}^3 \times 0.998 \text{ Btu/lb} \cdot ^\circ\text{F}} \\
 &= 105 + 21.8 \\
 &= 127^\circ\text{F}
 \end{aligned}$$

3. Calculation of T_w (Non-Boiling)

The wall temperature of the fuel element at any point may be determined once the bulk temperature and the heat flux (Q/A) at that point are known. The heat flux may be calculated directly from absolute thermal flux densities and fuel densities or may be determined from the power of the element, its total heat transfer surface and the ratio of the neutron flux at the point in question to the average neutron flux:

$$\frac{Q}{A} = \frac{F \frac{\phi}{\phi_{\text{ave}}} \Delta Q_{\text{total / element}}}{A} \quad (3-14)$$

where

F is the fraction of the total reactor power generated within the fuel plates. For the MTR, F has been determined as 0.90 (ref 8).

Example: The maximum neutron flux for MTR element 37, Cycle 106B, is 1.43 times the average neutron flux. The total heat transfer surface area for this element is 15.6 ft². What is the maximum probable heat flux for this element?

Solution: In the example in the preceding section, the total rate of heat generation by the element was found to be 6.42×10^6 Btu/hr:

$$\begin{aligned}
 \frac{Q}{A} &= \frac{0.90 \times 1.43 \times 6.42 \times 10^6 \text{ Btu/hr}}{15.6 \text{ ft}^2} \\
 &= 5.30 \times 10^5 \text{ Btu/hr} \cdot \text{ft}^2
 \end{aligned}$$

The wall temperature of the fuel element is determined by the equation:

$$T_w = T_b + \Delta T_f$$

where

ΔT_f , the film temperature drop, is calculated from equation (3-11):

$$\Delta T_f = \frac{Q/A}{h}$$

The heat transfer coefficient, h , must be determined from the physical properties of the system and fluid. There are many empirical equations based upon experimental data which may be used to calculate h . Of these, the modified Colburn equation has been specified for use at the MTR-ETR (ref 9) because of its accuracy and simplicity. The modified Colburn equation is:

$$\frac{hd}{k} = 0.023 \left(\frac{dV\rho}{\mu} \right)^{0.8} \left(\frac{C_p \mu}{k} \right)^{0.3} \quad (3-15)$$

The fluid properties are evaluated at the film temperature which is the average between the bulk coolant temperature and the wall temperature. Since it is necessary to know the film temperature before the wall temperature may be computed, and since the film temperature is a function of the wall temperature, the solution of the above equations is accomplished by trial and error.

Example: If the maximum heat flux occurs at the bottom of the fuel in MTR element 37, Cycle 106B, what is the wall temperature at this point? The equivalent diameter of the MTR fuel element flow channel is 0.222 in.

Solution: Assume that the film temperature is 155°F. From standard references we obtain the following data for water at 155°F:

$$\begin{aligned}
 C_p &= 1.00 \text{ Btu/lb} \cdot ^\circ\text{F} \\
 k &= 0.3848 \text{ Btu/hr} \cdot ^\circ\text{F} \\
 \mu &= 1.010 \text{ lb/hr} \cdot \text{ft} \\
 \rho &= 61 \text{ lb/ft}^3
 \end{aligned}$$

Thus the Reynolds number, $(dV\rho/\mu)$, is:

$$\frac{(0.222/12)\text{ft} \times (33 \times 3600)\text{ft/hr} \times 61\text{lb/ft}^3}{1.01\text{lb/ft-hr}} = 1.33 \times 10^5$$

The Prandtl number, $\frac{C_p\mu}{k}$ is:

$$\frac{1.00\text{ Btu/lb-}^\circ\text{F} \times 1.010\text{lb/hr-ft}}{0.3848\text{ Btu/hr-ft-}^\circ\text{F}} = 2.62$$

The heat transfer coefficient is obtained from the modified Colburn equation (3-15):

$$\frac{h(\text{Btu/hr-ft}^2\text{-}^\circ\text{F})0.222/12(\text{ft})}{0.3848(\text{Btu/hr-ft-}^\circ\text{F})} = 0.023 \times (1.33 \times 10^5)^{0.8} \times (2.62)^{0.3}$$

$$h = 8,000\text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$$

$$\begin{aligned}\Delta T_f &= \frac{Q/A}{h} \\ &= \frac{5.30 \times 10^5}{8,000} \frac{(\text{Btu/hr-ft}^2)}{(\text{Btu/hr-ft}^2\text{-}^\circ\text{F})} \\ &= 66^\circ\text{F}\end{aligned}$$

$$T_w = T_b + \Delta T_f = 127 + 66 = 193^\circ\text{F}$$

It is now necessary to calculate T_f to determine if the assumed value was correct.

$$T_f = \frac{T_b + T_w}{2} = \frac{127 + 193}{2} = 160^\circ\text{F}$$

Since the first assumed value of T_f was not correct, it is necessary to recalculate h using the new assumed value of T_f .

For

$$T_f = 160^\circ\text{F}$$

$$C_p = 1.00\text{ Btu/lb-}^\circ\text{F}$$

$$k = 0.3861\text{ Btu/hr-ft-}^\circ\text{F}$$

$$\mu = 0.970\text{ lb/hr-ft}$$

$$\rho = 60.9\text{ lb/ft}^3$$

$$\frac{DV\rho}{\mu} = \frac{0.222 \times 33 \times 3600 \times 60.9}{12 \times 0.970}$$

$$= 1.382 \times 10^5$$

$$\frac{C_p\mu}{k} = \frac{1.00 \times 0.970}{0.3861} = 2.51$$

$$\frac{h \times 0.222}{0.3861 \times 12}$$

$$= 0.023 \times (1.382 \times 10^5)^{0.8} \times (2.51)^{0.3}$$

$$h = 8190\text{ Btu/hr-ft}^2\text{-}^\circ\text{F}$$

$$\Delta T_f = \frac{5.30 \times 10^5}{8190} = 65^\circ\text{F}$$

$$T_w = 127 + 65 = 192^\circ\text{F}$$

$$T_f = \frac{127 + 192}{2} = 159.5^\circ\text{F}$$

Since this film temperature is approximately the same as the assumed film temperature, there is no need for further trial and error calculation. The wall temperature of 192°F , as calculated above, is the most likely temperature which will exist at the bottom of the fuel section in element 37.

For the reasons given below in paragraph 4-a, and since the MTR and ETR are non-boiling reactors, Reactor Safeguards has established the policy that the wall temperature, as calculated above, must be lower than the saturation temperature of the water. Exceptions may be made to this rule for experimental programs which are specifically designed to investigate problems involving boiling of the coolant.

If the wall temperature calculated by the above technique is more than a few degrees above the boiling point of water at the absolute pressure existing in the fuel element, then nucleate boiling will take place and the modified Colburn equation cannot be used to calculate the heat transfer coefficient.

4. Boiling Heat Transfer

If the heat flux is large enough to cause the wall temperature to rise a few degrees above the boiling temperature of the fluid, and if the bulk temperature of the fluid is still below the saturation temperature of the fluid, then a condition known as nucleate boiling is established. In nucleate boiling small bubbles of steam form on the surface of the metal at nucleation centers. These bubbles grow in size because of the continuing vaporization of fluid at the hot surface. The vapor bubbles then leave the surface and are carried into the cooler part of the fluid stream where the vapor condenses and the bubbles disappear. There is no net generation of steam in nucleate boiling. The formation and removal of bubbles tend to increase the degree of turbulence of the fluid and make heat transfer very efficient. Thus the heat flux may be increased by a large amount while the wall temperature is increased only a few degrees. This condition is shown as curve B-C in figure 11. As the heat flux is increased, the amount of the surface covered by steam bubbles increases until point C is reached. At this point, approximately 50 percent of the surface is covered by steam bubbles. If the heat flux is further increased, steam is generated at the surface faster than it can be removed by the bubbles moving into the cooler fluid. The surface becomes completely covered by a blanket or film of steam which acts as an insulator so the wall temperature quickly jumps to point E in figure 11. Since point E may represent a wall temperature of several thousand degrees Fahrenheit, and since the transition from point C to point E may occur in a few milliseconds, point C is called the burnout point. If point E is reached, and the heating surface has not melted, the heat transfer will be accomplished by conduction, radiation and convection through the insulating film as indicated by line DEF. This form of boiling is called "film boiling." The part of the curve represented by line C-D is called "unstable film boiling." While it is possible to operate

some types of equipment in the unstable film boiling region, nuclear reactors generally cannot be made to operate in this region. If a reactor surface temperature and heat flux were adjusted to operate at some point on curve C-D, the wall temperature would either decrease so that the operation falls into the nucleate boiling region (curve B-C), or increase so that the operation falls into the film boiling region (curve D-E).

a. Nucleate Boiling

As in the case of convective heat transfer, the wall temperature occurring during nucleate boiling may be calculated by means of empirical relationships. These relationships are found in most standard heat transfer texts. However, since the wall temperature varies only slightly with heat flux in this region, it is usually satisfactory to estimate that T_w is equal to the saturation temperature plus 20°F to 30°F.

Since nucleate boiling is such an efficient means of transferring heat energy, very high power densities may be obtained from liquid cooled reactors which operate in this region. However, there are several drawbacks to its use. These include increased corrosion rates, more control problems and less safety factors as burnout is approached.

When boiling is established, a certain fraction of the coolant is filled with voids (ie, steam bubbles). Since the coolant in the MTR and ETR is also the moderator, the introduction of voids into the coolant causes a change in the moderating ability of the water and a corresponding change in the reactivity of the reactor. Such changes must be followed closely and continually by the reactor controls to maintain constant power generation. Thus boiling, although a stable situation, creates additional control problems in the reactor.

b. Burnout

The close approach to burnout is a major problem in boiling experiments. If the burnout point is passed, the fuel element may melt, thereby destroying the core and releasing quantities of fission products into the coolant system. There is also the possibility of a violent chemical reaction between the molten metal and the water which would release addi-

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FIGURE 11
Modes of heat transfer
to subcooled water

tional great quantities of heat. Therefore, it is necessary to estimate accurately the heat flux at which burnout occurs and to design all experiments and reactor cores so that burnout cannot result.

Examples of equations useful in predicting burnout in the MTR and ETR are the Mirshak-Durant equation (ref 10) and the Gunther equation (ref 11) respectively. The Mirshak-Durant equation for light water is:

(3-16)

$$\left(\frac{Q}{A}\right)_{BO} = 479000 (1 + 0.0365 V)$$

$$\times (1 + 0.00507 \Delta T_{sub})(1 + .0131 P) \pm 16\%$$

The Gunther equation (for light water) is:

$$\left(\frac{Q}{A}\right)_{BO} = 7000 V^{\frac{1}{2}} \Delta T_{sub} \pm 12\% \quad (3-17)$$

where

$$\Delta T_{sub} = \text{subcooling } (^\circ\text{F}) = T_{sat} - T_b$$

Whenever an empirical equation is used to predict burnout, the range of all variables used to arrive at the equation should be investigated to determine whether the range of the equation coincides with the operating conditions under investigation. It is very poor practice to extrapolate such empirical equations.

*The accuracy claimed for the Gunther equation may be over-optimistic and a more conservative value should be used.

The maximum possible heat flux generated within any part of the reactor must always be less than the minimum possible burnout heat flux.

Example: Determine the minimum burnout heat flux for an MTR element, given the following data for the bottom of the fuel assembly.

$$V = 33 \text{ ft/sec} \pm 30\%$$

$$P = 21.4 \text{ psia}; T_{\text{sat}} = 236^\circ \text{F}$$

$$T_b (\text{Maximum}) = 160^\circ \text{F}$$

Solution: The Mirshak equation (3-16) is based on rectangular channels, heated only on the long sides. The range of variables covered by the equation is $V = 5$ to 45 ft/sec, $\Delta T = 9$ to 135°F , $P = 25$ to 85 psia and $d = 0.21$ to 0.46 in. Therefore, Mirshak's equation appears adequate for use in this calculation.

The minimum velocity expected is:

$$33 - \frac{30}{100} \times 33 = 23 \text{ ft/sec}$$

The minimum value of ΔT_{sub} is $236 - 160 = 76^\circ \text{F}$.

Mirshak's equation, with an uncertainty factor of 16 percent, is used to calculate the minimum burnout heat flux.

$$\left(\frac{Q}{A}\right)_{\text{BO}} = 0.84 \times 479,000 (1 + 0.0365 \times 23) \\ (1 + 0.00507 \times 76) (1 + 0.0131 \times 21.4) \\ = 1.31 \times 10^6 \text{ Btu/hr-ft}^2$$

5. Hot-Spot Hot-Channel Analysis

The above sample calculations indicate that the MTR will not boil and that it is safe

from burnout. However, these calculations are based in part on physical measurements and empirical equations all of which are subject to deviations which could change the actual values of T_w , T_{sat} and (Q/A) from those calculated. Therefore, it is necessary to analyze the complete set of calculations, data and equations to determine the worst probable conditions which could exist and what their effect would be on the T_w , T_{sat} and (Q/A) . This analysis is known as the "hot-spot hot-channel analysis," the general form of which is given in ref 9. The details of the analysis depend upon the method in which data were obtained, the allowable mechanical tolerances and the engineering judgment of the individual making the calculations.

Example: Is the "hot-spot" heat flux for MTR element 37 (Cycle 106B) lower than the minimum burnout flux?

Solution: The maximum probable heat flux was determined from the equation:

$$\frac{Q}{A} = \frac{\% \text{ core power}}{100} \frac{\phi_{\text{max}}}{\phi_{\text{avg}}} \\ \times \frac{\text{core power}}{\text{heat transfer area}} \times 0.90 \times 3.41 \times 10^6$$

An analysis of the method used to obtain percent core power and $\phi_{\text{max}}/\phi_{\text{avg}}$ indicates that the only errors associated with the product of these two terms arise from the random errors in flux measurements and from the deviation of fuel density from the nominal value. The random errors associated with flux measurements amount to plus or minus 11 percent (from ref 9) and could therefore increase the heat flux by a factor of 1.11. The total amount of fuel within a fuel plate is specified to plus or minus 2 percent and the fuel homogeneity and thickness combined are specified to within plus or minus 5 percent. Therefore, the uncertainties in percent core power $\times \phi_{\text{max}}/\phi_{\text{avg}}$ could increase the heat flux by a factor of $1.11 \times 1.02 \times 1.05 = 1.19$. In addition, a factor of 1.05 is estimated to allow for the horizontal flux variation within the element. We will assume that the total core power is measured and controlled

to plus or minus 5 percent which can multiply the heat flux by an additional factor of 1.05. From the drawing of the MTR fuel plate, the minimum allowable heat transfer area of the fuel assembly is found to be 14.89 ft². When compared to the nominal heat transfer area (15.6 ft²), this yields a "hot-spot" factor of 15.6/14.89 = 1.05. The 0.90 factor is the fraction of total power generated within the fuel plates and is assumed to be precise. The 3.41 x 10⁶ is a conversion factor between Mw and Btu/hr and contains no appreciable error. The total "hot-spot" factor on the heat flux is the product of all the factors listed above, ie:

$$1.19 \times 1.05 \times 1.05 \times 1.05 = 1.38$$

Therefore, the "hot-spot" heat flux is equal to the nominal heat flux times the "hot-spot" factor listed above:

$$\begin{aligned} \left(\frac{Q}{A} \right)_{HS} &= 1.38 \times 5.30 \times 10^5 \\ &= 7.31 \times 10^5 \text{ Btu/hr} \end{aligned}$$

The minimum burnout flux was found to be 1.31 x 10⁶. Thus, the "hot-spot" heat flux is 56 percent of the minimum burnout heat flux and the fuel element is safe from burnout.

A complete set of "hot-spot hot-channel" factors for the sample MTR calculations in this chapter is given in table 6. The values given for this analysis would not generally apply to calculations performed in a different manner or to different experiments, geometries or reactors. It is therefore necessary to perform an independent "hot-spot hot-channel" analysis for each new item inserted in the reactor.

It should be noted that the total effect of the variation of a parameter must be evaluated. In some instances this variation may have a detrimental effect on part of the calculation and a beneficial effect upon the remainder of the calculation. For example the fluid velocity in the fuel channel was measured at 33 ft/sec plus or minus 30 percent. Since ΔT_b is dependent inversely upon the fluid velocity, the "hot-channel" factor is:

$$\frac{33}{23} = 1.43$$

Since ΔT_f is dependent inversely upon the 0.8 power of the velocity, the associated "hot-spot" factor is:

$$\left(\frac{33}{23} \right)^{0.8} = 1.33$$

Both the above factors assume that in the "hot-channel" and at the "hot-spot," the velocity is 70 percent of the nominal velocity. The effect of this lower velocity upon the pressure at the "hot-spot" must also be determined. Considering only the applicable terms in equation (3-1), for the nominal and "hot-spot" cases, we find the change of pressure due to velocity change alone is given by:

$$\begin{aligned} -\Delta P &= \frac{\Delta V^2 \rho}{2g_c} \\ -\Delta P &= \frac{60.7 \text{ lb/ft}^3 (33^2 - 23^2) (\text{ft}^2/\text{sec}^2)}{2 \times 32.2 (\text{ft/sec}^2) \times 144 (\text{in.}^2/\text{ft}^2)} \\ &= 3.67 \text{ psi} \end{aligned}$$

Thus, the absolute pressure at the "hot-spot" is 3.67 psia greater than the pressure would be in the nominal channel. Water data show that this increases the saturation temperature of water at the "hot-spot" by 7.8°F. Therefore, as specified in ref 9, -7.8°F is included under ΔT_{add} .

When all the "hot-spot hot-channel" factors have been determined, the "hot-spot" wall temperature may be determined from the simple equation: (3-18)

$$\begin{aligned} T_w (\text{hot-spot}) &= T_i + \Delta T_{add} + \Pi (F_b) \Delta T_b \\ &\quad + \Pi (F_h) \Delta T_h \end{aligned}$$

However, the above equation will usually overestimate T_w (hot-spot) because it does not allow for the variation of h with temperature. A more refined calculation may be made using the following equation: (3-19)

$$T_w = T_i + \Delta T_{add} + \Pi (F_b) \Delta T_b + \Pi (F_h) \frac{Q/A}{h}$$

where

h is evaluated from the modified Colburn

Table 6

Hot-Spot and Hot-Channel Factors

	Hot-Channel Factor, F_b	Hot-Spot Factor, F_h	Additive Factors, T_{add}
A. Mechanical Tolerances (Hydraulic Effects)	1.10	1.01	Included in E-1 and in base calculations
(1) Effect on mass flow - External Effects (End boxes, etc.)			
(2) Effect on mass flow - Entrance and Exit Effects			
(3) Effect on mass flow - Channel Effects			
(4) Effect of D Variation			
B. Mechanical Distortion	1.0	1.0	
(1) Effect on mass flow - External Effects (End boxes, etc.)			
(2) Effect on mass flow - Entrance Effects			
(3) Effect on mass flow - Channel Effects			
(4) Effect of D Variation			
C. Correlation Equation on h	—	1.25	
D. Determination of Critical Parameters			
(1) Flow measurement	1.03	1.02	1.3°F 10°F
(2) Temperature measurements (metal)	—	—	
(3) Pressure measurements	1.01	1.01	
(4) Temperature measurements (coolant)	—	—	
(5) Reactor Power measurement	1.01	1.01	
(6) Weight U-235/Plate	1.02	1.02	
E. Flow Distribution Effects			-7.8°F
(1) Plenum Effects (velocity)	1.43	1.33	
(2) Edge Effects	1.0	1.0	
F. Neutron and Gamma Flux Distribution			
(1) Vertical Distribution	1.0	1.0	
(2) Horizontal Distribution	1.05	1.05	
(3) Internal Distribution	1.0	1.0	
G. Neutron and Gamma Flux Uncertainty			
(1) Unperturbed Flux source	1.11	1.11	
(2) Average Depression	—	—	
(3) Internal Distribution	—	—	
(4) Experimental Power Generation Variation Timewise	1.04	1.04	
H. Heat Flux Over Element Surface			
(1) Fuel Homogeneity	—	1.05	
(2) Mechanical Effects (heat and clad thickness) and Fuel Area	—	1.05	
I. Effects Peculiar to Experiment			
(1) None			
(2)			
(3)			
(4)	$\Pi(F_b) = 2.04$	$\Pi(F_h) = 2.38$	$\Delta T_{add} = 3.5$

equation at the "hot-spot" film temperature. The "hot-spot" film temperature is given by:

$$T_f = T_i + \Delta T'_{add} + \Pi (F_b) \Delta T_b + \frac{1}{2} \Delta T_f$$

(hot-spot) (3-20)

where

$\Delta T'_{add}$ is the summation of those ΔT_{add} terms which do not indicate a change in the saturation temperature of the fluid.

When this procedure is applied to the MTR case which has been illustrated, it is found that the T_w (hot-spot) for element 37, Cycle 106B is 280°F, or 44°F above the saturation temperature of the water. This indicates that there is a definite possibility of boiling occurring. However, it must be remembered that for boiling to occur, it would be necessary for most of the variables listed in the "hot-spot" analysis to have their worst possible values at the same time and same place. This is not a very probable occurrence. Therefore, the fact that T_w (hot-spot) is above the saturation temperature of the coolant is not reason for rejecting an element or experiment. Rather, it is reason to perform a critical evaluation to estimate how probable it is that the experiment will boil and to determine how undesirable it would be to have boiling occurring at a particular spot in the reactor. The probability that a given temperature will be exceeded in a reactor may be estimated in the procedure outlined in ref 20.

6. External Cooling

The very high heat generation rate in the core and the relatively small volume of water in the tank, with its correspondingly high flow rate and relatively low temperature, impose rather severe design considerations in the external cooling equipment. The schematic flow diagrams of the MTR and ETR primary coolant loops (figures 6 and 7) indicate that fairly conventional equipment is used. Both reactor primary coolant systems consist of pumping water at ~ 100°F by means of horizontal centrifugal pumps through the reactor, then through a heat removal device. In the ETR, the primary coolant is pumped through the tube side of shell and tube heat exchangers where the reactor heat is transferred to the cooler secondary stream which flows through the shell side of the heat

exchangers. In the MTR, the primary coolant flows into the flash evaporators where, because of the reduced pressure, a portion of the coolant evaporates. This phase change lowers the temperature in the evaporators to about 100°F. That portion of the coolant which evaporated is then condensed on the condenser tubes which are cooled by water from the secondary coolant loop. The reactor heat liberated in this condensation is thereby transferred to the secondary coolant loop.

The secondary coolant loops of both reactors reject the heat to the atmosphere through forced draft cooling towers. In these towers a portion of the secondary coolant is evaporated to the atmosphere. This phase change causes a decrease in the temperature of both the air and secondary coolant passing through the tower. Therefore, it is possible to cool the secondary coolant stream below the temperature of the outside air.

The operating parameters are within normal industrial practice and the engineering calculations are of conventional type for design and operation. Details of these calculations are available in the Reactor Engineering Handbook and other texts. Therefore, further details are not included in this chapter.

D. REACTOR FUELS

There are many different types of fuel systems in existence today and many more under design and development. The solid, or heterogeneous, fuel system is used in the MTR and ETR, as well as the two commercial* testing reactors, and this is also the fuel system proposed for the advanced type testing reactors. Since March, 1952, when the MTR first went critical, this reactor has operated successfully on the plate type fuel assembly. Although some minor changes have been made, these elements remain essentially the same today as when the reactor went critical.

Although a discussion of reactor fuels might logically appear in the chapter on metallurgy, it is covered here because in the application at the MTR-ETR many of the fuel problems have involved the engineering of hydraulics, stress and heat transfer. The operating phase of these reactors has concerned the test and operating performance of particular geometries rather than the metallurgy involved in the fuel formulation. In the

*GETR and WTR.

field of reactor fuels references are abundant in texts, in Geneva Conference Papers and currently in Technical Progress Reviews. Several suggested readings (ref 12, 13, 14 and 15) are listed at the end of this chapter. While no attempt is made here for completeness, it is appropriate to discuss some fuel considerations particularly pertinent to the existing test reactors.

1. Geometries

The plate type fuel elements used in the MTR-ETR have the advantage of being a proven design for which fabrication techniques are well established. An analysis of the hydraulic and heat transfer characteristics of parallel flow cooling indicates that for optimum operation every flow channel in the element should have the same hydraulic diameter. It can also be shown that for equivalent elements (ie, those occupying the same space, having the same equivalent diameters, the same clad thickness, and the same fuel content and composition) there is very little advantage of one geometrical design over any other, except those advantages directly caused by the elimination of waste space and of local "hot-spots." Waste space includes mechanical braces and spacers such as side plates, and those portions of the fuel element which do not contain fissionable or fertile material or which have extra thick cladding. Since fuel plates are weak structurally, heavy side plates are required to withstand the hydraulic forces within the reactor. In both MTR and ETR the fuel plate contains 15 mils of aluminum cladding on each side of the core which serves to confine the fission fragments to the fuel, to prevent corrosion of the fissionable material and to provide additional strength for the fuel plate. Improvements in plate designs have been developed through the use of a greater number of thinner plates, use of high strength aluminum alloys, slotted side plates, modified end boxes, mechanical assembly replacing brazing and non-uniform spacing of fuel plates to eliminate non-uniform flow caused by end boxes. Modification of the plate design has been investigated in the form of a tubular MTR fuel element. Circular segment and involute designs have been proposed for an advanced test reactor. Tests have also been performed on a tube bundle type fuel element.

2. Fuel Materials

a. Core

After the nuclear physicist has specified the critical mass of fuel in which the fission reaction will begin and sustain itself under controlled conditions, it falls to the metallurgist and the engineer to incorporate this fuel in the most promising form into the best geometry for each reactor application. In the case of the MTR and ETR, the fuel core or "meat" is composed of uranium metal about 93 percent enriched in the U-235 isotope, alloyed with high purity (99.7 percent) aluminum. The weight percent uranium in the alloy (14 to 18 percent) is in a range such that a homogeneous alloy can be prepared. Above 30 to 35 wt percent alloy difficulties are encountered in segregation with a resultant non-uniform distribution of uranium. These difficulties have been overcome by the use of fuel plate cores containing a mixture of uranium oxide and aluminum powder fabricated by powder metallurgy techniques. Fuel plates containing up to 60 wt percent uranium have been fabricated by this method. An MTR fuel element containing UO_2 has been fabricated and successfully tested in the reactor. Test fuel plates with 55 to 65 wt percent UO_2 are being tested in the MTR under Phillips Fuel Element Development Program.

The existing MTR-ETR fuel core compositions, therefore, represent no great difficulties in fabrication, nor do the 400 gram ETR nor the 250 gram MTR loadings. It is quite likely that 500 gram ETR and 300 gram MTR fuel elements could be fabricated by existing techniques. The core alloy, prepared by induction melting in the open atmosphere, is poured into graphite molds and allowed to cool. The cast alloy slabs are then hot (590°C) rolled to produce strip stock about 1/4 in. thick from which fuel plate cores ~ 2 in. by 2 25 in. are punched. These cores are jacketed by the picture frame technique and again hot rolled to just over finished thickness (0.053 in.) Final size is obtained by cold rolling (about 5 percent reduction) which provides improved size control and a better metal surface. For those situations where the additional strength available from cold work (a strength increase of 3 to 4 times for 1100 aluminum) is to be utilized, the cold rolling accounts for about 15 to 20 percent of the total reduction.

In the ETR fuel core, slightly less than 0.1 wt percent of boron has been added to serve as a burnable poison. (This application is discussed somewhat further in the following section.) Because boron is quite volatile at the melting temperature of the U-Al alloy, control of the amount of boron added is difficult. It has been estimated that to add boron directly in this manner leaves only about 1/3 of the original added quantity in the cast slab. A further complication arises in the very slight solid solubility of boron in aluminum. In the cooling process boron tends to segregate making control of uniform distribution difficult.

b. Burnable Poisons

Almost from the time the MTR first went critical, there has been some consideration of the use of a burnable poison in the core. This appears to be a simple expedient to extend the fuel reactivity over a longer burnup cycle without adding such an amount of initial excess reactivity that the reactor is difficult to control. The use of burnable poisons should also reduce the amount of shim rod movement, and thereby stabilize the neutron flux during a cycle. A somewhat simplified explanation of the operation of burnable poisons is shown in figures 12 and 13. In figure 12, the burnout curves are shown for fuel (U-235) and burnable poisons (boron and gadolinium). Also shown are the buildup curves for fission product poisons (xenon and samarium).

The boron burnup curve is a reasonable inverse approximation of the fuel burnup curve, and the gadolinium burnup curve is a fair inverse approximation of the combined xenon and samarium buildup curves. In other words the boron burnout balances the fuel burnout while the gadolinium burnout balances the fission product buildup in the fuel. It is thus theoretically possible to keep the reactivity of a fuel core constant by the proper selection and amounts of burnable poisons. The ideal, however, is probably not to be achieved.

Figure 13 indicates the combined effects of fuel and fission products shown in figure 12 and then further combines the effects of added burnable poisons. Although not the idealized flat reactivity curve, for at least one cycle of operation the change in reactivity does not exceed 1 percent $\Delta k/k$ and would reduce the

amount of shim rod movement to 1-1/2 in. from the "at power" position.

The application of burnable poisons for reactor use can be carried one step further. By proper selection of the loading of fuel and poison distributions, it is possible that a flattened vertical neutron flux condition can be made available to the experimenter and the safety factor or the power can be increased for the reactor operator. Particularly in the MTR, the existing vertical thermal flux gradients in experimental positions are apparently dependent on the vertical fast flux distributions existing in the core. By varying the fuel and poison concentrations non-uniformly in a vertical direction, an improvement in neutron flux and power distribution can be obtained. By reducing the existing peak to average power generation in a fuel element, a gain in power of 35 percent (14 Mw) is possible. Recent developments in fuel plate fabrication techniques have indicated that non-uniform distributions are feasible.

c. Cladding

The cladding on the standard MTR and ETR fuel plates consists of 0.015 in. of 1100 aluminum (except for the outside fuel plates in the MTR element which have claddings of 0.0225 in. thick). 1100 aluminum is the purest commercial aluminum alloy (98 percent minimum Al) and has the best corrosion resistance in the temperature range of the MTR-ETR. It is also one of the weakest aluminum alloys. In the annealed condition (which exists in brazed fuel assemblies) this material has a yield strength of about 5000 psi. By introducing some cold work strength the yield value can be increased from 14,000 to 21,000 psi. However, to retain this strength, a new means of assembly had to be developed. The roll swaging and pinning techniques were developed to improve the sub-standard early ETR elements made with annealed plates and brazed assembly. Other aluminum alloys are also considered for increased strength, but to date the problems associated with their use have restricted any wide acceptance of such alloys as 6061 and 5052 aluminum. The 6061 alloy in the T-6 (fully hardened) condition has a yield strength of 40,000 psi, but has a slightly higher neutron absorption cross section than 1100 and has been very difficult to bond to the U-Al core.

FIGURE 12
Reactivity vs ETR reactor time for
fuel, fission products and
various poisons

FIGURE 13
Total reactivity vs ETR reactor
time for fuel, fission products
and burnable poisons

For elevated temperature operation (such as proposed for advanced testing reactors), 1 percent Ni-Al alloys or aluminum to which has been added Al_2O_3 are proposed because of their superior strength at high temperatures.

A question has frequently been raised as to the need for 0.015 in. of cladding. This may appear a rather arbitrary selection when one considers that the corrosion on the normal MTR or ETR fuel element is less than the equivalent of 0.001 in. Unevenness in the thickness of the cladding, particularly where "dog-boning" of the fuel has taken place, and the ever present surface pitting and cracks force the fuel designer to use a minimum cladding thickness to provide some safety factor. In spite of this it is entirely possible that thinner claddings are feasible and would provide satisfactory performance. Claddings as thin as 0.010 in. aluminum are being tested.

Although other materials than aluminum may be used for claddings, the above discussion has been limited to aluminum because in the temperature range of the MTR and ETR this is an excellent material from the standpoint of performance as well as cost. It is improbable that aluminum will be replaced as a cladding material in research and testing reactors as long as the maximum temperatures do not exceed 450°F.

3. Acceptance Testing

Specifications for fuel element procurement are normally very detailed in dimensions and allowable tolerances. Most of these figures can be readily checked by shop personnel, on receipt of the elements, to be certain that the elements will fit properly in the grids. Checking of channel spacing is usually accomplished by the supplier. Samples of core melts are furnished by the supplier and can be used for additional checks on his chemical analysis. MTR fuel elements are scanned for total fuel content by neutron beam measurements. A portion of the ETR elements is similarly checked in the ETRC. Except for rather insensitive radiographs, no further check is made on total fuel or poison content or distribution, or on presence of voids or unbonded cladding. The voids and unbonded cladding can be determined non-destructively on single plates, but not on assemblies, by ultrasonic inspection in the equipment available at the MTR. Methods are currently being developed for scanning of fuel and poison distribution,

but this too will be on a single plate basis. Facilities are also available for hydraulic and heat transfer tests, but these are primarily for development or for proof testing the product of a new supplier rather than to be used as a routine acceptance test.

It has generally been the philosophy on fuel procurement that a careful survey of the fabricator's procedure and a thorough check of the first production terms are a reasonable test of acceptance of his product, supplemented of course by visual inspection, dimensional checks and fuel content measurements.

E. REACTOR STRUCTURAL MATERIALS

1. General Characteristics

The choice of materials for various components of the reactor system depends on the function of the part, the location with respect to the core and the type and operating conditions of the coolant. Many of the materials used in the construction of a reactor are the same as those used in other engineering construction. For purposes of illustrating the selection of reactor structural materials, we will consider here only such items as the reactor vessel, reflector and in-pile experimental tubes.

The effects of nuclear radiation on the structural character and physical properties of these items are of major importance and are discussed in a separate chapter on metallurgy. The material selected for a structural component in a reactor not only must withstand relatively high combined stresses (hydraulic, pressure, thermal, etc), have good corrosion and creep resistance under the reactor operating conditions, but also must have good nuclear properties, ie, a low absorption cross section for neutrons, exhibit no radioactivity on absorption of neutrons, or where there is a level of induced activity, the half life should be short.

2. Nuclear Properties

The absorption cross section, principal type and energy of induced radiation and the half life of some of the more common reactor structural materials are shown in table 7. Because a structural material in a reactor is a parasitic absorber of neutrons, its absorption cross section is of primary importance. Because in testing reactors it is often necessary

Table 7

Some Nuclear Properties of Possible Structural Materials

Material	Absorption Cross Section σ_a barns, 0.025 ev.	Principal Radiation		Half Life
		Type	Mev	
Al	0.22	β γ	3.01 1.80	2.3 min.
Be	0.01	β	0.56	2.7×10^6 yr.
Zr	0.18	γ β	0.92 0.39	65 day
Fe	2.4	γ β	1.3 0.4	46 day
Graphite	0.005	β	0.16	5600 yr.
18-8 ss	2.9	$\left\{ \begin{array}{l} \text{Fe } \gamma \\ \beta \end{array} \right.$ $\left\{ \begin{array}{l} \text{Cr } \gamma \\ \text{Ni } \gamma \end{array} \right.$	1.3 0.4 0.32 0.93	46 day 27 day 2.5 hr.

to do work within the reactor vessel and occasionally remove some of the structural components, the induced radioactivity and its half life are also important.

It is readily observed from table 7 that aluminum with its relatively low absorption cross section and very short half life is a good structural material for thermal reactor use. When one adds to these properties its low cost and good mechanical and physical properties, it becomes an excellent structural material for water cooled reactors and is limited only by the operating temperature. Iron and stainless steel are not as attractive nuclear-wise, but may be used in those situations where the operating temperature or the coolant medium eliminates the use of aluminum. The nuclear characteristics of zirconium and its alloys are better than those of stainless steel, but the zirconium is also more costly.

Beryllium and graphite are good nuclear materials because in reflector or moderator applications they often serve as supporting structures. They are listed in table 7. These are examples of materials whose induced activity has a long half life but, because of a

low absorption cross section and absence of γ - radiation, are excellent reactor materials.

3. Combined Stress

The structural loading of a reactor component is the resultant of all the forces acting on the component. The main forces are those due to load distribution, pressure and thermal stress. The latter stress may be calculated approximately using Hooke's law (assuming the material remains within its elastic limits):

$$\epsilon = \frac{S}{E} \quad (3-21)$$

which becomes:

$$S = C \Delta T \frac{E \alpha}{1-\nu} \quad (3-22)$$

in the three-dimensional situation where deformation is caused by thermal expansion. C is a constant less than 1 and greater than 1/2 which is dependent upon the geometry and upon the method of heating. (See ref 21.) This equation is useful in determining the thermal stress in various materials being

considered for a particular reactor application. The following example illustrates how one might select the material for a reflector piece based on the developed thermal stress. It should be noted that the illustrations in this section use simplified equations which are limited in scope. For a more rigorous approach to any particular application, it is advisable to refer to texts such as Timoshenko (ref 16) or Roark (ref 17).

Example: Calculate the thermal stress in a 3 in. square aluminum bar in a reactor reflector position. Assume a uniform heat generation rate of 4 watts/gm. What would be the corresponding stress in a similar bar of stainless steel? In a beryllium bar? (Use the average values of mechanical and physical properties given in table 8.)

Solution: The temperature difference from the center to the outside of a square bar is calculated by:

$$\Delta T = \frac{qt^2}{12K}$$

This equation for the square configuration is analogous to equation (3-8) for the cylindrical bar. The stress is determined from equation (3-22). For a square bar $c = \frac{5}{8}$ (ref 17).

$$S = \frac{5}{8} \Delta T = \frac{E\alpha}{1-\nu}$$

$$\Delta T = \frac{4(\text{watts/gm}) 2.70 \left(\frac{\text{gm}}{\text{cc}}\right) 3.41 \left(\frac{\text{Btu}}{\text{hr watt}}\right) (2.54)^3 \left(\frac{\text{cc}}{\text{in.}}\right) (3 \text{ in})^2 12 \left(\frac{\text{in.}}{\text{ft}}\right)}{12 \times 118 \frac{\text{Btu}}{\text{hr ft-}^\circ\text{F}}} = 46^\circ\text{F}$$

$$S = \frac{5}{8} \times \frac{46 (^\circ\text{F}) 10 \times 10^6 \left(\frac{\text{lb}}{\text{in.}^2}\right) 14 \times 10^{-6} \left(\frac{1}{^\circ\text{F}}\right)}{1 - 0.33} = 6000 \text{ psi}$$

The corresponding values for Be on stainless steel (SS) are:

$$\text{Be: } \frac{\Delta T}{40.5^\circ\text{F}} \quad \frac{S}{760 \text{ psi}}$$

$$\text{SS: } 1710^\circ\text{F} \quad 411,000 \text{ psi}$$

Thus, under the specified conditions beryllium thermal stresses will be below its yield strength, aluminum develops stresses about

equivalent to its yield strength for 1100 alloys, but below its yield strength for other alloys. Stainless steel develops stresses which greatly exceed its yield strength.

Example: Calculate the thermal stress in a cylindrical stainless steel pressure vessel 6 ft in diameter and 1 in. thick assuming a gamma heat value of 2.0 watts/cc. Assume the heat generation rate is uniform and the outside surface is insulated.

Solution: The temperature difference is obtained from equation (3-7) when $t = Z/2$.

$$\Delta T = \frac{qt^2}{2k} = \frac{2.0 \times 3.41 \times (2.54)^3 \times 1^2 \times 12}{2 \times 9.4} = 72^\circ\text{F}$$

$$S = \frac{2}{3} \frac{\Delta T \alpha E}{1-\nu} = \frac{2}{3} \times \frac{72 \times 29 \times 10^6 \times 9.3 \times 10^{-6}}{(1-0.3)}$$

$$= 18,000 \text{ psi}$$

Had aluminum been selected as the material of construction, the stress would have been

800 psi. It is apparent from these two values that it may be preferable to specify the use of aluminum rather than stainless steel where the temperature gradient is large due to high heat generation rate.

In the case of the reactor core vessel or experimental tubes, which are also pressurized, the stress in the material resulting from the combined pressure and thermal stresses

Table 8

Some Mechanical and Physical Properties of Structural Materials

Material	Al (1100)	Be	Graphite	SS
k Thermal Conductivity Btu/hr-ft-°F (at room temperature)	118	90	85	9.4
α Coefficient of Thermal Expansion	14×10^{-6}	6.7×10^{-6}	1.4×10^{-6}	9.3×10^{-6}
E Modulus of Elasticity	10×10^6	44×10^6	1.2×10^6	29×10^6
ν Poisson's Ratio	0.33	0.10	0.25	0.30
ρ Density g/cc	2.70	1.82	2.26	8.0
Yield Strength psi (at room temperature)	5,000	32,000	3,000	30,000

can be obtained by adding these two stresses vectorially. Where the stresses are colinear they may be added algebraically. The numerical maximum of this value may then be compared with the allowable stress values for the material.

Example: If the internal pressure is 300 psig in the above example, what is the combined pressure and thermal stress?

Solution: For this situation where the vessel wall thickness is small compared to the diameter and where the pressure is all applied internally, the hoop stress formula is adequate for the calculation of pressure stress.

$$S = \frac{\Delta P R}{t} \quad (3-23)$$

where

ΔP is the difference between the internal and external pressure:

$$S = \frac{300 \times 36}{1} = 10,800 \text{ psi}$$

$$\text{Combined stress} = 18000 + 10800 = 28800 \text{ psi}$$

In the case of experimental loop tubes where the internal pressure is large, the pressure stress is often the predominant factor, and for this reason high strength steels are usually preferred.

4. Creep

When the pressure, load and thermal stresses in a reactor exceed the elastic limit of a material, it may cause distortion or even rupture of a structural part. A somewhat more insidious phenomenon, which occurs over a longer period of time, is creep. The application of a constant stress will cause a progressive deformation in a material, the rate of which depends on the material, the temperature and the load. Although a considerable amount of creep data is now available, the understanding of the process is far from complete. Yet the importance of this phenomenon in reactor application cannot be underestimated. For example, extensive damage which occurred in the United Kingdom reactors, particularly to the fuel elements, has been attributed to creep.

A plot of creep (deformation vs time) generally consists of three major parts: (1) the initial extension; (2) a linear rate; and (3) a rising rate towards rupture. The creep strength of a material is defined as that stress which will cause a given amount of plastic deformation in the material during a fixed period of time. The creep rate (at constant temperature and high stresses) may be represented as:

$$u = ce^{\gamma S}$$

where

γ and c are constants and S is the stress. Even this number is valid only over a portion of the creep curve. Creep values for some of

Table 9

Creep Values for Typical Reactor Structural Materials

Material	Temperature °F	Stress psi	Creep Rate % hr
Aluminum	300	7,000	0.001
	800	350	0.001
Zirconium	500	20,000	0.001
	900	8,000	0.001
Beryllium	800	11,000	0.001
Graphite	3,500	2,000	0.001
SS (Type 304)	800	32,000	0.0001

the materials considered in this section are given in table 9. Since it was not possible to obtain all of these values on the same temperature and creep rate basis, some care must be exercised in drawing conclusions from the table. Also, differences in creep rate will be found even among specimens of the same material due to differences in impurities and metallurgical history.

In the low temperature range (300°F) of the MTR and ETR, aluminum has adequate creep strength, but in the 800°F range stainless steel is best; while at very high temperatures graphite is best.

5. Corrosion

a. Definition

The phenomenon of corrosion is generally described as the deterioration of a metal by reaction with its environment. It manifests itself in various forms, ie: (1) galvanic corrosion or pitting due to direct chemical action; (2) erosion due to the scouring action of flowing particles of the environment; (3) scaling in which layers of corrosion products are formed on the surface; (4) stress corrosion, in which the reaction is initiated and progresses by unrelieved stresses in the material; and (5) intergranular, wherein the corrosion begins at the surface grain boundaries and continues into the interior along the grain boundaries.

b. Aluminum Corrosion

The corrosion resistance of aluminum in a water environment is determined primarily by the purity and metallurgical condition of the metal, ie, the lower the purity and the higher the temperature, the lower the resistance. In aqueous systems, 1100 aluminum corrodes by pitting (galvanic) attack at susceptible points (ie, surface impurities) below 200°F, but this corrosion can be prevented by small additions of inhibitor to the water. Uniform attack occurs along with pitting and is the principal damage mechanism between 200°F and 400°F. Above 400°F, intergranular attack predominates. Additions of nickel, iron and silicon improve the high temperature corrosion resistance of aluminum.

c. Zirconium Corrosion

Dynamic tests in 400°F water have indicated that the corrosion resistance of zirconium is superior to stainless steel, beryllium and aluminum. However, the corrosion behavior of zirconium is dependent on the composition of the material, its surface condition, metallurgical history, the purity of the water and the amount of oxygen and nitrogen dissolved in the water. The zirconium-tin alloys and zirconium alloys containing iron, chromium and nickel exhibit increased corrosion resistance.

d. Beryllium Corrosion

Beryllium is inferior to zirconium in its corrosion resistance to water. By proper treatment, however, beryllium withstands the action of pure de-aerated water satisfactorily up to 500°F. In the presence of oxygen the corrosion rate is increased. Oxygen causes the surface of beryllium to tarnish rapidly but the oxide film, once formed, tends to retard further attack. The corrosion resistance is best when the pH of the water is below 6.5.

e. Stainless Steel Corrosion

The austenitic stainless steels generally have excellent corrosion resistance to water. Improper heat treatment of the unstabilized alloys can cause intergranular corrosion and incipient pitting attack.

6. Fatigue

When a steady state and transient conditions occur in a structure, the design should consider the possibility, ie, the tendency for the metal to break under repeated cycles of stressing at values well below its ultimate tensile strength. The maximum stress that a metal will withstand without failure for a specified (large) number of stress cycles is called its fatigue limit or endurance limit. Reactor applications occasionally result in combined stresses in various structures (especially loops) exceeding the proportional limit, yet the metal performs satisfactorily.

The design of structures and vessels subjected to repeated stressing is very complex. The structure size and material must be determined as a function of the mean stress, the maximum alternating stress and the number of cycles which the structure must withstand. Details of this type analysis, which are beyond the scope of this manual, may be found in ref 18.

If sufficiently high cyclic thermal strains are applied to a structure, subjected to internal pressure or mechanical loading, a cyclic growth will occur. This condition can be investigated according to the procedure of Miller (ref 19).

* * * *

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* * * *

Nomenclature

A	heat transfer area (ft ²)
a	total cross sectional area of flow (ft ²)
b	plate width (in.)
C	constant dependent upon geometry
C _p	heat capacity (Btu/lb-°F)
d	hydraulic diameter (ft). d is equal to the diameter of a cylindrical tube and is equal to (4 x cross sectional area of flow/wetted perimeter) for a non-circular flow channel.
E	modulus of elasticity (lb/in. ²)
F	fraction of power generated in the fuel plates
f	fanning friction factor
F _b	hot-channel factor
F _h	hot-spot factor
G	rate of flow (gal/min)
g	acceleration due to gravity (-32.2 ft/sec ²)
g _c	gravitational constant (32.2 lb _{mass} -ft/lb _{force} -sec ²)
h	heat transfer coefficient (Btu/hr-ft ² -°F)
K _m	minor friction loss coefficient
k	thermal conductivity (Btu/hr-ft-°F)
L	length of flow channel or heat transfer surface (ft)
P	pressure (lb/ft ²) or (lb/in. ²)
Q	heat generation rate (Btu/hr)
q	heat generation rate per unit volume (Btu/hr-ft ³)
R	radius (in.)
Re	Reynold's number $dV\rho/\mu$ (no dimensions)

S	shear stress (lb/in. ²)
T	temperature (°F)
T _{sub}	subcooling (°F) = (T _{sat} - T _b)
T _{sat}	saturation temperature of coolant (°F)
t	plate thickness (in.)
V	velocity of the fluid (ft/sec)
W	shaft work done by the system per unit mass of fluid flowing (ft-lb/lb): shaft work done on the system (eg, pumping) has a negative sign
x	distance from plate centerline (ft)
Z	elevation (ft)
z	thickness (ft)
α	coefficient of thermal expansion (1/°F)
Δ	the difference between the value of a given variable at two different points
ΔT _{add}	additive uncertainty factors (°F)
ε	deformation (in./in.)
μ	fluid viscosity (lb/ft-sec)
ν	Poisson's ratio
π	3.1416
Π(F _b)	the product of all hot channel factors
Π(F _h)	the product of all hot spot factors
ρ	density (lb/ft ³)
φ	thermal neutron flux (n/cm ² -sec)

* * * *

Subscripts

l	at the interface between the clad and the fuel
B O	at burnout
b	bulk
℄	at the centerline of the fuel
cl	clad
c	core
f	film (average between wall and bulk)
i	inlet
w	wall (the interface of the clad and coolant)

CHAPTER IV

REACTOR CHEMISTRY

A. INTRODUCTION

Despite the fact that most of the design and operation of nuclear reactors falls within the field of physics, extensive chemistry is involved. Ordinary water cannot be used to cool reactor elements because of the tremendous activity that would be induced in the normal raw water impurities. It is through chemical ion exchange that this water is purified. Further, piping, container vessels and fuel elements themselves are subject to chemical attack and corrosion if the pH of the water is not held within certain limits. These factors should be understood.

There are other chemistries which can be even more important as far as operating the reactors is concerned; they are contamination and corrosion. Despite best efforts, some atoms contaminate the cooling water and arise from chemical corrosion of the system components and from imperfect or failed test specimens. The latter can become serious in a short time so it is important to identify and remove the offending source as quickly as possible. Several monitoring systems are available to assist in this detective work. They will be discussed individually and then a typical rupture analysis will be described.

B. WATER CHEMISTRY

1. Make-up Water

Demineralized water rather than raw water is used in the primary and canal systems of the MTR and ETR because raw water is highly corrosive to aluminum and beryllium and because the dissolved solids in raw water would be activated by the neutron flux in the reactor.

The make-up and flush water for both MTR and ETR primary systems is supplied from the MTR demineralizer plant. Well water is deionized in a conventional two-bed demineralizer unit, and the demineralizer effluent water has a pH of 8 to 9 and a specific conductance of 1.5 to 2.5 micromhos. The pH of the demineralized water in the storage tanks is adjusted

to 6.0 ± 0.5 by the addition of nitric acid to the water in the storage tanks. The pH is adjusted to this range to reduce the corrosion of aluminum which would be severe at a pH of 8 to 9. The total solids in the demineralized water is 1 to 2 ppm.

2. Chemical Control

Since both the MTR and ETR reactor primary systems are composed largely of stainless steel and aluminum, chemical control of the water is very simple and consists of maintaining the coolant pH between 5.5 and 6.5. At the MTR this is done by adding small amounts of nitric acid to the primary coolant to reduce the pH. Addition of acid is done only when the cation resin bed is out of service since operation of this bed will lower the pH. The pH of the system naturally rises due to slight corrosion of the system if the resin bed is not in operation. With the resin bed operable, pH control of the coolant can be maintained by either of two methods: (1) the pH control of the coolant is permitted to increase to about 6.5 and then the resin bed is placed in operation which lowers the pH to about 5.5 at which time the resin bed is removed from service; (2) the pH control of the coolant is maintained by operating the bypass bed continuously and adding small quantities of sodium hydroxide to increase the pH.

At the ETR the primary coolant pH is controlled by resin bed operation exclusively without the use of nitric acid or sodium hydroxide. This is possible since the ETR demineralizer system consists of two cation units and two anion units. If the pH of the system is increasing, the cation resin bed flow is increased, and the anion bed flow is reduced. Conversely, if the pH of the primary coolant is decreasing, the cation bed flow is reduced, and the anion bed flow is increased.

In both systems the water quality is satisfactory and no attempt is made to improve the quality except as a secondary function of the

resin beds, which is primarily activity removal. The MTR coolant has a specific conductance of 2 to 3 micromhos while the ETR coolant has a specific conductance of 1 to 2 micromhos. The ETR coolant has less conductivity than the MTR coolant since both cation and anion beds are in operation in the ETR system.

The primary coolant in both reactors is routinely analyzed for iron, aluminum and beryllium by the Chemistry Section. Normally a sample of water is analyzed once per reactor cycle, and the sample is taken while the reactor is at power. It is questionable that the quantities of iron, aluminum and beryllium found are a measure of the corrosion in the systems, but the concentrations reported are an indication of the corrosion rates.

Table 10 lists some recent data on the quantities of iron, aluminum and beryllium determined in the primary coolant.

3. Radiolytic Decomposition

In MTR-ETR type reactors, decomposition of the water occurs because of the ionizing properties of protons and gammas on the water molecules. The ionizing radiation decomposes

Table 10
Corrosion Products in Primary Coolant

System	Date	Fe ppm	Al ppm	Be ppm
MTR	6-1-59	0.02	0.01	< 0.01
	6-22-59	0.03	0.01	< 0.01
	9-8-59	0.2	0.02	< 0.01
	9-14-59	0.7	----	----
	9-29-59	0.04	0.02	< 0.01
ETR	11-10-58	< 0.01	< 0.01	< 0.01
	12-1-58	< 0.01	0.01	< 0.01
	1-5-59	< 0.01	0.01	< 0.01
	5-27-59	0.01	0.02	< 0.01
	8-31-59	0.02	0.02	< 0.01

the water into hydrogen atoms and free hydroxyl radicals as follows:

The free radicals H and OH may then react as follows:

Since hydrogen peroxide, H_2O_2 , is unstable, some decomposition occurs as follows:

A reverse reaction also occurs:

Water is also reformed by a radiation-induced reaction as follows:

Thus, as decomposition of the water occurs in the reactor there is also a back reaction occurring which removes the decomposition products by recombination to form water. The net production of H_2 and O_2 is dependent on temperature, the radiation density and the impurities present in the water. These effects have never been investigated in the MTR or ETR because the amount of gases produced is small and has never been an operating problem.

In the MTR system the coolant is degassed after each pass through the reactor when the coolant passes through the flash evaporators. For several years after startup all of the primary coolant was degassed; however, at the present time only about 85 to 90 percent of the total flow passes through evaporators, and 10 to 15 percent of the water passes directly from the seal tank to the sump tank.

The coolant is degassed at a pressure of about 2 in. Hg absolute, and the total volume of gas from the evaporators is about 12 cfm at a reactor power of 40 Mw. The gas from the evaporators consists of about 35 percent hydrogen, 31 percent oxygen and 34 percent nitrogen. The hydrogen and some of the oxygen are formed from radiolytic decomposition of the coolant. The nitrogen and

remainder of the oxygen result from air leaks into the evaporators and associated piping and also from gases introduced by the make-up water which is saturated with air. The total gas concentration of the water entering the reactor is 0.2 to 0.4 cc/l and the gas concentration leaving the reactor is 3 to 5 cc/l.

In the ETR system, a side stream of 300 gpm is continuously reduced to atmospheric pressure in a degassing tank through which atmospheric air is purged so that hydrogen and oxygen formed by radiolytic decomposition are flashed to the atmosphere. During the initial reactor operation, the total gas content of the primary coolant was checked periodically to determine if the gas concentration would reach equilibrium. The maximum concentration of gas found was 40 to 50 cc/l which is well below saturation in the core. No analyses of the gas concentrations in the water have been made; however, the gases evolved from the water at atmospheric pressure were found to be 43 percent hydrogen, 26 percent oxygen, 30 percent nitrogen and less than 1 percent argon.

4. By-Pass Resin Beds

The by-pass resin beds are used primarily for activity removal from the primary coolant while the reactor is at power, but can also be used to clean up the primary water prior to discharge to the leaching pond.

The cation resins used are a polystyrene nuclear sulfonic acid type and are in the acid regenerated form. In the exchange process, cations dissolved in the water are exchanged with the hydrogen ions of the resin. Both radioactive and non-radioactive ions are exchanged in the process. The cation exchange process can be illustrated by the following chemical reaction:

where

R_z presents a polymeric anion which forms an insoluble salt.

The reverse reaction or regeneration is possible by treating the resin with a solution of 2 to 4 percent strong mineral acid. However, the by-pass cation units are never regenerated because of the large quantities of radioactivity concentrated in the bed. The cation resins are

removed from the unit to a disposable tank and transported to the burial ground. New resin is added to the exchanger unit to replace that which was removed.

The anion resin used is a quaternary amine type and is in the hydroxide regenerated form. In the exchange process soluble anions are exchanged with the hydroxyl ions of the resin. The anion exchange process can be illustrated by the following chemical reaction:

where

R₄N is the symbol for the complex anion exchanger radical. The reverse reaction is possible by treating the resin with a 4 percent solution of sodium hydroxide. The ETR primary system has two anion units, and these are regenerated because the activities collected on the resin have a short half life, and it is permissible to discharge these radioactive isotopes to the leaching pond.

Activity removal by the resin has been very satisfactory as indicated by table 11.

It should be noted that the ETR anion units are treating cation effluent and not primary coolant. Also, the gross activity determined in ETR water is made on evaporated samples, while gross activity in MTR water is made on liquid samples, so a direct comparison of the activities for the two systems cannot be made from table 11.

C. RADIOCHEMISTRY

In the operation of a reactor such as the MTR or ETR, many radioactive nuclides are formed. These are produced by neutron reactions on stable nuclides and from fission. They enter the primary coolant and find their way to all parts of the primary coolant system. The identification of these radioisotopes and determination of their radioactivity levels is important in determining when the reactor is operating normally. In the identification of these radioisotopes, radiochemistry plays a very important role.

Routine water samples are taken daily from the reactors and assayed for total gamma and alpha activity. Also, once a week a sample is routinely analyzed by radiochemical techniques to establish a normal operating level

Table 11

Gross Activity in Primary Coolant
and Resin Bed Effluents

System	Date	Primary Coolant c/min/ml	Resin Bed Effluent c/min/ml
MTR	11-27-59	26,000	13,000
	11-28-59	30,000	15,000
	11-29-59	31,000	14,000
	12-6-59	42,000	18,000
ETR	11-15-59	15,290	Cation 1 3,480
			Cation 2 3,310
			Anion 1 770
			Anion 2 750
	12-12-59	23,880	Cation 1 6,035
			Cation 2 7,668
			Anion 1 1,520
			Anion 2 1,865
	12-16-59	29,600	Cation 1 7,440
			Cation 2 7,140
			Anion 1 2,600
			Anion 1 2,480

for certain nuclides of interest. Among the chemical fractions isolated from the water sample are sodium, potassium, protactinium, neptunium, barium, strontium, iodine, ruthenium and zirconium.

Sodium (Na-24) activity is normally produced in the aluminum of the reactor by an $\text{Al-27} (n, \alpha) \text{Na-24}$ reaction. This activity enters the water coolant either by recoil, by corrosion or both. During normal operation Na-24 (along with Mn-56 which is produced mainly by neutron capture in stable Mn-55) accounts for ~ 90 percent of the radioactivity of the coolant stream. A sudden increase in the Na-24 in the primary water can occur upon rupture of a capsule containing Na or NaK. These two materials are often used as heat transfer media for capsules. (See figures 14 and 15.)

Protactinium (Pa-233) appears in the coolant when a thorium bearing capsule starts to leak. Thorium is often irradiated with neutrons in the reactor to produce U-233. Pa-233 is an intermediate isotope in the production chain and is the radioisotope most likely found when a thorium capsule breaks. (See figure 16.) The chain of reactions for producing U-233 is:

Neptunium (Np-239) is produced in large quantities when natural uranium is irradiated with neutrons. An increase in Np-239 activity would indicate a leak in a capsule containing natural, depleted or low enriched uranium. (See figure 17.) The nuclear reactions for this chain are:

The remainder of the elements mentioned previously are formed when U-235 undergoes fission. These elements are found in the water of an operating reactor even in the absence of a fission break. This is because all materials of construction used in the reactor contain as an impurity, small but measurable quantities of uranium. The uranium level in aluminum

for example is of the order of 10 to 20 ppm and its presence results in the formation of "background" fission products in the reactor coolant.

Routine samples are analyzed radiochemically to establish an operating norm for the various radioactive elements. Any sudden rise in any one of these radioactivities yields valuable information in deciding which of the many capsules or experiments in the reactor is a possible source of the observed radioactivity increase.

1. Gamma Spectrometry

Chemical fractions are counted for gamma activity with a Radiation Instrument Development Laboratory 256 Channel Gamma Spectrometer equipped with a 3 in. by 3 in. NaI(Tl) crystal. The NaI crystal in effect converts the short-wave length gamma photons into longer wave length (near visible) light pulses which are detected by a photomultiplier tube. The photomultiplier output current produces a voltage pulse that is proportional to the energy of an incident gamma ray which had undergone a photoelectric effect in the detector crystal. The spectrometer sorts these voltage pulses electronically and stores the information. The stored information is "read out" of the instrument at a later time and the information plotted as shown in figures 14 through 22. These figures represent normal routine gamma spectra. Table 12 has been prepared for the purpose of comparing the radioactive levels during normal operation of the MTR with those found during actual periods of abnormal operation. These data were obtained by analyses of the photopeaks in the gamma spectra. The important photopeaks are shown in the figures.

To identify a gamma as being definitely associated with a particular nuclide, an observed photopeak must meet two criteria: (1) it must have the proper energy; (2) it must decay with the proper half life. Both of these criteria are very important in establishing the purity of all the chemically separated samples.

2. Alpha Activity

When an increase in alpha activity in the water coolant is observed, the first step taken to identify the alpha emitter is to prepare a nearly weightless sample and measure the

Table 12

Comparative Radioactive Levels
During Normal MTR Operation

Radioisotope	Normal d/min/ml	Abnormal d/min/ml
Na-24	12,100	44,600
K-42	insignificant	1,510
Pa-233	229	138,200
Np-239	114	5,500
Ba-139	53	2,060
Ba-140	14	793
Sr-92	189	5,620
I-131	188	43,000
I-133	42	83,000
I-134	228	14,500
I-135	487	35,000
Ru-103	59	346
Ru-105	19	584
Zr-95	24	555
Zr-97	31	

energy of the alpha rays. To measure the energy of the alphas the Tracerlab Frisch Grid Chamber and Pulse Analyzer are employed. The alpha rays produce ionization in the gas chamber. The gas is at a sufficiently high pressure so that all the alpha energy is expended in the gas. The ions are collected and the current produced is proportional to the energy of the alpha. A plot of the data from a Frisch Grid Chamber results in a spectrum such as that shown in figure 23.

Alpha energies are helpful in selecting possible alpha emitting nuclide sources, but occasionally two different nuclides may emit alphas with approximately the same energy and cannot be resolved by alpha pulse analysis. An example of this is Np-237 (4.79 Mev α) and U-234 (4.78 Mev α). Here it is necessary to supplement the alpha pulse analysis data with specific chemistry to determine if the alphas observed are from only one of the possible nuclides or from a mixture.

There are many alpha emitting substances being irradiated in the reactor such as Cm-244, Pu-239, Th-230, Am-241 and Am-243. All of these are hazardous, but among the most hazardous isotopes are Pu-239 and Pu-240 since the maximum permissible body burdens of these nuclides is so low (< 0.04 μ c total Pu-239

FIGURE 14
Gamma spectra of Na^{24} in MTR
primary coolant water

FIGURE 15
Gamma spectra of K^{42} in MTR
primary coolant water

FIGURE 16
Gamma spectra of Pa^{233} in MTR
primary coolant water

FIGURE 17
Gamma spectra of Np^{239} in MTR
primary coolant water

FIGURE 18
Gamma spectra of Ba in MTR
primary coolant water

FIGURE 19
Gamma spectra of Sr in MTR
primary coolant water

FIGURE 20
Gamma spectra of I in MTR
primary coolant water

FIGURE 21
Gamma spectra of Ru in MTR
primary coolant water

PP CO D 2830

FIGURE 22
Gamma spectra of Zr in MTR
primary coolant water

FIGURE 23
Alpha spectra of Np^{237} , Am^{243} ,
 Am^{241} , Cm^{242} , and Cm^{244}

or Pu-240 per lifetime when fixed in the body). Therefore, it is essential to identify alpha activity in the process water, to pinpoint the source of activity and to evaluate the hazards.

3. Iodine Monitor

There are other systems which can be used to determine when fuel or a fuel bearing capsule in the reactor has ruptured. Among these are the iodine fission product monitor which is used at both the MTR and ETR. It utilizes a small anion exchange resin column preceded by a small cation resin column. A small stream of water is taken continuously from the bottom of the reactor and run first through the cation column and then through the anion column. The cation column takes out the majority of the cationic fission products and the anion column adsorbs the anions which still remain in the stream. Among the anions adsorbed on the anion column will be the fission product iodine isotopes I-131 (8d), I-132 (2.3h), I-133 (21h), I-134 (52 min) and I-135 (6.7h). This anion column is monitored for radioactivity by a NaI(Tl) scintillation crystal connected to a photomultiplier tube. The output of the phototube is monitored by a rate meter. Any sudden release of

fission products in the reactor will be indicated by a sudden rise of the rate meter. The rate of rise is an indication of the severity of the rupture. Background iodine activity in the process water will always cause a rise in the count rate for the first day or more after a major increase in power. However, the rate of rise after a power increase is relatively slow, and from past experience the rate of rise observed can be interpreted as being normal or abnormal.

4. Stack Gas Monitor

Another instrument very valuable in determining the type and severity of a rupture in the process water system is the stack gas monitor. At the MTR, the stack gas monitor is a large ionization chamber filled with helium at high pressure. A sample of the combined gases (prefiltered to remove particulate matter) which comes from the cooling air of the graphite zone of the reactor, from experimental gas cooling lines and from the reactor process water off-gas line, is passed through a two inch pipe in the ionization chamber. Gamma rays from the radioactive components of these gases produce ionization in the counting gas

which is in turn collected, amplified, counted and recorded. A fission break in any one of these gas streams will produce a high reading on the stack gas monitor because a fission break always releases noble radioactive fission gases. The action of the ETR stack gas monitor is very similar, except that the detecting element is a gamma scintillation crystal instead of an ionization chamber.

5. Vent Seal Monitor

Still another valuable instrument used in determining the source of activity in the MTR process water system is the vent seal monitor. This is a large ionization chamber that determines relative levels of gamma activity in the vicinity of the vent seal in the Process Water Building. A high reading on this instrument without a corresponding increase of activity on the stack gas monitor would indicate that a source of high activity existed in the process water system which did not come from a fission break. Conversely, a high reading on this instrument with a corresponding high reading on the stack gas monitor would confirm that a fueled structure in the process water system had ruptured.

6. Delayed Neutron Counter

Another fission break monitor used on loop experiments containing fuel is a delayed neutron counting apparatus. It consists essentially of a coolant holdup tank in which BF_3 -filled neutron counters are located. The BF_3 tubes detect neutrons by the reaction $\text{B-10}(\text{n}, \alpha) \text{Li-7}$. The neutrons counted are called "delayed" neutrons and are produced in the decay of several fission product elements. Among the group of fission elements which decay with the release of a neutron are 22 sec I-137, 56 sec Br-87, 16 sec Br-88 and 4.5 sec Br-89. The delayed neutron counting apparatus is used in conjunction with a gamma detector such as a GM detector. A simultaneous increase in activity in both the BF_3 tubes, and GM tube indicates release of short lived fission products to the loop stream. Often the GM tube will indicate a rise of gamma activity with no increase in delayed neutron activity. This condition occurs when a piece of an activated corrosion product breaks loose from the sidewalls of the loop and passes by the detectors. Delayed neutron counting sensitivities are very dependent upon the time of counting in relation to the time at which coolant stream leaves the high flux zone in the reactor. Since the reaction $\text{O-17}(\text{n}, \text{p}) \text{N-17}$ occurs in the water

in the reactor and this 4 sec N-17 decays by neutron emission, it will contribute delayed neutron activity to the background if counted too soon. The ideal time delay for counting the cooling streams for delayed neutrons is about 55 sec after fission. Further, because the neutron counters are sensitive to the high levels of 7.4 sec N-16 gamma activity present in the water upon discharge from the reactor, the use of a delay before counting minimizes the N-16 gamma background correction.

7. Rupture Analysis

An example of a case where radiochemistry was used in helping to pinpoint the type of capsule which had ruptured occurred on November 8, 1959. At 0845 Health Physics reported a large increase in the alpha activity in the routine water sample. Also, it was noted that the gamma activity in the water had increased. From gamma spectra taken on a gross water sample the gamma activity appeared to be due to ^{233}Pa (large gamma photopeak at 0.319 Mev). This led to the suspicion that a thorium containing capsule was involved. An alpha sample was prepared and pulse analyzed. This data showed the presence of two alphas, one of 5.28 Mev energy and the other at 4.64 Mev energy. Chemical tests showed that the activity carried on LaF_3 precipitate could not be separated from the LaF_3 by oxidation. Since Pu should have separated under these conditions, the possibility of a Pu capsule being broken was eliminated. Another possibility, Cm-244, was ruled out since it emits a 5.80 Mev alpha and no alphas of this energy were observed. Further chemical separations using a small anion resin column separated the alpha activity into two fractions. From chemical behavior and alpha energies, the observed alpha activities were assigned to ^{230}Th (4.68 Mev alpha) and ^{232}U (5.32 Mev alpha). This and the presence of ^{233}Pa led to the conclusion that a capsule containing both ^{230}Th and ^{232}Th must be involved. Such a group of capsules were in the reactor at the time this incident occurred. After removal of the capsules from the reactor, visual examination of the capsules in the MTR canal revealed that a number of capsules had been seriously damaged by vertical movement in their irradiation holder. One capsule actually had a hole worn through its side allowing the contents to leak out. This is an actual case which required the correlation of gamma spectra, alpha spectra and radiochemistry to confirm the source of the trouble and to eliminate the many other possibilities which existed.

CHAPTER V

REACTOR METALLURGY

A. INTRODUCTION

As in most operating plants throughout the country, there are many and varied metallurgical problems which have been generated by nuclear reactors and their supporting facilities. For this reason no attempt will be made in this chapter to point out specific metallurgical problems, but instead an effort will be made to impress the reader with the fact that problems exist far beyond those normally encountered in ordinary industry.

B. RADIATION DAMAGE

An important aspect of metallurgy as related to nuclear reactors is the changes in physical properties of metals when they are exposed to radiation. These changes are commonly referred to as "radiation damage" and are of major importance in regard to fuel element materials, reactor structural components and in-pile experimental facilities.

There are existing differences of opinion as to the exact mechanism of these property changes. Before discussing these effects, it may be well to briefly outline the basic concepts which have been proposed to explain the various property changes ascribed to neutron irradiation.

1. Displacement

Within a metal assume a neutron with high kinetic energy collides with an atom nucleus, displacing the atom within the lattice. The results are an interstitial atom and a vacancy at the original atom site. The displacement results in a "keying" effect on the crystal lattice, thus changing the physical characteristics of the metal. This can be thought of as a small interlock between crystal faces. It is this that changes the ductility, strength, etc., of the metal.

2. Thermal Spiking

During irradiation intense local heating ($1300^{\circ}\text{--}2200^{\circ}\text{K}$) results from bombarding neutron energy being lost to atoms adjacent to

the particle path. This temperature is sufficiently high to melt and possibly vaporize the metal locally. Rapid conduction of heat from these spike areas results in a local distortion when solidification re-occurs. These local distortion areas also have a keying effect on the crystal lattice.

3. Impurity Atoms

If a bombarding neutron is captured by the struck atom, a new atomic element is formed. This new element, or "impurity" atom, acts as a secondary component metal and creates a local pocket of alloy. The effects due to formation of impurity atoms are thought to be less significant than those caused by displacement.

Physical property changes in metals resulting from irradiation are similar to those produced by conventional metallurgical methods. However, in extensively cold-worked metals, radiation may produce effects similar to an annealing heat treatment which results in greater ductility, a decrease in hardness and a lowering of yield and ultimate strengths. This of course indicates that radiation damage is not completely analogous to cold work. For the most part, the effects of radiation are more analogous to solute hardening but the magnitude of changes in properties is much greater.

The general property changes which are normally found in metals when exposed to irradiation are as follows: (1) increase in hardness; (2) decrease in elongation and ductility; (3) increase in yield strength; and (4) increase in ultimate strength. An example of magnitude is shown by results of tests conducted on Inconel-702 in the MTR reactor as shown in table 13. As indicated in table 13, changes in properties occur most rapidly during early exposure and more gradually during subsequent long term irradiation. This factor has been quite beneficial from a design and operating standpoint due to the long periods of time required to obtain high net values on test specimens.

Table 13

Test Results for Irradiation of Inconel-702 in MTR Reactor

Nvt > 1 Mev	Rockwell Hardness	Tensile Strength, psi		% Elong. In 1 Inch
		Ultimate	Yield, 0.2% Offset	
0	B-82	94,900	42,600	67
5.0×10^{19}	B-96	113,100	101,100	50
1.8×10^{20}	B-100	121,500	120,300	41
2.9×10^{20}	B-101	130,000	128,800	36

Another interesting factor in regard to induced radiation effects on metals is that these property changes can be removed in varying degrees by holding the specimen at elevated temperatures. This process is called annealing although temperatures are considerably below normal annealing temperatures. These changes in properties can most likely be attributed to the return of interstitial atoms to nearby vacancies.

It should be pointed out that no attempt has been made locally to differentiate between effects of fast neutrons and their disturbance of the original crystal lattice, and the effects of thermal neutrons and the gradual addition of impurity atoms to a metal with long time exposures.

There are, however, programs in progress which are aimed at determining effects of radiation on the properties of materials used in construction of the MTR, the ETR and experimental equipment installed in the reactors. These programs are designated as the PAED-3, the PAED-44 and the PAED-45 radiation damage or surveillance programs. Through these programs, limits have been established on allowable life for in-reactor components, and it has been possible, in the case of the MTR, to safely extend the service lifetime of some in-reactor tubes from about three months to three years. The fast flux in many ETR positions exceeds that of the MTR, and data are not yet available which can verify safe operation of high pressure loaded components, within the reactor, beyond a few months. Under the above programs such data are being obtained. The materials under test include all of the metals and alloys used in construction of the MTR and ETR and new high temperature, high strength alloys being considered for experimental loops.

C. STRUCTURAL COMPONENTS

Metal materials of primary concern are the reactor structural components, fuel element alloys and materials in use in experimental facilities. As shown by table 14, the reactor and associated facilities are for the most part constructed of stainless steel and aluminum.

The materials listed in table 14 were chosen from a corrosion standpoint with the following considerations as a guide.

1. Material must be satisfactory for long service to provide a maximum of reliability without periodic inspection.

2. Material must be essentially unattacked by primary coolant.

- a. Hazards are involved in complete failure of reactor components. To assure a minimum of corrosion, two major items must be kept in mind at all times:

- (1) Careful pH control or primary coolant is an important factor in all corrosion control.

- (2) Materials such as copper alloys should not come into contact with primary coolant. Heavy metal ions can result in severe corrosion, particularly of aluminum and beryllium.

- b. Corrosion products could lead to excessively high radioactivity which would present accessibility problems.

Table 14
Reactor Structural Components

Material	Components	Reactor
347 SS	Upper and lower reactor tank sections	MTR
	Top and bottom plugs	MTR
	Lifting pins, shaft locating lugs	MTR
	Grid plate	ETR
	Primary coolant system (partially)	MTR
304 SS	Primary coolant system (partially)	MTR
	Primary coolant system	ETR
	Grid plate supports	ETR
Boron SS	Control rod	ETR
304 SS Clad Carbon Steel	Reactor tank	ETR
1100 Alum. (2S)	Center section reactor tank	MTR
	Fuel element cladding	MTR
	Control rods	MTR-ETR
Aluminum plus Boron	Fuel element cladding	ETR
Aluminum plus Uranium	Fuel element core material	MTR-ETR
6061 Al (61S)	Diffuser	ETR
	Control rod guide tube	ETR
63S Aluminum	Reflector	ETR
356 Aluminum	Upper and lower grids	MTR
Beryllium	Reflector	MTR-ETR
Cadmium	Control rod	MTR
Nickel	Control rod	ETR

c. There is danger of preferential deposition on heat transfer surfaces or in high flux zones.

D. GAMMA HEATING

Another problem which is unique in dealing with reactors and metals is that of gamma heating. This is a phenomenon of heat generation within a metal when exposed to high gamma flux levels. Aside from the cooling problem involved, there is one of thermal stresses. These stresses must be added to normal pressure stresses which result from temperature differentials across the material. This is particularly a problem when dealing with experimental in-pile tubes which run through the core of the ETR.

It would be most desirable to meet standard pressure vessel codes with in-pile pressure systems, but this is impossible because of gamma heating. Since the temperature drop through the tube wall increases exponentially with the thickness, it becomes very important to keep the wall as thin as possible to minimize thermal stress. Walls of this type are unable to meet standard pressure vessel codes. At the present time the Bureau of Ships is building vessels under standards which make allowances for this thermal stress problem. Locally, we have requested thermal stress tests on high pressure loops which have been installed in the ETR. These tests have been conducted on mock-up designs and materials corresponding to core sections of loops. Most have been conducted on a cyclic basis to duplicate changes in loop and reactor operating conditions. There are loops operating in the ETR which have been accepted on the basis of out-of-pile thermal stress tests.

E. HOT CELLS

In order to determine the effects of irradiation upon materials of construction, fuel elements, etc, it is necessary to be able to examine them visually, physically and metallurgically. To accomplish this, heavily shielded enclosures are constructed and provided with remote control equipment. These shielded enclosures are commonly called "hot cells."

The MTR-ETR "hot cells" are located inside the building which is situated between the MTR and ETR reactor buildings. The building contains a working (or "clean") area, a storage (or "dirty") area, office facilities, a darkroom, a change room and three "hot cells."

Two of the "hot cells" are constructed with four foot thick, 3.2 density concrete walls, and one is constructed with 2 ft 9 in. walls of 2.3 density concrete. Two of the cells have five viewing windows and one has four. Remote handling is accomplished by means of General Mills mechanical arms, light cranes and Argonne-type master slave manipulators which operate in power driven telescoping shields. Individual ventilating systems are provided for each cell which maintain negative pressures within to eliminate the indoor escape of radioactive particles or gases. The cells are provided with charging holes to allow transfer of "hot" experiments from shielded carriers.

A wide variety of remotely operable equip-

ment is also provided to accomplish the many and varied jobs required within the cells. A partial equipment list includes such items as a vertical milling machine, power hacksaw, microhardness tester (Tukon), macrohardness tester (Rockwell), tensile machine, impact tester, balance for weight and density determination, apparatus for measuring electrical resistivity, cut-off wheel, specimen mounting press, rough and fine polishing equipment, ultrasonic cleaner, electrolytic and acid etching equipment, macrophotographic equipment, metallograph, micrometers, profilometer, annealing oven, lathe and periscopes. In addition to these items it is common practice to design new equipment as necessary to accomplish specific jobs.

CHAPTER VI

BASIC INSTRUMENTATION

A. INTRODUCTION

The field of instrumentation is utilized in almost all of the scientific fields. In fact, in the highly specialized sciences of today, some type of instrumentation must be used to obtain tangible results. However, the main purpose of this chapter is to acquaint the reader with the basic fundamentals of instrumentation that are commonly used in industry on process control and specifically those instruments that are used within the MTR-ETR area.

This chapter will deal with the most common basic methods of detection and measurement of the parameters encountered in process control. To review all of the variations and types of measurements, as well as the shaded areas of in-between types, would be a laborious and lengthy process. The reader is expected to have a working knowledge of the basic principles of physics and engineering. With that in mind, this chapter will serve as a review. It is not intended to serve as a complete text of the complex field of instrumentation, but only to serve as a reference guide. Sources for additional information can be obtained through the references at the end of this chapter.

B. TEMPERATURE MEASUREMENTS

1. Thermocouples

The first discovery of the phenomenon of thermo-electric currents was made by Seebeck in 1821. At this time, it was observed that if wires of copper and iron were fused together at one end and the fused end heated, an emf was generated and a current would flow from the copper wire to the iron wire at the heated junction. It was later discovered that this flow of current was due to two causes, the Peltier effect and the Thompson effect.

The Peltier effect involves the liberation or absorption of heat at the warm junction of two dissimilar metals when a current flows across it. The rate at which heat is absorbed or liberated is proportional to the amount of

current flowing across the junction, its direction and the metals involved. The potential difference generated by two dissimilar metals varies with the temperature at the junction. The change may not be uniform, however, as it is dependent on the type of metals used in the junction.

The Thompson effect involves the generation of electromotive force when a temperature differential exists in a homogeneous metal. In other words, a potential will exist between the ends of a homogeneous wire if one end is at a higher temperature than the other. The theory required to further explain this phenomenon satisfactorily is complex and not entirely in agreement with the experimental facts. For more detailed studies of thermocouple thermodynamics, see the references.

a. Necessary Characteristics

The following four characteristics are the most desirable and are incorporated into the design and use of thermocouples in industry: (1) the direction of the Thompson emf in the two wires must be such as to be additive in the circuit and should vary directly with temperature; (2) the Peltier emf at the "hot" junction must develop a potential having the same sign as that produced by the Thompson emf. The Peltier emf must also vary directly with temperature; (3) the thermo-electric power of the thermocouple should be as great as possible; and (4) the metals should be capable of resisting corrosion and oxidation at elevated temperatures. Unfortunately, no known metal or combination of metals will fulfill all of these requirements although some closely approximate them. All emf temperature curves deviate from a straight line to a certain extent, and empirical equations have been worked out to satisfy the experimental results for each particular kind of thermocouple.

It should be noted that in all thermocouple circuits there are two junctions: the heated or "hot" junction and the cool end or "cold" junction. The difference in temperatures between the "hot" and "cold" junctions of the

FIGURE 24
Schematic diagram of a
thermocouple circuit

thermocouple produces the emf. Therefore, it can easily be seen that a change in temperature at the "cold" junction would alter the emf developed as readily as a change in the temperature of the "hot" junction. (This can be seen by referring to figure 24.)

The Thompson effect causes a flow of current from negative to positive; the Peltier effect or emf reinforces this flow. The "cold" junctions of the thermocouple are located at R. A and B are copper lead lines connecting the thermocouple to a galvanometer.

It should be noted that the emf of the thermocouple is dependent only on the difference in temperature between the "hot" and "cold" junctions and is independent of temperatures which may exist along the wires, provided there is homogeneity in the metals of the thermocouple wire.

It is difficult to maintain a constant temperature at the "cold" junction of a thermocouple if it is located near the "hot" junction. It is therefore desirable to remove the "cold" junction to a point some distance away, preferably near or in the measuring instrument. In order that the temperature of the "cold" junction will remain constant, constant temperature boxes are employed in many of the plant areas around ETR. When this is done, the measuring instrument must be so compensated as to indicate only the "hot" junction temperature.

Considerable research has been done in an attempt to discover the combination of alloys that will have as many of the desirable thermocouple characteristics as possible. The types of metals and their ranges frequently used throughout industry are listed below in table 15.

The accuracy that may be expected from a commercial thermocouple will depend on the temperature being measured, and the type of metal used; but in general it is possible to obtain base-metal thermocouples with an accuracy of plus or minus 15°F over the usual working range, but when necessary, thermocouples can be supplied with an accuracy of plus or minus 5°F. Platinum, platinum-rhodium thermocouples can be obtained with even greater accuracy.

b. Thermocouple Wiring Arrangements

Although the circuits will vary widely depending on applications, a few basic circuits will be covered here. The indicating instrument may be a millivoltmeter, a potentiometer, an electronic temperature recorder (or controller) or any other device for detecting and indicating small dc emf's. The connecting leads may be copper, provided that both junctions of the leads to the thermocouples are at the same temperature. If this is not possible, and it usually isn't, it is desirable to use thermocouple extension wire. This wire is made of the same material as the thermocouple wire or of a material having the same temperature emf relationship. (See figure 25.)

Many times, it is desirable to amplify the very small emf's generated at a low temperature. To do this a multiple thermocouple can be used. The "hot" junctions of a number of thermocouples are placed close together. Each thermocouple is in series with the other, and their "cold" junctions are at the same temperatures. The effect is similar to placing batteries in series; the emf generated by one is added to that generated by the second, etc; the total emf is the sum of the emf generated by each thermocouple.

To obtain an average temperature reading in a furnace or vessel, the thermocouples may be connected in parallel. As in the case of batteries in parallel, the resultant voltage will be the average of the voltages in parallel.

Another common thermocouple circuit is that having a number of thermocouples recorded by a single instrument by use of a rotary switch. There may be as many as three dozen thermocouples in a single system. In this case a common "cold" junction box is used which is located near the thermocouple "hot" junction and is kept at a constant temperature,

Table 15

Alloy Combinations Thermocouple Characteristics

Positive	Negative	App. Max. Temp. °F.	ISA Symbol	MV Full Range at 32°F Ref.
Chromel	Alumel	2400	K	53.01
Iron	Constantan	1500	J	46.53
Copper	Constantan	700	T	19.1
90% Platinum } 10% Rhodium }	Platinum	3100	S	17.94
87% Platinum } 13% Rhodium }	Platinum	3000	R	19.39
Chromel	Constantan	1800

FIGURE 25
Thermocouples in series,
voltage additive

usually above ambient temperature. Copper lead wire is then run to a recording instrument which uses an automatic selector switch to select and read the value of each thermocouple in the system. This particular method is used at ETR where the recording is accomplished by the data handling system in the control room.

c. Recording Instruments

The null balance potentiometer is the type of recording instrument most frequently used to measure the emf generated by a thermocouple. It consists basically of a Wheatstone bridge circuit with the thermocouple or resistance bulb acting on the bridge through a balancing amplifier. The bridge is supplied

with a uniform voltage source. The emf developed by the thermocouple is compared to this standard known voltage and the difference is amplified. The output of the amplifier powers a motor driven slide wire in the bridge circuit to a value which balances the bridge to match the emf of the couple. A recording pen is generally attached to the slide wire and records on a chart temperature that corresponds to the emf of the couple. The emf will be dependent on the temperature and type of thermocouple material used. A potentiometer measuring circuit is particularly suited to thermocouples because the resistance of the thermocouple lead wire becomes part of the bridge circuit and is balanced out. Therefore, distance from the "hot" junction to the recording instrument has no effect on the temperature reading.

Potentiometer recording instruments will be covered in a later section.

2. Temperature Thermometer Bulbs

The resistance thermometer depends upon the inherent characteristics of metals which change their electrical resistance when they undergo a change in temperature. Although industrial resistance thermometer bulbs are usually made of platinum, copper or nickel, the temperature resistance phenomenon also

FIGURE 26
Temperature thermometer
bulb characteristics

applies to semi-conductors. It is quite possible that as improved semi-conductor materials are developed, they will find an increased use in resistance thermometry.

Basically, the change in resistance of a material which corresponds to a change in temperature is termed the temperature coefficient of resistance of that material. The coefficient is expressed as a change in resistance per ohm per degree of temperature at a specified temperature. For most metals, the temperature coefficient is positive, for pure metals, the change in resistance with temperature is practically linear over a portion of the resistance temperature curve. The practical range of temperature measurement is governed by the materials used in the primary windings of the resistance bulb. Typical curves for platinum, copper and nickel are given in figure 26.

The primary design considerations in construction of resistance bulbs are as follows: (1) shape, (2) size; (3) material used in resistance windings, (4) installation of the windings, and (5) the type of winding. The windings must be in good thermal contact with the inner surface of the housing so that proper heat transfer from the medium being measured will occur. This will assure an acceptable speed response. For obvious reasons, the windings must be electrically insulated from the housing, thus, one of the fundamental design problems is acceptable time response and that of achieving the desired electrical insulation with minimum thermal-insulation.

a. Accuracy

The accuracy of calibration is a prime consideration in selecting a resistance bulb for a given application. Most commercially

available resistance bulbs have a guaranteed accuracy of plus or minus 0.5°F over the usual working range. Laboratory thermometer bulbs with careful standardization, and through the use of correction tables, may be relied upon to be accurate within plus or minus 0.2°F . This is considerably better than thermocouple measurements.

b. Stability

For reliability, resistance bulbs must not change appreciably in resistance over a long period of time, even though they may be subject to frequent extremes of temperature within their intended working range. In the process of drawing wire and winding it into a helical coil, stresses are set up in the wire resulting in an unstable condition. Therefore, temperature stability can be obtained only by using the highest quality wire which has been properly aged after winding.

c. Thermal Response

This is especially important where the temperature of the medium being measured is subject to rapid changes. Obviously, the temperature response of any type of temperature sensing element will be a function of the mechanical design and thermal conductivity of the materials of which the components are constructed. If the medium is at rest, such as still air, a state of equilibrium will exist between the temperatures of the bulb and the film of air immediately surrounding the bulb. With agitation, the thermal conductivity will be increased greatly. The term "time constant" is frequently used when describing thermal-response characteristics of resistance bulbs. A common expression of time constant is the time in seconds required for a change in resistance which corresponds to 90 percent of the change in temperature.

d. Self-Heating Error

Since the operation of resistance bulbs depends upon flow of current through the resistance winding, there is power (I^2R drop) through the winding and leads which produces some heat. This is called self-heating error because it causes false changes in resistance which are not indicative of the temperature change that is being measured. The self-heating error will vary with different types of con-

struction and material; therefore, no general statement can be given as to the maximum recommended value of current that should be used. The self-heating error will also vary considerably with the medium being measured.

The two main advantages in using resistance bulbs to measure temperature are: (1) the high degree of accuracy attainable compared to thermocouples; and (2) they require no "cold" junction compensation.

The method of recording the temperature signal from a resistance bulb is similar to that used for the thermocouple.

C. PRESSURE MEASUREMENTS

Measurement of absolute pressure, gauge pressure, vacuum (draft pressure) and differential pressure may be accomplished by two primary types of elements. One is a liquid column where the density and height of liquid are utilized to measure pressure. The other is a formed pressure element.

1. Liquid Column Elements

One of the common methods of measuring low pressures is with liquid column elements. These operate on the principle of displacing the liquids down or up a tube. Low pressures use light liquids and pressure up to 30 psig can be measured with mercury. The common liquids used range from mercury through heavy salt and organic solutions to liquids lighter than water. In all liquids, the densities vary with temperature, so in precise measurements this must be taken into consideration. The organic liquids have the greatest coefficients of expansion. Tables are available for temperature correction.

In liquid column pressure measurement there are at least seven general types of instruments. Only three will be discussed here, the common U-Tube Manometer, Well Manometer, and the Incline Tube Manometer. The others are similar in principle.

a. U-Tube Manometer

In this instrument, liquid partially fills both legs of a glass U. Pressure applied to the top of one side will force liquid to depress in that side and rise by the same amount in the other side. This double displacement is,

of course, proportional to the pressure and the tube liquid and independent of the tube cross-section. The difference in height of the two columns is what is used in calculation of pressure. For static balance the equation is:

$$P_2 - P_1 = dh$$

where

P_2 and P_1 are the pressure on the two columns, d is the density of the liquid and h is the differential height of the two columns in inches.

The U-tube manometer principle is used in industry in an instrument which has its tube legs expanded into chambers. A float rides on top of the liquid and transmits its position outside through a pressure tight bearing or torque tube. This arrangement is usually called a differential pressure manometer. An example of this type of instrument is the main coolant flow transmitter used at ETR.

b. Well Manometer

A second application of the liquid filled tube in industry is the well manometer. This unit has only a single column with its base in a liquid filled well. Pressure on the well surface forces liquids up the tube and the rise is indicative of the pressure applied.

c. Incline Tube Manometer

This device is an adaptation of the well type instrument wherein the tube is inclined at some angle so as to obtain greater sensitivity and increase the readability. Figure 27 is an illustration of an inclined manometer accompanied by interpretive equations.

FIGURE 27
Inclined tube manometer diagram

The following height-pressure relationships are commonly used in calculations: 27.7 in. of water = one psig, 2.05 in. of mercury = 1 psig, and one inch of water is equal to 0.0739 of mercury.

2. Mechanical-Pressure Elements

There are three main types of elastic pressure elements: (1) diaphragm; (2) bellows; and (3) bourdon tube. Basically, they are designed to follow the physical law that within the elastic limit, stress is proportional to strain (Hooks Law). That is, deflection is proportional to the pressure applied.

a. Metallic Diaphragms

A metallic diaphragm element is primarily a device for measuring relatively wide pressure ranges, minimum range 0 to 0.2 in. of water, maximum range 0 to 400 psig. It consists of one or several capsules rigidly connected together, so that upon pressure each capsule deflects. Metallic diaphragms are used in many industrial pressure measuring instruments such as differential pressure cells and pressure gauges.

b. Bellows Elements

The bellows element is a one-piece expandable and collapsible element. It is usually formed in one continuous operation from a thin seamless tube into a deeply folded or corrugated seamless unit by either a hydraulic or mechanical method. The bellows element is usually used in applications where a little wider and higher range is required. Minimum range 0 to 5 in. H₂O, maximum range 0 to 800 psig. The bellows can be used for applications similar to those for diaphragm elements, such as absolute-pressure gauges and differential-pressure gauges. The most common use in this area is in pneumatic miniature recorders and pressure switches.

c. Bourdon-Spring Elements

The bourdon tube can be described as a flattened tube bent into an arc and closed at one end. When pressure is applied at the open and stationary end, it tends to cause the flattened tube to become circular. This in turn tends to straighten out the tube and in so doing moves the free end an amount proportional to the pressure applied.

The bourdon tube can be used in many pressure measuring instruments that require a wide range at high pressure. The minimum range is 0 to 12 psig, and the maximum range is 0 to 100,000 psig. Many combinations of the bourdon principle are used in industrial instrumentation. They will most commonly be seen in high pressure gauges and high pressure switches where the movement of the free end activates a microswitch.

d. Electrical-Pressure Transducers

Although available in a large variety of types, the primary sensing element of an electrical pressure transducer generally takes the form of a (1) bourdon; (2) spiral; (3) helix; (4) diaphragm; (5) bellows; or (6) a modification of one of these.

Since the output of transducers is electrical, classification by the different electrical parameters offers a logical number of classes. Principle among them are: (1) strain; (2) resistive; (3) magnetic; (4) capacitive; (5) piezoelectric; and (6) oscillometric.

To cover in detail all the modifications possible within each class is beyond the scope of this chapter. If further information is required, use the references at the end of this chapter.

3. Strain Gauges

When a wire is stretched elastically, its length and diameter are altered, resulting in a change in its electrical resistance. Measurement of this change of resistance is the principle upon which the wire strain gauge is based. For example, stretching a wire 0.1 percent diminishes its diameter (with constant volume) by 0.03 percent, in accordance with Poisson's ratio, and theoretically should produce a change in resistance of 0.17 percent. In actual practice, for reasons unknown, measured gauge factors differ from the theoretical value. Change in resistance of a strain gauge is considered in terms of "change in strain" and is expressed as a ratio called strain sensitivity or gauge factor.

$$\text{Gauge Factor} = \frac{\Delta R/R}{\Delta L/L}$$

where

ΔR represents the resistance change in total resistance R , ΔL represents the change of length in the total length L .

a. Bonded

At present two basic forms of wire-resistance strain gauges are widely used: (1) the bonded; and (2) the unbonded type. The bonded type usually takes the form of a flat grid of wire ~ 0.001 in. in diameter. This grid is attached to a carrier sheet which may be a thin sheet of ceramic paper or a sheet of paper impregnated with plastic. The strain gauge is then cemented to the surface of the strain member. The cement must be strong to hold the grid firmly against the surface so that compressive as well as tensile strain can be transmitted. For measurement of pressure many methods are employed. One manufacturer uses a pressure-sensitive tube with strain gauge grid bonded to the outside surface. The pressure medium being measured causes the tube to expand and stretch the gauge grid, thus increasing its resistance. The grids are connected as one leg of a bridge circuit with a small voltage applied to the bridge. As the resistance changes, the bridge is unbalanced by an amount proportional to the applied pressure.

b. Unbonded

The unbonded strain gauge consists of a stationary frame which supports a movable armature through thin cantilever plates. The strain sensitive wire is strung under initial tension as a filament between pins located on the frame and armature (see figure 28) in the form of a resistance bridge. When the armature is displaced longitudinally, the filament is elongated which changes its resistance and unbalances the bridge circuit. This in turn unbalances the measuring circuit an amount proportional to the applied pressure. The normal bridge resistance is from 60 to 5000 ohms with input voltages from 5 to 60 volts ac or dc. These produce outputs of 6 to 175 mv full range, open circuit. The output voltage is directly proportional to the amount of movement of the armature which will be connected to almost any of the existing pressure sensing elements. Which one is determined by the range desired. A typical example is the Statham strain gauge transducer used to measure differential pressure across the ETR Reactor. The usual measuring system for strain gauges is the Wheatstone bridge circuit, as noted above.

Transducers that use resistive elements include the carbon pile, the moving-contact element and the temperature-resistance element (such as the heated wire). These types

FIGURE 28
Unbonded strain gauge diagram

are not widely used in this area at the present time. However, the need of such devices is increasing so we should expect to be using many variations of these basic principles in the future.

Magnetic pressure transducers employ the bourdon tube, diaphragm or bellows to change the inductance or reluctance in a magnetic circuit.

Piezoelectric pressure transducers use a crystal under strain to change the electrical properties of a measuring electrical circuit. Further information can be obtained on transducers in the references at the end of this chapter.

D. FLOW MEASUREMENTS

1. Head Flow Meters

The head meter is a subdivision of the more general classification of rate meter, which is distinguished by measuring the flow rate without dividing the fluid into isolated quantities. The head meter is further identified by the fact that its primary device creates a difference of pressure (differential head) which depends on the velocity and density of the fluid. The complete head meter consists of the primary element which produces the differential head, and the secondary element which measures the differential head. Many different devices have been used to create the differential head, but the most common primary elements are the sharp, square-edged, thin-plate orifice; the flow nozzle; and the venturi tube. Pitot tubes and other devices employing impact or suction effects and devices employing centrifugal force or frictional resistance are also used.

a. Theory of Head Flow Meters

Head flow meters operate on the principle of converting energy from one form to another. In the case of liquid flows only two forms are considered, namely, kinetic energy and energy due to static pressure. Orifice meters, venturi meters and similar types depend on the conversion of static pressure energy into the kinetic energy of velocity. In the Pitot tube conversion is from kinetic energy to pressure at the impact tube.

In the case of gas or vapor a third form of energy is involved in the interchange,

namely, the internal energy of the compressed gas. A close approach to an exact mathematical analysis, based on the laws of thermodynamics, may be made for venturi tubes and flow nozzles; for orifice plates and other primary devices in which the throat or jet area is indefinite, very little is gained over use of the hydraulic formula with experimentally determined correction factors.

In considering primary devices, it is apparent that the basic difference between a flow nozzle and a thin-plate, sharp-edged orifice is that the effective area of the stream at the down-stream tap of the orifice plate is not that of the bore of the primary device.

b. Connections

It is desirable for several reasons to locate the pressure taps at points where the change in pressure is small: (1) because there is less turbulence there; (2) because the exact location and size of the tap hole are less important at the point of steady pressure.

(1) Vena-Contracta Taps

From the foregoing it can be concluded that vena-contracta taps (downstream tap at point of minimum pressure), as well as the type with taps 2-1/2 pipe diameters upstream and 8 pipe diameters downstream (downstream tap at point of full pressure recovery) are best. But this slight theoretical advantage is counterbalanced by many practical considerations. Certainly tap locations at points of extreme pressure instability (such as between 3/4 and 3 pipe diameters downstream) should be avoided, but the difference between the stability at the vena-contracta and at the downstream flange-tap locations is small.

(2) Flange Taps

Probably the most commonly used tap connections in the United States are flange taps. These have the following advantages. They are located close to the face of the flange where they are accessible for inspection. They are located by the manufacturer of the flange and thus are largely independent of the human element at installation time. They are symmetrical and hence adapt themselves to measurement of flow in either direction. They also have been the subject

of a great deal of research to establish such factors as coefficients and installation limitations.

The fact that flange taps are not geometrically similar in the various pipe sizes is of little importance, since coefficients have been determined for all commonly used pipe diameters. Flange taps should not be used with high diameter ratios in sizes less than 2 in., because the downstream tap falls in a highly unstable pressure region.

2. Head-Flow Primary Metering Devices

a. Orifice Plate

Perhaps the most important characteristic of the orifice plate favoring its choice as a primary device for head meters is its ease of installation and replacement. The economy of manufacture and ease with which it may be reproduced are other important considerations. Because of its many advantages it has been the subject of a great deal of research work, and there is probably a greater mass of empirical test data and knowledge of performance for the orifice plate than for any other head-flow meter primary device. From the analysis of test results it would appear that, when used on clean fluids, its performance is at least as reproducible as that of any other device used to induce a differential head.

If the downstream pressure tap is located within a half pipe diameter or less of the orifice plate, a considerable portion of the measured pressure drop is recovered, particularly if the ratio of orifice diameter to pipe diameter is high. For the best results the piping upstream for 8 pipe diameters must be straight to prevent unnecessary turbulence at the orifice plate.

b. Venturi Tube

In the venturi tube the ultimate in pressure recovery is attained by the long cone exit, commonly designated in this country as the Herschel design and in Europe as the classical venturi tube design. This implies ~ a 10:1 taper or a 20:1 slope of the profile of the cone with respect to the axis of the venturi. A greater or lesser slope results in a greater percentage of pressure loss.

Since part of the pressure is recovered in the region of high velocity, a large part of the

possible pressure recovery can be attained by using a truncated recovery cone having an outlet diameter somewhat smaller than the inlet to the venturi. The shape or length of the outlet section has no effect on the coefficient of discharge of the tube.

In industrial flow measurement, quite frequently pressure-recovery characteristics are of no economic importance since any pressure that is recovered is dissipated in some form of regulation or control. It becomes important, however, when large volumes of fluid are being pumped at high velocities. In these cases power savings may in a short period of time pay the extra investment necessary for a venturi tube.

When power costs are not involved, the most common reason for choice of the venturi tube in preference to the orifice plate for industrial measurements is the better performance in the presence of solids in suspension in the flowing fluid. Suspensions very quickly deposit on the upstream side of an orifice plate and change its flow coefficient. The venturi tube, because of its streamlined entrance and exit, does not provide any dead pockets or impediments which permit these solids to settle out. Frequently, in the measurement of fluid catalysts, clay and oil mixtures, paper stock and similar fluids, single-pressure tap holes at the inlet and throat section are substituted for the usual piezometer rings to permit the solids to be purged back into the flow line by a continuous flow of clean purging liquid.

Either the flow nozzle or the venturi tube will permit accurate measurement of approximately two-thirds more fluid at the same differential head than the thin-plate, sharp-edged orifice of the same size. This is a frequent reason for the choice of the flow nozzle, especially on high-velocity steamlines.

c. Small Flows

The need for flow meters and flow controllers for pilot plants and plants manufacturing such materials as pharmaceuticals and rare chemicals, where extremely low flow rates are encountered, has aroused considerable interest in the investigation of small orifices and flow nozzles in recent years. Much of the work that has been done in the past on this subject is worthless because insufficient attention was paid to the ratio of thickness at the orifice edge to orifice diameter. Due to recent developments it is possible to measure low flow rates

by use of the small bore orifice plate. The theory involved will be covered in a later section.

d. Head-Flow Calculations

Experimentally determined factors must be used with basic equations to correct for the over simplification of theory and practice. This is especially true regarding nonideal fluids which are measured industrially. Working equations, which incorporate empirical constants, form the basis for practical calculations for flow rates with given orifice size or for orifice size with given flow conditions.

(1) Symbols

Following is a summary of equation symbols employed in flow metering and the common English units normally associated with the symbol.

- a = area of bore of primary device, sq in.
- A = area of pipe, sq in.
- C = coefficient of discharge
- d = diameter of venturi throat, nozzle throat or orifice, inches
- D = diameter of pipe at inlet tap, inches
- h = effective differential head, ft of flowing fluid
- K = discharge coefficient,

$$C / \sqrt{1 - (d/D)^4} = C \times 1 / \sqrt{1 - \beta^4}$$
- q = volume, ft³/sec
- w = rate of flow, lb/hr
 = diameter ratio (d/D)
 = specific weight, lb/cu ft

The basic equations for volume and weight rate of flow derived in terms of the discharge coefficient K are:

$$q = K a \sqrt{2gh}$$

$$w_s = K a \gamma \sqrt{2gh}$$

In terms of the coefficient of discharge $C = K \sqrt{1 - \beta^4}$, these equations are:

$$q = C a \sqrt{\frac{2gh}{1 - \beta^4}}$$

$$w = C a \sqrt{\frac{2gh}{1 - \beta^4}} (\gamma)$$

For further information regarding the mathematical analysis of flow see the references at the end of this chapter.

A useful generalization is that flow varies as the square root of the differential pressure across an orifice plate.

3. Head-Flow Measurement Instruments

a. Pneumatic

Pneumatic transmission of meter readings may be accomplished by either force-balance or position-balance systems. In the former, the conventional differential responsive member is replaced entirely by the transmitting mechanism.

The differential pressure acting directly against a flexible member (diaphragm, bellows, etc) creates a force which is conveyed to a force-balance mechanism, usually through a flexure tube or diaphragm, represented schematically by the pivot point. The force so conveyed is counterbalanced by pressure applied to a small bellows or other flexible capsule. This pressure is modulated by an air "feeler" mechanism usually consisting of a bleeder nozzle and a shut-off baffle, flapper or vane. When the baffle leaves the nozzle by a fraction of an inch, the pressure in the system is decreased. This pressure signal may be amplified through a relay to the bellows or diaphragm which furnishes the balancing force. Whenever the differential force changes, relative motion of the flapper and nozzle readjusts the pressure to restore the force balance. The regulated pressure is transmitted to a pressure-gauge receiver, which may record, indicate, control or integrate the flow. (See figure 29.)

On modern force-balance transmitters, the range span may be changed easily through a wide band to take care of change in range. When converted to use as a liquid level gauge for closed pressure-vessels, the wide adjustment of zero available on this instrument makes it possible to neutralize fixed liquid heads for a wide variety of vessel temperatures, seal liquids or tap locations, and thus place the operating span in the desired range. Modern designs also have over-range protection up to the full working pressure of the housing.

Since the motions involved in operating the force-balance mechanism are extremely small, careful manufacture and proper design are required to minimize changes in zero due to temperature and pressure distortion.

FIGURE 29
Pneumatic transducer functional diagram

b. Integral Differential Pressure

The integral differential pressure cell combines the small displacement characteristics of the force-balance instrument with an orifice in the differential pressure cell assembly, eliminating the troubles mentioned above. Standard orifices can be made in ranges from 8 down to 0.01 gpm of water or 32 down to 0.04 scfm of air at full scale.

c. Liquid Filled Manometers

Manometers, as discussed in the pressure measurement section, can also be used to indicate flow by reading the pressure drop across a flow device. Any of the various types may be used as long as their range is adequate.

It is sometimes required that a differential pressure be measured in a line of high pressure, a pressure above what the usual glass tube can stand. These line pressures may range as high as 5000 psig, but the differential pressure will be less than 30 psig so a mercury-filled manometer can span the range. The mercury is enclosed in a metal tube, and a float is moved by the changing mercury level. The main flow transmitter on the ETR primary coolant water is of this type.

The equation for balanced well-type manometers most often used in flow applications is:

$$P_2 - P_1 = \left(1 + \frac{A_2}{A_1}\right) d \gamma$$

where

P_2 = upstream pressure

P_1 = downstream pressure

γ = specific weight of mercury

A_2 = cross-sectional area of float chamber (high pressure leg)

A_1 = cross-sectional area of low pressure leg

d = float movement from zero position.

From this equation the fundamental characteristic of this type meter can be seen. That is, the float movement is directly proportional to the differential pressure. In order to obtain readings in terms of flow units, the recorder scale must be calibrated in square root units. Because of this, one should remember that an increase of two in flow requires an increase of four in the differential pressure. Range tubes are commonly designed to provide for increases in flow range in increments of about 25 percent.

d. Magnetic Flowmeters

The application of the principle of passing a magnetic flux through a conductive flowing fluid and measuring the generated voltage as a measure of rate of flow dates from the time of Faraday's work before 1832. It may seem

odd that a principle so simple was unused commercially until the last few decades, but the application of the principle was loaded with unexpected difficulties. The voltages generated with flux density obtainable at that time were so small that stray currents and external disturbances became serious drawbacks. On liquids of normal conductivity, the current flow through the electrodes had to be kept low enough to avoid loading down the voltage.

With the advent of liquid metal coolants to atomic piles, the principle of the electromagnetic meter found conditions to which it was ideally suited. The high conductivity of the liquid reduced the magnitude of the problem, and the unsuitability of conventional flow meters made the electromagnetic principle a good alternative. The development of this meter to a stage suitable for measuring liquids having only the conductivity of normal tap water, however, required further development, a high grade of engineering and an existing line of instruments suitable for measuring the signal. However, a model suitable for industrial use is now available. The simplified sketch in figure 30 omits the details necessary to obtain practical operation with fluids of low conductivity. For such service, a practical instrument uses a feedback to nullify the voltage generated by the flowing fluid. Any unbalanced voltage is amplified and used to reset the feed-back, the setting of which serves as a measure of the rate of flow.

The development of a magnetic meter design which would operate on fluids of low conductivities has opened up new possibilities for measuring flow with no pressure loss,

FIGURE 30
Magnetic flowmeter diagram

regardless of the viscosity, content of solid materials or size of conduit. Tubes have been made for sizes as small as 1/8 in. and as large as 72 in. Designs are available for much larger sizes. The principle is ideal for applications subject to sanitary regulations: ie, food products, pharmaceuticals; also for sewage, paper pulp, slurries and measurements made in radioactive "hot" areas.

The output of the magnetic tube is linearly proportional to the true weighted-average velocity for practically any flow pattern; hence, it is unaffected by Reynolds number or by swages and most other disturbing pipe fittings upstream from the point of measurement. Deposits on the wall of the tube do not affect the readings, provided they are of the same conductivity as the flowing fluid.

The magnetic meter may be made to perform any or all of the functions of the rate meter; that is, recording, integrating, telemetering or controlling. Magnetic flow meters are not, at the present time, used at the MTR-ETR. However, because of their future potential, the above details were given.

4. Late Developments in Flow Measurement

Some important advances in flow measurement have recently become commercially available.

a. The Dall Tube

This unit far outperforms the venturi tube in recovery of pressure. On high diameter ratios, the pressure loss may be as low as 2.5 percent of the measured pressure drop. The short length of the unit and its light weight make it easier to handle and to install than the venturi tube. Its use, however, should be confined to fluids which do not contain appreciable amounts of settleable solids.

b. The Target Meter

This unit, shown in cross-section in figure 31, is an application of the principle of the annular orifice. The target disk located centrally in the flow conduit is supported by the force bar of a balance transmitter. This arrangement allows for free flow of sediment or heavy liquids at the bottom of the pipe and for vapors or light liquids at the top of the pipe.

FIGURE 31
Target meter diagram

The difference in pressure on the two sides of the disk produces a force which is counteracted by the force of the transmitted signal pressure acting on a small bellows. The two forces are maintained in null balance by the usual flapper-and-nozzle position sensing mechanism.

c. Continuous Pneumatic Integrator

This unit operates from the 3 to 15 psig signal from a pneumatic transmitter. By balancing the force of the differential pressure against the force of a centrifugal governor mounted on a turbine rotor, a linear relationship between the speed of the flowing liquid and the speed of the turbine is obtained, thus eliminating the need for a square root cam.

5. Area Flow Meters

The area flow meter operates on the same basic principle as the orifice or other differential head meters. In the orifice meter, there is a fixed aperture and flow is indicated by a differential pressure. In the area meter, there is a variable orifice and a relatively constant pressure drop. Thus, in the area meter, flow is indicated by the area of the annular opening through which the fluid must pass. (The so-called adjustable orifice meter is not an area meter because the change in area does not respond automatically to the flow rate,

but can be adjusted externally only to certain set values.)

a. Types

(1) Rotameters

In the usual rotameter a tapered plummet weight is contained in an upright tapered tube with its large end up. The plummet is lifted to a position of equilibrium by the upward force of the fluid flowing past it. The higher it is lifted, the greater the surrounding annulus becomes, thus more flow can be measured. In smaller sizes the tube is made of glass which is graduated so that the flow can be read directly by observing the position of the plummet.

The term rotameter was derived from the fact that plummets were originally produced with slots to give them rotation for the purpose of centering and stabilizing the float. The present trend, however, is toward guided non-rotating floats. The rotation of such floats is now interpreted as an indication of an unfavorable entrance condition.

They are available in a number of different designs, the simplest consisting of a plummet in a tapered graduated Pyrex glass tube suitable for moderate pressure as indicated in table 16.

Table 16

Approximate Safe Working Pressures for Heavy-Wall
Pyrosilicate Glass Rotameter Tubes

Connection Pipe Size, in	Approx. Max. Safe Pressure, psi	Connection Pipe Size, in	Approx. Max. Safe Pressure, psi
1/8	550	1	200
1/4	450	1-1/2	100
3/8	375	2-1/2	80
1/2	300	3	65
3/4	250	4	50

At one time all glass metering tubes were made in Europe (principally Germany) from tapered scrap ends of drawn tubing. In modern practice, they are formed on a mandrel and annealed to prevent internal stresses so that strong, uniform tubes result. The mandrel method permits the forming of tubes with greater accuracy and interchangeability. It also assists in forming special shapes, such as nonconical tubes with curved elements designed to spread out the graduations at the lower end of the range, and thus give better readability at low flow rates. It is possible to modify the conical form slightly in order to give the exact linear relationship between aperture and plummet position which is not quite achieved with a purely conical tube.

The most important special shape is a modified conical section having internal beading or lands which serve to guide the plummet, and also to permit direct observation of the plummet through the thin film of liquid between the plummet and the lands. This shape can be used even with fairly dark liquids. Such tubes are now obtainable and are offered by some manufacturers as replacements for conventional conical tubes where desired.

The manufacturer's recommendations regarding maximum safe working pressures should always be followed. However, table 16 gives the parameters for heavy-wall metering tubes and can be used as a preliminary guide in tube selection. Improper handling or installation, or any of a number of other factors involving mechanical shock, may of course, cause breakage regardless of the working pressure.

Plummetts or floats can be made from many different materials for the purpose of

obtaining corrosion resistance or capacity modification. Ratings are usually stated in terms of meter capacity, using a stainless steel float, and if the only alteration in a meter is in the float material with the float dimensions remaining exactly constant, a correction factor based upon available equations will give reliable results.

(2) Piston-Type Meters

In these meters, the piston is accurately fitted inside a sleeve and is lifted by fluid pressure until sufficient port area in the sleeve is uncovered to permit an equilibrium passage of the flow. The flow is indicated by the position of the piston.

Both types of area meters are suitable for remote indication where electric, electronic or pneumatic transmitters are used.

(3) Orifice and Plug Type

This meter has some features of both of the foregoing types. It makes use of a tapered plunger inserted into a fixed orifice. The plunger is lifted axially by the fluid until the annular space between the orifice and plunger can handle the existing flow.

b. Performance Characteristics

(1) Linearity

The volume of flow through an area meter is essentially proportional to the area, and as a result, most of these meters have equal scale increments.

By comparison, the orifice used in conjunction with a simple manometer has a square root characteristic which is often unsatisfactory in the lower 20 percent of its range. The

area meter, therefore, is well adapted to the measurement of highly variable flow rates. Use of a 10:1 scale ratio is commonly accepted practice. The orifice meter, although available with a linear scale, must employ an intermediate device for extracting the square root of the differential head.

(2) Constant Differential

This is an important characteristic of the area meter, although a considerable increase in differential is observed as the flow approaches maximum. This increase in differential with increasing flow is due largely to frictional resistance within the meter body and varies from about 10 percent to more than 100 percent depending upon the size and type of meter relative to the flow rate. If this is an important consideration, the manufacturer should be consulted.

(3) Accuracy

Generally, the accuracy of area meters is within 2 percent at the maximum reading, but with individual calibration this accuracy can be increased considerably. There exists the possibility of mechanical resonance in the metering of gases. Unless damping is introduced, this results in excessive oscillation which makes the meter difficult to read.

(4) Other Characteristics

(a) Minimum Piping Requirements

An area meter usually can be installed without regard to the fittings or length of straight pipe preceding or following the meter.

(b) Corrosive or Difficult Liquids

These can often be handled successfully in an area meter. They include such materials as oil, tar, refrigerants, sulfuric acid, beverages and molten sulfur. In general, if the nature of the fluid does not permit the use of a conventional differential-pressure-type meter because the fluid is dirty, viscous or corrosive, certain area meters have an advantage over other types of meters.

(c) Small Sizes

Very small size area meters are obtainable which are suitable for the flow ranges below

those for which an orifice meter would be practical. Flow rates as low as 0.08 cc/min (water) can be measured by at least one area-meter manufacturer.

E. LIQUID LEVEL MEASUREMENT

The measurement of liquid level is one of the three fundamental measurements most frequently required in a chemical process, the other two being those of temperature and flow. Measurement of the liquid in a vessel may be made simply as a check on the supply of liquid to be delivered to a batch process, or as the primary measurement in a control system designed to maintain a certain level in a vessel that is part of a continuous process.

One of the first considerations in the problem of measuring liquid level is to determine the shape of the vessel containing the liquid. It is evident that the degree of accuracy with which the volume of liquid in a vessel may be determined by means of a liquid-level measurement is dependent in part on the shape of the vessel, since a tall vessel of small diameter can be more accurately measured than a flat vessel of large diameter. On the other hand where the liquid-level indicator is to be used to control the level in a vessel, it is usually more desirable to use a vessel that is low and large in diameter or long and cylindrical in shape and set in a horizontal position. This will give capacity to the controlled system. The shape of the vessel not only will affect the sensitivity of the liquid-level instrument as a volume-measuring device but will also dictate the type of instrument that is most desirable for the application.

The instruments used for measuring liquid level may be grouped as follows: (1) float-actuated instruments; (2) electrical-contact types; (3) static-pressure types; and (4) pressure-differential types. Examples of these will now be studied in some detail.

1. Visual Indicators

The simplest and most widely used method of measuring liquid is by means of a visual float-and-chain arrangement. These devices are cheap and easily maintained and will give reasonably accurate readings.

The sight glass consists of a glass tube mounted on the side of the vessel and connected

to it by means of pipes at the top and bottom. The liquid rises in the glass tube to the same level as that in the tank, and a graduated scale behind the glass permits the level to be read directly in feet, inches, percentage of full capacity or any other applicable units. This instrument is most frequently used on small tanks or where the liquid-level variations are comparatively small, since the length of the glass tube rarely exceeds 2 or 3 ft. Where large changes in level must be measured, a series of short tubes are usually mounted in such a manner as to encompass the range of the liquid level, with liquid being visible in one of the tubes at all times.

The sight glass is applicable only where the liquid is fluid at the ambient temperature and where it is not opaque and sticky enough to obscure the view of the liquid level. It can be used for the highest pressures if constructed with a steel chamber having a thick-glass opening at the front and back. This design is used for high-pressure boiler-drum, water-level indicators.

2. Float-Actuated Instruments

If a record of the liquid-level movement in a vessel over a period of time is desired, it is possible to operate a standard circular-chart recording mechanism either with a float and chain or by means of a float and lever arm. In chemical processing a record of this type is a valuable check on the operation of either a batch or a continuous system. A float and counterweight are connected by a cable which passes over a wheel mounted on the side of the recorder. This wheel has a pinion gear mounted on its shaft which meshes with a gear rack carrying the recording pen. As the wheel rotates, the pen moves across the chart face. The range of the instrument is determined by the ratio of the diameter of the cable wheel to the pitch diameter of the pinion gear. If a recording of the liquid level is needed over a long period of time, the instrument can be built with a strip chart which will give an unbroken record of the level over a period as long as 45 days.

3. Electrical-Contact Indicators

A third method of obtaining an indication of liquid level is by means of a series of electrodes fitted into the vessel in such a way as to close a separate electrical circuit

when the liquid rises to the height of each succeeding electrode. The circuits may be used to light a series of electric lights or any other mechanism that may be needed to control the liquid level. They are operated on a low voltage to eliminate danger to operators and to prevent arcing of the contacts which would destroy the electrodes in time.

This system has the advantage of being able to transmit the liquid-level indication to any desired location; it also can be used on closed vessels under pressure without the use of a pressure-tight shaft and bearing. However, it is limited by the fact that a continuous indication of level is not obtained unless a very large number of electrodes are used. Furthermore, it cannot be used where there is any possibility of an explosive atmosphere above the liquid level in the vessel.

A recording liquid-level indicator can be built using a suspended electrode in place of the float and cable previously described. In this case the recording mechanism is driven by a small reversible motor controlled by an electrode in contact with the liquid and suspended from an electric cable wound on a drum. The motor also drives the drum and is arranged to run in one direction while the electrode is in contact with the liquid and to reverse when the contact is broken. As a result, the motor "hunts" about the point of contact of the electrode and the liquid, and the pen arm driven by the drum records the correct liquid level. This method eliminates the friction inherent in the simple float-and-cable recorder and also eliminates any error due to changes in the weight of the float caused by material building up on the top. Likewise, no error results from the unbalanced weight of the cable.

4. Static-Pressure-Operated Indicators

The complications involved in maintaining the float type of liquid-level indicator have resulted in widespread use of the static-pressure method of measuring liquid level. Since the static pressure at the bottom of a vessel will vary directly with the head of the liquid level above it, all that is necessary to obtain an indication of the liquid level in a tank is to measure the static pressure existing at the base-point and to calibrate the measuring instrument in terms of the feet or inches of liquid represented by each increment of static-pressure. Several instruments working on this principle will now be described.

5. Manometer Type Indicator

If a simple U type or well type mercury manometer, as previously discussed, is connected to the base of a vessel filled with liquid, the pressure of the liquid in the vessel will produce a difference in level between the two legs of the manometer. In this case the ratio of the height of the liquid in the tank to the height of the mercury in the manometer will vary inversely as the ratio of the specific gravities of the liquid and the mercury.

The manometer connecting pipe is usually attached to the vessel about 2 in. above the bottom in order to avoid the trapping of sediment in the manometer body. If this is not done, the instrument may need frequent cleaning in order to maintain its accuracy, particularly if it is located below the tank bottom. In this case the sediment or dirt will build up in the connecting pipe and change the calibration.

6. Diaphragm Static-Pressure Indicator

It should be noted that the manometer type liquid-level indicator previously described must be located at or below the base of the vessel containing the liquid being measured. Often it is inconvenient if not impossible to install it at these points; where this is the case, a pressure bulb may be lowered into the vessel to measure the static pressure.

This type of liquid-level recorder is built with a metal cup ~ 6 in. in diameter connected to the end of a capillary tube, the other end of which is attached to a standard recording or indicating pressure gauge. Fastened to the bottom of the metal cup is a rubber diaphragm which traps a volume of air in the sealed system comprised of the diaphragm cup, the capillary tube and the bourdon-tube or bellow mechanism in the recorder. The pressure bulb or cup is installed at the bottom of the tank, and the capillary tubing run to a convenient location off the recorder. The diaphragm is sufficiently flexible to permit the air pressure inside the cup and the liquid pressure outside it to equalize without distortion. Therefore, the recorder will indicate the static pressure at the bottom of the tank, and the chart or dial may be calibrated directly in terms of liquid depth.

The pressure bulb is usually set in a bracket at the bottom of the vessel and may

be pulled up out of the liquid for inspection or repair at any time. It is also possible to install the pressure bulb in a small chamber located on the side of the vessel and connect it to a short pipe connection when it is inconvenient to run the capillary line inside. Calibration is achieved by applying a known pressure to the instrument system prior to installing the rubber diaphragm. The instrument may then be checked by immersing the bulb a certain depth in the liquid and observing the reading of the pen arm or dial pointer. The greatest care must be exercised to prevent any leak in the diaphragm or tubing, since this will destroy the calibration. Care must also be taken to be sure that the pressure bulb is not immersed in a liquid that will destroy the diaphragm material.

7. Air-Bubble Indicator

Because of the possibility of losing the air in the airtrap type pressure bulb as a result of absorption by the liquid or by condensate forming in the tubing, it is often desirable to use an auxiliary air supply to assure that the liquid-air surface is at the base of the tank. In this arrangement air is forced into the manometer or recording instrument at a point along the tubing leading from the instrument to the pressure bulb, which is of the open type without diaphragm. This air bubbles out of the pressure bulb and prevents the liquid from rising in the tube. Regardless of the pressure existing at the point of air supply, the pressure at the instrument will be that caused by the static pressure of the liquid, since if the air pressure exceeds this value, it will escape from the system and bubble to the surface of the vessel. This system permits the indicator or recorder to be located at a considerable distance from the vessel. In the case of the simple air-trap instrument or the instrument using the diaphragm-sealed pressure bulb, it is necessary to build the air trap large enough to provide sufficient air to prevent the tube system from filling with liquid at the maximum static pressure. Naturally, if the connecting tubing is long and the static pressure high, the tubing must be small in diameter to reduce its capacity. This in turn multiplies the danger of the tube system being clogged. With the air-bubble system the connecting tube may be larger, and the escaping air will tend to keep it clean.

8. Pressure-Differential Indicator

In most cases where the liquid-level measurement is made primarily to give direction

to a controlling mechanism, the range of liquid-level movement is comparatively small. When a liquid-level water-pressure head of 200 in. or less is developed in the vessel, it is possible to use a standard recording or indicating flow-meter manometer to measure the liquid level. This is advantageous not only because it permits the use of an instrument that is produced in large quantities and consequently at a low price, but also because these instruments are available for any working pressure. The fact that a flow-meter manometer can be used is important in the case of liquid-level control in distillation processes operating under high temperatures and pressures.

F. INDICATING AND RECORDING INSTRUMENTS

Because of the magnitude and variations of the measuring instruments in use today, it would be impractical to go into anything but basic fundamentals of the three major types of measuring instruments.

The three main parameters of interest in industry are temperature, pressure and flow. Only the fundamentals involved in measuring these three parameters will be investigated here. For further information refer to any of the source books referenced at the end of this chapter.

1. Temperature Instruments

The "null" or "balance" type potentiometer is becoming more widely used in industry as a means of measuring the emf of thermocouples. There are three advantages of the potentiometer pyrometer. (1) the accuracy of the calibration does not depend on the constancy of jewel bearings, springs, the level of the instrument, etc; (2) the instrument is more easily designed as a recording instrument; and (3) the accuracy of the instrument is practically independent of variations in the resistance of the thermocouple or lead lines. All these reasons are important, but probably the most important is the fact that the measurements are unaffected by variations in the external resistance of the circuit. The fundamental theory of the potentiometer circuit will clearly show the inherent advantage of the potentiometer as a thermoelectric measuring device.

a. Potentiometer Circuit

A null balance type of potentiometer may be defined as a device for measuring a potential

difference (such as that developed by a thermocouple). The potential difference is placed in the circuit in such a way that it is applied in opposition to a known emf developed by a standard cell. The fundamental potentiometer circuit in its simplest form (shown in figure 32) consists of a slide-wire AB with a contact C which moves freely along the wire. In most potentiometers this contact "C" is driven by a split phase motor.

A battery "M" supplies the constant potential drop along the slide-wire from A to B. The unknown emf developed by the thermocouple is connected, as shown, with an amplifier in series with it and attached to the slide-wire circuit in such a way as to have the potential oppose that of the battery. If the potential drop of the slide-wire is greater than the thermocouple emf, the amplifier will drive the motor which will position the contact "C" of the slide-wire to develop a potential drop equal and opposite to that developed by the thermocouple which will bring the amplifier input to zero and the motor will stop. The contact "C" is also connected to a pen or pointer which will indicate on a scale or chart the equivalent temperature for the type of thermocouple used.

The mathematical equation that satisfies these conductors can be found in the reference section. It will be noted that the resistance of the thermocouple lead wire does not affect the reading of the circuit as the difference in emf's is the value measured. This accounts for the inherent accuracy of this type of pyrometer, since it will be remembered that it is the emf of the thermocouple that varies with temperature.

The null balance potentiometer was described because of its wide use in the MTR-ETR area. Other types of temperature measuring instruments are also employed in the area, each type depending on the primary signal device used.

2. Pressure Instruments

Measuring instruments that involve the recording or indication of pressure are dependent on the type of primary element used. The types of elements that can be used have been described in the section on pressure. Instrumentation to be used with them can be separated into two main sections: electrical and pneumatic.

FIGURE 32
Potentiometer circuit diagram

a. Electrical

Measuring instruments that record pressure electrically usually involve the use of a strain gauge primary element, although there are other types. The pressure being measured causes a proportional electrical variation. The change is of low magnitude and can be adapted to the same type of electrical null balance potentiometer as used in thermocouple measurements.

b. Pneumatic

Measuring instruments that record pressure pneumatically use primary elements such as described in the section on pressure measurements. For the most part, pneumatic recorders use metallic bellows as secondary elements. The bellows are rigidly fixed at one end to the instrument case, and the other end is attached to a pen. As the pressure in the bellows increases, the free end expands and moves the linkage, which positions a pen or pointer to indicate on a scale calibrated for the range of the input pressure signal. The use of bellows as a secondary element limits the range of the recorder to the limits of the bellows. Other type elements are employed to increase the pressure range.

3. Flow Instruments

Measuring instruments that indicate flow are classified into the same two types that pressure instruments are: electric and pneumatic.

a. Electrical Type

The electric recorder is similar to the thermocouple potentiometer in that it records a small millivolt signal from an electrical pressure transducer which is connected to a flow primary element. A recorder connected to the output of a magnetic flow meter can use a low emf signal and works on the same principle as the other potentiometers described above.

b. Pneumatic Type

The pneumatic flow recorder is almost identical to that used in the recording of pressure. In actual practice they are the same except for the type of chart or indicating scale connected to the instrument. A pneumatic signal proportional to the differential pressure across a flow primary element (such as a sharp edge orifice) is transmitted to the recording instrument. Since the flow is proportional to the square root of the

FIGURE 33
Simplified heat exchanger diagram

differential pressure across an orifice plate, the scale must be a square root scale.

c. Other Types

There are other types of flow measuring instruments. Among the most frequently used in this area is the manometer that takes the differential pressure directly into the instrument. The signal pressure in the mercury chamber moves a float arrangement attached to a mechanical linkage which positions a pen on a square root scale proportional to the differential head. There are also types of instruments that extract the square root of the input signal and the pen reads on a linear chart or scale. The square root extraction is accomplished by use of a special mechanical cam. Except for the use of the cam the measuring instrument will be the same as the other type.

G. AUTOMATIC PROCESS CONTROL

The subject of automatic control is relatively easy to comprehend. In many ways, automatic is much like manual control; however, the automatic controller does not necessarily duplicate what the human operator does by hand. Automatic controllers can compute and remember, but they cannot reason from new conditions, nor can they forecast beyond the data which are built into them. Thus, these are basic differences between human and automatic control means—factors which govern automatic controller design and use.

1. Typical Process

To illustrate this clearly, the simple heat-exchange process of figure 33 is used.

The term "process" as used here, signifies the function and operations used in the treatment of material. In figure 33 for example, the operation of adding heat energy to water is a process. The steam coils, tank, pipes and valves comprise the plant within which the heating process is accomplished. The parts and characteristic behavior of this typical process will now be analyzed to see what effect these factors have on the controllability of the process. Although simple, this example serves as a good fundamental starting point in the development of automatic control principles.

2. Function of Automatic Control

Most processes, whether controlled manually or automatically, will perform well and efficiently only when certain variables are held within given limits. These process variables will change unless energy (or material) "input" equals "output." Thus, the fundamental function of process control is to manipulate the energy (or material) input-to-output relationship so as to keep the process variables within desired limits.

An automatic controller can be defined as a mechanism that measures the value of a process variable and operates to limit the deviation of that variable from a desired value. A process variable that is held within

limits is termed a controlled variable. The automatic controller regulates the controlled variable by making corrections to another variable of the process which is termed the manipulated variable. In figure 33, the controlled or desired variable is a specific temperature of hot water. The manipulated variable is the rate of steam flow. Any change in steam valve opening as dictated by the automatic controller comprises a correction to the manipulated variable. Thus, it is possible to hold or change the output hot-water temperature by manipulating the balance of energy input to energy output.

3. Process Time Lags

At first glance control of the water temperature in figure 33 may seem quite easy. All that appears necessary is to observe the hot-water thermometer and to correct the steam valve opening so as to hold or change the water temperature to the desired value. However, all processes have a normal time delay in their response, and this characteristic greatly increases the difficulty of control. These delays and retardations generally are termed process time lags.

Time lags, in addition to inertia, are caused by three properties of the process, namely: (1) capacitance; (2) resistance; and (3) transportation time.

a. Capacity and Resistance

Those parts of a process which have the ability to store energy (or a quantity of material) are termed capacities. In figure 33 for example, the walls of the steam coils and the water in the tank can store heat energy. This energy-storing property retards change. For instance, if the incoming steam temperature increases, it requires some time for enough energy to be added to the water in the tank so as to bring it up to a new temperature.

Those parts of a process which resist transfer of energy (or material) between capacities are termed resistances. In figure 33, the walls of the steam coils and the insulating effect of the layers of steam and water on either side of them resist the transfer of heat energy between the steam in the coils and the water on the outside. The combined effect of supplying a capacity through a resistance produces time retardations in the transfer of energy between capacities. Such resistance-capacity (R-C) time retardations are often called capacity lags or transfer lags.

It is desirable to classify processes by the number and arrangement of their R-C pairs, since processes containing a similar number and arrangement of R-C pairs tend to exhibit similar behavior characteristics. In order to classify processes in this manner, it is necessary to assume that their resistance-capacities are separated into discrete units. This method of analyzing a process is called the lumped-capacity (lumped-parameter) method. In some processes, resistance and capacity are distributed. These are termed distributed-capacity (distributed-parameter) processes. A natural gas pipeline is an example where resistance and capacity are distributed over many miles of pipe.

b. Transportation Time

A third property found in processes which contributes to time lag is that time required to carry a change from one point to another point in the process. If the temperature (see figure 33) of the incoming cold water drops, some time will elapse before this colder water can travel through the tank and reach the thermometer bulb at point A. This time lag is not just a slowing down or retardation of the change, but it is a definite time delay (or dead time) during which no change whatever occurs. The length of this dead time depends both on the velocity and on the distance over which it is carried. Thus, dead time is also termed distance-velocity lag or transportation lag.

Figure 34 is a block diagram of the water heater shown in figure 33, with the capacities for heat storage, resistances to heat flow and dead time identified. This shows how the amount of dead time depends upon the distance and the velocity with which temperature changes are transmitted. For example, if the hot water flows with a velocity of 1 fps with the bulb at point "A" (2 ft from the tank), the dead time is 2 sec. But, if the water velocity is dropped to 0.5 fps the dead time becomes 4 sec to bulb "A" and 20 sec to bulb "B."

4. Problems of Control

a. Closed-Loop Control

A correction for a disturbance in the system cannot be made until the disturbance is detected, which means that all processes have time lags and that some time must pass before

FIGURE 34
Water heater block diagram

the effect of disturbance can be known. Also, sometime is required to measure the deviation and make the correction. Process time lags further require that time pass before the effect of the correction can be known. Meanwhile, the controlled variable continues to deviate from the desired value. In short the problem of control is to overcome the effect of time lags that occur around the closed loop of control.

b. Process Reaction Curves

(1) Single Capacity Processes

Processes dominated by one resistance-capacity (R-C) pair are easier to control because they begin to react immediately to load changes, so deviations can be known and corrections applied without delay (figure 35), and corrections are immediately effective in reducing deviations (figure 36).

FIGURE 35
Water temperature vs time,
single capacity process

(2) Multicapacity Processes

Processes having two or more resistance-capacity (R-C) pairs of comparable size are more difficult to control because they do not begin to react to load changes immediately. As a result, deviations do occur and corrections can be applied only after some time has passed. Also, corrections are not immediately effective in reducing deviations. Further, dead time (transportation lag) increases control difficulty wherever it is found.

FIGURE 36
Water temperature vs time,
multicapacity process

5. Basic Control Methods

In the manually controlled process the operator measures the output temperature, compares it to his desired value, computes

how much to open the steam valve and makes that correction to the steam supply. Thus, the functions performed by the operator in manual control are: (1) measurement; (2) comparison; (3) computation; and (4) correction.

These then are the basic control functions that must be provided in any automatic control system if it is to perform the function of the human operator.

a. Proportional-Speed Floating Method

In this method the position of the final control element (control valve) is changed at a rate which is proportional to deviation. In other words, the greater the deviation becomes, the faster the valve moves. For example, when the deviation is maximum, the "speed" of valve travel is also maximum. The factor of primary importance in connection with the proportional-speed floating method is that as long as deviation continues, the correction to valve position continues. Proportional-speed floating control responds to the amount of time duration of the deviation. Thus, this control continues to operate until it does produce an exact correction for any load change. This is a unique advantage of "floating mode" over any other method, and is why it is so often used to produce the popular proportional-plus-reset control.

b. Proportional-Position Method

In proportional-position method control (throttling or modulating mode), there is a continuous linear relation between the value of the controlled variable and the position of the final control element (within the proportional band). In other words the valve moves the same amount for each unit of deviation.

Figure 37 shows the relation between valve position and controlled variable characteristic of proportional-position control. Note that the valve position changes in exact proportion to deviation. Note that at 2 min and 4 min, when the temperature was returned to its initial value, the valve also returned to its initial value; in other words there was no valve correction without deviation.

(1) Proportional Band (Throttling Range)

This is the change in value of the controlled variable that is necessary to cause full travel

FIGURE 37
Valve position vs temperature
for two different proportional bands

of the final control element. Figure 37 shows the relation between valve position and temperature for two different proportional bands.

The proportional band of a particular instrument is usually expressed as a percent of its full range. For example, if the full range of the instrument is 200°F and it takes a 50°F change in temperature to cause full valve travel, the percent proportional band is 50°F in 200°F, or 25 percent. Figure 38 helps explain the concept of percent proportional band. Notice that proportional bands over 100 percent cannot cause full valve travel even for full-range change in controlled variable.

(2) Gain (Sensitivity)

Another concept for expressing proportionality is gain or sensitivity. These terms describe the ratio between the input and output to and from a control device. Mathematically, gain and sensitivity are reciprocal to proportional band. The proportionality adjustment of some control instruments is calibrated in gain or sensitivity instead of in proportional band.

(3) Offset (Droop)

It is fundamental that any change in the process load requires a new valve position

FIGURE 38
Percent proportional band graph

to correct for the change in the process load. But, as shown in figure 37, the proportional-position method mode requires a change in deviation in order to produce the new valve position. Figure 39 shows the load-temperature and valve position for the heat-exchanger process of figure 33. In the proportional-position method a step load change occurs. Note that the temperature is not returned to the desired value of 100°F by proportional-position. Note in figure 39 that the offset from time units of 3 to 8 min was 5°F.

(4) Manual Reset

This offset deviation can be eliminated by manually resetting the set point. In figure 39 at 8 min the set point was raised manually to bring the temperature to the desired value of 100°F.

It must be emphasized, however, that such manual reset eliminates offset only for a single value of load. The factor of primary importance in connection with proportional-position method is that it is powerfully stabilizing, capable of wide adjustment and application, but it has the undesirable characteristic of offset error.

c. Proportional-Plus-Reset Method

It has been noted that floating methods have the unique advantage of continuing to correct valve position until no deviation remains. Thus, by adding floating control to proportional-position control, the undesirable offset characteristic of proportional-position can be overcome. The addition of the floating method does automatically what manual reset does by hand. Therefore, this combined system is usually called automatic reset or simply "reset." The best way to explain the operation of combined control is to picture separately the components of valve motion due to each method and observe what each contributes to the resultant valve position.

FIGURE 39
Load-temperature and valve position
for the heat-exchanger process graph

FIGURE 40
Valve-position components
under actual process control graph

Figure 40 shows an analysis of valve-position components under actual process control. At time zero, a step increase in load occurs. Because of proportional-position control, a large valve correction is imposed (area C, figure 40) as the temperature deviates and later is withdrawn completely as the temperature is returned to the setpoint. But it should be noted that the final exact correction is applied wholly by the proportional-speed floating-mode component. The cross-hatched area A under the proportional component curve, represents the energy added by proportional-position mode. Area B represents the energy added by proportional-speed floating mode. Area C, under the resultant valve position, represents the correction in excess of that required which was applied and withdrawn by proportional-position control.

The facts of primary importance in connection with proportional-plus-reset control are that control without offset error is possible under all load conditions, but reset mode alone does not contribute to the stability of the control loop.

(1) Reset Rate

The reset-rate adjustment changes the rate of floating of the proportional-speed floating component. Reset rate usually is expressed in repeats per minute. This term denotes the

number of times per minute that valve motion, due to proportional-position mode, is repeated by proportional-speed floating mode. Figure 41 explains this open loop system graphically. At time zero, a step change in temperature occurs. The valve immediately changes position by amount A because of the proportional-position component. The valve also begins to move at a constant rate because of the proportional-speed floating mode. Figure 41 gives curves for the reset component at three different reset rates. The reset rate can be easily calculated. For example, in curve A, the proportional-position mode valve motion A was repeated by proportional-speed floating mode in 1/2 min. Thus, this reset rate is 2.0 repeats/min. In curve B valve motion A was repeated by proportional-speed floating component in 1 min. Thus, this reset rate is 1.0 repeats/min. Some industrial control instruments are calibrated in min/repeat which is the mathematical reciprocal of repeats/min.

d. Proportional-Plus-Rate Method (Derivative)

In this method there is a continuous linear relation between rate of change (that is, first derivative) of the controlled variable and position of the final control element. In other words the valve motion is proportional to the speed at which the temperature is changing; the faster the temperature changes, the more the

FIGURE 41
Open loop system functional graph

valve is moved. Again the easiest way to explain what proportional-plus-rate mode does is to picture separately the components of valve motion due to each as in figure 42.

Note that the size of rate-mode correction is proportional to the rate of change (slope) of the controlled-variable curve. When the variable is changing fastest (at time zero), the rate correction is largest. When the variable is reversed (time 0.4) its rate of change is zero, so the rate-component is again zero. When the variable is changing away from the setpoint, rate mode adds the energy represented by area A to oppose this change. When the variable is changing toward the setpoint, rate mode subtracts the energy represented by area B to oppose this change. Thus, rate mode has a great stabilizing effect on control. Notice, however, that after the variable stops changing (time 1.0) only valve correction due to proportional-position is left. Rate mode has no direct effect on offset error.

In summary then, by opposing all change, the rate method has a great stabilizing effect on control, but it does not eliminate the undesirable offset characteristic of the proportional-position control method.

e. Proportional-Plus-Reset-Plus-Rate Method

The three previously described methods of control can be combined in one instrument to obtain all their advantages. Figure 43 shows separately the components of valve motion produced by each method following a step change in load at time zero. Figure 43 also sums up the previous discussion.

(1) The proportional-position mode component corrects valve position by an amount proportional to the deviation, and thus produces a temporary increased energy input represented by area A.

(2) The proportional-speed floating-mode (reset) component corrects valve position as a rate proportional to deviation, and thus produces a permanent increased energy input represented by area D.

(3) The rate-mode component corrects valve position by an amount proportional to the rate of change of the controlled variable. Rate correction first added the energy represented by area B and then subtracted the energy represented by area C.

FIGURE 42
Proportional-Plus-Rate
functional graph

FIGURE 43
Proportional-Plus-Reset-Plus-Rate
functional graph

The resultant valve-position curve shows that, first, an excess correction is applied to oppose the change of the variable away from the desired value. This excess energy input is represented by area E. Then less than an exact correction is applied as the variable is returned to the desired value. This less-than-exact energy input is represented by area F. Note that ultimately neither proportional-position mode nor rate mode produced any correction. It was the proportional-speed floating mode only which produced the new

valve correction that exactly satisfied the new load condition.

H. CHEMICAL COMPOSITION MEASUREMENT

The process industries usually involve material operation that includes changes in physical state or chemical composition. The changes which occur in the physical or chemical state involve atomic and molecular interaction. It can be readily seen that the techniques and apparatus for the measurement of these microscopic variables should be a matter of great practical importance. The field of measuring the chemical composition of solution has a wide scope; one that involves too much for a detailed investigation at this time. Therefore, we shall limit our discussion to the area of the field that is most widely used throughout the MTR-ETR area.

1. pH Measuring Systems

One of the most important applications of solutions measurement involves the determination of the effective concentration of acids and bases. Because acids and bases are of vital significance in our water treatment, measurement of the concentration and strength of acids and bases should be understood.

All water solutions of acids and bases owe their chemical activity to their relative hydrogen (H^+) and hydroxyl (OH^-) ion concentration.

In water, the equilibrium product of the hydrogen-hydroxyl ion concentration is a constant 10^{-14} at 22°C , and the pH scale uniquely applies to water at this temperature.

The pH scale covers the range of both acid and alkaline solutions. Pure water is neither acidic nor basic. Acid solutions increase in strength as the pH value falls below seven. Alkaline solutions increase in strength as the pH rises above seven. The scale is not linear with concentration. A change in one unit of pH represents a ten fold change in the effective strength of acid or base. A solution of pH-3 is ten times as strong in acid concentration as a solution of pH-4.

It is important to recognize that pH measures only the concentration of hydrogen ions actually dissociated in a solution, and not the total acidity or alkalinity. It is this factor that is responsible for the pH change in pure water with a change in temperature. As the water temperature increases, the amount of disassociation increases and the quantity of hydrogen and hydroxyl ions is increased equally. Unless the relationship between disassociation constant and temperature is known, it is not possible to predict the pH of a solution at a temperature from a known pH reading at some other temperature.

The chemical equations involved in dissolved acids or alkaline solution will not be covered here. If additional information along this line is desired, refer to the "Process Instrument Handbook" listed in the references at the end of this chapter.

Two methods are in general use for the direct determination of pH in chemical systems; chemical indication and potentiometric instruments (pH meters).

a. Chemical

Chemical indication is a method that is most commonly used under lab conditions and involves the various colored indicators which change with the hydrogen ion concentration.

b. Potentiometric

The pH meter or "potentiometric instrument" consists of a pH responsive electrode such as a glass or antimony membrane; a reference electrode, usually calomel or silver-

silver chloride; and a potential measuring device, usually some form of vacuum tube voltmeter circuit.

(1) Glass Electrodes

The glass electrode or pH responsive electrode consists of a glass bulb, the lower end of which has a glass membrane as shown in figure 44. The voltage response of the glass electrode is due to change in the hydrogen ion concentration of the solution contacting the outside of the membrane. The voltage follows the equation $E = E_0 - 0.0591 \log H$ (at 25°C) where E_0 is the voltage of the particular glass electrode at pH zero. It is dependent upon the nature of the buffer solution in contact with the metallic electrode and one side of the glass membrane.

(2) Reference Electrode

Voltage measurement using pH sensitive electrodes must be made with a reference to some stable electrode which does not change with the pH of the sample. A calomel electrode is most generally used. The hydrogen electrode is classically the standard reference but it was found to be inconvenient for everyday use. Reference electrodes generally owe their stability to the use of a saturated solution of soluble salt which produces a constant ion concentration at a given temperature. The low ion concentration provided by the sparingly soluble salt does not permit any appreciable diffusion of test solution ions into the salt bridge through which contact is made with the second electrode. A saturated solution of KCL is generally used as the salt bridge to insure a stable Cl concentration in solution and therefore, a stable potential.

It should be kept in mind that when pH voltage is measured, the electro-chemical system includes the reference electrode, the salt bridge connecting this electrode to the sample solution, the glass-electrode membrane, the internal buffer solution and the internal electrode. The measured voltage is the algebraic sum of all the voltages existing across the system. This voltage is a reliable measurement of pH because chemical conditions are set which insure that all voltages due to pH difference (other than that across the glass membrane) are either negligibly small or remain constant.

To summarize, the voltage across the glass membrane is proportional to the change in pH. This voltage is sensed by the vacuum tube voltmeter circuit and is amplified and read out on the pH scale which is usually 0 to 14 with 7 at midscale. For further theory concerning the pH meter or measurement, check the references at the end of this chapter.

2. Electrical Conductivity Measurement

Industrial interest in the measurement of

electrolytic conductivity arises chiefly from its usefulness as a measure of ion concentration in water solutions. Also, the necessary equipment is simple and inexpensive. Pure water is a very poor conductor. The very purest water has a conductivity approaching the theoretical minimum of 1/20 micromho/cm at 25°C which is due to the dissociation products of water itself. The conductivity of water is almost exclusively due to dissolved salts rather than to water ions, and thus a

FIGURE 44
pH responsive electrode diagram

criterion for electrolytic concentration can be established.

In conductivity measurement, current is usually introduced and leaves the system through metallic electrodes where chemical reactions occur. The conductivity of a solution depends upon the concentration and mobility of all ions present; this depends upon ion size and charge, as well as the dielectric constant of the solution at its temperature. Positive ions (cations) will move toward the negative cathode where reduction takes place, and negative ions (anions) will move toward the positive anode where oxidation takes place.

Two electrodes are inserted into the solution with the leads connected to a bridge circuit which is powered with one to ten volts ac. The electrodes are usually made of platinum where the chemical reaction takes place. The output of the bridge circuit is read on a meter and is calibrated in micromhos which are the usual units for measuring conductivity.

I: INSTRUMENTATION FLOW SYMBOLS

Throughout the industry there is the general tendency for each manufacturer to establish a separate set of drawing symbols as their standard. This practice did not cause many large problems in the past, as it did not take long for company engineers to become familiar with the drawing symbols. As the exchange of information and subcontracting became more common, the difference in drawing symbols became more of a problem. Here at MTR-ETR, the problem becomes ever greater where we are involved with many different companies as experimenters.

The Instrument Society of America (ISA) has been aware of this growing problem and as a result has made an attempt to set up universal symbols as standards. It would be unreasonable to list all the symbols used by the industry. We have hope that sometime in the future the ISA symbols will be accepted as standard in all engineering groups. Typical symbols, their codes and their use are illustrated on MTR-E-4593 at the end of Chapter I, Volume 4.

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CHAPTER VII

REACTOR KINETICS AND REACTOR CONTROL

A. INTRODUCTION

The subject of reactor controls can logically be divided into two parts: safety controls and automatic or "servo" controls. The role of safety controls will be discussed under the section entitled Reactor Safety, but a few words on that topic are in order here. The safety controls built into a reactor are designed to respond to situations occurring in the operation of a reactor that could endanger the personnel operating the reactor or cause damage to the reactor itself. The safety system is composed of numerous devices which continuously monitor such quantities as reactor power level, reactor periods, temperatures and pressures in the core. As long as these quantities remain within certain prescribed limits considered safe, the safety system does nothing. When the safe limits are exceeded by any one or more than one of these quantities, the safety system acts to eliminate the hazard to equipment and personnel. The safety system is capable at all times of "overriding" the automatic control system.

Since the safety system acts only when a dangerous condition exists, it is desirable as well as necessary to have some type of control under normal operating conditions. Under normal conditions the reactor can be controlled manually by an operator or automatically by electro-mechanical devices. In general it can be said that the automatic control system performs the same type of corrective actions as an operator but it performs them faster, more smoothly and it is not subject to errors in judgment. The purpose of the automatic control system, then, is to insure that the reactor operates in some prescribed manner under normal conditions.

The choice of the automatic control scheme is dictated by the type of reactor to be controlled. Different types of reactors would be testing reactors (MTR-ETR), power reactors (PWR), experimental reactors (EBR), measurement facilities (RMF), etc. Each of these

types of reactors imposes different requirements on the automatic control system, consequently the control system of the RMF would not be satisfactory for the PWR.

In addition to the differences in types of reactors, there are differences between similar types that cause one reactor to possess different kinetic behaviors from another of the same type. Among these differences are such basic quantities as neutron energies (fast or thermal), neutron generation times or lifetimes, temperature coefficients and effects of fission product poisons. These quantities also influence the design of the automatic control system although they do not necessarily alter the control scheme.

Designing an automatic control system for a nuclear reactor is much the same as designing a control system for an airplane. The same rules apply. The same analytical techniques are used.

Once the dynamic behavior without controls of the system, device or process to be controlled is known, the task of designing a suitable control system is a straightforward one. The "rules of the game" have been compiled by many authors (ref. 1,2,3) and are commonly known as the "Theory of Servomechanisms." M. A. Schultz has applied the theory to nuclear reactors in his book "Control of Nuclear Reactors and Power Plants" (ref 4).

B. ZERO POWER REACTOR KINETICS

1. Kinetics Equations

The following assumptions are basic to the derivation of the differential equations describing reactor kinetics: (1) the time behavior of the reactor is separable from the spatial behavior; (2) all neutrons have the same energy; (3) there are two groups of neutrons considered - prompt and delayed; and (4) the reactor is a bare (unreflected) reactor.

On the basis of the above assumptions it is possible to write the following differential equations:

$$\frac{dn}{dt} = \frac{K(1-\beta)-1}{\ell} n + \sum_{i=1}^6 \lambda_i C_i \quad (7-1)$$

$$\frac{dC_i}{dt} + \lambda_i C_i = \frac{K\beta_i}{\ell} n \quad i = 1, 2, \dots, 6$$

where as usual:

- t = time in seconds
 n = number of neutrons per cc or neutron density
 K = effective multiplication of the reactor
 ℓ = neutron lifetime in seconds
 β = total effective fraction of delayed neutrons
 λ_i = decay constant of i^{th} group of delayed neutron emitters in seconds⁻¹
 C_i = number of atoms per cc of delayed neutron emitters of the i^{th} group
 β_i = effective fraction of delayed neutron emitters of the i^{th} group

Typical values for λ_i, β_i for U-235 as fuel are: (ref. 5)

Table 17

Typical Values for λ_i, β_i for U-235 Fuel

i	λ_i	β_i (%) $\beta = 0.755$ %
1	0.01244	0.025
2	0.03051	0.165
3	0.1114	0.148
4	0.3014	0.298
5	1.136	0.087
6	3.014	0.032
		$0.755 \% = \sum_{i=1}^6 \beta_i$

Reference 5 also gives values of λ_i, β_i for U-233 and Pu-239.

For thermal or slow reactors, the neutron lifetime ℓ lies in the range 10^{-3} to 10^{-5} sec. For instance, in the MTR, $\ell \approx 10^{-4}$ sec. In the ETR, $\ell \approx 5 \times 10^{-5}$ sec.

For intermediate and fast reactors $\ell < 10^{-5}$ sec.

Although equation (7-1) gives the neutron density n as a function of time, it is customary to use these same equations to calculate neutron flux ϕ or reactor power P by simply replacing n by ϕ or P as required.

It is also customary to consider the reactor parameters ℓ, λ_i and β_i as constants. Any changes in n are assumed to result from changes in the effective multiplication K . For use in reactor control studies, it is convenient to use the quantity ρ , defined as the reactivity of the reactor, rather than K . The reactivity ρ is defined by equation (7-2).

$$\rho = \frac{K-1}{K}, \text{ or } K = \frac{1}{1-\rho} \quad (7-2)$$

Combining equation (7-2) with equation (7-1) yields equation (7-3):

$$\left. \begin{aligned} \frac{dn}{dt} &= \frac{\rho-\beta}{\ell^*} n + \sum_{i=1}^6 \lambda_i C_i \\ \frac{dC_i}{dt} + \lambda_i C_i &= \frac{\beta_i}{\ell^*} n; i = 1, 2, \dots, 6 \end{aligned} \right\} \quad (7-3)$$

where

$$\ell^* = \ell(1-\rho) \approx \ell$$

In practice ℓ^* is considered a constant, implying that $\rho < 1$. This is a valid assumption for control system studies. In fact, the difference between the automatic control system and the safety system may be stated in terms of ρ . The automatic control system is designed to operate in the region $\rho < \beta$. The safety system operates in the event that $\rho \geq \beta$. Thus, since $\beta = 0.00755$ for U-235 and less for the other possible fuels, the approximation $\ell^* = \ell$ is a good one for use in control system studies.

A reactor is said to be exactly critical when $\rho = 0$ or $K = 1$. This means physically that for each neutron absorbed one is produced so that the neutron density n is a constant. Then since n and C_i are constants, say n_0 and C_{i0} and $\rho = 0$, the conditions for criticality or steady state operation of the reactor can be obtained from equation (7-3).

$$\frac{\beta}{\rho^*} n_0 = \sum_{i=1}^6 \lambda_i C_{i0}; \rho = 0, \frac{dn}{dt} = \frac{dC_i}{dt} = 0, \quad (7-4)$$

or

$$C_{i0} = \frac{\beta_i}{\rho^* \lambda_i} n_0, \quad i = 1, 2, \dots, 6$$

When $\rho < 0$, the reactor is subcritical. This condition means, physically, that neutrons are being absorbed faster than they are being produced. Thus a subcritical reactor is one in which the neutron density tends to decrease.

When $\rho > 0$, the reactor is said to be supercritical. In this condition neutrons are produced faster than they are absorbed and the neutron density will tend to increase. It will be shown that the neutron level is a very sensitive function of reactivity. In normal reactor operation, ρ changes continuously due to three principal effects: fuel burnup, temperature variations in the core and accumulation of fission product poisons. These effects will be discussed in detail later. They are mentioned now to show that it is very difficult to maintain $\rho = 0$ without constant adjustments of regulating rods which are designed so that ρ can be increased or decreased as needed.

2. Effect of Neglecting Delayed Neutrons

For future reference we shall briefly discuss the reactor kinetics equations without delayed neutrons. Assuming that delayed neutrons do not exist, equation (7-3) reduces to the single equation:

$$\frac{dn}{dt} = \frac{\rho}{\ell^*} n \quad (7-5)$$

The solution for a step reactivity where the stable reactor period T is defined as ℓ^* / ρ is:

$$\frac{n}{n_0} = e^{t\rho/\ell^*} = e^{t/T} \quad (7-6)$$

To illustrate the importance of delayed neutrons, consider the following example.

$$\begin{aligned} n_0 &= 1 \text{ Mw} \\ \ell^* &= 10^{-4} \text{ sec} \\ \rho &= 0.005, \text{ step} \end{aligned}$$

then

$$n = e^{50t}, \text{ Mw}$$

where the stable period is 0.020 sec.

At the end of 0.1 sec, $n = 148 \text{ Mw}$; at 1.0 sec, $n = 5.18 \times 10^{21} \text{ Mw}$.

With delayed neutrons, the stable period is two sec, and the power level one sec after the step is 4.1 Mw.

For extremely large positive reactivity steps, equation (7-5) is reasonably accurate when properly modified. When the reactor is "super prompt critical," $\rho > \beta$, the power increase is so rapid that the amount of delayed neutrons emitted into the system may be neglected. (Note "prompt critical" describes the condition $\rho = \beta$.) In equation (7-3), $\sum \lambda_i C_i$ represents the rate of supply of delayed neutrons, and in this case may be neglected. Thus:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\ell^*} n, \text{ where } \rho > \beta \quad (7-7)$$

This is basically the same equation as equation (7-5), with β now included in the reactivity term. The quantity $(\rho - \beta)$ is called "prompt excess reactivity," ρ_{pe} . The solution to equation (7-7) for a step reactivity is equation (7-6) with ρ replaced by ρ_{pe} .

As an example, consider:

$$\begin{aligned} n_0 &= 1 \text{ Mw} \\ \ell^* &= 10^{-4} \text{ sec} \\ \rho &= 0.01255, \text{ step} \\ \beta &= 0.00755 \end{aligned}$$

Excess prompt reactivity, ρ_{pe} , is 0.005 and the solution is the same as the previous example.

$$n = e^{50t}, \text{ Mw}$$

A step reactivity of 1.25 percent does produce a stable period in the order of 20 milliseconds. Thus under the special condition of $\rho > \beta$, the seven kinetics equations may be reduced to a single simple equation.

3. Subcritical Kinetics

When a reactor core is being assembled into a critical configuration on a cold startup, a technique of "source multiplication" is used to build up the pile neutron level. Measurements

of source multiplication throughout an initial startup are also used to predict the final critical mass.

In order to simplify the description of subcritical kinetics, delayed neutrons will be neglected. Adding a source term to equation (7-1) and neglecting delayed neutrons gives:

$$\frac{dn}{dt} = \frac{K-1}{\ell} n + \frac{S}{\ell} \quad (7-8)$$

where

S is considered a constant neutron source with the same units as n . For a step change in K , the solution is:

$$\frac{n}{n_0} = \frac{1}{K-1} \left[\frac{-S}{n_0} + \left(\frac{S}{n_0} + K-1 \right) e^{\frac{K-1}{\ell} t} \right] \quad (7-9)$$

In a subcritical reactor $K < 1$. Thus $K-1$ is negative. The transient portion of the solution decays very rapidly leaving as a steady-state solution:

$$n = \frac{-S}{K-1} \quad (7-10)$$

As K approaches unity, that is as fuel elements are added to the core, $(K-1)$ decreases and the over-all neutron level increases. Thus, if criticality is approached with small reactivity additions, the neutron level will be multiplied to many times the original source strength.

As the critical mass is approached by the addition of more fuel, the reciprocal multiplication, $(\frac{1}{n})$, approaches zero. This property of subcritical multiplication is used to predict the final critical mass as illustrated in figure 45.

The curves obtained by this method usually are not straight lines due to reactor geometry, locations of the detector and source, etc. Since the exact shape of the curve is variable, this method cannot be relied upon to predict an accurate value for the final assembly.

FIGURE 45

$\frac{1}{n}$ vs fuel mass graph

At this point it is convenient to make some simplifying assumptions that will make a study of the reactor kinetics much easier. Assume that the reactor behavior is being examined over a short interval of time, a few minutes. Over this time interval, fuel burnup and fission product poisoning can be neglected as will be shown later. If it is further assumed that during this time the temperatures in the core are held constant, then temperature effects on reactivity can also be neglected. It will now be assumed that ρ can arbitrarily be made positive or negative in the range $|\rho| < \beta$ by changing the position of a regulating rod. The reactivity effect of a regulating rod will hereafter be denoted by the symbol ρ_r .

4. Qualitative Discussion of Reactor Kinetics Equations

Under the assumptions stated above it is possible to obtain analytic solutions of equation (7-3) for step functions of $\rho = \rho_r$. Consider a reactor in steady state operation with $n(0) =$

$$n_0, C_i(0) = C_{i_0}, \rho_r = 0, \frac{dn}{dt}(0) = \frac{dC_i}{dt}(0) = 0.$$

It can be shown that if a step function ρ_r is applied at $t = 0$, equation (7-3) has the following solution:

$$n(t) = n_0 \sum_{j=1}^7 A_j e^{m_j t} \quad (7-11)$$

where

A_j and m_j are complicated functions of ℓ^* ,

β_i , λ_i and ρ_r . In general, since $n(0) = n_0$, then $\sum_{j=1}^7 A_j = 1$. It can be shown that for the

case $\rho_r > 0$, m_1, m_2, m_3, m_4, m_5 and m_6 are all negative and m_7 is positive: m_7 is the reciprocal of the positive stable reactor period. Thus for $\rho_r > 0$ all terms of equation (7-11) except the 7th vanish in time. The 7th term carries the neutron density $n(t)$ to higher and higher levels in an exponential manner since for $\rho_r > 0$, $A_7 > 0$. For the case $\rho_r < 0$, all the m_j 's are negative and the neutron level will decrease exponentially to lower levels. Two important features of equation (7-11) that are not obvious because of the complexity of the m_j 's and A_j 's concern the magnitudes of the reciprocal stable periods - both positive and negative. Qualitatively it can be said that for $\rho_r > 0$, m_7 increases as ρ_r increases and as ℓ^* decreases. The reciprocal negative stable period ($\rho_r < 0$) approaches a limit as ρ_r continues to increase. The limit is about $1/80 \text{ sec}^{-1}$.

5. The In-Hour Equation

The equation relating stable reactor period and reactivity is called the "in-hour equation."

$$\rho = \frac{\frac{\ell}{T} + \sum_{i=1}^6 \frac{\beta_i}{1 + \lambda_i T}}{1 + \frac{\ell}{T}} \quad (7-12)$$

Figure 46 shows positive reactivity vs stable reactor period calculated from the in-hour equation for different reactor lifetimes and delayed neutron fractions.

For large periods, $T > 100 \text{ sec}$, the ℓ/T terms in equation (7-12) may be neglected:

$$\rho \approx \sum_{i=1}^6 \frac{\beta_i}{1 + \lambda_i T}, \quad T > 100 \text{ sec.} \quad (7-13)$$

The in-hour equation is also valid for negative periods when they physically exist. The shortest possible negative period, -80 sec , is limited by the longest half lived delayed neutron emitter.

For $\rho < 0$, $-T > \frac{1}{\lambda_1}$, $\lambda_1 = 0.01244 \text{ sec}^{-1}$ thus $-T > 80 \text{ sec}$.

Since λ_1 is approximately the same for all fuels, the limiting negative period for all reactors is the same.

6. The One-Group Model

For a quantitative study of the reactor response to step functions in ρ_r , it will be assumed that the six separate and distinct groups of delayed neutrons can be "lumped" into one representative group. There is some loss of accuracy in making this approximation but there is no loss of generality as will be seen. Applying this approximation, equation (7-3) is transformed to:

$$\frac{dn}{dt} = \frac{\rho - \beta}{\ell^*} n + \bar{\lambda} C \quad (7-14a)$$

$$\frac{dC}{dt} + \bar{\lambda} C = \frac{\beta}{\ell^*} n, \quad (7-14b)$$

where

$$\frac{1}{\bar{\lambda}} = \frac{1}{\beta} \sum_{i=1}^6 \frac{\beta_i}{\lambda_i} \quad (7-15)$$

All quantities in equations (7-14a) and (7-14b) are defined as before except for C and $\bar{\lambda}$. C represents the atom density or concentration of all delayed neutron emitters. $\bar{\lambda}$ is the mean decay constant for all six groups - weighted according to equation (7-15). As before, it is assumed that $\rho = \rho_r$ is a step function applied at $t = 0$. The solution of equations (7-14a) and (7-14b) is obtained as follows using the techniques of operational calculus. Writing equations (7-14a) and (7-14b) in operational form:

$$\begin{cases} Dn = \frac{\rho_r - \beta}{\ell^*} n + \bar{\lambda} C \\ DC + \bar{\lambda} C = \frac{\beta}{\ell^*} n \end{cases}$$

where

$$D = \frac{d}{dt}$$

FIGURE 46
Inhour curves for five lifetimes
and four delayed fractions

Rearranging terms,

$$\left\{ \begin{array}{l} \left(D + \frac{\beta - \rho_r}{\ell^*} \right) n = \bar{\lambda} C \quad (7-16a) \\ \left(D + \bar{\lambda} \right) C = \frac{\beta}{\ell^*} n \quad (7-16b) \end{array} \right.$$

Solving (16b) for $\bar{\lambda} C$,

$$\bar{\lambda} C = \frac{\bar{\lambda} \beta}{\ell^*} \frac{n}{D + \bar{\lambda}} \quad (7-17)$$

Substituting (17) into (16a) gives:

$$\left(D + \frac{\beta - \rho_r}{\ell^*} \right) n = \frac{\bar{\lambda} \beta}{\ell^*} \frac{n}{D + \bar{\lambda}}$$

On rearranging,

$$\left[D^2 + \left(\bar{\lambda} + \frac{\beta - \rho_r}{\ell^*} \right) D - \frac{\bar{\lambda} \rho_r}{\ell^*} \right] n = 0 \quad (7-18)$$

Equation (7-18) is a linear homogeneous equation of 2nd order. It has a solution of the form:

$$n(t) = a_1 e^{m_1 t} + a_2 e^{m_2 t} \quad (7-19)$$

where

m_1 and m_2 are solutions of the auxiliary equation:

$$m^2 + \left(\bar{\lambda} + \frac{\beta - \rho_r}{\ell^*} \right) m - \frac{\bar{\lambda} \rho_r}{\ell^*} = 0 \quad (7-20)$$

It is readily shown that m_1 and m_2 are given by:

$$\left. \begin{array}{l} m_1 = -\frac{1}{2} \left(\bar{\lambda} + \frac{\beta - \rho_r}{\ell^*} \right) - \sqrt{\left[\frac{1}{2} \left(\bar{\lambda} + \frac{\beta - \rho_r}{\ell^*} \right) \right]^2 + \frac{\bar{\lambda} \rho_r}{\ell^*}} \\ m_2 = -\frac{1}{2} \left(\bar{\lambda} + \frac{\beta - \rho_r}{\ell^*} \right) + \sqrt{\left[\frac{1}{2} \left(\bar{\lambda} + \frac{\beta - \rho_r}{\ell^*} \right) \right]^2 + \frac{\bar{\lambda} \rho_r}{\ell^*}} \end{array} \right\} \begin{array}{l} \approx - \left(\frac{\beta - \rho_r}{\ell^*} \right) \\ \approx \frac{\bar{\lambda} \rho_r}{\beta - \rho_r} \end{array} \quad (7-21)$$

The constants a_1 and a_2 of equation (7-19) are obtained by imposing the proper initial conditions on equation (7-19). If ρ_r is a step function and $n(0) = n_0$, $dn/dt(0) = \rho_r/\ell^* n_0$,* it is readily shown that a_1 and a_2 are given by:

$$\left. \begin{array}{l} a_1 = \frac{n_0 (m_2 - \rho_r/\ell^*)}{m_2 - m_1} \\ a_2 = \frac{-n_0 (m_1 - \rho_r/\ell^*)}{m_2 - m_1} \end{array} \right\} \quad (7-22)$$

Thus equation (7-19) can be written as:

$$n(t) = n_0 \left[\frac{m_2 - \rho_r/\ell^*}{m_2 - m_1} e^{m_1 t} - \frac{m_1 - \rho_r/\ell^*}{m_2 - m_1} e^{m_2 t} \right] \quad (7-23)$$

This is of the form

$$n(t) = n_0 \sum_{j=1}^2 A_j e^{m_j t} \quad (7-24)$$

where

$$\left. \begin{array}{l} A_1 = \frac{m_2 - \rho_r/\ell^*}{m_2 - m_1} \approx \frac{-\rho_r}{\beta - \rho_r} \\ A_2 = \frac{-m_1 - \rho_r/\ell^*}{m_2 - m_1} \approx \frac{\beta}{\beta - \rho_r} \end{array} \right\} \quad (7-25)$$

The approximate solution for a step ρ_r is:

$$n(t) = n_0 \left[\frac{-\rho_r}{\beta - \rho_r} \exp \left(-\frac{\beta - \rho_r}{\ell^*} t \right) + \frac{\beta}{\beta - \rho_r} \exp \left(\frac{\bar{\lambda} \rho_r}{\beta - \rho_r} t \right) \right] \quad (7-26)$$

This solution can be examined term by term. This type of examination allows a graphical analysis of $n(t)$ and leads to a clear definition of certain effects important in reactor kinetics.

*Due to the discontinuity of ρ_r at $t = 0$, $dn/dt(0) \neq 0$. The correct value is given above.

Let:

$$n_1(t) = n_o \left(\frac{-\rho_r}{\beta - \rho_r} \right) \exp \left(-\frac{(\beta - \rho_r)}{l^*} t \right)$$

and

$$n_2(t) = n_o \left(\frac{\beta}{\beta - \rho_r} \right) \exp \left(\frac{\bar{\lambda} \rho_r}{\beta - \rho_r} t \right)$$

such that

$$n(t) = n_1(t) + n_2(t)$$

First consider the values of $n_1(t)$ and $n_2(t)$ at the instant the step ρ_r is applied. At $t = 0$,

$$n_1(0) = n_o \left(\frac{-\rho_r}{\beta - \rho_r} \right),$$

and

$$n_2(0) = n \left(\frac{\beta}{\beta - \rho_r} \right)$$

Now assume that ρ_r is positive and less than β in magnitude. In this event, $n_1(0)$ is negative and the exponent of e in $n_1(t)$ is negative for all positive values of t . As t increases, the value of the exponential decreases and $n_1(t)$ approaches zero as t becomes larger and larger. $n_1(t)$ is shown in figure 47.

Now consider $n_2(t)$ for the same ρ_r . Obviously $n_2(0)$ is a positive quantity. Note that the exponent of e is also a positive quantity. This means that the exponential term in $n_2(t)$ will increase without bounds as t increases. $n_2(t)$ is also shown in figure 47.

In order to have some feeling for the magnitudes of $n_1(0)$ and $n_2(0)$ and for the

rate of decrease of $n_1(t)$ and the rate of increase of $n_2(t)$, consider a reactor with the following properties:

$$\begin{aligned} l^* &= 10^{-4} \text{ sec} \\ \beta &= 0.00755 \text{ (U-235)} \\ \bar{\lambda} &= 0.0769/\text{sec} \\ n_o &= 1 \text{ Mw} \\ \rho_r &= +0.005 \end{aligned}$$

These values require that:

$$\begin{aligned} n_1(0) &= -1.96 \text{ Mw} \\ n_2(0) &= +2.96 \text{ Mw} \end{aligned}$$

Also,

$$\begin{aligned} n_1(t) &= - \left[1.96 e^{-25.5t} \right] \text{ Mw} \\ n_2(t) &= \left[2.96 e^{0.15t} \right] \text{ Mw} \end{aligned}$$

Table 18 gives values of $n_1(t)$, $n_2(t)$ and $n(t)$ for the stated values of t .

Table 18

Values of $n_1(t)$, $n_2(t)$ and $n(t)$ for Values of t .

t	$n_1(t)$	$n_2(t)$	$n(t) = n_1(t) + n_2(t)$
0.00 sec	-1.96 Mw	2.96 Mw	1.00 Mw
0.01	-1.53	2.96	1.44
0.05	-0.54	2.98	2.44
0.10	-0.15	3.00	2.85
0.50	0.00	3.19	3.19
1.00	0.00	3.44	3.44
2.00	0.00	4.00	4.00

FIGURE 47
 $n_1(t)$ curve

It should be noted from table 18 that for values of t greater than 0.10 sec $n_1(t)$ is zero while $n_2(t)$ has increased only slightly. As shown in figure 47, $n(t)$ (which can be obtained by graphical addition of the curves $n_1(t)$ and $n_2(t)$) increases rapidly from n_0 to approximately $n_2(0)$, and for times greater than about 0.10 sec, $n(t)$ is identical to $n_2(t)$.

The initial "jump" in $n(t)$ from n_0 to $n_2(0)$ is called the "prompt jump." It can be estimated for other values of ρ_r by use of equation (7-27).

$$\text{prompt jump} = n_0 \frac{\beta}{\beta - \rho_r} = n_2(0) \quad (7-27)$$

After $n_1(t)$ has died out, $n(t)$ is given by $n_2(t)$ only. In this time interval, the power is increasing exponentially as specified by $n_2(t)$ and the reactor is said to be on a "positive stable period." The stable period is in general given by:

$$T = \frac{\beta - \rho_r}{\bar{\lambda} \rho_r} = 6.7 \text{ sec.} \quad (7-28)$$

While on a stable period, $n(t)$ can be calculated according to:

$$n(t) = n_2(0) e^{t/T} = n_2(0) \exp\left(\frac{\bar{\lambda} \rho_r}{\beta - \rho_r} t\right) \quad (7-29)$$

The stable period gives a measure of how rapidly $n(t)$ is changing with time. Thus, the statement that the reactor is on a positive 6.7 sec period simply means that every 6.7 sec $n(t)$ is increased by a factor $e = 2.72$.

Equation (7-28) defining the stable period is only a "rough approximation" of the actual period to be expected for any given ρ_r . Exact periods should be calculated from the in-hour equation (7-12). Equation (7-28) is inaccurate because of the assumptions made in lumping six groups of delayed neutrons into one group.

The discussion above concerning the prompt jump and the stable period was limited to positive reactivities for convenience. Now consider a negative reactivity step of the same magnitude as before. Using the same data as before, the following values are obtained when $\rho_r = -0.005$

$$n_1(0) = +0.398$$

$$n_2(0) = +0.602$$

$$\begin{aligned} n(t) &= +0.398 e^{-125.5t} + 0.602 e^{-0.031t} \\ &= n_1(t) + n_2(t) \end{aligned}$$

It should be noticed that for negative ρ_r , both $n_1(0)$ and $n_2(0)$ are positive and both exponentials in $n(t)$ are negative. Because of the large difference in magnitude of the exponents, $n_1(t)$ dies out much more rapidly than $n_2(t)$. This type of behavior predicts a prompt jump "down" from $n(t) = 1\text{Mw}$ at $t = 0$ to $n(t) = n_2(0) = .602\text{ Mw}$ in about .05 sec. After that time the reactor will be on a negative stable period of about 32 sec which can be calculated from equation (7-28). It should be obvious that the negative stable period predicted by the equation for the period derived from the one group model is completely in error. It has been previously stated that negative periods cannot be shorter than 80 sec. This example should show the magnitudes of the errors to be expected in calculating periods by means of equation (7-28). The correct period can be obtained by using the in-hour equation.

The response (or time behavior) of a reactor following a "step reactivity insertion" can now be summarized.

First, the power level experiences a prompt jump (up for positive ρ_r , down for negative ρ_r).

Second, following the prompt jump, the reactor is on a stable period (positive for positive ρ_r and negative for negative ρ_r).

A detailed examination of the solution to the kinetics equations considering all six groups of delayed neutrons would have shown the same two effects - the prompt jump and the stable period. When accurate values of $n(t)$ are needed, the one group model solution for $n(t)$ cannot be used.

It must be emphasized that the prompt jump and the stable period occur only when step reactivities are assumed. Other types of reactivity insertions such as the ramp do not produce these effects.

It is seen that equation (7-24) is of the same form as the more general case of equation (7-11). Equation (7-11) and (7-24)

are often normalized to the initial neutron density by dividing through by n_0 . Thus, equation (7-24) becomes

$$\frac{n(t)}{n_0} = \sum_{j=1}^2 A_j e^{m_j t} \quad (7-30)$$

This normalization is a convenient one because of the direct dependence of $n(t)$ on n_0 (see equation 7-24). Consider a critical reactor with neutron density n_0 at $t = 0$. At t_1 sec after a positive step function of reactivity ρ_r is applied, the neutron density n_1 will be given by

$$n_1 = n_0 \sum_{j=1}^2 A_j e^{m_j t_1} \quad (7-31)$$

Now consider the same reactor and the same step function of reactivity but let this reactor be critical at a neutron density $2n_0$ or $n(0) = 2n_0$: t_1 sec after the reactivity is applied the neutron density for this case is given by

$$n_2 = 2n_0 \sum_{j=1}^2 A_j e^{m_j t_1} \quad (7-32)$$

Comparing equations (7-31) and 7-32), it is seen that at time t_1 , the neutron density in the second case has risen to twice that attained by the first case. This is an important effect

in reactor kinetics and the control system must be designed so that the initial neutron density or power level, as the case may be, does not affect the ability of the control system to contain power excursions within certain limits. How this is accomplished will be taken up later.

There is another important aspect of reactor kinetics that must be studied at this point. The point in question now is what happens when a critical reactor is subjected to a step function increase or decrease in reactivity for a time τ and then at the end of the τ sec ρ_r is returned to zero stepwise. Assuming that prior to the application of a positive step function ρ_r , the reactor is just critical with neutron density n_0 . Figure 48 shows the time history of ρ_r and $n(t)$ prior to and after the application of the step changes in ρ_r . It is hereby contended that $n(t)$ will not return to n_0 but instead to a higher level n_1 . The new level n_1 is given by

$$\lim_{t \rightarrow \infty} n(t) = n_1 > n_0 \text{ if } \rho_r > 0 \quad (7-33)$$

Similarly, had ρ_r been a negative step, then n_1 would be less than n_0 .

This effect is useful in changing from one neutron density or power level to another. Say the reactor is at n_0 and it is desired to go to a new level n_1 . It is only necessary to move the rod out (or in) and then back to its

FIGURE 48
Time history of ρ_r and $n(t)$
prior to, and after application of,
step changes in ρ_r

original position. The reactor will then eventually level itself out at the new level n_1 .

This effect complicates the problem of holding the reactor at some constant level, n_0 . If some reactivity ρ_E is inadvertently applied step-wise, the reactor level will change according to equation (7-24). Exact cancellation of ρ_E by repositioning of the regulating rod will not return n to n_0 . The regulating rod must "overshoot" ρ_E before cancelling it in order to return n to n_0 . A reactor operator must develop a "feel" for how much overshoot is necessary to return n to n_0 . If the overshoot is too large, n will tend to return to a value less than (or greater than) n_0 , and another repositioning with an overshoot of the regulating rod will be necessary. Thus it is seen that manual control of a reactor designed to operate at constant power could be a frustrating task for an operator. Fortunately the automatic control system takes care of this effect very nicely.

It can be shown that for times greater than τ sec (figure 48), $n(t)$ is given by:

$$n(t) = \frac{n(\tau)}{m_3} \left[\bar{\lambda} + \frac{\beta}{\ell^*} e^{-m_3(t-\tau)} \right] + \bar{\lambda} \frac{c(\tau)}{m_3} \left[1 - e^{-m_3(t-\tau)} \right] \quad (7-34)$$

where $t > \tau$ sec

$$n(\tau) = n_0 \left[\frac{m_2 - \rho_r/\ell^*}{m_2 - m_1} e^{m_1\tau} - \frac{m_1 - \rho_r/\ell^*}{m_2 - m_1} e^{m_2\tau} \right] \quad (7-35)$$

$$c(\tau) = n_0 \left[\frac{\beta}{\ell^* \bar{\lambda}} \frac{m_2}{m_2 - m_1} e^{m_1\tau} - \frac{m_1}{m_2 - m_1} e^{m_2\tau} \right] \quad (7-36)$$

$c(\tau)$ = delayed neutron emitter concentration at $t = \tau$.

According to equation (7-33) the new level n_1 is:

$$n_1 = \frac{\bar{\lambda}}{m_3} \left[n(\tau) + c(\tau) \right] \quad (7-37)$$

7. Parametric Study of Step Response

Having examined several effects peculiar to the step response in terms of the one group

model, it is now appropriate to investigate the effect on the step response of various ρ_r insertions, and various neutron lifetimes (ℓ).

The responses shown in figures 49 and 50 were obtained by solving the reactor kinetics equations (equation 7-1) by using an analog

FIGURE 49
Response curve for three values
of ρ_r of a zero power reactor

computer. While these equations could have been solved manually, the labor and time required would have been excessive.

Figure 49 shows the response of a zero power reactor to three values of ρ_r assuming $\beta = 0.0075$ (U-235) and $\ell = 10^{-4}$ sec.

Figure 50 shows the response of three reactors with different neutron lifetimes (ℓ) to the same reactivity step. For this calculation $\beta = 0.00755$.

FIGURE 50
Response curve to the same
reactivity steps of three reactors
with different neutron lifetimes

In addition to the step function insertion of reactivity, there are three other functions that are normally of interest. They are the ramp function ($\rho_r = R_1 t$), the quadratic or parabolic function ($\rho_r = R_2 t^2$) and the sinusoidal function ($\rho_r = \rho_0 \sin \omega t$). The ramp and quadratic functions are primarily of interest in reactor accident studies. The sinusoidal function is probably the most useful of the four with respect to automatic control system design and analysis. For this reason the sinusoidal variation of reactivity and its effect on reactor neutron density or power level will be considered in some detail.

8. The Perturbation Equations

Consider a reactor operating at a constant neutron density n_0 . If along with a constant neutron density, it is required that $\frac{dn}{dt} = 0$,

$$\frac{dc}{dt} = 0, \quad \rho_r = 0, \text{ and } c_0 = \frac{\beta}{\ell^* \bar{\lambda}} n_0$$

then the reactor is said to be in a state or condition of equilibrium. If the reactivity is changed in some arbitrary fashion such that n and c are changed by a small amount away from the equilibrium values, the reactor is said to be in a perturbed condition. Mathematically this amounts to

$$\left. \begin{aligned} n(t) &= n_0 + \delta n(t) \\ c(t) &= c_0 + \delta c(t) \\ \rho_r &= \rho_r(t) \end{aligned} \right\} \quad (7-38)$$

where

$n(t)$ = instantaneous value of n
 n_0 = equilibrium value of n (a constant)
 $\delta n(t)$ = perturbation of n or change in n away from n_0

$c(t)$, c_0 , $\delta c(t)$ are similarly defined.

The correct interpretation of equation (7-38) is that they are the first two terms of the Taylor series expansion of $n(t)$ and $c(t)$ about n_0 and c_0 respectively.

Combining equations (7-38) with equations (7-14a) and (7-14b):

$$\left. \begin{aligned} \frac{d\delta n}{dt} &= \frac{\rho_r(t) - \beta}{\ell^*} (n_0 + \delta n) + \bar{\lambda} (c_0 + \delta c) \\ \frac{d\delta c}{dt} + \bar{\lambda} (c_0 + \delta c) &= \frac{\beta}{\ell^*} (n_0 + \delta n) \end{aligned} \right\} \quad (7-39)$$

Note that $\frac{dn_0}{dt} = \frac{dc_0}{dt} = 0$ and that the dependence of δn and δc on t is not explicitly shown. Nevertheless δn and δc are functions of time.

On rearrangement of equation (7-39):

$$\begin{aligned} \frac{d\delta n}{dt} &= \frac{n_0}{\ell^*} \rho_r(t) - \frac{\beta}{\ell^*} \delta n - \frac{\beta}{\ell^*} n_0 \\ &\quad + \frac{\rho_r(t)}{\ell^*} \delta n + \bar{\lambda} c_0 + \bar{\lambda} \delta c \end{aligned} \quad (7-40)$$

$$\frac{d\delta c}{dt} + \bar{\lambda} c_0 + \bar{\lambda} \delta c = \frac{\beta}{\ell^*} n_0 + \frac{\beta}{\ell^*} \delta n$$

Since at equilibrium, $\delta n = \delta c = \rho_r(t) = 0$, the condition of equilibrium is obtained from equation (7-40) as

$$\left. \begin{aligned} 0 &= -\frac{\beta}{\ell^*} n_0 + \bar{\lambda} c_0, \\ \text{and} \\ \bar{\lambda} c_0 &= \frac{\beta}{\ell^*} n_0 \end{aligned} \right\} \quad \begin{array}{l} \text{Equilibrium} \\ \text{equation} \end{array} \quad (7-41)$$

Imposing the equilibrium condition equation (7-41) on equation (7-40) yields the following equations:

(7-42a)

$$\frac{d\delta n}{dt} = \frac{n_0}{\ell^*} \rho_r(t) - \frac{\beta}{\ell^*} \delta n + \bar{\lambda} \delta c + \frac{\rho_r(t)}{\ell^*} \delta n$$

$$\frac{d\delta c}{dt} + \bar{\lambda} \delta c = \frac{\beta}{\ell^*} \delta n \quad (7-42b)$$

It has been asserted that δn and δc are to be "small" changes in n and c . Therefore, the last term on the right hand side of equation (7-42a) may be neglected with respect to the first term. With this last approximation, the reactor perturbation equations have been derived.

$$\left. \begin{aligned} \frac{d\delta n}{dt} &= \frac{n_0}{\ell^*} \rho_r(t) - \frac{\beta}{\ell^*} \delta n + \bar{\lambda} \delta c \\ \frac{d\delta c}{dt} + \bar{\lambda} \delta c &= \frac{\beta}{\ell^*} \delta n \end{aligned} \right\} \text{Perturbation equations} \quad (7-43)$$

The perturbation equations will now be solved assuming $\rho_r(t)$ to be a sinusoidal function of time.

9. The Reactor Frequency Response

Consider a reactor in the equilibrium condition. Let the regulating rod be moved sinusoidally with small amplitude. Then the reactivity variations produced by the rod will also be sinusoidal. The resulting variation in neutron density δn will also be sinusoidal.* Under these conditions,

$$\left. \begin{aligned} \rho_r(t) &= \bar{\rho}_r e^{j\omega t} \\ \delta n(t) &= \bar{\delta n} e^{j\omega t} \\ \delta c(t) &= \bar{\delta c} e^{j\omega t} \end{aligned} \right\} \quad (7-44)$$

where

$\bar{\rho}_r$ = amplitude of reactivity oscillation

$\bar{\delta n}$ = amplitude of neutron density oscillation

$\bar{\delta c}$ = amplitude of delayed neutron emitter concentration oscillation

*The "initial" reaction of the reactor is not truly sinusoidal, but if the sinusoidal motion of the rod is sustained, the changes in neutron density will eventually become sinusoidal.

ω = angular frequency of oscillation

$$j = \sqrt{-1}$$

Combining equations (7-44) with (7-43) leads to:

$$\left. \begin{aligned} j\omega \bar{\delta n} &= \frac{n_0}{\ell^*} \bar{\rho}_r - \frac{\beta}{\ell^*} \bar{\delta n} + \bar{\lambda} \bar{\delta c} \\ j\omega \bar{\delta c} + \bar{\lambda} \bar{\delta c} &= \frac{\beta}{\ell^*} \bar{\delta n} \end{aligned} \right\} \quad (7-45)$$

Solving for $\bar{\delta n}$,

$$\bar{\delta n} = \frac{1}{j\omega \ell^*} \frac{(j\omega + \bar{\lambda})}{[j\omega + (\bar{\lambda} + \beta/\ell^*)]} n_0 \bar{\rho}_r \quad (7-46)$$

Equation (7-46) gives the reactor frequency response. This equation is similar in form to those obtained in the analysis of electrical circuits driven by sinusoidal voltages or currents.

The quantity $\left| \frac{\bar{\delta n}}{n_0 \bar{\rho}_r} \right|$ is termed the amplitude frequency response and is given by:

$$\left| \frac{\bar{\delta n}}{n_0 \bar{\rho}_r} \right| = \frac{1}{\omega \ell^*} \sqrt{\frac{\omega^2 + \bar{\lambda}^2}{\omega^2 + (\bar{\lambda} + \beta/\ell^*)^2}} \quad \text{amplitude frequency response} \quad (7-47)$$

The amplitude frequency response gives the relative change in amplitude of the neutron density $\bar{\delta n}$ as the frequency of the driving function of amplitude $\bar{\rho}_r$ is varied.

The peaks of the ρ_r and δn oscillations do not occur at the same time. The δn peaks usually occur after the ρ_r peaks. The amount of lag between peaks depends on the frequency of ρ_r and is conveniently given by the "phase angle" of the frequency response. The phase angle is derived from equation (7-46) and is denoted by the symbol θ .

(7-48)

$$\theta \text{ (radians)} = -\pi/2 + \tan^{-1} \frac{\omega}{\bar{\lambda}} - \tan^{-1} \frac{\omega}{\bar{\lambda} + \beta/\ell^*}$$

Figure 51 shows how $\left| \frac{\bar{\delta n}}{n_0 \bar{\rho}_r} \right|$ and θ vary with

frequency. Note that the amplitude response is plotted in db (decibels), ie:

$$\left| \frac{\bar{\delta n}}{n_o \bar{\rho}_r} \right| \text{ db} = 20 \log \left| \frac{\bar{\delta n}}{n_o \bar{\rho}_r} \right|$$

FIGURE 51
Amplitude frequency response
and phase angle variation with frequency

The phase angle θ is plotted in degrees. Qualitatively, there is correlation between the reactor response to a step ρ_r and the frequency response. The high frequencies correspond to small times and the low frequencies to large times. Thus, the rapid increase of $\left| \frac{\bar{\delta n}}{n_o \bar{\rho}_r} \right|$ with decreasing frequency corresponds to the rapid increase of $\frac{n}{n_o}$ as time increases after the application of the step ρ_r . The plateau of $\left| \frac{\bar{\delta n}}{n_o \bar{\rho}_r} \right|$ corresponds with the plateau in the response to a step ρ_r . The decrease of $\left| \frac{\bar{\delta n}}{n_o \bar{\rho}_r} \right|$ with increasing w corresponds to the decrease of $\frac{n}{n_o}$ as $t \rightarrow 0$. The exact shape of the curves in figure 51 depends strongly on l^* from about $w = 1$ on up. From about $w = 1$ on down, both curves are independent of l^* .

Usually the reactor frequency response in itself is of little use. It is necessary for the application of the theory of servo-mechanisms to reactor control systems. This theory combines the frequency response of the reactor with that of the control system hardware and other physical processes (temperature variations, poisoning, etc) and, on the basis of the frequency response of the entire system, predicts stable or unstable operation. In addition to determining the stability of the reactor, it is also possible to predict (in the

stable case) the transient behavior of the reactor under control.

C. REACTOR CONTROL

1. The Transfer Function Concept

A function very closely related to the frequency response function is the transfer function. The frequency response is, in fact, a special case of the transfer function. Rigorous derivation of transfer functions requires the use of the mathematical quantity known as the Laplace Transform (see ref. 1, 2 or 3). Application of the Laplace Transformation to a differential equation transforms the variables in the differential equations from functions of time (t) into functions of the Laplace Transform variable, S . The resulting equations can be handled algebraically. Herein lies the usefulness of the Laplace Transform. As an example, consider the perturbation equations (7-42a) and (7-42b). After application of the Laplace Transformation, these equations become:

$$\left. \begin{aligned} S \delta n(S) &= \frac{n_o}{l^*} \rho_r(S) - \frac{\beta}{l^*} \delta n(S) + \bar{\lambda} \delta c(S) \\ S \delta c(S) + \bar{\lambda} \delta c(S) &= \frac{\beta}{l^*} \delta n(S) \end{aligned} \right\} \quad (7-49)$$

Solving these equations algebraically for $\delta n(S)$ will yield the "reactor transfer function":

$$\delta n(S) = \frac{1}{l^* S} \frac{(S + \bar{\lambda})}{S + (\bar{\lambda} + \beta/l^*)} n_o \rho_r(S) \quad (7-50)$$

Notice that replacing S by jw in equation (7-50) reduces it to equation (7-46) which is the reactor frequency response function. Within the scope of this text, it will be permissible to replace jw by S to obtain the transfer function.

It must be emphasized that the frequency response function is a special case of the transfer function. In general, $S \neq jw$. In equation (7-50), $\rho_r(S)$ is the Laplace Transform of $\rho_r(t)$. $\rho_r(t)$ can be a step function, ramp function or any function of time for which a Laplace Transform exists. In a great many cases, a transfer function is of the form:

$$y(S) = \frac{N(S)}{\Delta(S)} X(S) \quad (7-51)$$

where

$y(S)$ is considered the output and $X(S)$ the input or forcing function. $N(S)$ is a polynomial in S . $\Delta(S)$ is a polynomial in S of equal or greater degree than $N(S)$.

The equation $\Delta(S) = 0$ is called the characteristic equation. By solving for the values of S that satisfy the characteristic equation, it is possible to describe, qualitatively, the nature of the solution (in time) of the differential equations from which the transfer function was derived. The variable S is, in general, a complex quantity. It is composed of a real part R and an imaginary part I .

$$S = R + jI$$

The values of R and I determine the nature of the transient response as outlined for the following cases:

a. $I = 0; R < 0$

The transient response is non-oscillatory, ie, over-damped or critically damped, (fig 52a).

b. $I \neq 0; R < 0$

The transient response is under-damped (fig 52b).

c. $I = 0; R > 0$

The transient response is unstable. The output increases exponentially (fig 52c).

d. $I \neq 0; R > 0$

The amplitude of oscillations increases exponentially (fig 52d).

e. $I \neq 0; R = 0$

This condition defines the frequency response; that is, $I = w$ (fig 52e). The response consists of oscillations of constant amplitude and frequency w .

f. $I = R = 0$

The response corresponding to this case depends on the number of roots of $\Delta(S) = 0$ that are located at the origin ($S = 0$).

To illustrate this last case, consider the reactor transfer function (equation 7-50). It is of the form of equation (7-51) if δ_n is considered the output, and ρ_r the input. The

FIGURE 52a
Transient response, non-oscillatory,
(over-damped or critically damped)

FIGURE 52b
Transient response, under-damped

FIGURE 52c
Transient response, unstable
(output increases exponentially)

FIGURE 52d
Exponential increase in amplitude
of oscillations

FIGURE 52e
Frequency response, oscillations
of constant amplitude and frequency

characteristic equation of the reactor transfer function is:

$$\Delta(S) = S \left[S + (\bar{\lambda} + \beta/\rho^*) \right] = 0$$

The values of S that satisfy the characteristic equation are $S = 0$ and $S = -(\bar{\lambda} + \beta/\rho^*)$. The root $S = 0$ corresponds to case f. This characteristic equation contains only one root at the origin. For a single root at the origin, the characteristic equation predicts a linearly increasing output if the input is a step function. The other root corresponds to case a and stability is predicted. The zero root (case f) would dominate in this case and the response is considered unstable. It is known that the reactor response to a step function of reactivity leads to the neutron level increasing "exponentially." The characteristic equation predicts that the neutron density will increase "linearly." This discrepancy is due to the approximate nature of the perturbation equations (equation 7-43) from which the reactor transfer function was derived. At any rate the characteristic equation correctly predicts instability as it should if the reactor transfer function is to be at all useful in the design of an automatic control system for the reactor.

2. The Temperature Coefficient of Reactivity

As mentioned earlier, variations in temperature in the reactor core can produce changes in reactivity. There are three main reasons why this occurs. They are: (1) thermal expansion changes the density of reactor materials, and thus the atom densities of these materials; (2) the neutron temperature in thermal reactors is related to that of the medium; and (3) interactions between neutrons and nuclei vary with nuclear agitation. These three effects combine to produce one net effect. The effective multiplication factor of the reactor is changed, and therefore the reactivity also changes. The "temperature coefficient of reactivity" is a measure of this effect. It is defined by the relation:

$$c_T = \left. \frac{d\rho}{dT} \right|_0 \quad (7-52)$$

where

c_T = temperature coefficient of reactivity

T = temperature of core (usually of moderator)

$\left. \frac{d\rho}{dT} \right|_0$ = rate of change of ρ with respect to T evaluated at equilibrium conditions.

The reactivity ρ_T produced by a temperature change δT in the core is given by:

$$\rho_T = c_T \delta T \quad (7-53)$$

In general c_T can be positive or negative. A negative temperature coefficient will tend to make a reactor self-regulating.

A positive temperature coefficient will add to the instability inherent in the reactor.

To see why this is so, consider the reactor to be in equilibrium and assume that the perturbation equations (equation 7-43) will apply after allowance is made for the temperature effect. Let ρ_r be replaced by ρ and define ρ as:

$$\rho(t) = \rho_r(t) + \rho_T(t) \quad (7-54)$$

$\rho(t)$ is, then, the actual reactivity acting to disturb the equilibrium. $\rho_r(t)$ is, as before, the reactivity produced by motion of the regulating rod. $\rho_T(t)$ is the reactivity produced by temperature changes.

If $\rho_r(t)$ is changed in some fashion, the reactor power or n will change. Following the power change, the core temperature will change. The change in temperature will produce ρ_T through the temperature coefficient and finally $\rho(t)$ will change because of the change in ρ_T . It is now necessary to make some assumption as to how temperatures change when power changes. For simplicity let the temperature change be directly proportional to the power change. This is an unrealistic case because it neglects the heat capacity of the fuel plates, moderator and coolant. It will be useful only in showing the self-regulating effect of the negative temperature coefficient. It is now possible to write the differential equations that describe the behavior of a reactor having a temperature coefficient c_T assuming, as before, small deviations from the state of equilibrium.

$$\frac{d\delta n}{dt} = \frac{n_0}{\ell^*} \rho(t) - \frac{\beta}{\ell^*} \delta n + \bar{\lambda} \delta c \quad (7-55a)$$

$$\frac{d\delta c}{dt} + \bar{\lambda} \delta c = \frac{\beta}{\ell^*} \delta n \quad (7-55b)$$

$$\rho(t) = \rho_r(t) + \rho_T(t) \quad (7-55c)$$

$$\rho_T(t) = c_T \delta T(t) \quad (7-55d)$$

$$\delta T(t) = A \delta n \quad (7-55e)$$

A = a positive reactor constant.

The reactor transfer function is the same as before (replace ρ_r by ρ). Equations (7-55c) through (7-55e) can be transformed simply by replacing t by S .

$$\delta n(S) = \frac{n_o}{\ell^* S} \frac{S + \bar{\lambda}}{[S + (\bar{\lambda} + \beta/\ell^*)]} \rho(S) \quad (7-56a)$$

$$\rho(S) = \rho_r(S) + \rho_T(S) \quad (7-56b)$$

$$\rho_T(S) = c_T \delta T(S) \quad (7-56c)$$

$$\delta T(S) = A \delta n(S) \quad (7-56d)$$

coupling
equations

Solving equations (7-56a) through (7-56d) algebraically for $\delta n(S)$ in terms of $\rho_r(S)$,

$$\delta n(S) = \frac{n_o}{\ell^*} \left[\frac{S + \bar{\lambda}}{S^2 + (\bar{\lambda} + \frac{\beta}{\ell^*} - A c_T \frac{n_o}{\ell^*}) S - A c_T \bar{\lambda} \frac{n_o}{\ell^*}} \right] \rho_r(S) \quad (7-57)$$

Equation (7-57) is the "reactor transfer function with temperature feedback." It has the characteristic equation:

$$\Delta(S) = S^2 + \left(\bar{\lambda} + \frac{\beta}{\ell^*} - A c_T \frac{n_o}{\ell^*} \right) S - A c_T \bar{\lambda} \frac{n_o}{\ell^*} = 0 \quad (7-58)$$

Figure 53 shows qualitatively the effect of the temperature coefficient on the response of a reactor to a positive step function of reactivity ρ_r .

FIGURE 53
Response curves of a reactor
to a positive step function of reactivity ρ_r .

To be more realistic in considering the effects of temperature feedback necessitates rewriting the coupling equations (equation 7-56) to include the effects of heat transfer rates for fuel, moderator and coolant. The complexity of the corresponding reactor transfer function with temperature feedback will depend on the number of equations required to accurately describe all the heat transfer processes.

In general, $\delta T(S) = A \delta n(S)$ is given by the transfer function,

$$\delta T(S) = \frac{N_T(S)}{\Delta_T(S)} \delta n(S) \quad (7-59)$$

where

$N_T(S)$ and $\Delta_T(S)$ are polynomials in S . This equation is derived from the differential equations describing the heat transfer processes in the core, heat exchangers, cooling towers, etc, in the same manner that the reactor transfer function was derived from the differential equations of reactor kinetics.

In the notation of servo theory* the interaction between temperature effects and reactor power is described by the block diagram of figure 54.

FIGURE 54
Interaction between temperature effects
and reactor power block diagram

*Popular name for theory of servomechanisms.

The function $\frac{\delta n(S)}{\rho(S)}$ is the reactor transfer function (equation 7-56a). $\frac{\delta T(S)}{\delta n(S)}$ is the temperature feedback transfer function. The symbol

states that $\rho_r(S)$ adds to $\rho_T(S)$ to produce $\rho(S)$. This is obviously a pictorial representation of equation (7-56b). The order of interaction, reaction or signal flow occurs in the direction of the arrows. The interpretation of the block diagram could be the following: (1) ρ_r changes and immediately ρ changes; (2) the change in ρ acts through the reactor transfer function $\delta n(S)/\rho(S)$ to produce δn ; (3) δn acts through $\delta T(S)/\delta n(S)$ to produce δT ; (4) δT acts through the temperature coefficient c_T to produce ρ_T ; and (5) ρ_T adds to ρ_r to modify ρ .

This cycle is repeated until the reactor achieves a new equilibrium. If c_T is negative, or in the case that c_T is positive δn will continue to increase without bounds.

The block diagram of figure 54 can be decomposed into the following set of equations:

$$\rho(S) = \rho_r(S) + \rho_T(S) \quad (7-60a)$$

$$\delta n(S) = \left[\frac{\delta n(S)}{\rho(S)} \right] \rho(S) \quad (7-60b)$$

$$\delta T(S) = \left[\frac{\delta T(S)}{\delta n(S)} \right] \delta n(S) \quad (7-60c)$$

$$\rho_T(S) = c_T \delta T(S) \quad (7-60d)$$

In addition to temperature changes produced by reactor power changes, it is obvious that the core temperature would change if the temperatures of the circulating coolant were to change. Also, variations in coolant flow would be felt as temperature changes. These two temperature variations would then act through the temperature coefficient to disturb the equilibrium. If these two effects are lumped and denoted by δT_e (the subscript e indicates extraneous changes), figure 54 can be modified to include this effect. The modified block

diagram is shown in figure 55. The reactor depicted in figure 55 is considered to have two forcing functions or inputs: the reactivity input ρ_r due to the regulating rod, and the extraneous temperature change δT_e due to coolant temperature or flow changes.

FIGURE 55
Interaction between temperature effects
and reactor power block diagram

It can be shown in this case that the reactor transfer function with temperature feedback can be written as: (7-61)

$$\begin{aligned} \delta n(S) = & \frac{\left[\frac{\delta n(S)}{\rho(S)} \right]}{1 - c_T \left[\frac{\delta T(S)}{\delta n(S)} \right] \left[\frac{\delta n(S)}{\rho(S)} \right]} \rho_r(S) \\ & + \frac{c_T \left[\frac{\delta n(S)}{\rho(S)} \right]}{1 - c_T \left[\frac{\delta T(S)}{\delta n(S)} \right] \left[\frac{\delta n(S)}{\rho(S)} \right]} \delta T_e(S) \end{aligned}$$

The theorem of superposition holds for transfer functions. This means that the response to ρ_r and δT_e can be calculated separately. The response due to simultaneous applications of ρ_r and δT_e is then obtained by simple addition of the two separate responses.

In the discussions to follow, it will be assumed that $\delta T_e = 0$.

3. Fission Product Poisoning

The effects of fission product poisoning and fuel depletion or burnup can be analyzed in the same manner as the temperature effects. Both these processes depend on reactor power. The end effect of both is production of reactivity which then affects reactor power. For this reason they are considered to be feedback mechanisms. They are incorporated into the analysis as shown in figure 56.

FIGURE 56
Reactor feedback block diagram

The following definitions apply:

ρ_P = reactivity due to poisoning

ρ_B = reactivity due to burnup

$\frac{\rho_B(s)}{\delta n(s)}$ = burnup feedback transfer function

$\frac{\rho_P(s)}{\delta n(s)}$ = poisoning feedback transfer function

The block diagram of figure 56 can be reduced to the equivalent block diagram of figure 57.

FIGURE 57
Reactor feedback block diagram
simplified

The transfer function $\frac{\delta n(s)}{\rho_r(s)}$ must now

include the effects of temperature, poisoning

and burnup. It can be shown that $\frac{\delta n(s)}{\rho_r(s)}$ is given by:

$$\delta n(s) = \frac{\left[\frac{\delta n(s)}{\rho(s)} \right]}{1 - \left[\frac{\delta n(s)}{\rho(s)} \right] \left\{ C_T \left[\frac{\delta T(s)}{\delta n(s)} \right] + \left[\frac{\rho_P(s)}{\delta n(s)} \right] + \left[\frac{\rho_B(s)}{\delta n(s)} \right] \right\}} \rho_r(s) \quad (7-62)$$

Equation (7-62) is the transfer function that should properly be used to represent the reactor because it contains the effects of the inherent feedback processes of the reactor.

The poisoning and burnup transfer functions are derived from the differential equations representing these processes.

Non-fissioning nuclei with large absorption cross sections for thermal neutrons are called poisons because they will capture neutrons that would normally be available for fissioning of the fuel. The reduction of thermal neutrons available for fission has the effect of reducing the effective multiplication factor of the reactor. It then follows that negative reactivity is produced by the presence of poisons. The reactivity due to any poison is defined by:

$$\rho_P(t) = -f \frac{\Sigma_P(t)}{\Sigma_{af}} = -f \frac{P(t)\sigma_P}{\Sigma_{af}} \quad (7-63)$$

where

f = thermal utilization factor in absence of poisons

Σ_{af} = total macroscopic absorption cross section of fuel

σ_P = microscopic absorption cross section of poison

$P(t)$ = number of atoms/unit volume of poison present in reactor at time t .

While many poisons are formed during the operation of a reactor, there are two isotopes (Xe-135 and Sm-149) which are present in sufficient quantities to appreciably affect the operation of the reactor. The Xe-135 cross section is about 3.5×10^6 barns. The Sm-149 cross section is about 5.3×10^4 barns. Of these two poisons, Xe-135 is the most important. The effects of Sm-149 will not be analyzed in this brief treatment of the poisoning problem. However, Sm-149 does have an effect that should be included in a more complete analysis.

The presence of Xe-135 is approximately governed by the following sequence of events.

Production of Xe-135

- (1) Each U-235 fission produces a fraction γ_I of I-135 and a fraction γ_X of Xe-135 ($\gamma_I = 0.061$, $\gamma_X = 0.002$).
- (2) The radioactive I-135 decays to Xe-135 with decay constant $\lambda_I = 2.88 \times 10^{-5}/\text{sec}$.

Loss of Xe-135

- (3) The radioactive Xe-135 decays to Cs-135 (not a poison) with decay constant $\lambda_x = 2.1 \times 10^{-5}/\text{sec}$.
- (4) Xe-135 captures a neutron and goes to Xe-136 (not a poison).

At any time t the concentration of Xe-135, $P_x(t)$, is governed by the differential equations:

$$\left. \begin{aligned} \frac{dI}{dt} &= \gamma_I \Sigma_f \phi(t) - \lambda_I I \\ \frac{dP_x}{dt} &= \gamma_x \Sigma_f \phi(t) + \lambda_I I - \lambda_x P_x - \sigma_x P_x \phi(t) \end{aligned} \right\} \quad (7-64)$$

where

- I = concentration of I-135 atoms
- P_x = concentration of Xe-135
- Σ_f = macroscopic fission cross section of fuel
- ϕ = thermal neutron flux
- γ_I, γ_x = fission yields
- λ_I, λ_x = decay constants
- σ_x = Xe-135 absorption cross section

Two special cases of equation (7-64) are normally of interest. The first case assumes that the reactor starts up from $\phi = 0$ ($P_x = I = 0$) and is taken to operating power (ϕ_0) very quickly (step-wise). ϕ_0 is sustained until equilibrium between production and loss of P_x is attained. The first question is now asked: How much negative reactivity has been

produced? At equilibrium $\frac{dI}{dt} = \frac{dP_x}{dt} = 0$.

Solving equation (7-64) for P_x under this condition and $\phi(t) = \phi_0$ gives:

$$P_{x_0} = \frac{\Sigma_f (\gamma_I + \gamma_x) \phi_0}{\lambda_x + \sigma_x \phi_0}$$

The total reactivity produced by poisoning at equilibrium is then:

$$\rho_{P_{eq}} = -f \frac{\Sigma_f}{\Sigma_{af}} \frac{(\gamma_I + \gamma_x) \sigma_x \phi_0}{\lambda_x + \sigma_x \phi_0} \quad (7-65)$$

It is apparent that very high flux reactors will have approximately no more poisoning than:

$$\rho_{P_{eq}} \Big|_{\max} \approx -f \frac{\Sigma_f}{\Sigma_{af}} (\gamma_I + \gamma_x) \quad (7-66)$$

If the fuel is U-235, $\Sigma_f / \Sigma_{af} = 0.84$, f may be in the range of 0.8 to 1.0. For very high fluxes $\rho_{P_{eq}} \Big|_{\max}$ will be in the range of 0.043 to 0.053.

If a reactor is to achieve criticality at the flux ϕ_0 , it must have, built in, at least $\rho_{P_{eq}}$ positive reactivity.

The time history of $\phi(t)$ and $\rho_P(t)$ from startup to equilibrium is shown in figure 58.

FIGURE 58
Time history from reactor startup to equilibrium

Next consider that the reactor has been in operation at power (ϕ_0) for at least 50 hours. The poisoning has then very nearly attained equilibrium. At some later time, τ , the reactor is shut down. The second problem concerning poisons then arises. Figure 59 shows how $\rho_P(t)$ varies with time after shutdown of the reactor.

FIGURE 59
Variations of $\rho_P(t)$ with time after reactor shutdown

Figure 59 can be obtained by plotting the solutions of equation (7-64) for the regions $t < \tau$ and $t > \tau$.

The increase in poisoning is explained by the following sequence of events: (1) just prior to time τ , there are as many Xe-135 atoms being produced by fission and decay of I-135 as there are being lost by decay of Xe-135 and by neutron absorption; (2) just after time

τ , the flux is zero and the loss of Xe-135 by neutron absorption as well as the production by fission no longer exists; and (3) production by I-135 decay and loss by Xe-135 decay continues. Since λ_I is greater than λ_X the production will temporarily exceed the loss.

Depending on the value of ϕ_0 prior to shutdown, the peak will occur at about 10 to 12 hours after shutdown and may be many times the value $\rho_{P_{eq}}$. The height of the peak is very sensitive to ϕ_0 . For this reason, Xe-135 poisoning is not a very serious problem in low flux reactors. Conversely, in high flux reactors this is a serious problem. If it is desired to bring the reactor back up to power before 50 hours after shutdown, the reactor has to have enough positive reactivity built in to "override" the poisoning. For an equilibrium flux of 2×10^{14} , the peak after shutdown is worth about 50 percent in negative reactivity. Building this much positive reactivity into a reactor poses some serious safety problems.

The MTR has enough positive reactivity built in to override poisoning within about 30 min after shutdown. If the reactor has not achieved criticality before that time, it is usually necessary to refuel or wait until the poisoning has returned to a level that can be overridden.

Starting with equations (7-63) and (7-64), it is possible to derive a poisoning feedback transfer function $\frac{\rho_p(s)}{\delta_n(s)}$. The procedure would

be identical with that used in deriving the reactor transfer function. First, perturbation equations are obtained, then the transfer function is derived from the perturbation equations. The effect of Xe poisoning feedback on reactor stability will be discussed qualitatively. Depending on power level, a reactor with negative temperature coefficient may be unstable because of Xe poisoning. At power levels where instability exists, the power would tend to oscillate with limited amplitude. The saving feature of this effect is that the frequency of oscillation would be about one to two cycles per "day." The extremely low frequency of oscillation and the limited amplitude make it possible to neglect the effect of Xe in control system analysis. (For a more detailed discussion of the Xe stability problem, see ref 4 at the end of this chapter.)

4. Burnup or Fuel Depletion

The depletion of fuel in a reactor may be approximately represented by the following equations if production of Pu-239 and other fissionable isotopes is neglected.

$$\frac{dU}{dt} = -U \sigma_f \phi \quad (7-67a)$$

$$P = \frac{V}{3.1 \times 10^{10}} U \sigma_f \phi \quad (7-67b)$$

where

$$3.1 \times 10^{10} \text{ fissions/sec} = 1 \text{ watt power}$$

U = number U-235 atoms/unit volume

σ_f = fission cross section of U-235

ϕ = thermal neutron flux

P = reactor power in watts

V = volume of reactor

Solving equation (7-67b) for $U \sigma_f \phi$ and substituting in equation (7-67a)

$$\frac{dU}{dt} = \frac{-3.1 \times 10^{10} P}{V}$$

but since

$$U = \frac{m}{V} \frac{A_0}{235}$$

where

A_0 = Avogadro's number

m = mass of fuel

$$\frac{dm}{dt} = \frac{-3.1 \times 10^{10} \times 235}{A_0} P$$

Integrating

(7-68)

$$\delta m = m - m_0 = -1.2 \times 10^{-11} \int_0^t P(t) dt$$

δm represents the loss in fuel after operating for a time t at power P watts. The reactivity associated with this amount of fuel depletion will be denoted by ρ_B and is proportional to δm

$$\rho_B = -B \int_0^t P dt \quad (7-69)$$

where

B is a constant of proportionality.

A burnup feedback transfer function $\frac{\rho_B(S)}{\delta n(S)}$ can be derived from equation (7-69). It is unnecessary however, because depending on the reactor power, fuel depletion is generally a slow process. All that is required to control this process is a very slow continuous motion of the regulating rod. This is easily accomplished and introduces no difficult control problems. For this reason the effects of burnup need not be included in the reactor transfer function.

Having accounted for poisoning and burnup, the reactor transfer function to be used in control system design is equation (7-57). Notice that the temperature feedback effect has been retained.

5. Power Level Control System

An automatic control system designed to maintain a constant power level would resemble that shown in figure 60. It works according to the following schemes.

As long as the power level desired is n_0 , the demand signal change, δn_{DEMAND} , will be zero. The reactor output will be n_0 and the change in output δn_{OUT} will be zero. It will be the job of the control system to keep δn_{OUT} equal to zero, even if extraneous processes produce reactivity ρ_e which tends to change the reactor power. If ρ_e is introduced, the power will change according to the reactor transfer function by an amount δn_{OUT} . This change will be detected by the detector and passed on to the comparator. The comparator will compare δn_{DEMAND} with $\delta n_{\text{MEASURED}}$. If they are not equal, a voltage ϵ will be generated. ϵ will be amplified to the value V. The voltage V drives the amplidyne, which in turn drives the motor. The regulating rod driven by the motor will move in such a way as to produce a reactivity ρ_r that will return δn_{OUT} to zero and force the condition $\rho = \rho_r + \rho_e = 0$.

It may now be desired to change the power level from n_0 to $n_0 + \delta n_{\text{DEMAND}}$. In this case the demand signal to the control system is δn_{DEMAND} . Prior to the application of the demand, δn_{OUT} is zero, and also $\delta n_{\text{MEASURED}} = 0$. Upon application of the demand the comparator notices the difference between demand and output. A signal ϵ is generated

FIGURE 60
Automatic control system
maintaining constant power level
block diagram

and amplified to V , and a ρ_r is produced. The reactor responds to the new ρ_r by changing from the power n_0 to $n_0 + \delta n_{OUT}$. This change is measured and compared with δn_{DEMAND} . If they are not equal, a new ϵ is generated and the cycle continues until $\delta n_{OUT} = \delta n_{DEMAND}$. At this time the output will be at the desired power level $n_0 + \delta n_{DEMAND}$. In addition the regulating rod has returned to its original position, and the condition $\rho = \rho_r + \rho_e = 0$ is still satisfied.

Although these two schemes have been described as a sequential operation, most power level control systems are designed for continuous operation. By this it is meant that very small changes are continuously detected and corrected. For this reason the regulating rods on such systems can be seen to be in continuous motion. This motion is erratic in nature and is superimposed on the motion required for compensation of poisoning and burnup. The reason for this erratic motion of the regulating rod is that the control system is trying to compensate for the statistical fluctuations of the reactor power.

Some power level control systems are on-off devices. On-off control differs from continuous control in that no corrective measures are taken until the difference between demand and output equals or exceeds a preset value. When this threshold is reached, the regulating rod is driven at full speed in the proper direction to bring $\delta n_{DEMAND} - \delta n_{OUT}$ back below the threshold value. Obviously, on-off control is a "jerky" process by nature. For applications where smoothness of operation is not a prime requirement, an on-off controller would be satisfactory and would generally cost less to build and operate than a continuous controller.

a. Control System Analysis and Design

Before undertaking the actual design of a control system, the designer must determine what performance is required of the system. M. A. Schultz (ref 4) suggests that various systems and components be examined by four criteria in order to arrive at the best control configuration for the problem. The four criteria are: (1) absolute stability - exponentially increasing oscillations are not allowed; (2)

relative stability - how many oscillations occur before a transient has essentially died out; (3) maximum power level excursion - the peak power attained for a given disturbance; and (4) restoring time of the system - the time for the transient to die out essentially.

With these criteria in mind, the designer begins the analysis of various possible systems.

There are two avenues of approach open to the designer. He can simulate the various systems on an analog computer and compare the transient responses directly according to the four criteria. This approach has the advantage that it enables him to see, on a time basis, exactly how the controlled reactor will behave. It has the disadvantage that it is sometimes difficult to see what changes should be made to obtain the desired results. The extent of this disadvantage, of course, depends on the experience of the designer and on his ability to predict the effect on the system of whatever changes he deems necessary to achieve the desired results.

The other avenue of approach is to perform the analysis by application of servo theory. Servo theory allows a systematic analysis of the problem. According to servo theory, the transient response of a system is closely related to the frequency response; therefore, the usual procedure is to plot the frequency response of the various components - alone and in combination - and to study the effects of changing such properties of the components as gains and time constants on the shape of the frequency response curves. Some short cuts exist for plotting amplitude and phase angle response curves. If the system transient response is to be the desired one, the amplitude and phase angle curves must possess certain specified features. The big advantage of this technique is that the necessary changes in components can usually be readily predicted. The big disadvantage of this technique is that it is not applicable to non-linear systems. This is particularly true if the system possesses more than one non-linearity. In this event the best approach is simulation on the analog computer. For an example of the application of servo theory in the analysis of a control system, see ref 6.

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CHAPTER VIII

REACTOR SAFETY

A. INTRODUCTION

During the initial critical experiment of the first nuclear reactor ever assembled, there were two all-important questions that had to be answered. The first was one of the uncertainty of obtaining a self-sustaining chain reaction, and was of prime importance. The second was one of safety. Could the reaction be stopped after it had been started? Then there was also the question of how fast the reaction could be stopped. The answers to these questions were known qualitatively on the basis of the information obtained prior to the criticality experiment in Chicago on December 2, 1942. It is remarkable that the behavior of the reactor was predicted as well as it was when the preliminary experiments were re-examined. The average prompt neutron lifetime is an elusive quantity to evaluate; the really precise measurements of the delayed neutron yields and their associated decay constants were not made until recently. The outcome of the Chicago experiment was no doubt modified a great deal by the fact that very small reactivities and extremely long periods were encountered. This was very comforting to Fermi and his associates. All on the project were relieved at the seemingly benign behavior of the reactor. We now know that the reactor periods and reactivities were in the range where the lifetime is relatively unimportant and in the range where the longer-lived delayed precursors are important. The precursors with long half lives were and still are easier to measure reliably.

Reactor safety is still a most important consideration. The Atomic Energy Commission supports a large reactor transients program of which the Spert Complex operated by Phillips Petroleum Company is a part. Through this program much information, both experimental and theoretical, is being obtained on the behavior of reactors during large transients. Even so, it continues to be the policy to establish most experimental reactor plants away from highly populated areas.

There is practically no chance of a large nuclear detonation involving a thermal reactor core, and very little chance of the detonation of a fast reactor as the latter is not specifically designed to be a weapon. The primary effects of the initial power excursion, should it occur, are not necessarily the main concern of those interested in over-all personnel protection on-and off-site. Reactors which operate at high power levels always have within their fuel assemblies a large concentration of fission products. It is this fission product inventory that is of major concern should some incident result in scattering of the fission products around the country. Containment vessels are answers in many cases to this particular problem.

Although the practice of reactor safety extends to the protection of off-site personnel, this chapter is devoted almost entirely to protection of the reactor and experiments.

B. REACTOR SAFETY PHILOSOPHY

The philosophy is extended to cover three basic situations in operating reactor plants: (1) incidents which are detected and corrected through the neutron level channels; (2) incidents which involve the reactor plant (external) directly and the reactor core indirectly; and (3) natural catastrophies or incidents caused by direct military action or sabotage. These will be discussed in order.

1. Accidents Involving Neutron Level

These represent the largest class of nuclear incidents. Accidents which could involve the core parameters, and hence the excess reactivity of the reactor are generally of immediate concern. It has been stated that there is little, if any, danger of detonating a thermal reactor. There is, however, a finite chance of a large reactivity incursion producing a power excursion large enough to damage the reactor. In all such credible accidents related to the MTR and ETR, injury to personnel would probably not occur.

Since the neutron level itself is the indirect cause of damage in an abnormal excursion, the safety instrumentation is designed to protect the core function by sensing the reactor neutron flux. In effect it is equivalent to monitoring the reactor power, since the long-term disproportionality may be compensated by the occasional re-positioning of the level-sensing chambers.

2. Accidents Which Involve the Reactor Plant Directly and Reactor Core Indirectly

An accident which fits this category is one in which the primary reactor coolant pumps fail. In this accident the removal rate of the heat produced in the core is reduced considerably. If nothing is done to modify the reactor power, the core and adjacent components would be destroyed or irreparably damaged. This would be much worse if, for example, there is an accompanying loss of tank pressure. To protect the reactor from damage resulting from failure in the external system monitoring devices are employed which are capable of modifying the reactor operation - usually by reducing power. The design of these devices is not simple, and each instrument is designed for its specific purpose. An example of a particular consideration is that of assigning the proper power reduction to a loss-of-flow signal. Most reactors which are cooled by a large amount of water flow have a finite time from pump power-failure to thermal burnout. Depending on the time available, the requirements of the monitoring system may be more or less severe. Therefore, the detecting and actuating devices must be designed for the job. In addition, these devices are "backed up" by another related signal which does not originate from the same source. Tank ΔP , for example would be a valid back-up for primary coolant flow.

3. Natural Catastrophes and Military Action

In this category there are usually few if any corrective measures possible. It is handled in the MTR partially by providing "seal-loops" in the primary water system so that a break in the main coolant line (which is underground) would not result in draining the reactor tank. This is a problem even if the reactor power were shut down at the instant the break occurred. The power as a function of time after shutdown follows the expression:

$$P \approx 0.07 P_0 t^{-0.2}$$

where

P_0 is the normal operating power in watts, and t is the time after shutdown in seconds.

Thus it can be seen that if even at shutdown the coolant were removed there is still enough heat production from the fission-product decay to melt a high-flux reactor core. Both the ETR and MTR have provisions for dumping water other than process water into the primary system if it becomes necessary.

C. STARTUP ACCIDENT

In the various phases of reactor operation the approach to criticality is one of the most sensitive to accident conditions. This is especially true for high-power reactors where the startup neutron level is many decades below the operating level.

Perhaps the most dangerous time in a reactor's history is the initial startup. There are no experimental measurements to rely upon: control rods are not calibrated; calculated critical loadings are only approximate; and background neutron levels are extremely low. Even with extraneous or built-in neutron sources many decades in neutron level must be covered to reach operating power. The real danger lies in the ability to detect and evaluate accurately this tremendous range in neutron level. All low neutron levels detectors are usually positioned within the core to give optimum sensitivity. Even so, to achieve reasonable statistics in the absolute neutron level or counting rate, the time response of the detecting circuit must be slow. Should the reactor suddenly become supercritical in the startup range, the power may increase to very high levels before any control can be applied.

The basis for the so called "startup accident" is as follows: (1) with the reactor at or near the startup level, positive reactivity is continuously inserted into the core: this presupposes some type of malfunction in the control rod drive mechanisms; (2) criticality is achieved at low neutron levels where neither the operator nor the instruments observe the rapidly changing multiplication; (3) all safety devices such as period indicators or low power level scrams are

assumed to be inoperative. The only shutdown mechanism is a high level scram (usually 120 percent to 150 percent of full power). Other inherent safety devices such as the temperature coefficient are assumed to be zero; (4) as the power level passes through the high level scram, the signal to scram is generated. The actual shutdown is not initiated until some delay time later. This delay actually exists in the electronic, magnetic and mechanical releases; and (5) assuming some form of negative reactivity to be in a useful position (ie, safety or "black" rods withdrawn), the scram proceeds according to the particular mechanical arrangement in question. In the MTR or ETR this would be a gravity drop of one or more safety rods. The acceleration on the rods is a function of the coolant flow. For full flow conditions it is ~ 3 g; for no flow, the coolant (water) slows the rod drop to an effective 0.7 g acceleration.

With the possible exception of having the reactor supercritical with no shutdown reactivity available, the "startup accident" is considered the most serious theoretical accident possible.

As defined in the previous chapter, the reactor kinetics equations are:

$$\frac{dn}{dt} = \frac{K(1-\beta) - \lambda}{\ell} n + \sum \lambda_i C_i ,$$

$$\frac{dC_i}{dt} = \frac{K\beta_i}{\ell} n - \lambda_i C_i , \quad i = 1, 2, \dots, 6$$

These equations may be solved using a known $k(t)$ input by numerical integration methods. The only drawback is the amount of time and labor involved to grind out the many thousands of arithmetical operations necessary. For practical applications, these solutions are computed using digital computers. If the transients do not cover many decades, the fastest solutions are obtained from analog computers.

As examples of startup accident calculations consider the following cases: Case A compares transients with the delayed neutron fraction as the parameter; Case B compares the effect of reactivity input rates; and Case C compares exact numerical calculations to approximate calculations.

1. Comparison of the MTR Fueled with U-235, U-233 and Pu-239

In August, 1958, the MTR received a full core of Pu-239. The delayed neutron fraction from Pu-239 is $\sim 1/3$ of that from U-235, which presents control and safety problems. Comparative "startup accidents" were calculated to help in this respect.

Table 19 summarizes these calculations. Figure 61 is a plot of the calculated power transients.

In general a set of calculated transients using β_T as the only parameter would be expected to show peak powers and total transient times to be inversely related to β_T and the minimum periods directly related to β_T . The Pu-239 and U-233 curves show these effects.

The values of the parameters listed in table 19 were chosen from MTR data to produce the worst possible conditions. For the experimental Pu-239 core startup, the scram level was reduced from 60 Mw to 40 Mw (n_f). Figure 18 shows the Pu-239 transient to be less dangerous than the "standard" U-235 transient if the scram level is set at n_f and the reactivity insertion limited to $2.22 \times 10^{-3} \delta k/\text{sec}$. Inspection of figure 61 shows the integrated energy and peak power are greatest for the U-235 transient. From these results startup of the Pu-239 loading was assumed safe in comparison to the normal U-235 core.

2. MTR Double Speed Rod Drives

In a thermal reactor such as the MTR, the neutron flux-density is high enough to enable xenon growth to become a problem. It manifests itself as a loss of operating time in the MTR which must operate with regularly programmed shutdowns for maximum efficiency. After equilibrium conditions have been obtained in the MTR core, a power reduction will upset the balance between the production of iodine-135 and the burnup of xenon-135, its daughter product. Enough of the gross fission products are iodine-135 so that its decay (after shutdown) produces Xe-135 at a rate which will eventually reach the equivalent of about 50 percent reactivity in the MTR core. Since at startup only about 18 percent excess reactivity is built in the MTR core, there is never enough excess reactivity to overcome xenon poisoning for much longer than a few minutes after a shutdown. This becomes worse as the fuel is burned up, so that near the end of a scheduled cycle there is virtually no time

FIGURE 61
MTR startup accident for U-233,
U-235 and Pu-239

Table 19
MTR Calculations for U-235, Pu-239 and U-233 Transients

Fuel	U-235	Pu-239	U-233
$\beta_T(3)$	0.00755	0.00247	0.00305
l (sec)	2×10^{-4}	1×10^{-4}	2×10^{-4}
Withdrawal Rate (in./min)	5	5	5
$R \delta k/\text{sec}^*$	2.22×10^{-3}	2.22×10^{-3}	2.22×10^{-3}
Scram Acceleration**	-3 g	-3 g	-3 g
n_0 (Megawatts)	4×10^{-6}	4×10^{-6}	4×10^{-6}
n_1 (Megawatts)	60	40	40
τ (seconds)	0.015	0.015	0.015
n_2 (Megawatts)	76.0	57.5	51.8
T min (seconds)	0.063	0.041	0.058
n_{max} (Megawatts)	88.6	70.5	62.0

* The entire shim rod bank worth 40 percent was used.

** Full water flow.

for any power manipulation. Practically all experiments in the MTR core area are tied into the reactor safety system and are capable of various kinds of power reductions from very slow rates to a full-fledged scram. These scrams are unpredictable, of course, and when they happen it is sometimes impossible to operate the reactor without refueling. The alternative is to wait 60 hours for the xenon to decay. Since, for other reasons it is impractical to build in 50 percent excess reactivity (the k_∞ of the present MTR is about 1.6), the alternative is to reduce the rod recovery time. The superintendent of the MTR estimates that saving only 3 minutes in rod recovery will reduce the number of unrecoverable scrams by 10 percent. A method which is studied here involves doubling the rod rundown time which will save the 3 minutes. Such a study always includes the evaluation of an accidental withdrawal - though not permitted - which is described in this chapter. Two basic calculations were necessary: (1) a comparison of the "double speed reverse" to a normal reverse; and (2) comparative startup accident calculations.

Tables 20 and 21 summarize these calculations in conjunction with figures 62 and 63.

In the "reverse" calculations, three cases were considered: (1) normal speed reverse

(4.5 in./min) using all rods (40 percent δk_{TOT}); (2) "jr scram," that is one rod (3 percent δk_{TOT}) dropped with 3 g acceleration; and (3) the double speed reverse (9 in./min) with all rods (40 percent δk_{TOT}).

As expected the double speed reverse is intermediate between the jr scram and normal setback. (See figure 62.)

The startup accident calculations involved three cases: (1) double speed withdrawal (9 in./min) of all rods (40 percent δk_{TOT}) to level scram set at 60 Mw. The scram is produced by dropping one rod (10 percent δk_{TOT}) with 3 g acceleration (full water flow); (2) normal speed withdrawal (4.5 in./min) to compare with the above; and (3) double speed withdrawal involving a period scram at 0.5 sec where one rod is released with 0.7 g acceleration (no water flow).

In the actual calculations the period scram was not initiated at 0.5 sec, but rather at the instrumental power of 40 watts. The 0.5 sec period was reached when the power was 27 watts - too low for instrument operation.

All curves were numerically integrated to give the energy release. It is of special interest to note that the energy release of the two level scrams, 12.6 Mw-sec for the double

Table 20

MTR Power Reverse Data

	Normal Reverse	Double Speed Reverse	Jr. Scram
$\beta_T(3)$	0.00755	0.00755	0.00755
l (sec)	2×10^{-4}	2×10^{-4}	2×10^{-4}
Setback Rate (in./min)	-4.5	-9.0	
No. of Rods*	All (40% δk_T)	All (40% δk_T)	One (3% δk_T)
Scram Acceleration**			-3g
n_0 (relative)	1.0	1.0	1.0

* The reactivity was used in the form

$$k_{ex} = \frac{\delta k_T}{2} \left[\cos \frac{\pi}{2} \frac{x}{x_m} - \cos \frac{\pi}{2} \frac{x_0}{x_m} \right]$$

$$x_0 = 26 \text{ in.}, \quad x_m = 31 \text{ in.}$$

** Full water flow.

Table 21

MTR Double Speed Startup Accident

	Double Speed		Normal
	Period Scram	Level Scram	Level Scram
Rods Withdrawn	All (40% δk_T)	All (40% δk_T)	All (40% δk_T)
Scram Acceleration	-0.7g	-3g	-3g
Rods Scrammed	One (10% δk_T)	One (10% δk_T)	One (10% δk_T)
n_0 (Mw)	4×10^{-6}	4×10^{-6}	4×10^{-6}
n_1 (Mw)	4×10^{-5}	60.0	60.0
τ (sec)	(0.5 sec period) 0.015	0.015	0.015
n_{max} (Mw)	4.54×10^{-5}	120.2	89.1
Energy (Mw-sec)	3.24×10^{-5}	12.6	12.2

FIGURE 62
Reverse comparison
MTR double speed rod drives

FIGURE 63
 Startup accident
 MTR double speed rod drives

Table 22
ETR Power Overshoots

	No Flow	Full Flow
$\beta_T(3)$	0.00755	0.00755
l , sec	5×10^{-5}	5×10^{-5}
Withdrawal rate in./min	16.363	16.363
R (δk /sec)	2.105×10^{-3}	2.105×10^{-3}
Scram Accel- eration	-0.7 g	-3 g
n_0 (Mw)	5.25	5.25
n_1 (Mw)	263	263
τ (sec)	0.040	0.030
$n_{\max}$	347	317
Energy (Mw-sec) with Delayed Neutrons	142	125
Energy (Mw-sec) no Delayed Neutrons	104	54

speed and 12.2 Mw-sec for the normal speed withdrawal, show no excessive danger for the double speed case.

3. ETR Startup Accident

The ETR has an operating power of 175 Mw. In the operation of the reactor there is a need for both a low level and a high level scram. This is accomplished by allowing a contact on the log-level recorder to reduce the input sensitivity on the linear level chambers by a factor of ten when the power level reaches $3 n_L$ (5.25 Mw). If this change is a quick one, it is possible that the low level scram could be negated before the signal actually reaches the low level scram. The calculations described below reveal that the premature switching must be prevented. The method now in effect puts a deliberate delay in the range switching device so that for short periods the low level scram can prevent a large power overshoot. (See figure 63.)

The calculations are based on the accidental withdrawal of one black rod and 12 gray rods at high speed when the reactor is initially critical at 3 percent of full power (5.25 Mw). Two conditions were investigated: (1) no coolant flow, $\tau = 0.040$ sec, and $a = 0.7$ g; (2) full water flow, $\tau = 0.030$ sec, and $a = 3$ g. The data and results are listed in table 22.

For comparison the power overshoots were also calculated assuming no delayed neutrons. As seen in figure 64 these transients underestimate the energy release.

4. Startup Control

For low power startup many restrictions on the modes of operation are employed to prevent a "startup accident." Since a reactor may become critical at any power level, one good approach, especially for unknown systems, is to let the subcritical multiplication build up the neutron level to a comparatively high level before making the final approach to criticality. The main disadvantage to this procedure is the amount of time required, in the order of hours.

Under special conditions the maximum possible rate of reactivity insertion can be reduced. Also various methods of on-off withdrawal are used to reduce the hazard of a "startup accident."

For protection devices the reactor relies upon some form of power-level and period detection. Since period measuring instruments are basically differentiating circuits, their output can be very noisy especially for low level statistics. Period measurements are usually secondary safety devices when combined with a good neutron-power level scram arrangement.

One such arrangement is the multiple-fixed scram level system. Here the scram levels are imposed at each decade of power. As the startup proceeds, the operator manually observes the reactor conditions. If everything seems normal as a particular scram level is approached, he manually bypasses the scram permitting the power to increase to the next scram level. If the reactor runs away, an early scram will occur.

Another system is the variable power level scram. The scram level is constantly set at an amount above the actual power for slow increases. The time response of the

FIGURE 64
ETR power overshoot study

associated circuit is slow enough so that if the power level changes quickly, the scram would still be encountered.

5. Temperature Coefficient

So far very little has been said about the reactor temperature coefficient. Normally the temperature coefficient is assumed to be zero in a "startup accident" calculation. Except for high powers this assumption is fairly good. The kinetic behavior, due to the temperature coefficient, is strongly dependent on the type of reactor in question. For homogeneous reactors the effect of the temperature coef-

ficient may be considered practically instantaneous with a power change. For heterogeneous systems this is not true. In a reactor such as the MTR the heat is developed in a fuel plate and then transferred to the coolant-moderator. One would then expect to see some sort of delay time for heat generation to the effect of the temperature coefficient. The actual functional form for this delay may be very complicated. In the extreme accidents experimental evidence shows most reactors are self-regulating either through the effect of a negative temperature coefficient or expulsion of moderator.

6. Cold Water Accident

Assuming the reactor displays a large negative temperature coefficient, accident conditions will exist if the coolant temperature suddenly drops. The sudden drop in coolant temperature introduces positive reactivity into the system. As an example assume the temperature coefficient of a reactor is $-5.0 \times 10^{-4} \rho/^{\circ}\text{F}$ and the coolant drops 50°F , then the resultant reactivity is 2.5 percent. This is an extreme example in view of some of the SPERT transients. In all probability the reactor would be destroyed before power generation could heat the coolant and introduce negative reactivity.

7. Control Rod Position

The position of the control rods in a reactor is very important for safety considerations. The rod effectiveness varies as $\int \sin^2 x \, dx$ with position. If the safety rods are completely withdrawn when a scram is initiated, the actual reactivity input will be very small for a relatively large time. This condition produces the same effect as a delay time at the scram level. For example if the rod is required to fall 4 in. under a 1 g acceleration before a detectable amount of reactivity is introduced, the effective delay time will be 144 milliseconds. Compared to figure 61, this additional

delay could be disastrous. If the reactor were on a 50-millisecond period, the power would increase by an additional factor of 20.

8. Reactor Startup at High Neutron Levels

The basic difference between high power startup compared to the low power startup is the range of power covered, and thus there is a difference in minimum possible reactor periods.

When a reactor scrams after high-power operation the neutron level decays on an 80-sec period. This effect, due to the delayed neutron emitters, combined with the inherent neutron background of an old operational reactor produces a neutron startup level considerably higher than the initial startup level. Thus the minimum period, peak power and energy release of high-power startup is not as severe as the cold-clean "startup accident."

For normal operations scram levels are usually set at 120 percent to 150 percent of full power. The calculated accident transients using various reactivity insertion rates yield a family of curves where the peak power and energy release are criteria of the reactor safety. From such calculations maximum rod drive speeds may be determined.

* * * *

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