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Elastic Wide-Angle Hadronic Scattering

by

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The most recent ISR elastic pp data is compared with predictions of the SVNVM (gluon) model. The model is used to examine and predict qualitative features of high-energy pp, πp , $p\bar{p}$ elastic, $\gamma p \rightarrow \pi^+ n$, and $F_{\pi}(t)$ at very large momentum transfers.

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Recent experiments on elastic, hadronic scattering at wide-angles¹ require a theoretical analysis qualitatively different from Regge descriptions valid for forward and backward peaks. One such "gluon" model has previously been employed to produce qualitative agreement with proton electromagnetic form factor,² proton-proton scattering,³ and proton-neutron scattering⁴ data, at pre-ISR machine energies. In view of the latest ISR pp experiments, it becomes appropriate to evaluate the formulae of reference 3 at ISR energies, and to make qualitative predictions for previous and anticipated^{5,6} πp , $p\bar{p}$, pion form factor, and photopion production experiments at large momentum transfers.

The multiple exchanges which appear to be necessary at larger angles are here given by soft, virtual, neutral, vector mesons (SVNVM, e.g., ω and ρ_0) exchanged between the external legs of specific hadronic scattering amplitudes. For forward (or backward) directions, this field-theoretic picture must be amended to include eikonal models of increasing complexity, as the exchanged NVM are themselves allowed to possess structure.⁷ For non-forward (-backward) directions, not all the exchanged vector mesons may be considered soft, and a development may be made⁸ of the entire amplitude in terms of an increasing number of non-soft, or "hard" NVM. A single hard meson exchange, plus all possible SVNVM exchanges, have been combined into an approximate, wide-angle formula of form³

$$\frac{d\sigma}{dt}(s,t) = \left(\frac{d\sigma}{dt}\right)_H \cdot S(s,t) \quad (1)$$

in which $\left(\frac{d\sigma}{dt}\right)_H$ denotes the differential cross section calculated from the Born approximation for single (hard) meson exchange, while

$S(s, t)$ represents the strong damping effect of all exchanged SVNVM. For pp scattering, it is given by

$$S = \exp \left[\gamma \left(F(t) + F(u) - F(4m^2 - s) \right) \right] \quad (2)$$

where $F(x) = 1 - (2x+1) [x(x+1)]^{-\frac{1}{2}} \ln [\sqrt{x} + \sqrt{x+1}]$, $x \equiv -t/4m^2$,

and γ is a parameter combining nucleon-NVM coupling constant g , exchanged NVM mass μ , and a soft-momentum cut-off μ_c .⁹

$$\gamma \approx \frac{g^2}{8\pi^2} \ln \left(1 + \mu_c^2/\mu^2 \right). \quad (3)$$

Here, both ω and ρ_0 exchanges are possible, so that $g^2 \rightarrow g_{\rho\rho}^2 + g_{\omega\rho}^2$; in np scattering there is a distinction between soft isoscalar and isovector exchanges. In fitting previous data,^{2,3,4} γ was chosen to be ≈ 2.3 . In this note we shall continue to use the same μ_c/μ ratio for virtual mesons emitted by both nucleons and pions, but distinguish between the possible $g_{\rho\rho}$, $g_{\omega\rho}$, $g_{\rho\pi\pi}$ couplings.

In reference 4, curves have been plotted for a variety of s values (s, t, u in $(\text{Gev})^2$), $41.7 \geq s \geq 11.3$, and over corresponding regions of all wide-angle $|t|$ values, using two (almost equivalent) models for $\left(\frac{d\sigma}{dt} \right)_N$, of form ($s \gg 4m^2$)

$$\left(\frac{d\sigma}{dt} \right)_N = \frac{C}{s^2} \phi(s, t, u) \quad (4)$$

where (for $|t| \sim s/2$) $\phi \sim O[1]$ for a single hard vector exchange, $\phi \equiv 1$ for a single hard π^0 exchange, and C is an appropriate normalizing constant. With just two parameters, C and γ , one obtains a rather good reproduction of the experimental data, which appears to be bounded from below by $G^4(t)$. According to the new $s = 2800$ ISR data, after the first dip at $|t| \gtrsim 1$, one finds $\frac{d\sigma}{dt} \sim 10^{-2} \times G^4(t)$, up to the maximum measured $|t| \approx 4.3$. At present, the forward (shrinking) diffraction peak can only be described

phenomenologically; and it may be adjusted, by hand, to fit smoothly onto the wide-angle amplitude which, by itself, yields (1). According to (4), however, one expects a decrease of the wide-angle $\frac{d\sigma}{dt}$ in going (for example) from $s=38$ to $s=2800$ of amount $(38/2800)^2 \sim 2 \times 10^{-4}$. This is two orders of magnitude more than suggested by the data run at $|t|=4.3$. Since the region of applicability of (1) just begins at $|t| \sim 4$ - when the forward diffraction amplitude falls well below the (single hard, plus all soft exchanges of the) wide-angle amplitude - we expect a further decrease of $\frac{d\sigma}{dt}$ at larger $|t|$ by another two orders of magnitude. Unfortunately, this will probably be somewhat difficult to measure.

This estimate is based upon the use of the Born approximation for $(\frac{d\sigma}{dt})_h$, together with the observation that $S(s,t) \sim \exp[4\delta F(t)] \sim G^4(t)$ for $s \gg |t| \gg m^2$. If experiments which can be performed in this region show less of a fall off with increasing s , but continue to display the gentle, wide-angle curvature of $S(s,t)$ out to values of $|t|$ approaching $s/2$, it may be taken as an indication that a more complicated $(\frac{d\sigma}{dt})_h$ is necessary. Even at $|t| \sim 4$ it is difficult to match the experimental ratio of $\frac{d\sigma}{dt}$ for $s=2800$ to that of $s=38$ with just a single hard vector exchange. It is also possible to employ other, slightly more complicated forms for $S(s,t)$.⁹ But the clear deviation of $\frac{d\sigma}{dt}$ from $G^4(t)$ at $s=2800$ and $|t| \sim 4$ is by no means unexpected in the SVNVM model.

Based upon these ideas, it may be worthwhile to describe the essence of a similar calculation for large-angle, elastic πp scattering. The most recent $\pi^- p$ data at all angles is several years old, and at comparatively lower energies,⁵ but at $s \sim 18$ it does produce a picture (Figure 1) containing a sharp forward peak,

a dip at $|t| \sim 2$ followed by a slightly askew curve of points having a shallow minimum at $|t| \sim 13$, before rising to a backward peak several orders of magnitude smaller than the forward peak. One may again attempt to understand the qualitative behavior of the wide-angle region between forward and backward peaks as due to multiple SVNVM exchange between all the elastically scattered hadrons; the dips and bumps in this region may be thought of as "fine structure" of the Born, hard-exchange amplitudes. Again, we take the picture of a phenomenological diffractive amplitude dominant in the forward region, interfering with (in the region of the dip) and subsequently falling below a wide-angle amplitude, which by itself leads to a cross section of form (1). Here, however, the single hard exchange is more complicated than in the pp case, for there are clearly both s- and t- channel Born amplitudes to be considered. What we should like to observe is that, to within one-half an order of magnitude, easily provided by the more slowly varying $\left(\frac{d\sigma}{dt}\right)_*$, there is no difficulty in fitting the existing π^-p data to the dominant $S(s,t)$ dependence alone, over this entire region.

The curve given for $S(s,t)$ in Figure 1 is the simplest possible one which could be drawn, with χ parameters determined by reference to the pp calculation, and coupling constant ratios given by SU(3) and conserved isospin currents. Slight adjustment of these parameters can serve to bring $S(s,t)$ and a smoothed experimental $\frac{d\sigma}{dt}$ curve in the wide-angle region into almost perfect agreement; but this is perhaps premature, until $\left(\frac{d\sigma}{dt}\right)_*$ is specified in detail. Proceeding in exactly the same way as in references 2-4, one writes

$$\frac{1}{2} \ln S(s,t) = \gamma_{pp} F_{pp}(t) + \chi_{\pi\pi} F_{\pi\pi}(t) + 2\chi_{\pi p} [F_{\pi p}(u) - F_{\pi p}(2m^2 - s)] , \quad (5)$$

in which the pion mass m_π is neglected, compared to the proton mass m , whenever possible. Straightforward calculation shows that, while $F_{pp}(t)$ is given by the $F(t)$ of (2), $F_{\pi\pi}(t)$ may be represented (for $|t| \geq 1$) by its asymptotic value, $F_{\pi\pi}(t) \simeq -\ln(|t|/m_\pi^2)$; and that the combination $F_{pp}(u) - F_{pp}(2m^2 - s)$ has the approximate value $-\ln\left(1 - \frac{|t|}{s - m^2}\right)$, a simple form due to the smallness of the pion mass (which cancels out of the final combination). The cut-off ratio μ_c/μ is taken to be the same in every δ of (4), with differences between them given by the appropriate couplings. Thus, $\delta_{pp} = (g_{ppp}^2 + g_{\omega pp}^2) \cdot K$, while $\delta_{\pi\pi} = g_{\rho\pi\pi}^2 \cdot K$ and $\delta_{\pi p} = g_{\rho\pi\pi} \cdot g_{ppp} \cdot K$, corresponding to virtual ρ_0 emission by the charged pions, with additional ω exchange between the initial and final protons. From Gell-Mann's Table¹⁰ one determines $g_{\omega pp} = \sqrt{3} g_{ppp}$ and $g_{\rho\pi\pi} = +2 g_{ppp}$ for the emission of a ρ_0 by a π^- . Hence one obtains $\delta_{\pi\pi} = 2 \delta_{\pi p} = \delta_{pp} = 2.3$, with the numerical value of δ_{pp} taken from the previous fit to the proton data. A convenient and fairly accurate approximation to $F_{pp}(t)$ is given by $-\ln[1 + .4|t|]$, with $|t|$ in $(\text{Gev})^2$, and has been used in obtaining the $S(s, t)$ curve drawn in Figure 1.

The normalization of the curve was determined by matching $S(s, t)$ to the experimental $\frac{d\sigma}{dt}$ at $|t| = 6$. It is not difficult to see that the position of the minimum of this wide-angle curve occurs for

$$\frac{|t|}{s - m^2} = \frac{\delta_{pp} + \delta_{\pi\pi}}{\delta_{pp} + \delta_{\pi\pi} + 2 \delta_{\pi p}} \quad (6)$$

and can be varied somewhat by changing these parameters. On the other hand, for negative $\delta_{\pi p}$, $S(s, t)$ is monotonically decreasing as $|t|$ increases, over the entire wide-angle region; and hence one expects the experimental points for π^+p to fall much below those of π^-p , until the region of the backward peak is reached. In ef-

fect, the rise of the major portion of the backward peak in π^+p should be sharper than that of the corresponding backward peak in π^-p .

If the experimental, wide-angle π^+p curves at NAL energies follow a pattern similar to the lower-energy data of Figure 1, one may expect to be able to provide qualitative fits to $\frac{d\sigma}{dt}$ within the framework of the present model. For example, a Born amplitude corresponding to a single hard ρ_0 exchange should produce a $(\frac{d\sigma}{dt})_B$ of form (4), with $\phi \sim O[1]$; and hence one predicts a diminution of the complete differential cross section due to the explicit s^{-2} factor of (4).

Finally, we would like to make three brief remarks concerning related large-momentum transfer experiments which are just on the verge of being measured at asymptotic energies.

(1) The pion form factor may be thought of in a manner analogous to that of the proton, with an asymptotic behavior in part due to multiple SVNVM ρ_0 exchange,

$$F_\pi(t) = H(t) \exp[\gamma_{\pi\pi} F_{\pi\pi}(t)].$$

Here, $H(t)$ represents the non-soft part of the form factor, chosen as $1 - 2|t|/[m_\rho^2 + |t|]$, as in the proton case. Thus, with $\gamma = 2.3$ fixed by the proton form factor, and the choice $g_{\omega pp} = \sqrt{3} g_{\rho pp}$, one determines $\gamma_{\pi\pi} = 2.3$, and $F_{\pi\pi}(t) \sim |t|^{-2.3}$, the same asymptotic fall-off as that of the proton; for the choice $g_{\omega pp} = 3 g_{\rho pp}$, $\gamma_{\pi\pi} \approx .92$, and one finds a fall-off for $F_\pi(t)$ closer to that of the monopole type.

(2) Recent experiments¹¹ on elastic $p\bar{p}$ scattering at $s = 11.3$ indicate that $(\frac{d\sigma}{dt})_{p\bar{p}}$ falls well below the corresponding pp data over the entire wide-angle region, an effect predicted long ago² on the basis of the SVNVM model. Although the hard-exchange, Born approxi-

mation part of the amplitude has important s-channel contributions absent in the corresponding pp amplitude, the exponential factor of (2) now becomes

$$\exp \left[4\gamma (F(t) - F(u) + F(4m^2 - s)) \right] \quad (7)$$

which increases the $p\bar{p}$ damping relative to that of pp by a factor of $\exp \left[8\gamma (F(u) - F(4m^2 - s)) \right]$. Experimentally, the 90° cross sections are in a ratio of order 10^{-2} , while these soft exchanges alone, with $\gamma = 2.3$, lead to an effect of order 2×10^{-4} . However, the s-channel Born term for $p\bar{p}$ scattering may easily be imagined¹² to contribute an extra relative factor of order 10^2 , thereby increasing this ratio to its experimental value. A somewhat better test, less sensitive to the Born terms, is to measure the ratio of $\left(\frac{d\sigma}{dt}\right)_{p\bar{p}}$ at nearby wide-angle points, and to see how closely the soft exchanges match this experimental number. For example, using the $s = 11.3$ data, the ratio of (7) at $|t|=3$ to its value at $|t|=5$ is approximately 3×10^{-3} , while a value $\sim 10^{-2}$ is indicated by the experiments. It should be noted that at this energy there is still an extensive amount of dip-bump structure at wide-angles, which may well disappear as the energies increase.

(3) Photopion production experiments currently underway at SLAC¹³ exhibit the familiar wide-angle dependence on s and t. Applied to this reaction, one finds SVNVM dependence in $d\sigma/dt$ of form

$$\exp \left[2\gamma \left\{ a F_{pp}(t) - b (F_{\pi p}(u) - F_{\pi p}(2m^2 - s)) \right\} \right]$$

where $a = b = 1/2$ for $g_{wpp} = \sqrt{3} g_{ppp}$, and $a = 4/5$, $b = 1/5$ for $g_{wpp} = 3 g_{ppp}$. In this latter case, for example, one calculates the ratio of 90° cross sections at $s=15$ to that of $s=10$ as $(10/15)^{8\gamma/5} \sim .22$ for $\gamma = 2.3$. Multiplying this number by the ratio of anticipated s^{-2} Born approximation dependence, analogous to that

of (4), yields a factor of .44, thereby bringing this (asymptotic) theoretical ratio down to .098, which is of the same order as (just about double) the experimental ratio.

The simplicity of the SVNVM model invites comparison with a variety of wide-angle, high-energy processes, such as those discussed above. As the experimental energies increase, it will become appropriate to construct detailed fits to the data, of the same quality as that of reference 3. Applications of these ideas to hadronic bremsstrahlung processes have been given elsewhere.¹⁴

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8. H. M. Fried, "Functional Methods and Models in Quantum Field Theory", The MIT Press, Cambridge 1972; Chapter 9.

9. In references 2-4, the cut-off μ_c has been introduced in an arbitrary, but simple, covariant way which avoids the somewhat more complicated forms resulting from the retention of quadratic meson-momentum dependence in virtual nucleon propagators. The latter produce an $F(t) \sim -\ln^2 |t|$ rather than $\sim -\ln |t|$, which could conceivably be important for very large $|t|$.

10. M. Gell-Mann, "The Eightfold Way: A Theory of Strong Interaction Symmetry", Cal Tech Report CTSL-20 (1961), Table IV. Experimentally, $g_{\pi pp} \approx 3 g_{\rho pp}$, which shifts the $\pi^- p$ minimum to ^{slightly} larger values of $|t| \sim 13-14$.

11. V. Chabaud, et.al., "Elastic Scattering of Antiprotons on Protons at 5 GeV/c", CERN preprint 72-787.

12. Suppose there exists an s-channel π^0 contribution to $(\frac{d\sigma}{dt})_H$ constructed with coupling constant $g_{\pi pp}^2/4\pi \approx 15$. In the amplitude for such a γ_S/ρ_S interaction, there appears a factor $\sim \sqrt{S}$ at each vertex, multiplied by a pion propagator $\sim S^{-1}$; one is not allowed to argue that the πNN form factor will cut down the effective coupling, for in this model it is given by SVNVM exchanges, which must not be counted twice. The ratio $(g_{\pi pp}/g_{\rho pp})^2$ could then contribute the needed factor $\sim 10^2$.

13. We thank R. Anderson for permission to quote these results before publication.

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Figure Caption

Fig. 1. Qualitative fit of SVNVM exchange to wide-angle $\pi^- p$ scattering.

FIGURE 1