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GROWTH OF PHASE SPACE IN HIGH INTENSITY PROTON BEAMS
DUE TO NONUNIFORM RADIAL CHARGE DISTRIBUTION

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1. Introduction

With the increasing intensity and brightness requirements of ion beams - particularly protons - for injection into high energy accelerators the question of the influence of nonuniform radial distribution of charge in the beam on the growth of phase space or emittance area becomes increasingly important. Qualitative consideration was given to a special case of non-uniform charge distribution - namely that of electron trapping in the space-charge potential trough of a high intensity beam - by Turner.¹ Blewett² has recently investigated two cases of nonuniform charge distribution in proton beams drifting in free space and reaches the qualitative conclusions: a) that "any charge distribution in a dense beam other than a uniform one is undesirable", and b) that the "apparent emittance will increase for any non-uniform distribution".

This report summarizes the results of work aimed at establishing quantitative information on the rate of growth of apparent emittance area in the

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1. C.M. Turner, "Cosmotron Injection - Progress and Problems", BNL Accelerator Dept. (Cosmotron) Internal Report CMT-9, March 31, 1958.
 2. J.P. Blewett, "On the Rate and Type of Blowup of Dense Particle Beams", BNL Accelerator Dept. Internal Report AADD-98, February 10, 1966.

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drifting high intensity proton beams with nonuniform radial charge distributions considered by Blewett. These results clearly indicate that the effect is quantitatively significant, and is likely to be a factor contributing to the anomalous growth of emittance area in high intensity beams observed in the injection systems of the BNL Cosmotron, the BNL 33-BeV AGS, and the CERN proton synchrotron.

2. Techniques and Qualifications

Trajectory data. An iterative computer program was developed to compute trajectory data at desired radial intervals within the beam envelope, to provide the information necessary for drawing phase diagrams at any desired axial position in the drifting beam.

Definition and evaluation of "apparent emittance area". Since, by definition, the initial emittance area is zero, the phase diagrams are lines of zero width. For the purpose of this report "apparent emittance area" is defined as the area of the smallest ellipse required to enclose the phase diagram. Since the fitting of ellipses to a large number of phase diagrams is prohibitively time consuming, an alternative analytic technique was employed which makes use of the virtual source from which the individual trajectories appear to originate when projected backwards from the plane at which the emittance area is to be established. It can be shown (see Appendix) that the emittance area is equal to the product of the diameter of the virtual source and the angle of convergence of the virtual trajectories.

Trajectory crossovers. The computation of trajectories assumes no crossovers. If,

$$d/dr (E_r) \cong 0 \quad (1)$$

throughout the volume occupied by the beam, where E_r is the radial electric field, no crossovers can occur. This criterion shows that crossovers can occur in the outer portions of the triangular distributions with maximum intensity on the axis and zero intensity at the boundary - Cases IA and IB. These regions have been excluded from the evaluations of apparent emittance area.

Common initial conditions. The following initial conditions are common to all cases considered:

Ion species	-	Protons
Ion energy	-	780 keV*
Ion current	-	200 mA
Emittance area	-	0
Radial velocity	-	0

Program check. In order to establish the validity of the computer program, the case of uniform radial charge distribution was analyzed. The phase diagrams in this case should be straight lines for all drift distances since uniformly distributed space charge acts as a nonaberrant diverging lens. Figure 1 shows phase diagrams at various drift distances in a typical beam with uniform charge distribution.

3. Case I - Triangular Charge Distribution with Maximum on Axis

In this case the volume density of charge is a maximum on the axis and decreases linearly to zero at the boundary of the beam.

a) Case IA - 4.0 cm initial diameter. Figure 2 shows a family of phase curves at drift distances of 1, 2, 3, and 5 meters. The sharp hooks on the ends of these curves at the larger drift distances are due to trajectory cross-overs as noted above and these regions are excluded from the emittance area evaluations. Visual inspection clearly indicates a rapidly increasing apparent emittance area initially, but then it appears to decrease with the phase diagrams straightening out. It was not immediately clear whether this was a real or an apparent effect. However, analysis using the virtual source technique indicated that the effect was indeed real and that the apparent emittance area passed through a maximum at a drift distance of about 2 meters and decreased to near zero at 5 meters, as shown by Figure 3.

*780 keV rather than 750 keV was used in this work since the computer program used was initially written for another purpose and included a 30 keV ion source accelerating potential.

b) Case IB - 1.0 cm initial diameter. In order to magnify and hopefully clarify the puzzling effect of the emittance area going through a maximum, computations were next carried out with the initial beam diameter reduced to 1.0 cm with all other conditions remaining the same. Figure 4 shows the resulting phase diagrams and Figure 5 shows the variation of apparent emittance area with drift distance. The same characteristic behavior is observed but it develops more rapidly, with the emittance area increasing continuously after the minimum as far as the trajectories were computed. Reflection suggested that this type of behavior could result from the changing charge distribution with drift distance, since we are dealing with apparent, as distinguished from real, emittance area. Derivation of the charge distribution from the trajectory data at various drift distances shows that this was indeed the case, as shown by Figure 6. It is seen that the charge distribution which is initially peaked on the axis quickly develops a minimum on the axis. Aberrations associated with the minimum on axis counteract the effect of the aberrations associated with the maximum on axis. While this effect is a curious one it does not appear to have any particular significance to our present investigation and does not appear in Case II with the hollow distribution. The significant fact is that an emittance area large compared to that of the normalized ion source extraction emittance develops in drift distances comparable to those encountered in normal injection systems.

4. Case II - Hollow Distribution with Zero Intensity on Axis

In this case the volume density of charge is zero on axis and increases linearly to the boundary of the beam.

Figure 7 shows a family of phase diagrams for this case. A significant difference between these curves and those for Case I is the sharp break near the origin. This means that a much larger ellipse is required to enclose them and that the apparent emittance area is correspondingly greater for a given drift distance as indicated by Figure 8.

5. Case III - Beam with Negative Core

If, as assumed by Blewett, a beam is drifting in free space, we can neglect the consequences of interactions between protons and molecules of residual gas. However, unless we invest large sums in getter-ion pumping systems, or until accelerators can be assembled in orbit around the earth or on the moon, these interactions which give rise to free electrons within the volume of the beam cannot be neglected. Consider a free electron-ion pair created at some point within the volume of the beam. The radial space-charge field will accelerate the positive ion away from the axis and the electron toward the axis. The electron is constrained by the space-charge potential trough to execute simple harmonic motion across the axis unless or until the negative charge density is sufficient to give rise to self-limiting effects. Approximate analysis of the motion of trapped electrons shows that the average negative charge distribution is sharply peaked at the axis. Since a nonuniform distribution of trapped electron charge within a beam will have the same qualitative effect as nonuniform distribution of the positive charge on the growth of emittance area it cannot be neglected. From available range energy data it is easy to estimate the rate of growth of free electron charge within a drifting beam for a given pressure and composition of the residual gas. Such estimates show that at a residual pressure of 5×10^{-6} torr of H_2 the number of free electrons per unit length of beam becomes equal to the number of protons per unit length in about 50×10^{-6} sec. This effect may, or may not, be negligible depending on residual gas pressure and composition, and pulse length.

Figure 9 shows a family of phase curves in a 200 mA, 780 keV proton beam with uniform charge distribution over 4.0 cm initial diameter, with trapped electron charge equivalent to 50 mA of protons with a triangular distribution over 2.0 cm diameter. The effect here is similar to that of Case II except more pronounced - namely the offset of the phase curves at the origin. That portion of the beam lying inside the boundary between net negative and net positive charge quickly loses its optical coherence since it is drifting in a $1/r$ field, and that portion lying outside this boundary develops the characteristic of appearing to originate from a ring source,

as illustrated by Figure 10. If such a beam is refocused to image this virtual ring source, a hollow beam can be produced. Hollow beams have been observed on several occasions in the Cosmotron injection system.

Electron trapping can occur only in drift spaces free of electric fields other than the space-charge fields, and if proven to contribute significantly to loss of brightness it can be eliminated by the deliberate introduction of electric fields to continuously sweep out free electrons.

6. Summary and Discussion

Assuming that the initial conditions on which these calculations are based are physically reasonable, the most significant fact that emerges is that nonuniform distribution of space charge in a 780 keV proton beam acting over drift distances that can easily be encountered in injection systems can give rise to emittance areas large compared to that predicted by normalizing to 780 keV the emittance area observed at the ion source. The results indicate that hollow charge distributions are approximately ten times worse with regard to growth of emittance area than axially peaked distributions, presumably because the space-charge fields, being directed outward, cannot correct a charge deficit on the axis.

The surprisingly large magnitude of the growth of apparent emittance area in a 780 keV drifting proton beam raises several pertinent questions which must be clarified in order to provide a better understanding of the requirements for producing maximum brightness beams for injection purposes:

- a) If the ion extraction system delivers to the subsequent acceleration system a beam with nonuniform radial charge distribution, what is the magnitude of the growth of phase space during the course of acceleration to 750 keV?
- b) How is the growth of phase space during acceleration affected by the geometry and gradient of the acceleration system?
- c) What parameter should be optimized at the ion source extractor - initial emittance area or uniformity of radial charge distribution?
- d) What departure from uniformity of radial charge distribution is acceptable?

It is hoped to clarify some of these questions in further work using the above technique.

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APPENDIX

Emittance Area from Diameter of Real or Virtual Focus and Angle of Convergence

1. Introduction

We wish to demonstrate the validity of a simple technique for evaluating the apparent emittance area of an ion beam at any position along its trajectory in terms of the diameter of either a real or virtual focus and the angle of convergence of the rays toward this focus.

2. Parallel Beam of Finite Emittance

Consider a parallel beam of radius R and internal angular spread $\pm \theta$ as a consequence of its nonzero emittance (see Figure 11a). The corresponding emittance diagram for this beam is shown by Figure 11b. If E_a represents the emittance area, then from the geometry of Figure 11b we see that,

$$E_a = 2R \times 2\theta = 4R\theta \quad (2)$$

3. Focused Beam

Consider now the passage of the parallel beam of Figure 11 through a perfect lens L of focal length f as shown by Figure 12. The rays initially parallel to the axis will be brought to a point focus at a distance f beyond the lens. However, each of the initially parallel rays will be surrounded by a cone of rays of angular extent $\pm \theta$ due to the emittance angular spread. Hence for f large compared to $2R$, the diameter d of the focal spot is given by,

$$d = 2\theta f \quad (3)$$

The total included angle of convergence of the beam, ϕ , is given by,

$$\phi = 2R/f \quad (4)$$

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Taking the product of (3) and (4) above,

$$d \times \varphi = 2\theta f \times 2R/f = 4R\theta \quad (5)$$

Hence comparing (5) with (2) above we conclude that,

$$E_a = d \times \varphi \quad (6)$$

4. Equivalence of Technique for Real and Virtual Focus

Since the emittance area of a drifting beam remains constant (neglecting the effects of nonuniform distribution of space charge) the emittance area in the region after the focus in Figure 12 is the same as that before the focus. Hence if we wish to establish the emittance area by examining the beam in the region beyond the focus, we can do so by extrapolating the ion trajectories back to establish the diameter of the virtual focus.

5. Diameter of Virtual Focus in Terms of Spread of Trajectory Intersections with the Axis

In deriving emittance area from computer data, the diameter of the virtual focus must be established in terms of the axial spread of the points of intersection of individual trajectories with the axis. Referring to Figure 13, we wish to examine an axially symmetric beam at plane P moving to the right. First let us consider the case of a zero emittance beam shown by Figure 13a. Projecting trajectories backward they will intersect in a virtual point focus. Now, as shown by Figure 13b, let the aberration introduced by the nonuniform distribution of space charge in the beam be such as to progressively increase the distance to the virtual intersection with the axis with increasing radius in the beam. Let s represent the spread in the points of intersection with the axis of the displaced trajectories, L be the distance to the circle of least confusion, d be the diameter of the virtual focus, φ be the overall angle of convergence of the beam to the virtual focus, and D be the diameter of the beam at the plane of investigation. From the geometry of Figure 13b we see that, to close approximation,

$$d = s/2 \times \varphi = s/2 \times D/L \quad (7)$$

Combining equations (7) and (6),

$$E_a = d \times \phi = s/2 \times D/L \times D/L = s/2 \times D^2/L^2 \quad (8)$$

6. Check of Method by Ellipse Fitting

In order to further verify the validity of the virtual source technique, the value of emittance area given by it was compared to that given by ellipse fitting in one case. The values were found to agree within at least 10%. It is to be noted that no claim is made for high precision of the virtual source technique of evaluating emittance area. There is, however, little question but that it is sufficiently accurate for the purpose at hand where an explanation for gross discrepancies between theory and observation is being sought.

Initial charge distribution -
current - 200 mA, 780 keV protons

Figure 4. Phase diagrams vs. drift distance

Figure 6. Charge distribution vs. drift distance - 200 mA, 780 keV protons

Initial charge distribution -
current - 200 mA, 780 keV protons

Figure 7. Phase diagrams vs. drift distance

Figure 9. Phase diagrams vs. drift distance

Figure 10. Virtual ring source formed by negative core beam after 500 cm drift distance

Figure 11a

Figure 11b

Figure 12

Figure 13a

Figure 13b