BNWL-189

173-

AEC
RESEARCH
and
DEVELOPMENT
REPORT

UCOST - A COMPUTER CODE FOR CALCULATING THE COST
OF ENRICHED URANIUM

D. E. DEONIGI
E. A. ESCHBACH
M. F. KANNINEN

FEBRUARY, 1966

ROUTE TO	REV. NO.	LOCATION	FILE DATE
Drochem for	60013	271-16	APR 7 1966
2 F strips	61644	271-0	APR 11 1966
Super for	60013	703	MAY 1 1966
116 strips	57584	1760D	6-1-66

BATTELLE-NORTHWEST
BATTELLE MEMORIAL INSTITUTE / PACIFIC NORTHWEST LABORATORY

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

PACIFIC NORTHWEST LABORATORY

RICHLAND, WASHINGTON

operated by

BATTELLE MEMORIAL INSTITUTE

for the

UNITED STATES ATOMIC ENERGY COMMISSION UNDER CONTRACT AT(45-1)-1830

PRINTED BY/ FOR THE U. S. ATOMIC ENERGY COMMISSION

3 3679 00060 2062

BNWL-189
UC-80, Reactor Technology

FIRST
UNRESTRICTED

TID-4500

DISTRIBUTION
MADE

EDITION

UCOST - A COMPUTER CODE FOR CALCULATING THE COST
OF ENRICHED URANIUM

By

D. E. Deonigi
E. A. Eschbach
M. F. Kanninen

Fuel Cycle Analysis
Physics and Instruments Department

February, 1966

PACIFIC NORTHWEST LABORATORY
RICHLAND, WASHINGTON

Printed in USA. Price \$4.00. Available from the
Clearinghouse for Federal Scientific and Technical Information,
National Bureau of Standards
U.S. Department of Commerce
Springfield, Virginia

•

ABSTRACT

A major purpose of the UCOST code is to provide a means of calculating the cost of uranium of any enrichment level of U^{235} in U^{238} , based on a specified feed composition and cost, specified separative duty cost, and specified or optimum tails composition. The code can produce a schedule of costs, the theoretical burnup costs, and can provide toll enrichment costs for toll enrichment situations, as well as predict the present USAEC optimum price schedule. Examples of current and generalized price schedules are shown in tables and graphs in Appendixes.

TABLE OF CONTENTS

Summary.	1
Introduction	3
Basis for the Diffusion Cascade	4
Analysis of the Diffusion Cascade	16
The Cost Equation for Enriched Uranium	31
Optimum Tails Composition	33
The Atomic Energy Commission Formula	41
The Effect of a Nonoptimum Tails Composition on Nuclear Reactor Fuels Costs	43
Description of the Output	65
Appendix A: Derivation of the Value Function and the Separative Duty Requirements	67
Generalized Uranium Price Schedules	76
Appendix B: A Speculative U^{233} - U^{238} Price Schedule	81
Appendix C: Current Uranium Price Schedules	86
References	103

LIST OF TABLES

<u>Table</u>		<u>Page</u>
I	Components of Separative Duty Based on 0.253071 wt% Tails	24
II	Optimum Tails Compositions as a Function of the Price Schedule Parameters for Different Feed Compositions	37
III	Values of the Parameters Used in the Atomic Energy Commission Uranium Price Schedules	42
IV	Example Enriched Uranium Costs and Burnup Costs for an Arbitrary Nonoptimum Cascade Tails Composition	44
V	Nonoptimized Fuel Costs for Different Enrichments in a Boiling Water Reactor Simulation for Arbitrary Cascade Tails Compositions	48
VI	Minimized Fuel Costs in Three Typical Thermal Reactors as a Function of an Arbitrary Cascade Tails Composition	49
B-1	Sample Price Schedules for U^{233} - U^{238} Mixtures	85

LIST OF FIGURES

<u>Figure Number</u>	<u>Title</u>	<u>Page</u>
1*	Gaseous-Diffusion Cascade for Separation of $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$	8
2*	Unit, Stage, and Cascade	8
3*	Stage Flow Rates and Composition	10
4*	Simple Cascade, with no Reprocessing of Tails	10
5*	Countercurrent Recycle Cascade	13
6	Separative Duty	25
7*	Separation Potential	26
8	Cost of 2% Enriched Uranium as a Function of the Tails Composition	34
9	Cost of 90% Enriched Uranium as a Function of the Tails Composition in the Diffusion Cascade	35
10	Optimum Tails Composition as a Function of the Feed Composition	38
11	Ratio of the Optimum Tails Composition to the Feed Composition as a Function of the Price Schedule Parameters	39
12	Costs of Enriched Uranium as Affected by Assuming a Unit Tails Value	42
A-1	Separative Duty as a Function of Optimum Tails and Feeds	77
A-2	Separative Duty as a Function of Feed Costs for 93% Enriched Material	78
A-3	Separative Duty as a Function of Enrichment Feed Material Cost and Enrichment Uranium Price	79
A-4	Separative Duty as a Function of Percent Enrichment and of Enriched Uranium, at Constant Feed Materials Cost of \$32/kg	80

---

* From Nuclear Chemical Engineering by Benedict and
and Pigford. Copyright c 1957. Used by permission
of McGraw-Hill Book Company.

SUMMARY

The UCOST code provides a means of calculating the cost of a specified enrichment of uranium from the cascade, depending on the specified feed cost, feed composition, separative duty cost, and the tails composition. The code can produce a schedule of fuel costs and of the theoretical burnup costs. It can also predict the present USAEC optimum price schedule.

The theory of separation of U^{235} from U^{238} in the diffusion cascade is reviewed, including an analysis of the various flow streams and their relationships to separative costs. Selected figures display certain aspects of the cascade, and of the cost factors in enrichment. The generality of the UCOST formulas enables computation of enriched uranium costs for

- Either a specified nonoptimum tails composition or a calculated optimum tails composition
- A specified feed composition other than that of natural uranium
- A specified nonzero tails credit

These features can be used singly or in any combination to simulate all the probable situations inherent in toll enrichment.

Other features of the UCOST computation include

- The calculation of the corresponding burnup costs for any desired degree of burnup.
- The breakdown of the total cost into the feed cost and enrichment cost portions.
- Specification of the flow rate ratios for feed, tails and separative duty for enrichments, as both weight percent and atomic percent.

The code is programmed in FORTRAN for the 7090 Computer. A program listing and sample outputs for the various options are included.

Appendix A consists of a derivation of the Value Function and the Separative Duty, and illustrations of Generalized Uranium Price Schedules,

Appendix B is a simplified and somewhat speculative development of a price schedule for separating U^{233} from U^{238} . Such separations may be of potential value when the nuclear industry is large enough to justify highly sophisticated fuel cycles, including "crossed progeny" such as U^{233} enrichment of U^{238} and plutonium enrichment of thorium,

Appendix C is a computer calculation of two uranium price schedules as functions of enrichment, flow rate ratios and costs of feed and of separative work.

INTRODUCTION

The use of a wide range of U^{235} enrichments in U^{238} for nuclear fuels requires development of a system of equitable pricing for these various enrichment levels. Such pricing, as proposed by the Atomic Energy Commission, will depend upon a number of factors, which may be specified by or to the toll enrichment customer.

Natural uranium contains only about 0.7% of the fissile isotope U^{235} . Operation of a reactor requires that the fuel elements contain a higher percentage of this isotope. The higher percentages are produced in the diffusion cascade, at a considerable cost that depends upon a number of variables. Because customers wish to buy uranium with certain specified properties, including a wide range of U^{235} enrichments in U^{238} , and because other properties of the product will be specified to them, a fair pricing schedule is imperative so that each particular product bears its fair share of the cost of the cascade operation.

The UCOST code is particularly suited for analysis of this kind of a price schedule.

Spent uranium fed to the cascade will result in enrichment material with certain amounts of U^{236} and U^{232} . Because it will probably be some years before significant amounts of these isotopes appear in the process, little attention is paid to them here.

BASIS FOR THE DIFFUSION CASCADE

Enriched uranium is produced by the gaseous diffusion process. The separation of isotopes by this process is possible because the molecules of a gas have, on the average, the same kinetic energy

$$\text{K.E.} = 1/2 mv^2 \quad (1)$$

Solving for v

$$v = \sqrt{\frac{2(\text{K.E.})}{m}} \quad (2)$$

In a gas composed of a mixture of molecules, those with the lighter mass (m_1) will, on the average, have a higher velocity than those with the heavier mass (m_2) so that

$$v_1 = \sqrt{\frac{2(\text{K.E.})_1}{m_1}} \quad (3)$$

and

$$v_2 = \sqrt{\frac{2(\text{K.E.})_2}{m_2}} \quad (4)$$

and since

$$(\text{K.E.})_1 = (\text{K.E.})_2$$

then

$$\frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}} \quad (5)$$

and

$$v_1 = v_2 \sqrt{\frac{m_2}{m_1}} \quad (6)$$

The major fashion by which this law is exploited to effect isotope enrichment in the diffusion cascade can be seen as follows.

Let us assume that the walls of a container are porous* and allow molecular flow of gas in one direction only--out (that is, the outside surface is evacuated). The number of times a molecule strikes the wall will determine the rate at which the molecules will pass through the barrier. The probability that a gas molecule will strike the walls of the container is directly proportional to its velocity, and therefore the probability that a molecule will escape the vessel by passing through the porous barrier is inversely proportional to the square root of the mass of the molecule, as in Equation (2).

Unidirectional passage of gas through the porous barrier can be approached by maintaining gas pressures inside the vessel significantly higher than the gas pressure on the outside surface of the porous wall. Thus, a dynamic system is employed wherein a mixture of gaseous molecules of different atomic weights is fed to the high pressure side of the porous barrier and, on the reduced pressure side, larger fractions of the lighter molecules are maintained by pumping from this side. The gas on the high pressure side will exhibit depletion of the lighter molecules, and the effectiveness of separation will diminish unless this depleted material is removed continually and replaced by feed. Such a dynamic system can maintain a constant separations effectiveness by investment of pumping energy.

---

* An effective barrier is afforded by providing holes in a surface only large enough to allow passage of individual molecules--a phenomenon known as molecular effusion. The holes must not be so large as to allow general gas flow, under which circumstance gases of all masses would be "swept along."

For the gaseous diffusion process, the theoretical degree of separation α is the ratio of the probabilities of each type of molecule striking the wall and passing through the barrier, and is given by the inverse of the ratio of the square roots of the molecular weights, that is

$$\alpha = \frac{v_1}{v_2} = \sqrt{\frac{m_2}{m_1}} \quad (7)$$

The concentration of U^{235} with respect to U^{238} in natural uranium can be increased by subjecting UF_6 to the gaseous diffusion process. For this system:

$\alpha_{\text{theoretical}} = U^{235}$ theoretical degree of separation

$$= \sqrt{\frac{238.125 + 6 \times 19.004}{235.117 + 6 \times 19.004}} = 1.0043$$

It is apparent that a great many individual separating units are required to obtain a significant degree of enrichment. To achieve this, slightly enriched material from one unit is used as feed to the high pressure side of another vessel and so forth until the desired isotopic enrichment is obtained. There are many ways in which the units can be connected to achieve successive enrichment. A particularly effective method is to arrange the material flow so that the depleted stream of one vessel is a feed stream to the supply of the previous (lower enrichment level) vessel. The purposeful flow of depleted material from a higher stage to feed of a lower stage is termed "recycle" or "reflux." Within a stage, some eddy flow may occur, which in a solvent extraction plant is termed "back mixing" or "eddy diffusion." A number of units connected in a successive fashion are called a cascade. Hence, the name "Gaseous Diffusion Cascade" as shown in Figure 1.

The theoretical minimum number of stages in a uranium cascade is large indeed, as can be determined by the Fenske Equation (1) for total reflux.

$$\text{Minimum number of stages} = \frac{\ln \frac{X_p(1 - X_w)}{X_w(1 - X_p)}}{\ln \alpha} \quad (8)$$

For a product of 90% U^{235} and a tails of 0.3% U^{235} with a theoretical α of 1.0043

$$\begin{aligned} \text{Minimum number of stages} &= \frac{\ln \frac{(0.90)(1 - 0.003)}{(0.003)(1 - 0.90)}}{\ln 1.0043} \\ &= \frac{\ln \frac{(0.9)(0.997)}{(0.003)(0.1)}}{\ln 1.0043} = \frac{\ln \frac{0.8973}{0.0003}}{\ln 1.0043} \\ &= 1864 \end{aligned} \quad (9)$$

Terminology specific to analysis of isotope separation processes has been developed.⁽²⁻⁴⁾ While it is not actually necessary to be acquainted with isotope separation theory to utilize the UCOST code as a fuel cost component, a few terms will be paraphrased from these three references to provide some background for appreciating the structure of the uranium price schedule.

A separating unit is the smallest element of an isotope separations system that effects some separation of the isotope stream. A separations stage is a group of parallel-connected separations units operating with common feed and effluent streams. A separations cascade is a series-connected group of separations stages. Stages are usually connected in series to effect a greater degree of separations than can be achieved with a single stage. These relationships are clearly shown by Figure 2. The atomic fraction is a particularly useful term for describing

FIGURE 1

Gaseous-Diffusion Cascade for Separation of $^{235}\text{UF}_6$ and $^{238}\text{UF}_6$

FIGURE 2

Unit, Stage, and Cascade

binary isotopic mixtures of a single element and is defined as the ratio of the number of atoms of the desired isotope to the total number of atoms of the element. Similar, but different, is the abundance ratio, which is defined as the ratio of the number of atoms of the desired isotope to the number of atoms of the undesired isotopes.

The atom fraction, X , is

$$X = \frac{N_{\text{desired}}}{N_{\text{desired}} + N_{\text{undesired}}} \quad (10)$$

and the abundance ratio ξ is

$$\xi = \frac{N_{\text{desired}}}{N_{\text{undesired}}} \quad (11)$$

Since

$$XN_{\text{desired}} + XN_{\text{undesired}} = N_{\text{desired}}$$

$$XN_{\text{undesired}} = N_{\text{desired}} (1 - X)$$

$$N_{\text{undesired}} = N_{\text{desired}} \frac{(1 - X)}{X}$$

then

$$\xi = \frac{N_{\text{desired}}}{N_{\text{desired}} \frac{(1 - X)}{X}}$$

or

$$\xi = \frac{X}{1 - X} \quad (12)$$

Figure 3 shows the flow rates and composition of a separating unit.

As indicated in Figure 3, the feed stream of composition X is separated into a heads stream (measurably enriched in the

FIGURE 3

Stage Flow Rates and Composition

FIGURE 4

Simple Cascade, with no Reprocessing of Tails

desired isotope) and a tails stream (correspondingly depleted in the desired isotope). To measure the effectiveness of a separations stage, the term separation factor α is computed to indicate the difference in the light component concentration in the heads and the tails streams leaving the vessel. To simplify analysis of cascaded systems, "effective α " has been defined as the heads stream abundance ratio divided by the tails stream abundance ratio.

$$\alpha_{\text{effective}} = \frac{\xi'}{\xi''} = \left(\frac{X'}{X''} \right) \left(\frac{1 - X''}{1 - X'} \right) = \frac{\frac{X'}{1 - X'}}{\frac{X''}{1 - X''}} \quad (13)$$

Also useful in the heads separation factor, β , defined as the heads stream abundance ratio divided by the feed stream abundance ratio

$$\beta = \frac{\xi'}{\xi} = \frac{X'(1 - X)}{X(1 - X')} = \frac{\frac{X'}{1 - X'}}{\frac{X}{1 - X}} \quad (14)$$

For an effective plant β should lie between one and α .

Isotope separations stages can be connected into cascades in many fashions. The most direct, called the single cascade, is a series connection wherein the feed for one stage is the heads fraction from the lower stage and no attempt is made to reprocess the tails streams from each stage. This is shown in Figure 4.

If the tails have sufficient value, a countercurrent recycle cascade is used wherein the feed of an intermediate stage is then composed of the heads of the immediate lower stage and the tails of the immediate upper stage. Such a cascade logically can employ stages processing material, the desired isotopic composition of which is less than the feed, and the portion of the cascade performing this function is known as the stripping section. Conversely, the enriching section is the name of the portion of a cascade between the feed and product

end. These arrangements are shown in Figure 5, and a cascade with pumps in Figure 1. By virtue of the cost of natural uranium and the low value of α in the diffusion process, stripping is an important part of a U^{235} cascade and selection of the optimum waste composition has considerable impact on costs.

It is important to realize that the overall cascade has three streams: feed, product, and waste, and that at equilibrium material must be conserved. Thus, feed = product + waste. With the recycle cascade, however, enormous total flows must take place if the effective separations factor α is small, and if the desired product and waste compositions are to be significantly different. The smaller the separations factor α per stage, the larger must be the provision for recycle per stage to accomplish a given overall separations task. This is particularly true of the stripping operation, since the overall separation task includes production of an acceptably low U^{235} composition in the waste stream.

In the design of the recycle cascade, it is possible to interchange or "trade off," the number of stages with the amount of material recycled or "refluxed" from stage to stage. This is possible within limits because of the effective α of each stage. It heavily influences practical designs for a number of reasons: (1) specific barriers do not achieve ideal separations because depletion and backflow occur, (2) neutron criticality problems associated with pure U^{235} limit pipe and tube sizes, and (3) there is a significant volume and cost of unnecessary recycle equipment. The necessary recycle or reflux is minimized if the effective α for each stage is a maximum; and conversely, the effective α required for each stage can be minimized if the quantity recycled or refluxed is increased without limit. To those unfamiliar with the chemical industry, some of the terminology may be confusing. The terms "total reflux" and "minimum reflux" refer to the number of stages and the amount of reflux or recycle

FIGURE 5
Countercurrent Recycle Cascade

between stages. The dynamic characteristics of the cascade tend to mitigate against a quick assimilation of the ideas of the process. Each flow stream associated with one stage must go to or come from another stage. Thus, the relative size of the stages changes with recycle conditions. By adding stages, and requiring each stage to achieve less separation, the amounts of feed in and out can be reduced, particularly the recycle or reflux to the previous stage. The number of stages can be minimized by maximizing reflux or recycle. Thus, the minimum number of stages can be calculated by assuming no limit on reflux. This basis was used to compute the number of stages on page 8.

This minimum reflux or recycle can be calculated by assuming an unlimited number of stages. In practice, a compromise is made in which a reflux or recycle ratio is chosen that is greater than the minimum of the feed point. In many cases, this reflux ratio is maintained throughout the plant even though it could be smaller toward the product and waste ends. If the economic value of U^{235} were very low, a countercurrent recycle cascade would not be used and the tails of each stage could be rejected even though the composition were greater than the composition of feed to the plant. The economic necessity of recycling and the final rejection of tails material at an enrichment level well below the product stream enrichment levels inherently necessitate large recycle flows. Near the feed section, the recycled or refluxed flows can be 25,000 to 50,000 times the product flow.

In many operations, refluxing can be made constant at little cost, and the expense of engineering the plant more closely to optimum is not warranted. However, the cost and complexity of a U^{235} cascade are so great that close engineering is warranted, and a tapered plant is built. This plant has maximum reflux at the feed sections and correspondingly less and less toward the

two product streams--fully enriched uranium and cascade tails. Such a cascade designed for uranium separations by gaseous diffusion can be analyzed as an ideal cascade.

The theory of ideal cascades can be treated with relatively direct mathematical techniques as explained by K. Cohen.⁽³⁾ The low value of α and the large separations task of the UF_6 system involve cascades of several thousand stages which aid in making the actual AEC cascade approach an "ideal" cascade. The smoothly varying flow characteristics of the ideal cascade are not achieved in practice, but smoothness is approached by "squaring off" at appropriate intervals.

ANALYSIS OF THE DIFFUSION CASCADE

The objectives of the various cost analyses of diffusion cascades^(3,4) have been to produce a cost relationship that can be easily used, and at the same time to spare the user as many of the details of the analyses as possible. This section will briefly summarize results that have been obtained and will also venture some qualitative observations.

The specific objectives of the analyses are to describe the extremely complex flow sheet by an overall method so that a reasonable cost model can be prepared to provide equitable reimbursement of the feed and plant costs for sale of product of any U^{235} composition.

The result is that the cost of the product stream from a diffusion cascade is determined by summing the cost of operation, the cost of the feed, and the credit for the waste or tails stream. That is,

$$C_p P = C_s \Delta + C_f F - C_w W \quad (15)$$

where

- P = flow rate of the product stream
- F = flow rate of the feed stream
- W = flow rate of the tails stream
- Δ = separative duty
- C_p = unit cost of the product
- C_s = unit cost of the separative duty
- C_f = unit cost of the feed
- C_w = unit cost of the tails

For a unit amount of product, Equation (15) takes the form

$$C_p = \frac{C_s \Delta + C_f F - C_w W}{P} \quad (16)$$

The separative duty Δ describes the relative difficulty of accomplishing the particular isotopic separations task. Development of this term has grossly simplified representation of the total cost of enriched uranium. The cost of separative duty, C_s , includes the pumping costs and an equitable prorating of the associated operating and fixed costs of the cascade. The cost of separative duty is sometimes denoted as the pumping costs, which exclude fixed charges. It will be shown that this arises because the size of the cascade determines the number of pumps, associated fittings, other components, buildings, etc. The number of pumps, in turn, determines the pumping power. The separative duty Δ is expressed as

$$\Delta = W\phi(X_w) + P\phi(X_p) - F\phi(X_f) \quad (17)$$

where

W = flow rate of depleted material, kg uranium per day

P = flow rate of enriched material, kg uranium per day

F = flow rate of feed material, kg uranium per day

$$\phi(X_i) = 1 - 2X_i \ln \left[\frac{1 - X_i}{X_i} \right] \quad (18)$$

where

X_i = composition of the i th flow stream fraction

It is of particular importance to note that the functions $\phi(X_w)$, $\phi(X_p)$, and $\phi(X_f)$ are unique to each composition for waste, product, and feed, respectively. These compositions are determined by conventional mass balance relationships. As can be seen by the form of Δ , a simple unit definition cannot be composed because it involves three terms—waste, product, and feed. Furthermore, the magnitude of Δ can be altered by varying any of the six quantities, namely the composition of and the amount of waste, product, and feed.

The form of $\phi(X)$ is the most difficult concept to accept and will be discussed in some length and derived in Appendix A. The function $\phi(X)$ computed for three compositions of fuel, product, and tails, determines the value of Δ . At this point, it should be observed (Figure 7, page 26) that $\phi(X)$ vanishes at $X_i = 0.5$, that it is symmetrical about 0.5, and is unbounded as $X_i \rightarrow 0$, or as $X_i \rightarrow 1$. This implies that a separation job at 50% U^{235} in U^{238} is trivial, but 50% U^{235} - U^{238} must be produced. In effect, utilizing a basis of 50/50 composition allows the cascade to be analyzed as three distinct isotopic separations jobs: (1) production of product composition from 50/50 material, (2) production of waste from 50/50 composition, and (3) the production of 50/50 composition from feed (usually natural uranium). Since these are simultaneous operations, the feed stream is of opposite sign from the output streams of product and waste. Equation (17) says that the total separation job as represented by Δ is complicated by the consideration of having to produce waste and product from which the amount and composition of feed will reduce the size of the separation job. However, the cost of feed and the credit for waste will enter the cost of product shown in Equation (15) in accordance with mass balance relationships. The analysis to be described will also show how the total cost is minimized by balancing the cost of feed against the separative duty costs. The formulation of separative duty from the three components product, waste, and feed grossly simplifies its evaluation. This simplified approach is possible because the theory of an ideal cascade is an excellent approximation of the uranium diffusion cascade. The theory of the ideal cascade allows cancellation of the many intermediate feed and product terms, thereby leading to the overall simplification as represented by Equations (15) and (17).

Review of Analysis

Each stage within the cascade receives a feed stream which it separates into two output streams: (1) an "enriched" stream which contains a higher concentration of U^{235} than the feed stream, and (2) a "depleted" stream which contains a lower concentration of U^{235} than does the feed stream. A valuable measure of the degree of separation actually achieved in the stage is the ratio of the U^{235} abundance ratios in these two output streams. In practice, this ratio, α , must be less than the theoretical diffusion rate, 1.0043. A more convenient measure is the so-called heads separation factor β which is the ratio of the U^{235} abundance ratio in the enriched (or heads) output stream to the U^{235} abundance ratio in the feed stream. In reality, there are two feed streams to each stage since it is a mixture of the enriched output from the previous stage and the depleted output from a subsequent stage.

The mathematical analysis of a separations plant can be considered in two parts, the external and the internal. The external analysis consists of writing mass balances for all of the uranium and for the fissile components of the uranium. Six variables are included: the flow rate of the feed, the material product, and the waste, and also the composition of these three. These equations are

$$F = P + W \text{ (mass balance of total uranium)} \quad (19)$$

and

$$X_f F = X_p P + X_w W \text{ (mass balance of fissile uranium, } U^{235}) \quad (20)$$

where

F = flow rate of feed materials, kg/day

P = flow rate of product material, kg/day

W = flow rate of waste materials, kg/day

X_i = composition of flow stream, grams U^{235} /g uranium

from which

X_f = weight fraction of U^{235} in feed

X_p = weight fraction of U^{235} in product

X_w = weight fraction of U^{235} in waste

Statements (19) and (20) are more conveniently expressed as the ratios of the waste flow rate and the feed flow rate to the product flow rate. The ratios are functions only of the compositions.

$$\frac{W}{P} = \frac{X_p - X_f}{X_f - X_w} \quad (21)$$

$$\frac{F}{P} = \frac{X_p - X_w}{X_f - X_w} \quad (22)$$

Mass balance relations can be similarly used in the internal analysis but here an additional variable--the stage separations factor--is included in the flow rate equation. The internal variables are not usually of interest and it is desirable to eliminate them from the final result. The interstage flow rates and compositions can be eliminated by adopting a simplified model--the so-called "ideal" cascade. As previously indicated, the gaseous diffusion cascades operated by the AEC for the enrichment of uranium are well approximated by ideal cascades.

Ideal cascades are described by the following conditions:

1. The two feed streams to a stage (recycle from upper stage, and top of lower stage) have exactly the same composition, so that there is no loss of separative work when the streams are mixed.
2. Each stage has the same heads separation factor as every other stage.

From these conditions can be derived the key property of an ideal cascade, which is that the constant heads separation factor β is equal to the square root of the stage separations factor α . This condition excludes the internal flow variables from the final result.

The cost of acquiring and operating a cascade is directly tied to the necessary flow of material through the cascade. The larger the total flow, the larger the overall system. The flow of greatest importance to the unit cost of product is the interstage flow, that is, the recycle or refluxing. As additional refluxing is needed, the barrier area is directly increased, as are the pumping capacity and energy. For this reason the general shape of a uranium price schedule curve can be secured from an expression of the total flow rate and the total power rates for pumping which, in turn, is a sum of the flows or power rates for discrete parts of the plant. Summation of these and the criteria of the ideal cascade lead to a manageable equation of the following form^(3,4) for total flow rate of cascade:

$$J = \frac{\beta + 1}{(\beta - 1) \ln \beta} \underbrace{W(2X_w - 1) \ln \frac{X_w}{1 - X_w} + P(2X_p - 1) \ln \frac{X_p}{1 - X_p}}_{\text{OUTPUT}} - \underbrace{F(2X_F - 1) \ln \frac{X_F}{1 - X_F}}_{\text{INPUT}} \quad (23)$$

From this equation and the mass balances, the characteristics of the AEC uranium price schedule are developed. It is actually relatively simple, because the equation states that the total flow of the plant is the product of two factors--the

first involving the heads separations factor β , and the second involving the flow rates and composition of feed, product, and waste, where feed is the input, and product and waste are the output. The first term indicates the inherent difficulty of separations, because as β approaches 1 the job becomes enormous ($\beta = \sqrt{\alpha}$). For an ideal cascade the theoretical value of α for $U^{235}-U^{238}$ is 1.0043 and $\beta = 1.00215$. This makes the first factor, $\frac{(\beta + 1)}{(\beta - 1) \ln \beta}$, in Equation (23), = 433,000. In practice, the actual value of α for specific stages is nearer to 1.003, for which β is 1.0015 and the first term is 891,000.

The second factor, in brackets, tells how ambitious the desired separation job is, that is, the compositions and quantities of product and waste minus the feed. Note that the waste stream contributes to the flow and size of the cascade. If feed cost were low, each waste stream could be discarded and the flow and size of the cascade could be reduced. Benedict and Pigford⁽⁴⁾ discuss the second factor as follows: "...it is proportional to the throughput, and is large when product and waste differ substantially in composition from feed, and small when these compositions are nearly equal. The second factor has been termed the separative duty, because it has the same form as the heat duty of an evaporator or flash vaporizer. It equals: (1) the sum of two output terms, each the product of an output flow rate* and a function of the corresponding output condition, and (2) minus an input term which is the product of the feed rate and a function of the input condition."

The separative duty Δ merits further examination from Equation (23), where Δ is defined as the second factor of the first term

---

* W and P are waste and product flow rates.
 X_w and X_p are compositions of output.
 X_f is composition of feed.

$$\Delta = W(2X_w - 1) \ln \frac{X_w}{1 - X_w} + P(2X_p - 1) \ln \frac{X_p}{1 - X_p} - F(2X_F - 1) \ln \frac{X_F}{1 - X_F} \quad (24)$$

Note that each of the three terms in Δ has exactly the same form. Since

$$\phi_i(X) = (2X_i - 1) \ln \frac{X_i}{1 - X_i} \quad (25)$$

and by simple substitution

$$(2X_i - 1) \ln \frac{X_i}{1 - X_i} = (1 - 2X_i) \ln \frac{1 - X_i}{X_i} \quad (26)$$

the separative duty equation can now be written

$$\Delta = W\phi(X_w) + P\phi(X_p) - F\phi(X_f) \quad (27)$$

Table I shows the associated components of the separative duty Δ , based upon a waste composition from the cascade of 0.253071 wt%. Figure 6 shows a plot of these values.

The expression $\phi(X_i)$ is called the separation potential, or sometimes the value function, $V(X_i)$, and is symmetrical about $X = 0.5$; that is, $\phi(X) = \phi(1 - X)$ (Figure 7). Since

$$\phi(X) = (1 - 2X) \ln \frac{(1 - X)}{X} \quad (28)$$

then for

$$X = 1 - X$$

$$\begin{aligned} \phi(1 - X) &= (2X - 1) \ln \frac{X}{1 - X} \\ &= (1 - 2X) (-1) \ln \frac{X}{1 - X} \end{aligned} \quad (29)$$

TABLE I
 COMPONENTS OF SEPARATIVE DUTY
 BASED ON 0.253071 wt% TAILS

X_p	$W\phi(X_w)$	$P\phi(X_p)$	$W\phi(X_w) + P\phi(X_p)$	$F\phi(X_f)$	Δ
0.90	1158.2	1.8	1160.0	953.0	207.0
0.80	1028.5	0.8	1029.3	846.8	182.5
0.70	898.8	0.3	899.1	740.7	158.4
0.60	769.1	0.1	769.2	634.5	134.7
0.50	639.3	0.0	639.3	528.3	111.0
0.40	509.6	0.1	509.7	422.1	87.6
0.30	379.9	0.3	380.2	315.9	64.3
0.20	250.2	0.8	251.0	209.7	41.3
0.10	120.5	1.8	122.3	103.5	18.8
0.09	107.5	1.9	109.4	92.9	16.5
0.08	94.5	2.0	96.5	82.3	14.2
0.07	81.6	2.2	83.8	71.6	12.2
0.06	68.6	2.4	71.0	61.0	10.0
0.05	55.6	2.6	58.2	50.4	7.8
0.04	42.7	2.9	45.6	39.8	5.8
0.03	29.7	3.3	33.0	29.2	3.8
0.02	16.7	3.7	20.4	18.5	1.9
0.01	3.7	4.5	8.2	7.9	0.3
0.007115	0.0	4.9	4.9	-4.9	0.0
0.00253071	-5.9	5.9	0.0	0.0	0.0

$$\phi(X_f) = 4.868136$$

$$\phi(X_w) = 5.946471$$

$$W\phi(X_w) + P\phi(X_p) - F\phi(X_f) = \Delta$$

$$W(5.946471) + P\phi(X_p) - F(4.868136) = \Delta$$

FIGURE 6
Separative Duty

FIGURE 7
Separation Potential

or

$$(1 - 2X) \ln \frac{1 - X}{X}, \text{ which is } \phi(X) .$$

For reference

$$\frac{d\phi}{dX} = \phi'(X) = - \frac{(1 - 2X)}{X(1 - X)} - 2 \ln \frac{1 - X}{X} \quad (30)$$

and

$$\frac{d^2\phi}{dX^2} = \phi''(X) = \frac{1}{X^2(1 - X^2)} . \quad (31)$$

Since the value function vanishes for $X = 0.5$, if the feed material had this composition, little effort would be necessary to produce material of 0.4999 or 0.5001 composition. Note also that to produce 100% of either isotope requires an unbounded, that is, an infinite amount of work. The importance of the separations potential to the plant design can now be observed from Equation (23). The flow rate, and, hence, also the size of the cascade is determined by the value of the separations potential. The output terms w and p and the input term f directly involve the separations potential.

While Benedict and Pigford⁽⁴⁾ point out that the separative duty is analogous to the heat duty of an evaporator and that the separations potential is analogous to the enthalpy per mole of the streams entering and leaving on evaporation, to those not steeped in the engineering conventions of distillation, these analogies may be of little aid. Cohen⁽³⁾ references a Dirac treatise (a British MS) on a statistical approach to the problem, and a synopsis of Cohen's development is included in Appendix A.

A qualitative appreciation of the shape of the separations potential function can be gained as follows. Note that in a vessel containing two gases, A and B, the relative probability

of observing a given quantity of the A component is proportional to the relative rarity of A in A + B. Thus, the relative difficulty of observing A is the fraction

$$\frac{N_A + N_B}{N_A} \quad (32)$$

and this is a minimum for $N_A = N_B$ and is unlimited as $N_A \rightarrow 0$. The similar situation exists for B in A + B. Thus, the general shape of the separations potential plotted in Figure 7 can be appreciated from the above relation. One can construct more sophisticated models of this analogy, computing the time elapsed to remove a given quantity of gas A from the mixture of A + B and correcting successively as A is depleted in A + B. If the mixture is 50/50, comparatively no time passes before one molecule of A is removed. As one departs from this 50/50 mixture, an increasing amount of time is required to remove additional material, and the total time to alter the composition to a given value from the original 50/50 value can be estimated.

The real simplification that the value function contributes is that it measures the difficulty of separating specific compositions (that is, waste, product, and feed) as compared to the simplest separation job at 50/50. The separative duty reflects that waste is, in effect, a product, that is, production of appropriately pure U^{238} as waste is desirable because U^{235} is thus made available for the enriched product stream.

Benedict and Pigford⁽⁴⁾ note that most large isotope separations plants have sufficient flexibility that their separative duty can be kept very nearly constant under changing conditions. This is especially true if the separating units in series and in parallel are so arranged that mixing of streams of different compositions is avoided. This arrangement is important because

it allows steady withdrawal of partially enriched material, and also because it allows for provision of supplementary feed of various compositions.

With the total flow formulation in the cascade composed of a waste stream, product streams, and feed streams, it is important to inquire as to the relative sizing of the feed and waste streams. If feed costs little, such as water to a D_2O plant, little stripping would be warranted. However, natural uranium is relatively expensive, and an appropriate stripper is warranted. Of course, if the total cost of the stripping operation is greater than the cost of feed material, it is cheaper to purchase more feed. It is no surprise then, that analysis shows that the optimum tails composition is such as to just balance stripping costs against feed. With such a relationship, a cost of uranium of various enrichment can be readily developed for the ideal cascade and it is this relationship that is employed by the AEC, although weight fractions are employed rather than atom fractions.

The separative duty Δ is a most useful quantity because it is directly proportional both to the initial cost of the plant and to its operating cost. Thus, the cost of effecting a separation of isotopes, exclusive of the raw materials cost, can be expressed as the product of Δ with an empirical proportionality factor, C_s . This factor will reflect the capital cost of the plant, the plant overhead, and the unit cost of power and will be a constant so long as these do not change.

Therefore, C_s can be considered as the unit cost of separative duty. However, a unit of separative duty cannot be easily defined because it is the sum of three terms. While some of these terms can be flexible, namely, the amount and

composition of feed and the waste composition, a complex formulation for a unit of separative duty still involves different amounts of waste depending upon the amounts and compositions of product. The concept of the value of a unit of separative duty is simplified if one thinks in terms of the amount of product multiplied by the separations potential. The external mass balance equations and the separative duty formula, together with the concept of a unit separative duty cost, are sufficient to derive a formula for the cost of producing enriched uranium in a diffusion cascade. The equation that represents the AEC price schedule can be derived from the general equation by introducing certain arbitrary conditions. These relations will be derived in the following section.

THE COST EQUATION FOR ENRICHED URANIUM

Factors that contribute to the cost of enriched uranium are the cost of the raw materials, the cost of the enrichment process itself, and the value of the waste materials. The cost of the raw materials in the steady state is the product of the feed rate with the unit cost of the material when in a form that can be introduced into the cascade. The enrichment cost is the product of the separative duty and its unit cost as described in the preceding section. These costs are partially offset by a credit for the value of the waste material, which equals the product of the waste flow rate and its unit value. Thus, the cost of the product stream from a diffusion cascade is determined by summing the cost of the separative duty, the cost of the feed, and credit for the waste or tails stream, as shown by Equation (16) on page 18.

$$C_p = \frac{C_s \Delta + C_f F - C_w W}{P}$$

The mass balance relations, Equations (15), (21), and (22) can be combined with Equation (17) to give an expression for Δ/P in terms of the compositions only

$$\frac{\Delta}{P} = \phi(X_p) + \frac{X_p - X_f}{X_f - X_w} \phi(X_w) - \frac{X_p - X_w}{X_f - X_w} \phi(X_f) \quad (33)$$

Substituting Equations (22), (23) and (33) into Equation (17) gives the general uranium price schedule equation

$$C_p = C_s \left[\phi(X_p) + \frac{X_p - X_f}{X_f - X_w} \phi(X_w) - \frac{X_p - X_w}{X_f - X_w} \phi(X_f) \right] + C_f \left[\frac{X_p - X_w}{X_f - X_w} \right] - C_w \left[\frac{X_p - X_f}{X_f - X_w} \right] \quad (34)$$

This equation gives the unit cost of the product--enriched uranium--per unit mass of uranium. Note that the cost per unit of contained U^{235} can be obtained by dividing C_p by the product enrichment, X_p .

OPTIMUM TAILS COMPOSITION

In the design of the cascade, one must balance the contributions of feed and stripping to the total cost of product. In a D_2O plant where feed can cost nothing, no stripping is involved. However, in a U^{235} - U^{238} plant wherein feed is expensive, stripping the feed is of considerable importance. If the feed were stripped completely of U^{235} , feed costs would be minimized, but stripping costs and plant size would increase without limit. Making pure U^{238} is essentially as difficult as making pure U^{235} (Figure 7).

On the other hand, eliminating stripping reduces the plant size, but the feed costs become very excessive. Thus, designing for minimum cost production is warranted.

The practical aspects of an optimum tails composition can be demonstrated by calculating the cost of a given enrichment as a function of the tails composition. This was done for 2% enriched uranium, with natural uranium feed at \$23.50 per kilogram, separative duty costs of \$30 and \$37.29 per kilogram and a zero tails credit. The solutions to Equation (34) for this case are shown in Figure 8.

The curves do not have a sharply defined minimum point and increase only moderately in the vicinity of the minimum. The function increases steeply outside of this region, and approaches infinity at the two extreme X_w values--zero and the feed composition. Changing one of the price schedule parameters relocates the minimum point. Figure 9 shows the greater effect of tails composition on the cost of 90% enriched uranium.

The optimum tails composition is, by definition, the composition that minimizes the unit cost of the enriched product. This composition is found by equating the derivative (with respect to X_w) of Equation (34) to zero.

FIGURE 8

Cost of 2% Enriched Uranium as a Function of the Tails Composition
 (Cost of natural uranium feed = \$23.50/kg)

FIGURE 9
 Cost of 90% Enriched Uranium as a Function
 of the Tails Composition in the Diffusion Cascade
 ($C_f = \$23.50/\text{kg}$)

$$\phi'(X_w) = \frac{d\phi(X_w)}{dX_w}$$

$$\phi''(X_w) = \frac{d^2\phi(X_w)}{dX_w^2}$$

$$\begin{aligned} \frac{\partial C_p}{\partial X_w} = \frac{X_p - X_f}{(X_f - X_w)^2} \left\{ C_f - C_w - C_s [\phi(X_f) - \phi(X_w) \right. \\ \left. - (X_f - X_w) \phi'(X_w)] \right\} \end{aligned} \quad (35)$$

Then, if the optimum value of X_w is denoted as X_o , $\frac{\partial C_p}{\partial X_w}$ can be zero only if

$$\frac{C_f - C_w}{C_s} = \phi(X_f) - \phi(X_o) - (X_f - X_o) \phi'(X_o) \quad (36)$$

This defines X_o in terms of the price schedule parameters C_f , C_s , and C_w , and the feed composition X_f . Because this expression contains the ϕ functions, it cannot be solved explicitly for X_o . An iterative approach is required.

An iterative calculation is included in the UCOST code to solve Equation (36) for X_o , given the price schedule parameters and the feed composition. Results of this computation are shown in Table II and Figure 10. These data suggest that, to a good approximation, the quantity X_o/X_f is independent of X_f . Therefore, this ratio can be approximately represented as a function of the price schedule parameters as a single curve, as shown in Figure 11.

Although the tails composition cannot be eliminated from the general uranium cost equation, the optimum tails composition equation can be used to eliminate another parameter. It is

TABLE II

OPTIMUM TAILS COMPOSITIONS AS A FUNCTION OF THE PRICE SCHEDULE PARAMETERS*
FOR DIFFERENT FEED COMPOSITIONS

$C_f - C_w$ C_s	$X_f = 0.0040$		$X_f = 0.007115$		$X_f = 0.010$		$X_f = 0.020$	
	X_o	X_o/X_f	X_o	X_o/X_f	X_o	X_o/X_f	X_o	X_o/X_f
0.01	0.00349	0.871	0.00620	0.872	0.00872	0.872	0.01746	0.873
0.05	0.00296	0.741	0.00528	0.742	0.00742	0.742	0.01488	0.744
0.10	0.00264	0.660	0.00470	0.661	0.00662	0.662	0.01327	0.664
0.50	0.00170	0.425	0.00303	0.425	0.00426	0.426	0.00855	0.428
1.00	0.00127	0.318	0.00227	0.319	0.00319	0.319	0.00641	0.321
5.00	0.00050	0.124	0.00088	0.124	0.00124	0.124	0.00249	0.124
10.00	0.00029	0.074	0.00052	0.074	0.00074	0.074	0.00148	0.074
50.00	0.00007	0.018	0.00013	0.018	0.00018	0.018	0.00036	0.018
100.00	0.00004	0.009	0.00007	0.009	0.00009	0.009	0.00019	0.009

* Price schedule parameters are as follows:

C_f = cost of feed material as UF_6 , dollars per kilogram uranium

C_s = cost of separative duty, dollars per kilogram uranium

C_w = cost of tails composition, dollars per kilogram uranium

X_o = optimum tails composition

X_f = feed composition

FIGURE 10

Optimum Tails Composition as a Function of the Feed Composition
[Based on Equation (36)]

FIGURE 11

Ratio of the Optimum Tails Composition to the Feed Composition
as a Function of the Price Schedule Parameters

convenient to solve Equation (36) for C_f and then, by substitution into Equation (34),

$$C_p = C_s [\phi(X_p) - \phi(X_o) - (X_p - X_o)\phi'(X_o)] + C_w \quad (37)$$

This is the general uranium cost equation with the optimum tails composition.

THE ATOMIC ENERGY COMMISSION FORMULA

The formula used by the AEC to price enriched uranium is derived in the same general way as that used to obtain Equation (34). It is somewhat more specific because the tails are assumed to have no value, and because an optimum tails composition is specified. The formula can be obtained, therefore, by setting $C_w = 0$ in Equation (37). This gives the equation

$$C_p = C_s [\phi(X_p) - (X_p - X_o) \phi'(X_o)] \quad (38)$$

The optimum tails composition is determined by Equation (36) with $C_w = 0$ for use in this formula. The values of the feed cost, the separative duty cost, and the resulting tails composition used by the AEC to establish the present price schedules are shown in Table III.

It may seem as if the differences between the costs given by Equations (37) and (38), that is, with and without a tails credit, are negligible for reasonable values of C_w . This is not the case, because the tails credit strongly affects the optimum tails composition. In fact, the price schedule generated with a nonzero tails credit is very nearly identical to the schedule generated with a feed cost reduced by the value of C_w . For a fixed feed cost, therefore, increasing the tails credit will increase the optimum tails composition and decrease the resulting product costs.

The costs of producing enriched uranium per gram of contained U^{235} for different tails credits are shown in Figure 12. Because the optimum tails composition is the enrichment for which C_p is a minimum, the minimum points on the plotted curve, C_p/X , do not coincide with X_o .

TABLE III
VALUES OF THE PARAMETERS USED
IN THE ATOMIC ENERGY COMMISSION URANIUM PRICE SCHEDULES*

Schedule in Effect	C_F , Unit Cost of Natural Uranium as UF_6 , Dollars/kg Uranium	C_S , Unit Cost of Separative Duty, Dollars/kg Uranium	X_0 , Optimum Tail Composition
Prior to July 1, 1961	39.27	37.29	0.2214
From July 1, 1961 to July 1, 1962	23.50	37.29	0.2770
From July 1, 1962	23.50	30.00	0.2531

* X_0 is determined by the ratio of C_F to C_S with $C_W = 0$ as shown by Equation (36) and Figure 12.

FIGURE 12

Costs of Enriched Uranium
as Affected by Assuming a Unit Tails Value

THE EFFECT OF A NONOPTIMUM TAILS COMPOSITION
ON NUCLEAR REACTOR FUEL COSTS

The effect of toll enrichment may be to a large extent the result of having enrichment costs based on a nonoptimum tails composition. It is unlikely that a toll enrichment system could provide each customer with an optimum tails composition because (1) the prices paid for the feed material will vary widely, (2) the unit cost of separative duty might well depend on the size of the job, and (3) the value of the tails will vary. Thus, it is probable that the plant will be operated with a fixed number of stages in the stripping end which, given the feed composition, fixes the tails composition.⁽³⁾

It is not possible to evaluate the many different potential tolls separation cases. These may involve variations in the feed cost and composition, variations in the cost of separative duty and variations in the tails credit, although, as was shown previously, this is nearly the same as a reduction in the feed cost. It will suffice here to investigate the effect of a nonoptimum tails composition, with fixed feed and separative duty costs, on the fuel costs of some typical thermal reactors, thus demonstrating the applicability of the UCOST program.

The costs of enriching natural uranium to three different levels in cascades operated at five different tails compositions are shown in Table IV. The values assumed for the feed and separative duty costs correspond to the 1962 AEC price schedule and to the optimum tails composition in this schedule, 0.253071%. The costs constitute a uranium price schedule for each case. The burnup costs associated with each of these schedules are also shown. (The equations for the calculations are presented on page 50.)

TABLE IV
EXAMPLE ENRICHED URANIUM COSTS AND BURNUP COSTS
FOR AN ARBITRARY NONOPTIMUM CASCADE TAILS COMPOSITION

$C_f = \$23.50/\text{kg Uranium}$
 $C_s = \$30.00/\text{kg Uranium}$
 $C_w = 0$
 $X_f = 0.007115$

<u>Tails Composition, wt fraction</u>	<u>Production Cost, Dollars/kg Uranium</u>	<u>Production Cost, Dollars/g U²³⁵</u>	<u>Burnup Cost Dollars/g U²³⁵ Infinitesimal</u>	<u>Consumed 50%</u>
<u>Product Enrichment = 0.02</u>				
0.0005	194.53	9.73	14.10	13.51
0.0015	154.52	7.73	11.02	10.43
0.002531	146.52	7.33	10.40	9.81
0.0035	152.83	7.64	10.89	10.30
0.0045	176.65	8.83	12.72	12.13
<u>Product Enrichment = 0.05</u>				
0.0005	639.23	12.78	15.04	14.79
0.015	506.06	10.12	11.95	11.71
0.002531	479.42	9.59	11.34	11.10
0.0035	500.45	10.01	11.82	11.58
0.0045	579.70	11.59	13.66	13.42
<u>Product Enrichment = 0.90</u>				
0.0005	14,135.00	15.71	15.77	15.74
0.0015	11,362.00	12.63	12.68	12.65
0.002531	10,808.00	12.01	12.07	12.04
0.0035	11,246.00	12.50	12.55	12.52
0.0045	12,896.00	14.32	14.39	14.36

It may be seen that for a variation of tails from 0.0015 to 0.0035 weight fraction the burnup cost varies only from \$10.43 to \$9.81, and back to \$10.30 per gram for the case of 2% enrichment. This range of variation is not drastic, but it is significant, amounting to about 6%. If this effect on the burnup costs is interacted with other fuel cycle variables that can be adjusted to minimize the influence of nonoptimum tails, the variation becomes less than 6%, as will be shown in the paragraphs to follow. These data demonstrate the cost penalties incurred in nonoptimum operation of the cascade. But there is a region surrounding the minimum point, where the penalties are small, as was shown in Figure 10.

It is also worth noting that although reducing the tails composition below the optimum may increase costs slightly, it also decreases the amount of natural uranium required. This may be important in those instances wherein the amounts of natural uranium are considered to be in short supply.

U²³² and U²³⁶

It is well known that the neutron absorption does not always lead to fission even in the so-called fissile nuclei such as U²³³, U²³⁵, Pu²³⁹, and Pu²⁴¹, but that they also capture a neutron and produce higher isotopes--namely, U²³⁴, U²³⁶, Pu²⁴⁰, and Pu²⁴², respectively. Since U²³⁴ and Pu²⁴⁰ are fertile isotopes producing U²³⁵ and Pu²⁴¹, respectively, upon further neutron absorption, these are not normally considered parasitic in the sense that U²³⁶ and Pu²⁴² are considered parasitic. Further down the line Pu²³⁸ and Cm²⁴⁴ may in time have firm market values as decay isotopes for which U²³⁶ and Pu²⁴² are precursors. While uses of U²³⁴ and U²³⁶ as precursors may be of some value, these isotopes constitute a potential detriment to the most

straightforward application of the cascade, because their masses are closer to U^{235} than U^{235} is to U^{238} . Further, the U^{236} contamination introduced by one batch of feed will not be quantitatively recovered in the product and waste for that specific run but may appear in the cascade at a higher enrichment.

The price schedule developed in this report is based upon separation of a two component system--namely, U^{235} from U^{238} . Thus, the potential problem of separating U^{236} that may enter the cascade in feed uranium recovered from spent reactor fuels is not treated here. It will actually be some years before the total quantities of U^{236} fed to the cascade are significant compared to natural feed uranium fuels. Further, U^{234} from U^{233} may never enter the cascade unless intimately mixed U^{233} - U^{238} fuels are applied as discussed in Appendix B. There is one other potential problem of operating the cascade with uranium from spent reactor fuels. It is posed by the (n,2n) reaction on Np^{237} producing Pu^{236} , which then decays (2.85-yr half-life) to U^{232} . The U^{232} in turn decays (72 years) to Th^{228} , which then has an α decay sequence resulting in Tl^{208} , which emits a very hard gamma in decaying to Pb^{208} . The α activity and the gamma activity could complicate operation of the cascade. There is a very strict specification on the U^{232} content of feed to the cascade. If this limitation cannot be met with spent fuel from reactors, consideration will have to be given to altering the specification or to reducing the U^{232} content of the average feed. The equations developed in this text are limited then to the separation of U^{235} from U^{238} . Additional costs of one form or another might be involved if any significant amounts of other uranium isotopes were present.

Nuclear Reactor Simulations

The nuclear reactor simulations to be considered in this report are characterized in the MELEAGER burnup code⁽⁶⁾ mainly by the so-called parasitic absorption index, SNF. This parameter is the flux weighted absorption cross section of the non-fuel materials in the reactor per unit volume of fuel. The values selected correspond approximately to a D₂O moderated reactor, a boiling water reactor, and a pressurized water reactor. These are in order of increasing SNF.

The burnup calculations were performed for constant specific power irradiation in MELEAGER. These results were converted to uninteracted graded irradiation data by the PROTEUS code.⁽⁷⁾ Fuel costs and the determination of the minimum fuel cost were computed by a special version of the QUICK code⁽⁸⁾ which incorporated a portion of the UCOST code.

The fuel costs calculated for the boiling water reactor simulation are presented in Table V. These data show two effects.

First, like the burnup costs, the total fuel costs are at a minimum for the price schedule based on an optimum tails composition. However, because of the jacketing and separations charges, the effect is not as pronounced. Studies on higher jacketing costs, for example, show that the importance of optimum tails is reduced.

Second, the enrichment for minimum fuel cost increases as the tails composition approaches optimum from either the high or low side.

The effect of the nonoptimum tails composition on the minimized fuel costs in each of the three reactor types is shown in Table VI. These data show clearly that the optimum enrichment

TABLE V

NONOPTIMIZED FUEL COSTS FOR DIFFERENT ENRICHMENTS
IN A BOILING WATER REACTOR SIMULATION
FOR ARBITRARY CASCADE TAILS COMPOSITIONS
 Uranium Enrichment - Graded Irradiation

C_f = \$23.50 per kilogram uranium
 C_s = \$30.00 per kilogram uranium
 C_w = 0
 X_f = 0.007115 (weight fraction)
 FEFJ = \$20.00 per pound uranium
 Use Charge Rate = 4.75%
 Plutonium Credit = \$8.00 per gram

<u>Initial Enrichment</u>	<u>Exposure MWd/ton</u>	<u>Cascade Tails Composition</u>				
		<u>0.0005</u>	<u>0.0015</u>	<u>0.002531</u>	<u>0.0035</u>	<u>0.0045</u>
0.0123	5,800	2.45	2.12	2.06	2.11	2.30
0.0148	13,300	1.49	1.26	1.21	1.25	1.39
0.0198	23,400	1.28	1.05	1.01	1.04	1.18
0.0217	26,900	1.28	1.05	1.00	1.04	1.17
0.0247	31,700	1.30	1.06	1.02	1.05	1.20
0.280	36,900	1.36	1.10	1.05	1.09	1.24
0.0305	40,400	1.40	1.13	1.08	1.12	1.28

TABLE VI

MINIMIZED FUEL COSTS IN THREE TYPICAL THERMAL REACTORS
AS A FUNCTION OF AN ARBITRARY CASCADE TAILS COMPOSITION

Uranium Enrichment - Graded Irradiation

C_f = \$23.50 per kilogram uranium
 C_s = \$30.00 per kilogram uranium
 C_w = 0
 X_f = 0.007115 (weight fraction)
 $FEFJ$ = \$20.00 per pound uranium
 Use Charge Rate = 4.75%
 Plutonium Credit = \$8 per gram

D₂O Moderated Reactor Simulation

<u>X_w, Tails Composition</u>	<u>Initial Enrichment</u>	<u>Exposure, MWd/ton</u>	<u>Fuel Cost, mills/kWh_e</u>
0.0005	0.0155	21,400	1.03
0.0015	0.0164	23,100	0.87
0.002531	0.167	23,600	0.83
0.0035	0.0165	23,200	0.86
0.0045	0.0159	22,100	0.96

Boiling Water Reactor Simulation

0.0005	0.0206	24,900	1.27
0.0015	0.0214	26,200	1.05
0.002531	0.0215	26,400	1.00
0.0035	0.0214	26,200	1.04
0.0045	0.0209	25,400	1.17

Pressurized Water Reactor Simulation

0.0005	0.0298	30,000	1.65
0.0015	0.0295	29,600	1.33
0.002531	0.0297	29,900	1.27
0.0035	0.0296	29,700	1.32
0.0045	0.0295	29,600	1.51

is highest for a price schedule based on optimum tails composition, although the effect is not large. Again, this result would be modified by changing the jacketing costs.

In concluding this section, it should be emphasized that the foregoing analysis is not intended as a thorough investigation of a toll enrichment economy. Rather, it is an attempt to demonstrate the applicability of the UCOST program to problems in this area.

The Burnup Cost Calculation in UCOST

The burnup cost calculation is included in a subroutine of the UCOST program. This cost, which is calculated as a function of the initial enrichment and the burnup fraction, is given as dollars per gram U^{235} consumed. Thus, these values will always be at least as high as the price (in dollars per gram of U^{235} of the initial enrichment).

The equation is derived as follows:

Let

$$\begin{aligned} M &= U^{235} \text{ density, gram/cm}^3 \\ N &= U^{238} \text{ density, gram/cm}^3 \\ X &= \text{enrichment fraction} \end{aligned}$$

Then, the burnup cost per gram of U^{235} consumed, B, is:

$$\begin{aligned} B &= \frac{(M + N) C_p(X) - (M - \delta M + N - \delta N) C_p(X - \delta X)}{\delta M} \\ &= \left(\frac{M + N}{\delta M} \right) [C_p(X) - C_p(X - \delta X)] + \frac{\delta M + \delta N}{\delta M} C_p(X - \delta X) . \end{aligned}$$

(39)

Eliminating the variables M , N , δM , and δN in favor of X and δX will simplify the problem considerably. The relations between these are

$$X = \frac{M}{M + N}$$

and

$$X - \delta X = \frac{M - \delta M}{M - \delta M + N - \delta N} \quad (40)$$

A convenient parameter is R , the ratio of the grams of U^{238} consumed during the burnup to the grams of U^{235} consumed. That is, $R = \delta N / \delta M$. This definition, together with the above relations gives

$$\frac{\delta M}{M - N} = \frac{\delta X}{1 - (1 + R)(X - \delta X)} \quad (41)$$

Substituting Equation (41) into Equation (39) gives

$$B = [1 - (1 + R)(X - \delta X)] \frac{C_p(X) - C_p(X - \delta X)}{\delta X} + (1 + R) C_p(X - \delta X) \quad (42)$$

Equation (42) is the general uranium burnup cost equation as a function of X , the initial enrichment, and $\beta \equiv \delta X / X$, the burnup fraction. The parameter R is not considered an independent variable but instead as a function of X and δX as will be shown by development of Equation (45).

The form of Equation (21) suggests considering the limit as $\delta X \rightarrow 0$. This will give the so-called infinitesimal burnup cost, which is usually the maximum burnup cost for a given enrichment. Defining this quantity as B_0 gives

$$\begin{aligned}
 B_o(X) &= \lim_{\delta X \rightarrow 0} B(S, \delta X) \\
 &= [1 - (1 + R)X] \frac{dC_p(X)}{dX} + (1 + R) C_p(X) \quad (43)
 \end{aligned}$$

Using Equation (17) for $C_p(X)$ gives for the derivative

$$\frac{dC_p(X)}{dX} = C_s [\phi'(X_o) - \phi'(X_o)] \quad (44)$$

The parameter R is evaluated by assuming (1) that both U^{235} and U^{238} see the same neutron flux throughout the burnup, (2) that the ratio of the effective absorption cross sections is approximately equal to the ratio of the 2200 m/sec cross sections, and (3) that the amount of U^{236} produced is negligible. Under these conditions, for a small burnup

$$R = \frac{\delta N}{\delta M} = \frac{N\sigma_N}{M\sigma_M} = \left(\frac{1 - X}{X} \right) \sigma_R \quad (45)$$

where σ_R denotes the ratio of the U^{238} effective cross section to the U^{235} effective cross section. Therefore, for a finite burnup, the following equation is obtained

$$R = \frac{1}{\delta X} \int_X^{X + \delta X} \left(\frac{1 - \xi}{\xi} \right) \sigma_R d\xi \quad (46)$$

For σ_R constant, this leads to

$$R = \frac{\sigma_R}{\delta X} \left[\ln \xi - \xi \right] \Big|_X^{X + \delta X} \quad (47)$$

so that

$$R = \sigma_R \left[\frac{\ln \frac{X}{X - \delta X}}{\delta X} - 1 \right] \quad (48)$$

In the burnup section of the UCOST code, Equation (48) with $\sigma_r = 0.00468$ is used in Equation (42) to give the burnup costs associated with each enrichment and each burnup fraction specified. A sample computation is shown in Appendix B. In the burnup routine, Equation (45) is used in place of Equation (48) for the infinitesimal burnup case.

The UCOST Computer Code Listing

Following is the UCOST program as coded in FORTRAN and designed for the IBM 7090. The objective was to make the calculation as general as possible and still require only a small amount of input.

UCOST PROGRAM

* LIST8

* LABEL

CUCST UCOST - A PROGRAM TO CALCULATE URANIUM PRICE SCHEDULES

C

C

C

C

C

OPERATING INSTRUCTIONS

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

C

EACH PRICE SCHEDULE CALCULATION REQUIRES A MINIMUM OF ONE DATA CARD AND A MAXIMUM OF FIVE DATA CARDS. DATA CARD NUMBER 1, THE CONTROL INTEGERS CARD, MUST BE USED FOR THE FIRST CASE BUT MAY BE OMITTED IN SUBSEQUENT CASES BY APPROPRIATELY SETTING N5. DATA CARDS NUMBER 2, 3, AND 4 ARE ALWAYS OPTIONAL EXCEPT THAT THEY MUST FOLLOW DATA CARD NUMBER 1. DATA CARD NUMBER 5, THE PRICE SCHEDULE PARAMETERS CARD, IS REQUIRED IN EVERY CASE.

DATA CARDS NUMBER 2 AND 3 SPECIFY THE ENRICHMENTS TO BE CALCULATED AND ARE USED WHEN N2 AND N10 RESPECTIVELY ARE NONZERO. DATA CARD NUMBER 4, SPECIFIES THE BURNUP FRACTIONS AND IS USED WHEN N9 IS TWO OR GREATER. NOTE THAT IF THESE CARDS ARE NOT USED THE PROGRAM WILL USE A STANDARD SET OF PARAMETERS.

1. CONTROL INTEGERS CARD (FORMAT 10I5,F10.5)

N1=1, THE PRICE OF THE FEED MATERIAL, THE COST OF SEPARATIVE DUTY AND, THE TAILS VALUE ARE SPECIFIED. THE OPTIMUM TAILS COMPOSITION WILL BE CALCULATED.

=2, THE PRICE OF THE FEED MATERIAL, THE OPTIMUM TAILS COMPOSITION AND, THE TAILS VALUE ARE SPECIFIED. THE REQUIRED COST OF SEPARATIVE DUTY WILL BE CALCULATED.

=3, THE COST OF SEPARATIVE DUTY, THE OPTIMUM TAILS COMPOSITION AND, THE TAILS VALUE ARE SPECIFIED. THE REQUIRED FEED MATERIAL PRICE WILL BE CALCULATED.

=4, THE PRICE OF THE FEED MATERIAL, THE COST OF SEPARATIVE DUTY AND, THE OPTIMUM TAILS COMPOSITION ARE SPECIFIED. THE REQUIRED TAILS VALUE WILL BE CALCULATED.

=5, THE PRICE OF THE FEED MATERIAL, THE COST OF SEPARATIVE DUTY A TAILS COMPOSITION AND, THE TAILS VALUE ARE SPECIFIED. THE TAILS COMPOSITION WILL NOT NECESSARILY BE THE OPTIMUM.

N2=0, A STANDARD SET OF OUTPUT FORMAT PARAMETERS WILL BE USED.

=1, THE OUTPUT FORMAT PARAMETERS WILL BE SUPPLIED ON CARD NO. 2.

N3=0, INPUT DATA WILL BE INTERPRETED AS \$/KG.

=1, INPUT DATA WILL BE INTERPRETED AS \$/LB.

N4=0, INPUT DATA WILL BE INTERPRETED AS WEIGHT FRACTIONS.

=1, INPUT DATA WILL BE INTERPRETED AS ATOMIC FRACTIONS.

N5=0, CONTROL INTEGER CARD WILL BE USED IN THE NEXT CALCULATION.

= THE NUMBER OF CASES TO BE RUN WITH THIS CONTROL CARD.

N6=0, SCHEDULES WILL BE NUMBERED CONSECUTIVELY.

=1, SCHEDULE IDENTIFICATION WILL BE ON DATA CARD NUMBER 5.

N7=0, FOR PRICE SCHEDULE PRINTOUT ONLY.

=1, ITERATIVE TAILS COMPOSITION CALCULATION WILL BE PRINTED.

N8=0, PRICE SCHEDULES WILL BE WRITTEN ON A-3. OTHERWISE, TAPE NUMBER N8 WILL BE THE OUTPUT TAPE.

N9=0, BURNUP CALCULATION WILL BE OMITTED.

=1, URANIUM BURNUP COSTS FOR SIX DEGREES OF BURNUP WILL BE CALCULATED FOR EACH SPECIFIED ENRICHMENT.

=2-6, URANIUM BURNUP COSTS WILL BE CALCULATED FOR THE BURNUP FRACTIONS SPECIFIED ON DATA CARD 4.

N10 = THE NUMBER OF SPECIAL ENRICHMENTS TO BE CALCULATED.

XMIN = THE MINIMUM PRICED COMPOSITION USED IN THE BURNUP CALCULATION. IF XMIN IS ZERO OR NEGATIVE ITS ABSOLUTE VALUE WILL BE USED AS THE MINIMUM PRICE.

2. OUTPUT FORMAT PARAMETERS CARD (FORMAT 5I4,6F10.7)

THIS CARD WILL BE USED ONLY IF N2=1. THE INTEGERS REPRESENT THE NUMBER OF ENRICHMENTS IN EACH OF FIVE GROUPS. THE FIRST FIXED POINT VARIABLE REPRESENTS THE FIRST ENRICHMENT TO BE CALCULATED WHILE THE REMAINING FIVE VARIABLES ARE ENRICHMENT INCREMENTS IN EACH GROUP. THE UNITS ARE SPECIFIED BY N4.

IF N2=0, THE PARAMETERS WILL BE ASSIGNED INTERNALLY AS FOLLOWS

10	10	15	5	8	.001	.001	.001	.002	.01	.10

3. IRREGULAR ENRICHMENTS CARD (FORMAT 8F10.7)

IF N10=0 THIS CARD WILL NOT BE USED. OTHERWISE, N10 ENRICHMENTS THAT CANNOT BE CONVENIENTLY SPECIFIED BY THE OUTPUT FORMAT PARAMETERS WILL BE CALCULATED.

4. BURNUP FRACTIONS CARD (FORMAT 5F10.4)

IF N9 IS LESS THAN TWO THIS CARD WILL NOT BE USED. OTHERWISE, FROM ONE TO FIVE BURNUP FRACTIONS WILL BE CALCULATED DEPENDING ON THE MAGNITUDE OF N9.

IF N9=1, THE BURNUP FRACTIONS THAT WILL BE USED ARE

0.10	0.25	0.50	0.75	0.95

5. PRICE SCHEDULE PARAMETERS CARD FORMAT (5F10.4,5A6)

THIS CARD CONTAINS THE FOLLOWING FIVE PARAMETERS AND A 30 DIGIT IDENTIFICATION NUMBER OR PHRASE.

THE COST OF THE FEED MATERIAL
 THE COST OF SEPARATIVE DUTY
 THE OPTIMUM TAILS COMPOSITION
 THE VALUE OF THE TAILS COMPOSITION
 THE FEED COMPOSITION

NOTE THAT ONLY THREE OF THE FIRST FOUR PARAMETERS CAN BE SPECIFIED INDEPENDENTLY IF THE SCHEDULE IS TO BE BASED ON THE OPTIMUM

WILL BE IGNORED. NOTE THAT IF N1=5 ALL FOUR PARAMETERS WILL BE USED BUT THAT THE TAILS COMPOSITION WILL NOT NECESSARILY BE THE OPTIMUM. THE UNITS ARE DETERMINED BY N3 AND N4.

THE PROGRAM WILL INTERPRET A ZERO FEED COMPOSITION AS .007115.

MAIN VARIABLE NAMES

CF = COST OF FEED
 CS = COST OF SEPARATIVE DUTY
 XT = TAILS COMPOSITION
 CW = TAILS VALUE
 XM = MINIMUM PRICED COMPOSITION
 X = ENRICHMENT, WT FRACTION
 W = ENRICHMENT, ATOMIC FRACTION
 XF = FEED COMPOSITION
 VXT = VALUE FUNCTION EVALUATED AT THE FEED COMPOSITION
 VPXT = DERIVATIVE OF THE VALUE FUNCTION EVALUATED AT THE TAILS COMPOSITION
 BUPT = BURNUP FB ACTION
 DX = URANIUM COST
 FPT = FEED TO PRODUCT FLOW RATE RATIO
 SPR = SEPARATIVE DUTY TO FLOW RATE RATIO
 DEX = ENRICHMENT COST PORTION OF TOTAL COST
 DNX = FEED COST PORTION OF TOTAL COST

DIMENSION W(1000),N(12),M(5),F(6),A(5),BUFT(10)

COMMON W,A,CF,CS,XT,CW,XF,VXT,VXF,VPXT,MK,N,ID,XM,M,BUFT

ID=0
 READS DATA CARD NUMBER 1
 10 READ INPUT TAPE 2,1001,(N(J),J=1,10),XMIN
 I=N(1)
 IF (I) 980,990,11
 11 IF (I-5) 12,12,980
 12 IF (N(8)) 13,13,15
 13 MK=3
 GO TO 20
 15 MK=N(8)
 PRINT 1000
 PAUSE
 DETERMINES OUTPUT FORMAT
 CALL RDYTP (MK)
 20 IF (N(2)) 29,21,29
 21 M(1)=9
 M(2)=10
 M(3)=15
 M(4)=5
 M(5)=8
 F(1)=.001
 F(2)=.001
 F(3)=.001
 F(4)=.002
 F(5)=.01
 F(6)=.10
 READS DATA CARD NUMBER 2
 25 GO TO 30
 29 READ INPUT TAPE 2,1002,M,F
 M(1)=M(1)-1

```

31 M1=2
35 DO 60 J=1,5
40 IF (M(J)) 65,65,45
45 M2=M1+M(J)-1
50 DO 55 K=M1,M2
55 W(K)=W(K-1)+F(J+1)
60 M1=M2+1
65 M2=M1+N(10)-1
   N(11)=M2
C
C   READS DATA CARD NUMBER 3
70 IF (N(10)) 80,80,75
75 READ INPUT TAPE 2,1003,(W(J),J=M1,M2)
C
80 IF (N(4)) 95,95,85
85 DO 90 J= 1,M2
90 W(J)=W(J)/(1.012794-0.012794*W(J))
C
C   READS DATA CARD NUMBER 4
95 IF (N(9)-1) 100,100,96
96 K=N(9)
   READ INPUT TAPE 2,1003,(BUFT(J),J=2,K)
C
C
C   READS DATA CARD NUMBER 5
100 READ INPUT TAPE 2,1004,CF,CS,XT,CW,XF,A
   NPAGE=0
C   SETS INITIAL CONDITION FOR CASE - PRICE SCHEDULE PARAMETERS IN $/KG
C   AND ENRICHMENTS IN WEIGHT FRACTION
   NLINE=1
   K=0
   IF (CF+CS+XT+CW) 990,990,105
105 IF (N(3)) 115,115,110
110 CF=CF*2.20462
   CS=CS*2.20462
   CW=CW*2.20462
115 IF (N(4)) 125,125,120
120 XT=XT/(1.012794-0.012794*XT)
   XF=XF/(1.012794-0.012794*XF)
C
C   DETERMINES FEED COMPOSITION
125 IF (XF) 135,135,130
130 IF (XF-1.0) 140,135,135
135 XF=0.007115
140 VXF=(1.0-2.0*XF)*LOGF((1.0-XF)/XF)
C
150 GO TO (200,155,155,155,155),I
C   CALCULATES UNSPECIFIED PARAMETER FROM OPTIMUM TAILS EQUATION
155 IS = 1
   XTG=LOGF((1.0-XT)/XT)
160 VXT=(1.0-2.0*XT)*XTG
165 VPXT=(2.0*XT-1.0)/(XT*(1.0-XT))-2.0*XTG
170 RFS=VXF-VXT-(XF-XT)*VPXT
C
175 GO TO (200,180,185,190,300),I
180 CS=(CF-CW)/RFS
   GO TO 300
185 CF=CS*RFS+CW
   GO TO 300
190 CW=CF-CS*RFS
   GO TO 300
C   START OF OPTIMUM TAILS CALCULATION

```

```

200 IF (CS) 270,270,201
201 IF (CF-CW) 280,280,202
202 S=CS/(CF-CW)

```

```

C
  IF (N(7)) 203,205,203
203 WRITE OUTPUT TAPE 3,2001,S
205 AM=0.0
  IS=1
  AN=0.0
  IM=0
  IN=0
C
  INITIAL ESTIMATE OF TAILS COMPOSITION
  IF (S-5.0) 207,207,206

```

```

206 XT=XF*0.7
  GO TO 210
207 IF (S-0.010) 209,208,208
208 E=.0143+.5562*S-.3349*S**2+.1075*S**3-.0161*S**4+.00096*S**5
  1 -.000014*S**6
  XT=XF*E
  GO TO 210

```

```

209 XT=1.5*S
C
  ITERATIVE LOOP

```

```

C
210 XTLOG=LOGF(XT/(1.0-XT))
  VXT=(2.0*XT-1.0)*XTLOG
  VPXT=(2.0*XT-1.0)/(XT*(1.0-XT))+2.0*XTLOG
  RFS=VXF-VXT-(XF-XT)*VPXT
  Y=S*RFS
  IF (N(7)) 212,215,212
212 WRITE OUTPUT TAPE3,2009,AM,IM,AN,IN,XT,RFS ,Y
C
  CONVERGENCE CHECK

```

```

C
215 IF (Y-1.0) 220,300,225
220 IF (Y-0.999999) 230,230,300
225 IF (Y-1.000001) 300,235,235

```

```

C
230 AN=-1.0
  GO TO 240
235 AN=1.0
240 IF (IN) 245,245,250
245 AM=AN
  IN=1

```

```

C
250 IF (AM-AN) 255,260,255
255 IM=IM+1
  AM=0.0
260 XTD = 0.0001*AN/(2.0**IM)
  IF (XT+XTD) 255,255,261
261 XT=XT+XTD
  GO TO 210

```

```

C
C
270 IS=3
  XT=0.0
  VXT=0.0
  VPXT=0.0
  GO TO 300

```

```

C
280 IS=2
  XT=XF
  VXT=VXF
  VPXT=(2.0*XF-1.0)/(XF*(1.0-XF))+2.0*LOGF(XF/(1.0-XF))

```

GO TO 305

START OF SPECIFIC CALCULATION FOR THE CASE

300 $WXT = (VXF - VXT) / (XF - XT)$

305 $DF = CF / 2.20462$

$DS = CS / 2.20462$

$DW = CW / 2.20462$

HEADING PRINT OUT

$N(12) = IS$

IF (N(6)) 320,310,320

310 $ID = ID + 1$

315 WRITE OUTPUT TAPE MK,2002,ID

GO TO 324

320 WRITE OUTPUT TAPE MK,2003,A

324 GO TO (325,325,325,325,326),I

325 WRITE OUTPUT TAPE MK,2004,XF,XT,CF,DF,CS,DS,CW,DW

GO TO 327

326 WRITE OUTPUT TAPE MK,2012,XF,XT,CF,DF,CS,DS,CW,DW

327 WRITE OUTPUT TAPE MK,2007

335 $RX = (WXT - VPXT) * CS$

ENRICHMENT CALCULATION LOOP

350 DO 450 J=1,M2

$X = W(J)$

IF (X) 500,500,355

355 IF (X-1.0) 360,500,500

360 $Y = 1.012794 * X / (1.0 + 0.012794 * X)$

370 $VX = (2.0 * X - 1.0) * \text{LOGF}(X / (1.0 - X))$

URANIUM COST CALCULATION

GO TO (376,376,375),IS

375 $DX = X * (CF - CW) / XF + CW$

GO TO 385

376 GO TO (380,380,380,380,377),I

377 $DX = CS * (VX + (VXT * (X - XF) - VXF * (X - XT)) / (XF - XT)) + CF * (X - XT) / (XF - XT)$
 $- CW * (X - XF) / (XF - XT)$

GO TO 385

380 $DX = CS * (VX - VXT - (X - XT) * VPXT) + CW$

CALCULATION OF RELATED QUANTITIES

385 $DP = DX / 2.20462$

390 $DR = DX / (X * 1000.0)$

395 $DEX = CS * (VX - VXT - (X - XT) * WXT)$

400 $DNX = (CF * (X - XT) - CW * (X - XF)) / (XF - XT)$

405 $FPR = (X - XT) / (XF - XT)$

410 $SPR = VX - VXT + FPR * (VXT - VXF)$

415 WRITE OUTPUT TAPE MK,2005,X,Y,DX,DP,DR,FPR,SPR,DNX,DEX

IF (J-M2) 430,450,450

430 IF (J-M(NLINE)-K-1) 440,435,440

435 WRITE OUTPUT TAPE MK,2008

$NLINE = NLINE + 1$

$K = J - 1$

440 IF (J-20-NPAGE*35) 450,445,450

445 IF (N(6)) 447,446,447

HEADING PRINTOUT FOR NEW PAGE

446 WRITE OUTPUT TAPE MK,2002,ID

GO TO 448

447 WRITE OUTPUT TAPE MK,2003,A
 448 WRITE OUTPUT TAPE, MK,2007
 NPAGE=NPAGE+1

C

450 CONTINUE

C

ENTRANCE TO BURNUP CALCULATION ROUTINE

C

IF (N(9)) 500,500,455

455 IF (XMIN) 465,465,460

460 XM=XMIN

GO TO 475

C

465 GO TO (470,470,466),15

466 DXM=-XMIN

XM=XF*(DXM-CW)/(CF-CW)

GO TO 475

470 XM=-XMIN

CALL MINX

C

475 CALL BURNUP

C

500 N(5) = N(5)-1

IF (N(5)) 10,10,100

C

C

END OF JOB HOUSEKEEPING

980 WRITE OUTPUT TAPE 3,2011

990 IF (MK-3) 995,999,995

995 END FILE MK

REWIND MK

PRINT 2006

PAUSE

999 CALL EXIT

C

1000 FORMAT(39H MOUNT SPECIFIED TAPE, THEN PRESS START/////)

1001 FORMAT (10I5,F10.4)

1002 FORMAT (5I4,6F10.7)

1003 FORMAT (8F10.7)

1004 FORMAT (5F10.7,5A6)

2001 FORMAT (42H1

DIAGNOSTICS FOR S = F10.6//72H

1 AM IM AN IN XT VNAT

1 Y//)

2002 FORMAT (1H1///42X,32HURANIUM PRICE SCHEDULE NUMBER14/////)

2003 FORMAT (1H1///36X,25HURANIUM PRICE SCHEDULE 5A6/////)

2004 FORMAT (14X,19HFEED COMPOSITION =2PF8.5,4H W/O,19X,28HOPTIMUM TAI

1LS COMPOSITION =F9.6,4H W/O///

235X,22HFEED MATERIALS COST =OPF6.2,11H \$/KG OR F5.2,5H \$/LB//

335X,22HSEPARATIVE DUTY COST =F6.2,11H \$/KG OR F5.2,5H \$/LB//

435X,22HASSIGNED TAILS VALUE =F6.2,11H \$/KG OR F5.2,5H \$/LB/////)

2005 FORMAT (2PF8.3,F11.3,OPF17.4,F12.4,F10.4,F17.4,F14.4,F18.4,F12.4)

2006 FORMAT (1H020X,33H REMOVE OUTPUT TAPE - PRESS START ///)

2007 FORMAT (5X,10HENRICHMENT,54X,18HFLOW RATE RATIOS,9X,24HEXPENSE P

1ORTIONS, \$/KG U/30X,27HCALCULATED URANIUM PRICES/2X,17HWEIGHT

2 ATOMIC,49X,23HKG FEED KG SEP WORK,9X,19HCOST OF COST OF/

3120H PER CENT PER CENT \$/KG U \$/LB U \$/GM U235

4 PER KG PROD PER KG PROD FEED MAT SEP WORK ///)

2008 FORMAT (1H)

2009 FORMAT (F9.2,I7,F8.2,I7,F13.9,2F15.6)

2010 FORMAT (1H1//)

2011 FORMAT (1H0/10X,40HINPUT DATA IS INCORRECT - JOB TERMINATED)

2012 FORMAT (14X,19HFEED COMPOSITION =2PF8.5,4H W/O,19X,29HASSIGNED TA

1LS COMPOSITION =F9.6,4H W/O///

235X,22HFEED MATERIALS COST =OPF6.2,11H \$/KG OR F5.2,5H \$/LB//

435X,22HASSIGNED TAILS VALUE =F6.2,11H \$/KG OR F5.2,5H \$/LB/////)

END

* LABEL
* LIST8
* LABEL

C 214 BURNUP SUBROUTINE OF UCOST FEBRUARY 14, 1963

SUBROUTINE BURNUP

DIMENSION W(1000),A(5),N(12),B(6),C(6),D(10),M(5)

COMMON W,A,CF,CS,XT,CW,XF,VXT,VXF,VPXT,MK,N,ID,XM ,M,D

C

I=N(1)
M2=N(11)
IS=N(12)
D(1)=0.00
NPAGE=0
NLINE=1
NK=0

C

IF (XM) 200,200,201

200 DX=0.0

X=0.0

GO TO 5

C

201 X=XM

VX=(2.0*X-1.0)*LOGF(X/(1.0-X))

GO TO (2,2,1),IS

1 DX=X*(CF-CW)/XF+CW

GO TO 5

2 GO TO (4,4,4,4,3),1

3 DX=CS*(VX+(VXT*(X-XF)-VXF*(X-XT))/(XF-XT))+CF*(X-XT)/(XF-XT)

GO TO 5

4 DX=CS*(VX-VXT-(X-XT)*VPXT)+CW

5 DXM=DX

XM=X

C

WRITE OUTPUT TAPE MK,1000

VPXX=(2.0*XF-1.0)/(XF*(1.0-XF))+2.0*LOGF(XF/(1.0-XF))

IF (N(6)) 7,6,7

6 WRITE OUTPUT TAPE MK,1001,ID

GO TO 8

7 WRITE OUTPUT TAPE MK,1002,A

C

8 IF (N(9)-1) 9,9,10

9 N9=6

D(2)=0.10

D(3)=0.25

D(4)=0.50

D(5)=0.75

D(6)=0.95

GO TO 11

10 N9=N(9)

11 WRITE OUTPUT TAPE MK,2010,(D(K),K=2,N9)

D(N9+1)=1.00

C

DO 115 J=1,M2

X=W(J)

IF (X-XM) 110,15,15

15 VX=(2.0*X-1.0)*LOGF(X/(1.0-X))

VPX=(2.0*X-1.0)/(X*(1.0-X))+2.0*LOGF(X/(1.0-X))

R=0.004680*(1.0-X)/X

C

GO TO (25,25,20),IS

20 DX=X*(CF-CW)/XF+CW

```

      DPX=(CF-CW)/XF
      GO TO 40
25  GO TO (35,35,35,35,30),I
30  DX=CS*(VX+(VXT*(X-XF)-VXF*(X-XT))/(XF-XT))+CF*(X-XT)/(XF-XT)
      1 -CW*(X-XF)/(XF-XT)
      DPX=CS*(VPX+(VXT -VXF)/(XF-XT))+CF/(XF-XT)-CW/(XF-XT)
      GO TO 40
35  DX=CS*(VX-VXT-(X-XT)*VPXT)+CW
      DPX=CS*(VPX-VPXT)
C
40  DX=DX*0.001
      DPX=DPX*0.001
      DPG=DX/X
      B(1)=(1.0-(1.0+R)*X)*DPX+(1.0+R)*DX
C
C
      DO 90 K=2,N9
      Z=(1.0-D(K))*X
      R=0.004680*(LOGF(X/Z)/(X-Z)-1.0)
C
      IF (Z-XM) 44,44,45
44  Y=XM
      DY=DXM
      GO TO 70
C
45  Y=Z
      VY=(2.0*Y-1.0)*LOGF(Y/(1.0-Y))
C
      GO TO (55,55,50),IS
50  DY=Y*(CF-CW)/XF+CW
      GO TO 70
55  GO TO (65,65,65,65,60),I
60  DY=CS*(VY+(VXT*(Y-XF)-VXF*(Y-XT))/(XF-XT))+CF*(Y-XT)/(XF-XT)
      1 -CW*(Y-XF)/(XF-XT)
      GO TO 70
65  DY=CS*(VY-VXT-(Y-XT)*VPXT)+CW
C
70  DY=DY*0.001
      B(K)=(1.0-(1.0+R)*Z)*(DX-DY)/(X-Z)+(1.0+R)*DY
90  CONTINUE
C
97  DO 98 K=1,7
      IF (XM-X*(1.0-D(K))) 98,99,99
98  CONTINUE
C
99  GO TO (106,105,104,103,102,101,100,100),K
100 WRITE OUTPUT TAPE MK,2000,X,DPG,(B(L),L=1,N9)
      GO TO 110
101 WRITE OUTPUT TAPE MK,2001,X,DPG,(B(L),L=1,N9)
      GO TO 110
102 WRITE OUTPUT TAPE MK,2002,X,DPG,(B(L),L=1,N9)
      GO TO 110
103 WRITE OUTPUT TAPE MK,2003,X,DPG,(B(L),L=1,N9)
      GO TO 110
104 WRITE OUTPUT TAPE MK,2004,X,DPG,(B(L),L=1,N9)
      GO TO 110
105 WRITE OUTPUT TAPE MK,2005,X,DPG,(B(L),L=1,N9)
      GO TO 110
106 WRITE OUTPUT TAPE MK,2006,X,DPG,(B(L),L=1,N9)
C
110 CONTINUE
C

```

96 WRITE OUTPUT TAPE MK,2012
 NLINE=NLINE+1
 NK=J-1

95 IF (J-20-NPAGE*35) 115,91,115
 91 WRITE OUTPUT TAPE MK,2011,XM,DXM
 IF (N(6)) 93,92,93
 92 WRITE OUTPUT TAPE MK,1001,ID
 GO TO 94
 93 WRITE OUTPUT TAPE MK,1002,A
 94 WRITE OUTPUT TAPE MK,2010,(D(K),K=2,N9)
 NPAGE=NPAGE+1

115 CONTINUE
 WRITE OUTPUT TAPE MK,2011,XM,DXM
 120 RETURN

1000 FORMAT (1H1///38X,45HBURN-UP COSTS PER GRAM OF U235 CONSUMED
 1////////)
 1001 FORMAT (42X,32HURANIUM PRICE SCHEDULE NUMBER14)
 1002 FORMAT (35X,26HURANIUM PRICE SCHEDULE 5A6)
 2000 FORMAT (2PF9.3,0PF17.3,5X,6F13.3)
 2001 FORMAT (2PF9.3,0PF17.3,5X,6F13.3,2H *)
 2002 FORMAT (2PF9.3,0PF17.3,5X,5F13.3,2H *F11.3,2H *)
 2003 FORMAT (2PF9.3,0PF17.3,5X,4F13.3,2H *F11.3,2H *F11.3,2H *)
 2004 FORMAT (2PF9.3,0PF17.3,5X,3F13.3,2H *F11.3,2H *F11.3,2H *F11.3,
 1 2H *)
 2005 FORMAT (2PF9.3,0PF17.3,5X,2F13.3,2H *F11.3,2H *F11.3,2H *F11.3,
 1 2H *F11.3,2H *)
 2006 FORMAT (2PF9.3,0PF17.3,5X,F13.3,2H *F11.3,2H *F11.3,2H *F11.3,
 1 2H *F11.3,2H *F11.3,2H *)
 2010 FORMAT (1H /// 9H INITIAL,8X,13HINITIAL PRICE, 9X,
 1 64HRATIO OF THE ENRICHMENT DECREASE TO THE INITIAL
 2 ENRICHMENT X 100/
 311H ENRICHMENT/
 411H WT PERCENT,8X, 9H\$/GM U235 ,6X,13HINFINITESIMAL
 5 2PF9.1,4F13.1//)
 2011 FORMAT (1H // 10X,90H* DENOTES CASES IN WHICH THE FINAL ENRICHMENT
 1 IS LESS THAN THE MINIMUM PRICED COMPOSITION./
 2 12X,33HTHE MINIMUM PRICED COMPOSITION IS2PF7.3,27H PERCENT WHICH
 2IS PRICED AT0PF7.2,14H \$/KG URANIUM./1H1)
 2012 FORMAT (1H)

END
 * LIST8
 * LABEL

C 214 MINX SUBROUTINE OF UCOST
 SUBROUTINE MINX

DIMENSION W(1000),N(12),M(5),F(6),A(5),BUFT(10)

COMMON W,A,CF,CS,XT,CW,XF,VXT,VXF,VPXT,MK,N,ID,XM,M,BUFT

DXM=XM
 AM=0.0
 AN=0.0
 IM=0
 IN=0

X=XF
 VXF=(2.0*XF-1.0)*LOGF(XF/(1.0-XF))
 VX=VXF

T=XF-XT
 C=XT*(VXF-CF/CS)-XF*(VXT-CW/CS)

S=DXM*T/CS-C
R=VXT-CW/CS-VXF+CF/CS
CR=(CF-CW)/(XF-XT)
VR=(VXT-VXF)/(XF-XT)

C

210 Y=T*VX+R*X-S
IF (Y) 220,300,225
220 IF (Y+1.0E-07) 230,230,300
225 IF (Y-1.0E-07) 300,235,235

C

230 AN=-1.0
GO TO 240
235 AN=1.0
240 IF (IN) 245,245,250
245 AM=AN
IN=1

C

250 IF (AM-AN) 255,260,255
255 IM=IM+1
AM=0.0

C

260 DEL=1.0E-03*AN/(2.0**IM)
X=X-DEL
VX=(2.0*X-1.0)*LOGF(X/(1.0-X))
VPX=(2.0*X-1.0)/(X*(1.0-X))+2.0*LOGF(X/(1.0-X))
DPX=CS*(VPX+VR)+CR

C

IF (DPX) 280,300,210
280 IM=IM+1
AN=-1.0
GO TO 260

C

300 XM=X
RETURN
END

DESCRIPTION OF THE OUTPUT

The printed output for each price schedule in Appendix C shows the feed and the tails compositions in weight percent. It shows the feed materials cost, the separative duty cost and the assigned tails value in dollars per kilogram and per pound of uranium. If an optimum tails composition is calculated, it is identified as such.

The following quantities are printed for each product enrichment specified:

1. the equivalent enrichment in weight percent and in atomic percent
2. uranium prices in dollars per kilogram of uranium, dollars per pound of uranium, and dollars per gram of U^{235} contained
3. the flow rates of feed and separative work per kilogram of product
4. feed and separative work cost portions of the total cost in dollars per kilogram of uranium.

The formulas for these are as follows:

1. The Atomic Enrichment, X_a , is

$$X_a = \frac{AX}{1 + (A - 1)X} \quad (49)$$

where A is the ratio of the atomic weight of U^{238} to the atomic weight of U^{235} . This ratio is 1.01279.

2. The Uranium Price, C_p
  - (a) C_p in dollars per kilogram of uranium is given by Equation (34) on page 33 with C_s , C_f , and C_w in dollars per kilogram of uranium.
  - (b) C_p in dollars per pound of uranium is obtained by dividing the result in dollars per kilogram of uranium by the constant 2.20462.
  - (c) C_p in dollars per gram U^{235} is obtained by dividing the result in dollars per kilogram of uranium by 1000X.

3. Flow Rate Ratios

The ratio of the feed flow rate to the product flow rate, F/P , is given by Equation (23) on page 29. The ratio of the separative work to the product flow rate, Δ/P , is given by Equation (21) on page 22. The ratio of the tails flow rate to the product flow rate, W/P is simply $F/P - 1$, and is not calculated.

4. Feed Materials Cost

- (a) N , the portion of the total cost of the enriched uranium that is directly related to the materials cost, can be obtained from Equation (34).

$$N = \frac{C_f(X_p - X_w) - C_w(X_p - X_f)}{(X_f - X_w)} \quad (50)$$

Thus, it is not strictly a feed materials cost unless $C_w = 0$.

- (b) E , the portion of the total product cost that is directly related to the enrichment cost, is the remainder of Equation (34)

$$E = C_s \left[\phi(X_p) + \frac{(X_p - X_f) \phi(X_w) - (X_p - X_w) \phi(X_f)}{(X_f - X_w)} \right] \quad (51)$$

APPENDIX A

DERIVATION OF THE VALUE FUNCTION
AND THE SEPARATIVE DUTY REQUIREMENT

- - -

GENERALIZED URANIUM PRICE SCHEDULES

APPENDIX A

DERIVATION OF THE VALUE FUNCTION AND THE SEPARATIVE DUTY REQUIREMENT*

Suppose that G moles of material containing N mole fraction of a valuable component are fed to a separating element and a fraction θ of the G moles passes the barrier to the enriched stream. Let $V(N)$ be the value per mole of the feed, N' the mole fraction of the enriched stream, and N'' the mole fraction of the depleted stream. A material balance on the valuable component gives

$$N = \theta N' + (1 - \theta) N'' \quad (A-1)$$

The values, U , of each of the respective streams are the value per mole times the number of moles

$$U = GV(N)$$

$$U' = \theta GV(N')$$

$$U'' = (1 - \theta) GV(N'')$$

The change in value, δU , resulting from the passage of G moles of material through one separating element is the difference between the value of the exit streams and that of the entering stream

$$\delta U = \theta GV(N') + (1 - \theta) GV(N'') - GV(N) \quad (A-2)$$

If $V(N')$ and $V(N'')$ are expanded in a Taylor series about N

$$V(N') = V(N) + V'(N)(N' - N) + \frac{1}{2}V''(N)(N' - N)^2 + \dots (A-3)$$

$$V(N'') = V(N) + V'(N)(N'' - N) + \frac{1}{2}V''(N)(N'' - N)^2 + \dots (A-4)$$

---

* The nomenclature follows Cohen,⁽³⁾ and the derivation is essentially a rearrangement with the solution of the value function added.

Using only the first three terms and substituting these series into Equation (A-2)

$$\begin{aligned}\delta U = & \theta G[V(N) + V'(N)(N' - N) + \frac{1}{2}V''(N)(N' - N)^2] \\ & + (1 - \theta)G[V(N) + V'(N)(N'' - N) + \frac{1}{2}V''(N)(N'' - N)^2] \\ & - GV(N)\end{aligned}\quad (A-5)$$

Rearranging to collect coefficients of $V(N)$, $V'(N)$ and $V''(N)$

$$\begin{aligned}\delta U = & [\theta G + (1 - \theta)G - G]V(N) \\ & + [\theta G(N' - N) + (1 - \theta)G(N'' - N)]V'(N) \\ & + [\frac{1}{2}\theta G(N' - N)^2 + \frac{1}{2}(1 - \theta)G(N'' - N)^2]V''(N)\end{aligned}\quad (A-6)$$

$$\begin{aligned}\delta U = & (\theta)V(N) + [\theta GN' + (1 - \theta)GN'' - GN]V'(N) \\ & + \frac{G}{2}[\theta(N' - N)^2 + (1 - \theta)(N'' - N)^2]V''(N)\end{aligned}\quad (A-7)$$

But the coefficient of $V(N)$ vanishes and that of $V'(N)$ is just the material balance, Equation (A-1), and there is left

$$\delta U = \frac{G}{2}[\theta(N' - N)^2 + (1 - \theta)(N'' - N)^2]V''(N)\quad (A-8)$$

Before simplifying the coefficient of $V''(N)$ in Equation (A-8), the stage separation factor α is introduced and defined such that

$$\frac{N'}{1 - N'} = \alpha \frac{N}{1 - N}\quad (A-9)$$

Solving for α

$$\alpha = \frac{N'(1 - N)}{N(1 - N')}\quad (A-10)$$

and

$$\begin{aligned}\alpha - 1 &= \frac{N'(1 - N) - N(1 - N')}{N(1 - N')} \\ &= \frac{N' - N}{N(1 - N')}\end{aligned}\quad (A-11)$$

so that

$$N' - N = (\alpha - 1) N(1 - N') \quad (\text{A-12})$$

Since α is only slightly larger than 1, $\alpha - 1 \ll 1$, and $N' \approx N$

$$N' - N = (\alpha - 1) N(1 - N) \quad (\text{A-13})$$

Rewriting Equation (A-1) for the material balance as

$$\theta N' - N = -(1 - \theta)N'' \quad (\text{A-14})$$

The identity

$$(1 - \theta)N = (1 - \theta)N$$

may be added to get

$$\theta(N' - N) = -(1 - \theta)N'' - N \quad (\text{A-15})$$

Combining Equation (A-15) with Equation (A-8) gives

$$\delta U = \frac{G}{2} [\theta(N' - N)^2 + \frac{\theta^2}{1 - \theta} (N' - N)^2] V''(N) \quad (\text{A-16})$$

and by substituting Equation (A-13) into (A-16)

$$\delta U = \frac{G}{2} [\theta(\alpha - 1)^2 N^2(1 - N)^2 + \frac{\theta^2}{1 - \theta} (\alpha - 1)^2 N^2(1 - N)^2] V''(N) \quad (\text{A-17})$$

or

$$\delta U = \frac{\theta}{1 - \theta} (\alpha - 1)^2 \frac{G}{2} [N(1 - N)]^2 \frac{d^2 V(N)}{dN^2} \quad (\text{A-18})$$

To pump G moles through the separation element and achieve the separation indicated by the value of α , requires the same amount of work, regardless of the composition N . Consequently, the δU , the increment of value caused by the passage through this one separation element, should be independent of N .

Therefore

$$\frac{d^2 V(N)}{dN^2} [N(1 - N)]^2 = \text{a constant.} \quad (\text{A-19})$$

If the value of this constant is set at one, Equation (A-18) becomes

$$\delta U = \frac{\theta}{1 - \theta} \frac{G(\alpha - 1)^2}{2} . \quad (\text{A-20})$$

To find $V(N)$, which is the molar worth of any stream of composition N and $1 - N$, we must solve the linear, nonhomogeneous, second order differential equation,

$$\frac{d^2 V(N)}{dN^2} = \frac{1}{[N(1 - N)]^2} . \quad (\text{A-21})$$

The complete solution of Equation (A-21) requires the specification of two conditions to remove the constants of integration. No work needs to be done to the feed, which has the composition N_0 . Thus, $V(N_0) = 0$ is one condition. A positive amount of work must be done to change composition in either direction, consequently, $V(N_0)$ should be a minimum. This is assured if the second condition is taken to be

$$\frac{dV(N_0)}{dN} = 0, \quad (\text{A-22})$$

since, an inspection of Equation (A-21) will verify,

$$\frac{d^2 V(N_0)}{dN^2} \quad (\text{A-23})$$

is positive for any N .

A particular integral of Equation (A-11) can be found by the method of variation of parameters.⁽⁹⁾ This method consists of finding two independent solutions, V_1 and V_2 , to the homogeneous equation, $V'' = 0$, and two functions, U_1 and U_2 such that

$$V(N) = U_1 V_1 + U_2 V_2 . \quad (A-24)$$

Differentiating Equation (A-24) yields

$$(V'(N) = U_1 V'_1 + U'_1 V_1 + (U_2 V'_2 + U'_2 V_2) \quad (A-25)$$

If it is required that U_1 and U_2 satisfy the condition

$$U'_1 V_1 + U'_2 V_2 = 0 \quad (A-26)$$

then

$$V'(N) = U_1 V'_1 + U_2 V'_2 \quad (A-27)$$

and

$$V''(N) = (U_1 V''_1 + U'_1 V'_1) + (U_2 V''_2 + U'_2 V'_2) . \quad (A-28)$$

Equation (A-21) can now be rewritten, using Equation (A-28)

$$(U_1 V''_1 + U'_1 V'_1) + (U_2 V''_2 + U'_2 V'_2) = \frac{1}{N^2(1-N)^2} . \quad (A-29)$$

However, $V'_1 = V''_2 = 0$, since V_1 and V_2 are solutions to the homogeneous equation, $V'' = 0$.

Therefore

$$U'_1 V'_1 + U'_2 V'_2 = \frac{1}{N^2(1-N)^2} \quad (A-30)$$

and since

$$U'_1 V_1 + U'_2 V_2 = 0 \quad (A-26)$$

these can be solved for U'_1 and U'_2 .

$$U'_1 = \frac{-V_2}{V_1 V'_2 - V_2 V'_1} \cdot \frac{1}{N^2(1-N)^2} \quad (A-31a)$$

$$U'_2 = \frac{V_1}{V_1 V'_2 - V_2 V'_1} \cdot \frac{1}{N^2(1-N)^2} \quad (A-31b)$$

We know that, since $V'' = 0$, V_1 , V_2 , V'_1 and V'_2 can be expressed as

$$V_1 = a_1 N + v_2 \quad V_2 = a_2 N + b_2 \quad (\text{A-32a})$$

$$V'_1 = a_1 \quad V'_2 = a_2 \quad (\text{A-32b})$$

Substituting Equations (A-32a) and (A-32b) into Equations (A-31a) and (A-31b) yields

$$\begin{aligned} U'_1 &= \frac{-a_2 N - b_2}{(a_1 N + b_1) a_2 - (a_2 N + b_2) a_1} \cdot \frac{1}{N^2 (1 - N)^2} \\ &= \frac{-a_2 N - b_2}{a_2 b_1 - a_1 b_2} \cdot \frac{1}{N^2 (1 - N)^2} \end{aligned} \quad (\text{A-33})$$

and

$$U'_2 = \frac{a_1 N + b_1}{a_2 b_1 - a_1 b_2} \cdot \frac{1}{N^2 (1 - N)^2} \quad (\text{A-34})$$

Let

$$a_2 b_1 - a_1 b_2 = W \quad (\text{A-35})$$

The quantity, W , is the Wronskian of the solutions V_1 and V_2 , and thus cannot vanish if V_1 and V_2 are linearly independent, which is a condition imposed on V_1 and V_2 .

Then

$$W U'_1 = \frac{-a_2}{N(1 - N)^2} - \frac{b_2}{N^2(1 - N)^2} \quad (\text{A-36a})$$

and

$$W U'_2 = \frac{a_1}{N(1 - N)^2} + \frac{b_1}{N^2(1 - N)^2} \quad (\text{A-36b})$$

Equations (A-36a) and (A-36b) can be integrated to solve for U_1 and U_2 , or rather for $W U_1$ and $W U_2$.

$$W U_1 = \frac{-a_2 N + b_2(1 - 2N)}{N(1 - N)} + (a_2 + 2b_2) \ln \left(\frac{1 - N}{N} \right) + C_1 \quad (\text{A-37a})$$

$$W U_2 = \frac{a_1 N - b_1(1 - 2N)}{N(1 - N)} - (a_1 + 2b_1) \ln \left(\frac{1 - N}{N} \right) + C_2 \quad (\text{A-37b})$$

Now Equations (A-37a) and (A-37b) can be inserted into Equation (A-24), which is also multiplied by W.

$$\begin{aligned} W V(N) &= W U_1 V_1 + W U_2 V_2 \\ &= \{(a_1 N + b_1) [-a_2 N + b_2(1 - 2N)] \\ &\quad + (a_2 N + b_2) [a_1 N - b_1(1 - 2N)]\} / N(1 - N) \\ &\quad + [(a_2 + 2b_2)(a_1 N + b_1) - (a_1 + 2b_1) \\ &\quad \cdot (a_2 N + b_2)] \ln \left(\frac{1 - N}{N} \right) \\ &\quad + C_1(a_1 N + b_1) + C_2(a_2 N + b_2) \end{aligned} \quad (\text{A-38})$$

Collecting terms and making use of the definition of W in Equation (A-35) gives

$$\begin{aligned} W V(N) &= \frac{2 W(N^2 - N)}{N(1 - N)} + (-2WN + W) \ln \left(\frac{1 - N}{N} \right) \\ &\quad + (C_1 a_1 + C_2 a_2)N + (C_1 b_1 + C_2 b_2) \end{aligned} \quad (\text{A-39})$$

$$\begin{aligned} V(N) &= 2 + (1 - 2N) \ln \left(\frac{1 - N}{N} \right) \\ &\quad + \frac{(C_1 a_1 + C_2 a_2)N + (C_1 b_1 + C_2 b_2)}{a_2 b_1 - a_1 b_2} \end{aligned} \quad (\text{A-40})$$

Equation (A-40) is the particular integral of Equation (A-21). The constants can be eliminated by applying the boundary conditions. When $N_0 = 0.5$, for an equimolar mixture, the

conditions $V'(N_o) = 0$; can be applied or, differentiating Equation (A-40),

$$V'(N_o) = 0 = \frac{(1 - 2N_o)N_o}{1 - N_o} - 2 \ln \left(\frac{1 - N_o}{N_o} \right) + \frac{C_1 a_1 + C_2 a_2}{a_2 b_1 - a_1 b_2} \quad (A-41)$$

If Equation (A-41) is to be valid, then $C_1 a_1 + C_2 a_2 = 0$, and Equation (A-40) becomes

$$V(N) = -2 + (1 - 2N) \ln \left(\frac{1 - N}{N} \right) + \frac{C_1 b_1 + C_2 b_2}{a_2 b_1 - a_1 b_2} \quad (A-42)$$

Applying the other boundary condition, that $V(N_o) = 0$, to Equation (A-42) gives

$$V(N_o) = 0 = -2 + \frac{C_1 b_1 + C_2 b_2}{a_2 b_1 - a_1 b_2} \quad (A-43)$$

which implies that

$$\frac{C_1 b_1 + C_2 b_2}{a_2 b_1 - a_1 b_2} = 2$$

Finally, the solution to Equation (A-21) is

$$V(N) = (2N - 1) \ln \left(\frac{N}{1 - N} \right) = (1 - 2N) \ln \left(\frac{1 - N}{N} \right) \quad (A-44)$$

If N_o had not been set equal to 0.5 the complete solution would have been

$$V(N) = (2N - 1) \ln \frac{N(1 - N_o)}{N_o(1 - N)} + \frac{(N - N_o)(1 - 2N_o)}{N_o(1 - N_o)} \quad (A-45)$$

But $V(N)$ as given by Equation (A-44) is called the "value function" and is the value per mole of concentration N when the feed is valued at zero.

One separation element produces a change in value equal to δU and the number of elements required to produce a change in

value ΔU is equal to $\Delta U/\delta U$. This total work, ΔU , is the overall cost of the cascade which must be paid for and represents the real value of the different streams. For the whole cascade

$$\Delta U = U_p + U_w - U_o. \quad (A-46)$$

Thus, if $U_o = U_w = 0$, the total work for the cascade is

$$\frac{U_p}{G\delta U} = \frac{2PV(N_p)(1 - \theta)}{(\alpha - 1)^2 \theta} \quad (A-47)$$

where P is the product flow rate.

Another case arises from Equation (A-44) where the equimolar feed is assumed valueless. This function is called the "Separation potential" and represents the cost of separating an equimolar mixture, so that

$$\frac{U_p}{G\delta U} = \frac{2P(1 - \theta)}{(\alpha - 1)^2 \theta} (2N_p - 1) \ln \frac{N_p}{1 - N_p}. \quad (A-48)$$

The "separative duty" Δ is derived by a value balance which essentially assumes that this last function applies to each of the streams, and that W , P , and F are considered as starting from an equimolar mixture of zero value

$$\Delta' = \frac{GU_w}{\delta U} + \frac{-GU}{\delta F} \quad (A-49)$$

and

$$\begin{aligned} \Delta = & W(2N_w - 1) \ln \frac{N_w}{1 - N_w} + P(2N_p - 1) \ln \frac{N_p}{1 - N_p} \\ & - F(2N_F - 1) \ln \frac{N_F}{1 - N_F}. \end{aligned} \quad (A-50)$$

Where the definitions of Δ' and Δ differ by the factors which cancel, Δ has the dimensions of mass, although it connotes value. The cost of the work done and the work required for separation of a unit mass of material relate Δ back to its basic meaning.

A number of graphs show the relationships between separative duties (of \$15 to \$60) and feed costs, optimum tails, enrichment level and costs per gram. Figure A-1 shows the optimum tails as a function of separative duty and feeds. Figure A-2 shows the feed material cost of natural uranium for 93% enriched material varies as a function of feed materials costs and separative duty. Figure A-3 shows the uranium price for 2% and 3% enrichment for various feed and separative duty costs. Figure A-4 shows how the price of enriched uranium passes through the common neutral point of 0.7115% for natural uranium for various separative duty costs. Thus one observes the interesting anomaly that the higher the cost of separation the lower the cost of uranium below natural and above zero value tails composition.

FIGURE A-1

Optimum Tails as a Function of Feed Materials and Separative Duty

FIGURE A-2

Feed Cost of 93% Enriched Uranium as a Function of Natural Uranium Feed Material Cost and of Separative Duty

FIGURE A-3

Enriched Uranium Price as a Function of Natural Uranium Feed Material Cost and Separative Duty

FIGURE A-4

Price of Enriched Uranium as a Function of Percent Enrichment Separative Duty, at Constant Natural Uranium Feed Material Cost of \$32/kg

APPENDIX B

A SPECULATIVE U^{233} - U^{238} PRICE SCHEDULE

APPENDIX B

A SPECULATIVE U^{233} - U^{238} PRICE SCHEDULE

In certain circumstances, it may be attractive to enrich depleted uranium with the U^{233} bred in thorium. In order to calculate the burnup cost of such a fuel loading, the cost of separating the U^{233} from the U^{238} in the irradiated fuel must be determined; that is, a schedule of prices based on a specified price and enrichment and on a specified separative duty cost must be constructed. For simplicity, it will be assumed that the cascade will be operated with optimum tails composition, that the U^{234} , U^{235} , and U^{236} in the fuel can be ignored, and that the unit cost of separative duty is proportional to the relative difficulty of separating U^{233} from U^{238} compared with U^{235} from U^{238} . It is also assumed that U^{232} will not be a complicating factor. This may be the situation if U^{233} free of U^{232} can be used for enrichment of U^{238} . It may be that the (n,2n) reaction on U^{233} to form U^{232} will be negligible, but this needs confirmation.

One of the basic assumptions in the formulation of the AEC price schedule is that the energy required to effect a separation of isotopes in a diffusion cascade is directly proportional to the total flow rate within the cascade. As described in the text, a formula for the total flow rate in an ideal cascade with single waste, feed, and product streams is

$$J = \frac{\beta + 1}{(\beta - 1) \ln \beta} \left\{ W\phi(X_w) + P\phi(X_p) - F\phi(X_f) \right\} \quad (B-1)$$

where

β = heads separation factor

W, P and F = flow rates of the waste, product, and feed streams

X_w , X_p and X_f = compositions of waste, product, and feed
 $\phi(X)$ = separation potential, a mathematical function of enrichment defined as

$$\phi X = (2X - 1) \ln \frac{X}{1-X} \quad (B-2)$$

The bracketed quantity in Equation (B-1) is often called the separative duty and assigned the symbol Δ . The separative duty does not depend upon the isotopes being separated. The function of the heads separation factor that multiplies the separative duty in Equation (B-1) is strongly dependent upon the isotopes being separated. In fact, a key property of an ideal cascade is that

$$\beta = \sqrt{\alpha} \quad (B-3)$$

where α is the ratio of the square roots of the molecular weights of the isotopes being separated.

The ratio of the energy required to separate U^{233} from U^{238} to the energy required to separate U^{235} from U^{238} is, for β slightly greater than one

$$R = \frac{(\beta_1 + 1)(\beta_2 - 1) \ln \beta_2}{(\beta_2 + 1)(\beta_1 - 1) \ln \beta_1} \approx \left[\frac{\beta_2 - 1}{\beta_1 - \beta_2} \right]^2 \quad (B-4)$$

As the diffusing gas is UF_6 , the heads separation factor in each case is

$$\beta_1 = \sqrt[4]{\frac{238.1 + 6 \times 19.0}{233.1 + 6 \times 19.0}} = 4\sqrt[4]{1.01440} = 1.00358 \quad (B-5)$$

$$\beta_2 = \sqrt[4]{\frac{238.1 + 6 \times 19.0}{235.1 + 6 \times 19.0}} = 4\sqrt[4]{1.00859} = 1.00214 \quad (B-6)$$

so that

$$R \approx \left[\frac{0.00214}{0.00358} \right]^2 = 0.357 \quad (B-7)$$

Therefore, the energy required to effect a separation of U^{233} from U^{238} per unit of separative duty is roughly 1/3 of that required to separate U^{235} from U^{238} .

In formulating the uranium price schedule equation it is assumed that the cost of separation is the product $C_s \Delta$ where C_s is a constant called the unit cost of separative duty. If this formulation is to represent accurately the cost of enrichment, C_s must contain plant overhead which does not depend on the throughput nor on the unit cost of electrical power. In order to consider the most optimistic situation it could be assumed that the unit cost of separative duty in the U^{233} - U^{238} case is RC_s . That is, using the present value of \$30.00/kg, $RC_s = \$10.71/\text{kg}$.

One of the basic factors in the AEC uranium price schedule is that the energy required to effect a separation of isotopes in a diffusion cascade is directly proportional to the total flow rate within the cascade. A formula for this quantity of an ideal cascade with single waste, feed, and product streams is

$$J = \frac{\beta + 1}{(\beta - 1) \ln \beta} \Delta \quad (\text{B-8})$$

where

J = total flow rate in an ideal cascade, kg uranium/
unit time

Δ = separative duty, kg uranium/unit time

β = heads separation factor

The separative duty is a function of the composition and flow rates of the waste, feed, and product, and does not depend upon the isotopes being separated. Therefore, the ratio of the energies required for two different systems (denoted as 1 and 2) is for heads separation factors very nearly 1:

$$R = \frac{(\beta_1 + 1)(\beta_2 - 1) \ln \beta_2}{(\beta_1 - 1)(\beta_2 + 1) \ln \beta_1} \approx \left[\frac{\beta_2 - 1}{\beta_1 - 1} \right]^2 \quad (\text{B-9})$$

One of the key properties of an ideal cascade is that $\beta = \sqrt{\alpha}$ where α , the stage separations factor, equals the square root of the ratio of the molecular weights of the molecules being separated. Thus, for $\text{U}^{235}\text{-U}^{238}$ systems (as UF_6), $\beta = 1.00358$. Using these numbers in Equation (B-9) gives $R = 0.357$. This means that the separation of U^{235} from U^{238} requires roughly one-third of the energy required to separate U^{235} from U^{238} or, based on the present AEC separative duty cost of \$30/kg uranium, \$10.71/kg uranium.

A uranium price schedule is usually established by specifying a feed composition and unit cost, such as natural uranium at \$23.50/kg, a unit cost of separative duty, such as \$30/kg, and by specifying that the tails stream will be valueless and will have the optimum composition to minimize the product cost. An identical schedule can be derived by considering a product composition to be the feed composition, provided that the "feed" price is the same as would have been obtained in the former case. This principle can be used to derive a schedule for a $\text{U}^{233}\text{-U}^{238}$ system, given a value for the U^{233} bred in thorium. Table B-I contains schedules calculated by assigning different values to the U^{233} mixed with U^{238} up to a 2% enrichment. Also shown is the cost of a 50% fissile burnup in each schedule.

Several items are worthy of note from Table B-I. First, that similarly to the $\text{U}^{235}\text{-U}^{238}$ system, if U^{233} is mixed with U^{238} to produce 2% enrichment and then burned to 1%, the burnup costs are greater than the cost of U^{233} in 2% enriched fuel. This emphasizes the reduction in U^{233} value that must be compensated by some other factor if such fuel cycles are to be

advantageous. This points up that the chemical separability of plutonium from U^{238} is a significant asset. The second item to note is that extremely low tails compositions are involved compared to a U^{235} - U^{238} separations because of the low unit cost of separative duty for U^{233} . This low value may not be achievable in practice because of inventory and clean up problems involved with converting the cascade operation from U^{235} - U^{238} to U^{233} - U^{238} and back again.

TABLE B-I

SAMPLE PRICE SCHEDULES FOR U^{233} - U^{238} MIXTURES

(Unit Cost of Separative Duty = \$10.71/kg Uranium)

Percent Enrichment, U^{233} in U^{238}	<u>Unit Value of U^{233} at 2% Enrichment, Dollar/Gram</u>			
	\$7.85	\$10.00	\$12.63	\$15.00
	<u>Optimum Tails Composition, Percent</u>			
	0.108%	0.088%	0.072%	0.062%
	<u>Prices Per Gram of U^{233} Contained</u>			
0.2	1.26	2.42	4.07	5.71
0.5	4.50	6.32	8.62	10.75
1.0	6.48	8.52	11.04	13.33
2.0	7.85	10.00	12.63	15.00
99.0	10.00	12.26	15.00	17.45
	<u>Cost of Burning from 2% to 1%/Gram</u>			
	<u>Dollar/Gram U^{233} Destroyed</u>			
	9.13	11.46	14.18	16.52

APPENDIX C

Uranium Price Schedules

APPENDIX C

Uranium Price Schedules

Computer printouts are given in the following pages for two uranium price schedules. The feed composition for both schedules is constant at 0.71150 wt%, and the separative duty is constant at \$30/kg. The feed materials cost and the optimum tails composition vary as indicated.

URANIUM PRICE SCHEDULE NUMBER 1

FEED COMPOSITION = 0.71150 W/O

OPTIMUM TAILS COMPOSITION = 0.253071 W/O

FEED MATERIALS COST = 23.50 \$/KG OR 10.66 \$/LR

SEPARATIVE DUTY COST = 30.00 \$/KG OR 13.61 \$/LR

ASSIGNED TAILS VALUE = -0. \$/KG OR -0. \$/LR

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LR U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
0.100	0.101	9.7456	4.4205	9.7456	-0.3339	0.5864	-7.8467	17.5923
0.200	0.203	0.7730	0.3506	0.3865	-0.1158	0.1164	-2.7205	3.4935
0.300	0.304	0.4622	0.2097	0.1541	0.1024	-0.0648	2.4057	-1.9434
0.400	0.405	3.7055	1.6808	0.9264	0.3205	-0.1275	7.5319	-3.8264
0.500	0.506	8.9002	4.0371	1.7800	0.5386	-0.1253	12.6581	-3.7579
0.600	0.608	15.3317	6.9544	2.5553	0.7568	-0.0818	17.7843	-2.4526
0.700	0.709	22.6185	10.2596	3.2312	0.9749	-0.0097	22.9105	-0.2920
0.800	0.810	30.5325	13.8493	3.8166	1.1931	0.0832	28.0367	2.4958
0.900	0.911	38.9266	17.6568	4.3252	1.4112	0.1921	33.1629	5.7637
1.000	1.013	47.7002	21.6365	4.7700	1.6293	0.3137	38.2891	9.4111
1.100	1.114	56.7813	25.7556	5.1619	1.8475	0.4455	43.4153	13.3660
1.200	1.215	66.1170	29.9902	5.5098	2.0656	0.5859	48.5415	17.5755
1.300	1.316	75.6669	34.3219	5.8205	2.2837	0.7333	53.6677	21.9992
1.400	1.418	85.3994	38.7366	6.1000	2.5019	0.8869	58.7939	26.6056
1.500	1.519	95.2899	43.2228	6.3527	2.7200	1.0457	63.9201	31.3698
1.600	1.620	105.3180	47.7715	6.5824	2.9381	1.2091	69.0463	36.2717
1.700	1.721	115.4674	52.3752	6.7922	3.1563	1.3765	74.1725	41.2949
1.800	1.823	125.7245	57.0277	6.9847	3.3744	1.5475	79.2987	46.4258
1.900	1.924	136.0776	61.7238	7.1620	3.5925	1.7218	84.4249	51.6527
2.000	2.025	146.5173	66.4592	7.3259	3.8107	1.8989	89.5511	56.9662

87

BNWL-189

URANIUM PRICE SCHEDULE NUMBER 1

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LB U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
2.100	2.126	157.0351	71.2300	7.4779	4.0288	2.0786	94.6773	62.3578
2.200	2.228	167.6240	76.0331	7.6193	4.2470	2.2607	99.8035	67.8205
2.300	2.329	178.2778	80.8655	7.7512	4.4651	2.4449	104.9297	73.3481
2.400	2.430	188.9910	85.7250	7.8746	4.6832	2.6312	110.0559	78.9351
2.500	2.531	199.7590	90.6093	7.9904	4.9014	2.8192	115.1821	84.5769
2.600	2.632	210.5775	95.5164	8.0991	5.1195	3.0090	120.3083	90.2692
2.700	2.734	221.4428	100.4449	8.2016	5.3376	3.2003	125.4345	96.0083
2.800	2.835	232.3516	105.3930	8.2983	5.5558	3.3930	130.5607	101.7909
2.900	2.936	243.3009	110.3596	8.3897	5.7739	3.5871	135.6869	107.6140
3.000	3.037	254.2881	115.3433	8.4763	5.9920	3.7825	140.8131	113.4750
3.200	3.240	276.3666	125.3579	8.6365	6.4283	4.1767	151.0655	125.3011
3.400	3.442	298.5704	135.4294	8.7815	6.8646	4.5751	161.3179	137.2525
3.600	3.644	320.8856	145.5514	8.9135	7.3009	4.9772	171.5703	149.3153
3.800	3.847	343.3006	155.7187	9.0342	7.7371	5.3826	181.8227	161.4779
4.000	4.049	365.8056	165.9268	9.1451	8.1734	5.7910	192.0751	173.7305
4.200	4.251	388.3920	176.1718	9.2474	8.6097	6.2021	202.3275	186.0645
4.400	4.454	411.0526	186.4505	9.3421	9.0460	6.6158	212.5799	198.4727
4.600	4.656	433.7811	196.7600	9.4300	9.4822	7.0316	222.8323	210.9488
4.800	4.858	456.5720	207.0978	9.5119	9.9185	7.4496	233.0846	223.4873
5.000	5.061	479.4203	217.4617	9.5884	10.3548	7.8694	243.3370	236.0833
5.200	5.263	502.3219	227.8497	9.6600	10.7910	8.2911	253.5894	248.7325
5.400	5.465	525.2730	238.2601	9.7273	11.2273	8.7144	263.8418	261.4311
5.600	5.668	548.2700	248.6914	9.7905	11.6636	9.1392	274.0942	274.1757
5.800	5.870	571.3100	259.1422	9.8502	12.0999	9.5654	284.3466	286.9633
6.000	6.072	594.3902	269.6112	9.9065	12.5361	9.9930	294.5990	299.7911
6.200	6.274	617.5082	280.0973	9.9598	12.9724	10.4219	304.8514	312.6567
6.400	6.477	640.6617	290.5996	10.0103	13.4087	10.8519	315.1038	325.5578
6.600	6.679	663.8486	301.1170	10.0583	13.8449	11.2831	325.3562	338.4923
6.800	6.881	687.0671	311.6488	10.1039	14.2812	11.7153	335.6086	351.4584
7.000	7.083	710.3155	322.1941	10.1474	14.7175	12.1485	345.8610	364.4544
7.200	7.285	733.5922	332.7522	10.1888	15.1538	12.5826	356.1134	377.4787
7.400	7.488	756.8958	343.3226	10.2283	15.5900	13.0177	366.3658	390.5299
7.600	7.690	780.2250	353.9045	10.2661	16.0263	13.4536	376.6182	403.6067
7.800	7.892	803.5785	364.4975	10.3023	16.4626	13.8903	386.8706	416.7078
8.000	8.094	826.9552	375.1010	10.3369	16.8989	14.3277	397.1230	429.8321

URANIUM PRICE SCHEDULE NUMBER 1

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LB U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
9.000	9.105	944.1527	428.2610	10.4906	19.0802	16.5256	448.3850	495.7676
10.000	10.115	1061.7999	481.6249	10.6180	21.2616	18.7384	499.6470	562.1528
11.000	11.125	1179.8190	535.1575	10.7256	23.4429	20.9637	550.9090	628.9100
12.000	12.135	1298.1523	588.8327	10.8179	25.6243	23.1994	602.1710	695.9812
13.000	13.144	1416.7554	642.6302	10.8981	27.8057	25.4441	653.4330	763.3223
14.000	14.154	1535.5936	696.5344	10.9685	29.9870	27.6966	704.6950	830.8985
15.000	15.163	1654.6392	750.5326	11.0309	32.1684	29.9561	755.9569	898.6821
16.000	16.172	1773.8696	804.6147	11.0867	34.3497	32.2217	807.2189	966.6505
17.000	17.180	1893.2664	858.7722	11.1369	36.5311	34.4928	858.4809	1034.7853
18.000	18.188	2012.8140	912.9982	11.1823	38.7125	36.7690	909.7429	1103.0710
19.000	19.196	2132.4996	967.2867	11.2237	40.8938	39.0498	961.0049	1171.4945
20.000	20.204	2252.3119	1021.6327	11.2616	43.0752	41.3348	1012.2669	1240.0448
21.000	21.212	2372.2415	1076.0319	11.2964	45.2565	43.6237	1063.5289	1308.7124
22.000	22.219	2492.2802	1130.4806	11.3285	47.4379	45.9163	1114.7909	1377.4892
23.000	23.226	2612.4209	1184.9756	11.3584	49.6193	48.2123	1166.0529	1446.3678
24.000	24.233	2732.6573	1239.5140	11.3861	51.8006	50.5114	1217.3149	1515.3422
25.000	25.239	2852.9839	1294.0933	11.4119	53.9820	52.8136	1268.5769	1584.4068
26.000	26.245	2973.3958	1348.7113	11.4361	56.1634	55.1186	1319.8389	1653.5567
27.000	27.251	3093.8889	1403.3661	11.4588	58.3447	57.4263	1371.1008	1722.7878
28.000	28.257	3214.4591	1458.0559	11.4802	60.5261	59.7365	1422.3628	1792.0961
29.000	29.262	3335.1033	1512.7792	11.5004	62.7074	62.0493	1473.6248	1861.4782
30.000	30.268	3455.8182	1567.5347	11.5194	64.8888	64.3644	1524.8868	1930.9311
31.000	31.273	3576.6012	1622.3210	11.5374	67.0702	66.6817	1576.1488	2000.4521
32.000	32.277	3697.4498	1677.1370	11.5545	69.2515	69.0013	1627.4108	2070.0387
33.000	33.282	3818.3617	1731.9818	11.5708	71.4329	71.3230	1678.6727	2139.6886
34.000	34.286	3939.3350	1786.8544	11.5863	73.6142	73.6467	1729.9347	2209.4000
35.000	35.290	4060.3679	1841.7541	11.6011	75.7956	75.9724	1781.1967	2279.1709
36.000	36.293	4181.4587	1896.6801	11.6152	77.9770	78.3000	1832.4587	2348.9998
37.000	37.297	4302.6063	1951.6317	11.6287	80.1583	80.6295	1883.7207	2418.8853
38.000	38.300	4423.8089	2006.6084	11.6416	82.3397	82.9609	1934.9827	2488.8259
39.000	39.303	4545.0656	2061.6096	11.6540	84.5210	85.2940	1986.2446	2558.8206
40.000	40.305	4666.3753	2116.6348	11.6659	86.7024	87.6289	2037.5066	2628.8683
41.000	41.308	4787.7372	2171.6837	11.6774	88.8838	89.9656	2088.7686	2698.9682
42.000	42.310	4909.1502	2226.7557	11.6885	91.0651	92.3040	2140.0306	2769.1193
43.000	43.312	5030.6139	2281.8508	11.6991	93.2465	94.6440	2191.2926	2839.3209

URANIUM PRICE SCHEDULE NUMBER 1

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LB U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
44.000	44.313	5152.1274	2336.9685	11.7094	95.4279	96.9858	2242.5546	2909.5725
45.000	45.315	5273.6904	2392.1086	11.7193	97.6092	99.3291	2293.8165	2979.8735
46.000	46.316	5395.3025	2447.2710	11.7289	99.7906	101.6741	2345.0785	3050.2235
47.000	47.317	5516.9631	2502.4554	11.7382	101.9719	104.0207	2396.3405	3120.6222
48.000	48.317	5638.6721	2557.6617	11.7472	104.1533	106.3690	2447.6025	3191.0692
49.000	49.318	5760.4293	2612.8899	11.7560	106.3347	108.7188	2498.8645	3261.5643
50.000	50.318	5882.2345	2668.1399	11.7645	108.5160	111.0703	2550.1265	3332.1075
60.000	60.305	7102.9594	3221.8521	11.8383	130.3296	134.6737	3062.7464	4040.2124
70.000	70.266	8328.9862	3777.9691	11.8986	152.1432	158.4540	3575.3663	4753.6193
80.000	80.203	9562.0641	4337.2845	11.9526	173.9569	182.4692	4087.9862	5474.0771
90.000	90.114	10808.1362	4902.4940	12.0090	195.7705	206.9176	4600.6061	6207.5293
91.000	91.104	10934.1475	4959.6518	12.0155	197.9518	209.4093	4651.8681	6282.2785
92.000	92.093	11060.6083	5017.0135	12.0224	200.1332	211.9159	4703.1301	6357.4774
93.000	93.082	11187.6270	5074.6283	12.0297	202.3146	214.4411	4754.3920	6433.2340
94.000	94.071	11315.3601	5132.5671	12.0376	204.4959	216.9902	4805.6541	6509.7052
95.000	95.060	11444.0486	5190.9393	12.0464	206.6773	219.5711	4856.9160	6587.1317
96.000	96.049	11574.0922	5249.9261	12.0563	208.8586	222.1971	4908.1780	6665.9133
97.000	97.037	11706.2330	5309.8643	12.0683	211.0400	224.8931	4959.4400	6746.7922
98.000	98.025	11842.1207	5371.5020	12.0838	213.2214	227.7139	5010.7020	6831.4178
99.000	99.012	11986.9618	5437.2009	12.1080	215.4027	230.8332	5061.9640	6924.9969
0.711	0.721	23.5000	10.6594	3.3029	1.0000	0.0000	23.5000	0.0000
2.000	2.025	146.5173	66.4592	7.3259	3.8107	1.8989	89.5511	56.9662
3.000	3.037	254.2881	115.3433	8.4763	5.9920	3.7825	140.8131	113.4750
5.000	5.061	479.4204	217.4617	9.5884	10.3548	7.8694	243.3371	236.0833
93.000	93.082	11187.6282	5074.6288	12.0297	202.3146	214.4412	4754.3925	6433.2347

90

BNWL-189

BURN-UP COSTS PER GRAM OF U235 CONSUMED

URANIUM PRICE SCHEDULE NUMBER 1

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE \$/GM U235	RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100					
		INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
0.300	0.154	1.852	1.316	0.612 *	0.307 *	0.205 *	0.162 *
0.400	0.926	4.352	3.950	3.222	1.843 *	1.231 *	0.974 *
0.500	1.780	5.856	5.533	4.950	3.540 *	2.365 *	1.872 *
0.600	2.555	6.860	6.591	6.104	4.927	3.395 *	2.687 *
0.700	3.231	7.579	7.348	6.930	5.919	4.291 *	3.398 *
0.800	3.817	8.119	7.916	7.550	6.665	5.068 *	4.013 *
0.900	4.325	8.540	8.359	8.033	7.246	5.742 *	4.547 *
1.000	4.770	8.877	8.714	8.420	7.711	6.330 *	5.015 *
1.100	5.162	9.153	9.005	8.737	8.092	6.836	5.427 *
1.200	5.510	9.383	9.247	9.002	8.410	7.258	5.792 *
1.300	5.821	9.578	9.453	9.226	8.679	7.616	6.118 *
1.400	6.100	9.746	9.629	9.418	8.910	7.922	6.412 *
1.500	6.353	9.892	9.782	9.585	9.111	8.188	6.677 *
1.600	6.582	10.019	9.917	9.732	9.286	8.421	6.918 *
1.700	6.792	10.132	10.035	9.861	9.441	8.627	7.138 *
1.800	6.985	10.232	10.141	9.976	9.579	8.810	7.340 *
1.900	7.162	10.322	10.235	10.079	9.703	8.974	7.526 *
2.000	7.326	10.402	10.320	10.172	9.814	9.122	7.698 *

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION.
THE MINIMUM PRICED COMPOSITION IS 0.254 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

URANIUM PRICE SCHEDULE NUMBER 1

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE \$/GM U235	RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100					
		INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
2.100	7.478	10.476	10.397	10.256	9.915	9.255	7.857 *
2.200	7.619	10.542	10.467	10.332	10.007	9.377	8.006 *
2.300	7.751	10.603	10.532	10.402	10.091	9.488	8.144 *
2.400	7.875	10.659	10.591	10.466	10.168	9.590	8.273 *
2.500	7.990	10.711	10.645	10.525	10.239	9.684	8.394 *
2.600	8.099	10.759	10.695	10.580	10.304	9.771	8.508 *
2.700	8.202	10.803	10.741	10.631	10.365	9.851	8.615 *
2.800	8.298	10.844	10.785	10.678	10.421	9.926	8.716 *
2.900	8.390	10.882	10.825	10.721	10.474	9.995	8.812 *
3.000	8.476	10.918	10.862	10.762	10.523	10.060	8.902 *
3.200	8.636	10.983	10.931	10.837	10.612	10.178	9.070 *
3.400	8.781	11.040	10.991	10.903	10.691	10.282	9.221 *
3.600	8.913	11.091	11.045	10.961	10.761	10.375	9.359 *
3.800	9.034	11.137	11.093	11.014	10.824	10.458	9.485 *
4.000	9.145	11.179	11.137	11.061	10.880	10.533	9.600 *
4.200	9.247	11.216	11.176	11.104	10.932	10.601	9.707 *
4.400	9.342	11.251	11.212	11.143	10.978	10.662	9.805 *
4.600	9.430	11.282	11.245	11.179	11.021	10.719	9.896 *
4.800	9.512	11.311	11.275	11.212	11.060	10.771	9.981 *
5.000	9.588	11.337	11.303	11.242	11.097	10.818	10.060 *
5.200	9.660	11.362	11.329	11.270	11.130	10.863	10.134
5.400	9.727	11.384	11.353	11.296	11.161	10.903	10.203
5.600	9.791	11.406	11.375	11.320	11.190	10.942	10.267
5.800	9.850	11.425	11.396	11.343	11.217	10.977	10.326
6.000	9.907	11.444	11.415	11.364	11.242	11.010	10.382
6.200	9.960	11.461	11.433	11.384	11.266	11.041	10.433
6.400	10.010	11.478	11.451	11.402	11.288	11.070	10.482
6.600	10.058	11.493	11.467	11.420	11.309	11.098	10.528
6.800	10.104	11.507	11.482	11.436	11.328	11.124	10.571
7.000	10.147	11.521	11.496	11.452	11.347	11.148	10.612
7.200	10.189	11.534	11.510	11.467	11.364	11.171	10.650
7.400	10.228	11.546	11.523	11.481	11.381	11.193	10.687
7.600	10.266	11.558	11.535	11.494	11.397	11.214	10.721
7.800	10.302	11.569	11.547	11.506	11.412	11.233	10.754
8.000	10.337	11.580	11.558	11.518	11.426	11.252	10.785

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION.
THE MINIMUM PRICED COMPOSITION IS 0.254 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

URANIUM PRICE SCHEDULE NUMBER 1

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE \$/G4 U235	RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100					
		INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
9.000	10.491	11.626	11.606	11.571	11.488	11.334	10.921
10.000	10.618	11.663	11.645	11.613	11.538	11.399	11.030
11.000	10.726	11.694	11.677	11.648	11.580	11.453	11.120
12.000	10.818	11.720	11.704	11.677	11.615	11.499	11.194
13.000	10.898	11.742	11.728	11.703	11.644	11.537	11.258
14.000	10.969	11.761	11.748	11.724	11.670	11.570	11.313
15.000	11.031	11.778	11.766	11.743	11.692	11.600	11.361
16.000	11.087	11.793	11.781	11.760	11.712	11.625	11.403
17.000	11.137	11.807	11.795	11.775	11.730	11.648	11.440
18.000	11.182	11.819	11.808	11.789	11.746	11.668	11.473
19.000	11.224	11.829	11.819	11.801	11.760	11.687	11.503
20.000	11.262	11.839	11.829	11.812	11.773	11.703	11.530
21.000	11.296	11.848	11.839	11.822	11.785	11.719	11.555
22.000	11.329	11.857	11.848	11.831	11.796	11.733	11.577
23.000	11.358	11.865	11.856	11.840	11.806	11.745	11.598
24.000	11.386	11.872	11.863	11.848	11.815	11.757	11.617
25.000	11.412	11.879	11.870	11.855	11.823	11.768	11.635
26.000	11.436	11.885	11.877	11.862	11.831	11.778	11.651
27.000	11.459	11.891	11.883	11.869	11.839	11.788	11.666
28.000	11.480	11.896	11.888	11.875	11.846	11.797	11.680
29.000	11.500	11.901	11.894	11.881	11.853	11.805	11.694
30.000	11.519	11.906	11.899	11.886	11.859	11.813	11.706
31.000	11.537	11.911	11.904	11.891	11.865	11.821	11.718
32.000	11.555	11.916	11.908	11.896	11.870	11.828	11.729
33.000	11.571	11.920	11.913	11.901	11.876	11.834	11.740
34.000	11.586	11.924	11.917	11.905	11.881	11.841	11.749
35.000	11.601	11.928	11.921	11.910	11.886	11.847	11.759
36.000	11.615	11.932	11.925	11.914	11.890	11.852	11.768
37.000	11.629	11.935	11.928	11.917	11.895	11.858	11.777
38.000	11.642	11.939	11.932	11.921	11.899	11.863	11.785
39.000	11.654	11.942	11.935	11.925	11.903	11.868	11.793
40.000	11.666	11.945	11.939	11.928	11.907	11.873	11.800
41.000	11.677	11.948	11.942	11.931	11.910	11.878	11.807
42.000	11.688	11.951	11.945	11.935	11.914	11.882	11.814
43.000	11.699	11.954	11.948	11.938	11.918	11.887	11.821

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION.
THE MINIMUM PRICED COMPOSITION IS 0.254 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

URANIUM PRICE SCHEDULE NUMBER 1

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE \$/GM U235	RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100					
		INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
44.000	11.709	11.957	11.951	11.941	11.921	11.891	11.827
45.000	11.719	11.960	11.954	11.944	11.924	11.895	11.833
46.000	11.729	11.963	11.956	11.947	11.927	11.899	11.839
47.000	11.738	11.965	11.959	11.949	11.931	11.903	11.845
48.000	11.747	11.968	11.962	11.952	11.934	11.906	11.851
49.000	11.756	11.970	11.964	11.955	11.936	11.910	11.856
50.000	11.764	11.973	11.967	11.957	11.939	11.913	11.861
60.000	11.838	11.995	11.989	11.980	11.965	11.944	11.907
70.000	11.899	12.016	12.009	12.000	11.987	11.971	11.945
80.000	11.953	12.037	12.029	12.021	12.009	11.998	11.981
90.000	12.009	12.066	12.055	12.046	12.037	12.030	12.021
91.000	12.016	12.070	12.058	12.049	12.041	12.035	12.026
92.000	12.022	12.074	12.062	12.053	12.045	12.039	12.032
93.000	12.030	12.079	12.065	12.057	12.050	12.044	12.038
94.000	12.038	12.084	12.070	12.062	12.055	12.050	12.044
95.000	12.046	12.090	12.075	12.067	12.061	12.057	12.052
96.000	12.056	12.098	12.081	12.074	12.068	12.064	12.060
97.000	12.068	12.107	12.088	12.082	12.077	12.074	12.071
98.000	12.084	12.120	12.099	12.093	12.090	12.087	12.085
99.000	12.108	12.141	12.117	12.113	12.111	12.109	12.108
0.711	3.303	7.649	7.421	7.010	6.016	4.387 *	3.473 *
2.000	7.326	10.402	10.320	10.172	9.814	9.122	7.698 *
3.000	8.476	10.918	10.862	10.762	10.523	10.060	8.902 *
5.000	9.588	11.337	11.303	11.242	11.097	10.818	10.060 *
93.000	12.030	12.079	12.065	12.057	12.050	12.044	12.038

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION.
THE MINIMUM PRICED COMPOSITION IS 0.254 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

URANIUM PRICE SCHEDULE NUMBER 2

FEED COMPOSITION = 0.71150 W/O

OPTIMUM TAILS COMPOSITION = 0.280571 W/O

FEED MATERIALS COST = 18.30 \$/KG OR 8.30 \$/LB

SEPARATIVE DUTY COST = 30.00 \$/KG OR 13.61 \$/LB

ASSIGNED TAILS VALUE = -0. \$/KG OR -0. \$/LB

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LB U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
0.100	0.101	11.6888	5.3020	11.6888	-0.4190	0.6452	-7.6682	19.3570
0.200	0.203	1.5480	0.7022	0.7740	-0.1870	0.1657	-3.4216	4.9696
0.300	0.304	0.0692	0.0314	0.0231	0.0451	-0.0252	0.8251	-0.7559
0.400	0.405	2.1443	0.9726	0.5361	0.2771	-0.0976	5.0717	-2.9274
0.500	0.506	6.1708	2.7990	1.2342	0.5092	-0.1049	9.3183	-3.1475
0.600	0.608	11.4342	5.1865	1.9057	0.7413	-0.0710	13.5650	-2.1308
0.700	0.709	17.5528	7.9618	2.5075	0.9733	-0.0086	17.8116	-0.2588
0.800	0.810	24.2987	11.0217	3.0373	1.2054	0.0747	22.0583	2.2404
0.900	0.911	31.5247	14.2994	3.5027	1.4374	0.1740	26.3049	5.2197
1.000	1.013	39.1301	17.7491	3.9130	1.6695	0.2860	30.5516	8.5785
1.100	1.114	47.0431	21.3384	4.2766	1.9015	0.4082	34.7982	12.2449
1.200	1.215	55.2106	25.0432	4.6009	2.1336	0.5389	39.0449	16.1658
1.300	1.316	63.5923	28.8450	4.8917	2.3657	0.6767	43.2915	20.3008
1.400	1.418	72.1568	32.7298	5.1541	2.5977	0.8206	47.5381	24.6186
1.500	1.519	80.8791	36.6862	5.3919	2.8298	0.9698	51.7848	29.0943
1.600	1.620	89.7391	40.7050	5.6087	3.0618	1.1236	56.0314	33.7076
1.700	1.721	98.7203	44.7788	5.8071	3.2939	1.2814	60.2781	38.4422
1.800	1.823	107.8092	48.9015	5.9894	3.5259	1.4428	64.5247	43.2845
1.900	1.924	116.9942	53.0678	6.1576	3.7580	1.6074	68.7714	48.2229
2.000	2.025	126.2658	57.2733	6.3133	3.9901	1.7749	73.0180	53.2477

95

BNWL-189

URANIUM PRICE SCHEDULE NUMBER 2

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LB U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
2.100	2.126	135.6155	61.5142	6.4579	4.2221	1.9450	77.2646	58.3508
2.200	2.228	145.0362	65.7874	6.5926	4.4542	2.1175	81.5113	63.5249
2.300	2.329	154.5218	70.0900	6.7183	4.6862	2.2921	85.7579	68.7639
2.400	2.430	164.0669	74.4196	6.8361	4.9183	2.4687	90.0046	74.0623
2.500	2.531	173.6667	78.7740	6.9467	5.1503	2.6472	94.2512	79.4155
2.600	2.632	183.3171	83.1513	7.0507	5.3824	2.8273	98.4979	84.8192
2.700	2.734	193.0142	87.5499	7.1487	5.6145	3.0090	102.7445	90.2697
2.800	2.835	202.7549	91.9682	7.2412	5.8465	3.1921	106.9911	95.7637
2.900	2.936	212.5361	96.4048	7.3288	6.0786	3.3766	111.2378	101.2982
3.000	3.037	222.3551	100.8587	7.4118	6.3106	3.5624	115.4844	106.8706
3.200	3.240	242.0973	109.8136	7.5655	6.7747	3.9373	123.9777	118.1195
3.400	3.442	261.9648	118.8254	7.7048	7.2389	4.3165	132.4710	129.4938
3.600	3.644	281.9438	127.8877	7.8318	7.7030	4.6993	140.9643	140.9794
3.800	3.847	302.0225	136.9953	7.9480	8.1671	5.0855	149.4576	152.5649
4.000	4.049	322.1912	146.1436	8.0548	8.6312	5.4747	157.9509	164.2402
4.200	4.251	342.4413	155.3289	8.1534	9.0953	5.8666	166.4441	175.9971
4.400	4.454	362.7656	164.5479	8.2447	9.5594	6.2609	174.9374	187.8281
4.600	4.656	383.1578	173.7977	8.3295	10.0235	6.6576	183.4307	199.7270
4.800	4.858	403.6124	183.0757	8.4086	10.4877	7.0563	191.9240	211.6883
5.000	5.061	424.1245	192.3799	8.4825	10.9518	7.4569	200.4173	223.7071
5.200	5.263	444.6898	201.7081	8.5517	11.4159	7.8593	208.9106	235.7791
5.400	5.465	465.3045	211.0588	8.6168	11.8800	8.2634	217.4039	247.9006
5.600	5.668	485.9653	220.4304	8.6780	12.3441	8.6689	225.8971	260.0680
5.800	5.870	506.6690	229.8215	8.7357	12.8082	9.0759	234.3904	272.2785
6.000	6.072	527.4129	239.2308	8.7902	13.2723	9.4843	242.8837	284.5291
6.200	6.274	548.1946	248.6572	8.8418	13.7364	9.8939	251.3770	296.8175
6.400	6.477	569.0118	258.0997	8.8908	14.2006	10.3047	259.8703	309.1414
6.600	6.679	589.8624	267.5574	8.9373	14.6647	10.7166	268.3636	321.4987
6.800	6.881	610.7446	277.0294	8.9815	15.1288	11.1296	276.8568	333.8876
7.000	7.083	631.6567	286.5150	9.0237	15.5929	11.5435	285.3501	346.3065
7.200	7.285	652.5972	296.0134	9.0639	16.0570	11.9585	293.8434	358.7536
7.400	7.488	673.5645	305.5241	9.1022	16.5211	12.3743	302.3367	371.2276
7.600	7.690	694.5574	315.0463	9.1389	16.9852	12.7909	310.8300	383.7272
7.800	7.892	715.5746	324.5796	9.1740	17.4494	13.2084	319.3233	396.2512
8.000	8.094	736.6150	334.1234	9.2077	17.9135	13.6266	327.8166	408.7983

URANIUM PRICE SCHEDULE NUMBER 2

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LB U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
9.000	9.105	842.1311	381.9847	9.3570	20.2340	15.7283	370.2830	471.8479
10.000	10.115	948.0968	430.0500	9.4810	22.5546	17.8449	412.7494	535.3472
11.000	11.125	1054.4346	478.2840	9.5858	24.8752	19.9739	455.2159	599.2185
12.000	12.135	1161.0864	526.6606	9.6757	27.1958	22.1135	497.6823	663.4039
13.000	13.144	1268.0080	575.1595	9.7539	29.5163	24.2620	540.1487	727.8591
14.000	14.154	1375.1648	623.7650	9.8226	31.8369	26.4183	582.6151	792.5494
15.000	15.163	1482.5289	672.4646	9.8835	34.1575	28.5816	625.0816	857.4470
16.000	16.172	1590.0779	721.2481	9.9380	36.4780	30.7510	667.5480	922.5296
17.000	17.180	1697.7933	770.1070	9.9870	38.7986	32.9259	710.0144	987.7785
18.000	18.188	1805.6595	819.0343	10.0314	41.1192	35.1059	752.4809	1053.1782
19.000	19.196	1913.6636	868.0242	10.0719	43.4397	37.2905	794.9473	1118.7159
20.000	20.204	2021.7945	917.0716	10.1090	45.7603	39.4793	837.4137	1184.3803
21.000	21.212	2130.0426	966.1722	10.1431	48.0809	41.6721	879.8802	1250.1620
22.000	22.219	2238.3999	1015.3223	10.1745	50.4015	43.8684	922.3466	1316.0528
23.000	23.226	2346.8591	1064.5187	10.2037	52.7220	46.0682	964.8130	1382.0456
24.000	24.233	2455.4141	1113.7584	10.2309	55.0426	48.2711	1007.2795	1448.1341
25.000	25.239	2564.0592	1163.0391	10.2562	57.3632	50.4771	1049.7459	1514.3128
26.000	26.245	2672.7897	1212.3585	10.2800	59.6837	52.6859	1092.2123	1580.5769
27.000	27.251	2781.6013	1261.7147	10.3022	62.0043	54.8974	1134.6787	1646.9220
28.000	28.257	2890.4902	1311.1059	10.3232	64.3249	57.1115	1177.1452	1713.3444
29.000	29.262	2999.4529	1360.5306	10.3429	66.6454	59.3280	1219.6116	1779.8407
30.000	30.268	3108.4864	1409.9874	10.3616	68.9660	61.5469	1262.0780	1846.4077
31.000	31.273	3217.5879	1459.4751	10.3793	71.2866	63.7681	1304.5444	1913.0428
32.000	32.277	3326.7550	1508.9925	10.3961	73.6072	65.9915	1347.0108	1979.7435
33.000	33.282	3435.9856	1558.5387	10.4121	75.9277	68.2169	1389.4773	2046.5076
34.000	34.286	3545.2774	1608.1127	10.4273	78.2483	70.4444	1431.9437	2113.3330
35.000	35.290	3654.6289	1657.7138	10.4418	80.5689	72.6739	1474.4101	2180.2180
36.000	36.293	3764.0384	1707.3411	10.4557	82.8894	74.9054	1516.8766	2247.1610
37.000	37.297	3873.5044	1756.9941	10.4689	85.2100	77.1387	1559.3430	2314.1606
38.000	38.300	3983.0255	1806.6722	10.4816	87.5306	79.3738	1601.8094	2381.2153
39.000	39.303	4092.6009	1856.3747	10.4938	89.8511	81.6108	1644.2758	2448.3242
40.000	40.305	4202.2291	1906.1013	10.5056	92.1717	83.8495	1686.7422	2515.4860
41.000	41.308	4311.9095	1955.8516	10.5169	94.4923	86.0900	1729.2087	2582.7000
42.000	42.310	4421.6411	2005.6251	10.5277	96.8128	88.3322	1771.6751	2649.9651
43.000	43.312	4531.4233	2055.4215	10.5382	99.1334	90.5760	1814.1415	2717.2809

URANIUM PRICE SCHEDULE NUMBER 2

ENRICHMENT		CALCULATED URANIUM PRICES			FLOW RATE RATIOS		EXPENSE PORTIONS, \$/KG U	
WEIGHT PER CENT	ATOMIC PER CENT	\$/KG U	\$/LB U	\$/GM U235	KG FEED PER KG PROD	KG SEP WORK PER KG PROD	COST OF FEED MAT	COST OF SEP WORK
44.000	44.313	4641.2555	2105.2406	10.5483	101.4540	92.8216	1856.6079	2784.6466
45.000	45.315	4751.1371	2155.0821	10.5581	103.7746	95.0687	1899.0744	2852.0618
46.000	46.316	4861.0676	2204.9458	10.5675	106.0951	97.3175	1941.5408	2919.5259
47.000	47.317	4971.0469	2254.8316	10.5767	108.4157	99.5680	1984.0072	2987.0386
48.000	48.317	5081.0744	2304.7393	10.5856	110.7363	101.8200	2026.4736	3054.5998
49.000	49.318	5191.1501	2354.6689	10.5942	113.0568	104.0736	2068.9401	3122.2090
50.000	50.318	5301.2739	2404.6203	10.6025	115.3774	106.3289	2111.4065	3189.8663
60.000	60.305	6405.1844	2905.3463	10.6753	138.5831	128.9704	2536.0708	3869.1123
70.000	70.266	7514.3969	3408.4772	10.7349	161.7888	151.7887	2960.7351	4553.6603
80.000	80.203	8630.6602	3914.8063	10.7883	184.9945	174.8420	3385.3994	5245.2592
90.000	90.114	9759.9180	4427.0296	10.8444	208.2002	198.3284	3810.0637	5949.8524
91.000	91.104	9874.2478	4478.8888	10.8508	210.5208	200.7239	3852.5301	6021.7158
92.000	92.093	9989.0272	4530.9520	10.8576	212.8413	203.1343	3894.9966	6094.0287
93.000	93.082	10104.3644	4583.2681	10.8649	215.1619	205.5633	3937.4630	6166.8995
94.000	94.071	10220.4161	4635.9083	10.8728	217.4825	208.0162	3979.9294	6240.4848
95.000	95.060	10337.4231	4688.9819	10.8815	219.8031	210.5008	4022.3958	6315.0254
96.000	96.049	10455.7854	4742.6702	10.8914	222.1236	213.0307	4064.8623	6390.9211
97.000	97.037	10576.2449	4797.3097	10.9033	224.4442	215.6305	4107.3287	6468.9141
98.000	98.025	10700.4510	4853.6487	10.9188	226.7648	218.3551	4149.7951	6550.6538
99.000	99.012	10833.6106	4914.0490	10.9430	229.0853	221.3782	4192.2615	6641.3470
0.711	0.721	18.3000	8.3008	2.5720	1.0000	0.0000	18.3000	0.0000
2.000	2.025	126.2658	57.2733	6.3133	3.9901	1.7749	73.0180	53.2477
3.000	3.037	222.3551	100.8587	7.4118	6.3106	3.5624	115.4844	106.8706
5.000	5.061	424.1245	192.3799	8.4825	10.9518	7.4569	200.4173	223.7071
93.000	93.082	10104.3655	4583.2686	10.8649	215.1619	205.5633	3937.4633	6166.9002

98

BNWL-189

BURN-UP COSTS PER GRAM OF U235 CONSUMED

URANIUM PRICE SCHEDULE NUMBER 2

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE \$/GM U235	RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100					
		INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
0.300	0.023	0.691	0.228 *	0.091 *	0.046 *	0.031 *	0.024 *
0.400	0.536	3.191	2.788	2.061	1.066 *	0.712 *	0.564 *
0.500	1.234	4.694	4.371	3.788	2.454 *	1.640 *	1.298 *
0.600	1.906	5.698	5.428	4.942	3.765	2.532 *	2.004 *
0.700	2.508	6.416	6.185	5.767	4.757	3.330 *	2.637 *
0.800	3.037	6.956	6.753	6.387	5.502	4.033 *	3.194 *
0.900	3.503	7.376	7.196	6.870	6.083	4.650 *	3.683 *
1.000	3.913	7.713	7.550	7.257	6.548	5.193 *	4.114 *
1.100	4.277	7.989	7.841	7.574	6.929	5.674 *	4.496 *
1.200	4.601	8.219	8.084	7.838	7.246	6.095	4.837 *
1.300	4.892	8.415	8.289	8.062	7.516	6.453	5.142 *
1.400	5.154	8.582	8.465	8.255	7.747	6.759	5.418 *
1.500	5.392	8.727	8.618	8.421	7.947	7.025	5.667 *
1.600	5.609	8.855	8.752	8.568	8.122	7.258	5.895 *
1.700	5.807	8.967	8.871	8.697	8.277	7.464	6.103 *
1.800	5.989	9.068	8.976	8.812	8.415	7.647	6.294 *
1.900	6.158	9.157	9.071	8.915	8.539	7.810	6.471 *
2.000	6.313	9.238	9.156	9.007	8.650	7.958	6.634 *

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION.
THE MINIMUM PRICED COMPOSITION IS 0.282 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

URANIUM PRICE SCHEDULE NUMBER 2

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE \$/GM U235	RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100					
		INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
2.100	6.458	9.311	9.233	9.091	8.751	8.092	6.786 *
2.200	6.593	9.378	9.303	9.168	8.843	8.213	6.927 *
2.300	6.718	9.439	9.367	9.238	8.927	8.324	7.059 *
2.400	6.836	9.495	9.426	9.302	9.003	8.426	7.182 *
2.500	6.947	9.547	9.480	9.361	9.074	8.520	7.298 *
2.600	7.051	9.594	9.530	9.416	9.140	8.607	7.407 *
2.700	7.149	9.638	9.577	9.466	9.200	8.687	7.509 *
2.800	7.241	9.679	9.620	9.513	9.257	8.762	7.606 *
2.900	7.329	9.718	9.660	9.557	9.309	8.831	7.698 *
3.000	7.412	9.753	9.698	9.598	9.358	8.896	7.785 *
3.200	7.566	9.818	9.766	9.672	9.447	9.014	7.945 *
3.400	7.705	9.876	9.827	9.738	9.526	9.118	8.091 *
3.600	7.832	9.927	9.880	9.797	9.596	9.211	8.223 *
3.800	7.948	9.973	9.929	9.849	9.659	9.294	8.344 *
4.000	8.055	10.014	9.972	9.896	9.716	9.368	8.456 *
4.200	8.153	10.052	10.011	9.939	9.767	9.436	8.558 *
4.400	8.245	10.086	10.047	9.978	9.814	9.498	8.653 *
4.600	8.330	10.117	10.080	10.014	9.856	9.554	8.741 *
4.800	8.409	10.146	10.111	10.047	9.896	9.606	8.823 *
5.000	8.482	10.172	10.138	10.077	9.932	9.654	8.900 *
5.200	8.552	10.197	10.164	10.105	9.965	9.698	8.972 *
5.400	8.617	10.220	10.188	10.131	9.996	9.739	9.039 *
5.600	8.678	10.241	10.210	10.156	10.025	9.777	9.102 *
5.800	8.736	10.261	10.231	10.178	10.052	9.812	9.162
6.000	8.790	10.279	10.250	10.199	10.077	9.846	9.217
6.200	8.842	10.296	10.269	10.219	10.101	9.877	9.269
6.400	8.891	10.313	10.286	10.238	10.123	9.906	9.318
6.600	8.937	10.328	10.302	10.255	10.144	9.933	9.364
6.800	8.982	10.343	10.317	10.271	10.164	9.959	9.407
7.000	9.024	10.356	10.331	10.287	10.182	9.983	9.448
7.200	9.064	10.369	10.345	10.302	10.200	10.006	9.486
7.400	9.102	10.381	10.358	10.316	10.216	10.028	9.522
7.600	9.139	10.393	10.370	10.329	10.232	10.049	9.557
7.800	9.174	10.404	10.382	10.342	10.247	10.069	9.590
8.000	9.208	10.415	10.393	10.354	10.261	10.087	9.621

100

BNWL-189

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION, THE MINIMUM PRICED COMPOSITION IS 0.282 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

URANIUM PRICE SCHEDULE NUMBER 2

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE \$/GM U235	RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100					
		INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
9.000	9.357	10.461	10.441	10.406	10.323	10.169	9.757
10.000	9.481	10.498	10.480	10.448	10.374	10.234	9.865
11.000	9.586	10.529	10.512	10.483	10.415	10.288	9.955
12.000	9.676	10.555	10.539	10.512	10.450	10.334	10.030
13.000	9.754	10.577	10.563	10.538	10.479	10.372	10.093
14.000	9.823	10.596	10.583	10.559	10.505	10.406	10.148
15.000	9.884	10.613	10.601	10.578	10.527	10.435	10.196
16.000	9.938	10.628	10.616	10.595	10.547	10.460	10.238
17.000	9.987	10.642	10.630	10.610	10.565	10.483	10.275
18.000	10.031	10.654	10.643	10.624	10.581	10.503	10.308
19.000	10.072	10.664	10.654	10.636	10.595	10.522	10.338
20.000	10.109	10.674	10.664	10.647	10.608	10.539	10.365
21.000	10.143	10.684	10.674	10.657	10.620	10.554	10.390
22.000	10.175	10.692	10.683	10.667	10.631	10.568	10.412
23.000	10.204	10.700	10.691	10.675	10.641	10.580	10.433
24.000	10.231	10.707	10.698	10.683	10.650	10.592	10.452
25.000	10.256	10.714	10.705	10.691	10.658	10.603	10.470
26.000	10.280	10.720	10.712	10.697	10.666	10.613	10.486
27.000	10.302	10.726	10.718	10.704	10.674	10.623	10.501
28.000	10.323	10.731	10.723	10.710	10.681	10.632	10.515
29.000	10.343	10.737	10.729	10.716	10.688	10.640	10.529
30.000	10.362	10.741	10.734	10.721	10.694	10.648	10.541
31.000	10.379	10.746	10.739	10.726	10.700	10.656	10.553
32.000	10.396	10.751	10.743	10.731	10.705	10.663	10.564
33.000	10.412	10.755	10.748	10.736	10.711	10.669	10.575
34.000	10.427	10.759	10.752	10.740	10.716	10.676	10.585
35.000	10.442	10.763	10.756	10.745	10.721	10.682	10.594
36.000	10.456	10.767	10.760	10.749	10.725	10.687	10.603
37.000	10.469	10.770	10.763	10.752	10.730	10.693	10.612
38.000	10.482	10.774	10.767	10.756	10.734	10.698	10.620
39.000	10.494	10.777	10.770	10.760	10.738	10.703	10.628
40.000	10.506	10.780	10.774	10.763	10.742	10.708	10.635
41.000	10.517	10.783	10.777	10.766	10.745	10.713	10.642
42.000	10.528	10.786	10.780	10.770	10.749	10.717	10.649
43.000	10.538	10.789	10.783	10.773	10.753	10.722	10.656

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION, THE MINIMUM PRICED COMPOSITION IS 0.282 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

URANIUM PRICE SCHEDULE NUMBER 2

INITIAL ENRICHMENT WT PERCENT	INITIAL PRICE		RATIO OF THE ENRICHMENT DECREASE TO THE INITIAL ENRICHMENT X 100				
	\$/GM U235	INFINITESIMAL	10.0	25.0	50.0	75.0	95.0
44.000	10.548	10.792	10.786	10.776	10.756	10.726	10.662
45.000	10.558	10.795	10.789	10.779	10.759	10.730	10.668
46.000	10.568	10.798	10.791	10.782	10.762	10.734	10.674
47.000	10.577	10.800	10.794	10.784	10.766	10.738	10.680
48.000	10.586	10.803	10.797	10.787	10.769	10.741	10.686
49.000	10.594	10.805	10.799	10.790	10.771	10.745	10.691
50.000	10.603	10.808	10.802	10.792	10.774	10.748	10.696
60.000	10.675	10.830	10.824	10.815	10.800	10.779	10.742
70.000	10.735	10.851	10.844	10.835	10.822	10.806	10.780
80.000	10.788	10.872	10.864	10.856	10.844	10.833	10.816
90.000	10.844	10.901	10.890	10.881	10.872	10.865	10.856
91.000	10.851	10.905	10.893	10.884	10.876	10.869	10.861
92.000	10.858	10.909	10.897	10.888	10.880	10.874	10.867
93.000	10.865	10.914	10.900	10.892	10.885	10.879	10.873
94.000	10.873	10.919	10.905	10.897	10.890	10.885	10.879
95.000	10.881	10.925	10.910	10.902	10.896	10.892	10.887
96.000	10.891	10.933	10.916	10.909	10.903	10.899	10.895
97.000	10.903	10.942	10.923	10.917	10.912	10.909	10.906
98.000	10.919	10.955	10.934	10.928	10.925	10.922	10.920
99.000	10.943	10.976	10.952	10.948	10.946	10.944	10.943
0.711	2.572	6.486	6.258	5.847	4.853	3.416 *	2.704 *
2.000	6.313	9.238	9.156	9.007	8.650	7.958	6.634 *
3.000	7.412	9.753	9.698	9.598	9.358	8.896	7.785 *
5.000	8.482	10.172	10.138	10.077	9.932	9.654	8.900 *
93.000	10.865	10.914	10.900	10.892	10.885	10.879	10.873

* DENOTES CASES IN WHICH THE FINAL ENRICHMENT IS LESS THAN THE MINIMUM PRICED COMPOSITION.
THE MINIMUM PRICED COMPOSITION IS 0.282 PERCENT WHICH IS PRICED AT 0.00 \$/KG URANIUM.

REFERENCES

1. M. R. Fenske. "Fractionation of Straight-run Pennsylvania Gasoline," Ind. Eng. Chem., vol. 24, p. 482-485. 1932.
2. R. E. Wilson. "Testimony on Private Ownership of Special Nuclear Materials," Hearings of Joint Committee on Atomic Energy. July 29, 1963.
3. K. Cohen. The Theory of Isotope Separations, McGraw-Hill New York, 1951.
4. M. Benedict and T. H. Pigford. Nuclear Chemical Engineering, McGraw-Hill, New York, 1957.
5. R. Stephenson. Introduction to Nuclear Engineering, McGraw Hill, New York, 1958. 2nd ed., p. 382.
6. J. R. Triplett and G. J. Busselman. MELEAGER - A Burnup Code for Fuel Cycle Analysis, HW-68100, General Electric, Richland, Washington. March 1961.
7. M. F. Kanninen. PROTEUS - A Computer Code to Extend the Burnup Data from a Point Burn Up Code, HW-73536, General Electric, Richland, Washington. November 1965
8. E. A. Eschbach, D. E. Deonigi, and S. Goldsmith. QUICK - A Simplified Fuel Cost Code, HW-71811. December 1961.
9. C. R. Wylie, Jr. Advanced Engineering Mathematics, McGraw-Hill, New York, 1960. 2nd ed. p. 99.

ONSITE DISTRIBUTIONCopy NumberPacific Northwest Laboratory

1	F. W. Albaugh
2	C. A. Bennett
3	J. B. Burnham, Jr.
4	F. G. Dawson, Jr.
5	D. E. Deonigi
6	R. F. Dickerson
7 - 152	E. A. Eschbach
153	S. L. Fawcett
154	H. Harty
155	E. D. Jones, Jr.
156	B. B. Lane
157	L. W. Lang
158	E. T. Merrill
159	J. N. Morgan
160	J. H. Nail
161	W. I. Neef
162	R. S. Paul
163	J. G. Rake
164	W. A. Reardon
165	R. L. Reynolds
166	W. D. Richmond, Jr.
167	E. A. Schnaible
168	J. R. Sheff
169	R. E. Thomas
170	R. E. Thompson
171	C. R. Tipton, Jr.
172	M. T. Walling, Jr.
173	Technical Information Files
174 - 175	Technical Publications, 700 Area

176 - 180

Douglas United NuclearGeneral Electric Company, Richland

181 - 185

GETA File Copies

186 - 190

M. C. Leverett (NRD)

191 - 195

IsochemRichland Operations Office

196

R. K. Sharp

197

Technical Information Library

198 - 200

P. G. Holsted

OFFSITE DISTRIBUTION (SPECIAL)No. of Copies

59

Division of Reactor Development
AEC, Washington, D. C.