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VIBRATION OF MONOLITHIC GRID-STIFFENED PLATES

C. Omid'varan ⁽¹⁾ and W. Delagarza ⁽²⁾

The steady state frequencies of vibration of grid-stiffened rectangular plates are formulated. The plate and the grid are assumed to have a monolithic connection at the node points.

INTRODUCTION

A knowledge of dynamic characteristics of the conventional grid-stiffened plates is essential to rational design of these systems subjected to dynamic perturbations. Historical notes concerning the applications of grid-stiffened plates, the available literature, and the known techniques for determining the response of grid-stiffened plates are explored in a recent publication by the author[1].

This paper represents an extension of the information presented in above reference by assuming a monolithic connection at the node points between the plate and the grid, and by accounting for the shear as well as normal force interactions between these structural components. The application of earlier solution[1] to systems with shear bond will result in approximate solutions. The contribution of the top plate to the system stiffness is similar to the contribution of flange to the stiffness of a T beam in the presence of shear bond. The numerical examples which proceed this presentation further illustrate the significance of accounting for the

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membrane action of the top plate produced by shear bond.

SOLUTION

The macro-mechanic technique [1,2] is employed for this derivation. The grid and the plate will be isolated from each other. Each will be analysed for the behavior at the nodes, based on a discrete field solution. The compatibility requirements at the nodal points are employed to formulate the frequencies of vibration. Differences between the application of the technique here and the application in earlier publication [1] are to accommodate accounting for the membrane action of the plate and in plane forces acting on the grid due to shear bond.

Figure 1. Composite plate grid construction.

THE GRID

A simply supported $N \times M$ grid will be assumed to serve as the stiffener for the composite construction. The grid element stiffness are assumed different in orthogonal directions. Harmonic time dependent interaction

forces F_r , F_s and F_z will be acting on the grid at the nodal point (r,s) , [Fig 2].

Fig (2). Grid joint isolated

The shearing interaction forces F_r and F_s acting along the top surface of the joint will produce concentrated moments M_r^0 and T_s^0 acting on the joints (r,s) in addition to in plane concentrated force effects.

$$M_r^0 = F_r h, \quad T_s^0 = F_s \bar{h} \quad (1)$$

h and $\bar{h}$ denote distance from top fibers to the centroids of stiffener members in r and s directions, respectively. (Fig 2).

GRID DEFORMATIONS

The time dependent displacements of top fibers of the nodal point (r,s) of the grid subjected to interaction forces will be designated as $U_{r,s}$, $V_{r,s}$, and $W_{r,s}$ in r , s and z directions respectively.

For composite systems with thin plates; it will be assumed that the contribution to the displacements of the grid top fibers U , V and W due to the grid member axial elongation is small compared with the top fiber displacement components due to grid bending. For a grid with rigid joint construction, the joint displacements $U(r,s)$ and $V(r,s)$ are related to angles of rotations $\alpha(r,s)$ and $\varphi(r,s)$ at the node points.

$$U_{r,s} = \alpha(r,s) h, \quad V_{r,s} = \varphi(r,s) \bar{h}, \quad W_{r,s} = \rho(r,s) a \quad (2)$$

a and b refer to typical stiffener lengths in r and s directions. (Fig 3). The discrete analysis technique and calculus of finite differences will be employed to formulate the grid displacements.

The designations employed to represent shears and moments at the joint (r,s) are shown in Figure 3.

Figure 3. Moment and shear designations. $\odot$, Shear directed out of plane; $\otimes$, shear directed into plane.

For a torsionless grid, neglecting the joint rotational inertias, consideration of the free body diagram of joint (r,s) (Fig. 2,3) will produce:

$$\begin{aligned} M_{r+1,s} + M'_{r,s} &= F_r h \\ T_{r,s+1} + T'_{r,s+1} &= F_s \bar{h} \\ V_{r+1,s} - V'_{r,s} + Q_{r,s+1} - Q'_{r,s} + m \frac{\partial^2 W_{r,s}}{\partial t^2} &= F_z \end{aligned} \quad (3)$$

m denotes the grid mass concentrated at the node point.

FORCE DEFORMATION RELATIONS:

The slope deflection equations for the grid elements comprise the force-deformation relations:

$$\begin{aligned} M_{r+1,s} &= K_r \left[(4 + 2E_r) \alpha_{r,s} - 6\Delta_r \rho_{r,s} \right] \\ M'_{r,s} &= K_r \left[(2E_r^{-1} + 4) \alpha_{r,s} - 6\nabla_r \rho_{r,s} \right] \\ T_{r,s+1} &= K_s \left[(4 + 2E_s) \varphi_{r,s} - 6\frac{a}{b} \Delta_s \rho_{r,s} \right] \\ T'_{r,s} &= K_s \left[(2E_s^{-1} + 4) \varphi_{r,s} - 6\frac{a}{b} \nabla_s \rho_{r,s} \right] \end{aligned} \quad (4)$$

in which K_r and K_s denote the grid element flexural rigidities, E_r and E_s denote difference operators in r and s direction and Δ and ∇ refer to 1st forward and backward difference operators.

$$\Delta_r \rho_{r,s} \equiv \rho_{r+1,s} - \rho_{r,s} ; \quad \nabla_s \rho_{r,s} \equiv \rho_{r,s} - \rho_{r,s-1} \quad (5)$$

Shear-deformation relations are derived from consideration of the following shear-moment relations:

$$\begin{aligned} a \quad V_{r+1,s} &= M_{r+1,s} + M'_{r+1,s} \\ b \quad Q_{r,s+1} &= T_{r,s+1} + T'_{r,s+1} \end{aligned} \quad (6)$$

In view of harmonic nature of motion, we have $\left(\frac{\partial^2 W_{r,s}}{\partial t^2} = \omega^2 W_{r,s} \right)$.

ω denotes the frequency of vibration. Introduce force-deformation relations into dynamic equations (3), considering (2) we find:

$$\begin{bmatrix} \frac{2K_r}{h^2} (\Delta \nabla_r + 6) & 0 & -\frac{12K_r}{ah} \Lambda_r \\ 0 & \frac{2K_s}{h^2} (\Delta \nabla_s + 6) & -\frac{12K_s}{bh} \Lambda_s \\ \frac{12K_r}{ah} \Lambda_r & \frac{12K_s}{bh} \Lambda_s & -\frac{12K_r}{a^2} \Delta \nabla_r - \frac{12K_s}{b^2} \Delta \nabla_s - m\omega^2 \end{bmatrix} \begin{bmatrix} U_{r,s} \\ V_{r,s} \\ W_{r,s} \end{bmatrix} = \begin{bmatrix} F_r \\ F_s \\ F_z \end{bmatrix} \quad (7)$$

$\Delta \nabla_r$ denotes second central difference operator and Λ_r denotes the mean central difference operator.

$$\Delta \nabla_r U_{r,s} \equiv U_{r+1,s} - 2U_{r,s} + U_{r-1,s} ; \quad \Lambda_s V_{r,s} \equiv \frac{1}{2} (V_{r,s+1} - V_{r,s-1}) \quad (8)$$

The set of three partial difference equations (7) represent the mathematical model for the grid.

For simply supported end conditions, the following finite series solutions are justified:

$$\begin{aligned} U_{r,s} &= \sum_{i=0}^M \sum_{j=1}^{N-1} \bar{U}_{ij} \cos \frac{i\pi r}{M} \sin \frac{j\pi s}{N} \sin \omega t \\ V_{r,s} &= \sum_{i=1}^{M-1} \sum_{j=0}^N \bar{V}_{ij} \sin \frac{i\pi r}{M} \cos \frac{j\pi s}{N} \sin \omega t \\ W_{r,s} &= \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} \bar{W}_{ij} \sin \frac{i\pi r}{M} \sin \frac{j\pi s}{N} \sin \omega t \end{aligned} \quad (9)$$

The field of interaction forces at the nodal points will be represented by the following finite series:

$$\begin{aligned}
 F_r &= \sum_{i=0}^M \sum_{j=1}^{N-1} R_{ij} \cos \frac{i\pi r}{M} \sin \frac{j\pi s}{N} \sin \omega t \\
 F_s &= \sum_{j=1}^{M-1} \sum_{j=0}^N S_{ij} \sin \frac{i\pi r}{M} \sin \frac{j\pi s}{N} \sin \omega t \\
 F_z &= \sum_{i=1}^{M-1} \sum_{j=1}^{N-1} Z_{ij} \sin \frac{i\pi r}{M} \sin \frac{j\pi s}{N} \sin \omega t
 \end{aligned} \tag{10}$$

Introduce (9) and (10) into (7). A solution to displacement coefficients is obtained by matching the terms of the series:

$$\begin{bmatrix} \bar{u}_{ij} \\ \bar{v}_{ij} \\ \bar{w}_{ij} \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{12} & d_{22} & d_{23} \\ d_{13} & d_{23} & d_{33} \end{bmatrix}_{ij} \begin{bmatrix} z_{ij} \\ R_{ij} \\ S_{ij} \end{bmatrix} \tag{11}$$

Eq. (11) represents the grid force-deformation relations. The grid flexibility coefficients d_{ij} are given in Appendix I.

THE PLATE

The top plate will be assumed simply supported along four edges against deflection and will be subjected to the field of in-plane and lateral plate-grid discrete interaction forces.

The plate is a continuum. We are interested here in a field solution to the finite number of displacements of the plate at the nodal points, i.e., the discrete field solution to the finite number of ordinates of the continuous functions representing the plate deflection as well as the plate membrane action are developed.

The interaction of the bending and membrane action will be neglected. The governing equation for plate deflection is classical, and in terms of dimensionless parameters r and s ($r = x/a$, $s = y/b$) is given by:

$$\left[D_r^4 + 2\left(\frac{a}{b}\right)^2 D_r^2 D_s^2 + \left(\frac{a}{b}\right)^4 D_s^4 - \frac{a^4 \rho}{D} \omega^2 \right] W(r,s,t) = \frac{a^4}{D} q(r,s,t) \quad (12)$$

ρ and D denote the plate mass density and flexural stiffness respectively. q denotes the lateral load intensity.

Neglecting the in-plane inertia of the plate, the membrane action is governed by:

$$\begin{bmatrix} D_r^2 + \gamma_{11} D_s^2 & \gamma_{12} D_r D_s \\ \gamma_{12} D_r D_s & \left(\frac{a}{b}\right)^2 D_s^2 + \frac{1-\nu}{2} D_r^2 \end{bmatrix} \begin{bmatrix} U \\ V \end{bmatrix} = \gamma_{13} \begin{bmatrix} Q_r \\ Q_s \end{bmatrix} \quad (13)$$

Q_r and Q_s refer to in-plane load intensities in r and s directions, and

$$\gamma_{11} = \frac{1-\nu}{2} \left(\frac{a}{b}\right)^2, \quad \gamma_{12} = \frac{1+\nu}{2} \left(\frac{a}{b}\right), \quad \gamma_{13} = \frac{1-\nu^2}{Eh} a^2 \quad (14)$$

Based on the calculus of finite differences, the finite number of ordinates of continuum displacement functions are represented by finite series. For integer values of r and s ($0 \leq r \leq M$; $0 \leq s \leq N$).

The field of interaction forces are discrete by nature. Assuming a contact area A at the nodal points the plate-grid action reaction will imply

$$F_r = \frac{1}{2} A Q_r, \quad F_s = -A Q_s, \quad F_t = -A q(r,s,t) \quad (15)$$

For integer values of r and s , the suitable series for representation of displacements are the same as those given for the grids and is given by eq. (9). Similar series for presentations for forces F_r , F_s and F_t are given by eq. (10).

A solution to the field of plate displacements at the nodal points is obtained by introducing (9), (10) and (15) into (12) and (13). Matching the coefficients of series obtained in this process will give:

$$\begin{bmatrix} \bar{U}_{ij} \\ \bar{V}_{ij} \\ \bar{W}_{ij} \end{bmatrix} = H_{ij} \begin{bmatrix} C_{11} & C_{12} & 0 \\ C_{12} & C_{22} & 0 \\ 0 & 0 & C_{33} \end{bmatrix} \begin{bmatrix} R_{ij} \\ S_{ij} \\ Z_{ij} \end{bmatrix} \quad (16)$$

In this notation

$$C_{11} = a^2 \pi^2 \left[\left(\frac{j}{N} \right)^2 + \frac{1-\nu}{2} \left(\frac{ib}{Ma} \right)^2 \right], \quad C_{12} = ab \pi^2 \left(\frac{ij}{MN} \right) \left(\frac{1+\nu}{2} \right) \quad (17)$$

$$C_{22} = b^2 \pi^2 \left[\left(\frac{i}{M} \right)^2 + \frac{1-\nu}{2} \left(\frac{ja}{Nb} \right)^2 \right] \quad (18)$$

$$C_{33} = -a^4 / \left\{ AD \pi^2 \left[\left(\frac{i}{M} \right)^2 + \left(\frac{aj}{bN} \right)^2 \right]^2 - \rho a^4 A \omega^2 \right\} \quad (19)$$

$$H_{ij} = (1-\nu^2) a^2 b^2 / AEh \left[C_{11} C_{22} - \left(\frac{1+\nu}{2} \right)^2 \left(\frac{ijba}{MN} \right)^2 \right] \quad (20)$$

Composite Construction

The compatability of deformations at the nodal points will require that the $\bar{U}_{ij}$, $\bar{V}_{ij}$ and $\bar{W}_{ij}$ coefficients be identical for the plate and the grid.

Referring to equations (11) and (16), setting displacement coefficients equal, we obtain a set of three homogeneous equations for R_{ij} , S_{ij} and Z_{ij} . Letting the matrix of coefficients of the set of linear equations equal zero will give:

$$\omega^2 = \omega_{pg}^2 + \frac{m}{m+A\rho} \frac{\alpha_2 + 2\alpha_3 + \alpha_4}{\alpha_3^2 \alpha_2 \alpha_4} \quad (21)$$

Equation (21) is the frequency equation for composite construction. ω_{pg} refers to the frequencies of vibration in the absence of any shear bond [Reference 1]. In terms of ω_g , the grid frequency independently and ω_p , the plate frequency; we have [1]

$$\omega_{pg}^2 = \omega_g^2 + \frac{A\rho}{m+A\rho} (\omega_p^2 - \omega_g^2) \quad (22)$$

The grid and the plate frequencies are given by:

$$\rho \omega_p^2 = D\pi^4 \left[\left(\frac{i}{L} \right)^2 + \left(\frac{j}{L} \right)^2 \right]^2 \quad (23)$$

$$m\omega_g^2 = \frac{12K_r (\cos \frac{i\pi}{M} - 1)^2}{a^2 (\cos \frac{i\pi}{M} + 2)} + \frac{12K_s (\cos \frac{j\pi}{N} - 1)^2}{b^2 (\cos \frac{j\pi}{N} + 2)} \quad (24)$$

Referring to corrective additive to ω_{pg} , Eq (21), the α coefficients are defined as follows:

$$\alpha_2 = \frac{mah d_{11}}{12K_r d_{12} \sin \frac{i\pi}{M}} - m H_{ij} C_{11} \left(\frac{d_{11}}{d_{12}} \right)^2$$

$$\alpha_3 = m C_{12} H_{ij} \frac{d_{11}}{d_{22}} \quad (25)$$

$$\alpha_4 = m \left(1 - \frac{d_{12}}{d_{13}} C_{14} \right) \left(\frac{d_{11}}{d_{23}} \right)$$

Numerical Example:

Consider the case of a square aluminum plate 60" x 60", stiffened by an aluminum grid with bars spaced 5" apart in each direction. The grid bars are assumed to be 1" deep and 1/16" wide. For a 1/8" thick plate with a contact area of 1.5 in², we find the frequency of vibration in the first mode ($i=1, j=1$) is 282 rad./sec. This compares with a frequency of 167 rad/sec. for the grid alone and with a frequency of 29.5 rad/sec. for the plate alone [1].

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- [1] Omid'varan, C. *Free Vibration of Grid-Stiffened Plates*", Journal of Sound & Vibration, Vol. 19, No. 4, 1971, pp 463-472.
- [2] Dean, D. L., and Rao, G., 1970, American Society of Civil Engineering, Journal of Engineering Mechanics, 96, Em4, 377-395, *Macro Approach to Discrete Field Analysis*.

APPENDIX I

The grid stiffness coefficients (eq. 11) are as follows:

$$d_{11} = 1 / \left[k_r \eta_1 + k_s \gamma_1 + \bar{k}_r + \bar{k}_s + m\omega^2 \right], \quad A-1$$

$$d_{22} = 1 / \left[-k_r \gamma_2 + 12 \frac{K_r}{h^2} + \frac{a}{h^2} \frac{\bar{k}_r}{6} \right] \quad A-2$$

$$d_{33} = 1 / \left[-K_s \eta_2 + 12 \frac{K_s}{h^2} + \frac{b^2}{h^2} \frac{\bar{k}_s}{6} \right] \quad A-3$$

$$d_{12} = \gamma_2 d_{22}, \quad d_{13} = \eta_2 d_{33}, \quad d_{23} = \eta_1 d_{13} \quad A-4$$

In above notation we have:

$$k_r = \frac{12K_r}{ah} \sin \frac{j\pi}{M} \quad ; \quad k_s = \frac{12K_s}{bh} \sin \frac{j\pi}{N} \quad A-5$$

$$\bar{k}_r = \frac{24K_r}{a^2} \left(\cos \frac{j\pi}{M} - 1 \right) \quad ; \quad \bar{k}_s = \frac{24K_s}{b^2} \left(\cos \frac{j\pi}{N} - 1 \right) \quad A-6$$

$$\gamma_1 = \frac{3h \sin \frac{j\pi}{M}}{b \left(\cos \frac{j\pi}{N} + 2 \right)} \quad ; \quad \eta_1 = \frac{3h \sin \frac{j\pi}{M}}{a \left(\cos \frac{j\pi}{N} + 2 \right)} \quad A-7$$

$$\gamma_2 = \frac{-K_r}{k_s \eta_1 + \bar{k}_r + \bar{k}_s + m\omega^2} \quad ; \quad \eta_2 = \frac{-k_s}{k_s \eta_1 + \bar{k}_r + \bar{k}_s + m\omega^2} \quad A-8$$