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Parametric Decay Near the Lower Hybrid Resonance

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ABSTRACT

Spatial parametric decay of a wave with frequency near the lower hybrid frequency is investigated. A three-waves problem is considered with special emphasis on the effect of the mismatch of the wave vectors on the saturation amplitude of the excited waves and the characteristic length of conversion.

Recent experiments¹ on nonlinear decay of a wave with frequency $\omega_1 \approx \omega_L$, the lower hybrid frequency, into a wave with frequency $\omega_2 \approx \omega_1$ and a ion wave with $\omega_3 = \omega_1 - \omega_2 \ll \omega_1$, have shown that the process has a very low threshold. This is in agreement with the theoretical prediction.² An important question for the possible application of this process to ion heating in fusion devices is the saturation amplitude of the excited waves. The aim of this note is to investigate the influence of the hybrid resonance and

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the inevitable mismatch of the wave vectors on the final amplitude of the waves.

We consider a steady state situation and take $\partial/\partial t = 0$ in the nonlinear equations. For propagation in the (x, z) plane, where x is the direction of the density and temperature gradients and z that of the confining magnetic field, we write

$$\vec{E}(\vec{r}, t) = \frac{1}{2} \sum_{j=1}^3 \vec{E}_j \exp i \left(\int^x k_{jx}(x) dx + k_{jz}z - \omega_j t \right) + c.c. , \quad (1)$$

where $n_{jz}^2 = (ck_{jz}/\omega_j)^2 \gg 1$, $|k_{jz}| \ll |k_{jx}|$ and k_{jx} is obtained from

$$\epsilon(\vec{k}_j, \omega_j) = 1 + \sum_{\alpha} \frac{k_{D\alpha}^2}{k_j^2} \left\{ 1 - \sum_n \frac{\omega_j I_n(\rho) \exp(-\rho)}{\omega_j - n\omega_{\alpha}} [1 + W(\zeta_n)] \right\} , \quad (2)$$

where $W(z) = -1 + 2z \exp(-z^2) \int_0^z \exp t^2 dt$ and

$$k_{D\alpha}^2 = 4\pi N_0 q_{\alpha}^2 / T_{\alpha} , \quad v_{\alpha}^2 = 2T_{\alpha} / m_{\alpha} , \quad \rho = k_{jx}^2 T_{\alpha} / m_{\alpha} \omega_{\alpha}^2 ,$$

$$\zeta_n = (\omega_j - n\omega_{\alpha}) / k_{jz} v_{\alpha} , \quad \omega_{\alpha} = q_{\alpha} B_0 / m_{\alpha} c .$$

Using the usual perturbation approach^{3,4} we get

$$\frac{d}{dx} \left(\frac{\partial \epsilon}{\partial k_x} \right)_j E_j = \frac{4\pi}{\omega_j} \frac{\vec{k}_j}{k_j} \cdot \vec{j}_j^{(2)} , \quad (3)$$

where E_j is defined by the relation $\vec{E}_j = (\vec{k}_j/k_j) E_j$. In Eq. (3) we have not considered the z variation of E_j . In principle there is no reason to do so. On the other hand it is not possible

to take into account this variation in the framework of the perturbation theory for the essential assumption should be $k_{jz} \gg \partial E_j / E_j \partial z$. Now, k_{jz} is usually small and determined by finite geometry conditions and therefore it is difficult to visualize the z -dependence of E_j . It is also worth mentioning that for the waves we consider the energy flux is orthogonal to $\vec{k} \approx k_x \vec{e}_x$ and the group velocity along z is much greater than the x -component.

The nonlinear current $\vec{J}_j^{(2)}$ is obtained from

$$\frac{\partial f_\alpha^{(2)}}{\partial t} + \vec{v} \cdot \frac{\partial f_\alpha^{(2)}}{\partial \vec{r}} + \frac{q_\alpha}{m_\alpha c} \vec{v} \times \vec{B}_0 \cdot \frac{\partial f_\alpha^{(2)}}{\partial \vec{v}} = - \frac{q_\alpha}{m_\alpha} \vec{E}(\vec{r}, t) \cdot \frac{\partial f_\alpha^{(1)}}{\partial \vec{v}}. \quad (4)$$

In calculating $\vec{J}_1^{(2)}$ we use the fact that $\omega_i \ll \omega_1 \ll |\omega_e|$ and $\omega_1 \gg k_{1z} v_\alpha$ and therefore we neglect the $\vec{v} \cdot [\partial f_\alpha^{(2)} / \partial \vec{v}]$ term, hence

$$N_0 \vec{J}_{\alpha,1}^{(2)} \approx \bar{\sigma}_\alpha(\omega_1) \cdot \frac{1}{2} [\vec{E}_2 n_{\alpha,3}^{(1)} + n_{\alpha,2}^{(1)} \vec{E}_3] \exp i\psi,$$

where $\omega_1 = \omega_2 + \omega_3$, $k_{1z} = k_{2z} + k_{3z}$ and $\psi = \int^x (k_{2x} + k_{3x} - k_{1x}) dx$ and

$$\sigma_{\alpha xx} = - \frac{\omega_1}{4\pi i} \frac{\omega_{p\alpha}^2}{\omega_1^2 - \omega_\alpha^2}, \quad \sigma_{\alpha zz} = - \frac{\omega_1}{4\pi i} \frac{\omega_{p\alpha}^2}{\omega_1^2}.$$

Using Poisson's equation $n_{i,j}^{(1)} - n_{e,j}^{(1)} = (i/4\pi q_i) \vec{k}_j \cdot \vec{E}_j$ we get

$$2N_0 \vec{J}_1^{(2)} = \bar{\sigma}(\omega_1) \cdot [\vec{E}_2 n_{e,3}^{(1)} + n_{e,2}^{(1)} \vec{E}_3] \exp i\psi + \frac{i}{4\pi q_i} \bar{\sigma}_i(\omega_1) \cdot (\vec{E}_2 \vec{k}_3 \cdot \vec{E}_3 + \vec{E}_3 \vec{k}_2 \cdot \vec{E}_2) \exp i\psi. \quad (5)$$

In calculating $n_{e,j}^{(1)}$ we note that $\omega_j \ll |\omega_e|$ and $k_{jx}^2 T_e / m_e \omega_e^2 \ll 1$, thus

$$n_{e,j}^{(1)} = i \frac{q_e}{m_e} E_{zj} \int_{-\infty}^{\infty} dv_z \frac{\partial f_{oe} / \partial v_z}{k_{jz} v_z - \omega_j},$$

which yields $n_{e,2}^{(1)} \approx 0$ and $n_{e,3}^{(1)} \approx -i(q_e/T_e) N_0 (E_{z3}/k_{3z})$ for $\omega_2 \gg k_{2z} v_e$ and $\omega_3 \ll k_{3z} v_e$. Moreover, $k_{2x}^2 \lambda_D^2 \ll 1$ and $k_{3x}^2 \lambda_D^2 \ll 1$, hence

$$\frac{4\pi}{\omega_1} \frac{\vec{k}_1}{k_1} \cdot \vec{j}_1^{(2)} \approx \frac{q_i}{2T_e} \frac{E_2 E_3}{k_1 k_2 k_3} [\vec{k}_1 \cdot \vec{\epsilon}(\omega_1) \cdot \vec{k}_2 - \vec{k}_1 \cdot \vec{k}_2] \exp i\psi,$$

where $\epsilon_{ij}(\omega_1) = \delta_{ij} + (4\pi i/\omega_1) \sigma_{ij}(\omega_1)$. From Eq. (2) with $j=1,2$ we have

$$k_{jx}^2 = -k_{jz} \epsilon_{zz}(\omega_j) / \epsilon_{xx}(\omega_j) = k_{jz}^2 \left(\frac{m_i}{m_e} \right) \frac{\omega_L^2}{\omega_j^2 - \omega_L^2}, \quad (6)$$

where $\omega_L^2 = \omega_{pi}^2 / [1 + (\omega_{pe}^2 / \omega_e^2)]$ and we must have $|\Delta k| = |k_{2x} + k_{3x} - k_{1x}| \ll |k_x|$. Now, we have $(\omega_1/k_{1x})(\partial\omega/\partial k_x)_1 < 0$, and $(\omega_2/k_{2x})(\partial\omega/\partial k_x)_2 < 0$ and usually $(\omega_3/k_{3x})(\partial\omega/\partial k_x)_3 > 0$. We consider the case $(\partial\omega/\partial k_x)_j > 0$ hence $k_{1x} = -|k_{1x}|$, $k_{2x} = -|k_{2x}|$ and $k_{3x} = |k_{3x}|$ [note that parametric decay takes place if $(\partial\omega/\partial k_x)_2 (\partial\omega/\partial k_x)_3 > 0$]. Using Eq. (6)

$$\Delta k(x) = k_{3x} - \left(\frac{m_i}{m_e} \right)^{1/2} \omega_L \left(\frac{|k_{2z}|}{(\omega_2^2 - \omega_L^2)^{1/2}} - \frac{|k_{1z}|}{(\omega_1^2 - \omega_L^2)^{1/2}} \right). \quad (7)$$

Equation (7) suggests that the small Δk condition is more likely fulfilled if we take $k_{1z} = -k_{2z}$. Now

$$\vec{k}_1 \cdot \vec{\epsilon}(\omega_1) \cdot \vec{k}_2 \approx k_{1x}^2 \epsilon_{xx} - k_{1z}^2 \epsilon_{zz} = 2k_{1x}^2 \epsilon_{xx}(\omega_1) ,$$

and Eq. (3) becomes

$$\frac{d}{dx} \left(\frac{\partial \epsilon}{\partial k_x} \right)_1 E_1 = -H E_2 E_3 \exp i\psi , \quad (8)$$

where $H \approx (q_i/2T_e k_3) [1 - 2\epsilon_{xx}(\omega_1)]$. The corresponding equations for E_2 , E_3 are obtained by using the symmetry of the nonlinear coefficients^{5,6} and we get

$$\frac{d}{dx} \left(\frac{\partial \epsilon}{\partial k_x} \right)_2 E_2^* = H E_1^* E_3 \exp i\psi , \quad \frac{d}{dx} \left(\frac{\partial \epsilon}{\partial k_x} \right)_3 E_3^* = H E_1^* E_2 \exp i\psi . \quad (9)$$

Equations (8) and (9) are difficult to handle because of the complicated x -dependence of ψ , H and $(\partial \epsilon / \partial k_x)_j$. An analytical solution can be obtained for the homogeneous plasma. We consider this case and for $\omega_{p\alpha}^2$ we take a constant value which is representative of the actual situation, at least in a limited range of x . Letting $-H = h \exp i\psi$, $E_j = E_j \exp i\theta_j$, $E_1(x=0) = E_0$ and $E_2(x=0) = E_3(x=0) \approx 0$, we get^{5,7}

$$E_1^2(x) = E_M^2 + E_0^2 \left[1 - \left(\frac{\Delta k \ell}{2} \right)^2 \right] \tanh^2 \frac{x_0 - x}{\ell} , \quad (10)$$

$$\left(\frac{\partial \epsilon}{\partial k_x} \right)_2 E_2^2(x) = \left(\frac{\partial \epsilon}{\partial k_x} \right)_3 E_3^2(x) = \left(\frac{\partial \epsilon}{\partial k_x} \right)_1 E_0^2 \left[1 - \left(\frac{\Delta k \ell}{2} \right)^2 \right] \operatorname{sech}^2 \frac{x_0 - x}{\ell} , \quad (11)$$

where

$$E_M^2 = \left(\frac{\Delta k}{2h} \right)^2 \left(\frac{\partial \epsilon}{\partial k_x} \right)_2 \left(\frac{\partial \epsilon}{\partial k_x} \right)_3 , \quad E_M^2 / E_0^2 = \left(\frac{\Delta k \ell}{2} \right)^2 ,$$

$$\ell = \frac{\ell_0}{[1 - (\Delta k \ell_0 / 2)^2]^{1/2}} \quad \text{and} \quad \ell_0 = [(\frac{\partial \epsilon}{\partial k_x})_2 (\frac{\partial \epsilon}{\partial k_x})_3]^{1/2} / h E_0. \quad (12)$$

Equations (10), (11) and (12) show the effect of a finite Δk on the final values of E_2 and E_3 and on the characteristic length ℓ for total conversion. The maximum value of energy flux $S_{3x} = v_{3x} (1/16\pi) (\partial \omega \epsilon / \partial \omega)_3 E_3^2 = -(\omega_3 / 16\pi) (\partial \epsilon / \partial k_x)_3 E_3^2$ is given by

$$S_{3x} = \frac{\omega_3}{\omega_1} S_{1x}(x=0) [1 - (\frac{\Delta k \ell_0}{2})^2]. \quad (13)$$

For $\omega_3 \approx \omega_i$

$$k_{3x}^2 = -k_{3z}^2 \epsilon_{zz}(\omega_3) / \epsilon_{xx}(\omega_3) \approx \frac{\omega_3^2 - \omega_i^2}{T_e / m_i},$$

where $\epsilon_{xx}(\omega_3) = 1 - (\omega_{pi}^2 / \omega_3^2 - \omega_i^2) + (\omega_{pe}^2 / \omega_e^2) \approx -(\omega_{pi}^2 / \omega_3^2 - \omega_i^2)$ and $\epsilon_{zz}(\omega_3) = 1 - (\omega_{pi}^2 / \omega_3^2) + (m_i \omega_{pi}^2 / k_{3z}^2 T_e)$, thus

$$\ell_0 = 4 \left(\frac{T_e}{q_1 \phi_0} \right) \frac{c}{\omega_1} \frac{|(k_{3x} / k_{2x}) \epsilon_{xx}(\omega_2) \epsilon_{xx}(\omega_3)|^{1/2}}{q_1 |1 - 2 \epsilon_{xx}(\omega_1)|}, \quad (14)$$

where $\phi_0 = E_0 / |k_{1x}|$ and $q_1^2 = n_{1z}^2 (m_i / m_e) (\omega_L^2 / \omega_1^2 - \omega_L^2)$. Now, total or appreciable conversion occurs if ℓ_0 is less than the transverse dimensions of the plasma. Since $(T_e / q_1 \phi_0)$ is much larger than one we must have $\omega_1 \approx \omega_2 \approx \omega_L$, i.e. $\epsilon_{xx}(\omega_1) \approx \epsilon_{xx}(\omega_2) \ll 1$ and $q_1 \sim q_2 = (c / \omega_2) k_{2x} \gg 1$. On the other hand for $\omega_1, \omega_2 \rightarrow \omega_L$, it may be difficult to make $\Delta k \ell_0 \ll 1$ because of the resonant dependence of Δk on $\omega_1^2 - \omega_L^2$. It must also be noted that ℓ_0 increases with T_e and for high temperature plasmas ω_1 and ω_2 must be taken too close to ω_L to make ℓ_0 acceptable.

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