MASTER

Statistical Analysis of Fuel Rod Scans

Martin S. Zucker
Lester G. Epel

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Technical Support Organization
Brookhaven National Laboratory
Upton, New York 11973

June 1971

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

Statistical Analysis of Fuel Rod Scans

By: Martin S. Zucker
Lester G. Epel

Technical Support Organization
Brookhaven National Laboratory

Abstract

Gamma scanning of fuel rods containing plutonium has gained acceptance as a way of determining the uniformity of fuel loading and, to a lesser extent, the total amount of material. The usual electronics for making these measurements includes an analogue ratemeter driving a strip chart recorder. The resulting strip chart recording is hard to interpret in more than a qualitative manner. The use, recently, of a digital ratemeter in a rod scanning apparatus has made statistical analysis of the data much more convenient and prompted such an analysis to be made. The following analyses were performed: (1) A test to see whether at least one segment of the rod differed significantly (i.e., with some confidence limit) from the rest of the rod, (2) Assuming that the rod failed test (1) for homogeneity, the segments were ranked into groups which are statistically homogeneous within themselves, (3) An attempt to find periodicities in the rod due to its internal structure.

Additionally, use was made of the gamma spectra of the whole rod as recorded by a multichannel analyzer during the rod scans to try to determine whether significant differences in isotopic composition could be found. Estimates were made of the contribution to the counting rate coming from that part of the rod shadowed by the scanning slits.

Introduction

Scanning the nuclear radiation emitted by fuel rods has gained acceptance as a way of determining the uniformity of the loading and, to a lesser extent, the total amount of material present. Such apparatus can and has been used to indicate gross defects, e.g., missing pellets, or pellets of completely abnormal composition. Attempts to be quantitative have been made but with uncertain results, due to the difficulties involved in interpreting the strip chart recording in anything more than a semi-quantitative manner. A basic problem is that the strip chart recording, in the form of a continuous analogue signal, has folded into it such effects as the time constant of the analogue ratemeter, the recorder slewing response, and the random statistical fluctuations in the count rate, and these are difficult to disentangle from the response due to a possible non-uniformity in the fuel rod. A practical difficulty with the usual strip chart recording lies in the tediousness of extracting numerical information that one can use to do statistical analysis.

In a recent rod scanning exercise, the electronic equipment included a digital ratemeter rather than the usual analogue device. (1) This removed some of the above theoretical and practical difficulties and made data acquisition so (relatively) easy that an attack on a question of long standing was prompted;

namely, what quantitative statistical meaning can be placed on such scanning measurements.

The feature of digital ratemeters which facilitated the present measurements is that it sums up all the counts received in a given preset time interval while making available for readout the results of the previous time interval. The fact that the readout represents the sum of the counts over a finite time interval gets rid of most of the instantaneous fluctuations characteristic of analogue ratemeters without the necessity of introducing a large time constant, which might cause a defect in the rod to be obscured. In principle the counts per time interval (equivalent to the counts received from a corresponding length of rod) could be read out automatically with some printing device. In the present instance, the values were read out visually and recorded by hand; this did limit the total amount of data which could be taken.

The analysis of the data and its interpretation are tentative and is presented here on that basis in order to expose it to comment.

Description of the Experiment

Before discussing these results the actual apparatus will be described. The rod scanner is illustrated in Fig. 1. The rods themselves, consisting of sintered $\text{PuO}_2\text{-UO}_2$ pellets (~10g each, consisting of ~20% PuO_2 in depleted UO_2 , $\text{Pu}^{239}/\text{Pu} \sim .9$) in stainless tubing and having dimensions as indicated in Fig. 2, were held in an aluminum angle carrier and moved at a constant speed by a synchronous motor driven rack and pinion past a photomultiplier with an interposed 2 in.-thick lead collimator. The photomultiplier was shielded from the rest of the rod and from room background by additional shielding, not shown. The individual pellets were approximately $5/8$ " long by $7/8$ " in diameter, somewhat dished at each end, but otherwise cylindrical. The rod was moved at $1.2 \text{ in/min.} = 0.02 \text{ in/s.}$ The slit width was $\sim 3\text{mm} \sim .12 \text{ in.}$ Approximately $1/5$ of the length of a pellet moved past the slit (whose width was itself about $1/5$ of a pellet in length) in 6 seconds (the integration time chosen for the digital ratemeter). Thus, even aside from the effect of penumbra, there is a certain amount of "running averaging" as the scan proceeds, which tends to deemphasize any differences between neighboring pellets. This point will be of particular significance later.

The electronics used in this experiment are functionally

Figure 1

* CONSISTS OF INDIVIDUAL PELLETS $\frac{3}{8}$ LG.
NOTE: DIMENSIONS IN INCHES.

FUEL LOADING DISTRIBUTION

Figure 2

as indicated in Fig. 3. The equipment is essentially the same as has been used before with the same scanning apparatus but with the noted exception of the digital rather than analogue ratemeter. The single channel analyzer condition preceding the ratemeter was set to encompass both the 208 keV peak and the "380 keV complex," while the multichannel analyzer included the 60 keV Am^{241} and some X-ray peaks as well. As mentioned before, the digital ratemeter was set to sum up its input counts over successive 6 second intervals ($\sim .12$ in. of rod), while the multichannel analyzer accumulated spectrum from the whole fuel rod, being manually gated on and off in accordance with the strip chart trace indicating the beginning and end of the fuel rod. This latter operation could be done with a precision of $\sim 1/300$. A run indicated the background to be about (4-5)% in the vicinity of the 208 keV peak and the highest portions of the 380 keV complex.

The strip chart record reveals in a crudely quantitative way several features of the scanning apparatus which have a bearing on interpreting the statistical results to be discussed.

The general nature of the strip chart trace is shown in Fig. 4, which is of a dummy rod with one pellet deliberately replaced with a depleted UO_2 pellet. Except in the neighborhood of the dummy pellet, the trace seems flat except for

BLOCK DIAGRAM OF ELECTRONICS USED IN FUEL ROD SCAN

Figure 3

Figure 4

statistical fluctuations. Corresponding to the missing pellet the trace dips $(7.4-1.8)/(7.4-.2) = 78\%$ from its average level, and the full width at half maximum ($\sim .6''$) corresponds well to the length of a pellet. It is not clear from this alone whether the remainder above background is due to the rest of the rod shining through the shielding. Fig. 5a shows the trace of a normal fuel rod at the end of the loading; Fig. 5b shows the scan of the same rod near the central region where there is no fuel loading. Studying these two traces it is possible to estimate that the rest of the rod contributes something like $(4.2-3.7)/(4.2-.6) \sim 14\%$ to the count received from the pellet just in front of the slit. Judging from the way the contributions add together in the region of the central unloaded section, only about two inches of rod on either side of the slit contribute to the counting rate received from that part of the rod immediately in front of the slit.

For comparison with the above (which could be called "digital" strip chart recordings), Fig. 6 shows the more usual "analogue" strip chart scan of the same type of rod, in the neighborhood of the unloaded middle and with some deficient pellets. While not identical, the experiment resulting in this plot is quite comparable to the present one. (2) The difficulties in reducing this record to data suitable for analysis

(a) START OF LOADED SECTION (b) CENTRAL UNLOADED SECTION

SCAN OF ONE END AND THE MIDDLE OF ROD 016

Figure 6

is evident. Even the answer to the question of the number of deficient pellets would be uncertain because of the way statistical fluctuations in the count rate are tied in with the time constant of the response of the ratemeter and the recorder. If it were not for the fact that several deficient regions occur in the same rod even an intercomparison (which leads to estimates of 4, 2, and 1 deficient pellets) could not have been made.

Statistical Analyses

1. One Way Analysis of Variance

The digital ratemeter had a visual output which enabled the counts collected during each six-second count interval to be (manually) recorded for successive intervals of time over appreciable lengths of the fuel rod being scanned. The sequence of such numbers were divided into groups, each corresponding to a segment of fuel rod of equal length. Each measurement of a particular group was considered a sampling from that segment of the rod corresponding to the group (assuming each segment to be uniform in itself). The background, being small and constant was not subtracted. Thus the measurements from a segment of rod constitutes a sample which was compared with the samples corresponding to the other segments of the rod to test for uniformity along the rod. In making such comparisons, measurements of segments of the rod known a priori to suffer from various effects extraneous to the fuel loading itself were excluded: for example, the regions adjacent to the ends and the unloaded middle portion of the rod.

The test used to make this comparison was the "one way analysis of variance" (ANOVA) in which the variance of the sample means is compared to the mean of the sample variances. The hypothesis tested is that the k samples (segments of fuel rod),

each with n measurements (the six-second integrated counts) all have equal means. Although the procedure is predicated on the assumption that the samples are normally distributed and have equal variances, the method is fairly "robust" and insensitive to these conditions. (Actually there is no reason to suspect that the data doesn't meet these prerequisites anyway.)

Data mainly from the "standard" rod 016, but also from rods 005 and 023, were used. Since a contiguous sequence of approximately 125 measurements were made along the rod, the groupings into k samples containing n measurements each could be accomplished in several ways. Because 5 adjacent readings constitute a pellet width, one "natural" grouping consisted of 25 samples of 5 observations each, though actually, there wasn't any convenient way to precisely synchronize the count with the actual location of each pellet, and as indicated previously, there is a fair amount of "averaging in" of adjacent readings represented in any one measurement. The 5 by 25 and eight other partitions of the data shown in Table I were analyzed. The results of the ANOVA computation and the critical values of the f distribution at the 5% level of significance (95% confidence level) are also listed there. The "between variance" (the variance of the k sample means) divided by the "within variance"

Table I

ANOVA Results for Rod Scan Experiments
(one way classification of variance test)

<u>Rod</u>	<u>Sample size (n)</u>	<u>Number of samples (k)</u>	<u>Critical* value of f</u>	<u>Computed value of f</u>	
016	2	62	1.53	1.53	consecutive segments
016	3	41	1.56	1.99	
016	4	31	1.60	2.47	
016	5	25	1.64	3.06	
016	6	20	1.67	3.15	
016	7	17	1.76	4.05	
016	8	15	1.80	4.36	
016	9	13	1.85	4.28	every third segment
016	10	12	2.13	4.13	
016	6	7	2.38	6.82	
005	5	45	1.40	1.56	consecutive segments
023	5	43	1.42	1.83	

* for $\alpha = .05$ (95% confidence limits).

(which is the average variance of the n observations within each of the samples) gives the f factor, $f = (s_B^2/s_W^2)$. This value computed from the data was then compared with the theoretical value of the random variable having the so called f distribution with $(k-1)$ and $k(n-1)$ degrees of freedom at a confidence level of $(1-\alpha)$. If the means of all the samples were statistically equivalent (i.e., all segments of the rod essentially the same) then the calculated f would be smaller than that corresponding to the confidence level $(1-\alpha)$. It is clear however from Table I that (except for the $n = 2, k = 62$ case) the ratio of between variance to the within variance is always well into the critical region, and thus the segments into which the rod were divided cannot be considered to all have the same mean.

As mentioned in the previous section, the observations of count rates for adjacent points are not entirely independent, because of the effect of neighboring points contributing to the particular point being observed at any time. This effect can only serve to decrease the between variance compared to the within variance and reduce the computed value of f . An example illustrating this can be seen in the single case shown in Table I (7 samples of 6 observations each) where only every third reading from the digital ratemeter was used in the analysis. That is,

two intermediate observations between readings were discarded and the remaining observations were grouped into 7 samples in which the adjacent observations were essentially independent determinations. It is seen that the computed f value is further into the critical region for this case than in those cases where the consecutive readings correspond to physically adjacent points on the rod. Based on this consideration and the results shown in Table I it may be said that the standard rod is not composed of statistically homogeneous segments along its entire length.

As shown in Table I, similar tests were conducted on two other rods. On one rod the observations were partitioned into 45 samples of 5 observations each; on the other rod the data was grouped into 43 samples of 5 observations each. These results also indicate that these rods are similarly composed of segments whose means are different in a statistically significant sense.

2. χ^2 Test of Observed Variances

The successive readings taken in a rod scan contain whatever variability the rod may have mixed in with whatever variance is induced by the equipment; in the present situation, drift in the "electronics" was chiefly suspect. To study this effect, the rod transport was stopped at an arbitrary point on the rod and successive counts for a fixed time interval recorded. The average count rate for 40 observations was found to be 1176 while the variance in these observations was 1367. If counting statistics were solely involved (Poisson process) then the variance would be equal to the mean. It is thus tempting to attribute this difference to the contribution to the measured variance arising from instabilities in the electronics. However, the observed difference is not statistically significant since the χ^2 value computed from the two estimates of the variance was 45, and 95% of χ^2 values with $40-1=39$ degrees of freedom lie between 23 and 58.

3. The Multiple Range Test

The ANOVA test is indicative of whether there is inhomogeneity in the sample means, in this case each representing a segment of the rod, but it alone gives no indication of how many or which ones are not equal or by how much they fail to be equal. Duncan's multiple range test (DMR) will take a population

previously proven inhomogeneous (as has been done in this case by the ANOVA test previously described) and separate the samples into groups that are homogeneous within themselves, although not with one another, with some specified degree of confidence. The DMR test limits the difference allowed between any two sample means for them to belong to the same homogeneous population. This allowed range is a function of the sample variance, the desired level of significance, and the number of data points involved. The allowed range (the "least significant range") is in fact the so called "least significant studentized range" at the desired level of significance multiplied by the standard error of the means being tested for inclusion in the same subset. Using this range, a list of the sample means ordered as to size can be separated into subsets that are statistically speaking homogeneous within themselves.

Table II is a listing of the ordered sample means from rod 016, grouped as indicated by the brackets, assuming a confidence level of 95%.

Other groupings would result if different confidence levels were demanded, or if certain of the means were omitted from consideration. As an example of the latter point, if the lowest 4 means are discounted, all of the data can be reduced to only two internally homogeneous subsets. If, additionally, the two highest means are discarded, the means

Table II
Duncan's Multiple Range Test Applied to Rod 016

<u>Segments in order of rank</u>	<u>Average over segment</u>	<u>Grouping and means within groups</u>
1	1087	} 1115
2	1112	
3	1116	
4	1127	
5	1133	} 1138
6	1139	
7	1140	
8	1143	
9	1144	} 1147
10	1145	
11	1148	
12	1148	
13	1150	} 1158
14	1150	
15	1165	
16	1166	
17	1167	} 1163
18	1167	
19	1179	
20	1179	
21	1180	
22	1182	
23	1184	
24	1189	
25	1191	

for the remaining 19 samples (segments) can be considered equal at the 95% confidence level. Discarding only the one lowest mean yields 4 subsets. While the first two examples mentioned above were for the sake of illustration, the last has a valid point to it, in that the segment giving rise to this low value happens to be closer to the spacer in the rod than the others in the set, and thus might be expected to indicate low. (No other correlations with position were noticed.) The 24 remaining means then fall into 4 self-homogeneous subsets. The value of the highest mean of these four subsets is 2% higher than the mean of the lowest subset.

Thus the combination of ANOVA and DMR tests enable a reasonably quantitative statement to be made about the uniformity of this particular fuel rod: A scan of about half the fuel loading indicates that with 95% assurance, the rod was not homogeneous to the extent that the 24 segments that this part of the rod was divided into, could be lumped into four self-homogeneous groups such that the difference in means between the highest and lowest was about 2%. Something about pellet homogeneity could be inferred from this. Unfortunately the present apparatus did not allow a direct correlation between pellet identity and position along the rod, otherwise a stronger statement would be possible.

4. Attempt to Find Periodicities in the Scan

As previously mentioned the pellets are known to have slightly dished ends so that in principle it might be expected that a periodicity show up in the counts received during the six second count intervals, namely, every fifth reading should be somewhat depressed. In practice, it was expected that self-shielding might mask the effect at the level of counting statistics used (~1200 cts/6 sec); however, because of curiosity the attempt was made anyway.

An autocorrelation function of the form

$$C(\tau) = \sum_1^{n-\tau} f(t)f(t+\tau)$$

was used. Here, $f(t)$ is the counting rate at position (six second count interval) t along the rod, and τ , the "lag," is a shift τ time intervals along the rod. Considering C as a function of τ , relative maxima are expected to occur at values of τ which are integral multiples of any periodicity in the function $f(t)$; in the present instance a periodicity of 5 might be expected.

The function $C(\tau)$ was computed on the basis of some data points obtained from rod 016; 100 values were used to compute C for lags of 0 through 10, each sum always generated by the same number of terms. The 11 sums thus obtained were

averaged and the deviations from this average, $C(\tau) - C(\tau)_{ave}$, plotted as a function of lag (Fig. 7, curve 1). There is no evidence of any periodicity. To estimate how much of an effect could have been detected if present, percentages of 10% and 20% were deducted from every fifth count and the calculation $C(\tau)$ repeated, giving curves 2 and 3, respectively. Deducting 10 and 20%, respectively, for every third count resulted in curves 4 and 5. The periodicities of 5 and 3 artificially induced in this way are clearly visible, indicating that a periodic deficiency (or enhancement) of as little as 10% and probably appreciably less could have been detected in this way. That no periodicity was detected presumably is due to both the self-shielding, and the lack of sufficient statistical precision on the individual measurements compared to the total number available for the analyses.

5. Statistical Analysis of the Total Rod Spectrum

The multichannel analyzer (MCA) collected a spectrum characteristic of each whole rod scanned. A spectrum from the standard rod 016 run 2, shown in Fig. 8 is typical, yet there were seeming differences between other runs of the same rod and of different rods. To decide whether these differences were real, a preliminary data reduction for the net counts in the 60 keV Am^{241} , the 208 keV U^{237} , the 380 keV complex,

$$C(\tau) = \sum_{t=0}^{n-\tau} f(t)f(t+\tau); f(t) \text{ IS THE CT. RT. AT POSITION } t$$

(1) UNTOUCHED DATA

(2), (3) EVERY FIFTH $f(t)$ REDUCED BY 10%, 20%

(4), (5) EVERY THIRD $f(t)$ REDUCED BY 10%, 20%

Figure 7

Figure 8

and the 208 keV + 380 keV complex was made for five of the runs, as listed in Table III.

Using the 208 keV or the 380 keV complex, or both (as a gross count) as indications of Pu content, there is seen to be no reason to suspect any differences in total Pu content; especially considering that there was, as mentioned previously, a 1/300 uncertainty in the timing of the MCA runs.

However this cannot be said for the differences in Am^{241} content that are revealed by analysis of the total counts in the 60 keV Am^{241} peak. The statistical question posed was whether the average of the 60 keV Am^{241} peaks in the three runs made with 016 was significantly different from the 60 keV results with rods 023 and 005. The use of a Student t test indicated that this average was indeed different than the values for the other two rods for a two-tailed test at the 95% confidence level. (At the 99% confidence level, however, this average was significantly different from that of rod 023 only.) The t test was mandated by the small amount of data available and was implemented by comparing in turn the measured values for rods 023 and 005 with the mean of the three measurements of 016. Since there were only 3-1=2 degrees of freedom, the allowed deviations were quite large---yet even these were exceeded.

Table III
STATISTICAL ANALYSIS OF COUNTS IN PEAKS

Run No.		2	3	4	5	6
		016	016	023	005	016
(208 + 300) keV (complex)	gross	383,063	383,231	382,994	384,359	384,065
	rel. error	.00162	.00162	.00162	.00161	.00161
	abs. error	618.9	619.1	618.9	620.0	619.7
208 keV	gross	126,811	126,775	126,740	128,411	127,894
	back.	76,461	76,577	74,372	76,419	74,529
	net	50,350	50,198	52,378	51,992	53,365
	rel. error	.00895	.00898	.00856	.00870	.00843
	abs. error	450.6	450.8	448.4	452.3	449.9
300 keV (complex)	gross	201,038	201,036	200,801	201,046	201,333
	back.	99,093	95,408	98,222	98,858	95,643
	net	102,045	105,628	102,589	102,188	106,690
	rel. error	.00537	.00515	.00533	.00536	.00511
	abs. error	548.0	544.0	546.8	547.7	545.2
60 keV	gross	798,546	804,456	777,142	794,636	805,719
	back.	222,631	225,330	223,052	223,408	229,509
	net	575,915	579,126	554,090	571,228	576,210
	rel. error	.00175	.00175	.00180	.00177	.00177
	abs. error	1007.9	1001.9	997.4	1011.1	1019.9

This result, indicating differences in Am^{241} content was surprising since the material for the pellets was known to come from one master blend. The explanation may be that while the original feed material was thoroughly mixed, the precipitation of the oxide from the nitrate solution was carried out in separate batches, and that small differences in some process parameter caused somewhat different amounts of Am^{241} to be carried down with the precipitate. A simpler explanation is simply the lack in uniformity of wall thickness of the cladding from one rod to the next. The 60-keV peak intensity is, of course, very sensitive to differences in wall thickness.

Use of a high resolution Ge detector would make a more detailed isotopic analysis possible but the trade-off in counting rate might be too severe to be practical.

Summary

The basic point of this paper is the desirability and even necessity of applying statistical analysis to the data resulting from fuel rod scans in order to extract the maximum amount of meaningful information. Some methods are tentatively proposed (and data from a real industrial situation used as an example) to stimulate discussion of this subject.

A method (ANOVA-DMR) has been proposed to enable rod scanning data to be used to determine the degree of homogeneity of fuel rod loadings, thus augmenting the usual qualitative or semi-quantitative inspection of the strip chart recording. The high statistical precision of the MCA total rod spectrum can be used not only to measure total fuel loading but also to detect certain isotopic differences, possibly of interest for process control if for no other reason. The possibility of using an autocorrelation technique to search for imperfections or artifacts of a periodic nature in the fuel rod is indicated although this probably is of limited utility.

Finally, experimental data has been used to illustrate the estimation of the effectiveness of the slits in defining the portion of the fuel rod being scanned.

Acknowledgements

We wish to acknowledge the cooperation of NFS, and in particular Messrs. John Crawford and Ralph Jones, who enabled us to make certain of the referred-to measurements.

Footnotes

- (1) The basic equipment was as assembled by Mr. John Crawford, of NFS, Erwin.
- (2) This recording of a defective fuel rod was not taken during the present exercise, but stems from a much older experiment.
- (3) The statistical tests to be described here are standard and can be found in many sources; the authors have found the simple treatment in "Introduction to Statistics" by R. E. Walpole, Macmillan (1968) to be convenient.