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MASTER

ANISOTROPIC PHOTOELASTIC ANALYSIS OF
FIBER REINFORCED COMPOSITE MATERIALS

Charles Eugene Knight, Jr.

UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT

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ANISOTROPIC PHOTOELASTIC ANALYSIS OF
FIBER REINFORCED COMPOSITE MATERIALS

Charles Eugene Knight, Jr.

This report is based on a study by the author as partial fulfillment of requirements for the degree of Master of Science in Engineering Mechanics from The University of Tennessee.

Oak Ridge, Tennessee

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ABSTRACT

A theoretical and experimental investigation was made of the photoelastic effect in a transparent anisotropic material. The material is a fiber glass reinforced epoxy resin, and vacuum impregnation techniques were developed to render the material transparent. Isochromatics and isoclinics in the material when subjected to both uniaxial and biaxial stresses were observed and compared to the predicted data. Isochromatic data proved to be representable by an orthotropic stress-optic law. Isoclinics were predictable, but at a given point in a stressed model the isoclinic angle depends on both the angle of principal stress directions and the ratio of principal stress magnitudes. Thus, the isoclinics do not directly give the principal stress directions. At this point, the anisotropic photoelasticity solution cannot be determined.

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CHAPTER I

INTRODUCTION

One of the newest engineering materials in wide use is fiber glass reinforced plastics. Its most obvious advantages are: high strength and a high strength-to-weight ratio. However, another distinct advantage is its ability to be tailored structurally to meet strength and rigidity requirements. This basic anisotropy of the material also makes it more difficult to design and analyze a structure accurately.

Fiber glass reinforced plastics can be made optically transparent by special fabricating techniques. By the use of a special glass and clear plastics, the structure can be made almost water clear. This property makes the use of photoelasticity practical for the analysis of fiber glass reinforced plastic structures during their loading. However, there exists both mechanical and optical anisotropy in this type of material which makes the analysis very difficult.

The objectives of this study were: (1) to develop a transparent, anisotropic photoelastic material from a fiber glass reinforced plastic; (2) evaluate the photoelastic effects in this material, and (3) to solve anisotropic stress distributions in a model.

In order to obtain transparency, the composite must be free of air bubbles. This condition was accomplished by proper vacuum impregnation techniques of the

fiber glass roving with a certain choice of plastic resins. Also, a close match of the indices of refraction between the fiber and resin was desirable for optimum transparency. The photoelastic evaluation concerned both the isochromatic and isoclinic patterns. In order to understand the optical behavior of this type of material, an analysis of a chosen mathematical model of the fiber-resin composite was made. First uniaxial and then biaxial loading tests were performed to establish the optical behavior. The experimental isochromatics and isoclinics were compared with those predicted from the mathematical model, and fair agreement was obtained. From these photoelastic data, an assumed stress-optic law was essentially verified. The main objective was the solution of a stress problem in an anisotropic model, but this was not realized in this study.

This program represents an attempt to extend the photoelastic method to deal with anisotropic materials. At this time a complete photoelastic analysis on anisotropic materials cannot be made because the observed isoclinic does not represent the principal stress directions. However, it is felt that significant progress has been made on predicting the optical behavior and on the verification of a proposed stress-optic law for anisotropic materials.

CHAPTER II

THEORETICAL BACKGROUND

The concept of fiber reinforced materials is not new, but the recent availability of ultra high-strength fibers has brought about a new interest in their design and application. When fibers are incorporated into a matrix which is of lower strength and stiffness, a material is formed which has the desired anisotropic strength and stiffness properties. Thus, the stress distributions derived for isotropic materials will not apply to fiber-reinforced anisotropic materials. A plot of the typical properties for a fiber glass-epoxy composite is presented in Figure 1. As seen in the plot, there is a large variation in the strength properties depending on the fiber orientation. This fact makes it imperative that stresses be distributed with the maximum stress in the fiber direction.

Several investigators have used constituent material properties to predict the overall elastic properties of the composite. Axial properties, modulus of elasticity, and major Poisson's ratio have been predicted by a "law of mixtures" relation by nearly all investigators. The transverse and shear stiffness have been treated in a variety of ways. A strength-of-materials approach was taken by Shaffer.^{1*} He used a series-parallel arrangement of equivalent bars of matrix

*Numbers refer to citations in the References, Page 75.

FIGURE 1

VARIATION OF STRENGTH AND STIFFNESS
WITH FIBER ORIENTATION

and fiber to predict the transverse stiffness. Hashin and Rosen² used variational principles of the theory of elasticity to establish the upper and lower bounds on the elastic properties. Tsai³ introduced a contiguity factor which related the upper and lower bounds with a linear equation in order to correlate the experimental data with the theoretical equations. Recent work by Pickett⁴ and Adams, et al⁵ has been directed toward an elasticity solution of the multiple inclusion problem. One of the models, shown in Figure 2, is chosen and the elasticity equations are written for a typical element. Numerical techniques are used to solve the equations. Very good experimental verification has been obtained for this approach. In general, methods for predicting elastic constants are well under control.

For the analysis of composite materials, the only difference in the elasticity solutions is Hooke's law. Hooke's law expressed in indicial notation is:⁶

$$\sigma_{ij} = C_{ijkl} e_{kl} \quad , \quad (1)$$

and

$$e_{ij} = S_{ijkl} \sigma_{kl} \quad , \quad (2)$$

where:

σ_{ij} represents the stress tensor,

e_{ij} the strain tensor,

S_{ijkl} the compliance tensor, and

C_{ijkl} the stiffness tensor.

FIGURE 2

FIBER-MATRIX MODELS FOR ELASTICITY SOLUTION OF TRANSVERSE
AND SHEAR STIFFNESSES

The fourth-rank stiffness and compliance tensors have, for the most general case, 81 components; but, by symmetry, these reduce to 21 independent elastic constants in the general case. From material symmetry considerations for the fiber reinforced systems where three axes of symmetry exist, the number reduces to nine. This substance is known as an orthotropic material. Of course, for the isotropic material they reduce to two.

In this thesis, two-dimensional states of stress are of primary interest. In the case of plane strain or plane stress, either the stress or strain in the third dimension is dependent upon the properties of the two-dimensional system. Then, the number of elastic constants reduces to four. The stress-strain relations are given in the equations that follow for the plane stress case which is being studied here.⁶

$$\begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix} = \begin{bmatrix} E_{11}/(1-\nu_{12}\nu_{21}) & \nu_{21}E_{11}/(1-\nu_{12}\nu_{21}) & 0 \\ \nu_{21}E_{11}/(1-\nu_{12}\nu_{21}) & E_{22}/(1-\nu_{12}\nu_{21}) & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \begin{bmatrix} e_{11} \\ e_{22} \\ e_{12} \end{bmatrix}, \quad (3)$$

and

$$\begin{bmatrix} e_{11} \\ e_{22} \\ e_{12} \end{bmatrix} = \begin{bmatrix} 1/E_{11} & -\nu_{21}/E_{22} & 0 \\ -\nu_{12}/E_{11} & 1/E_{22} & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}, \quad (4)$$

where:

G_{12} represents the shear modulus,

- E_{11} the axial stiffness (parallel to fiber),
 E_{22} the transverse stiffness (transverse to fiber),
 ν_{12} the major Poisson's ratio (transverse contraction due to axial extension), and
 ν_{21} the minor Poisson's ratio (axial contraction due to transverse extension).

These relations are for a state of stress aligned with the axes of material symmetry. When the stiffness or compliance tensor is transformed to some off-axis orientation, terms will appear in the matrix where the zeroes are in the present matrix. These terms are known as shear coupling coefficients.

The orthotropic stress-strain relations are incorporated into the equations of elasticity. The governing equations are then combined and reduced to a single equation using the familiar Airy's stress function:⁷ (neglecting body sources),

$$\begin{aligned}
 &\beta_{22} \frac{\partial^4 F}{\partial x^4} - 2\beta_{26} \frac{\partial^4 F}{\partial x^3 \partial y} + (2\beta_{12} + \beta_{66}) \frac{\partial^4 F}{\partial x^2 \partial y^2} - 2\beta_{16} \frac{\partial^4 F}{\partial x \partial y^3} + \\
 &\beta_{11} \frac{\partial^4 F}{\partial y^4} = 0
 \end{aligned} \tag{5}$$

where:

- F represents the Airy's stress function,
 $\beta_{11} = 1/E_{xx}$,

$$\beta_{12} = -\nu_{xy}/E_{xx},$$

$$\beta_{22} = 1/E_{yy}, \text{ and}$$

$$\beta_{66} = 1/G_{xy}.$$

β_{16} and β_{26} are the shear coupling coefficients.

The elastic properties referred to the x-y axes are obtained by transformation from the 1-2 material symmetry axes. This relationship may be compared to the field equations in Airy's stress function form for isotropic materials; (neglecting body forces)

$$\nabla^4 F = 0. \quad (6)$$

As can be seen, the elasticity solutions for isotropic materials are independent of the material properties, while the solutions for orthotropic materials are very much dependent on material properties.

The stress-optic law in two-dimensional photoelasticity has been expressed by Frocht⁸ as:

$$(p-q) = fN/t, \quad (7)$$

where:

p, q represent the principal stresses at the point under consideration,

f the normal stress material fringe value,

N the fringe order, and

t the model thickness.

This equation is experimentally verified for isotropic transparent materials by an observation of the specimens under known states of stress. The general stress-optic law is derived by Maxwell⁹ from analogy to the stress-strain relations for the material. Expressing them in three dimensions gives:

$$e_1 = \frac{1}{E} [\sigma_1 - \nu(\sigma_2 + \sigma_3)], \quad (8)$$

and, similarly, for e_2 and e_3 . The elongation can be related to strain by:

$$\frac{l_1 - l_0}{l_0} = e_1, \quad (9)$$

where:

l_0 and l_1 represent original and final lengths of an infinitesimal element.

From Equations 8 and 9 we obtain:

$$l_0 e_1 = l_1 - l_0 = \frac{l_0}{E} \sigma_1 - \frac{\nu l_0}{E} (\sigma_2 + \sigma_3). \quad (10)$$

By analogy, the difference of the indices of refraction can be expressed as:

$$n_1 - n_0 = c_1 \sigma_1 - c_2 (\sigma_2 + \sigma_3), \quad (11)$$

(Similarly, relations were also used for $n_2 - n_0$ and $n_3 - n_0$.)

where:

n_1 , n_2 , or n_3 represent the principal indices of refraction in a stressed transparent material, and

n_0 the index of refraction in unstressed material.

c_1 and c_2 represent the stress optical constants.

For two-dimensional stress conditions we have:

$$n_1 - n_0 = c_1 \sigma_1 - c_2 \sigma_2, \text{ and} \quad (12)$$

$$n_2 - n_0 = c_1 \sigma_2 - c_2 \sigma_1. \quad (13)$$

By subtraction, we obtain:

$$n_1 - n_2 = (c_1 + c_2) (\sigma_1 - \sigma_2) = c(\sigma_1 - \sigma_2), \quad (14)$$

which is similar to the form of Equation 7 for two dimensions. This derivation of the stress-optic law will be used later to develop an orthotropic stress-optic law.

CHAPTER III

MATERIAL AND MODEL FABRICATION

The primary requirement for a photoelastic material is that it be transparent. Various materials have been used in photoelastic research, namely: glass, celluloid, bakelite, and polyester and epoxy resins. The recently developed polyester and epoxy resins are the most commonly used materials today. All these materials are essentially isotropic and, as such, are used to analyze the stress distribution in isotropic materials. Most of the composite materials coming into use are highly anisotropic; therefore, the photoelastic model used for the analysis of such materials should have similar material properties.

One of the most popular composite materials has two transparent materials as its components—fiber glass and a plastic resin, either polyester or epoxy. This composite, however, is normally not transparent due to two factors: First, it is most difficult to completely impregnate the fiber glass with resin. (Usually minute air bubbles are left in the composite during the impregnation process, making the product completely opaque. However, special impregnating techniques have been developed which eliminate air bubbles.) Second, the difference in the index of refraction between fiber and resin. The fibers are very small (0.00035" dia.) and, therefore, the optical path through the thickness of a structure must pass through

both the fiber and resin elements a large number of times. Because of this requirement, even small differences in the refractive indices will cause the material to be translucent. If the indices of refraction of the two components can be matched, the transparency will be further improved.

Not all fiber glass materials have the same degree of transparency. One type that is particularly good is W-1 glass manufactured by Owens-Corning Fiberglas, Inc. This material is produced in roving form with either a polyester or epoxy compatible finish. The manufacturer's numbers of the finish are P678 for polyester and K895 for epoxy compatibility. The glass is a modified "E" glass, but the modification is so slight it retains most of the physical properties of "E" glass listed in Table I.

Many composite-material structures are made up of unidirectional composite elements. The elements are made by aligning the fibers all in one direction within the resin matrix. The roving of W-1 glass contains 2,040 filaments. The process is shown schematically in Figure 3. Roving is pulled from the spool through the impregnating tank onto the rotating mandrel. The roving spool is mounted on a tensioning device consisting of a prony brake arrangement on the rotating shaft. The vacuum impregnating tank is a special system designed to impregnate the roving without air bubbles. First, the chamber is filled with resin to a level that covers about half of the dip pulley. The roving enters the tube through a slitted bushing and

TABLE I
PHYSICAL PROPERTIES OF W-1 FIBER GLASS

Modulus of Elasticity	10×10^6 psi
Index of Refraction	1.549
Specific Gravity	2.54
Poisson's Ratio	0.22
Fiber Tensile Strength	450,000 - 500,000 psi

FIGURE 3

MATERIAL FABRICATION PROCESS

air is evacuated as the roving bundle travels toward the resin dip. (A vacuum of -25 inches of mercury or better has proven to be sufficient for complete impregnation.) During the roving dip, the epoxy impregnates the bundle and excessive resin is removed by the wiping rods. As the roving leaves the vacuum tank, any lack of impregnation is filled due to the atmospheric pressure on the evacuated voids. The roving is then wound on a flat-plate mandrel with a side-by-side lead. The winding process also entraps air between the layers. This air is removed by evacuating the winding in a bell jar preferably between the winding of each additional layer. Mylar polyester film is used on the mandrel and on the outer surface of the finished winding. A flat plate is lightly clamped on the surface to give a smooth finish and the part rotates until the resin is gelled. The Mylar film is stripped when the material is separated from the mandrel after curing.

This vacuum impregnation process is limited primarily by the roving speed and the pot life of the resin system. The highest winding speed was limited to approximately 10 feet per minute for proper impregnation using the low-viscosity resins in this study. Several different resin systems were tried. The main requirements were a long pot life and a low viscosity. Another requirement was imposed during this study. Residual stresses due to differences in the thermal expansion coefficients between glass and resin demanded a room-temperature curing system. As mentioned before, the resin should have the same index of refraction as the glass.

The best match of the indices of refraction was obtained with the Maraglas 655 epoxy heat curing system, manufactured by the Marbelette Corporation. The systems used during this study were room-temperature curing resins, Maraglas 656 epoxy, and Epon 815 (manufactured by the Shell Chemical Company)/TETA/ally glycidal ether (100/12/2.5) epoxy. The pot life of these two systems is approximately 1 1/2 hours. This period usually provides only sufficient time to wind a six-inch by six-inch plate to 0.125 inch in thickness. The impregnation is good in these systems, but the difference in the refractive indices makes it translucent. Since the material is relatively insensitive photoelastically, two or more of these plates can be bonded together to make thicker specimens. Because of the translucence of the specimen, the light transmission was not high enough to use the photometer on hand to determine the fringe orders; all data had to be taken by visual observation.

Specimens used in this study were fabricated by the use of a high-speed router. Figure 4 shows the mounted router along with a template and a specimen. The router head rotates at 25,000 r.p.m. A guiding-pin hole was bored in the base plate concentric with the router. Guide pins are inserted in the hole to guide the template. A range of pin diameters allows successive cuts to be made on the specimens to remove a small amount of material with each cut. Figure 5 presents a schematic of the specimen machining process. The specimen was first roughed out with a band saw to within 1/8 inch of the finished contour, then mounted on the

FIGURE 4

ROUTER, SPECIMEN, AND TEMPLATE

FIGURE 5

SCHEMATIC OF THE SPECIMEN MACHINING PROCESS

template with doublefaced tape and hand fed into the rotating cutter. Tungsten carbide cutters were used exclusively because of the extreme abrasiveness of the fiber glass composite. Extreme care and a solid grip of the specimen is necessary to prevent the specimen edges from splintering. Also, when cutting into the "grain" at an angle, the cutter will have a tendency to grab the specimen. A combination of both a light cut and a slow material feed is the best preventative for these problems. No evidence of machining stresses has been found in any specimens so prepared.

CHAPTER IV

FRINGE VALUE DETERMINATION AND ISOCLINICS FOR UNIAXIAL LOADING

The first step toward understanding the optical behavior of fiber glass reinforced epoxy is to study specimens under a uniaxial loading. A simple analysis has been performed for the case of uniaxial loading on various fiber orientations. The analysis was performed on the mathematical model shown in Figure 6. The model consists of two parallel bars, one of fiber, and the other of matrix. The widths of the bars are taken as 1/2 of unity each. The thickness of the fiber bar is $2 A_f$, and of the matrix is $2 A_m$, where A_f , A_m are the fiber and matrix area fractions. This condition gives a total cross sectional area of unity. Since the lengths of bars are equal, the area fraction equals the volume fraction, V .

For the case of loading parallel to the fiber, the basic assumption is that both the fiber and the resin are strained equally, leading to the compatibility equation:

$$\frac{\sigma_{fa}}{E_f} = \frac{\sigma_{ma}}{E_m} , \quad (15)$$

where:

σ_{fa} represents the fiber stress parallel to the fiber,

FIGURE 6

MODEL OF A FIBER-MATRIX COMPOSITE FOR PREDICTING
OPTICAL BEHAVIOR

E_f the fiber modulus of elasticity,

σ_{ma} the matrix stress parallel to the fiber, and

E_m the matrix modulus of elasticity.

The equilibrium equation then leads to:

$$\sigma_{fa} (2A_f) (1/2) + \sigma_{ma} (2A_m) (1/2) = \sigma_a , \quad (16)$$

where:

σ_a represents the applied stress parallel to the fiber.

Solving these two equations simultaneously yields:

$$\sigma_{fa} = \frac{\sigma_a}{[V_f + E_m (1 - V_f)/E_f]} , \text{ and} \quad (17)$$

$$\sigma_{ma} = E_m \sigma_{fa} / E_f . \quad (18)$$

For transverse loading, the average applied load is assumed to transmit in series through the fiber and the matrix elements. The stress is proportionately higher or lower in matrix and fiber depending on the volume fraction of each element in the composite. Then, the transverse stresses are:

$$\sigma_{ft} = \sigma_t / 2A_f , \quad (19)$$

$$\sigma_{mt} = \sigma_t / 2A_m = \sigma_t / 2(1 - A_f) , \quad (20)$$

where:

σ_{ft} represents the transverse fiber stress,

σ_{mt} the transverse matrix stress, and

σ_t the applied transverse stress.

The shear load is assumed to be uniform within each element with a perfect bond at the interface. The stress is proportionately higher or lower in each element, similar to the transverse loading case. These stresses are:

$$\tau_f = \tau / 2A_f, \text{ and} \quad (21)$$

$$\tau_m = \tau / 2A_m = \tau / 2 (1 - A_f), \quad (22)$$

where:

τ represents the applied shear stress,

τ_f the fiber shear stress, and

τ_m the matrix shear stress.

Now, consider a specimen with a fiber orientation angle, θ , under uniaxial applied stress, σ , as shown in Figure 7. (The average applied stress is assumed to be a uniaxial stress with constant magnitude.) Then, the transformation equations can be used to resolve the uniaxial stress into components parallel and transverse to the fiber and interfiber shear stresses. Equations 17 through 22 are used to calculate the state of stress in both the fiber and matrix. The stresses are converted to fringe orders by use of the appropriate fringe value for fiber and matrix. Then, the optical effect of the three stress components in each state of stress can be added algebraically, since they are parallel, to give the total optical effect at the point in the specimen. Since the optical effect is given in terms of fringe orders

FIGURE 7

UNIAXIAL LOADING OF COMPOSITE SPECIMEN AT
A θ FIBER ORIENTATION ANGLE

or relative retardation, the normal stress fringe value of the composite can be computed as:

$$f_{\theta} = \frac{\sigma t}{N}, \quad (23)$$

where:

f_{θ} represents the normal stress fringe value at the fiber orientation angle, θ ,

σ the uniaxial stress,

t the model thickness, and

N the relative retardation in fringes.

The isoclinic angle of the optical effect can also be obtained by the transformation equations. Of course, the principal stress direction is at 0 degree, but the optical isoclinic, as determined by the computations just derived, is variable with the fiber orientation angle, θ .

The analysis just given is not intended in any way to be a rigorous treatment of the multiple inclusion elasticity problem, it is only presented as an approximate and practical method to predict the optical observations when loading the composite specimens in the polariscope. Although the treatment is not rigorous, the stresses calculated from the mathematical model should be close to the average stresses in each component of the composite. Since the optical effect in the polari-

scope is an averaging process itself, the predictions should at least approximately agree with the experimental data.

Compression tests were carried out on fiber glass reinforced Maraglas 656 resin specimens (1/2 inch in width and 2 inches long) fabricated by the aforementioned machining techniques. Specimens were cut at a 0, 22 1/2, 45, 67 1/2, and 90-degree fiber orientation angle, θ , as shown in Figure 7. The normal calibration procedure used on isotropic photoelastic materials was followed to determine the material fringe value at each fiber orientation angle. The data for these specimens are plotted in Figure 8, along with the theoretical data generated from the computations based on the mathematical model. Further tests were performed on one-inch-square specimens fabricated from fiber glass reinforced Epon 815 resin which were also used for the biaxial loading. Theoretical and experimental data for the one-inch-square specimens are plotted in Figure 9. Data on the physical properties of the fiber and matrix used for the computations are listed in Table II for the two resin systems used in this study. In Figure 10, the theoretical and experimental data for the isoclinics versus fiber orientation angle are plotted. The principal strain directions determined from the orthotropic stress-strain relations and computed elastic constants are also plotted in Figure 10. It can be observed from the curves that the correlation between the theoretical and experimental data is not perfect. However, the theoretical analysis does predict the optical behavior

FIGURE 8

NORMAL STRESS FRINGE VALUE FOR UNIAXIAL
LOADING OF FIBER GLASS REINFORCED
MARAGLAS 656 RESIN

FIGURE 9

NORMAL STRESS FRINGE VALUE FOR UNIAXIAL
LOADING OF FIBER GLASS REINFORCED
EPON 815 RESIN

TABLE II
DATA ON THE PHYSICAL PROPERTIES
OF COMPOSITE MATERIALS

<u>W-1 Fiber Glass Properties</u>	
Modulus of Elasticity	10×10^6 psi
Normal Stress Fringe Value	1200 psi/fringe/in.
Poisson's Ratio	0.22
<u>Maraglas 656 Epoxy Resin Properties</u>	
Modulus of Elasticity	0.5×10^6 psi
Normal Stress Fringe Value	50 psi/fringe/in.
Poisson's Ratio	0.35
Volume Per Cent Fiber in the Composite	40.3
<u>Epon 815 Epoxy Resin Properties</u>	
Modulus of Elasticity	0.5×10^6 psi
Normal Stress Fringe Value	80 psi/fringe/in.
Poisson's Ratio	0.35
Volume Per Cent Fiber in the Composite	44.6

FIGURE 10

OPTICAL ISOCLINICS FOR UNIAXIAL LOADING OF FIBER
GLASS REINFORCED EPON 815 RESIN

of the composite to an extent such that the curves are of a similar shape. Correlation of the material fringe value is off by approximately a factor of two. Perhaps a refined mathematical model would improve this correlation. The theoretical and experimental isoclinics have fairly good agreement. Correlation between the experimental isoclinics and the principal strain directions is not as good as the theoretical isoclinics.

CHAPTER V

BIAXIAL TESTS

The tests in uniaxial loading provide an insight into the averaging effect photoelastic behavior of the composite; however, in the majority of practical cases, the state of stress is expected to be biaxial. In order to evaluate the realistic optical effect, specimens must be subjected to biaxial stresses.

The analysis developed in Chapter IV can be easily extended to biaxial loading by using the proper transformation equations. Then, by similar methods, the theoretical material fringe value and isoclinics can be predicted.

To load the specimens biaxially, there must be two independent loading systems: For the vertical load, the loading frame in the polariscope was used; the horizontal load was provided by an air cylinder. The biaxial loading arrangement is shown in Figure 11. The available air system provided a pressure of 70 p.s.i. and, with a regulator, the load can be controlled. A four-inch air cylinder was used which provided a loading capacity of 900 lbs. A strain-gage load cell was made by bonding two biaxial strain-gage rosettes to the loading rod surface and using a BLH strain indicator to read the load. The load cell was calibrated with a Dillon force gage.

FIGURE 11
BIAXIAL LOADING SYSTEM

Application of the load to the specimen was transmitted by steel loading blocks. The block has a V groove on the back side so that the force was transmitted through a pin lying in the groove, thus eliminating bending stresses. Latex rubber pads, 0.020 inch thick, were bonded to the specimen edges so that local irregularities on the specimen could not influence the loading uniformity.

The relative retardation data will be discussed in the next chapter. If the data were plotted with fringe order versus $(p-q)$, then the resulting material fringe value can have any value between zero and infinity depending on the biaxial stress ratio. Because this material has a material fringe value which depends on the fiber orientation angle in uniaxial loading, the apparent fringe value is zero when $(p-q)$ equals zero; it is infinity when $(p-q)$ has some magnitude and the fringe order is zero.

Isoclinics for the cases of biaxial loading for a given fiber orientation also depend upon the ratio of the two loads. The analysis developed in Chapter IV is used to predict the optical isoclinic. In Figure 12, the theoretical curve for the isoclinic angle is plotted along with the experimental points. The values of the abscissa vary from the cases of vertical uniaxial loading to horizontal uniaxial loading. The isoclinic angle is measured from vertical with the positive angle taken in the same direction as the fiber orientation angle. This plot is for a fiber orientation of 15 degrees. Figure 13 shows a similar plot for a 30-degree fiber

FIGURE 12

OPTICAL ISOCLINICS FOR BIAxIAL LOADING OF FIBER GLASS
REINFORCED EPON 815 AT A 15-DEGREE FIBER ORIENTATION

FIGURE 13

OPTICAL ISOCLINICS FOR BIAXIAL LOADING OF FIBER GLASS REINFORCED
EPON 815 AT A 30-DEGREE FIBER ORIENTATION

orientation. The principal strain directions are also plotted in these figures. In the case of 0-degree fiber orientation, the optical isoclinic is also 0 degrees both theoretically and experimentally for all biaxial load ratios. Thus, the 0-degree isoclinic in the structure will correspond to a 0-degree principal stress direction when the fiber is oriented at 0 degrees. The experimental data verify the computed curves in the two cases of a 15-degree and 30-degree fiber orientation. The theoretical optical isoclinics of all fiber orientations and a spectra of the biaxial load ratios between the two extreme cases were calculated and plotted in Figure 14. Optical isoclinics are plotted using the fiber orientation with respect to the principal stress directions as the independent variable and the principal stress magnitude ratio as a parameter. Curves for the principal strain directions are similar. A fair correlation with the experimental data is obtained for both the computed isoclinics and the principal strain direction. However, in either case both the isoclinic angle and the principal stress ratio must be known to determine the principal stress direction.

FIGURE 14

OPTICAL ISOCLINICS VERSUS FIBER ORIENTATION FOR THE
WHOLE RANGE OF BIAxIAL STRESS RATIOS AT A
0-DEGREE MECHANICAL ISOCLINIC

CHAPTER VI

ORTHOTROPIC STRESS-OPTIC LAW

Since the material has a different optical sensitivity in different directions, the relative retardation produced by the same state of stress, uniaxial or biaxial, depends on the fiber orientation. The procedure for the derivation of the general stress-optic law in Chapter II (which was based on the isotropic stress-strain relations) will be used in conjunction with the orthotropic stress-strain relations to derive an orthotropic stress-optic law.

From Chapter II, an analogy with the stress-strain law was used to derive the stress-optic law. The orthotropic stress-strain relations in two dimensions are:

$$e_1 = \frac{\sigma_1}{E_{11}} - \frac{\nu_{21} \sigma_2}{E_{22}}, \text{ and} \quad (24)$$

$$e_2 = \frac{\sigma_2}{E_{22}} - \frac{\nu_{12} \sigma_1}{E_{11}}, \quad (25)$$

where:

- 1, 2 represent the principal stress directions,
- σ_1, σ_2 the principal stresses, and
- e_1, e_2 the strains in the 1-2 directions.
- $E_{11}, E_{22}, \nu_{12}, \nu_{21}$ are the elastic constants referenced to the 1-2 axes.

The strain-deformation relations are:

$$e_1 = \frac{\ell_1 - \ell_0}{\ell_0} \text{ and } e_2 = \frac{\ell_2 - \ell_0}{\ell_0}. \quad (26)$$

Using Equations 24 and 26 we obtain:

$$\ell_1 - \ell_0 = \frac{\ell_0}{E_{11}} \sigma_1 - \frac{\ell_0 \nu_{21}}{E_{22}} \sigma_2, \quad (27)$$

then using the analogy, we have:

$$n_1 - n_0 = \frac{C}{E_{11}} \sigma_1 - \frac{C \nu_{21}}{E_{22}} \sigma_2. \quad (28)$$

Similarly,

$$n_2 - n_0 = \frac{C}{E_{22}} \sigma_2 - \frac{C \nu_{12}}{E_{11}} \sigma_1. \quad (29)$$

From Equations 28 and 29,

$$n_1 - n_2 = \frac{C(1 + \nu_{12})}{E_{11}} \sigma_1 - \frac{C(1 + \nu_{21})}{E_{22}} \sigma_2, \quad (30)$$

where:

C represents the material constant.

The relative retardation, R , is given by:

$$R = \frac{t}{\lambda} (n_1 - n_2), \quad (31)$$

where:

t represents the material thickness, and
 λ the light wavelength.

Let:

$$f = \frac{\lambda}{C} , \quad (32)$$

where:

f represents the orthotropic material fringe constant.

The form of the stress-optic law is then:

$$R = \frac{t}{f} (C_1 \sigma_1 - C_2 \sigma_2). \quad (33)$$

C_1 and C_2 are the principal stress coefficients which are dependent upon the angle between the principal stresses and the fiber directions. The coefficients, C_1 and C_2 , may be computed by calculating the elastic constants referred to the principal stress directions by the transformation equations. The transformation equations are derived from the properties of a fourth-rank tensor⁶. The coefficients are, from Equation 30.

$$C_1 = \frac{1 + \nu_{12}}{E_{11}} \quad \text{and} \quad C_2 = \frac{1 + \nu_{21}}{E_{22}} . \quad (34)$$

These coefficients are plotted in Figure 15 for the W-1 fiber and Epon 815 resin data in Table II (Page 30). Specimens from this material are used throughout the biaxial

FIGURE 15 -
PRINCIPAL STRESS COEFFICIENTS FOR FIBER GLASS REINFORCED
EPON 815 RESIN

tests. Figures 16 through 19 show experimental fringe-order data plotted against $(C_1\sigma_1 - C_2\sigma_2)$ using the computed values of C_1 and C_2 . In these plots, P_1 is the vertical stress and P_2 the horizontal stress. The vertical direction was used as a reference for both the fiber orientation and isoclinic angle. Each plot shows the vertical and horizontal uniaxial loading as special cases along with a biaxial case for various fiber orientations. The 30-degree fiber orientation was chosen for the study of a range of biaxial stress ratios. The results are presented in Figure 20. The average slope of all the calibration curves is used to determine the orthotropic fringe constant, f , for each specimen.

The graphs of Figures 16 through 20 indicate that the form of the stress-optic law may be correct, but the accuracy is not quite acceptable. The principal stress coefficients, C_1 and C_2 , can also be experimentally determined by using uniaxial loading data. The same fringe-order data used in Figures 16 through 20 were used to determine C_1 and C_2 experimentally by using the average value of $f = 2.5(10^{-4})$ 1/fringe/inch obtained from these figures. The experimentally determined coefficients are plotted in Figure 21 and compared to the theoretical values. Figures 22 through 26 are similar to the previous plots, but using the experimental coefficients. These curves show that the orthotropic fringe constant, f , is practically independent of the biaxial stress ratios in the majority of cases.

FIGURE 16

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES
FOR FIBER GLASS REINFORCED EPON 815 RESIN
AT A 0-DEGREE FIBER ORIENTATION

FIGURE 17.

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES
FOR FIBER GLASS REINFORCED EPON 815 RESIN
AT A 15-DEGREE FIBER ORIENTATION

FIGURE 18

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES
 FOR FIBER GLASS REINFORCED EPON 815 RESIN
 AT A 30-DEGREE FIBER ORIENTATION
 (FOR BIAXIAL STRESS RATIOS)

FIGURE 19

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES
FOR FIBER GLASS REINFORCED EPON 815 RESIN
AT A 45-DEGREE FIBER ORIENTATION

FIGURE 20

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES FOR FIBER GLASS
REINFORCED EPON 815 RESIN AT A 30-DEGREE FIBER ORIENTATION
(FOR A RANGE OF BIAxIAL STRESS RATIOS)

FIGURE 21

THEORETICAL AND EXPERIMENTAL PRINCIPAL STRESS
COEFFICIENTS FOR FIBER GLASS REINFORCED
EPON 815 RESIN

FIGURE 27

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES FOR FIBER GLASS
REINFORCED EPON 815 RESIN AT A 0-DEGREE FIBER ORIENTATION
(EXPERIMENTAL COEFFICIENTS)

FIGURE 23

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES FOR FIBER GLASS
REINFORCED EPON 815 RESIN AT A 15-DEGREE FIBER ORIENTATION
(EXPERIMENTAL COEFFICIENTS)

FIGURE 24

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES FOR FIBER GLASS
REINFORCED EPON 815 RESIN AT A 30-DEGREE FIBER ORIENTATION
(EXPERIMENTAL COEFFICIENTS: FOR BIAXIAL STRESS RATIOS)

FIGURE 25

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES FOR FIBER GLASS
REINFORCED EPON 815 RESIN AT A 45-DEGREE FIBER ORIENTATION
(EXPERIMENTAL COEFFICIENTS)

FIGURE 26

ORTHOTROPIC STRESS-OPTIC LAW CALIBRATION CURVES FOR FIBER GLASS
REINFORCED EPON 815 RESIN AT A 30-DEGREE FIBER ORIENTATION
(EXPERIMENTAL COEFFICIENTS; FOR A RANGE OF BIAXIAL STRESS RATIOS)

It has been demonstrated from this series of experiments that a reasonably good stress-optic law for orthotropic materials is established. It should be noted that in orthotropic materials, a hydrostatic stress will produce fringe orders and isoclinics. The worst deviations from the proposed stress-optic law were those approaching hydrostatic stress cases.

CHAPTER VII

OBSERVATIONS OF AN ORTHOTROPIC CIRCULAR DISC

One of the simplest models which can be used to study the general biaxial stress conditions is a circular disc under diametral compression. It is well suited for this study since it presents an anisotropic elasticity stress distribution. The disc is in a state of plane stress and the governing differential equation is as presented in Chapter II, Equation 5. The orthotropic elasticity solution of this problem is not available.

The specimens were manufactured from the fiber glass reinforced Epon 815 epoxy resin using the method described in Chapter III. The dimensions were 2.5 inches in diameter by 0.250 inch in thickness. Two plates, of 0.125 inch thickness, were bonded together to make the 0.250-inch-thick specimens. The disc was loaded diametrically by a loading frame in the polariscope.

Several photographs have been taken of the fringe patterns at various fiber orientation angles. These pictures are presented in Figures 27 through 30. The fringe patterns are all fairly symmetrical along the horizontal centerline, but the cases with fiber orientations other than at 0 or 90 degrees have distorted fringe patterns near the loading point. The patterns indicate similar stress distributions

FIGURE 27

2.5-INCH-DIAMETER BY 0.250-INCH-THICK CIRCULAR DISC
UNDER DIAMETRAL COMPRESSION OF 530 POUNDS
AT A 0-DEGREE FIBER ORIENTATION

FIGURE 28

2.5-INCH-DIAMETER BY 0.250-INCH-THICK CIRCULAR DISC
UNDER DIAMETRAL COMPRESSION OF 390 POUNDS
AT A $22\frac{1}{2}$ -DEGREE FIBER ORIENTATION

FIGURE 29

2.5-INCH-DIAMETER BY 0.250-INCH-THICK CIRCULAR DISC
UNDER DIAMETRAL COMPRESSION OF 390 POUNDS
AT A 45-DEGREE FIBER ORIENTATION

FIGURE 30

2.5-INCH-DIAMETER BY 0.250-INCH-THICK CIRCULAR DISC
UNDER DIAMETRAL COMPRESSION OF 390 POUNDS
AT A 90-DEGREE FIBER ORIENTATION

along the horizontal centerline for different fiber orientations; however, the distributions look more concentrated at the center for the vertical fiber orientation and they gradually widen as the fiber is orientated toward the horizontal, seemingly a logical pattern for the stress distribution. Since the material elastic modulus is higher in the fiber direction than in the transverse direction, it would follow that the stress would be higher near the center for vertical fiber orientation. A plot of the fringe orders versus distance from the center for the vertical and horizontal fiber orientations is presented in Figure 31. Since the material is not sufficiently transparent to use the photometer for fringe-order readings directly, photographs were taken of the disc with the polariscope set up to correspond to $1/4$, $1/2$, $3/4$, and even-order fringes in the Tardy method of compensation 8. The fringe orders used in Figure 31 were taken directly from the photographs.

Contrast-process panchromatic film was used to take the photographs. For the determination of fringe orders, the negatives of the fringe pattern were placed in the polariscope field and the location of the minimum intensity was found by moving the loading-frame horizontal feed with the help of a photometer. This plot does not give the separate x and y normal stress distributions; therefore, the plot does not prove the idea that the stress distribution is more concentrated for vertical fiber orientation.

FIGURE 31

FRINGE-ORDER PLOT FOR 2.5-INCH-DIAMETER BY 0.250-INCH-THICK
CIRCULAR DISC UNDER DIAMETRAL COMPRESSION AT A 0
AND 90-DEGREE ORIENTATION

As discussed in Chapter V, isoclinic patterns do not represent the directions of principal stresses in the composite, but an average of the principal stress directions in fiber and resin. However, there were some unusual and interesting isoclinic patterns for the circular disc and they are presented here. Figures 32 and 33 show the typical cross pattern for the 0-degree isoclinic on the vertical and horizontal fiber orientations. It should be noted that in the case of the vertical fiber orientation, a large portion of the area near the outer boundary is almost black. This condition indicates that low stresses exist in that area. The other isoclinics for vertical or horizontal orientation are similar to the case of the isotropic disc. Figure 34 shows the 0-degree isoclinic for a $22\frac{1}{2}$ -degree fiber orientation. The isoclinic pattern is the small loops extending from the load points. Figure 35 shows the $67\frac{1}{2}$ -degree isoclinic for the same fiber orientation. This isoclinic covers most of the specimen field. No explanation can be offered for these patterns except to say that they are caused by the average of the isoclinic in the fiber and resin elements and they are not representative of the anisotropic stress principal directions.

FIGURE 32

0-DEGREE ISOCLINIC FOR 2.5-INCH-DIAMETER BY 0.250-INCH-THICK
CIRCULAR DISC UNDER DIAMETRAL COMPRESSION AT A
0-DEGREE FIBER ORIENTATION

FIGURE 33

0-DEGREE ISOCLINIC FOR 2.5-INCH-DIAMETER BY 0.250-INCH-THICK
CIRCULAR DISC UNDER DIAMETRAL COMPRESSION AT A
90-DEGREE FIBER ORIENTATION

FIGURE 34

0-DEGREE ISOCLINIC FOR 2.5-INCH-DIAMETER BY 0.250-INCH-THICK
CIRCULAR DISC UNDER DIAMETRAL COMPRESSION AT A
22 1/2-DEGREE FIBER ORIENTATION

FIGURE 35

67 1/2-DEGREE ISOCLINIC FOR 2.5-INCH-DIAMETER BY 0.250-INCH-THICK
CIRCULAR DISC UNDER DIAMETRAL COMPRESSION AT A
22 1/2-DEGREE FIBER ORIENTATION

CHAPTER VIII

CONCLUSIONS AND DISCUSSION

The following conclusions have been reached as a direct result of the work described in this thesis:

1. The fiber glass-epoxy composite can be made transparent by the elimination of air bubbles during the manufacturing process. This treatment alone will produce a translucent composite material but a proper matching of the indices of refraction of fiber and resin will produce glass-like transparency. However, it may take a perfect match of indices of refraction to produce this transparency since there are a large number of fiber-resin interfaces through the material thickness. A good match was obtained with Maraglas 655 epoxy resin, but it is a heat curing system and it left a residual stress too high to be usable as a photoelastic material. Epon 815 epoxy resin has the best matching among the room-temperature curing systems tried.
2. The simple analyses developed in this study can be used to predict the average optical behavior of transparent composite materials. The predicted values of the isoclinics have better agreement with the experimental data than the isochromatics. The accuracy of the prediction was

not of immediate concern since an understanding of the behavior was the main goal of this investigation. Possibly more accurate predictions could be obtained by using the elasticity solutions for the multiple inclusion problems, but these are rather lengthy and tedious for practical use. Since an average optical effect is obtained in the photoelastic tests anyway, the chosen mathematical model may be a fair representation of the average effect.

3. Isochromatics in the anisotropic material are related to stress in the material by a stress-optic law of the form presented in Chapter VI. This law was derived from similar techniques employed for the general stress-optic law for isotropic materials. The law was checked using theoretically determined principal stress coefficients and although it seemed to be fairly independent of the biaxial stress ratio, the scattering of the data was too wide to be conclusive. Uniaxial test data were used to determine the experimental principal stress coefficients. The data were plotted using these experimental coefficients and the majority of calibration curves verified the proposed stress-optic law.
4. Optical isoclinics are dependent on both the biaxial stress ratio and fiber orientation with respect to the principal stresses. This fact is

illustrated vividly in Figure 14, Page 39. It is observed that the isoclinic angle does not give the principal stress direction. The biaxial stress ratio must be known in conjunction with the isoclinic angle to give the principal stress directions. Then, in order to analyze an anisotropic model by the photoelastic method, another relation must be developed which will give the biaxial stress ratio. It is not known at this time what this relation is; therefore, the photoelastic method is not yet applicable to the general solution of the overall anisotropic stress distribution.

CHAPTER IX

AREAS OF FUTURE RESEARCH

Four main areas of research are of immediate need for developing the photoelasticity of anisotropic materials. They are:

1. Determination of the principal stress directions in a model. A method is needed for determining the principal stress ratio to be used with the isoclinic angle for this determination. As an alternate approach, there might be another way of determining the principal stress directions.
2. All of the biaxial tests were run with compressive loadings. Tension tests need to be performed to verify the results obtained here. Suitable tensile biaxial loading specimens will have to be developed to meet this objective.
3. From the fiber-resin element mathematical model analyses, it appears that optical isotropy in conjunction with mechanical anisotropy might be obtained. The computations reveal that for a fiber and a resin with the same fringe value, the composite-material fringe value is independent of the fiber orientation. Also, the optical isoclinic coincides with the principal stress direction and is independent of the biaxial stress ratio. This conclusion must be

verified experimentally. The main problem will be to find a fiber and a resin with equal fringe values and yet have different moduli of elasticity so that the material anisotropy will be preserved.

4. There is a fair correlation between the principal strain direction and the experimental data. This fact needs further investigation including more isoclinic data and experimental verification of the principal strain direction. If the isoclinic coincides with the principal strain direction, then strain-optic relations should be developed for the isochromatics. The strain problems can then be solved and the stress problem solutions obtained by the stress-strain relations.

If Item 3 can be fulfilled, it may eliminate Items 1 and 2, since the normal photoelastic methods may be employed for the analysis. When a method for the photoelastic analysis of anisotropic materials is developed, several experimental solutions of known anisotropic elasticity problems should be made and checked against the existing theoretical solutions.

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APPENDIX

PHOTO 1

```

C   THIS PROGRAM COMPUTES THE UNIAXIAL FRINGE VALUE AND OPTICAL
C   ISOCLINIC ANGLE FOR UNIAXIAL LOADING OF SPECIMENS WITH 0 FIBER
C   ORIENTATION ANGLE
      READ (50, 100) V, E1, E2, F1, F2
C   V = FIBER VOLUME FRACTION, E1 = FIBER MODULUS, E2 = RESIN MODULUS,
C   F1 = FIBER FRINGE VALUE, F2 = RESIN FRINGE VALUE
      WRITE (51, 101)
      DO 10 IA = 0, 90, 5
        A = IA
        A = A/57.296
        S1 = (COS(A))**2*(V/(V + (E2/E1)*(1.-V))**F1)
        S2 = (SIN(A))**2/(2.**F1)
        T1 = SIN(A)*COS(A)/(2.**F1)
        S3 = S1*(F1/V)*(E2/E1)*(1.-V)/F2
        S4 = (SIN(A))**2/(2.**F2)
        T2 = SIN(A)*COS(A)/(2.**F2)
        S5 = -(S1 + S3)
        S6 = -(S2 + S4)
        T3 = T1 + T2
        P = 2.**SQRT(((S6 - S5)/2.)**2 + T3**2)
        P1 = ((ATAN(2.**T3/(S6 - S5)))/2.)**57.296
        A = A**57.296
        P2 = A - P1
        F3 = 1./P

```

```
10 WRITE (51, 102) IA, F3, P2
100 FORMAT (F6.0, 2E10.0, 2F6.0)
101 FORMAT (1H1, 5HFIBER, 11X, 6HFRINGE, 10X, 7HOPTICAL
112H ORIENTATION, 5X, 5HVALUE, 11X, 9HISOCLINIC)
102 FORMAT (1H I3, 13X, F6.1, 10X, F6.1)
CALL EXIT
END
```

PHOTO 2

```

C   THIS PROGRAM COMPUTES THE PRINCIPAL STRESS COEFFICIENTS GIVEN
C   THE COMPONENT MATERIAL PROPERTIES AND RESIN WEIGHT PERCENT
      READ (50, 100) D5, E5, N5, D6, E6, N6, K, C, W
C   D5 = FIBER SPECIFIC GRAVITY, E5 = FIBER MODULUS, N5 = FIBER POISSON'S
C   RATIO, D6 = RESIN SPECIFIC GRAVITY, E6 = RESIN MODULUS, N6 = RESIN
C   POISSON'S RATIO, K = FIBER MISALIGNMENT FACTOR, C = CONTIGUITY
C   FACTOR, W = RESIN WEIGHT PERCENT
      WRITE (51, 101)
      V = (D5/D6)/((100./W) + (D5/D6)-1.)
      K5 = E5/(2.*(1.-N5))
      K6 = E6/(2.*(1.-N6))
      G5 = E5/(2.*(1. + N5))
      G6 = E6/(2.*(1. + N6))
      E1 = K*(E5-(E5 - E6)**V)
      M1 = 2.*(1.-N5 + (N5 - N6)**V)
      M2 = K5*(2.*K6 + G6) - G6*(K5 - K6)**V
      M3 = 2.*K6 + G6 + 2.*(K5 - K6)**V
      M4 = K5*(2.*K6 + G5) + G5*(K6 - K5)**V
      M5 = 2.*K6 + G5 - 2.*(K6 - K5)**V
      E2 = M1*((1.-C)**M2/M3 + C**M4/M5)
      P1 = K5*N5*(2.*K6 + G6)*(1.-V)
      P2 = K6*N6*(2.*K5 + G6)**V
      P3 = K5*(2.*K6 + G6) - G6*(K5 - K6)**V
      P4 = K6*N6*(2.*K5 + G5)**V

```

```

P5 = K5*(2.*K6 + G5)*(1.-V)
P6 = K5*(2.*K6 + G5) + G5*(K6 - K5)*V
P7 = (2.*G5 - (G5 - G6)*V)/(2.*G6 + (G5 - G6)*V)
P8 = ((G5 + G6) - (G5 - G6)*V)/((G5 + G6) + (G5 - G6)*V)
N1 = (1.-C)*(P1 + P2)/P3 + C*(P4 + P5)/P6
G1 = (1.-C)*G6*P7 + C*G5*P8
DO 10 IA = 0, 90, 5
  A = IA
  A = A/57.296
  M = COS(A)
  N = SIN(A)
  E3 = 1./(M**4/E1 + (1./G1 - 2.*N1/E1)*M**2*N**2 + N**4/E2)
  E4 = 1./(N**4/E1 + (1./G1 - 2.*N1/E1)*M**2*N**2 + M**4/E2)
  N2 = (E3/E1)*(N1 - (1. + 2.*N1 + E1/E2 - E1/G1)*M**1*N**2)
  N3 = E4*N2/E3
  C1 = (1. + N2)/E3
  C2 = (1. + N3)/E4
  A = A*57.296
10 WRITE (51, 102) IA, E3, E4, C1, C2
100 FORMAT (F6.0, E10.0, 2F6.0, E10.0, 4F6.0)
101 FORMAT (1H1, 11HORIENTATION/1H , 14HANGLE, DEGREES, 2X, 7HE1, PS1,
  19X, 7HE1, PS1, 9X, 2HC1, 14X, 2HC2)
102 FORMAT (1H , 13, 12X, 4(E10.0, 6X))
  CALL EXIT
  END

```

PHOTO 3

```

C      THIS PROGRAM COMPUTES THE ISOCLINIC ANGLE FOR BIAxIAL LOADING
C      OF A SPECIMEN WITH A KNOWN FIBER ORIENTATION ANGLE
      READ (50, 100) V, E1, E2, F1, F2
C      V = FIBER VOLUME FRACTION, E1 = FIBER MODULUS, E2 = RESIN
C      MODULUS, F1 = FIBER FRINGE VALUE, F2 = RESIN FRINGE VALUE
      WRITE (51, 101)
5 READ (50, 102) P1, P2, A
C      P1 = VERTICAL STRESS, P2 = HORIZONTAL STRESS, A = FIBER
C      ORIENTATION ANGLE
      A = A/57.296
      S1 = (P1**COS(A)**2 + P2**SIN(A)**2)*V/((C + (E2/E1)*(1. - V))*F1)
      S2 = (P1**SIN(A)**2 + P2**COS(A)**2)/(2.*F1)
      T1 = (P2 - P2)*SIN(A)*COS(A)/(2.*F1)
      S3 = S1*(F1/V)*(E2/E1)*(1. - V)/F2
      S4 = (P1**SIN(A)**2 + P2**COS(A)**2)/(2.*F2)
      T2 = (P1 - P2)*SIN(A)*COS(A)/(2.*F2)
      S5 = -(S1 + S3)
      S6 = -(S2 + S4)
      T3 = T1 + T2
      P3 = ((ATAN(2.*T3/(S6 - S5)))/2.)*57.296
      A = A*57.296
      P4 = A - P3
      P5 = P1/P2
      WRITE (51, 103) A, P5, P4
      GO TO 5

```

```
100 FORMAT (F6.0, 2E10.0, 2F6.0)
101 FORMAT (1H1, 5HFIBER, 11X, 7HBIAXIAL, 9X, 7HOPTICAL/
11H , 11HORIENTATION, 5X, 10HRATIO, V/H, 6X, 9HISOCLINIC)
102 FORMAT (3F6.0)
103 FORMAT (3(F6.0, 10X))
CALL EXIT
END
```