238
ANL-7562

1092
ANL-7562

11-18-69
MASTER

Argonne National Laboratory

THE DYNAMIC PLASTIC RESPONSE OF A TUBE TO AN IMPULSIVE RING LOAD OF ARBITRARY PULSE SHAPE

by

Carl K. Youngdahl

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

The facilities of Argonne National Laboratory are owned by the United States Government. Under the terms of a contract (W-31-109-Eng-38) between the U. S. Atomic Energy Commission, Argonne Universities Association and The University of Chicago, the University employs the staff and operates the Laboratory in accordance with policies and programs formulated, approved and reviewed by the Association.

MEMBERS OF ARGONNE UNIVERSITIES ASSOCIATION

The University of Arizona	Kansas State University	The Ohio State University
Carnegie-Mellon University	The University of Kansas	Ohio University
Case Western Reserve University	Loyola University	The Pennsylvania State University
The University of Chicago	Marquette University	Purdue University
University of Cincinnati	Michigan State University	Saint Louis University
Illinois Institute of Technology	The University of Michigan	Southern Illinois University
University of Illinois	University of Minnesota	University of Texas
Indiana University	University of Missouri	Washington University
Iowa State University	Northwestern University	Wayne State University
The University of Iowa	University of Notre Dame	The University of Wisconsin

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or

B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

Printed in the United States of America

Available from

Clearinghouse for Federal Scientific and Technical Information
National Bureau of Standards, U. S. Department of Commerce
Springfield, Virginia 22151

Price: Printed Copy \$3.00; Microfiche \$0.65

ARGONNE NATIONAL LABORATORY
9700 South Cass Avenue
Argonne, Illinois 60439

THE DYNAMIC PLASTIC RESPONSE OF A TUBE
TO AN IMPULSIVE RING LOAD
OF ARBITRARY PULSE SHAPE

by

Carl K. Youngdahl

Reactor Engineering Division

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:
A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.
As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

April 1969

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

THIS PAGE
WAS INTENTIONALLY
LEFT BLANK

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT	5
INTRODUCTION	5
GENERAL EQUATIONS	7
SOLUTIONS	13
RESULTS AND CONCLUSIONS	16
APPENDIXES	
A. Closed-form Solution for Hinge-band Deformation	22
B. Local Series Solution of Differential Equations	25
C. Solution Behavior in the Vicinity of Instantaneous Jumps in ψ	30
D. Computer Programs	33
ACKNOWLEDGMENTS	58
REFERENCES	58

LIST OF FIGURES

<u>No.</u>	<u>Title</u>	<u>Page</u>
1.	Stress Resultants on Element of Shell	7
2.	Circular Cylindrical Shell Subjected to an Impulsive Ring Load	7
3.	Limited-interaction Yield Condition	9
4.	Final Deformation as a Function of Peak Load for Rectangular, Triangular, Linear-decay, and Exponential-decay Pulses	16
5.	Final Deformation as a Function of Effective Load for the Same Pulse Shapes as Fig. 4	17
6.	Duration of Plastic Flow as a Function of Effective Load for the Same Pulse Shapes as Fig. 4	17
7.	Problem a: Load ψ , Hinge-circle and -band Location ξ and η , and Central Deformation u_0	19
8.	Problem b: Load ψ , Hinge-circle and -band Location ξ and η , and Central Deformation u_0	19
9.	Problem c: Load ψ , Hinge-circle and -band Location ξ and η , and Central Deformation u_0	20
10.	Problem a: Radial Deformation as a Function of Axial Position for Various Times	20
11.	Problem b: Radial Deformation as a Function of Axial Position for Various Times	21
12.	Problem c: Radial Deformation as a Function of Axial Position for Various Times	21

THE DYNAMIC PLASTIC RESPONSE OF A TUBE
TO AN IMPULSIVE RING LOAD
OF ARBITRARY PULSE SHAPE

by

Carl K. Youngdahl

ABSTRACT

The problem of the deformation of a long, circular, cylindrical shell made of a rigid, perfectly plastic material and subjected to an impulsive ring load is solved for arbitrary pulse shapes. The principal result of the investigation is that the dynamic effect of the pulse shape is almost completely characterized by its associated impulse and effective load (defined as the impulse divided by twice the mean time of the pulse). This is to say that details of the pulse shape and, in particular, its peak value are comparatively unimportant in determining the history of the plastic deformation and the final shape of the shell. The implications for experimental work are encouraging, since the impulse and effective load are easily computed from a pressure-time plot and are not strongly influenced by experimental inaccuracies.

INTRODUCTION

In nuclear-reactor safety analyses it is frequently important to estimate the plastic deformation of pipes and tubes in the reactor produced by small chemical, mechanical, or nuclear explosions. For example, proof tests of fuel-element prototypes are carried out in experimental loops in special testing reactors in which the fuel elements to be tested are deliberately caused to fail. It is necessary to predict the extent to which the resultant localized release of energy will distort the test loop if damage to the reactor itself is to be avoided. Another example of reactor analysis pertaining to plastic distortion of tubes is the case of a coolant-flow blockage producing a localized fuel-element failure. The impulsive pressure may damage the fuel-assembly can, and this may lead to a more serious accident or, at the least, may jam the can into its neighbors, creating repair and removal problems.

The nature of the energy-release phenomena in these applications is usually only crudely known, and out-of-pile simulations mockup only part

of the complex nuclear and structural configurations of a reactor. Consequently, it would be advantageous to know what properties of the time-shape of an impulsive load are important in determining the final plastic deformation of a tube; in particular, for experimental applications, one should know what scaling and equivalence laws may be used and how detailed and accurate pressure-time measurements must be.

For a free-free beam made of a rigid-plastic material, Symonds¹ compared the final deformations produced by triangular, sinusoidal, and rectangular pulse shapes and postulated that pulses with the same total impulse and peak value had approximately the same effect. Hodge,² however, found for a reinforced circular cylindrical shell that this equivalence was reasonably adequate if the yield load was greatly exceeded, but that the final deformations were strongly dependent on the pulse shape for "medium" values of the load, which are of the same order of magnitude as the collapse load. In particular, he showed the deformation produced by an exponentially decaying pulse, which is a reasonable approximation to an actual explosive load, differed significantly from that produced by a rectangular pulse with the same impulse and peak value.

This report shows that, for a long, circular cylinder made of a rigid plastic material and acted on by an impulsive ring load, the final plastic deformation is almost completely determined by the impulse and effective load associated with the pulse. The effective load is defined as the impulse divided by twice the time-mean of the pulse (the time-mean being the length of the interval between the time at which yielding occurs and the centroid of the pulse shape). This correlation has been found to be valid for exponentially decaying, triangular, rectangular, ramp, multisaw-toothed, and other widely varying pulse shapes. Both parameters are easily determined from experimental pressure-time graphs, in contrast to the measurement of the peak load, which is strongly influenced by transducer inaccuracies.

The dynamic plasticity problem treated here has been solved by Eason and Shield³ for the special cases of a rectangular pulse, for which they obtained a closed-form solution, and a triangular pulse, which they treated numerically. Their work is extended here to the consideration of arbitrary pulse shapes. An elaborate computer program was devised to numerically solve the resulting sets of coupled nonlinear differential equations for any pulse-shape input. However, if the error involved in applying the two-parameter correlation is acceptable (and it is apparently less than the errors made in idealizing the material behavior), a semigraphical procedure may be used in place of the computer analysis.

The basic equations for the plastic deformation of a long tube acted on by an impulsive ring load are derived in Ref. 3 in considerable detail. A condensed version of this derivation is given in the next section to make this report reasonably self-contained.

GENERAL EQUATIONS

Consider a long, circular, cylindrical tube, made of a rigid, perfectly plastic material and loaded by an internal pressure $P(Z, T)$, where Z and T are axial coordinate and time. The usual shell-theory assumptions are employed such that stress distributions across the shell thickness are replaced by their resultants per unit circumferential length as in Fig. 1. The equations of motion of the shell can be expressed in terms of the axial bending moment M , the radial shear force Q , the circumferential force N , the radial velocity V , and the radial displacement U . Define dimensionless quantities by

$$\left. \begin{aligned} z &= \frac{Z}{(RH)^{1/2}}, \quad t = \frac{T}{T_0}, \quad u = \frac{UR\rho}{\sigma_y HT_0^2}, \quad v = \frac{VR\rho}{\sigma_y HT_0}, \quad m = \frac{4M}{\sigma_y H^2}, \\ n &= \frac{N}{\sigma_y H}, \quad q = \frac{QR^{1/2}}{\sigma_y H^{3/2}}, \quad \text{and} \quad p = \frac{PR}{\sigma_y H}, \end{aligned} \right\} \quad (1)$$

where R and H are the radius and thickness, of the shell as shown in Fig. 2, σ_y and ρ are its yield stress and surface density, and T_0 is a time interval characteristic of the problem under discussion.[†] The equations of motion are then

$$\frac{\partial q}{\partial z} = p - n - \frac{\partial v}{\partial t}, \quad \frac{\partial m}{\partial z} = 4q, \quad \text{and} \quad \frac{\partial u}{\partial t} = v, \quad (2)$$

with all dependent variables being functions of z and t .

113-1880

Fig. 1. Stress Resultants on Element of Shell

113-1885

Fig. 2. Circular Cylindrical Shell Subjected to an Impulsive Ring Load

[†]Any value of T_0 may be chosen, since the results are plotted so as to eliminate this quantity.

To obtain a concentrated ring load, let the region of application of the internal pressure shrink to the plane $Z = 0$, maintaining the value of the resultant force as a function of time. Then

$$P(Z, T) = \Psi(T) \delta(Z), \quad (3)$$

where δ is the Dirac delta function. The dimensionless ring-load magnitude $\psi(t)$ is defined by

$$\psi = \frac{\Psi R^{1/2}}{2\sigma_y H^{3/2}}. \quad (4)$$

Some quantities of interest associated with $\Psi(T)$ are the impulse I per unit circumferential length and the time mean T_m of the pulse, defined by

$$I = \int_{T_y}^{T_f} \Psi(T) dT,$$

and

$$T_m = \frac{1}{I} \int_{T_y}^{T_f} (T - T_y) \Psi(T) dT, \quad (5)$$

where T_y and T_f are the times at which plastic deformation begins and ends. It is proposed that the parameter that, along with I , best characterizes the effect of the pulse, is the associated effective load Ψ_e defined by

$$\Psi_e = \frac{I}{2T_m}. \quad (6)$$

Let the dimensionless counterparts of I , T_m , Ψ_e , T_y , and T_f be given by

$$\left. \begin{aligned} i &= \frac{IR^{1/2}}{2\sigma_y T_0 H^{3/2}} = \int_{t_y}^{t_f} \psi(t) dt, \\ t_m &= \frac{T_m}{T_0} = \frac{1}{i} \int_{t_y}^{t_f} (t - t_y) \psi(t) dt, \\ \psi_e &= \frac{IR^{1/2}}{4T_m \sigma_y H^{3/2}} = \frac{i}{2t_m}, \\ t_y &= \frac{T_y}{T_0}, \text{ and } t_f = \frac{T_f}{T_0}. \end{aligned} \right\} \quad (7)$$

Assume the yield condition in m, n space to be given by the limited interaction curve of Fig. 3. (The relation of this ideal square curve to the

113-1883

true yield condition, is discussed by Drucker⁴ and Hodge.⁵) The relevant points of the yield curve for this problem are on the side AB, including the end points, such that $n = +1$ throughout the plastic region. Since the strain-rate vector, which has components proportional to $\partial^2 v / \partial z^2$ and v , must be normal to the yield curve, it follows that the radial velocity v must be positive and that

$$\frac{\partial^2 v}{\partial z^2} = 0 \quad (8)$$

Fig. 3. Limited-interaction Yield Condition

at shell sections that correspond to interior points of AB. Hinge circles occur at sections of the shell for which $m = \pm 1$; they correspond to points A and B where the strain-rate vector can have any direction between the limiting normals, implying the possibility of a discontinuity in the velocity slope $\partial v / \partial z$. If a hinge circle spreads over a finite axial region of the shell, a hinge band is formed in which m has the corresponding extremum value and $\partial^2 v / \partial z^2$ may have any value compatible with the yield condition...

The static collapse load for a long shell acted on by a circumferential ring force is found in Ref. 4 to be

$$\Psi_S = \frac{2\sigma_y H^{3/2}}{R^{1/2}} \quad (9)$$

For a dynamically applied load whose maximum value exceeds the collapse load, plastic deformation begins when (by Refs. 4 and 9) ψ first exceeds unity. Hinge circles initially occur at $z = 0$ and $z = \pm 1$. During the subsequent motion, the outer hinge circles occupy the positions $z = \pm \xi(t)$ and, for some load-time functions, may expand into hinge bands in the regions $\xi(t) \leq |z| \leq \eta(t)$. We consider first the deformation history up until the time when hinge bands first appear.[†]

In the region $0 < z < \xi(t)$, $t \geq t_y$, we have (from Eq. 8 and Fig. 3) that

$$v(z, t) = v_0(t) \left[1 - \frac{z}{\xi(t)} \right], \text{ and } n(z, t) = 1, \quad (10)$$

[†]As shown in the section on Solutions, a hinge band cannot appear instantaneously at $t = t_y$ even if an instantaneous jump in the load occurs there.

where $v_0(t)$ is the radial velocity at $z = 0$. The first of Eqs. 2 can then be written

$$\frac{\partial q}{\partial z} = zC_1(t) - C_2(t), \quad (11)$$

with C_1 and C_2 defined by

$$C_1 = \frac{d}{dt} \left(\frac{v_0}{\xi} \right), \text{ and } C_2 = 1 + \frac{dv_0}{dt}. \quad (12)$$

The integrations of Eq. 11 and the second of Eqs. 2 with respect to z result in

$$\left. \begin{aligned} q &= \frac{1}{2} z^2 C_1(t) - z C_2(t) + C_3(t), \\ \text{and} \\ m &= \frac{2}{3} z^3 C_1(t) - 2z^2 C_2(t) + 4z C_3(t) + C_4(t), \\ \text{for } 0 \leq z \leq \xi(t), \quad t \geq t_y. \end{aligned} \right\} \quad (13)$$

The boundary conditions on m are (using Fig. 3)

$$m(0, t) = -1, \quad m(\xi, t) = +1, \quad (14)$$

while $q = 0$ at $z = \xi$ where m has its maximum and q has a discontinuity of 2ψ at $z = 0$ because of the concentrated load there. Applying these boundary conditions to Eqs. 13 gives four relations for C_1 , C_2 , C_3 , and C_4 , whose solution is

$$\left. \begin{aligned} C_1 &= \frac{6}{\xi^3} (\psi \xi - 1), \\ C_2 &= \frac{1}{\xi^2} (4\psi \xi - 3), \\ C_3 &= \psi, \\ \text{and} \\ C_4 &= -1. \end{aligned} \right\} \quad (15)$$

Using Eqs. 10, 12, and 15 and the third of Eqs. 2, we arrive at the set of coupled nonlinear differential equations

$$\left. \begin{aligned}
 \frac{dv_0}{dt} &= \frac{4\psi\zeta - 3 - \zeta^2}{\zeta^2}, \\
 \frac{d\zeta}{dt} &= \frac{-2\psi\zeta + 3 - \zeta^2}{v_0\zeta}, \\
 \text{and} \\
 \frac{\partial u}{\partial t} &= v_0 \left(1 - \frac{z}{\zeta}\right), \quad 0 \leq z \leq \zeta(t), \\
 &= 0, \quad z > \zeta(t),
 \end{aligned} \right\} \quad (16)$$

subject to the initial conditions

$$v_0(t_y) = 0, \quad \zeta(t_y) = 1, \quad \text{and} \quad u(z, t_y) = 0. \quad (17)$$

Since m is a cubic in z , satisfies the boundary conditions (Eqs. 14), and has a horizontal tangent at $z = \zeta$, necessary and sufficient conditions that $|m| < 1$ in the interval $0 < z < \zeta$ are that

$$\frac{\partial m}{\partial z} \geq 0 \quad (18)$$

at $z = 0$, and

$$\frac{\partial^2 m}{\partial z^2} \leq 0 \quad (19)$$

at $z = \zeta$. The slope of m at the origin is $\psi(t)$. Since $\psi(t)$ is assumed to be nonnegative, the first of these conditions is always satisfied. This implies that a hinge band[†] cannot form in the vicinity of $z = 0$. By Eqs. 13 and 15, the Inequality 19 is equivalent to

$$\psi\zeta \leq 3/2. \quad (20)$$

For some load histories, the deformation behavior is such that at some time $t = t_1$, say, while ψ is increasing,

$$\psi\zeta = 3/2. \quad (21)$$

At this instant, the hinge circle at $\zeta(t)$ begins to broaden into a hinge band in the region $\zeta(t) \leq z \leq \eta(t)$. The hinge band continues to broaden until $t = t_M$ when ψ reaches a local maximum ψ_M . As ψ decreases, the hinge band contracts, until ζ and η coincide at some time t_2 when Eqs. 16 are again applicable. If ψ has a number of peaks, the hinge band may not

[†]It is shown in Ref. 3 that a hinge band can form at the origin if the load is axially distributed, rather than concentrated.

completely disappear between adjacent maxima, but begin to expand as ψ passes through the intervening minimum.

During the existence of the hinge band, the shell radial velocity in the region $0 \leq z \leq \xi(t)$ is still linear in z , but is given by

$$v(z,t) = v_0(t) - [v_0(t) - v_\xi(t)] \frac{z}{\xi(t)}, \quad (22)$$

where $v_\xi(t)$ is the radial velocity at $z = \xi$. The solution to the first two equations of Eqs. 2 is still given by Eqs. 13 and 15, where now C_1 and C_2 are defined as

$$C_1 = \frac{d}{dt} \frac{v_0 - v_\xi}{\xi}, \text{ and } C_2 = 1 + \frac{dv_0}{dt}. \quad (23)$$

In the region $\xi(t) \leq z \leq \eta(t)$,

$$m = 1, \quad q = 0, \quad \text{and} \quad n = +1, \quad (24)$$

so that the solution to the first two equations of Eqs. 2 is

$$v(z,t) = -t + \Omega(z), \quad (25)$$

where $\Omega(z)$ is an arbitrary function determined by the deformation. Since the radial velocity at the outer end of the hinge band vanishes, we have that

$$\Omega(\eta) = t, \quad (26)$$

while by definition of v_ξ ,

$$v_\xi = -t + \Omega(\xi). \quad (27)$$

It is shown in Ref. 3 that in the interval $t_1 \leq t \leq t_M$ the inner edge of the hinge band ξ is related to ψ through Eq. 21. The pair of nonlinear differential equations obtained from Eqs. 23 and 15 can then be solved for $v_0(t)$ and $v_\xi(t)$. Moreover, since ψ is increasing in this time interval, ξ must be decreasing; that is, the inner end of the hinge band moves toward the origin. As it does so, the function Ω is generated through Eq. 27. The outer end of the hinge band η is then determined by Eq. 26. In the interval $t_M \leq t \leq t_2$, ξ and ψ are no longer related by Eq. 21, but ξ now increases with time so that it sweeps back through positions z for which Ω is known. Consequently Eqs. 23, 15, and 27 give three equations for the determination of ξ , v_0 , and v_ξ , while η is still found from Eq. 26.[†] Since ξ is

[†] It is easily shown that Ω is a decreasing function of z so that, by Eq. 26, η must be a decreasing function of time. Consequently, η passes through positions z previously occupied by ξ for which $\Omega(z)$ has been determined.

increasing and η is decreasing, they eventually coincide and the hinge band reverts to a hinge circle.

A function Ω must be determined for each hinge band if more than one occurs for a ψ function with several peaks. If between adjacent peaks the hinge band begins expanding again, rather than degenerating to zero length, the function $\Omega(z)$ must be redefined at locations that are passed through by ξ a second time.

A closed-form solution for v_0 , v_ξ , ξ , η , and Ω during hinge formation and motion for arbitrary[†] $\psi(t)$ is derived in Appendix A. However, in obtaining results for the shell deformation, we decided that solving the differential equations numerically was probably no more difficult than numerically evaluating the transcendental equations of the closed-form solution and could be adapted more readily to the solution of multiple-peak problems.

SOLUTIONS

A numerical solution of the coupled set of nonlinear differential equations (Eqs. 16) and the set, for hinge-band motion, given by Eqs. 15, 22, 23, 25, 26, and 27 was programmed for the CDC-3600 computer for an arbitrary load-time function $\psi(t)$. The program and auxiliary subroutines are given in Appendix D. The inputs to the program are the function ψ and the time intervals and axial positions at which results are desired; the outputs are radial velocities and displacements and hinge-band locations. The Bulirsch-Stoer method of extrapolation by rational functions was used,⁶ an initial power series being employed to facilitate the start of the solution. (See Appendix B.)

A closed-form solution may be obtained for a rectangular pulse shape. Let ψ be given by

$$\left. \begin{aligned} \psi &= \psi_M, & 0 \leq t \leq \tau; \\ &= 0, & t > \tau. \end{aligned} \right\} \quad (28)$$

At $t = 0$, the hinge-circle location jumps instantaneously from its yield location at $z = 1$ to the position z_0 . During the interval $0 < t \leq \tau$, the hinge-circle location remains fixed while v_0 is a linear function of time. From Eqs. 16, we have, for $0 < t \leq \tau$,

$$\xi(t) = z_0, \text{ and } v_0(t) = Kt, \quad (29)$$

[†]The particular case of this solution for ψ a triangular peak is presented in Ref. 3.

with

$$\left. \begin{aligned} z_0 &= \left(\psi_M^2 + 3 \right)^{1/2} - \psi_M, \\ \text{and} \\ K &= \frac{3(1 - z_0^2)}{z_0^2}. \end{aligned} \right\} \quad (30)$$

The solution to Eqs. 16 for any $\psi(t)$ that vanishes after some time τ is given by

$$\xi = A \frac{t}{\tau} + B, \quad v_0 = \frac{(3 - \xi^2) \tau}{A \xi}, \quad t \geq \tau, \quad (31)$$

where A and B are found from the values of ξ and v_0 at τ . By Eqs. 29 and 30, for the rectangular pulse,[†]

$$A = \frac{(3 - z_0^2) z_0}{3(1 - z_0^2)}, \quad \text{and } B = z_0 - A. \quad (32)$$

From Eqs. 16 and 29-32, the radial displacement is found to be, for $0 \leq t \leq \tau$,

$$\left. \begin{aligned} u(z, t) &= \frac{1}{2} K \left(1 - \frac{z}{z_0} \right) t^2, \quad 0 \leq z \leq z_0, \\ &= 0, \quad z > z_0, \end{aligned} \right\} \quad (33)$$

while, for $t > \tau$,

$$\left. \begin{aligned} \frac{u(z, t)}{\tau^2} &= \frac{1}{2} K \left(1 - \frac{z}{z_0} \right) + \frac{1}{A^2} \left\{ \frac{1}{2} [z_0 - \xi(t)] [z_0 + \xi(t) - 2z] \right. \\ &\quad \left. + 3 \log \frac{\xi(t)}{z_0} + 3z \left[\frac{1}{\xi(t)} - \frac{1}{z_0} \right] \right\}, \quad 0 \leq z \leq z_0; \\ &= \frac{1}{A^2} \left\{ -\frac{1}{2} [\xi(t) - z]^2 + 3 \log \frac{\xi(t)}{z} + 3 \left[\frac{z}{\xi(t)} - 1 \right] \right\}, \\ &\quad z_0 < z \leq \xi; \\ &= 0, \quad z > \xi, \end{aligned} \right\} \quad (34)$$

[†]The results (Eqs. 29-32) are presented in Ref. 3, in somewhat different form.

with $\zeta(t)$ and A given by Eqs. 31 and 32. The plastic deformation ends when v_0 vanishes, which, by Eqs. 31, occurs at a time t_f when

$$\zeta(t_f) = \sqrt{3}. \quad (35)$$

Letting u_{of} be the radial displacement at $z = 0$ and $t = t_f$, we have, from Eqs. 29-32, 34, 35, and 7, that

$$\left. \begin{aligned} \frac{t_f}{i} &= \frac{\sqrt{3} - B}{A\psi_M}, \\ \text{and} \\ \frac{u_{of}}{i^2} &= \frac{1}{\psi_M^2 A^2} \left(z_0 A + 3 \log \frac{\sqrt{3}}{z_0} \right). \end{aligned} \right\} \quad (36)$$

Note that, for the rectangular pulse,

$$\psi_e = \psi_M. \quad (37)$$

It is easily shown that

$$\psi_M z_0 < 3/2, \quad (38)$$

so that no hinge band is produced by a rectangular pulse. The same is also true for any pulse shape that increases instantaneously from zero to its maximum value ψ_M and then decreases monotonically from that point on. The initial instantaneous value of ζ will be z_0 , given in Eqs. 30, for which Eq. 38 holds, and a hinge band can only be initiated when ψ is increasing. However, if the pulse shape is such that the shell is already in motion when a positive jump in ψ occurs, a hinge band may appear instantaneously. Consider a time t_1 at which ζ and v_0 have the values ζ_1 and v_{01} and at which ψ jumps from ψ_1 to ψ_M , where $\psi_M \zeta_1$ exceeds $3/2$. Let primes denote quantities just after the jump. Then,

$$\left. \begin{aligned} \eta_1' &= \zeta_1, \quad \zeta_1' = \frac{3}{2\psi_M}, \quad v_{01}' = v_{01}, \quad v_{\zeta_1}' = v_{01} \left(1 - \frac{\zeta_1'}{\zeta_1} \right), \\ \Omega(\zeta_1) &= t_1, \quad \text{and} \quad \Omega(\zeta_1') = t_1 + v_{\zeta_1}', \end{aligned} \right\} \quad (39)$$

and Ω is a linear function of z in the range $\zeta_1' \leq z \leq \zeta_1$. In Appendix C, these results are obtained by considering the series solution to the differential equations at some time t_1 and finding the appropriate limits as Δt vanishes with $\Delta\psi$ remaining fixed.

RESULTS AND CONCLUSIONS

Figures 4-6 show the results of some of the parameter studies. The solution for the rectangular pulse shape was discussed in the previous section; the linear-decay, triangular, and exponential-decay pulse shapes are given by

Linear Decay

$$\left. \begin{aligned} \psi &= \psi_M \left(1 - \frac{t}{\tau}\right), & 0 \leq t \leq \tau; \\ &= 0, & t > \tau. \end{aligned} \right\} \quad (40)$$

Triangular

$$\left. \begin{aligned} \psi &= 2\psi_M \frac{t}{\tau}, & 0 \leq t \leq \frac{1}{2}\tau; \\ &= 2\psi_M \left(1 - \frac{t}{\tau}\right), & \frac{1}{2}\tau < t \leq \tau; \\ &= 0, & t > \tau. \end{aligned} \right\} \quad (41)$$

113-1881

Fig. 4. Final Deformation as a Function of Peak Load for Rectangular, Triangular, Linear-decay, and Exponential-decay Pulses. Points a, b, and c are the results of problems a, b, and c.

113-1887

Fig. 5. Final Deformation as a Function of Effective Load for the Same Pulse Shapes as Fig. 4

113-1882

Fig. 6. Duration of Plastic Flow as a Function of Effective Load for the Same Pulse Shapes as Fig. 4

Exponential Decay

$$\psi = \psi_M \exp(-t/\tau), \quad 0 \leq t < \infty. \quad (42)$$

In Figs. 4 and 5, the final radial displacement at $z = 0$ divided by the square of the impulse is plotted as a function of the peak load ψ_M and the effective load ψ_e , respectively. Figure 4 shows that equating the effects of pulses with the same impulse and peak load would lead to large relative errors, particularly when ψ_M is close to the yield load of unity. On the other hand, the bunching of the curves in Fig. 5 shows that pulse shapes with the same impulse and effective load have essentially the same effect, to well within the errors inherent in the assumptions of the problem. Moreover, the curves converge closely as the yield load is approached.

The integrals in Eqs. 7 have as their limits t_y and t_f , the times when plastic deformation begins and ends. The initial yield time is easily determined from a knowledge of $\psi(t)$; however, t_f is not known a priori since the motion may cease before the end of the pulse.[†] Figure 6 indicates that $(t_f - t_y)/i$ is a weak function of ψ_e . Consequently, given an arbitrary pulse shape, t_f can be estimated and i , t_m , and ψ_e computed using Eqs. 7. For these values of i and ψ_e , a new value of t_f can be found from Fig. 6. Then revised values of i , t_m , and ψ_e can be computed and the procedure repeated until convergence is attained. Very few iteration steps have been found to be necessary, because of the insensitivity of the value of $(t_f - t_y)/i$ to ψ_e . Having obtained i and ψ_e , we can find the final plastic deformation approximately from Fig. 5 and the appropriate deformation history from the closed-form, rectangular-pulse results (Eqs. 29-36).

Since the semilog plot of the final deformation for a rectangular pulse is very nearly a straight line (as shown in Figs 4 and 5), we can approximate Eqs. 36 for rough calculations by

$$\left. \begin{aligned} \frac{u_{of}}{i^2} &\approx 1.405 \log \psi_M \\ &\approx 1.405 \log \psi_e \end{aligned} \right\} \quad (43)$$

Figures 7, 8, and 9 show ψ , ξ , η , and u_0 (the radial displacement at $z = 0$) for three irregular pulse shapes, which produce extensive hinge-band motion. The final radial displacements and duration of plastic flow for the problems are identified in Figs. 4, 5, and 6 as points a, b, and c. These results and the results for many other problems not discussed here cluster close to the curves in Figs. 5 and 6.

[†] In the results included here, this occurs for all the exponential-decay pulses, of course, and for the linear-decay and triangular cases with ψ_e close to unity.

Fig. 7

Problem a: Load ψ , Hinge-circle and -band Location ζ and η , and Central Deformation u_0 ($i = 4.833$, $\psi_M = 3$, $\psi_e = 1.589$)

113-1886

Fig. 8

Problem b: Load ψ , Hinge-circle and -band Location ζ and η , and Central Deformation u_0 ($i = 27$, $\psi_M = 6$, $\psi_e = 3.627$)

113-1879

Fig. 9

Problem c: Load ψ , Hinge-circle and -band Location ζ and η , and Central Deformation u_0 ($i = 24.875$, $\psi_M = 7.5$, $\psi_e = 4.527$)

113-1878

Figures 10-12 show the radial deformation shapes as a function of dimensionless axial position at various times for problems a, b, and c, respectively. On the figures, τ_f is the time at which the pulse ends.

We have, from Eqs. 1 and 7, that

$$\left. \begin{aligned} \frac{t}{i} &= \frac{2\sigma_y H^{3/2}}{\sqrt{R}} \frac{T}{I}, \\ \frac{v}{i} &= 2\rho\sqrt{RH} \frac{V}{I}, \\ \text{and} \\ \frac{u}{i^2} &= 4\rho\sigma_y H^2 \frac{U}{I^2}. \end{aligned} \right\} \quad (44)$$

Therefore, scaling the variables by the appropriate power of the impulse eliminates the arbitrary multiplicative factor in the time scale, represented by T_0 in the nondimensionalization and τ in the problem statements.

113-1884

Fig. 10. Problem a: Radial Deformation as a Function of Axial Position for Various Times ($t_y/i = 0.0690$)

113-1888

Fig. 11. Problem b: Radial Deformation as a Function of Axial Position for Various Times ($t_y/i = 0$)

113-1889

Fig. 12. Problem c: Radial Deformation as a Function of Axial Position for Various Times ($t_y/i = 0$)

APPENDIX A

Closed-form Solution for Hinge-band Deformation

A closed-form solution to the differential equations that determine the hinge-band motion in the vicinity of a peak of the pulse can be obtained as follows:

Assume a hinge band begins to form at some instant t_1 when ψ , ξ , and v_0 have the values ψ_1 , ξ_1 , and v_{01} . The pulse increases to a peak value ψ_M at t_M and then decreases to some value ψ_2 at t_2 when the band degenerates to a hinge circle. Let ψ^{-1} denote the inverse of ψ for $t_1 \leq t \leq t_M$. Define

$$\mu(x) = \psi^{-1}\left(\frac{3}{2x}\right), \quad \frac{3}{2\psi_M} \leq x \leq \frac{3}{2\psi_1}. \quad (\text{A.1})$$

The differential equations to be solved are found from Eqs. 23 and 15 to be

$$\left. \begin{aligned} \frac{dv_0}{dt} &= \frac{1}{\xi^2} (4\psi\xi - 3) - 1, \\ \text{and} \quad \frac{d}{dt} \left(\frac{v_0 - v_\xi}{\xi} \right) &= \frac{6}{\xi^3} (\psi\xi - 1). \end{aligned} \right\} \quad (\text{A.2})$$

1. Interval $t_1 \leq t \leq t_M$

Since ξ is related to ψ through Eq. 21 in this interval, ξ can be eliminated from Eqs. A.2, resulting in simple linear differential equations for v_0 and v_ξ ; their solution is

$$\left. \begin{aligned} v_0(t) &= v_{01} - (t - t_1) + \frac{4}{3} \int_{t_1}^t \psi^2 dt, \\ \text{and} \quad v_\xi(t) &= v_0(t) - \frac{1}{\psi(t)} \left[\psi_1 v_{01} + \frac{4}{3} \int_{t_1}^t \psi^3 dt \right], \end{aligned} \right\} \quad (\text{A.3})$$

making use of the initial conditions at t_1 . Since ξ and v_ξ are known from Eqs. 21 and A.3, Ω is found from Eqs. 27 to be

$$\Omega(z) = t_1 + v_{01} \left(1 - \frac{z}{\xi_1} \right) + \frac{4}{3} \int_{t_1}^{\mu(z)} \psi^2 dt - \frac{8z}{9} \int_{t_1}^{\mu(z)} \psi^3 dt. \quad (\text{A.4})$$

A transcendental equation for η is then obtained from Eqs. 26 and A.4.

2. Interval $t_M < t \leq t_2$

The function Ω , as given by Eq. A.4, is still used with Eq. 26 to find η and, with Eq. 27, yields the following relation between v_ξ and ξ :

$$v_\xi(t) = -(t - t_1) + v_{01} \left(1 - \frac{\xi}{\xi_1}\right) + \frac{4}{3} \int_{t_1}^{\mu(\xi)} \psi^2 dt - \frac{8\xi}{9} \int_{t_1}^{\mu(\xi)} \psi^3 dt. \quad (\text{A.5})$$

The function v_ξ can be eliminated from Eqs. A.2, resulting in a pair of nonlinear equations for v_0 and ξ . Define an auxiliary function F through

$$F = \xi^2 v_0 + (\xi^2 - 3) t. \quad (\text{A.6})$$

From Eqs. A.2, A.5, and A.6 we arrive at

$$\frac{dF}{dt} = \left[2(t_1 + v_{01}) \xi + \frac{8\xi}{3} \int_{t_1}^{\mu(\xi)} \psi^2 dt \right] \frac{d\xi}{dt}, \quad (\text{A.7})$$

which can be integrated to give

$$F = \xi^2 (t_1 + v_{01}) - 3\mu(\xi) + \frac{4\xi^2}{3} \int_{t_1}^{\mu(\xi)} \psi^2 dt + C. \quad (\text{A.8})$$

Solving Eq. A.6 for v_0 and using Eq. A.8 with the continuity conditions at t_M , we obtain

$$v_0(t) = v_{01} - (t - t_1) + \frac{3}{\xi^2} [t - \mu(\xi)] + \frac{4}{3} \int_{t_1}^{\mu(\xi)} \psi^2 dt. \quad (\text{A.9})$$

The substitution of Eq. A.9 into the first of Eqs. A.2 then gives the following differential equation for ξ :

$$\frac{3}{\xi^2} [t - \mu(\xi)] \frac{d\xi}{dt} + 2\psi - \frac{3}{\xi} = 0. \quad (\text{A.10})$$

Equation A.10 can be written as a perfect differential having the standard form

$$X(\xi, t) d\xi + Y(\xi, t) dt = 0, \quad (\text{A.11})$$

with

$$X = \frac{3}{\xi^2}[t - \mu(\xi)], \quad Y = 2\psi(t) - \frac{3}{\xi}, \quad \text{and} \quad \frac{\partial X}{\partial t} = \frac{\partial Y}{\partial \xi}. \quad (\text{A.12})$$

Consequently, following the usual integration procedure and applying the continuity requirements at t_M , we arrive at the transcendental equation relating ξ and t ,

$$3[\mu(\xi) - t] + 2\xi \int_{\mu(\xi)}^t \psi \, dt = 0. \quad (\text{A.13})$$

APPENDIX B

Local Series Solution of Differential Equations

The computer program RINGLOAD uses series solutions to the coupled nonlinear differential equations (Eqs. 16) in the vicinity of t_y and t_f , since v_0 vanishes at these times and the second of Eqs. 16 becomes indeterminate. The series solution at $t = t_y$ is used to start the computer computation by calculating results at $t = t_y + \delta t$. The subroutine DIFSUB, which uses the Bulirsch-Stoer extrapolation method, is used to carry the computation forward until either the pulse ends and the closed-form solution (Eqs. 31) is applicable or the velocity v_0 becomes small. In the latter case, a series solution is then used to bring the calculation up to the final time t_f at which the motion stops.

Consider a time t^* when all the dependent variables are known quantities, and let $t^* + \delta t$ be a time when values of the dependent variables are desired. Assume δt is small enough that power series solutions of Eqs. 16 in δt are convergent. Define the auxiliary function v_{aux} by

$$v_{aux}(t) = \frac{v_0(t)}{\xi(t)}, \quad (B.1)$$

so that

$$v(z,t) = v_0(t) - zv_{aux}(t), \quad 0 \leq z \leq \xi(t). \quad (B.2)$$

Let

$$\left. \begin{aligned} \psi(t^* + \delta t) &\approx \sum_{j=0}^n p_j(\delta t)^j, \\ \xi(t^* + \delta t) &\approx \sum_{j=0}^n a_j(\delta t)^j, \\ v_0(t^* + \delta t) &\approx \sum_{j=0}^n b_j(\delta t)^j, \end{aligned} \right\} \quad (B.3)$$

and

$$v_{aux}(t^* + \delta t) \approx \sum_{j=0}^n c_j(\delta t)^j,$$

where

$$p_0 = \psi(t^*), \quad a_0 = \zeta(t^*), \quad b_0 = v_0(t^*), \quad \text{and} \quad c_0 = v_{\text{aux}}(t^*) = \frac{b_0}{a_0}. \quad (\text{B.4})$$

From Eqs. B.1 and B.3 and the third of Eqs. 16, the series for the radial displacement is then given by

$$\left. \begin{aligned} u(z, t^* + \delta t) &= u(z, t^*) + \sum_{j=0}^n (b_j - z c_j) \frac{(\delta t)^{j+1}}{j+1}, \\ 0 \leq z &\leq \zeta(t^* + \delta t), \end{aligned} \right\} \quad (\text{B.5})$$

and

$$u(z, t^* + \delta t) = u(z, t^*), \quad z > \zeta(t^* + \delta t).$$

The series for ψ is assumed to be known from the given load-time function. From Eqs. B.1 and B.3, the coefficients of the series for v_{aux} are given in terms of the coefficients of the series for ζ and v_0 by

$$c_n = \frac{1}{a_0} \left(b_n - \sum_{j=1}^n a_j c_{n-j} \right). \quad (\text{B.6})$$

Substituting from Eqs. B.3 for ψ , ζ , and v_0 into the first of Eqs. 16 and equating coefficients of like powers of δt , we obtain the following relations:

$$\left. \begin{aligned} a_0 b_0 a_1 + a_0^2 + 2a_0 p_0 - 3 &= 0, \\ 2a_0 b_0 a_2 + b_0 a_1^2 + a_0 a_1 b_1 + 2a_0 a_1 + 2a_1 p_0 + 2a_0 p_1 &= 0, \\ 3a_0 b_0 a_3 + 3b_0 a_1 a_2 + 2a_0 b_1 a_2 + a_1^2 b_1 + a_0 a_1 b_2 + 2a_0 a_2 \\ &+ a_1^2 + 2a_2 p_0 + 2a_1 p_1 + 2a_0 p_2 = 0, \\ 4a_0 b_0 a_4 + 4b_0 a_1 a_3 + 3a_0 b_1 a_3 + 2b_0 a_2^2 + 3a_1 b_1 a_2 + 2a_0 a_2 b_2 \\ &+ a_1^2 b_2 + a_0 a_1 b_3 + 2a_0 a_3 + 2a_1 a_2 + 2a_3 p_0 + 2a_2 p_1 \\ &+ 2a_1 p_2 + 2a_0 p_3 = 0, \end{aligned} \right\} \quad (\text{B.7})$$

and

$$\left. \begin{aligned} 5a_0 b_0 a_5 + 5b_0 a_1 a_4 + 4a_0 b_1 a_4 + 5b_0 a_2 a_3 + 4a_1 b_1 a_3 + 3a_0 b_2 a_3 \\ &+ 2b_1 a_2^2 + 3a_1 a_2 b_2 + 2a_0 a_2 b_3 + a_1^2 b_3 + a_0 a_1 b_4 + 2a_0 a_4 \\ &+ 2a_1 a_3 + a_2^2 + 2a_4 p_0 + 2a_3 p_1 + 2a_2 p_2 + 2a_1 p_3 + 2a_0 p_4 = 0. \end{aligned} \right\}$$

The application of the same procedure to the second of Eqs. 16 gives the following relations:

$$\left. \begin{aligned}
 a_0^2 b_1 + a_0^2 - 4a_0 p_0 + 3 &= 0, \\
 a_0^2 b_2 + a_0 a_1 b_1 + a_0 a_1 - 2a_1 p_0 - 2a_0 p_1 &= 0, \\
 3a_0^2 b_3 + 4a_0 a_1 b_2 + 2a_0 b_1 a_2 + a_1^2 b_1 + 2a_0 a_2 + a_1^2 - 4a_2 p_0 \\
 - 4a_1 p_1 - 4a_0 p_2 &= 0, \\
 2a_0^2 b_4 + 3a_0 a_1 b_3 + 2a_0 a_2 b_2 + a_1^2 b_2 + a_0 b_1 a_3 + a_1 b_1 a_2 + a_0 a_3 \\
 + a_1 a_2 - 2a_3 p_0 - 2a_2 p_1 - 2a_1 p_2 - 2a_0 p_3 &= 0, \\
 5a_0^2 b_5 + 8a_0 a_1 b_4 + 6a_0 a_2 b_3 + 3a_1^2 b_3 + 4a_0 a_3 b_2 + 4a_1 a_2 b_2 \\
 + 2a_0 b_1 a_4 + 2a_1 b_1 a_3 + b_1 a_2^2 + 2a_0 a_4 + 2a_1 a_3 + 2a_2^2 \\
 - 4a_4 p_0 - 4a_3 p_1 - 4a_2 p_2 - 4a_1 p_3 - 4a_0 p_4 &= 0.
 \end{aligned} \right\} \quad (B.8)$$

and

At the beginning of the deformation, when $t^* = t_y$, the shell is at rest, so that $v_0(t_y) = 0$. If ψ passes gradually through the yield value of unity, the initial hinge-circle location is $\zeta(t_y) = 1$. However, if ψ jumps instantaneously through yield to a value ψ_M greater than unity, then $\zeta(t_y) = z_0$ given by Eqs. 30. Consequently, the two possible initial cases for $t^* = t_y$ are

$$b_0 = c_0 = 0, \quad a_0 = p_0 = 1, \quad (B.9)$$

and

$$b_0 = c_0 = 0, \quad p_0 = \psi_M, \quad a_0 = \sqrt{\psi_M^2 + 3} - \psi_M \quad (B.10)$$

The remaining a_j and b_j coefficients are found from Eqs. B.7 and B.8 to be

$$\left. \begin{aligned}
 b_1 &= \frac{-1}{a_0^2} (a_0^2 - 4a_0 p_0 + 3), \\
 a_1 &= \frac{2a_0 p_1}{k_1}, \\
 b_2 &= \frac{-1}{a_0^2} [a_0 a_1 (b_1 + 1) - 2(a_1 p_0 + a_0 p_1)],
 \end{aligned} \right\} \quad (B.11)(\text{Contd.})$$

$$\begin{aligned}
a_2 &= \frac{1}{k_2} [a_0 a_1 b_2 + a_1^2 (b_1 + 1) + 2(a_1 p_1 + a_0 p_2)], \\
b_3 &= \frac{-1}{3a_0^2} [4a_0 a_1 b_2 + (2a_0 a_2 + a_1^2)(b_1 + 1) - 4(a_2 p_0 + a_1 p_1 + a_0 p_2)], \\
a_3 &= \frac{1}{k_3} [a_0 a_1 b_3 + (2a_0 a_2 + a_1^2) b_2 + a_1 a_2 (3b_1 + 2) \\
&\quad + 2(a_2 p_1 + a_1 p_2 + a_0 p_3)], \\
b_4 &= \frac{-1}{2a_0^2} [3a_0 a_1 b_3 + (2a_0 a_2 + a_1^2) b_2 + (a_0 a_3 + a_1 a_2)(b_1 + 1) \\
&\quad - 2(a_3 p_0 + a_2 p_1 + a_1 p_2 + a_0 p_3)], \\
a_4 &= \frac{1}{k_4} [a_0 a_1 b_4 + (2a_0 a_2 + a_1^2) b_3 + 3(a_0 a_3 + a_1 a_2) b_2 \\
&\quad + (2a_1 a_3 + a_2^2)(2b_1 + 1) + 2(a_3 p_1 + a_2 p_2 + a_1 p_3 + a_0 p_4)],
\end{aligned}
\tag{B.11}$$

and

$$\begin{aligned}
b_5 &= \frac{-1}{5a_0^2} [8a_0 a_1 b_4 + 3(2a_0 a_2 + a_1^2) b_3 + 4(a_0 a_3 + a_1 a_2) b_2 \\
&\quad + (2a_0 a_4 + 2a_1 a_3 + a_2^2)(b_1 + 1) \\
&\quad - 4(a_4 p_0 + a_3 p_1 + a_2 p_2 + a_1 p_3 + a_0 p_4)],
\end{aligned}$$

where

$$k_n = \frac{1}{a_0} [a_0^2 (n-2) - 2a_0 p_0 (2n+1) + 3n]. \tag{B.12}$$

If the initial conditions are given by Eqs. B.9, the first of Eqs. B.11 yields $b_1 = 0$, so that the $(\delta t)^2$ terms are the first nonvanishing terms in the power series for the velocity. Consequently, the numerical integration of the second of Eqs. 16 is initially rather slowly convergent if ψ gradually passes through the yield value. On the other hand, $b_1 \neq 0$ for the initial conditions (Eqs. B.10), so that problems with an instantaneous jump at t_y are easier to integrate numerically near t_y . This difference in the rapidity of convergence near t_y shows up as shorter running times for problems with an initial jump.

If the shell is in motion at t^* , then $b_0 \neq 0$ and the five equations in each of Eqs. B.7 and B.8 are easily solved for $a_1, \dots, a_5$ and $b_1, \dots, b_5$, respectively, in terms of values of the functions at t^* .

In extrapolating to $t = t_f$, we use the numerical integration scheme until v_0 becomes small compared to its maximum during the deformation. Let t^* be the time at which this small value is attained and at which the power-series coefficients are evaluated. Let

$$\left. \begin{array}{l} t_f = t^* + \delta t, \\ \text{and} \\ v_0(t_f) = 0 \approx \sum_{j=0}^n b_j(\delta t)^j, \end{array} \right\} \quad (\text{B.13})$$

where δt is to be determined. We can approximate δt by

$$\delta t \approx - \frac{2b_0}{b_1 + \sqrt{b_1^2 - 4b_0b_2}}. \quad (\text{B.14})$$

The value of v_0 at $t^* + \delta t$ is found from the power series, and if less than some selected constant ϵ_v , the computation is concluded. If v_0 is greater than ϵ_v , the extrapolation procedure is repeated starting from the new value.

APPENDIX C

Solution Behavior in the Vicinity of Instantaneous Jumps in ψ

In the vicinity of instantaneous jumps in ψ , there is the possibility of corresponding instantaneous jumps in the values of some of the dependent variables. It would be advantageous then to use the numerical-integration scheme up to the jump, make the appropriate change in the values of the variables, and continue on with the numerical integration, rather than trying to integrate through the jump, where the convergence of the solution may be poor. A study of the local power-series solution for the problem reveals that the dependent variables are continuous functions if no hinge-band formation occurs. If a hinge band already exists or is produced by the jump in ψ , then a corresponding jump in ξ occurs.

At $t = t^*$, let ψ be such that

$$\begin{aligned}\psi(t^* + \delta t) &= p_0 + p_1 \delta t \\ &= \psi(t^*) + \delta \psi.\end{aligned}\tag{C.1}$$

We wish to determine what happens to the solution of the differential equations as $\delta t \rightarrow 0$ with $\delta \psi$ kept constant.

Consider first the case when no hinge band exists and Eqs. 16 are appropriate. The series solutions for ξ and v_0 are given in Appendix B. By Eqs. B.7 and B.8, we can write

$$\text{and } \left. \begin{aligned} a_n &= a_{n0}p_0 + a_{n1}p_1, \\ b_n &= b_{n0}p_0 + b_{n1}p_1, \end{aligned} \right\} \tag{C.2}$$

where a_{n0} , a_{n1} , b_{n0} , and b_{n1} are independent of p_0 and p_1 . From Eqs. B.7 and B.8,

$$a_{01} = a_{11} = b_{01} = b_{11} = 0, \tag{C.3}$$

so that the series for ξ and v_0 can be written

$$\xi(t^* + \delta t) = \xi(t^*) + a_{10}\delta t + \sum_{j=2}^n (a_{j0}p_0 + a_{j1}p_1)(\delta t)^j,$$

and

$$v_0(t^* + \delta t) = v_0(t^*) + b_{10}\delta t + \sum_{j=2}^n (b_{j0}p_0 + b_{j1}p_1)(\delta t)^j. \tag{C.4}$$

It is apparent then that

$$\xi(t^* + \delta t) \rightarrow \xi(t^*), \text{ and } v_0(t^* + \delta t) \rightarrow v_0(t^*) \text{ as } \delta t \rightarrow 0, \quad (C.5)$$

keeping $p_1 \delta t$ constant. It is assumed in this analysis that

$$[\psi(t^*) + \delta \psi] \xi(t^* + \delta t) \leq 3/2. \quad (C.6)$$

If Inequality C.6 does not hold, that is, if the instantaneous jump produces a hinge band, the instantaneous jump may be decomposed into two parts by

$$\left. \begin{aligned} \psi(t^* + \delta t) &= p_0 + p_1' \delta t' + p_1'' \delta t'', \\ \delta t &= \delta t' + \delta t'', \end{aligned} \right\} \quad (C.7)$$

and

with $\delta t'$ determined from

$$(p_0 + p_1 \delta t') \xi(t^*) = 3/2. \quad (C.8)$$

Letting $\delta t' \rightarrow 0$ carries the deformation up to the point of producing a hinge band. The remainder of the instantaneous jump, produced by letting $\delta t'' \rightarrow 0$ with $p_1'' \delta t''$ kept constant, then occurs while a hinge band exists. This latter case will be considered next.

The governing differential equations when a hinge band exists are Eqs. A.2. If ψ is increasing, ξ is found from Eq. 21 and Ω is determined from Eq. 27; if ψ is decreasing, Ω is now a known function and Eq. 27 gives a relation between ξ and v_ξ . Define v_{aux} now by

$$v_{aux}(t) = \frac{v_0(t) - v_\xi(t)}{\xi(t)}, \quad (C.9)$$

and, as in Eq. C.1, let a jump of $\delta \psi$ occur in ψ at $t = t^*$. Employing a similar series analysis to that used previously, we find for $\delta \psi$ positive that v_0 , v_{aux} , and η are continuous at t^* , while ξ jumps to $3/\{2[\psi(t^*) + \delta \psi]\}$ and the corresponding jump in v_ξ is found from Eq. C.9. The portion of Ω generated by the jump in ψ is given by

$$\left. \begin{aligned} \Omega(z) &= v_0(t^*) - v_{aux}(t^*) z + t^*, \\ \text{for } \xi(t^*) &\geq z \geq \frac{3}{2[\psi(t^*) + \delta \psi]}. \end{aligned} \right\} \quad (C.10)$$

On the other hand, if $\delta\psi$ is negative, then v_0 , v_{aux} , v_ξ , ξ , and η are continuous at t^* .

Since η , the outer edge of the hinge band, is always a continuous function, the length of the region of plastic deformation is also a continuous function of time and does not vary abruptly, even with severe load changes.

APPENDIX D

Computer Programs

The dynamic plastic deformation of a tube acted on by a ring load of arbitrary pulse shape is computed by the CDC-3600 program RINGLOAD and its auxiliary subroutines DIFSUB,⁶ DIFFUN, OMEGA, OMEGINV, ZETAEQ, PULSEINF, PSIFUN, and PSICOEF. The subroutines PULSEINF, PSIFUN, and PSICOEF are specialized to the particular form of the pressure pulse; RINGLOAD and the other subroutines do not depend on the pulse shape. Two versions of the set PULSEINF, PSIFUN, and PSICOEF are given here. The Piecewise Linear Version treats pulses that are specified by a number of t, ψ pairs and uses linear interpolation to obtain values of ψ for times between the prescribed points. This version can be used to handle any pulse shape by specifying a sufficient number of data points, and its use is recommended for most problems. If, however, a parameter study is to be made for a number of problems involving the same general pulse form and this form is tedious to approximate by a piecewise-linear function, it may be advantageous to write special subroutines for the pulse form. This was done for the case of a pulse that rises linearly to its maximum and then decays exponentially, that is,[†]

$$\left. \begin{aligned} \psi &= \psi_M \frac{(t - \tau_0)}{(\tau_M - \tau_0)}, & \tau_0 \leq t \leq \tau_M; \\ &= \psi_M e^{-(t - \tau_M)/\tau}, & \tau_M < t \leq \tau_F; \\ &= 0, & t > \tau_F. \end{aligned} \right\} \quad (D.1)$$

The subroutines PULSEINF, PSIFUN, and PSICOEF for a pulse defined by Eqs. D.1 are given as the Exponential Decay Version. Problems of this type can also be handled by the Piecewise Linear Version if an adequate number of points on the exponential decay portion of the pulse are used as input data.

The input needed for a computation consists of two parts: input for the program RINGLOAD and input for the subroutine PULSEINF.

1. Input for RINGLOAD

Card 1. FORMAT (10A8)

Problem name or identifier (Anything may be punched on this card and will be printed at the beginning of the output.)

[†]The quantities τ_0 and τ_M may be made equal to produce an instantaneous jump to the pulse maximum.

Card 2. FORMAT (I2, 2X, I2, 2X, I2)

ISCALE: If zero, results are not scaled. If one, results are scaled by the appropriate power of the impulse i , as explained below.

NTSINT: $\Delta t/i = 1/\text{NTSINT}$ is the scaled time interval at which the output is printed. NTSINT must be at least 10; 20 is a convenient value for most problems.

NZINT: $\Delta z = 1/\text{NZINT}$ is the axial interval at which radial displacements are printed. NZINT must not exceed 25; 10 or 20 gives an adequate number of points for a graph. If NZINT is zero, then $z = 0$ is the only location at which the displacement is computed.

2. Input for PULSEINF. Piecewise Linear Version

Card 3. FORMAT (I2)

NJ: Number of input data points. NJ must not exceed 99.

Cards 4, 5, 6, ..., NJ + 3 FORMAT (2E15.7)

TJ, PSIJ: Input data pairs t_j, ψ_j .

3. Input for PULSEINF. Exponential Decay Version

Card 3. FORMAT (5E15.7)

TAUO, TAUM, TAUF, TAU, PSIM: Parameters in pulse shape, $\tau_0, \tau_M, \tau_F, \tau, \psi_M$, as in Eqs. D.1.

The printed output of the program includes first some general parameters associated with the pulse shape, such as t_y, t_m, i, ψ_e , and the total area under the pulse curve. The values of t_m, i , and ψ_e that are printed are based on integrals of the pulse from t_y to the end of the pulse. If the motion stops before the end of the pulse,[†] it is advisable to rerun the problem, chopping off the pulse input values at approximately t_f as given by the first run;^{††} then the integrals will be taken over the interval t_y to t_f as required in Eqs. 7. For many problems in which the motion stops before the end of the pulse, the pulse has decayed enough that the difference in the values of t_m, i , and ψ_e between integrating to the end of the pulse and integrating to the end of the motion is negligible.

[†]A statement announcing this is printed at the end of the output.

^{††}If the pulse data input is cut off at exactly t_f , there will be convergence problems in the final time step of the second problem run. Chopping at a time value equal to t_f to the first few significant figures avoids this difficulty and is accurate enough for the parameter evaluations.

The output that gives the history of the deformation is t , $\psi(t)$, $u_0(t)$, $v_0(t)$, $\xi(t)$, $\eta(t)$, NOFNS, and $u(n\Delta z, t)$, $n = 1, 2, 3, \dots$, if ISCALE = 0; the printouts are at intervals of Δt plus intermediate times when results are calculated for program requirements. The quantity NOFNS is the number of evaluations of the right sides of the differential equations and is an indication of the difficulty of convergence of the numerical integration procedure.

If ISCALE = 1, the quantities printed are t/i , ψ , u_0/i^2 , v_0/i , $\xi(t)$, $\eta(t)$, NOFNS, and $u(n\Delta z, t)/i^2$. The scaling by powers of i is such that the arbitrary time constant T_0 appearing in Eqs. 1 is eliminated, as well as the arbitrary factor in the time scale of the pulse, as indicated in Eqs. 44.

From Eqs. 16 we see that $\partial u / \partial t$ is found differently, depending on whether z is greater or less than ξ . In RINGLOAD, if a time step carries ξ past an axial position z where the deformation is desired, the program interpolates back to find the time step for which ξ and this z -location coincide; then each z at which the deformation is computed is either inside or outside of ξ for the time step, and Eqs. 16 can be applied.

The function $\Omega(t)$ mapped out through the use of Eq. 27 is stored as a pair of vectors ZL(JZ), OMEGAL(JZ). Linear interpolation is used to obtain values intermediate to those contained in the lists; that is, Ω is assumed to be a linear function of z between the stored values.

```

C      PROGRAM RINGLOAD
C
C      DYNAMIC PLASTIC DEFORMATION OF A CIRCULAR CYLINDRICAL SHELL
C      PRODUCED BY A RING LOAD WITH AN ARBITRARY PULSE SHAPE, PSI(T).
C
C      AUXILIARY PROGRAMS REQUIRED ARE ANL LIBRARY SUBROUTINE DIFSUB
C      AND SPECIAL SUBROUTINES PULSEINF, PSIFUN, PSICOE, DIFFUN,
C      OMEGA, OMEGINV, AND ZETAEQ.
C
C      RINGLOAD, DIFSUB, DIFFUN, OMEGA, OMEGINV, AND ZETAEQ ARE
C      GENERAL PROGRAMS APPROPRIATE TO ANY PULSE SHAPE, WHILE
C      PULSEINF, PSIFUN, AND PSICOE MUST BE WRITTEN FOR THE
C      PARTICULAR FORM OF THE PULSE. GENERAL PURPOSE VERSIONS OF
C      PULSEINF, PSIFUN, AND PSICOE HAVE BEEN PROGRAMMED FOR THE
C      CASE WHERE PULSE VALUES ARE PRESCRIBED AT A SUFFICIENT NUMBER
C      OF TIME POINTS SUCH THAT LINEAR INTERPOLATION BETWEEN VALUES
C      IS PERMISSIBLE.
C
C      REQUIRED INPUT FROM DATA CARDS.
C      CARD 1.  FORMAT(10A8)
C      NPROB -- PROBLEM NAME OR IDENTIFIER.
C      CARD 2.  FORMAT(I2,2X,I2,2X,I2)
C      ISCALE -- IF ZERO, RESULTS ARE NOT SCALED. IF ONE, RESULTS
C      ARE SCALED BY THE APPROPRIATE POWER OF THE PLASTIC IMPULSE.
C      NTSINT -- 1/NTSINT IS THE SCALED TIME INTERVAL AT WHICH
C      OUTPUT IS DESIRED. NTSINT MUST BE .GE. 10.
C      NZINT -- 1/NZINT IS THE AXIAL INTERVAL AT WHICH DISPLACEMENTS
C      ARE DESIRED. NZINT MUST BE .LE.25. IF NZINT=0, THEN Z=0 IS
C      THE ONLY LOCATION AT WHICH THE DISPLACEMENT IS COMPUTED.
C
C      PLUS INPUT REQUIRED FOR SUBROUTINE PULSEINF.
C
C      PRINTED OUTPUT
C      T -- TIME.
C      PSI(T) -- PULSE VALUE.
C      U(0,T) -- RADIAL DISPLACEMENT AT Z=0.
C      V(0,T) -- RADIAL VELOCITY AT Z=0.
C      ZETA(T) -- LOCATION OF HINGE CIRCLE OR OUTER EDGE OF HINGE BAND.
C      ETA(T) -- LOCATION OF INNER EDGE OF HINGE BAND.
C      NOFNS -- NUMBER OF USES OF SUBROUTINE DIFFUN. (GIVES AN
C      INDICATION OF THE RAPIDITY OF CONVERGENCE OF THE SOLUTION.)
C      U(Z,T) -- RADIAL DISPLACEMENT AT Z=N/NZINT, N=1,2,3,....
C
C      IN THE BODY OF THE PROGRAM, PHASE 1 REFERS TO DEFORMATION
C      WITH A HINGE CIRCLE AT ZETA, WHILE PHASE 2 REFERS TO
C      DEFORMATION WITH A HINGE BAND BETWEEN ZETA AND ETA.
C
C      DIMENSION TMAX(50),PSIMAX(50),TMAXS(50),U(50),US(50)
C      DIMENSION F(4),DF(4),FMAX(4),FF(4),RLTVER(4),NPROB(10)
C      DIMENSION TJMP(50),PSIJMP(50),TJMPS(50),UF(50),SPSI(4)
C      COMMON/COMDIF/NOFNS,KPHASE,TAUJMP,PJMP
C      COMMON/COMOMEG/JOM,OMEGAL(1001),ZL(1001)
1  READ 170,NPROB
   IF(EOF,60)2,3
2  STOP
3  PRINT 171,NPROB
   READ 172,ISCALE,NTSINT,NZINT
   CALL PULSEINF(TO,TF,TYIELD,TOTIMP,PLASIMP,TMEAN,NOMAX,TMAX,
1  PSIMAX,INSTJMP,NOJMP,TJMP,PSIJMP,KSTOP)
   IF(KSTOP)4,5
4  PRINT 173,KSTOP
   GO TO 1

```

```

C***** DATA INPUT, PARAMETER DETERMINATIONS, AND ASSOCIATED PRINTOUTS
5 IF(INSTJMP)6,7
6 PRINT 174,INSTJMP
7 EFFLOAD=PLASIMP/(2.0*TMEAN)
PRINT 175,TO,TF,TYIELD,TOTIMP,PLASIMP,TMEAN,EFFLOAD
8 TOS=TO/PLASIMP
TFS=TF/PLASIMP
TYIELDS=TYIELD/PLASIMP
TMEANS=TMEAN/PLASIMP
PRINT 176,TOS,TFS,TYIELDS,TMEANS
9 DO 10 K=1,NOMAX
10 TMAXS(K)=TMAX(K)/PLASIMP
PRINT 177,(K,TMAX(K),PSIMAX(K),TMAXS(K),K=1,NOMAX)
IF(NOJMP.EQ.0) GO TO 16
DO 15 K=1,NOJMP
15 TJMPS(K)=TJMP(K)/PLASIMP
PRINT 180,(K,TJMP(K),PSIJMP(K),TJMPS(K),K=1,NOJMP)
16 CONTINUE
DTS=1.0/NTSINT
DT=DTS*PLASIMP
IF(NZINT)11,12
11 DZ=1.0/NZINT
GO TO 13
12 DZ=0.0
13 PRINT 178,DT,DTS,DZ
C**** CHECK SIZE OF SELECTED DTS
IF(NTSINT.GE.10) GO TO 17
PRINT 179
GO TO 1
C**** CHECK SIZE OF SELECTED DZ
17 IF(NZINT.LE.25) GO TO 18
PRINT 181
GO TO 1
C**** PRINT OUTPUT HEADINGS
18 IF(ISCAL)19,20
19 PRINT 182
IF(NZINT) PRINT 183
GO TO 21
20 PRINT 184
IF(NZINT) PRINT 185
21 CONTINUE
C***** INITIALIZATION OF U(Z,T)
NZTOT=1.73205*NZINT
DO 22 IZ=1,NZTOT
U(IZ)=0.0
US(IZ)=0.0
22 CONTINUE
C***** INITIAL VALUES AT TYIELD
UO=UOS=VO=VOS=UAUX=VAUX=0.0
NOFNS=0
PSIO=PSIFUN(TO)
ZETA0=1.0
IF(ISCAL) GO TO 23
PRINT 187,TO,PSIO,UO,VO,ZETA0,NOFNS
GO TO 24
23 PRINT 187,TOS,PSIO,UOS,VOS,ZETA0,NOFNS
24 PSI=PSIFUN(TYIELD)
IF(INSTJMP) GO TO 25
ZETA=1.0
GO TO 26
25 ZETA=SQRTF(3.0+PSI**2)-PSI
26 IF(ISCAL) GO TO 27

```



```

PRINT 187,TYIELD,PSI,UO,VO,ZETA,NOFNS
PRINT 190,(U(IZ),IZ=1,NZTOT)
GO TO 28
27 PRINT 187,TYIELDS,PSI,UOS,VOS,ZETA,NOFNS
PRINT 190,(US(IZ),IZ=1,NZTOT)
28 CONTINUE
C***** SOLUTION BETWEEN TYIELD AND TI, TI NEAR TYIELD
C**** USE POWER SERIES SOLUTION OF DIFFERENTIAL EQUATIONS NEAR TYIELD.
C**** CALCULATE COEFFICIENTS IN POWER SERIES EXPANSIONS.
C**** ZETA = AC + A1*D + A2*D**2 +..., D = T-TYIELD
C**** VO = B0 + B1*D + B2*D**2 +...
C**** VAUX = C0 + C1*D + C2*D**2 +...
C**** PSI = P0 + P1*D + P2*D**2 +...
CALL PSICOF(TYIELD,SPSI)
PG=PSI
P1=SPSI(1)
P2=SPSI(2)
P3=SPSI(3)
P4=SPSI(4)
AQ=ZETA
B0=C0=0.0
B1=-(AQ**2+3.0-4.0*AQ*PG)/AQ**2
C1=B1/AQ
A1=-2.0*AQ*P1/(B1*AQ+2.0*P0+2.0*AC)
B2=-(B1*A1*AQ-2.0*A1*P0-2.0*AQ*P1+A1*AQ)/AQ**2
C2=(B2-C1*A1)/AQ
A2=-(B2*A1*AQ+B1*A1**2+2.0*A1*P1+2.0*AQ*P2+A1**2)/(2.0*(B1*AQ+P0
1 +AQ))
B3=-(4.0*B2*A1*AQ+2.0*B1*A2*AQ+B1*A1**2-4.0*A2*P0-4.0*A1*P1
1 -4.0*AQ*P2+2.0*A2*AQ+A1**2)/(3.0*AQ**2)
C3=(B3-C2*A1-C1*A2)/AQ
A3=-(B3*A1*AQ+2.0*B2*A2*AQ+B2*A1**2+3.0*B1*A2*A1+2.0*(A2*P1+A1*P2
1 +AQ*P3+A2*A1))/(3.0*B1*AQ+2.0*P0+2.0*AQ)
B4=-(3.0*B3*A1*AQ+2.0*B2*A2*AQ+B2*A1**2+B1*A3*AQ+B1*A2*A1
1 -2.0*(A3*P0+A2*P1+A1*P2+AQ*P3)+A3*AQ+A2*A1)/(2.0*AQ**2)
C4=(B4-C3*A1-C2*A2-C1*A3)/AQ
A4=-(B4*A1*AQ+B3*A1**2+2.0*B3*A2*AQ+3.0*B2*A2*A1+3.0*B2*A3*AQ
1 +2.0*B1*A2**2+4.0*B1*A3*A1+2.0*(A3*P1+A2*P2+A1*P3+AQ*P4)
2 +2.0*A3*A1+A2**2)/(2.0*(2.0*B1*AQ+P0+AQ))
B5=-(8.0*B4*A1*AQ+6.0*B3*A2*AQ+3.0*B3*A1**2+4.0*B2*A3*AQ
1 +4.0*B2*A2*A1+2.0*B1*A4*AQ+2.0*B1*A3*A1+B1*A2**2-4.0*(A4*P0
2 +A3*P1+A2*P2+A1*P3+AQ*P4)+2.0*A4*AQ+2.0*A3*A1+A2**2)/
3 (5.0*AQ**2)
C5=(B5-C4*A1-C3*A2-C2*A3-C1*A4)/AQ
C**** FIND TIME TI AT WHICH PSI DIFFERS FROM PSI AT TYIELD BY AT MOST
C**** THREE PERCENT.
EPSSER=0.03
DELT=0.01*TMEAN
37 TI=TYIELD+DELT
PSII=PSIFUN(TI)
EPSI=ABS(PSII/P0-1.0)
IF(EPSI.LE.EPSSER) GO TO 38
DELT=DELT*EPSSER/EPSI
GO TO 37
C**** EVALUATE POWER SERIES AT TI
38 ZETA=AQ+A1*DELT+A2*DELT**2+A3*DELT**3+A4*DELT**4
VO=B0+B1*DELT+B2*DELT**2+B3*DELT**3+B4*DELT**4+B5*DELT**5
VAUX=C0+C1*DELT+C2*DELT**2+C3*DELT**3+C4*DELT**4+C5*DELT**5
UO=U0+B0*DELT+B1*DELT**2/2.0+B2*DELT**3/3.0+B3*DELT**4/4.0
1 +B4*DELT**5/5.0+B5*DELT**6/6.0
UAUX=UAUX+C0*DELT+C1*DELT**2/2.0+C2*DELT**3/3.0+C3*DELT**4/4.0
1 +C4*DELT**5/5.0+C5*DELT**6/6.0

```

```

C**** PRINT RESULTS AT TI
      IF(ISCALE) GO TO 39
      PRINT 188,TI,PSII,UO,VO,ZETA,0
      GO TO 40
39  TIS=TI/PLASIMP
      VOS=VO/PLASIMP
      UOS=UO/PLASIMP**2
      PRINT 188,TIS,PSII,UOS,VOS,ZETA,0
40  IF(NZINT)41,50
C**** CALCULATION OF U(Z,TI)
41  NZ=NZINT
42  IF(NZ*DZ.LE.ZETA) GO TO 43
      NZ=NZ-1
      GO TO 42
43  DO 44 IZ=1,NZ
44  U(IZ)=UO-IZ*DZ*UAUX
      IF(ISCALE) GO TO 45
      PRINT 189,(U(IZ),IZ=1,NZTOT)
      GO TO 50
45  DO 46 IZ=1,NZ
46  US(IZ)=U(IZ)/PLASIMP**2
      PRINT 189,(US(IZ),IZ=1,NZTOT)
50  CONTINUE
C***** SCHEDULED PRINTOUTS AT TMAX(K), TJMP(K), TF, AND INTEGER
C***** MULTIPLES OF DT
      DO 51 JMAX=1,NOMAX
      IF(TMAX(JMAX).GT.TI) GO TO 52
51  CONTINUE
      JMAX=NOMAX+1
52  DO 53 JJMP=1,NOJMP
      IF(TJMP(JJMP).GT.TI) GO TO 54
53  CONTINUE
      JJMP=NOJMP+1
54  DO 55 IT=1,1000
      IF(IT*DT.GT.TI) GO TO 56
55  CONTINUE
      GO TO 1
56  CONTINUE
C***** SOLUTION BETWEEN TI AND TF
C**** USE SUBROUTINES DIFSUB AND DIFFUN
C**** F(1)=VO, F(2)=ZETA, F(3)=UOINC, F(4)=UAINC
C**** INITIALIZATIONS FOR DIFSUB
      F(1)=VO
      F(2)=ZETA
      F(3)=F(4)=FF(3)=FF(4)=0.0
      NEQ=4
      NOFNS=0
      MORD=6
      METH=0
      KKSTP=+1
      KPHASE=0
      DTMIN=0.000001*PLASIMP
      EPSERR=0.00001
      EPSZ=0.00001
      EPSPR=0.000001
      EPSRV=0.001
      EPSV1=0.01
      EPSV2=0.000001
      NZZ=NZ
      DO 57 I=1,4
57  FMAX(I)=F(I)
      T=TI

```

C**** SELECTION OF NEXT SCHEDULED PRINTOUT TIME TPR

```

58 CONTINUE
   IF(JJMP.GT.NOJMP) 59,60
59 TAUJMP=2.0*PLASIMP
   PJMP=0.0
   GO TO 61
60 TAUJMP=TJMP(JJMP)
   PJMP=PSIJMP(JJMP)
61 IF(JMAX.GT.NOMAX) 62,63
62 TAUMAX=2.0*PLASIMP
   PMAX=0.0
   GO TO 64
63 TAUMAX=TMAX(JMAX)
   PMAX=PSIMAX(JMAX)
64 TDT=IT*DT
   TPR=AMIN1(TDT,TAUJMP,TAUMAX)
   IF(TPR.GT.TF) TPR=TF

```

C**** INTEGRATION OF NONLINEAR DIFFERENTIAL EQUATIONS USING DIFSUB

```

70 H=TPR-T
71 TT=T
   FF(1)=F(1)
   FF(2)=F(2)
   F(3)=F(4)=0.0
   CALL DIFSUB(NEQ,T,F,DF,H,DTMIN,EPSERR,MORD,METH,FMAX,RLTVER,KKSTP)
   PSI=PSIFUN(T)
   IF(T.EQ.TAUJMP) PSI=PSI-PJMP
   IF(KKSTP.GE.0) 73,72
72 PRINT 191,T
   GO TO 1
73 VO=F(1)
   ZETA=F(2)
   UOINC=F(3)
   UAINC=F(4)
   TS=T/PLASIMP
   VOS=VO/PLASIMP
   IF(NZINT.EQ.0) GO TO 74
   IF(ZETA.LE.NZ*DZ) GO TO 90
   IF(ZETA.GE.(NZ+1)*DZ) GO TO 95
74 IF(PSI*ZETA.GT.1.5) GO TO 122
   IF(VO.LT.0.0) GO TO 100
75 UO=UO+UOINC
   UOS=UO/PLASIMP**2
   IF(NZINT) 76,78
76 DO 77 IZ=1,NZ
   U(IZ)=U(IZ)+UOINC-IZ*DZ*UAINC
   US(IZ)=U(IZ)/PLASIMP**2
77 CONTINUE
78 NZ=NZZ
   IF(ABSF(TPR-T).LE.EPSPR) GO TO 82

```

C**** INTERMEDIATE PRINTOUT

```

   IF(ISCALE) GO TO 80
   PRINT 188,T,PSI,UO,VO,ZETA,NCFNS
   PRINT 189,(U(IZ),IZ=1,NZTOT)
   GO TO 81
80 PRINT 188,TS,PSI,UOS,VOS,ZETA,NCFNS
   PRINT 189,(US(IZ),IZ=1,NZTOT)
81 IF(VO/FMAX(1).LE.EPSRV) 100,70

```

C**** SCHEDULED PRINTOUT

```

82 IF(ISCALE) GO TO 83
   PRINT 187,T,PSI,UO,VO,ZETA,NCFNS
   PRINT 190,(U(IZ),IZ=1,NZTOT)
   GO TO 85

```

```

83 PRINT 187,TS,PSI,UOS,VOS,ZETA,NOFNS
PRINT 190,(US(IZ),IZ=1,NZTOT)
85 IF(TPR.EQ.TF) GO TO 110
IF(VO/FMAX(1).LE.EPSRV) GO TO 100
IF(TPR.EQ.TAUJMP.AND.(PSI+PJMP)*ZETA.GT.1.5) GO TO 222
IF(TPR.EQ.TAUJMP) JJMP=JJMP+1
IF(TPR.EQ.TAUMAX) JMAX=JMAX+1
IF(TPR.EQ.TDT) IT=IT+1
GO TO 58
C**** ZETA HAS CROSSED NZ*DZ FROM RIGHT TO LEFT IN THIS TIME INTERVAL
90 NZZ=NZ-1
91 UOP=UO+UCINC
UOPS=UOP/PLASIMP**2
IF(ISCALE) GO TO 92
PRINT 188,T,PSI,UOP,VO,ZETA,NOFNS
GO TO 93
92 PRINT 188,TS,PSI,UOPS,VOS,ZETA,NOFNS
93 H=(T-TT)*(NZ*DZ-FF(2))/(ZETA-FF(2))
DO 94 I=1,4
94 F(I)=FF(I)
T=TT
CALL DIFSUB(NEQ,T,F,DF,H,DTMIN,EPSERR,MORD,METH,FMAX,RLTVER,KKSTP)
PSI=PSIFUN(T)
IF(T.EQ.TAUJMP) PSI=PSI-PJMP
IF(KKSTP.LT.0) GO TO 72
VO=F(1)
ZETA=F(2)
UOINC=F(3)
UAINC=F(4)
TS=T/PLASIMP
VOS=VO/PLASIMP
IF(ABSF(ZETA-NZ*DZ).LE.EPSZ) 74,91
C**** ZETA HAS CROSSED (NZ+1)*DZ FROM LEFT TO RIGHT IN THIS TIME INTERVAL
95 NZZ=NZ+1
96 UOP=UO+UCINC
UOPS=UOP/PLASIMP**2
IF(ISCALE) GO TO 97
PRINT 188,T,PSI,UOP,VO,ZETA,NOFNS
GO TO 98
97 PRINT 188,TS,PSI,UOPS,VOS,ZETA,NOFNS
98 H=(T-TT)*((NZ+1)*DZ-FF(2))/(ZETA-FF(2))
DO 99 I=1,4
99 F(I)=FF(I)
T=TT
CALL DIFSUB(NEQ,T,F,DF,H,DTMIN,EPSERR,MORD,METH,FMAX,RLTVER,KKSTP)
PSI=PSIFUN(T)
IF(T.EQ.TAUJMP) PSI=PSI-PJMP
IF(KKSTP.LT.0) GO TO 72
VO=F(1)
ZETA=F(2)
UOINC=F(3)
UAINC=F(4)
TS=T/PLASIMP
VOS=VO/PLASIMP
IF(ABSF(ZETA-(NZ+1)*DZ).LE.EPSZ) 74,96
C***** DEFORMATION STOPS BEFORE TF
100 IF(ABSF(VOS).LE.EPSV1) GO TO 101
C**** EXTRAPOLATE TO VO=C, APPROXIMATELY
H=(T-TT)*VO/(FF(1)-VO)
GO TO 71
101 IF(ABSF(VOS).GT.EPSV2) GO TO 102
PRINT 192

```

```

      GO TO 1
102 CONTINUE
C**** USE POWER SERIES TO IMPROVE APPROXIMATION TO VO=0
C**** COEFFICIENTS
      CALL PSICOF(T,SPSI)
      P0=PSI
      P1=SPSI(1)
      P2=SPSI(2)
      P3=SPSI(3)
      P4=SPSI(4)
      A0=ZETA
      B0=VO
      C0=B0/A0
      A1=(3.0-2.0*P0*A0-A0**2)/(A0*B0)
      B1=(4.0*P0*A0-A0**2-3.0)/A0**2
      C1=(B1-C0*A1)/A0
      A2=(B0*A1**2+B1*A1*A0+2.0*(P0*A1+P1*A0+A1*A0))/(-A0*B0)
      B2=- (B1*A1*A0-2.0*(P0*A1+P1*A0)+A1*A0)/A0**2
      C2=(B2-C1*A1-C0*A2)/A0
      A3=- (3.0*B0*A2*A1+2.0*B1*A2*A0+B1*A1**2+B2*A1*A0+2.0*(P0*A2+P1*A1
1      +P2*A0+A2*A0)+A1**2)/(3.0*A0*B0)
      B3=- (4.0*B2*A1*A0+2.0*B1*A2*A0+B1*A1**2-4.0*(P0*A2+P1*A1+P2*A0)
1      +2.0*A2*A0+A1**2)/(3.0*A0**2)
      C3=(B3-C2*A1-C1*A2-C0*A3)/A0
      A4=- (4.0*B0*A3*A1+3.0*B1*A3*A0+2.0*B0*A2**2+3.0*B1*A2*A1
1      +2.0*B2*A2*A0+B2*A1**2+B3*A1*A0+2.0*(P0*A3+P1*A2+P2*A1+P3*A0
2      +A3*A0+A2*A1))/(4.0*A0*B0)
      B4=- (3.0*B3*A1*A0+2.0*B2*A2*A0+B2*A1**2+B1*A3*A0+B1*A2*A1
1      -2.0*(P0*A3+P1*A2+P2*A1+P3*A0)+A3*A0+A2*A1)/(2.0*A0**2)
      C4=(B4-C3*A1-C2*A2-C1*A3-C0*A4)/A0
      A5=- (5.0*B0*A4*A1+4.0*B1*A4*A0+5.0*B0*A3*A2+4.0*B1*A3*A1
1      +3.0*B2*A3*A0+2.0*B1*A2**2+3.0*B2*A2*A1+2.0*B3*A2*A0+B3*A1**2
2      +B4*A1*A0+2.0*(P0*A4+P1*A3+P2*A2+P3*A1+P4*A0+A4*A0+A3*A1)
3      +A2**2)/(5.0*A0*B0)
      B5=- (8.0*B4*A1*A0+6.0*B3*A2*A0+3.0*B3*A1**2+4.0*B2*A3*A0
1      +4.0*B2*A2*A1+2.0*B1*A4*A0+2.0*B1*A3*A1+B1*A2**2-4.0*(P0*A4
2      +P1*A3+P2*A2+P3*A1+P4*A0)+2.0*A4*A0+2.0*A3*A1+A2**2)/
3      (5.0*A0**2)
      C5=(B5-C4*A1-C3*A2-C2*A3-C1*A4-C0*A5)/A0
C**** EVALUATE POWER SERIES
      XDELTA=-4.0*B2*B0/B1**2
      DELTA=XDELTA*B1/(2.0*B2)/(SQRT(1.0+XDELTA)+1.0)
      T=T+DELTA
      PSI=PSIFUN(T)
      ZETA=A0+A1*DELTA+A2*DELTA**2+A3*DELTA**3+A4*DELTA**4+A5*DELTA**5
      VO=B0+B1*DELTA+B2*DELTA**2+B3*DELTA**3+B4*DELTA**4+B5*DELTA**5
      UOINC=B0*DELTA+B1*DELTA**2/2.0+B2*DELTA**3/3.0+B3*DELTA**4/4.0
1      +B4*DELTA**5/5.0+B5*DELTA**6/6.0
      UAINC=C0*DELTA+C1*DELTA**2/2.0+C2*DELTA**3/3.0+C3*DELTA**4/4.0
1      +C4*DELTA**5/5.0+C5*DELTA**6/6.0
      TS=T/PLASIMP
      VOS=VO/PLASIMP
      GO TO 75
C***** COASTDOWN CALCULATION AFTER PULSE ENDS, T.GT. TF
110 PRINT 193
      ZETA=ZETA
      VOF=VO
      UOF=UO
      IF(NZINT.EQ.0) GO TO 109
      DO 111 IZ=1,NZTOT
111 UF(IZ)=U(IZ)
109 PSI=0.0

```

```

      AF=(3.0-ZETAF**2)/(VOF*ZETAF)
      BF=ZETAF-AF*TF
      TEND=(SQRTF(3.0)-BF)/AF
      IT=0
112  IT=IT+1
      T=IT*DT
      IF(T.LE.TF) GO TO 112
113  ZETA=AF*T+BF
      VO=(3.0-ZETA**2)/(AF*ZETA)
      UO=UOF+((ZETAF**2-ZETA**2)/2.0+3.0*LOGF(ZETA/ZETAF))/AF**2
      IF(ISCALE) GO TO 114
      PRINT 194,T,PSI,UO,VO,ZETA
      GO TO 115
114  TS=T/PLASIMP
      VOS=VO/PLASIMP
      UOS=UO/PLASIMP**2
      PRINT 194,TS,PSI,UOS,VOS,ZETA
115  IF(NZINT.EQ.0) GO TO 121
C**** CALCULATE U(Z,T)
      IZ=0
116  IZ=IZ+1
      Z=IZ*DZ
      IF(Z.GT.ZETAF) GO TO 117
C**** Z IS BETWEEN ZERO AND ZETAF
      U(IZ)=U(IZ)+((ZETAF-ZETA)*(ZETAF+ZETA-2.0*Z)/2.0
1      +3.0*LOGF(ZETA/ZETAF)+3.0*Z*((1.0/ZETA)-(1.0/ZETAF)))/AF**2
      GO TO 116
C**** Z IS BETWEEN ZETAF AND ZETA
117  IF(Z.GT.ZETA) GO TO 118
      U(IZ)=U(IZ)+(-(ZETA-Z)**2/2.0+3.0*LOGF(ZETA/Z)+3.0*(Z/ZETA-1.0))
1      /AF**2
      GO TO 116
118  IF(ISCALE) GO TO 119
      PRINT 190,(U(IZ),IZ=1,NZTOT)
      GO TO 121
119  DO 120 IZ=1,NZTOT
120  US(IZ)=U(IZ)/PLASIMP**2
      PRINT 190,(US(IZ),IZ=1,NZTOT)
121  IF(T.EQ.TEND) GO TO 1
      IT=IT+1
      T=IT*DT
      IF(T.LE.TEND) GO TO 113
      T=TEND
      GO TO 113
C***** MOTION GOES INTO PHASE 2 TYPE DEFORMATION PATTERN
C**** FIND TIME TAU12 AT WHICH MOTION GOES INTO PHASE 2
122  CONTINUE
      EPSPH2=0.00001
      VO2=VO
      ZETA2=ZETA
      T2=T
      PSI2=PSI
      VO1=FF(1)
      ZETA1=FF(2)
      T1=TT
      PSI1=PSIFUN(T1)
124  UOP=UO+UCINC
      UOPS=UOP/PLASIMP**2
      IF(ISCALE) GO TO 125
      PRINT 188,T,PSI,UOP,VO,ZETA,NOFNS
      GO TO 126
125  PRINT 188,TS,PSI,UOPS,VOS,ZETA,NOFNS

```

```

126 H=(T2-T1)*(1.5-PSI1*ZETA1)/(PSI2*ZETA2-PSI1*ZETA1)
DO 127 I=1,4
127 F(I)=FF(I)
T=T1
CALL DIFSUB(NEQ,T,F,DF,H,DTMIN,EPSERR,MORD,METH,FMAX,RLTVER,KKSTP)
PSI=PSIFUN(T)
IF(KKSTP.LT.0) GO TO 72
VO=F(1)
ZETA=F(2)
UOINC=F(3)
UAINC=F(4)
TS=T/PLASIMP
VOS=VO/PLASIMP
IF(ABSF(PSI*ZETA-1.5).LT.EPSPH2) GO TO 130
IF(PSI*ZETA.GT.1.5) GO TO 129
T1=T
ZETA1=ZETA
PSI1=PSI
DO 128 I=1,4
128 FF(I)=F(I)
GO TO 124
129 T2=T
ZETA2=ZETA
PSI2=PSI
GO TO 124
130 CONTINUE
UO=UO+UOINC
UOS=UO/PLASIMP**2
DO 131 IZ=1,NZ
U(IZ)=U(IZ)+UOINC-IZ*DZ*UAINC
US(IZ)=U(IZ)/PLASIMP**2
131 CONTINUE
TAU12=T
PRINT 195,T,TS
ETA=ZETA
IF(ISCALE) GO TO 132
PRINT 196,T,PSI,UO,VO,ZETA,ETA,NOFNS
PRINT 190,(U(IZ),IZ=1,NZTOT)
GO TO 133
132 PRINT 196,TS,PSI,UOS,VOS,ZETA,ETA,NOFNS
PRINT 190,(US(IZ),IZ=1,NZTOT)
133 CONTINUE
KPHASE=1
K21=0
K22=0
F(1)=VO
F(2)=VAUX=VO/ZETA
JOM=1001
OMEGAL(JOM)=TAU12
ZL(JOM)=ZETA
GO TO 229
C**** AN INSTANTANEOUS JUMP INTO PHASE 2 OCCURS AT T
222 TAU12=T
PRINT 199,T,TS
ZETA1=ZETA2=ETA2=ETA=ZETA
PSI2=1.5/ZETA2
K21=K22=0
PSI=PSI+PJMP
ZETA=1.5/PSI
IF(ISCALE) GO TO 223
PRINT 196,T,PSI2,UO,VO,ZETA2,ETA2,NOFNS
PRINT 190,(U(IZ),IZ=1,NZTOT)

```

```

PRINT 196,T,PSI,UO,VO,ZETA,ETA,NOFNS
PRINT 190,(U(IZ),IZ=1,NZTOT)
GO TO 224
223 PRINT 196,TS,PSI2,UOS,VOS,ZETA2,ETA2,NOFNS
PRINT 190,(US(IZ),IZ=1,NZTOT)
PRINT 196,TS,PSI,UOS,VOS,ZETA,ETA,NOFNS
PRINT 190,(US(IZ),IZ=1,NZTOT)
224 VAUX2=VAUX=VO/ZETA2
VZETA2=0
VZETA=VO-VAUX*ZETA
OMEGAL(1001)=TAU12
ZL(1001)=ZETA2
OMEGAL(1000)=VZETA+TAU12
ZL(1000)=ZETA
JOM=1000
F(1)=VO
F(2)=VAUX
JJMP=JJMP+1
IF(TAU12.EQ.TDT) IT=IT+1
IF(TAU12.EQ.TAUMAX) GO TO 226
KPHASE=1
GO TO 229
226 JMAX=JMAX+1
KPHASE=2
229 CONTINUE
C***** PHASE 2 DEFORMATION
C**** INTEGRATION TIME STEPS AND PRINTOUT TIMES
NT2=10
NT3=2
DT2=DT/NT2
DT3= DT2/NT3
TDT=IT*DT
DO 231 IT2=1,NT2
TDT2=TDT-(NT2-IT2)*DT2
IF(TDT2.GT.TAU12) GO TO 232
231 CONTINUE
GO TO 1
232 DO 233 IT3=1,NT3
TDT3=TDT2-(NT3-IT3)*DT3
IF(TDT3.GT.TAU12) GO TO 235
233 CONTINUE
GO TO 1
235 IF(JJMP.GT.NOJMP) 236,237
236 TAUJMP=2.0*PLASIMP
PJMP=0.0
GO TO 238
237 TAUJMP=TJMP(JJMP)
PJMP=PSIJMP(JJMP)
238 IF(JMAX.GT.NOMAX) 239,240
239 TAUMAX=2.0*PLASIMP
PMAJ=0.0
GO TO 241
240 TAUMAX=TMAX(JMAX)
PMAJ=PSIMAX(JMAX)
241 TDT=IT*DT
242 TDT2=TDT-(NT2-IT2)*DT2
243 TDT3=TDT2-(NT3-IT3)*DT3
TPR=AMIN1(TDT2,TAUJMP,TAUMAX)
244 TCAL=AMIN1(TDT3,TAUJMP,TAUMAX)
C**** USE SUBROUTINES DIFSUB AND DIFFUN
246 T1=T
PSI1=PSI

```



```

ZETA1=ZETA
ETA1=ETA
VO1=VO
VAUX1=VAUX
249 F(3)=F(4)=0.0
250 H=TCAL-T
CALL DIFSUB(NEQ,T,F,DF,H,DTMIN,EPSERR,MORD,METH,FMAX,RLTVER,KKSTP)
PSI=PSIFUN(T)
IF(T.EQ.TAUJMP) PSI=PSI-PJMP
IF(KKSTP.LT.0) GO TO 72
IF(ABSF(TCAL-T).GT.EPSPR) GO TO 250
VO=F(1)
VAUX=F(2)
UOINC=F(3)
UAINC=F(4)
TS=T/PLASIMP
VOS=VO/PLASIMP
IF(KPHASE.EQ.2) GO TO 253
C**** T IS BETWEEN TAU12 AND TAUMAX (KPHASE=1)
252 ZETA=1.5/PSI
VZETA=VO-VAUX*ZETA
JOM=JOM-1
OMEGAL(JOM)=VZETA+T
ZL(JOM)=ZETA
ETA=OMEGINV(T)
GO TO 265
C**** T IS BETWEEN TAUMAX AND TAU21 (KPHASE=2)
253 ZETA=ZETAEQ(VAUX,VO+T)
ETA=OMEGINV(T)
VZETA=VO-VAUX*ZETA
IF(K21) GO TO 255
IF(K22) GO TO 258
IF(ETA.LE.ZETA) GO TO 254
IF(PSI*ZETA.GT.1.50001) GO TO 257
GO TO 265
C**** T IS IN THE VICINITY OF TAU21
254 K21=1
255 IF(ABSF(ETA-ZETA).LE.EPSZ) GO TO 259
TCAL=(T*(ETA1-ZETA1)+T1*(ZETA-ETA))/(ETA1-ZETA1+ZETA-ETA)
T=T1
F(1)=VO1
F(2)=VAUX1
GO TO 249
C**** INTERPOLATE TO PSI*ZETA=1.5
257 K22=1
258 IF(ABSF(PSI*ZETA-1.5).LE.EPSZ) GO TO 259
TCAL=((PSI*ZETA-1.5)*ZETA1*T1+(1.5-PSI1*ZETA1)*ZETA*T)/
1 ((PSI*ZETA-1.5)*ZETA1+(1.5-PSI1*ZETA1)*ZETA)
T=T1
F(1)=VO1
F(2)=VAUX1
GO TO 249
259 TPR=T
GO TO 265
C**** DEFORMATION CALCULATIONS
265 UO=UO+UOINC
UOS=UO/PLASIMP**2
DO 268 IZ=1,NZTOT
Z=IZ*DZ
IF(Z.GT.ZETA) GO TO 266
U(IZ)=U(IZ)+UOINC-Z*UAINC
GO TO 267

```

```

266 IF(Z.GT.ETA) GO TO 268
    U(IZ)=U(IZ)+(-0.5*(T+T1)+OMEGA(Z))*(T-T1)
267 US(IZ)=U(IZ)/PLASIMP**2
268 CONTINUE
C**** PRINTOUTS
    IF(ABSF(T-TPR).LE.EPSPR) T=TPR
    IF(T.EQ.TPR) 269,272
269 IF(ISCALE) GO TO 270
    PRINT 196,T,PSI,UO,VO,ZETA,ETA,NOFNS
    PRINT 190,(U(IZ),IZ=1,NZTOT)
    GO TO 275
270 PRINT 196,TS,PSI,UOS,VOS,ZETA,ETA,NOFNS
    PRINT 190,(US(IZ),IZ=1,NZTOT)
    GO TO 275
C**** BRANCHING
272 IT3=IT3+1
    GO TO 243
275 IF(K21) GO TO 277
    IF(K22) GO TO 280
    IF(T.EQ.TAUJMP) GO TO 285
    IF(T.EQ.TAUMAX) GO TO 295
    IF(T.EQ.TDT2) GO TO 297
    GO TO 1
C**** T EQUALS TAU21 (RETURN TO PHASE 1 CALCULATION)
277 KPHASE=0
    PRINT 197,T,TS
    IF(T.GE.TF) GO TO 279
    F(1)=VO
    F(2)=ZETA
    NZ=NZZ=NZTOT
278 IF(NZ*DZ.LE.ZETA) GO TO 70
    NZ=NZZ=NZ-1
    GO TO 278
279 TF=T
    GO TO 110
C**** T EQUALS A NEW TAU12 (HINGE BAND BEGINS TO EXPAND AGAIN)
280 TAU12=T
    KPHASE=1
    K22=0
    PRINT 198,T,TS
    DO 281 JZ=JOM,1000
    IF(ZETA.GE.ZL(JZ).AND.ZETA.LE.ZL(JZ+1))GO TO 282
281 CONTINUE
    JOM=JOM-1
    OMEGAL(JCM)=VZETA+T
    ZL(JOM)=ZETA
    GO TO 242
282 OM=OMEGA(ZETA)
    JOM=JZ-1
    OMEGAL(JZ)=OM
    OMEGAL(JCM)=VZETA+T
    ZL(JZ)=ZL(JOM)=ZETA
    GO TO 242
C**** T EQUALS TAUJMP
285 JJMP=JJMP+1
    PSI1=PSI
    PSI=PSI+PJMP
    IF(PJMP.LE.0.0) GO TO 292
    IF(KPHASE.EQ.2) GO TO 286
    JOM=JCM-1
    ZETA=1.5/PSI
    VZETA=VO-VAUX*ZETA

```

```

      OMEGAL(JCM)=VZETA+T
      ZL(JOM)=ZETA
      GO TO 292
286  IF(PHI*ZETA.LT.1.5) GO TO 292
      PSI2=1.5/ZETA
      ZETA2=ZETA
      VZETA2=VZETA=VO-VAUX*ZETA
      PRINT 198,T,TS
      IF(ISCAL) GO TO 287
      PRINT 196,T,PSI2,UO,VO,ZETA2,ETA,NOFNS
      PRINT 190,(U(IZ),IZ=1,NZTOT)
      GO TO 288
287  PRINT 196,TS,PSI2,UOS,VOS,ZETA2,ETA,NOFNS
      PRINT 190,(US(IZ),IZ=1,NZTOT)
288  TAU12=T
      DO 289 JZ=JCM,1000
      IF(ZETA2.GE.ZL(JZ).AND.ZETA2.LE.ZL(JZ+1)) GO TO 290
289  CONTINUE
      JOM=JOM-1
      OMEGAL(JCM)=VZETA2+T
      ZL(JOM)=ZETA2
      GO TO 291
290  OM=OMEGA(ZETA2)
      JOM=JZ-1
      OMEGAL(JZ)=OM
      OMEGAL(JOM)=VZETA2+T
      ZL(JZ)=ZL(JOM)=ZETA2
291  ZETA=1.5/PSI
      VZETA=VO-VAUX*ZETA
      KPHASE=1
      JOM=JOM-1
      OMEGAL(JCM)=VZETA+T
      ZL(JOM)=ZETA
292  IF(ISCAL) GO TO 293
      PRINT 196,T,PSI,UO,VO,ZETA,ETA,NOFNS
      PRINT 190,(U(IZ),IZ=1,NZTOT)
      GO TO 294
293  PRINT 196,TS,PSI,UOS,VOS,ZETA,ETA,NOFNS
      PRINT 190,(US(IZ),IZ=1,NZTOT)
294  IF(T.NE.TAUMAX) GO TO 296
295  JMAX=JMAX+1
      KPHASE=2
296  IF(T.EQ.TOT2) 297,235
297  IT3=1
      IF(IT2.EQ.NT2) GO TO 298
      IT2=IT2+1
      GO TO 235
298  IT2=1
      IT=IT+1
      GO TO 235
C***** FORMATS
170  FORMAT(10A8)
171  FORMAT(1H1,7X*PROGRAM RINGLOAD*//8X,10A8)
172  FORMAT(I2,2X,I2,2X,I2)
173  FORMAT(1H0,7X*INPUT ERROR*/8X*KSTOP ==I2)
174  FORMAT(1H0,7X*INSTJMP ==I2
1      *, INDICATING AN INSTANTANEOUS JUMP AT TYIELD*)
175  FORMAT(1H0,7X*PULSE PARAMETERS*/12X
1      *PULSE BEGINS AT TAUO ==E12.5/12X
2      *PULSE ENDS AT TAUJ ==E12.5/12X
3      *PLASTIC FLOW STARTS AT TY ==E12.5/12X
4      *TOTAL IMPULSE TOTIMP ==E12.5/12X

```

```

5      *PLASTIC IMPULSE (IMPULSE AFTER TY) PLASIMP = I ==E12.5/12X
6      *MEAN TIME OF PULSE TM ==E12.5/12X
7      *EFFECTIVE LOAD PSIE ==E12.5)
176 FORMAT(1H0,7X*SCALED PULSE PARAMETERS*/12X
1      *TAUC/I ==E12.5/12X*TAUF/I ==E12.5/12X
2      *TY/I ==E12.5/12X*TM/I ==E12.5)
177 FORMAT(1H0,7X*LOCATIONS OF RELATIVE MAXIMA OF PULSE*/3X*K*7X
1      *TMAX*12X*PSIMAX*11X*TMAX/I*/(I4,3E17.7))
178 FORMAT(1H0,7X*THE SELECTED OUTPUT INTERVALS ARE*/12X
1      *TIME INTERVAL DT ==E12.5/12X
2      *SCALED TIME INTERVAL DTS = DT/I ==E12.5/12X
3      *AXIAL INTERVAL DZ ==E12.5)
179 FORMAT(1H0,7X*SELECTED SCALED TIME INTERVAL IS TOO LARGE,*
1      /8X*I.E., NTSINT IS SMALLER THAN TEN.*)
180 FORMAT(1H0,7X*INSTANTANEOUS JUMPS IN PULSE SHAPE*/3X*K*7X
1      *TJMP*12X*PSIJMP*11X*TJMP/I*/(I4,3E17.7))
181 FORMAT(1H0,7X*SELECTED AXIAL INTERVAL FOR DISPLACEMENT PRINTOUT *
1      *IS TOO SMALL,*/8X*I.E., NZINT IS LARGER THAN 25*)
182 FORMAT(1H1,8X*T/I*12X*PSI(T)*9X*U(0,T)/I/I*8X*V(0,T)/I*10X
1      *ZETA(T)*10X*ETA(T)*11X*NOFNS*)
183 FORMAT(1H0,37X*U(1DZ,T)/I/I*5X*U(2DZ,T)/I/I*5X*U(3DZ,T)/I/I*5X
1      *U(4DZ,T)/I/I*5X*U(5DZ,T)/I/I*/38X*U(6DZ,T)/I/I*5X
2      *U(7DZ,T)/I/I*5X*U(8DZ,T)/I/I*5X*U(9DZ,T)/I/I*5X
3      *U(10DZ,T)/I/I*)
184 FORMAT(1H1,9X*T*13X*PSI(T)*11X*U(0,T)*11X*V(0,T)*11X*ZETA(T)*10X
1      *ETA(T)*11X*NOFNS*)
185 FORMAT(1H0,39X*U(1DZ,T)*9X*U(2DZ,T)*9X*U(3DZ,T)*9X*U(4DZ,T)*9X
1      *U(5DZ,T)*40X*U(6DZ,T)*9X*U(7DZ,T)*9X*U(8DZ,T)*9X*U(9DZ,T)*
2      9X*U(10DZ,T)*)
187 FORMAT(1H0,/5E17.7,17X,I10)
188 FORMAT(1H0,/2X,5E17.7,17X,I10)
189 FORMAT(1H0,35X,5E17.7/(36X,5E17.7))
190 FORMAT(1H0,33X,5E17.7/(34X,5E17.7))
191 FORMAT(1H0,7X*NO CONVERGENCE IN STEP TO T ==E12.5)
192 FORMAT(1H0,7X*DEFORMATION STOPS BEFORE END OF PULSE*)
193 FORMAT(1H0,7X*DEFORMATION CONTINUES AFTER END OF PULSE*)
194 FORMAT(1H0,/5E17.7)
195 FORMAT(1H0,//8X*A HINGE BAND FORMS AT T ==E12.5*, T/I ==E12.5)
196 FORMAT(1H0,/6E17.7,I10)
197 FORMAT(1H0,//8X*THE HINGE BAND REDUCES TO A HINGE CIRCLE AT T ==
1      E12.5*, T/I ==E12.5)
198 FORMAT(1H0,//8X*THE HINGE BAND BEGINS TO EXPAND AGAIN AT T ==
1      E12.5*, T/I ==E12.5)
199 FORMAT(1H0,//8X*A HINGE BAND FORMS INSTANTANEOUSLY AT T ==
1      E12.5*, T/I ==E12.5)
      END

```

```

      SUBROUTINE DIFFUN(T,F,DF)
C
C      SUBROUTINE WHICH PROVIDES RINGLOAD PROGRAM WITH RIGHTHAND SIDES OF
C      DIFFERENTIAL EQUATIONS
C
      DIMENSION F(4),DF(4),SPSI(4)
      COMMON/CCMDIF/NOFNS,KPHASE,TAUJMP,PJMP
      PSI=PSIFUN(T)
      IF(T.EQ.TAUJMP) PSI=PSI-PJMP
C**** F(1)=VO, F(3)=UOINC, F(4)=UAINC
      IF(KPHASE-1)400,401,402
C**** MOTION IS IN PHASE 1 (KPHASE=0)
400 VO=F(1)
      ZETA=F(2)
      DF(1)=(4.0*PSI-3.0/ZETA-ZETA)/ZETA
      DF(2)=(-2.0*PSI+3.0/ZETA-ZETA)/VO
      DF(3)=VC
      DF(4)=VC/ZETA
      GO TO 404
C**** MOTION IS IN PHASE 2 (KPHASE=1)
401 VO=F(1)
      VAUX=F(2)
      ZETA=1.5/PSI
      GO TO 403
C**** MOTION IS IN PHASE 2 (KPHASE=2)
402 VO=F(1)
      VAUX=F(2)
      ZETA=ZETA*EQ(VAUX,VO+T)
403 DF(1)=(4.0*PSI-3.0/ZETA-ZETA)/ZETA
      DF(2)=6.0*(PSI*ZETA-1.0)/ZETA**3
      DF(3)=VC
      DF(4)=VAUX
404 NOFNS=NOFNS+1
      END

```

```

      FUNCTION OMEGA(Z)
C
C      FINDS OMEGA(Z) BY LINEAR INTERPOLATION IN THE TABLE OMEGAL
C
      COMMON/CCOMEG/JOM,OMEGAL(1001),ZL(1001)
      DO 500 JZ=JOM,1000
      IF(ZL(JZ).EQ.ZL(JZ+1)) GO TO 500
      IF(Z.GE.ZL(JZ).AND.Z.LE.ZL(JZ+1)) GO TO 501
500 CONTINUE
      IF(Z.GT.ZL(1001)) 506,507
506 PRINT 510,Z,(JZ,ZL(JZ),OMEGAL(JZ),JZ=JOM,1001)
      STOP
507 CONTINUE
      IF(Z.LT.ZL(JOM)) JZ=JOM
501 OMEGA=(OMEGAL(JZ+1)*(Z-ZL(JZ))+OMEGAL(JZ)*(ZL(JZ+1)-Z))
      1 / (ZL(JZ+1)-ZL(JZ))
510 FORMAT(1H0,7X*OMEGA STOP, Z =*E12.5/(8X,16,2E17.7))
      END

```

```

FUNCTION OMEGINV(X)
C
C SOLVES X = OMEGA(Z) FOR Z
C
COMMON/CCMOMEG/JOM,OMEGAL(1001),ZL(1001)
DO 550 JZ=JOM,1000
IF(OMEGAL(JZ).EQ.OMEGAL(JZ+1)) GO TO 550
IF(X.LE.CMEGAL(JZ).AND.X.GE.CMEGAL(JZ+1)) GO TO 551
550 CONTINUE
IF(X.LT.CMEGAL(1001)) 556,557
556 PRINT 560,X,(JZ,ZL(JZ),OMEGAL(JZ),JZ=JOM,1001)
STOP
557 CONTINUE
IF(X.GT.CMEGAL(JOM)) JZ=JOM
551 OMEGINV=(ZL(JZ+1)*(X-OMEGAL(JZ))+ZL(JZ)*(OMEGAL(JZ+1)-X))
1 / (OMEGAL(JZ+1)-OMEGAL(JZ))
560 FORMAT(1H0,7X*OMEGINV STOP, T =*E12.5/(8X,I6,2E17.7))
END

```

```

FUNCTION ZETAEQ(A,B)
C
C SOLVES THE EQUATION OMEGA(Z) = B - A*Z FOR Z, WHERE
C OMEGA(Z) = AOM(J)*Z + BOM(J)
C
COMMON/CCMOMEG/JOM,OMEGAL(1001),ZL(1001)
DO 600 JZ=JOM,1000
IF(ZL(JZ).EQ.ZL(JZ+1)) GO TO 600
AOM=(OMEGAL(JZ+1)-CMEGAL(JZ))/(ZL(JZ+1)-ZL(JZ))
BOM=(OMEGAL(JZ)*ZL(JZ+1)-OMEGAL(JZ+1)*ZL(JZ))/(ZL(JZ+1)-ZL(JZ))
IF(ABS(A+AOM).GT.G.000001) GO TO 603
ZETAEQ=ZL(JOM)
RETURN
603 CONTINUE
Z=(B-BOM)/(A+AOM)
IF(Z.GE.ZL(JZ).AND.Z.LE.ZL(JZ+1)) GO TO 601
600 CONTINUE
AOM=(OMEGAL(JOM+1)-OMEGAL(JOM))/(ZL(JOM+1)-ZL(JOM))
BOM=(OMEGAL(JOM)*ZL(JOM+1)-OMEGAL(JOM+1)*ZL(JOM))
1 / (ZL(JOM+1)-ZL(JOM))
Z=(B-BOM)/(A+AOM)
IF(Z.GT.ZL(JOM)) 606,601
606 PRINT 610,Z,(JZ,ZL(JZ),OMEGAL(JZ),JZ=JOM,1001)
STOP
601 ZETAEQ=Z
610 FORMAT(1H0,7X*ZETAEQ STOP, Z =*E12.5/(8X,I6,2E17.7))
END

```

```

SUBROUTINE PULSEINF(TO,TF,TYIELD,TOTIMP,PLASIMP,TMEAN,NOMAX,
1  TMAX,PSIMAX,INSTJMP,NOJMP,TJMP,PSIJMP,KSTOP)
C
C  SUBROUTINE WHICH PROVIDES RINGLOAD PROGRAM WITH BASIC INFORMATION
C  ON PULSE SHAPE.
C
C  PIECEWISE LINEAR VERSION.
C  THE PULSE SHAPE IS DESCRIBED BY GIVING PSI VALUES AT A NUMBER OF
C  TIME POINTS AND USING LINEAR INTERPOLATION.
C
C  REQUIRED INPUT FROM DATA CARDS
C  CARD 3.  FORMAT(I2)
C  NJ (NUMBER OF INPUT DATA POINTS)
C  CARDS 4,5,6....  FORMAT(2E15.7)
C  TJ(J) AND PSIJ(J) (DATA POINT PAIRS)
C
C  OUTPUT TO RINGLOAD
C  TO (TIME AT WHICH PULSE BEGINS)
C  TF (TIME AT WHICH PULSE ENDS)
C  TYIELD (TIME AT WHICH PLASTIC FLOW FIRST OCCURS, PSI=1.0)
C  TOTIMP (TOTAL IMPULSE)
C  PLASIMP (IMPULSE AFTER TYIELD)
C  TMEAN (TIME MEAN OF PLASTIC IMPULSE MEASURED FROM TYIELD)
C  NOMAX (NUMBER OF RELATIVE MAXIMA)
C  TMAX(K) AND PSIMAX(K) (LOCATIONS AND VALUES OF RELATIVE MAXIMA)
C  INSTJMP (IF POSITIVE,PULSE HAS INSTANTANEOUS JUMP AT TYIELD)
C  NOJMP (NUMBER OF TIMES AT WHICH PSI HAS AN INSTANTANEOUS JUMP)
C  TJMP(K) AND PSIJMP(K) (LOCATIONS AND VALUES OF JUMPS)
C  KSTOP (IF POSITIVE, THERE IS AN INPUT ERROR)
C
C  COMMON WITH FUNCTION PSIFUN AND SUBROUTINE PSICOEF
C  NJ, TJ(J), PSIJ(J)
C
C  IF AN INSTANTANEOUS JUMP OCCURS AT TJ, THE VALUE OF PSI TO
C  THE LEFT OF THE JUMP IS RETURNED.
C
C  DIMENSION TJS(99),TJMP(50),PSIJMP(50),TMAX(50),PSIMAX(50)
C  COMMON/PULSE/NJ,TJ(99),PSIJ(99)
C***** TITLE, DATA INPUT, AND INITIALIZATIONS
  READ 286,NJ
  READ 285,(TJ(J),PSIJ(J),J=1,NJ)
  PRINT 290
  KSTOP=0
  INSTJMP=0
C***** CHECK ORDERING OF INPUT CARDS
  DO 201 J=2,NJ
    IF(TJ(J).LT.TJ(J-1)) GO TO 280
  201 CONTINUE
C***** TO AND TF
  TO=TJ(1)
  TF=TJ(NJ)
C***** TYIELD AND INSTJMP
  IF(PSIJ(1).GT.1.0) GO TO 204
  DO 203 J=2,NJ
    IF(PSIJ(J).GT.1.0.AND.PSIJ(J-1).LE.1.0) GO TO 205
  203 CONTINUE
  GO TO 281
  204 INSTJMP=1
  JYIELD=1
  TYIELD=TO
  GO TO 209
  205 JYIELD=J

```

```

      IF(TJ(J).EQ.TJ(J-1)) GO TO 206
      TYIELD=(TJ(J-1)*(PSIJ(J)-1.0)+TJ(J)*(1.0-PSIJ(J-1)))/(PSIJ(J)-
1      PSIJ(J-1))
      GO TO 209
206 INSTJMP=1
      TYIELD=TJ(J)
209 CONTINUE
C***** TOTIMP, PLASIMP, TMEAN
      TOTIMP=0.0
      DO 210 J=2,NJ
210 TOTIMP=TOTIMP+(PSIJ(J)+PSIJ(J-1))*(TJ(J)-TJ(J-1))/2.0
      IF(INSTJMP) GO TO 212
      PLASIMP=(PSIJ(JYIELD)+1.0)*(TJ(JYIELD)-TYIELD)/2.0
      AUXTM=(2.0*PSIJ(JYIELD)+1.0)*(TJ(JYIELD)-TYIELD)**2/6.0
      GO TO 213
212 PLASIMP=0.0
      AUXTM=0.0
213 JJ=JYIELD+1
      DO 214 J=JJ,NJ
      PLASIMP=PLASIMP+(PSIJ(J)+PSIJ(J-1))*(TJ(J)-TJ(J-1))/2.0
      AUXTM=AUXTM+(PSIJ(J)*(2.0*TJ(J)+TJ(J-1)-3.0*TYIELD)+PSIJ(J-1)*
1      (TJ(J)+2.0*TJ(J-1)-3.0*TYIELD))*(TJ(J)-TJ(J-1))/6.0
214 CONTINUE
      TMEAN=AUXTM/PLASIMP
      EFFWIDTH=2.0*TMEAN
C***** NOMAX, TMAX(K), AND PSIMAX(K)
      NOMAX=0
      IF(PSIJ(2).GT.PSIJ(1)) GO TO 221
      NOMAX=1
      PSIMAX(1)=PSIJ(1)
      TMAX(1)=TJ(1)
221 DO 223 J=3,NJ
      IF(PSIJ(J-1).GT.PSIJ(J-2).AND.PSIJ(J).LE.PSIJ(J-1))222,223
222 NOMAX=NOMAX+1
      PSIMAX(NOMAX)=PSIJ(J-1)
      TMAX(NOMAX)=TJ(J-1)
223 CONTINUE
      IF(PSIJ(NJ).GT.PSIJ(NJ-1))224,225
224 NOMAX=NOMAX+1
      PSIMAX(NOMAX)=PSIJ(NJ)
      TMAX(NOMAX)=TJ(NJ)
225 CONTINUE
C***** SCALING OF TIMES BY PLASTIC IMPULSE
      DO 230 J=1,NJ
230 TJS(J)=TJ(J)/PLASIMP
C***** NOJMP, TJMP(K), AND PSI JMP(K)
      NOJMP=0
      DO 241 J=2,NJ
      IF(TJ(J).EQ.TJ(J-1)) 240,241
240 NOJMP=NOJMP+1
      TJMP(NOJMP)=TJ(J)
      PSI JMP(NOJMP)=PSIJ(J)-PSIJ(J-1)
241 CONTINUE
C***** PRINTOUTS OF PULSE DATA AND ERROR MESSAGES
      PRINT 294,(J,TJ(J),PSIJ(J),TJS(J),J=1,NJ)
      RETURN
280 KSTOP=1
      PRINT 291
      PRINT 292,(J,TJ(J),PSIJ(J),J=1,NJ)
      RETURN
281 KSTOP=1
      PRINT 293

```



```

      PRINT 292,(J,TJ(J),PSIJ(J),J=1,NJ)
      RETURN
C***** FORMATS
285 FORMAT(2E15.7)
286 FORMAT(I2)
290 FORMAT(1H0,7X*SUBROUTINE PULSEINF IS PIECEWISE LINEAR VERSION*/8X
1    *THE PULSE SHAPE IS DESCRIBED BY GIVING VALUES OF PSI(J)*/8X
2    *AT NJ POINTS OF TIME T(J) AND USING LINEAR INTERPOLATION*)
291 FORMAT(1H0,7X*INPUT CARDS ARE OUT OF ORDER*)
292 FORMAT(1H0,7X*PULSE INPUT DATA*/3X*J*8X*T*15X*PSI*/(I4,2E17.7))
293 FORMAT(1H0,7X*NO PLASTIC FLOW OCCURS (PSI NEVER EXCEEDS 1.0)*)
294 FORMAT(1H0,7X*PULSE INPUT DATA*/3X*J*8X*T*15X*PSI*14X*T/I*/
1    (I4,3E17.7))
      END

```

```

      FUNCTION PSIFUN(T)
C
C      SUBROUTINE WHICH PROVIDES RINGLOAD PROGRAM WITH VALUES OF PSI.
C
C      PIECEWISE LINEAR VERSION.
C      THE PULSE SHAPE IS DESCRIBED BY GIVING PSI VALUES AT A NUMBER OF
C      TIME POINTS AND USING LINEAR INTERPOLATION.
C
C      INPUT
C      T (TIME AT WHICH PSI IS WANTED)
C
C      OUTPUT
C      PSI (PULSE MAGNITUDE AT T)
C
C      COMMON WITH SUBROUTINE PULSEINF
C      NJ, TJ(J), PSIJ(J)
C
C      COMMON/PULSE/NJ,TJ(99),PSIJ(99)
C      PSIFUN=0.0
C      IF(T.LT.TJ(1).OR.T.GE.TJ(NJ)) RETURN
C      DO 304 J=2,NJ
C      IF(T.GE.TJ(J-1).AND.T.LT.TJ(J)) GO TO 305
304  CONTINUE
305  PSIFUN=(PSIJ(J-1)*(TJ(J)-T)+PSIJ(J)*(T-TJ(J-1)))/(TJ(J)-TJ(J-1))
      END

```

```

SUBROUTINE PSICOF(T,SPSI)
C
C SUBROUTINE WHICH PROVIDES RINGLOAD PROGRAM WITH THE COEFFICIENTS
C OF THE POWER SERIES EXPANSION OF PULSE SHAPE PSI.
C
C PIECEWISE LINEAR VERSION.
C THE PULSE SHAPE IS DESCRIBED BY GIVING PSI VALUES AT A NUMBER OF
C TIME POINTS AND USING LINEAR INTERPOLATION.
C
C INPUT
C T (TIME)
C
C OUTPUT
C SPSI(N) (NTH DERIVATIVE OF PSI DIVIDED BY N FACTORIAL AND
C EVALUATED AT T FOR N = 1,2,3,4)
C
C COMMON WITH SUBROUTINE PULSEINF
C NJ, TJ(J), PSIJ(J)
C
C DIMENSION SPSI(4)
C COMMON/PULSE/NJ,TJ(99),PSIJ(99)
C SPSI(1)=SPSI(2)=SPSI(3)=SPSI(4)=0.0
C IF(T.LT.TJ(1).OR.T.GE.TJ(NJ)) RETURN
C DO 354 J=2,NJ
C IF(T.GE.TJ(J-1).AND.T.LT.TJ(J)) GO TO 355
354 CONTINUE
355 SPSI(1)=(PSIJ(J)-PSIJ(J-1))/(TJ(J)-TJ(J-1))
C END

SUBROUTINE PULSEINF(TAUO,TAUF,TYIELD,TOTIMP,PLASIMP,TMEAN,NOMAX,
1 TMAX,PSIMAX,INSTJMP,NOJMP,TJMP,PSIJMP,KSTOP)
C
C SUBROUTINE WHICH PROVIDES RINGLOAD PROGRAM WITH BASIC INFORMATION
C ON PULSE SHAPE.
C
C EXPONENTIAL DECAY VERSION.
C THE PULSE IS A LINEAR RISE FOLLOWED BY AN EXPONENTIAL DECAY
C PSI=PSIM*(T-TAUO)/(TAUM-TAUO), TAUM.LE.T.LE.TAUM
C PSI=PSIM*EXP(-((T-TAUM)/TAU)), TAUM.LT.T.LE.TAUF
C
C REQUIRED INPUT FROM DATA CARD AND COMMON WITH PSIFUN AND PSICOF
C TAUC,TAUM,TAUF,TAU,PSIM
C
C DIMENSION TMAX(1),PSIMAX(1),TJMP(1),PSIJMP(1)
C COMMON/PULSE/TO,TM,TF,TAU,PSIM
C ***** DATA INPUT AND TITLE
C READ 290,TO,TM,TF,TAU,PSIM
C PRINT 291,TO,TM,TF,TAU,PSIM
C TAUO=TO
C TAUF=TF
C KSTOP=0
C IF(PSIM.GT.1.0) GO TO 203
C KSTOP=1
C PRINT 292
C RETURN
C ***** TYIELD, INSTJMP,NOMAX,TMAX,PSIMAX,NOJMP,TJMP,PSIJMP
203 IF(TM.GE.TO.AND.TF.GE.TM) GO TO 205

```

```

      KSTOP=1
      PRINT 293
      RETURN
205  NOMAX=1
      PSIMAX(1)=PSIM
      TMAX(1)=TM
      IF(TM.GT.TO) GO TO 208
      INSTJMP=NOJMP=1
      TJMP(1)=TYIELD=TO
      PSIIMP(1)=PSIM
      GO TO 210
208  INSTJMP=NOJMP=0
      TYIELD=TO+(TM-TO)/PSIM
C***** TOTIMP AND PLASIMP
210  EXPINT=PSIM*TAU*(1.0-EXP((TM-TF)/TAU))
      IF(TM.GT.TO) GO TO 212
      TOTIMP=PLASIMP=EXPINT
      GO TO 215
212  TOTIMP=PSIM*(TM-TO)/2.0+EXPINT
      PLASIMP=PSIM*(TM-TYIELD)*(TM+TYIELD-2.0*TO)/(2.0*(TM-TO))+EXPINT
C***** TMEAN
215  TEXPINT=PSIM*TAU*(TM+TAU-TYIELD-(TF+TAU-TYIELD)*EXP((TM-TF)/TAU))
      IF(TM.GT.TO) GO TO 217
      TMEAN=TEXPINT/PLASIMP
      GO TO 219
217  TTINT=PSIM*(2.0*TM**3-3.0*(TYIELD+TO)*TM**2+6.0*TYIELD*TO*TM
1      +(TYIELD-3.0*TO)*TYIELD**2)/(6.0*(TM-TO))
      TMEAN=(TTINT+TEXPINT)/PLASIMP
219  CONTINUE
      RETURN
C***** FORMATS
290  FORMAT(5E15.7)
291  FORMAT(1H0,7X*SUBROUTINE PULSEINF IS EXPONENTIAL DECAY VERSION*/8X
1      *THE PULSE IS A LINEAR RISE FOLLOWED BY AN EXPONENTIAL DECAY*
2      /13X*PSI = (PSIM)(T-TAUO)/(TAUM-TAU), TAUO.LE.T.LE.TAUM,*,
3      /13X*PSI = (PSIM)EXP(-(T-TAUM)/TAU), TAUM.LT.T.LE.TAUF,*,
4      /13X*TAUO ==E12.5*, TAUM ==E12.5*, TAUF ==E12.5
5      *, TAU ==E12.5*, PSIM ==E12.5)
292  FORMAT(1H0,7X*NO PLASTIC DEFORMATION OCCURS SINCE PSI NEVER *
1      *EXCEEDS 1.0*)
293  FORMAT(1H0,7X*ERROR IN INPUT DATA*)
      END

```

```

      FUNCTION PSIFUN(T)
C
C      SUBROUTINE WHICH PROVIDES RINGLOAD PROGRAM WITH VALUES OF PSI.
C
C      EXPONENTIAL DECAY VERSION.
C      THE PULSE IS A LINEAR RISE FOLLOWED BY AN EXPONENTIAL DECAY.
C
      COMMON/PULSE/TO,TM,TF,TAU,PSIM
      PSIFUN=0.0
      IF(T.LT.TO.OR.T.GT.TF) RETURN
      IF(T.GE.TO.AND.T.LT.TM) 304,305
304  PSIFUN=PSIM*(T-TO)/(TM-TO)
      RETURN
305  PSIFUN=PSIM*EXP((TM-T)/TAU)
      END

```

```

C      SUBROUTINE PSICDEF(T,SPSI)
C
C      SUBROUTINE WHICH PROVIDES RINGLOAD PROGRAM WITH THE COEFFICIENTS
C      OF THE POWER SERIES EXPANSION OF PULSE SHAPE PSI.
C
C      EXPONENTIAL DECAY VERSION.
C      THE PULSE IS A LINEAR RISE FOLLOWED BY AN EXPONENTIAL DECAY.
C
C      INPUT
C          T (TIME)
C
C      OUTPUT
C          SPSI(N) (NTH DERIVATIVE OF PSI DIVIDED BY N FACTORIAL AND
C          EVALUATED AT T, FOR N = 1,2,3,4)
C
C      DIMENSION SPSI(4)
C      COMMON/PULSE/TO, TM, TF, TAU, PSIM
C      DO 350 I=1,4
350  SPSI(I)=0.0
C      IF(T.LT.TO.OR.T.GT.TF) RETURN
C      IF(T.GE.TO.AND.T.LT.TM) 354,355
354  SPSI(1)=PSIM/(TM-TO)
C      RETURN
355  E=PSIM*EXP(-((TM-T)/TAU))
C      SPSI(1)=-E/TAU
C      SPSI(2)=E/(2.0*TAU**2)
C      SPSI(3)=-E/(6.0*TAU**3)
C      SPSI(4)=E/(24.0*TAU**4)
C      END

```

ACKNOWLEDGMENTS

I wish to thank G. S. Rosenberg of the Reactor Engineering Division for suggesting this problem and P. G. Hodge, Jr., Professor of Mechanics, Illinois Institute of Technology, for his comments.

REFERENCES

1. P. S. Symonds, Dynamic Load Characteristics in Plastic Bending of Beams, J. Appl. Mech. 20 (1953), pp. 475-481.
2. P. G. Hodge, Jr., The Influence of Blast Characteristics on the Final Deformation of Circular Cylindrical Shells, J. Appl. Mech. 23 (1956), pp. 617-624.
3. G. Eason and R. T. Shield, Dynamic Loading of Rigid-plastic Cylindrical Shells, J. Mech. Phys. Solids 4 (1956), pp. 53-71.
4. D. C. Drucker, "Limit Analysis of Cylindrical Shells under Axially Symmetric Loading," Proceedings of the First Midwest Conference on Solid Mechanics, pp. 158-163 (1953).
5. P. G. Hodge, Jr., Limit Analysis of Rotationally Symmetric Plates and Shells, Prentice-Hall, Inc., Englewood Cliffs, N. J. (1963), Ch. 3.
6. N. W. Clark, DIFSUB, 3600 FORTRAN Subroutine Which Performs One Integration Step for a System of First Order Ordinary Differential Equations, ANL Library Subroutine ANL-D250 (1966).