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Informal Report

Luminosity for Unbunched Beams in the ISA

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ABSTRACT

The determination of the luminosity is based on a set of superimposed constraints. One must consider, for example, the beam-beam tune shifts, including the long range contributions, the amount of free space around the crossing point (related to the maximum allowable β -value consistent with lattice stability), the collision interaction length, and the limitations on particle line density due to single beam phenomena. Because of the many constraints, the determination of the accelerator parameters giving a design luminosity must be the result of a complex optimization process. A simplified treatment is given here. Assuming rectangular, uniform density, unbunched beams, a simplified optimization is performed. A few examples of sets of design parameters are given.

1. Introduction

The determination of the luminosity is based on a set of superimposed constraints. The most stringent of these is that imposed by the beam-beam interaction including the long range effect.¹ However, there are other constraints. One might, for example, put a maximum on the collision interaction length. There are also constraints on the free space (i.e. no magnetic lattice elements) around the crossing point. The expanding beam envelope places an upper limit on this free length. This is due to the fact that the presence of high β -values tends to lower the stability of the lattice. On the other hand, considerations relative to experimental design in the collision region might place a lower limit on the free space. In other words, the available free space could prove to be a limitation on the type of experiments that can be performed. And, of course, there may be a limit on the particle line density due to single beam phenomena, e.g. space charge ν -shift, transverse and longitudinal collective instabilities, and beam interaction with gas molecules and ions in the surrounding environment.

It is thus clear that the determination of a design luminosity must be the result of a complex optimization process.² We will not attempt a complete analysis here. Rather, we will take a simplified situation consisting

of rectangular, uniform density, unbunched beams and perform a somewhat simplified optimization process. We will then give a few examples to show how the process works, thus arriving at a few sets of design parameters.

2. Optimization Process

The parameters for the optimization process are as follows:

- L : the luminosity in units ($\text{cm}^{-2} \text{ sec}^{-1}$),
- $\Delta\nu_x$: the horizontal beam-beam tune shift,
- $\Delta\nu_y$: the vertical beam-beam tune shift,
- l : the total free space around the interaction point,
- β_0 : the value of β at the crossing point,
- α : the crossing angle for vertical crossing, and
- λ : the line density (number of particles per unit length of circumference).

We assume that the horizontal and vertical emittances, ϵ_H and ϵ_V respectively, are given, that the energy γ (in units of proton mass) is fixed, and that the horizontal and vertical β -functions in the free space around the crossing point are the same. We also consider vertical crossing, although our results apply equally well for horizontal crossing, with the proper change in the meanings of the parameters.

The simplified optimization process we will use consists of the following steps:

- a. Fix l .
- b. From a given $\beta_{\max}$, determine β_0 from

$$\beta_{\max} = \beta_0 + \frac{l^2}{4\beta_0} \quad (2.1)$$

- c. Fix λ (or α).

d. Determine α (or λ) so that $\Delta\nu_x$ is at the beam-beam limit. For vertical crossing, $\Delta\nu_y$ is considerably less than $\Delta\nu_x$ and so need not be considered. The conventional beam-beam limit, obtained from electron colliding beam results is 0.025. The validity of this limit to proton colliding beams is not clear.²⁻⁴ We will here arbitrarily choose the value 0.01, although even this might be too high. It should also be noted that since we use uniform density beams, the computed tune can be expected to be somewhat lower

than for more realistic beam shapes (i.e. Gaussian). Thus, the choice of 0.01 may not be conservative at all.

e. Given l , β_0 , α , λ , compute L .

3. Luminosity and Tune Shifts for Uniform, Rectangular Beams

We will not give the derivations of the expressions for the luminosity and the tune shifts here. The tune shifts are obtained in a manner analogous to that used in Ref. (1). For the luminosity, we use the usual expression for unbunched beams,²

$$L = 2c \int \rho_1 \rho_2 dx dy ds, \quad (3.1)$$

where s is the azimuthal distance from the crossing point, ρ_1 and ρ_2 are particle densities, c is the velocity of light, x and y refer to the horizontal and vertical dimensions respectively, and the integration is to be done over the entire interaction volume.

We therefore can write for the luminosity, L , and the tune shifts Δv_x and Δv_y (including the long range interaction),

$$L = L_0 K(\xi, \eta), \quad (3.2)$$

$$\Delta v_x = \Delta' v_0 l I_x(\xi, \eta), \quad (3.3)$$

$$\text{and} \quad \Delta v_y = \Delta' v_0 l I_y(\xi, \eta). \quad (3.4)$$

The quantity L_0 is the maximum luminosity for head-on collisions,

$$L_0 = \frac{\pi c \lambda^2}{2\sqrt{\epsilon_H \epsilon_V}}, \quad (3.5)$$

while $\Delta' v_0$ is the tune shift per unit length for circular beams colliding head-on,

$$\Delta' v_0 = -\frac{\lambda r_p}{4\gamma \sqrt{\epsilon_H \epsilon_V}}, \quad (3.6)$$

where r_p is the classical proton radius ($= 1.54 \times 10^{-18}$ m).

The form factors K , I_x and I_y are functions of the derived variables,

$$\xi = \frac{l}{2\beta_0}, \quad (3.7)$$

$$\eta = \frac{\alpha \sqrt{\beta_0}}{\sqrt{\epsilon_V}}, \quad (3.8)$$

and are given by,

$$K(\xi_m, \eta) = \frac{2}{\pi} \int_0^{\xi_m} \frac{dt}{(1+t^2)^{3/2}} \left[\sqrt{1+t^2} - \frac{1}{2}\eta t \right], \quad (3.9)$$

with

$$\xi_m = \text{minimum} \left[\xi, \frac{2}{\sqrt{\eta^2 - 4}} \right], \quad (3.10)$$

$$I_x(\xi, \eta) = \frac{2}{\pi\xi} \int_0^\xi dt \left[\tan^{-1} X_+ + \tan^{-1} X_- \right], \quad (3.11)$$

$$I_y(\xi, \eta) = \frac{2}{\pi\xi} \int_0^\xi dt \left[\tan^{-1} Y_+ + \tan^{-1} Y_- \right], \quad (3.12)$$

with

$$X_\pm = \sqrt{\frac{\epsilon_V}{\epsilon_H}} \left\{ \frac{\sqrt{1+t^2} \pm \eta t}{\sqrt{1+t^2}} \right\}, \quad (3.13)$$

$$Y_\pm = \sqrt{\frac{\epsilon_H}{\epsilon_V}} \left\{ \frac{\sqrt{1+t^2}}{\sqrt{1+t^2} \pm \eta t} \right\}. \quad (3.14)$$

The parameters ξ and η have simple physical meanings¹ in the case where $l > 2\beta_0$. Since in this case,

$$l \approx 2\sqrt{\beta_{\max}\beta_0}, \quad (3.15)$$

and the distance of the beam centers, d , at $\pm l/2$ is given by

$$d \approx \frac{1}{2} \alpha l \approx \alpha \sqrt{\beta_{\max}\beta_0}, \quad (3.16)$$

we have

$$\xi = \sqrt{\frac{\beta_{\max}}{\beta_0}}, \quad (3.17)$$

and

$$\eta = \frac{d}{\sqrt{\beta_{\max}\epsilon_V}}. \quad (3.18)$$

In other words, ξ is the ratio of the beam size at the end of the interaction region to that at the center. ξ^2 is the corresponding ratio for β -functions. η is a measure of the beam separation at the ends of the interaction region (note that the interaction region is not synonymous with the collision region unless the crossing angle is sufficiently small). Specifically, η is the number of beam $\frac{1}{2}$ widths of separation at the ends of the interaction region.

The parameter ξ_m is related to the collision length, ℓ_{coll} . For large crossing angles, i.e., large η ,

$$\ell_{coll} \approx \frac{4\beta_0}{\eta} \quad . \quad (3.19)$$

For the ISA, $\gamma = 200$, $\epsilon_H = 0.076 \times 10^{-6}$ rad m, $\epsilon_V = 0.56 \times 10^{-6}$ rad m. Therefore,

$$L_0 = 2.2 \times 10^{34} \hat{\lambda}^2 \text{ sec}^{-1} \text{ cm}^{-2} \quad , \quad (3.20a)$$

$$\Delta' v_0 = - 0.003 \hat{\lambda} \text{ m}^{-1} \quad , \quad (3.20b)$$

where $\hat{\lambda}$ is the line density in units of 3.1×10^{11} per m. We have used this unit since it corresponds to a current of 15 A, or 7.5×10^{14} protons stacked in a ring of circumference 2400 m.

Thus, we have

$$L = 2.2 \times 10^{34} \hat{\lambda}^2 K(\xi_m, \eta) \quad , \quad (3.21)$$

$$\Delta v_x = - 0.003 \hat{\lambda} \ell I_x(\xi, \eta) \quad , \quad (3.22)$$

$$\Delta v_y = - 0.003 \hat{\lambda} \ell I_y(\xi, \eta) \quad , \quad (3.23)$$

4. Examples

We will follow the simplified optimization procedure outlined in section 2. We will give three examples, setting

$$\begin{aligned} \ell &= 20, 60, 100(\text{m}) \quad , \\ \beta_{\max} &= 52, 156, 260(\text{m}) \quad . \end{aligned} \quad (4.1)$$

$$\text{This gives} \quad \beta_0 = 2, 6, 10(\text{m}) \quad . \quad (4.2)$$

The ξ parameter, which is approximately the ratio of the square root of $\beta_{\max}$ to β_0 is then,

$$\xi = 5, 5, 5 \quad . \quad (4.3)$$

If we now fix the current at 15 A by putting $\hat{\lambda} = 1$, we can find η from (3.22), setting

$$\Delta v_x = - 0.003 \hat{\lambda} \ell I_x(\xi, \eta) = - 0.01 \quad , \quad (4.4)$$

since ξ , ℓ , and $\hat{\lambda}$ are given.

The solution for η can be done numerically [with I_x given by (3.11)].

since,
$$I_x = 0.167, 0.056, 0.033, \quad (4.5)$$

we obtain

$$\eta = 3.4, 8.0, 13.0 \quad . \quad (4.6)$$

For these values of η , we can compute from (3.9),

$$K = 0.19, 0.08, 0.05 \quad . \quad (4.7)$$

The luminosities are thus

$$L = \begin{cases} 4.1 \times 10^{33} \text{ cm}^{-2} \text{ sec}^{-1} \\ 1.8 \times 10^{33} \\ 1.1 \times 10^{33} \end{cases} \quad (4.8)$$

Since the luminosity varies with the square of the charge density, while the tune shift goes linearly with the charge density and roughly linearly with the crossing angle, an increase in the luminosity can be attained if we increase both charge density and crossing angle (i.e. η) proportionately. If we do this, the tune shift remains constant. However, since the luminosity decreases only linearly with the crossing angle, the luminosity will increase linearly with the density. Thus, in the first case above, we can triple the crossing angle and roughly triple the current (i.e. 45 A) to obtain

$$\eta = 10.2 \quad ,$$

which leads to

$I_x = 0.044$ (actually a decrease of more than a factor of 3);
and keeping $\hat{\lambda} I_x$ constant:

$$\hat{\lambda} I_x = 0.167 \quad , \quad (4.9)$$

we have

$$\hat{\lambda} = 3.8 \text{ (57 A)} \quad .$$

The phenomena limiting the attainment of such a current are the single beam effects, including collective instabilities and instabilities relating to the vacuum. These must be avoided in the design of the storage rings.

Using $\eta = 10.2$ and $\hat{\lambda} = 3.8$, we obtain for the luminosity

$$L = 2.0 \times 10^{34} \text{ cm}^{-2} \text{ sec}^{-1} \quad . \quad (4.10)$$

The four cases we have considered here are summarized in Table I.

TABLE I
Design Parameters for Intersection Regions
p-p Storage Rings with Unbunched Beams

Parameter		Case			
		1	2	3	4
Luminosity ($\text{cm}^{-2}\text{sec}^{-1}$)	L	4.1×10^{33}	1.8×10^{33}	1.1×10^{33}	2.0×10^{34}
Beam-beam ν -shift (Hor)	$\Delta \nu_x$	- 0.01	- 0.01	- 0.01	- 0.01
Beam-beam ν -shift (Ver)	$\Delta \nu_y$	+ 0.002	+ 0.001	+ 0.001	+ 0.001
Line density (m^{-1})	λ	3.1×10^{11}	3.1×10^{11}	3.1×10^{11}	11.8×10^{11}
Current (A)		15	15	15	57
Interaction Region Length (m)	ℓ	20	60	100	20
Collision Region length (m)	ℓ_{coll}	2.4	3.0	3.1	0.8
β -value at crossing (m)	β_o	2	6	10	2
β -value at $\pm \frac{1}{2} \ell$ (m)	β_{max}	52	156	260	52
Crossing angle (mrad)	α	1.8	2.4	3.1	5.4
Beam size variation factor	ξ	5	5	5	5
Beam separation factor	η	3.4	8.0	13.0	10.2
Energy	γ	200			
Horizontal emittance ($\mu\text{rad m}$)	ϵ_H	0.076			
Vertical emittance ($\mu\text{rad m}$)	ϵ_V	0.56			

3. Conclusions

Using a simplified optimization procedure, we have derived a few sets of parameters characterizing the interaction regions. Our limited objective was to see what range of free length around the crossing point could be attained and what the corresponding luminosities would be. From the results of Table I, we can see that a wide range of free lengths can be attained. The range in our examples was a total length, ℓ , of 20 to 100 meters free. The corresponding luminosities decreased an order of magnitude, from $2 \times 10^{34} \text{ cm}^{-2}\text{sec}^{-1}$ to 1×10^{33} for the longest free length.

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