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REACTOR PHYSICS QUARTERLY REPORT

JULY, AUGUST, SEPTEMBER 1969

November 1969

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REACTOR PHYSICS QUARTERLY REPORT
JULY, AUGUST, SEPTEMBER 1969

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REACTOR PHYSICS QUARTERLY REPORT
JULY, AUGUST, SEPTEMBER 1969

FOREWORD

The objective of the Reactor Physics Quarterly Report is to inform the scientific community in a timely manner of the technical progress made on the many phases of reactor physics work within the laboratory. The report contains brief technical discussions of accomplishments in all areas where significant progress has been made during the quarter. The results presented herein should be considered preliminary, and do not constitute final publication of the work. A list of publications and papers issued during the last quarter is included in the report. Anyone desiring additional information concerning the work reported herein is encouraged to contact the author directly.

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BNWL-534	April, May, June	1967

BNWL-634	July, August, September	1967
BNWL-685	October, November, December	1967
BNWL-775	January, February, March	1968
BNWL-887	April, May, June	1968
BNWL-921	July, August, September	1968
BNWL-985	October, November, December	1968
BNWL-1053	January, February, March	1969
BNWL-1150	April, May, June	1969

TABLE OF CONTENTS

LIST OF FIGURES	ix
LIST OF TABLES	xi
1.0 SUMMARY	1.1
Reactor Theory and Code Development.	1.1
Thermal Reactors	1.1
Fast Reactors	1.2
Critical Mass Physics	1.3
2.0 REACTOR THEORY AND CODE DEVELOPMENT.	2.1
Compressible Fluid Flow (BNW Program Gas)	2.1
Constraints in Nonlinear Least-Squares	
Program Learn	2.2
Definition of the Infinite Neutron Multiplication	
Factors Measured by the PCTR Poisoned and	
Unpoisoned Technique	2.3
Program Bufit: A Least Squares Code for the	
Analysis of Burnup Data.	2.16
3.0 THERMAL REACTORS ,	3.1
Multiplication Dependence on Energy and Angular	
Detail in Transport Theory Calculations.	3.1
Heterogeneity Correction to Fast-Fissions	3.6
An Equivalence Approximation for Use in Applying	
the Dunn Flux Peaking or Dipping Correction	
to Pin Activities	3.12
Batch Core Burnup Data	3.19
PRTR Batch Core - Critical Test No. 4	3.20
ZODIAC-ALTHAEA Comparison with Pu	
Enriched Fuels	3.22
4.0 FAST REACTORS	4.1
Neutron Spectrum Studies in the PCTR	
Fast Neutron Cavity.	4.1
A Multichannel Approach to Fast Reactor	
Safety Analysis	4.9

	Fast Reactor Disassembly Calculation	
	Utilizing a Temperature-Density Dependent	
	Equation of State.	4.12
	ETOX	4.15
	RSYN	4.15
	Beam Bending Code Development.	4.17
	Model Formulation - Beam Bending Code.	4.18
	Code Design - Beam Bending Code	4.23
	Impact of Swelling Model on FTR Core Design	4.28
5.0	CRITICAL MASS PHYSICS.	5.1
	Critical Parameters for 0 to 7% PuO ₂ -U(NAT)O ₂	
	Rods in Water.	5.1
	Reflector Savings for Uranyl Nitrate	
	of Low Enrichment.	5.10
6.0	PUBLICATIONS AND PRESENTATIONS	6.1
	Publications	6.1
	Papers Accepted for Presentation at the Winter	
	Meeting of the American Nuclear Society,	
	San Francisco, California,	
	November 30-December 4, 1969	6.1
	Papers Accepted for Publication	6.2
	Papers Presented	6.2

LIST OF FIGURES

4.1	Comparison of Proton-Recoil Spectrometer and Multiple Foil Activation Results	4.2
4.2	Neutron Flux Spectrum in the PCTR Fast Neutron Cavity	4.6
4.3	Reactivity and Power Versus Time for a Hypothetical Pipe Rupture Accident	4.11
4.4	Pressure-Density Variations at the Central Node for a Hypothetical 100\$/sec Ramp in a FTR Type Core	4.14
4.5	Pseudotemperature Gradients Across Flow Ducts	4.32
4.6	Flow Duct Displacement Due to Swelling	4.33
4.7	Unrestrained Duct Displacement Due to Swelling	4.34
4.8	Comparison of Creep Effects	4.36
4.9	Maximum Restrained Duct Deflection from Differential Swelling	4.38
4.10	Maximum Unrestrained Duct Deflection Versus Burnup	4.39
4.11	Duct Dimensional Change	4.41
4.12	Dimensional Change in Pin Bundle	4.43
5.1	Minimum Critical Mass as a Function of Fuel Enrichment	5.4
5.2	Minimum Critical Volume as a Function of Fuel Enrichment	5.5
5.3	Minimum Critical Cylinder Diameter as a Function of Fuel Enrichment	5.6
5.4	Minimum Critical Number of Rods in Cylindrical Geometry as a Function of Fuel Enrichment	5.7
5.5	Minimum Critical Slab Thickness as a Function of Fuel Enrichment	5.8

5.6	Minimum Critical Areal Density as a Function of Fuel Enrichment	5.9
5.7	Reflector Savings for a 2.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere	5.12
5.8	Reflector Savings for a 3.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere	5.13
5.9	Reflector Savings for a 4.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere	5.14
5.10	Reflector Savings for a 5.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere	5.15
5.11	Reflector Savings as a Function of Reflector K_{∞} for a 50.80 cm Thick Uranyl Nitrate Reflector Surrounding a Spherical Uranium Lattice	5.16

LIST OF TABLES

3.1	Description of the Reactors	3.1
3.2	Calculated Values of k_{eff} for Different Multi-Group Structures	3.2
3.3	Group Structure (Lower Energy Boundaries in eV)	3.3
3.4	Effects of Using Higher Order S_n Approximation	3.4
3.5	Comparison Between Results of Transport Theory and Diffusion Theory Calculations	3.4
3.6	Heterogeneity Corrections to Fast-Fissions	3.11
3.7	The Results of Measurements of β/λ	3.21
3.8	ZODIAC-ALTHAEA Comparison	3.23
4.1	Infinitely Dilute Saturated Activities	4.5
4.2	Summary of Component Bending Reflection	4.29
4.3	Summary of Component Swelling in Radial Dimension	4.29
5.1	Comparison of HAMMER System Calculations with Experimental Measurements	5.2
5.2	Minimum Critical Parameters for 0 to 7 wt% PuO ₂ -UO ₂ Rods in Water (5% ²⁴⁰ Pu; natural uranium; 95% theoretical density)	5.3
5.3	Core Configurations Used in Calculations	5.11

1.0 SUMMARY

REACTOR THEORY AND CODE DEVELOPMENT

Battelle-Northwest Program Gas, a gas loop analysis program, has been modified to treat gases with temperature-dependent specific heats.

A general method for imposing explicit equality constraints on the functional parameters in the nonlinear least-squares program LEARN-I-A has been developed and demonstrated.

Major changes have been made to the least-squares code, which is used to extract one group cross section information from isotopic concentration burnup data. The resulting new programs are called BUFIT (burnup fitting code) and a double precision version DBUFIT.

The definition of the infinite multiplication factor has been clarified and related to the parameter usually calculated by theory. In addition, leakage terms required for using an unpoisoned technique have been derived.

THERMAL REACTORS

The fourth interim critical tests were conducted on the 55 fuel element Batch Core loading at the PRTR at an average core burnup of 5330 MWd/MTM. Tests were also conducted on two smaller loadings of 31 and 19 fuel elements.

Destructive burnup data from samples cut from PRTR rods have been evaluated for consistency and processed through the ISODIL burnup analysis code.

Calculations of the fast effect for uranium fueled-H₂O lattices have shown that the error incurred in assuming a homogeneous cell is less than 0.5% in k_{eff} . Thus, if all other sources of error were eliminated in the calculations, values of $k_{eff} \leq 1.005$ should be calculated for the UO₂ and the UO₂-PuO₂ fueled water lattices.

An analytical study was made to determine the sensitivity of calculated values of k_{eff} to the amount of energy and angular detail used in a transport theory solution for a thermal reactor problem. The results show that a 10-group S-8 solution is accurate to within $\pm 0.2\%$ in k_{eff} .

An equivalence approximation has been developed for use in applying the Dunn flux peaking or dipping correction to pin activations.

A comparison was made between the results of burnup calculations obtained using the ZODIAC G and ALTHAEA codes. As a result of this comparison, it is concluded that the ALTHAEA code can be used in studies of reactor operation and behavior for plutonium fueled cores as well as uranium fueled cores.

FAST REACTORS

Neutron spectrum studies have been made in the Physical Constants Test Reactor Fast Neutron Cavity using both a proton-recoil spectrometer and a multiple foil activation method. Comparison of the two experimental methods shows that information obtained from each supplements and confirms the results of the other. Composite results of these measurements provide a valuable check of both reactor theory and analytical technique for developing calculational methods for fast-thermal coupled reactor cores.

Comparisons of the group cross sections generated from three codes (ETOX-1DX, PIC², and GARFAR) were compared for the Fast Reactor ZPR III Assembly 52.

The Beam Bending Code Development Program has progressed to the point where major decisions are being made for the model formulation and code computing procedures. The code to be developed will be a modification of AXISOL-AXICRP. This general finite element procedure will permit the creep bending analysis

of the flow ducts using the same structural idealization as used for elastic bending analyses. A matrix decomposition scheme that has been developed for the code's application simplifies solution of the equations governing the beam behavior.

Differential swelling in the fuel assemblies result in duct bending which is the major fuel lifetime-limiting item. Additionally, fuel pin packing within the duct may occur between 30 and 80,000 MWd/MTM, depending upon the metal swelling models used.

CRITICAL MASS PHYSICS

The HAMMER system has been found to calculate, with reasonable accuracy, material bucklings measured experimentally for 1.5, 2.0, 4.0 and 6.6 wt% $\text{PuO}_2/\text{PuO}_2\text{-UO}_2$ rods in water.

Values of reflector savings are presented for uranyl nitrate of low enrichment (<5 wt% ^{235}U) reflecting a spherical lattice of uranium rods in water. Curves of reflector savings as a function of k_∞ for a reflector of fixed thickness are nearly independent of the ^{235}U enrichment. Calculations were performed by HFN computer program.

2.0 REACTOR THEORY AND CODE DEVELOPMENT

COMPRESSIBLE FLUID FLOW (BNW PROGRAM GAS)

B. H. Duane

Battelle-Northwest Program Gas constructs one-dimensional analytic streamtube integrals for the supersonic or subsonic flow of a real gas (having temperature-variant specific heats) through an open-loop or closed-loop succession of nozzles, oblique-shock diffusers, normal shocks, bypass flows with pressure-matched re-entries, heat exchangers, burners, turbines, compressors, shock pumps, and other such elements simply characterizable in terms of mass, energy, momentum, and entropy transfer. Analysis versatility includes coupled variation of design specifications to meet performance requirements.

Physics Representation

$$\text{Mass Balance: } \nabla \cdot \rho \underline{v} = Q_1 \quad (1)$$

$$\text{Energy Balance: } \nabla \cdot \rho \underline{v} (h + v^2/2) = Q_2 \quad (2)$$

$$\text{Momentum Balance: } \nabla \cdot (\rho \underline{v} \underline{v} + p) = Q_3 \quad (3)$$

$$\text{Entropy Balance: } \nabla \cdot \rho \underline{v} s = Q_4 \quad (4)$$

$$\text{Equations of State: } p = r \rho T \quad (5)$$

$$c_P(T) = c_V(T) + r \quad (6)$$

$$c_V(T) = r \sum_m (N_m/N) \left\{ 3/2 + R_m/2 + \sum_n \left[(T_{mn}/2T) \operatorname{csch}(T_{mn}/2T) \right]^2 \right\} \quad (7)$$

Here, ρ is fluid density (mass/volume), $\underline{v}$ is fluid velocity, h is fluid enthalpy/mass, p is fluid pressure, s is fluid entropy/mass, T is fluid absolute temperature, $r = p/\rho T$ is the gas constant, $c_P(T) = (\partial h/\partial T)_P$ is specific heat/mass at constant pressure, $c_V(T) = T(\partial s/\partial T)_V$ is specific heat/mass at constant volume, and N_m/N is the fraction of molecules having

rotational degrees of excitation freedom R_m and internal vibration characteristic temperatures T_{mn} . The flexible source-sinks Q_1 , Q_2 , Q_3 , and Q_4 provide for piecewise variant transfer of mass, energy, momentum, and entropy respectively.

These equations are integrated analytically over each streamtube segment by conversion to surface integrals to provide firm numeric reliability for any segmentation mesh size. The transcendental integrated equations are solved exactly, by iterating second-order expansion logic to solid convergence. Traveling or static oblique shocks are generated as Galilean velocity transforms of static normal shocks.

The source deck consists of a 2087-card, 7-link overlay chain, formulated in the machine-independent dialectal intersection of Univac-1108 Fortran-V-CSC and IBM-7090 Fortran-IV-13, and designed for both modular growth versatility and freedom from obsolescence. Input consists, in essence, of gas compositions (r , N_m/N , R_m , T_{mn}), shock intensities and angles, implicit or explicit transverse areas, implicit or explicit source-sinks (Q_1 , Q_2 , Q_3 , Q_4) for any three of the four balance equations, and design-variation specifications, all fed in via teletype, cards, bulk peripheral storage, remote transmitter, or fast memory input linkage. Output consists of thermodynamically complete portrayals of both design and performance, fed out via plotter, teletype, line printer, remote transmitter, or fast memory output linkage.

CONSTRAINTS IN NONLINEAR LEAST-SQUARES PROGRAM LEARN

D. A. Kottwitz

A general method for imposing explicit equality constraints on the functional parameters in nonlinear least-squares program LEARN-I-A has been developed and demonstrated. It is a natural generalization of a previously reported scheme for holding a subset of parameters fixed at constant values.⁽¹⁾ It has been designed for incorporation in existing LEARN program packages

with a minimum of reprogramming. The current general version is exact only for linear constraints. To avoid excessive waste of computer time and storage, nonlinear equality constraints should be handled on an individual basis.

The following is a brief description of the logic of the method. At execution time the user specifies which parameters in his fitting function are free, fixed, or constrained. For each constrained parameter A_c he supplies (in the constraint subroutine) an explicit function of the free and fixed parameters $A_c = f_c(A_{\text{free}}, A_{\text{fix}})$ and its first partial derivative $\partial A_c / \partial A_{\text{free}}$. During execution the program uses the constraint equations and their derivatives to modify the partial derivatives of the free parameters. The iterative corrections of the free parameters only are then calculated in the usual manner. At various times during execution the parameter list is collapsed to the free set or expanded to the entire set, depending on which is currently required.

In its present form, the scheme is implemented by three new subroutines (CALA, EXPAND, WRITEA), a slightly modified package subroutine (STATUS), and several standard batches of cards to be included in three of the package routines (PUTIN, THEORY, PUTOUT).

Reference

1. D. A. Kottwitz. "The Shape of Peaks in Gamma-Ray Spectra from Ge(Li) Detectors," *Reactor Physics Quarterly Report*, April, May, June 1969, BNWL-1150. Battelle-Northwest, Richland, Washington, August 1969.

DEFINITION OF THE INFINITE NEUTRON MULTIPLICATION FACTORS MEASURED BY THE PCTR POISONED AND UNPOISONED TECHNIQUE

E. P. Lippincott, C. R. Richey, D. D. Lanning

Recently questions have arisen regarding the definition of the infinite neutron multiplication factors (k_{∞}^*) as measured by

the well known Physical Constants Test Reactor (PCTR) "poisoned" technique⁽¹⁾ and the more recently developed "unpoisoned" method. The unpoisoned technique was originally proposed by Heineman⁽²⁾ as a means of measuring k_{∞}^* in systems where the buckling was not adjusted to zero. It is the purpose of this report to clarify the definition of the measured k_{∞}^* and relate it to the parameter usually calculated by theory. In addition, some questions, which have been raised concerning the adequacy of the leakage terms in the original derivation of k_{∞}^* for the unpoisoned technique,⁽²⁾ are answered by a new derivation.

Definition of k_{∞}^*

The PCTR technique for measuring k_{∞}^* of a neutron multiplying medium is based primarily upon determining the excess neutron production of a test sample. This excess neutron production is deduced from the measured amount of neutron absorber that (1) must be added to the test sample to obtain a null reactivity change when the test sample is compared with a void and (2) reduces the buckling of the poisoned central cell to zero. The k_{∞}^* measured in the PCTR poisoned technique is consistent with the usual two group critical equation⁽³⁾

$$k_{\infty}^* = (1 + B^2\tau)(1 + B^2L^2), \quad (1)$$

where τ and L^2 are defined as

$$\tau = \frac{D_1}{\Sigma_1^a + \Sigma_1^R} \text{ and } L^2 = \frac{D_2}{\Sigma_2^a},$$

where Σ_1^a and Σ_2^a are the absorption cross sections for epithermal and thermal neutrons, and Σ_1^R is the cross section for removal of neutrons from the epithermal group by scattering to the thermal group. Equation (1) is obtained from the usual condition that the two group flux equations,

$$-D_1 B^2 \phi_1' - \Sigma_1^a \phi_1' - \Sigma_{1 \rightarrow 2} \phi_1' + \frac{k_\infty^*}{p} \Sigma_2^a \phi_2' = 0 \quad (2)$$

$$-D_2 B^2 \phi_2' - \Sigma_2^a \phi_2' + \Sigma_{1 \rightarrow 2} \phi_1' = 0 \quad (3)$$

have a nontrivial solution, i.e. the determinant of the coefficients shall vanish. One particular point needing emphasis is that ϕ_1' and ϕ_2' are the equilibrium fluxes in the test region in the fundamental mode (just critical) environment.

For Equation (2) to explicitly account for all neutron production, epithermal as well as thermal fissions, k_∞^* must be defined as

$$k_\infty^* = \frac{\nu_1 \Sigma_1^f \phi_1' + \nu_2 \Sigma_2^f \phi_2'}{\Sigma_2^a \phi_2'} \times p \quad (4)$$

where

$$p = \frac{\Sigma_1^R}{\Sigma_1^R + \Sigma_1^a} = \frac{(\Sigma_2^a + D_2 B^2) \phi_2'}{(\Sigma_2^a + D_2 B^2) \phi_2' + \Sigma_1^a \phi_1'} \quad (5)$$

Definition of k'_∞

The infinite neutron multiplication factor calculated by various theoretical models is defined as

$$k'_\infty = \frac{\text{Total neutron production}}{\text{Total neutron absorption}} = \frac{\nu_1 \Sigma_1^f \phi_1' + \nu_2 \Sigma_2^f \phi_2'}{\Sigma_1^a \phi_1' + \Sigma_2^a \phi_2'}$$

where ϕ_1' and ϕ_2' are equilibrium fluxes in the fundamental mode problem (a just critical sphere of the test material). This definition leads to a different expression for Equation (2), i.e.,

$$-D_1 B^2 \phi_1' - \Sigma_1^a \phi_1' - \Sigma_{1 \rightarrow 2} \phi_1' + k'_\infty (\Sigma_1^a \phi_1' + \Sigma_2^a \phi_2') = 0. \quad (2a)$$

The critical equation obtained from the coefficients of Equation (2a) and Equation (3) is

$$k_{\infty}' = (1 + B^2 L^2) (1 + B^2 \tau) / [1 + (1 - p) B^2 L^2] \quad (6)$$

Thus, the relationships between the infinite neutron multiplication factor measured in the PCTR (k_c) and that usually quoted from theory (k_{∞}') is

$$k_{\infty}^* = k_{\infty}' [1 + (1 - p) B^2 L^2] = \frac{k_{\infty}'}{p} \left(1 + \frac{\Sigma_1^a \phi_1'}{\Sigma_2^a \phi_2'} \right)^{-1} \quad (7)$$

It is also readily apparent that if the thermal leakage is negligible ($D_2 B^2 \phi_2 \rightarrow 0$), then the escape-to-thermal probability becomes

$$p = \left(1 + \frac{\Sigma_1^a \phi_1'}{\Sigma_2^a \phi_2'} \right)^{-1} \quad (8)$$

and k_{∞}^* becomes identical with k_{∞}' . The advantage of the PCTR technique is that it measures $k_{\infty}^* - 1$. Therefore it is appropriate to look at the difference between k_{∞}' and k_{∞}^* and compare this difference with $k_{\infty}' - 1$. Rewriting Equation (6),

$$k_{\infty}' = (1 + L^2 B^2) (1 + B^2 \tau) - \frac{(1 + \tau B^2) (1 + L^2 B^2) (1 - p) L^2 B^2}{1 + (1 - p) L^2 B^2} \quad (9)$$

and thus

$$\begin{aligned} k_{\infty}' &= k_{\infty}^* - \frac{(1 + \tau B^2) (1 + L^2 B^2) (1 - p) L^2 B^2}{1 + (1 - p) L^2 B^2} \\ &\approx k_{\infty}^* - (1 - p) L^2 B^2. \end{aligned} \quad (10)$$

It may be seen that in cases with thermal leakage and p significantly less than 1, this difference can be a significant correction to $k_{\infty}' - 1$.

Physically, the difference between k_{∞}' and k_{∞}^* arises because k_{∞}^* does not take into account the fact that neutrons absorbed before being thermalized do not undergo thermal leakage. The value of $k_{\infty}' - 1$ represents the true number of neutrons that leak out of a bare critical reactor for each neutron (of any energy) absorbed in the reactor.

Unpoisoned Technique

The unpoisoned technique is a means for obtaining k_{∞}^* from the ratio of the reactivity coefficients of a neutron multiplying sample and a normalizing thermal neutron absorber. The method makes use of the general perturbation expressions derived by Heineman(2)

$$\chi_{\Delta\rho} = \sum_i \int_S dS \phi_i^+ J_i' \quad (11)$$

where

$$\chi = \sum_i \int_{V_o} dV \phi_i^+ \phi_i' \quad (12)$$

and S is the surface enclosing the test sample, V_o is the reactor volume exterior to the test sample, and J' is the outward normal current at the surface of the test sample.

Contracting the general perturbation equation to two groups, Heineman obtained for the reactivity coefficient for the test sample,

$$\chi_{\Delta\rho_{\text{cell}}} = \phi_1^+ D_1 B^2 \phi_1' V_{\text{cell}} + \phi_2^+ D_2 B^2 \phi_2' V_{\text{cell}} \quad (13)$$

and for a small poison sample inserted in the test void

$$\chi_{\Delta\rho_p} = -\phi_1^+ \Sigma_1^p V_p \phi_1^{p'} - \phi_2^+ \Sigma_2^p V_p \phi_2^{p'}. \quad (14)$$

Thus on dividing Equation (1) by Equation (2) and rearranging

$$-\frac{\Delta\rho_{\text{cell}}}{\Delta\rho_p} \frac{\Sigma_2^p \phi_2^{p'} v_p \left\{ 1 + \frac{\Sigma_1^p \phi_1^+ \phi_1^{p'}}{\Sigma_2^p \phi_2^+ \phi_2^{p'}} \right\}}{(\Sigma_2^a \phi_2^{p'} v)_{\text{cell}}} = \frac{D_2 B^2}{\Sigma_2^a} + \frac{D_1 B^2}{\Sigma_2^a} \frac{\phi_1^+ \phi_1^{p'}}{\phi_2^+ \phi_2^{p'}} \quad (15)$$

In the original work⁽²⁾ the adjoint equations were defined by the interchange of rows and columns of the operator matrix defined by Equation (2) and Equation (3) i.e.,

$$-D_1 B^2 \phi_1^+ - \Sigma_1^a \phi_1^+ - \Sigma_1^R \phi_1^+ + \Sigma_1^R \phi_2^+ = 0 \quad (16a)$$

$$\frac{k_\infty^*}{p} \Sigma_2^a \phi_1^+ - D_2 B^2 \phi_2^+ - \Sigma_2^a \phi_2^+ = 0 \quad (16b)$$

Since k_∞^* is a function of both $\phi_1^{p'}$ and $\phi_2^{p'}$, more correct adjoint equations would be

$$-D_1 B^2 \phi_1^+ - \Sigma_1^a \phi_1^+ - \Sigma_1^R \phi_1^+ + v_1 \Sigma_1^f \phi_1^+ + \Sigma_1^R \phi_2^+ = 0 \quad (17a)$$

$$v_2 \Sigma_2^f \phi_1^+ - D_2 B^2 \phi_2^+ - \Sigma_2^a \phi_2^+ = 0 \quad (17b)$$

Using this form of the two group adjoint expressions with the two group flux equations, Equation (2) and Equation (3), the following expression for k_∞^* in the unpoisoned method is obtained.

$$k_\infty^* = 1 - \left(\frac{\Delta\rho_{\text{cell}}}{\Delta\rho_p} \right) \frac{\Sigma_2^p v_p \phi_2^{p'}}{(\Sigma_2^a v \phi_2^{p'})_{\text{cell}}} \left(1 + \frac{\phi_1^+ \Sigma_1^p \phi_1^{p'}}{\phi_2^+ \Sigma_2^p \phi_2^{p'}} \right) + \frac{\tau B^2 (1 + B^2 L^2) [\tau B^2 - v_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)]}{1 + \tau B^2 - v_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)} \quad (18)$$

The details of the derivation of Equation (18) are given on page 2.11. This expression differs from that originally derived by Heineman only in the leakage term. Heineman's original expression is(2)

$$k_{\infty}^* = 1 - \left(\frac{\Delta \rho_{\text{cell}}}{\Delta \rho_p} \right) \frac{\Sigma_2^p V_p \phi_2^{p'}}{(\Sigma_2^a V \phi_2')_{\text{cell}}} \left(1 + \frac{\phi_1^+}{\phi_2^+} \frac{\Sigma_1^p \phi_1^{p'}}{\Sigma_2^p \phi_2^{p'}} \right) + \frac{B^4 \tau^2}{1 + B^2 \tau} \quad (19)$$

It is seen that Equation (18) reduces to Equation (19) if the epithermal fissions are negligible and if the factor $(1 + B^2 L^2)$ is set equal to unity. Using Equation (10) and a substitution of $\eta_1 f_1 \Sigma_1^a$ for $v_1 \Sigma_1^f$

$$\begin{aligned} k_{\infty}' = 1 - \left(\frac{\Delta \rho_{\text{cell}}}{\Delta \rho_p} \right) \frac{(\Sigma_2^p V_p \phi_2^{p'})}{(\Sigma_2^a V \phi_2')_{\text{cell}}} & \left\{ 1 + \frac{\phi_1^+}{\phi_2^+} \frac{\Sigma_1^p \phi_1^{p'}}{\Sigma_2^p \phi_2^{p'}} \right\} \\ & - \frac{(1 + \tau B^2)(1 + L^2 B^2)(1 - p)L^2 B^2}{1 + (1 - p)L^2 B^2} \\ & + \frac{\tau B^2(1 + L^2 B^2) [\tau B^2 - \eta_1 f_1 (1 - p)]}{1 + \tau B^2 - \eta_1 f_1 (1 - p)} \end{aligned} \quad (20)$$

It should be noted that due to the inadequate treatment of leakage in the definition of k_{∞}^* , the value of this quantity will vary slightly with the energy boundary between groups 1 and 2 (cadmium cutoff). Thus, in Equation (18), as the upper energy limit of thermal neutrons is raised and more absorptions are included in the thermal group, the value of $(\Sigma_2^a \phi_2' V)_{\text{cell}}$ is raised, thereby lowering k_{∞}^* . The value of k_{∞}' however, remains relatively constant under the same conditions, because such a change results in a higher value of p in the correction term to counteract the decrease in the first

term (of course k'_{∞} will not remain exactly constant with the boundary change because of limitations of the two-group theory).

Application of Leakage Correction to HTGR Experimental Lattice

For the HTGR test lattice [$^{235}\text{UC}_2$ - ThO_2 -C] experiments in the High Temperature Lattice Test Reactor (HTLTR),⁽⁴⁾ the leakage correction is

$$\frac{\tau B^2(1 + B^2 L^2) [\tau B^2 - \eta_1 f_1(1 - p)]}{1 + \tau B^2 - \eta_1 f_1(1 - p)} = \sim 0.002 \pm 0.002$$

where an error of 10% was assumed for the calculated quantities of $\eta_1 f_1$, τ , and L^2 . The estimated uncertainties for the escape-to-thermal probability and buckling were 5 and 20%, respectively. This results in an experimental value of k_{∞}^* for the HTGR test lattice of

$$k_{\infty}^* = 1.136 \pm 0.002$$

where the assigned uncertainty is that due only to the computed leakage correction. The resulting value of k'_{∞} then becomes

$$k'_{\infty} = k_{\infty}^* [1 + (1 - p) B^2 L^2]^{-1} = 1.127 \pm 0.003.$$

Applying an **endcap** correction, for graphite plugs on each end of the fuel rods, obtained from a graphite coefficient measurement in the HTLTR reduces⁽⁴⁾ k'_{∞} to

$$k'_{\infty} = 1.114 \pm 0.003$$

A corresponding end cap correction computed with a two group, two dimensional diffusion theory model of the HTLTR gave a corrected value of

$$k'_{\infty} = 1.095 \pm 0.003.$$

This value compares favorably with the theoretical value calculated by Richey and Hanson: (4)

$$k_{\infty}' = 1.096.$$

Details of the Derivation of Equation (18)

The unpoisoned technique was originally proposed by Heineman⁽²⁾ as a means of extending the previous null reactivity method to systems where the buckling was not adjusted to zero. Some steps in his derivation will be repeated here for clarity and his multigroup expressions will be reduced to the two-group approximation.

The definition of the two groups used is arbitrary, but it should be remembered that using only two groups is an approximation and that the physics will often dictate a best way of dividing the two. In the case of a copper absorber,

$$\Delta\rho_p = \frac{1}{\chi} \left(-\phi_1^+ \Sigma_1^P V_p \phi_1' - \phi_2^+ \Sigma_2^P V_p \phi_2' \right) \quad (21)$$

where $\Delta\rho_p$ is the measured reactivity of the sample, χ is a normalizing integral over the whole reactor, ϕ^+ is the unperturbed importance, and ϕ' is the perturbed flux averaged over the poison volume V_p . A reasonable definition of the two groups is to include in Group 1 all neutrons with energies above the lowest resonance in the fuel so that ϕ_1^+ and ϕ_2^+ will closely approximate the values obtained from the adjoint equations [Equation (28)]. This means, in most cases, neutrons absorbed in the copper resonances will be counted as Group 1, and neutrons absorbed in the $1/v$ region above the cadmium cut-off will be counted as thermal neutrons. Thus

$$-\chi\Delta\rho_p = \phi_2^+ (\Sigma_2^P \phi_2 V_p) \left\{ 1 + \frac{1}{CdR - 1} \left(\frac{\Sigma_{\text{epi-Cd}}^{\text{Cu}} - \Sigma_{\text{ex}}^{\text{Cu}}}{\Sigma_{\text{epi-Cd}}^{\text{Cu}}} + \frac{\phi_1^+ \Sigma_{\text{ex}}^{\text{Cu}}}{\phi_2^+ \Sigma_{\text{epi-Cd}}^{\text{Cu}}} \right) \right\} \quad (22)$$

where $(\Sigma_2^P \phi_2 V_2)$ may be taken as the thermal (subcadmium) reaction rate and CdR is the copper cadmium ratio for the thickness of poison used. The total epithermal cross section, $\Sigma_{\text{epi-Cd}}^{\text{Cu}}$, has been broken up into two parts: $\Sigma_{\text{ex}}^{\text{Cu}}$ due to resonance contributions, and the remainder due to $1/v$ contributions. In practice, the contribution of copper absorptions above the cadmium cutoff is usually about 10%, and the difference between different methods of assigning importance to these neutrons is small. Values of $\Sigma_{\text{epi-Cd}}^{\text{Cu}}$ and $\Sigma_{\text{ex}}^{\text{Cu}}$ have been tabulated by Bennett⁽⁵⁾ for ^{63}Cu , and the assumption is usually made that ^{65}Cu has similar behavior.

From Heineman's⁽²⁾ derivation,

$$\chi \Delta \rho_{\text{cell}} = \int_{\text{cell surface}} dS (\phi_1^+ J_1' + \phi_2^+ J_2') \quad (23)$$

where J' is the outward neutron current and ϕ^+ is the unperturbed importance (i.e., in void) and ϕ' is the perturbed flux. Using the wave equation,

$$\nabla^2 \phi_1' + B^2 \phi_1' = 0 \quad (24)$$

and Gauss' theorem, and taking the fluxes as suitable volume averages over the cell, we obtain

$$\frac{\chi \Delta \rho_{\text{cell}}}{V_{\text{cell}}} = \phi_1^+ D_1 B^2 \phi_1' + \phi_2^+ D_2 B^2 \phi_2' \quad (25)$$

where D_1 and D_2 are the epithermal and thermal diffusion coefficients, respectively. In the case of the cell, it is usually most practical to use the cadmium cutoff as the division between Groups 1 and 2.

If we abbreviate the bracket term in Equation (22) as $(1 + a)$ and combine with Equation (5), we get:

$$- \frac{\Delta \rho_{\text{cell}}}{\Delta \rho_p} \frac{(\Sigma_2 \phi_2' V) (1 + a)}{(\Sigma_2 \phi_2' V)_{\text{cell}}} = \frac{D_2 B^2}{\Sigma_2} + \frac{D_1 B^2}{\Sigma_2} \frac{\phi_1^+}{\phi_2^+ + \phi_2'} \quad (26)$$

The left hand side of this equation contains measured quantities (except for a small part of the correction (a), and the right-hand side remains to be interpreted.

The two-group equations in the steady state, including fissions in both groups, and the adjoint equations are as follows

$$-D_1 B^2 \phi_1 - \Sigma_1^a \phi_1 - \Sigma_1^R \phi_1 + \nu_1 \Sigma_1^f \phi_1 + \nu_2 \Sigma_2^f \phi_2 = 0 \quad (27a)$$

$$\Sigma_1^R \phi_1 - D_2 B^2 \phi_2 - \Sigma_2^a \phi_2 = 0 \quad (27b)$$

$$-D_1 B^2 \phi_1^+ - \Sigma_1^a \phi_1^+ - \Sigma_1^R \phi_1^+ + \nu_1 \Sigma_1^f \phi_1^+ + \Sigma_1^R \phi_2^+ = 0 \quad (28a)$$

$$\nu_2 \Sigma_2^f \phi_1^+ - D_2 B^2 \phi_2^+ - \Sigma_2^a \phi_2^+ = 0 \quad (28b)$$

where Σ_1^R is the cross section for removal from Group 1 by scattering to Group 2. Since the buckling, B^2 , is not zero, the ϕ and ϕ^+ defined by these equations are for a finite, just-critical reactor. For comparison with the poisoned technique, k_∞^* shall be defined as the infinite multiplication factor for a system with these fluxes. We define k_∞^* by:

$$k_\infty^* = \frac{\text{neutrons produced}}{\text{thermal neutrons absorbed}} \times p = \frac{\nu_1 \Sigma_1^f \phi_1^1 + \nu_2 \Sigma_2^f \phi_2^1}{\Sigma_2^a \phi_2^1} \times p \quad (29)$$

which is consistent with the parameter measured in the poisoned technique.

We also define,

$$\tau = \frac{D_1}{\Sigma_1^R + \Sigma_1^a} \quad (30)$$

$$L^2 = \frac{D_2}{\Sigma_2^a} \quad (31)$$

$$p = \frac{\Sigma_1^R}{\Sigma_1^R + \Sigma_1^a} \quad (32)$$

where p is the escape-to-thermal probability defined with cross sections consistent with Equation (7), i.e., in the system with leakage. From the condition that for a nontrivial solution the determinant of the coefficients for Equation (27a) and Equation (27b) shall vanish, the critical equation for k_∞^* is

$$k_\infty^* = (1 + B^2 L^2) (1 + B^2 \tau) \quad (33)$$

Going back to the right-hand side of Equation (26), and using Equations (27b) and (28a) we obtain

$$\begin{aligned} \frac{D_2 B^2}{\Sigma_2^a} + \frac{D_1 B^2}{\Sigma_2^a} \frac{\phi_1^+}{\phi_2^+} \frac{\phi_1}{\phi_2} &= L^2 B^2 + \frac{\tau B^2 (1 + B^2 L^2)}{1 + \tau B^2 - \nu_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)} \\ &= (1 + L^2 B^2) (1 + \tau B^2) - 1 - \frac{\tau B^2 (1 + B^2 L^2) [\tau B^2 - \nu_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)]}{1 + \tau B^2 - \nu_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)} \\ &= k_\infty^* - 1 - \frac{\tau B^2 (1 + B^2 L^2) [\tau B^2 - \nu_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)]}{1 + \tau B^2 - \nu_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)} \end{aligned} \quad (34)$$

Therefore, the expression for k_∞^* in the unpoisoned technique becomes

$$\begin{aligned} k_\infty^* &= 1 - \left(\frac{\Delta \rho_{\text{cell}}}{\Delta \rho_p} \right) \frac{(\Sigma_2^a \phi_2^V)_p}{(\Sigma_2^a \phi_2^V)_{\text{cell}}} \left\{ 1 + \frac{\phi_1^+}{\phi_2^+} \frac{(\Sigma_1^a \phi_1)_p}{(\Sigma_2^a \phi_2)_p} \right\} \\ &\quad + \frac{\tau B^2 (1 + B^2 L^2) [\tau B^2 - \nu_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)]}{[1 + \tau B^2 - \nu_1 \Sigma_1^f / (\Sigma_1^R + \Sigma_1^a)]} \end{aligned} \quad (35)$$

Values obtained using this equation may be directly compared with values obtained via the poisoned technique.

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PROGRAM BUFIT: A LEAST SQUARES CODE FOR THE ANALYSIS OF
BURNUP DATA

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Major changes have been made to the least squares code which is used to extract one group cross section information from isotopic concentration burnup data. The resulting new programs are called BUFIT (burnup fitting code) and DBUFIT, a double precision version. They replace the DUBLIK code⁽¹⁾ which was previously used to perform this type of analysis. Both the DUBLIK and BUFIT codes are framed around the LIKELY⁽²⁾ general least squares code. The changes fall into four categories:

1. An error in subroutine CHANCE has been corrected. This subroutine which is used to form the first and second derivatives of the least square misfit was incorrectly programmed in the DUBLIK code.
2. Improvements have been made in subroutine THEORY which cause the iterative procedure used in BUFIT to approach the least squares minimum in a more direct fashion. The procedure is now less likely to stop at a false saddle type solution. This is brought about by casting the initial isotopic concentrations as parameters instead of being lumped together with the cross-section ratios as constants of integration which were previously used as parameters. This change greatly reduces the correlation between the parameters that are used in the THEORY equations. The net result seems to be a more reliable iterative path to the solution of the least squares burnup problem.

An additional equation that describes the ^{239}Pu fissions has been included in the THEORY subroutine for U-Pu

systems. This extra equation provides a better determination of the capture-to-fission ratio for ^{239}Pu , and again, reduces the effort required to arrive at the least squares best fit. Experimental data for ^{239}Pu fissions is required by the BUFIT program; however, these can be obtained from burnup measurements of the total fissions.

3. The iterative scheme used to arrive at the least squares best fit solution has been altered. The new scheme will not proceed to the next iteration unless it has a lower misfit (Q-value) than does the current iteration. Several step types⁽³⁾ are used in an attempt to obtain a lower misfit. The parameter changes predicted by a given step type are used to generate a new Q-value. If this Q-value is lower than that of the current iteration, the parameters predicted by the step type are used for the next iteration. If the new Q-value is larger than or equal to the current Q-value, the next type of step is tried. The iterative process continues until either the change in the Q-value is less than one part in 10^5 and the parameter variance-covariance matrix is positive definite, or none of the step types yield a lower Q-value.

If the change in Q estimated from the matrix step is negative, the sequence of steps that are tried is

(1) matrix step, (2) half matrix step, (3) gradient step, (4) one fifth of the gradient step, and (5) one twentieth of the matrix step. If the change in Q that is estimated from the matrix step is positive, it is assumed that the parameter changes indicated by the matrix step are in the wrong direction. Therefore, the parameter changes estimated for the half matrix step are reversed and the resulting Q-value computed. If this reversed half matrix step fails, the gradient and succeeding steps are tried.

This series of step types has performed well in the problems that it has been tested on. In general, the matrix half step and the gradient step (and occasionally the reversed matrix half step) are prominent in bringing the iterative process under the influence of the minimum on the Q-value surface where the matrix step becomes dominant. The guaranteed drop in Q-value from one iteration to the next tends to save time by reducing the numbers of false steps that are taken during the iterative process.

4. A new subroutine has been included in the BUFIT program. It calculates the cross section ratios and their one standard deviation errors from the fitted parameters and their variance-covariance matrix.

The single precision BUFIT is a little faster and a little less reliable than the double precision DBUFIT code. The DBUFIT code typically requires 3 seconds per iteration for a uranium-plutonium case including data from six spent fuel samples. Starting with rough initial parameter estimates, it converges to the best fit parameters in from 10 to 25 iterations. The BUFIT and DBUFIT codes have been encoded in FORTRAN V language for use on the UNIVAC-1108 computer. These codes are available upon request.

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3.0 THERMAL REACTORS

MULTIPLICATION DEPENDENCE ON ENERGY AND ANGULAR DETAIL IN TRANSPORT THEORY CALCULATIONS

S. R. Dwivedi*

Predicting the leakage from clean, critical assemblies is one of the potential causes of inaccuracy in calculated results.

The results of previous analytical studies on selected thermal reactor systems^(1,2) show that a systematic difference in calculated values of k_{eff} occurs using diffusion theory and transport theory. *A priori*, it is expected that transport theory is more accurate and that any differences occurring between diffusion and transport theory results is attributable to inaccuracies in diffusion theory.

The object of this study was to determine the sensitivity of calculated values of k_{eff} to the amount of energy and angular detail used in a transport theory solution for a thermal reactor problem. Two homogeneous spherical, critical reactors⁽³⁾ were selected for the study to: (1) minimize the approximations associated with reactor geometry, and (2) have a high leakage system. The systems were aqueous plutonium nitrate solutions contained in a stainless steel vessel with one reactor bare and the other reflected with H_2O . A brief description of the systems are given in Table 3.1.

TABLE 3.1. *Description of the Reactors*⁽¹⁾

	<u>Critical Mass, kg</u>	<u>H/Pu, atom ratio</u>	<u>Solution Diameter, ID, cm</u>
Plutonium Nitrate Bare Sphere	1.18	668	38.608
Plutonium Nitrate Reflected Sphere (Reflector thick- ness = 31.5 cm)	1.17	553	38.608

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The calculations were performed using the HRG⁽⁴⁾ and BATTELLE-REVISED THERMOS⁽⁵⁾ codes to compute the multigroup spectra and group constants along with DTF-IV⁽⁶⁾ code to calculate the system leakage and multiplication, k_{eff} .

The dependence of k_{eff} on energy group detail was calculated using the S-4 approximation in DTF-IV with different multigroup structures, and the results are presented in Table 3.2. The number of groups was varied from four to twenty with varying group boundaries. The group structures used are shown in Table 3.3.

TABLE 3.2. Calculated Values of k_{eff} for Different Multi-Group Structures

Total No. of Groups	No. of Groups in the Energy Range, 10 to 0.1 MeV	k_{eff} (a)	
		Bare Sphere	Reflected Sphere
20	10	1.0179	1.0334
18	8	1.0211	1.0373
12	8	1.0196	1.0349
12	2	1.0246	1.0404
<u>10</u>	<u>6</u>	<u>1.0196</u>	<u>1.0354</u>
8	4	1.0241	1.0404
8	2	1.0255	1.0414
6	2	1.0258	1.0420
4	0	1.0839	1.1059

a. Experimental $k_{eff} = 1.000$

Since the maximum number of groups at high energies is limited by the minimum lethargy width of 0.25 in HRG, the value of k_{eff} obtained from the 20-group calculation can be assumed to be the most accurate. Relative to this value of k_{eff} , it is seen that the multiplication is sensitive to both the number of energy groups and the group structure. From a

practical standpoint, 10-group model with six high energy groups is sufficient, in that k_{eff} is accurate to within 20.2%. The 10-group model was obtained by arranging the levels in the energy range 0.1 to 10 MeV, such that nearly equal number of fission neutrons are generated in each group, i.e., the fission spectrum is almost constant. It is also seen from the results in Table 3.3 that the multiplication is relatively insensitive to energy detail below this range (i.e., <0.1 MeV).

TABLE 3.3. *Group Structure (Lower Energy Boundaries in eV)*

<u>20</u> <u>Groups</u>	<u>18</u> <u>Groups</u>	<u>12</u> <u>Groups</u>	<u>10</u> <u>Groups</u>	<u>8</u> <u>Groups</u>	<u>6</u> <u>Groups</u>	<u>4</u> <u>Groups</u>
7.79×10^6	--					
6.07×10^6	--			--		
4.72×10^6	--		4.72×10^6			
3.68×10^6	--					
2.87×10^6	--		2.87×10^6	--		
2.23×10^6						
1.74×10^6	--		1.74×10^6			
1.35×10^6	--	--	1.35×10^6	--	--	--
4.98×10^5	--		4.98×10^5			
1.83×10^5	--	--	1.83×10^5		--	--
2.48×10^4	--	--	1.17×10^4	--	--	--
3.36×10^3	--	--			--	
4.54×10^2	--	--				
1.01×10^2	--	--			--	
37.3	--	--				
13.7	--	--				
5.04	--	--				
1.86	--	--	2.38	--	--	--
0.683	--	--	0.683	--	--	--
0.0	--	--	0.0	--	--	--

The effects of higher order S_n approximations on the calculated results are presented in Table 3.4. These results show a $\leq 0.04\%$ effect on k_{eff} in going from S-16 to S-8 and a $\leq 0.2\%$ effect in going from S-8 to S-4 independent of number of groups. Thus, it is concluded that a 10-group S-8 calculation is sufficient in accuracy.

TABLE 3.4. *Effects of Using Higher Order S_n Approximation*

Total No. of Groups	Sn-order	k_{eff}	
		Bare Sphere	Reflected Sphere
20	4	1.0179	1.0334
20	8	1.0154	1.0314
20	16	1.0151	1.0310
10	4	1.0196	1.0354
10	8	1.0171	1.0335
10	16	1.0168	1.0331

A comparison between values of k_{eff} obtained using diffusion theory (HFN) ⁽⁷⁾ and transport theory is made in Table 3.5. The table shows that diffusion theory gives results which are insensitive to number of groups whereas transport theory results are sensitive. Also, the diffusion theory results agree better with experiment. This conclusion does not indicate that the lower order diffusion theory is more accurate, but probably that compensating errors exist in the calculational model. It will be interesting to pinpoint those errors and to investigate the region where diffusion theory will not be able to compensate those errors.

TABLE 3.5. *Comparison Between Results of Transport Theory and Diffusion Theory Calculations*

No. of Groups	Calculational Mode	k_{eff}	
		Bare Sphere	Reflected Sphere
18	Transport	1.0211	1.0373
	Diffusion	1.0140	1.0194
4	Transport	1.0839	1.1059
	Diffusion	1.0133	1.0192

Based upon the results presented here the following conclusions can be drawn:

- Transport theory results are very sensitive to the number of multigroups used to describe the fission spectrum, and the best estimate of k_{eff} can be obtained when the fission spectrum is uniformly distributed in the fast energy groups.
- Using the 10-group structure as described in Table 3.3, the value of k_{eff} can be calculated within the accuracy of 0.2% (in comparison to 20-group model).
- For high leakage systems as described above, the S-8 approximation is sufficient. S-4 theory overestimates the eigenvalue by 0.2%.
- Diffusion theory is insensitive to the number of multigroups used in the high energy region. Compensating errors must exist in the diffusion theory calculations which give better values of multiplication factors than transport theory. These compensatory errors are different from the one customarily encountered, i.e., compensatory errors due to basic cross-sections and spectrum model.
- The most elaborate transport theory calculations overestimate k_{eff} by about 1.5% for the bare system and about 3% for reflected systems. The reasons for these errors are not known; however, the first area to attack seems to be the spatial-spectral model (e.g., perhaps the reflected sphere needs to be described by multiple core and reflector regions with different group constants).

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HETEROGENEITY CORRECTION TO FAST-FISSIONS

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In the analysis of light-water moderated, enriched uranium or plutonium lattices, the fast fission i ^{238}U is usually obtained from multigroup calculations of the fast neutron spectrum of the finite homogenized lattice. In this usual approach one neglects a heterogeneity correction (spatial effect) in the fast energy range. This effect is relatively small because of the long mean free path for neutrons, but it is not negligible. The object of this study was to determine how large an error is introduced in the calculated k_{eff} of typical enriched UO_2 lattices by neglecting the spatial detail. Multigroup calculations for the homogeneous cells were calculated using the HRG code.⁽¹⁾

The unit cell was assumed to consist of the fuel region and moderator region, and the collision equations were solved using a constant fission source in the fuel region. The

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reaction rate in ^{238}U nuclei was obtained using a well known variational principle. The calculations were performed only for neutrons having energies greater than 1.35 MeV.

The neutron balance equation can be written as

$$\begin{aligned}\Sigma_1^t \phi_1 &= \left(\beta v \Sigma_1^f + \Sigma_1^s \right) \phi_1 P_{11} + \Sigma_2^s \phi_2 P_{21} + S P_{11} \\ \Sigma_2^t \phi_2 &= \left(\beta v \Sigma_1^f + \Sigma_1^s \right) \phi_1 P_{12} + \Sigma_2^s \phi_2 P_{22} + S P_{12}\end{aligned}\quad (1)$$

where

$$\Sigma_i^s = V_i \sum_j N_j \sigma_j^s$$

$$i = 1, 2$$

$$j = \text{all the nuclei in region } i$$

$$V_i = \text{volume fraction of } i^{\text{th}} \text{ region}$$

$$\sigma_j^s = \text{microscopic scattering cross-section with neutron energy remaining above 1.35 MeV after scattering}$$

$$N_j = \text{nuclei density}$$

$$\beta = \text{fraction of fission neutrons generated above 1.35 MeV}$$

$$v \Sigma_i^f = V_i \sum_j N_j v_j \sigma_j^f$$

$$v_i \sigma_i^f = \text{number of neutrons generated due to fast fissions in nuclei } j$$

$$\Sigma_i^t = \Sigma_i^{\text{rem}} + \Sigma_i^s$$

$$\Sigma_i^{\text{rem}} = V_i \sum_j N_j \sigma_i^{\text{rem}}$$

$$\sigma_j^{\text{rem}} = \text{removal cross-section of } j^{\text{th}} \text{ nuclei}$$

$$S = \text{uniformly distributed source in region 1}$$

$$P_{ij} = \text{probability of a neutron generated uniformly in region } i \text{ to have its first collision in region } j.$$

Equation (1) can be written in a matrix form as

$$[H] [\phi] + [S] = 0,$$

where

$$= \begin{bmatrix} (\beta v \Sigma_1^f + \Sigma_1^s) P_{11} - \Sigma_1^t & \Sigma_2^s P_{21} \\ (\beta v \Sigma_1^f + \Sigma_1^s) P_{12} & \Sigma_2^s P_{22} - \Sigma_2^t \end{bmatrix}$$

$$[\phi] = \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}$$

$$[S] = \begin{bmatrix} SP_{11} \\ SP_{12} \end{bmatrix}$$

We are interested in the reaction rate in ^{238}U due to the fission source S . If Σ_u^f is the fission cross section of ^{238}U , then the required quantity is

$$\begin{bmatrix} \Sigma_u^f, 0 \end{bmatrix} \times \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}.$$

The variational principle which can be used to calculate this quantity is given by

$$L \left[\phi, \phi^*, \Sigma_u^f \right] = [S^*]^T [\phi] + [\phi^*]^T [H] [\phi] + [\phi^*]^T [S], \quad (2)$$

where the superscript T represents the transpose of the matrix and $[\phi^*]$ is the solution of the adjoint equation

$$[H^*] [\phi^*] + [S^*] = 0, \quad (3)$$

where

$$[H^*] = \begin{bmatrix} (\beta v \Sigma_1^{\bar{f}} + \Sigma_1^s) P_{11} - \Sigma_1^t & (\beta v \Sigma_1^{\bar{f}} + \Sigma_1^s) P_{12} \\ \Sigma_2^s P_{21} & \Sigma_2^s P_{22} - \Sigma_2^t \end{bmatrix}$$

$$[\phi^*] = \begin{bmatrix} \phi_1^* \\ \phi_2^* \end{bmatrix}$$

$$[S^*] = \begin{bmatrix} \Sigma_u^f \\ 0 \end{bmatrix}.$$

The first order approximation to the direct and adjoint fluxes to be used in equation (2) can be obtained from the solution for the corresponding homogeneous lattice cell. The stationary behavior of the variational principle will cause a second order error in the uranium reaction rate. The neutron balance equation for a homogeneous cell can be formed by substituting P_{ij}^o (the collision probabilities in homogeneous mixture) in place of P_{ij} in equations (1) and (3). The solutions to these equations will be given by

$$\phi_1^o = \phi_2^o = \frac{S}{\Sigma_1^t + \Sigma_2^t - (\beta v \Sigma_1^f + \Sigma_1^s + \Sigma_2^s)}$$

$$\text{and } \phi_1^{*o} \quad \phi_2^{*o} = \frac{\Sigma_u^f (\Sigma_1^t + \Sigma_2^t) (1 - \Sigma_2^s / \Sigma_2^t)}{\Sigma_1^t (\Sigma_1^t + \Sigma_2^t - \beta v \Sigma_1^f - \Sigma_1^s - \Sigma_2^s)} \quad (4)$$

where the superscript o represents the homogeneous solution.

Using these values of direct and adjoint fluxes in the variational principle (2) the ratio of reaction rates in ^{238}U nuclei in heterogeneous to the homogeneous case can be given by

$$\gamma = 1 + \delta P_c \left(1 + \frac{\Sigma_2^t}{\Sigma_1^t}\right)^2 \left(1 - \frac{\Sigma_2^s}{\Sigma_2^t}\right)^2 \times \frac{\Sigma_1^t}{(\Sigma_1^t + \Sigma_2^t - \beta v \Sigma_1^f - \Sigma_1^s - \Sigma_2^s)} \quad (5)$$

where δP_c is the difference in first flight collisions in uranium due to the heterogeneity of the lattice.

Using the Glasstone and Edlund⁽²⁾ definition of ϵ (the fast-fission factor), the heterogeneity contribution to the eigenvalue can be given by

$$\frac{\Delta k}{k} = \frac{\Delta \epsilon}{\epsilon} = \frac{\gamma - 1}{1 + \frac{\Sigma_1^t + \Sigma_2^t - (\beta \bar{\nu} \Sigma_1^f + \Sigma_1^s + \Sigma_2^s)}{\beta (\bar{\nu}_{28} - 1) \Sigma_u^f}} \quad (6)$$

where $\bar{\nu}_{28}$ = average ν for ^{238}U above 1.35 MeV. The following equations were used in deriving equation (6)

$$\epsilon_{28}^{\text{hom}} = \frac{\beta \bar{\nu}_{25} \Sigma_u^f}{\Sigma_1^t + \Sigma_2^t - (\beta \bar{\nu} \Sigma_1^f + \Sigma_1^s + \Sigma_2^s)}$$

$$(\epsilon - 1)_{\text{hom}} = \frac{\bar{\nu}_{28} - 1}{\bar{\nu}_{25}} \delta^{28}$$

$$\frac{(\epsilon - 1)_{\text{het}}}{(\epsilon - 1)_{\text{hom}}} = \frac{\delta_{\text{het}}^{28}}{\delta_{\text{hom}}^{28}} =$$

$$\delta^{28} = \text{fast-fission ratio of } ^{238}\text{U}$$

The error in k_{eff} caused by neglecting heterogeneity in the calculation of fast fission in ^{238}U was calculated for five enriched uranium lattices with different moderator-to-fuel volume ratios. The macroscopic cross sections were obtained from HRG,⁽¹⁾ and p_c^{het} , the first flight collision probability with uranium nuclei, was calculated from the following relation given by Nordheim⁽³⁾

$$p_c^{\text{het}} = 1 - \frac{P_o}{1 + \frac{C}{1 - C} \bar{\ell} \Sigma_o P_o} .$$

In this equation, C is the Dancoff correction factor calculated with the code DASQHE;⁽⁴⁾ $\bar{\lambda}$ is the mean chord length in the rod; Σ_0 is the macroscopic total cross-section of UO_2 ; and P_0 is the escape probability given by Case, deHoffman and Placzek:⁽⁵⁾

$$P_0(x) = \frac{2}{3} \times \left\{ 2x \left[K_1(x) I_1(x) + K_0(x) I_0(x) \right] - 2 + \frac{1}{x} K_1(x) I_1(x) - K_0(x) I_1(x) + K_1(x) I_0(x) \right\}$$

where $x = r \cdot \Sigma_0$, r = radius of the rod. The results of the calculations are presented in Table 3.6. The error in k , due to neglecting heterogeneity in the calculation of fast fissions in ^{238}U is always less than 5 mk.

TABLE 3.6. *Heterogeneity Corrections to Fast-Fissions*

<u>Lattice, Enrichment</u>	<u>Rod Radius, cm</u>	<u>V_m/V_f, Vol. Ratio</u>	<u>γ</u>	<u>$\Delta k/k$, %</u>
(2.734%)	0.389	1.048	1.0353	0.22
(3.037%)	0.5634	1.15	1.0599	0.31
(1.328%)	0.5042	1.43	1.054	0.28
(1.328%)	0.5042	2.40	1.100	0.36
(2.35%)	0.5588	2.41	1.137	0.45

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AN EQUIVALENCE APPROXIMATION FOR USE IN APPLYING THE DUNN FLUX
PEAKING OR DIPPING CORRECTION TO PIN ACTIVITIES

C. L. Bennett and D. R. Oden

In performing lattice parameter measurements by foil activation techniques, it is important to correct for the peaking or dipping of the flux in the foil with respect to the surrounding media, especially in the thermal energy range. One such method that is both reasonably accurate and simple to apply is the Dunn approximation.⁽¹⁾ This method has been routinely applied to foil (mostly round and square shaped) activities in the analysis of PCTR experiments since about 1967.⁽²⁾ In addition to the standard foil data in such experiments, extensive pin activation data has usually been obtained also. The usual procedure in the past has been to only use the pin data (uncorrected) for inferring cell spatial shape detail and as backup for the standard foil data. Therefore, in order to increase the usefulness of this data, some sort of a foil peaking or depression correction method was also needed for pin activities.

We decided first to directly apply the Dunn approximation by obtaining an approximate equivalence relationship between finite slabs and cylinders and then possibly at some later date, either take the more obvious direct approach of explicitly deriving a first order pin correction or use some other more accurate correction method.

First, this note briefly outlines the Dunn approximation, then presents the determination of the approximate equivalence relationships (actually two are involved) required to apply the correction to pins, and finally it summarizes the resulting procedure for actually using this method.

The Dunn Approximation

If A is the activation of a square or round foil of thickness t and A_0 is the activation that would occur if the flux in the foil were the same as in the surrounding medium then Dunn's approximation can be written as

$$\frac{A_0}{A} = 1 - \left(\Sigma_a^{\text{med.}} - \Sigma_a^{\text{foil}} \right) t \ln \frac{\beta}{t},$$

$$\beta = \begin{cases} L(\text{width}), & \text{square foils} \\ D(\text{diameter}), & \text{round foils} \end{cases} \quad (1)$$

where edge effects have been neglected.

Equivalence Relationships

As previously mentioned, two equivalence relationships are needed to directly apply the Dunn approximation to pin activities. The first, establishing a cylindrical equivalent to β/t , is obtained using mean chord length arguments; the second, obtaining explicitly the cylindrical equivalent to the slab thickness t , is obtained primarily using self-shielding arguments.

β/t Equivalence

The mean chord lengths for infinite slabs and cylinders are, respectively:

$$\bar{\ell}_{\text{slab}}^m = 2t \quad (2)$$

$$\text{and } \bar{\ell}_{\text{cyl}}^\infty = 2r. \quad (3)$$

If the ratio of these mean chord lengths is defined as C_∞ , then

$$C_\infty \equiv \frac{\bar{\ell}_{\text{slab}}^m}{\bar{\ell}_{\text{cyl}}^\infty} = \frac{t}{r} \quad (4)$$

Keeping this definition in mind, we will consider both round and square foils, since β can refer to either.

Square Foils

The mean chord lengths for finite slabs (square foils) and cylinders (pins) are, respectively:

$$\bar{\ell}_{\text{slab}} = \frac{4t L^2}{2 L^2 + 4Lt} = \frac{2t}{1 + \frac{2t}{L}} \quad (5)$$

where L is the width and t the thickness,

$$\text{and } \bar{\ell}_{\text{cyl}} = \frac{4\pi r^2 h}{2\pi rh + 2\pi r^2} = \frac{2r}{1 + \frac{r}{h}} \quad (6)$$

where h is the height and r the radius.

If the ratio of the finite mean chord lengths is now defined as C , then

$$C \equiv \frac{\bar{\ell}_{\text{slab}}}{\bar{\ell}_{\text{cyl}}} = \frac{t}{r} \left[\frac{1 + \frac{r}{h}}{1 + \frac{2t}{L}} \right]. \quad (7)$$

In the limit as h and L go to infinity, C becomes C_∞ . If r/h and $2t/L$ are both small, C_∞ can be used as a reasonable approximation to C . Therefore, replacing C by C_∞ in Equation (7) and solving for L/t results in

$$\frac{L}{t} = \frac{2h}{r}. \quad (8)$$

Round Foils

The mean chord length for a disk of diameter D and thickness t is

$$\bar{\ell}_{\text{disk}} = \frac{4t \pi (D/2)^2}{2\pi (D/2)^2 + 2\pi (D/2)t} = \frac{2t}{1 + \frac{2t}{D}}. \quad (9)$$

If $\bar{\lambda}_{\text{disk}}$ is used instead of $\bar{\lambda}_{\text{slab}}$ in an expression similar to Equation (7), i.e.,

$$C' \equiv \frac{t}{r} \left[\frac{1 + \frac{r}{h}}{1 + \frac{2t}{D}} \right], \quad (10)$$

and if, as before, C' is approximated by C_{∞} (C_{∞} remains unchanged since $\bar{\lambda}_{\text{disk}}^{\infty} = \bar{\lambda}_{\text{slab}}^{\infty}$), the resulting equation can be solved for D/t , giving

$$\frac{D}{t} = \frac{2h}{r} \quad (11)$$

The RHS of Equation (11) is the same as the RHS of Equation (8), so for a given thickness, D is equivalent to L . This is the same assumption Dunn used. Therefore, the first equivalence says

$$\frac{\beta}{t} = \frac{2h}{r} \quad (12)$$

Foil Thickness Equivalence

For small τ ($\tau \equiv \bar{\lambda}_{\infty} \Sigma_t/2$), the self-shielding factors for infinite slabs and cylinders immersed in an isotropic flux can be approximated (neglecting scattering effects) as follows:⁽³⁾

$$f_{\text{cyl}} \approx 1 - \frac{1}{2} \tau_{\text{cyl}} \left[\frac{8}{3} - \tau_{\text{cyl}} (\ln \tau_{\text{cyl}} + 1.365931) \right] \quad (13)$$

$$f_{\text{slab}} \approx 1 - \frac{1}{2} \tau_{\text{slab}} \left[.922784 - \ln \tau_{\text{slab}} + \frac{1}{3} \tau_{\text{slab}} \right] \quad (14)$$

where $\tau_{\text{slab}} = t \Sigma_t^{\text{foil}}$ and $\tau_{\text{cyl}} = r \Sigma_t^{\text{foil}}$. Therefore, given an infinite cylinder of radius r , the equivalent thickness, t , of an infinite slab of the same material, which gives the same self shielding factor as the cylinder, can be found by simply equating Equations (13) and (14). The resulting expression

$$\begin{aligned} & \tau_{\text{cyl}} \left[\frac{8}{3} - \tau_{\text{cyl}} (\ln \tau_{\text{cyl}} + 1.365931) \right] \\ &= \tau_{\text{slab}} \left[.922784 - \ln \tau_{\text{slab}} + \frac{2}{3} \tau_{\text{slab}} \right] \end{aligned} \quad (15)$$

also holds true if the usual scattering correction⁽⁴⁾ is made to the self-shielding factors

$$f_{\text{corr}} = \frac{f_0}{1 - C(1 - f_0)} \quad (16)$$

where $C = (\Sigma_s/\Sigma_t)^{\text{foil}}$. Equating the corrected self-shielding factors, i.e.,

$$\frac{f_{\text{cyl}}}{1 - C(1 - f_{\text{cyl}})} = \frac{f_{\text{slab}}}{1 - C(1 - f_{\text{slab}})} \quad (17)$$

and using Equations (13) and (14), respectively, for f_{cyl} and f_{slab} , results identically in Equation (15). Equation (15) could be solved by iteration for τ_{slab} and hence t except that the resulting thickness would refer to an equivalent infinite slab instead of a finite one. For this reason a slightly different approach is used. Equations (13) and (14) are assumed to also hold for finite slabs and cylinders, which is in keeping with our previous assumptions of small r/h and $2t/L$. Equations (13) through (15) are, therefore, unchanged except for the definition of τ which would be

$$\tau = \frac{1}{2} \bar{k} \Sigma_t. \quad (18)$$

More specifically,

$$\tau_{\text{cyl}} = \frac{r \Sigma_t^{\text{pin}}}{1 + \frac{r}{h}} \quad (19)$$

$$\text{and } \tau_{\text{slab}} = \frac{t \Sigma_t^{\text{pin}}}{1 + \frac{2t}{\beta}} \quad (20)$$

Substituting our first equivalence, Equation (12) into Equation (20) gives

$$\tau_{\text{slab}} = \frac{t \Sigma_t^{\text{pin}}}{1 + \frac{r}{h}} \quad (21)$$

Solving Equation (21) for t results in our second equivalence expression

$$t = \frac{\left(1 + \frac{r}{h}\right) \tau_{\text{slab}}}{\Sigma_t^{\text{pin}}} \quad (22)$$

Pin Activity Correction

The desired result (an approximate flux peaking or dipping correction for pin shaped foils) is obtained by substituting Equations (12) and (22) into Equation (1), which then becomes

$$\left(\frac{A_o}{A}\right)_{\text{pin}} = 1 - \left(\frac{\Sigma_a^{\text{med}} - \Sigma_a^{\text{pin}}}{\Sigma_t^{\text{pin}}}\right) \tau_{\text{slab}} \left(1 + \frac{r}{h}\right) \ln \left(\frac{2h}{r}\right) \quad (23)$$

Finally, the following summarizes the simple procedure leading to $A_{\text{pin}}^{\text{corr}}$:

1. Determine Σ_a^{pin} , Σ_t^{pin} , and Σ_a^{med} .
2. Calculate τ_{cyl} from Equation (19).

3. Calculate the LHS of Equation (15) and then solve for τ_{slab} by iteration.
4. Calculate $(A_o/A)_{\text{pin}}$ from Equation (23).
5. Correct the measured activity via $A_{\text{pin}}^{\text{corr}} = A_{\text{pin}}^{\text{meas.}} (A_o/A)_{\text{pin}}$.

Added Comments

Since the results here are based on monoenergetic considerations, Equation (23) actually expresses the energy dependent ratio of the unperturbed to the perturbed flux, $\phi_o(E)/\phi(E)$. The proper spectrum weighted form of Equation (23) can be shown to be

$$\frac{A_o}{A} = \left[\left\langle \frac{1}{1 - \left(1 + \frac{r}{h}\right) \ln \left(\frac{2h}{r}\right) \beta(E)} \right\rangle^{\text{act}} \right]^{-1} \quad (24)$$

where

$$\left\langle \frac{1}{1 - \left(1 + \frac{r}{h}\right) \ln \left(\frac{2h}{r}\right) \beta(E)} \right\rangle^{\text{act}} = \frac{\int \left[1 - \left(1 + \frac{r}{h}\right) \ln \left(\frac{2h}{r}\right) \beta(E) \right]^{-1} \phi^{\text{med}}(E) \Sigma_a^{\text{pin}}(E) dE}{\int \phi^{\text{med}}(E) \Sigma_a^{\text{pin}}(E) dE} \quad (25)$$

and $\beta(E) \equiv \frac{\tau_{\text{slab}}(E)}{\Sigma_t^{\text{pin}}(E)} \left(\Sigma_a^{\text{med}}(E) - \Sigma_a^{\text{pin}}(E) \right).$ (26)

For estimating purposes, the simple five step procedure can be retained by using unperturbed spectrum average cross sections.

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BATCH CORE BURNUP DATA

R. P. Matsen

Destructive analysis data from samples cut from PRTR rods have been received, plotted for consistency checks, and processed through the ISODIL burnup analysis code. Twenty-four samples were cut from rods extracted from the reactor during the first, second, and third interim shutdowns. When the basic data were received from analytical chemistry, it was plotted in order to discover inaccurate measurements. A much lower percentage of these inaccurate measurements exists among this data than for any set received to date. The few questionable results were submitted for reanalysis. In addition, several samples of unirradiated fuel have been submitted for analysis in order to ensure that the data for the zero burnup point is as reliable as that from the other samples.

Results from the ISODIL code reveal that the maximum sample exposure was approximately 9000 MWd/MTM for (as expected) a sample cut from the middle of a replaceable rod taken from a ring-one element during the third interim shutdown.* Input data for a least squares burnup analysis code BUFIT was

* See R. P. Matsen's "Program BUFIT: A Least Squares Code for the Analysis of Burnup Data" in this report.

punched via the ISODIL code for fourteen samples taken from replaceable rods that were irradiated in ring three of the core. The least squares analysis of this data, in order to extract cross section ratio information, has begun.

PRTR BATCH CORE - CRITICAL TEST NO. 4

J. H. Lauby and L. D. Williams

The fourth interim critical tests were conducted on the 55 fuel-element Batch Core loading at the PRTR at an average core burnup of 5330 MWd/MTM. Tests were also conducted on two smaller loadings of 31 and 19 fuel elements. Moderator level worth measurements were made using critical approach and positive period techniques. Data were obtained at three different moderator levels (100, 95, and 90 in.) for each of the loadings. Reactor oscillator measurements were also made for each configuration at the high moderator level.

The ^{10}B concentrations for each critical configuration were determined by reactivity comparisons made in the Thermal Test Reactor (TTR) using standard samples. Analysis of the ^{10}B concentration measurements is in progress.

Moderator level reactivity coefficients for each loading and boron concentration were consistent with previous measurements. Critical moderator height and one-standard deviation uncertainties for each critical loading were determined from the positive period measurements using program LEARN. Final analysis of the moderator level reactivity coefficients and critical moderator height is awaiting the ^{10}B results.

Analysis of the reactor oscillator experiments is complete. The results of measurements of β/λ , the ratio of the delayed neutron fraction to the prompt neutron lifetime, are summarized in Table 3.7 as a function of burnup for the Batch Core experiment.

TABLE 3.7. *The Results of Measurements of β/λ*

Average Core Burnup, MWd/MTM	Core Size, Fuel Elements	^{10}B Conc.,	
		wppm ^{10}B	$\beta/\lambda, \text{sec}^{-1}$
0	55	20.4	17.9 ± 0.5
~10	77 (250 watt)	21.8	20 ± 1
~10	77 (1 MW)	22.5	18.1 ± 0.9
3560	55	13.8	15.1 ± 0.6
5330	55	10.6	14.3 ± 0.3
6139	31	5.9	10.4 ± 0.5
6512	19	2.3	8.0 ± 0.2

ZODIAC-ALTHAEA COMPARISON WITH Pu ENRICHED FUELS

E. T. Merrill, E. A. Schnaible

A comparison was made between the results of burnup calculations obtained using the ZODIAC G⁽¹⁾ and ALTHAEA⁽²⁾ codes. The ALTHAEA code uses a simplified model for spectral averaging cross sections and as a result uses only about 1/400 of the computing time required for a corresponding ZODIAC G solution. There are numerous burnup calculations required in the studies of using plutonium in a typical boiling water reactor (Oyster Creek). Thus, there is a considerable savings realized in using ALTHAEA rather than ZODIAC G for these studies.

A comparison of ZODIAC and ALTHAEA calculations covering three different plutonium enriched fuels is shown in Table 3.8. For each fuel the reactivity and concentrations of uranium and plutonium isotopes are compared at three fuel exposures. The adjustable parameters in the ALTHAEA code were set at the same values for all fuels. The comparison showed that isotopic concentrations agreed within 1% with the exception of the ²⁴²Pu concentration in the low density fuel. Values of k_{∞} checked within 0.5% for all exposures and all fuels calculated. As a result of this comparison, we have concluded that the ALTHAEA code can be used in studies of reactor operation and behavior for plutonium-fueled cores as well as uranium-fueled cores.

TABLE 3.8. *ZODIAC-ALTHAEA Comparison*

		Exposure MWD/MT	K _∞	Isotope Concentration, g/MT						
				²³⁵ U	²¹⁸ U	Total Pu	²⁹³ Pu	²⁴⁰ Pu	²⁴¹ Pu	²⁴² Pu
Enrichment 3.6 wt %	ZODIAC	4,000	1.095	6,313.0	949,958.0	39,317.2	22,446.2	10,139.5	5,014.4	1,717.2
	% Variation		0.27	0.02	-0.001	-0.02	-0.08	-0.03	0.21	0.10
	ZODIAC	8,000	1.079	5,827.8	947,252.0	38,041.4	20,755.3	10,047.7	5,421.5	1,816.9
	% Variation		0.27	0.01	-0.002	-0.01	-0.11	-0.14	0.49	0.30
	ZODIAC	10,786	1.069	5,501.5	945,336.0	37,161.1	19,669.5	9,952.3	5,642.0	1,897.3
	% Variation		0.19	0.01	0.001	-0.01	-0.12	-0.28	0.68	0.50
Enrichment 2.7 wt %	ZODIAC	4,000	1.075	6,249.2	962,321.0	27,000.1	15,171.7	7,071.0	3,523.7	1,233.7
	% Variation		-0.19	0.02	-0.000	0.07	0.13	0.13	-0.26	-0.09
	ZODIAC	8,000	1.054	5,622.5	959,436.0	26,117.7	13,839.8	7,069.4	3,843.3	1,365.2
	% Variation		-0.28	0.06	-0.004	0.20	0.38	0.13	-0.25	-0.05
	ZODIAC	10,777	1.041	5,208.6	957,375.0	25,523.7	13,026.2	7,028.1	4,001.8	1,467.6
	% Variation		-0.48	-0.08	-0.006	0.28	0.55	0.06	-0.17	0.07
Enrichment 4.3 wt %	ZODIAC	4,000	1.130	3,927.1	603,219.0	30,481.2	16,858.4	8,162.6	4,041.9	1,418.3
	% Variation		-0.18	0.07	-0.004	0.05	-0.06	0.18	0.04	0.50
Low fuel density	ZODIAC	8,000	1.104	3,533.3	601,087.0	28,637.0	14,613.4	8,096.7	4,364.7	1,562.2
	% Variation		-0.27	0.08	-0.010	0.16	0.01	0.20	0.24	1.13
	ZODIAC	10,748	1.086	3,269.4	599,581.0	27,389.2	13,217.7	7,988.1	4,506.1	1,677.3
	% Variation		-0.28	0.10	-0.015	0.27	0.14	0.12	0.36	1.65

4.0 FAST REACTORS

NEUTRON SPECTRUM STUDIES IN THE PCTR FAST NEUTRON CAVITY

D. F. Newman and L. C. Davenport

Introduction

Neutron spectrum measurements were made in the interior of the Fast Neutron Cavity assembled inside the central zone of the Physical Constants Testing Reactor (PCTR). An inter-comparison between two experimental methods for determining the neutron spectrum was obtained using a proton-recoil spectrometer and a multiple foil activation method. The results of these neutron spectrum measurements provide a valuable check of both reactor theory and analytical technique for developing calculational methods applicable to fast-thermal coupled reactor cores.

Proton-Recoil Spectrometer Measurements

Proton-recoil counter measurements using both hydrogen-gas-filled and methane-gas-filled detectors were made in the center of the Fast Neutron Cavity, following the method of E. F. Bennett.⁽¹⁾ The neutron spectrum obtained from the proton-recoil counter data, shown in Figure 4.1, is the composite of several measurements at the same reactor flux level. Count rates during all measurements were kept below 920 counts per sec. The proton-recoil spectrum was observed in consecutive overlapping energy intervals, obtained by varying the gas multiplication in each proportional counter. The methane detector spanned energies between 1.4 MeV and 160 keV in three intervals. Neutron energies between 335 keV and 9.6 keV were observed in six intervals using the hydrogen detector. Corrections for systematic errors in the recoil-proton spectrum including wall-and-end effect, inhomogeneous electric field effect (tip effect), and heavy nuclei recoil distortion were made in the data analysis. The fast neutron flux spectrum was

FIGURE 4.1. *Comparison of Proton-Recoil Spectrometer and Multiple Foil Activation Results*

unfolded by differentiating the proton recoil-spectrum using computer program PSNS.⁽²⁾ The spectrometer was calibrated using the $^{14}\text{N}(n,p)^{14}\text{C}$ reaction which releases ionization equivalent to a proton with energy of 615 keV when thermal neutrons are incident. Both detectors, containing 4.0% nitrogen, were calibrated in the PCTR thermal column⁽³⁾ with the reactor at a 10 watt power level. The averaged composite neutron spectral results from the proton-recoil measurements, shown in Figure 4.1, consist of inverse variance weighted data from the individual measurements within the neutron energy interval 1.4 MeV to 9.6 keV. Neutron energy resolution was narrower than 15% (full-width at half-maximum) over the energy interval 1 MeV to 40 keV and increased to about 23% at 10 keV.

Multiple Foil Activation Measurements

Analysis of multiple foil activation measurements in the empty Fast Neutron Cavity (FNC) was made to obtain a set of saturated activities for 26 foil reactions. Six of these reactions are threshold reactions that respond only to neutrons with energies in the fast range. The remaining twenty reactions were obtained from bare foils, foils covered by 0.1 cm thick cadmium filters, and foils covered by 1.0 cm thick ^{10}B filters. Neutron filters altered the energy responses of the foils such that activation data spanning three different neutron energy ranges were obtained.

Activation shelf-shielding corrections were measured by irradiating several foil thicknesses of the same material simultaneously and extrapolating the specific activities to zero thickness. The absolute counter efficiency calibration for a NaI(Tl) gamma-ray scintillation spectrometer was measured with a set of IAEA standard sources of known activity. The sources included: ^{241}Am (60 keV), ^{57}Co (122 keV), ^{203}Hg (279 keV), ^{22}Na (511, 1275 keV), ^{137}Cs (622 keV), ^{54}Mn (835 keV), ^{88}Y (898, 1836 keV), and ^{60}Co (1173, 1332 keV). Infinitely

dilute saturated activities for the $^{115}\text{In}(n,\gamma)$, $^{197}\text{Au}(n,\gamma)$, and $^{63}\text{Cu}(n,\gamma)$ reactions were determined from foils irradiated in the PCTR thermal column. Non- $1/v$ correction factors for the 2200 m/sec cross sections were calculated for the thermal column with THERMOS.⁽⁴⁾ The thermal flux in the PCTR thermal column at 100 Watts was determined to be 1.07×10^8 neutrons per $\text{cm}^2\text{-sec}$ ($\pm 2.6\%$). This flux level and the THERMOS calculated non- $1/v$ factors for the thermal column were used to determine saturated activities of other reactions in the FNC from activity measurements relative to the thermal column. Saturated activities for the $^{235}\text{U}(n,f)$, $^{239}\text{Pu}(n,f)$, $^{232}\text{Th}(n,\gamma)$, and $^{238}\text{U}(n,\gamma)$ reactions were determined by this method. Infinitely dilute saturated activities for 26 foil reactions measured in the FNC are listed in Table 4.1. The average uncertainty in all the saturated activity measurements was less than 3%.

The infinitely dilute saturated activities obtained from foil measurements were analyzed using the SAND-II computer code⁽⁵⁾ to unfold the neutron spectrum. Initial trial functions for the neutron spectrum in the SAND-II analysis included: a transport theory calculation⁽⁶⁾ from DTF-IV,⁽⁷⁾ a constant $\phi(u)$, a Gaussian function centered at 100 keV, and a Gaussian function centered at 1 MeV. The best SAND-II representation for the neutron spectrum, shown in Figures 4.1 and 4.2, was obtained from a weighted average of the iterative neutron spectrum solutions resulting from different initial trial functions. A weighting factor proportional to the inverse variance in the measured-to-calculated activity ratios was used in the averaging procedure. The standard deviation in the multiple foil activation neutron spectrum was inferred from the width, $\Delta\phi(E)_{\text{Max}}/\phi(E)_{\text{Ave}}$, of the envelope of all SAND-II solutions obtained from measured activities using different initial trial functions.

TABLE 4.1. *Infinitely Dilute Saturated Activities*

Reaction	Filter	Infinitely Dilute Saturated Activity, disintegrations/sec-nucleus	Nominal 90% Activity Limits, MeV	
			Lower Energy	Upper Energy
$^{27}\text{Al}(n,\alpha)^{24}\text{Na}$	Bare	$5.62 \times 10^{-20} (\pm 3.3\%)$	6.600	12.500
$^{27}\text{Al}(n,p)^{27}\text{Mg}$	Bare	$3.06 \times 10^{-19} (\pm 2.8\%)$	3.300	9.400
$^{58}\text{Ni}(n,p)^{58}\text{Co}$	Bare	$1.65 \times 10^{-17} (\pm 3.9\%)$	2.200	6.900
$^{115}\text{In}(n,n')^{115\text{m}}\text{In}$	Bare	$2.45 \times 10^{-17} (\pm 2.3\%)$	0.720	4.900
$^{103}\text{Rh}(n,n')^{103\text{m}}\text{Rh}$	Bare	$4.73 \times 10^{-16} (\pm 3.9\%)$	0.300	3.800
$^{238}\text{U}(n,f)\text{F.P.}$	Bare	$3.48 \times 10^{-17} * (\pm 3.4\%)$	1.500	5.500
$^{235}\text{U}(n,f)\text{F.P.}$	Bare	$5.52 \times 10^{-15} * (\pm 2.8\%)$	6.00×10^{-8}	0.475
	Cadmium	$4.03 \times 10^{-15} * (\pm 2.9\%)$	9.20×10^{-7}	0.600
	Boron-10	$1.27 \times 10^{-15} * (\pm 2.7\%)$	0.012	1.400
$^{239}\text{Pu}(n,f)\text{F.P.}$	Bare	$1.06 \times 10^{-14} * (\pm 2.9\%)$	9.20×10^{-8}	0.320
	Cadmium	$4.44 \times 10^{-15} * (\pm 2.7\%)$	8.00×10^{-7}	0.690
	Boron-10	$2.42 \times 10^{-15} * (\pm 3.7\%)$	0.023	2.100
$^{238}\text{U}(n,\gamma)^{\beta-239}\text{Np}$	Bare	$5.36 \times 10^{-16} (\pm 3.8\%)$	1.80×10^{-6}	0.400
	Cadmium	$5.36 \times 10^{-16} (\pm 3.8\%)$	4.00×10^{-6}	0.400
	Boron-10	$1.56 \times 10^{-16} (\pm 4.2\%)$	5.75×10^{-3}	0.760
$^{232}\text{Th}(n,\gamma)^{\beta-233}\text{Pa}$	Bare	$1.60 \times 10^{-15} (\pm 2.6\%)$	5.75×10^{-7}	0.400
	Cadmium	$1.59 \times 10^{-15} (\pm 2.6\%)$	1.80×10^{-6}	0.425
	Boron-10	$9.05 \times 10^{-16} (\pm 2.6\%)$	1.35×10^{-2}	0.660
$^{63}\text{Cu}(n,\gamma)^{64}\text{Cu}$	Bare	$1.22 \times 10^{-16} (\pm 2.2\%)$	1.60×10^{-7}	0.280
	Cadmium	$1.13 \times 10^{-16} (\pm 2.2\%)$	1.35×10^{-6}	0.300
	Boron-10	$0.41 \times 10^{-16} (\pm 2.2\%)$	2.00×10^{-2}	0.500
$^{197}\text{Au}(n,\gamma)^{198}\text{Au}$	Bare	$6.31 \times 10^{-15} (\pm 2.2\%)$	6.30×10^{-7}	5.75×10^{-2}
	Cadmium	$6.18 \times 10^{-15} (\pm 2.2\%)$	2.20×10^{-6}	0.063
	Boron-10	$3.47 \times 10^{-16} (\pm 2.2\%)$	8.80×10^{-4}	0.475
$^{115}\text{In}(n,\gamma)^{116\text{m}}\text{In}$	Bare	$2.78 \times 10^{-14} (\pm 2.8\%)$	9.20×10^{-7}	2.00×10^{-6}
	Cadmium	$2.71 \times 10^{-14} (\pm 2.8\%)$	1.05×10^{-6}	2.10×10^{-6}

* $\frac{\text{Fissions}}{\text{sec-nuc Zeus}}$

$$\text{Average Error} = \frac{\sqrt{\sum_{i=1}^N \sigma_i^2}}{N} = 2.977\%$$

FIGURE 4.2 *Neutron Flux Spectrum in the PCTR Fast Neutron Cavity*

Discussion and Comparison of Results

The integral flux in the neutron energy interval 9.32 keV to 1.17 MeV was obtained from the SAND-I1 analysis of multiple foil activations within +3% uncertainty from all iterative spectrum solutions. This permitted normalization of proton-recoil spectrometer results to multiple foil activation results, as shown in Figures 4.1 and 4.2. In addition to permitting direct comparison between the two experimental methods, this normalization was used as an absolute flux level calibration for the proton-recoil spectrometer. The envelope of all SAND-I1 solutions exhibited poor resolution in the interval where the proton-recoil counter results were obtained. The activation cross sections were small and similar in shape over this neutron energy interval. Activation integrals were relatively insensitive to the shape of the neutron spectrum in the region of the flux peak. Agreement of the results from the two methods was encouraging. Large flux depressions at about 7 eV and between 20 and 40 eV, shown in Figure 4.2, were the result of ^{238}U resonance absorption in the FNC buffer. The apparent depression in the SAND-I1 spectrum at approximately 2 MeV is believed to be due to errors in the cross sections and the activation measurements for the $^{115}\text{In}(n,n')^{116\text{m}}\text{In}$ and $^{58}\text{Ni}(n,p)^{58}\text{Co}$ reactions.

Energy degradation of fission spectrum neutrons into the few hundred keV energy region, as shown both by experimental results and calculations,⁽⁶⁾ was the result of ^{238}U inelastic scattering in a uranium buffer zone. This 5.4-in. uranium buffer zone completely surrounded a 1/2-in. thick stainless steel box (29.0 in. by 29.0 in. by 24.7 in. inner dimensions) forming the large Fast Neutron Cavity.⁽⁸⁾ The uranium fuel in the FNC absorbed incident thermal neutrons and produced fast neutrons by the fissioning process such that a flux of predominantly fast neutrons entered the central cavity.

Neutron spectrum results from multiple foil activation measurements were obtained over the entire energy spectrum, 10^{-10} to 18 MeV. Detailed neutron spectrum structure was measured by proton-recoil spectrometer methods in the energy interval, 9.6 keV to 1.4 MeV, comprising 80% of the integrated neutron flux. Thus, each of these two experimental methods supplemented and confirmed the information obtained from the other. Using both methods permitted the absolute value of the neutron flux spectrum to be obtained in the center of the FNC with the PCTR operating at a 100 Watt power level.

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A MULTICHANNEL APPROACH TO FAST REACTOR SAFETY ANALYSIS

Alan E. Waltar and Andrew Padilla, Jr.

As the construction of large sodium-cooled fast reactors approaches reality, increased efforts are being focused on methods to better understand core behavior under postulated accident conditions.⁽¹⁻⁴⁾ Since the interaction of Doppler, sodium voiding, and fuel relocation reactivity effects -- particularly near prompt critical -- can greatly influence the consequences of major accidents, a multichannel analysis is necessary to properly couple the various reactivity effects.

Because a detailed multichannel code could easily entail such long execution times that the necessary parametrics could not be performed, the present approach has been to construct a flexible, fast running code initially containing several rather simplified yet fairly realistic models in order to provide a reasonable basis for assurance that all major feedback mechanisms, and their interaction, can be accounted for. The program is designed to easily accommodate more sophisticated models as support experimental data become available.

The code, entitled MELT-11, is capable of computing temperatures in 10 radial regions in the core and each pin (representative of a radial region) can be subdivided into 20 axial segments and 15 radial nodes. Since an implicit heat transfer scheme is employed, a variable time step method is used wherein the only restriction to step size is the rate of feedback. For loss of flow accidents, a zero neutron lifetime approximation has been successfully formulated to further extend the time step size.

Three sodium voiding models and two fuel relocation models are currently available in MELT-11. For loss of flow accidents which result in sodium boiling, a slug flow model can be used where the expulsion pressure is based on the clad

temperature at the point of initial boiling. For severe power transients, the expulsion of sodium can be determined by pressures generated from a molten fuel-sodium interaction. In both the loss of flow and power transient conditions, sodium voiding can be effected by the release and expansion of the fission gases contained within the fuel pin. For fuel relocation, one model assumes a gravity collapse following clad failure and the other model assumes fuel squirting to relieve internal pin pressures.

As an indication of the possible interaction of various reactivity effects, Figure 4.3 shows the power and reactivity changes after initial clad failure during a hypothetical guillotine pipe rupture accident, without protective action, in a fast reactor patterned after the FTR. Since for this case, clad failure at the top core-reflector interface preceded coolant boiling, release of the fission gas was assumed to be the sodium voiding mechanism. Although the maximum positive sodium voiding "dome" was worth about \$3.30 for this example, the radial channel voiding incoherence for the six channels involved was such that the entire core plus the axial reflectors ultimately voided without raising fuel temperatures to the point where core disassembly, in the traditional Bethe-Tait sense, was possible. However, some cases have been observed to lead to an eventual secondary excursion due to fuel collapse. Furthermore, for the time scales involved, it may be important to consider the consequences associated with coolant re-entry.

Acknowledgement

The authors are greatly indebted to Dr. Richard J. Shields, Computer Sciences Corporation, for valuable programming assistance.

FIGURE 4.3. Reactivity and Power Versus Time for a Hypothetical Pipe Rupture Accident

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FAST REACTOR DISASSEMBLY CALCULATION UTILIZING A TEMPERATURE-DENSITY DEPENDENT EQUATION OF STATE

Alan E. Waltar

In attempting to analyze a complete spectrum of major fast reactor accidents which are assumed to terminate by core disassembly, it is important to account for density dependence in the equation of state of the core. This is especially true for situations in which a considerable sodium inventory is present in the core.⁽¹⁾ Exceptionally large pressures for such cases can be generated relatively early in a transient as the result of a small degree of fuel expansion. However, even an infinitesimal amount of core expansion is often adequate to return the driving pressure back to the two-phase vapor equilibrium state. Neglect of these effects can cause large errors in the history of a disassembly accident.

The equation of state used for the present study is basically that developed by Harris.⁽²⁾ The approach is to use existing density-temperature data for UO_2 to determine the temperature at which the departure point into the single phase liquid region is attained, and then compute the pressure-temperature relationship in this region based on the theorem of corresponding states. For the numerical example reported below, Menzies critical constants⁽³⁾ were employed. Sodium compressibility is accounted for by reducing $\partial P / \partial T$ in the single phase region and adjusting the effective departure point temperature.

The temperature-density-dependent equation of state as formulated above has been successfully incorporated into the VENUS computer program.⁽⁴⁾ It is to be noted that one of the characteristics of this type of formulation is that the pressure can oscillate between two-phase and single-liquid-phase regions with little density change. This certainly provides a severe test of the program from the numerical stability

standpoint, although experience to date has indicated no numerical instability for the problems of interest. The VENUS program employs a mid-point finite difference scheme⁽⁵⁾ in the hydrodynamics which is a modified version of Kolsky's method.⁽⁶⁾ Shock waves are handled by the "shock smearing" technique developed by Von Neumann and Richtmyer.⁽⁷⁾

Figure 4.4 illustrates the typical pressure-density behavior with a temperature-density-dependent equation of state for sodium-in conditions. The particular example chosen was a rapid power excursion in an FTR type core in which the core (radius ~60 cm, height 90 cm) was divided into 11 radial and 18 axial nodal points. Values indicated are for the node in the core center after the transient reaches the disassembly phase. If the compressibility of sodium is ignored, the solid lines indicate a sharp rise in pressure when the fuel density drops to the point where all the void space available to the fuel is filled. Nodal expansion quickly follows, causing a density decrease. This relieves the liquid pressure allowing the pressure buildup in adjacent nodes to cause recompression. High pressure again follows and the process oscillates until enough overall core expansion has occurred to terminate the excursion. If sodium compressibility is taken into account, peak pressures are lower because of the increased volume available to the fuel. For the particular case chosen, including the compressibility effect increased the amount of energy contained in the molten fuel by about 10%.

Acknowledgement

The author is indebted to Dr. William T. Sha, Argonne National Laboratory, for his keen interest and collaboration in this study.

FIGURE 4.4. *Pressure-Density Variations at the Central Node for a Hypothetical 100\$/sec Ramp in a FTR Type Core*

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ETOX

J. L. Baker, R. E. Schenter, and D. P. Taylor

Group cross sections generated from the three computer codes: ETOX-1DX, MC², and GAFGAR, using the ENDF/B library as input, were compared for the fast reactor ZPR III assemblies 52 and 50 and zebra assembly 6A. Results for these calculations, for assembly 52, show that in the energy region having significant fast reactor flux, the group cross sections agree very closely. Discrepancies observed in the overlap regions were eliminated by improving the interpolation algorithms in ETOX. The algorithms have been revised between the resolved and unresolved regions and between the unresolved and smooth regions. Discrepancies in these regions are now less than 1%. Modifications also have been added to improve calculations in the neighborhood of the three resonances in the resolved region for ²³Na.

The severe disagreements between the codes in the resolved resonance region casts doubt on the use of any of the codes for analyzing effects due to reactions in the resolved region. Consequently, work is underway to resolve these discrepancies.

RSYN

K. B. Stewart

RSYN (Reactor SYNthesis) solves the three-dimensional, multigroup reactor diffusion equations by the method of flux synthesis. The two-dimensional diffusion code 2DB is used to generate the trial functions in the horizontal plane, and the full three-dimensional solution is then synthesized. In

the current version, only the (X,Y) geometry option of 2DB has been incorporated into RSYN. Subsequent versions will be extended to include the (r, θ) and hexagonal geometry capabilities of 2DB in generating trial functions. Downscattering from any group to all groups lower in energy is permitted.

RSYN will allow up to 50 energy groups. The maximum number of X and Y intervals is 2500. The number of Z intervals in mesh used to describe the Z-axis of the reactor must not exceed 50.

The total number of axial layers used must not exceed 10 and the total number of trial functions used to describe these layers cannot exceed 15 per group. Due to variable dimensioning it is not possible to state categorically which combinations will fit in core at any one time.

The calculation of the leakage term involves $\text{grad } \phi(X,Y,Z)$. The initial programming effort found that the derivatives could not be calculated across material boundaries and that the surface integral term included in the leakage calculation was especially sensitive to the partial derivative calculations. Originally, the derivatives were calculated assuming $\text{grad } \phi$ for the trial functions was continuous across material boundaries, but a sample problem with a diffusion coefficient on the order of 10^3 barns pointed out the difficulty of correctly calculating a partial derivative over the boundaries.

The second major area of difficulty was the surface integral contribution to the leakage. The effect was much larger than anticipated (approximately 2%) and because of this, the programming necessary to calculate the surface integral was included in RSYN. The calculation of k_{eff} is still affected by this calculation but the effect is on the order of 0.5%.

The next phase of programming will utilize the fact that $D(X,Y) \text{ grad } \phi(X,Y)$ is continuous across material boundaries for the individual trial function in the partial derivative calculation, and the calculation of the surface integral contribution should subsequently be improved.

RSYN can be used on a general basis; however, some experience should be accrued on sample problems before it is used to analyze a complete reactor. Generally, the synthesis calculations require about the same amount of computer time as one of the 2DB calculations. Also, the synthesis direction should be the one of least complexity.

BEAM BENDING CODE DEVELOPMENT

V. B. Watwood and W. H. Sutherland

The following decisions have been made with regard to the creep bending code task.

- 1) The element model will be formulated with the same degree of sophistication as the planar model in AXICRP and in fact the development will parallel that model very closely.
- 2) The geometrical generality will initially be restricted to the framework of a two dimensional beam formulation with axial deformation plus bending in only a single plane. Although restrictive from a completely general point of view, this constraint is completely in accord with the present knowledge of loading (radial variation only) and the practical observation that little interaction will occur duct to duct.
- 3) The code to be employed will be a modification-rewrite of AXISOL-AXICRP. SAMIS was considered in this regard but was judged much too cumbersome for efficient modification.

All of these decisions were made with the objectives in mind to (a) produce useful results as quickly as possible, (b) check the basic validity of the presently envisioned concept of treating creep behavior in beams.

Although the geometrical generality will be restricted in this initial program, consideration will be given to the full three dimensional beam formulation and stiffness merge problems with the aim of making the extension later as easy as possible.

MODEL FORMULATION - BEAM BENDING CODE

W. H. Sutherland

The objective of this work is to develop a finite element formulation for beams with arbitrary cross section which will permit the incorporation of any arbitrary creep law of the general functional form,

$$\Delta \epsilon_e^C = F(\sigma_e, \epsilon_e^C, \phi, T, \Delta t)$$

$$\Delta \epsilon_e^C = \text{incremental effective creep strain}$$

$$\sigma_e = \text{effective stress}$$

$$\epsilon_e^C = \text{effective creep strain}$$

$$\phi = \text{flux}$$

$$T = \text{temperature}$$

$$\Delta t = \text{increment of time}$$

into the analysis. Furthermore, it is desirable to use the same finite element structural idealization that has been used for the elastic analysis of beams. These objectives can be achieved with the following formulation.

Using the principle of virtual displacements the following identity is obtained.

$$\int_V \epsilon_{ij}^* \sigma_{ij} dU = \int_{S_T} \bar{T}_i u_i^* dS$$

where $\epsilon_{ij}^* = \frac{1}{2} (u_{i,j}^* + u_{j,i}^*)$

and u_i^* are the virtual displacements which must satisfy the prescribed displacement boundary conditions.

S_T represents the surface on which the tractions ($\bar{T}_i$) are prescribed.

The stress-strain relations are given by

$$\sigma_{ij} = C_{ijmn} \epsilon_{mn}^e.$$

In the analysis of the flow duct the total strains (ϵ_{mn}) are given by the following sum

$$\epsilon_{mn} = \epsilon_{mn}^e + \epsilon_{mn}^c + \epsilon_{mn}^T + \epsilon_{mn}^S$$

where ϵ_{mn}^e = elastic strains

ϵ_{mn}^c = creep strains

ϵ_{mn}^T = thermal strains

ϵ_{mn}^S = swelling strains.

Substitution of the elastic strains into the identity derived from the principle of virtual displacements gives

$$\int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^e dU - \int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^c dU - \int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^T dU$$

$$- \int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^S dU = \int_{S_T} \bar{T}_i u_i^* dS.$$

The stiffness matrices for an arbitrary beam can be obtained from the above integrals by appropriate minimization with respect to generalized nodal displacements. The resultant stiffness matrices are represented symbolically as follows.

$$\underline{K} \underline{x} = \underline{R}^C + \underline{R}^T + \underline{R}^S + \underline{R}^a = \underline{R}^i + \underline{R}^a$$

where $\underline{K}$ represent the stiffness matrix for the beam element

$\underline{x}$ = generalized nodal displacement vector

$\underline{R}^C$ = force vector resulting from creep strains

$\underline{R}^T$ = force vector resulting from thermal strains

$\underline{R}^S$ = force vector resulting from swelling strains

$\underline{R}^a$ = force vector resulting from applied tractions

$\underline{R}^i$ = force vector resulting from initial strains.

The stiffness matrix is derived from the first integral and is the same stiffness matrix derived for elastic beam analysis. The force vectors derived from the thermal and swelling strains are computed in the same manner as for the elastic analysis. The formulation permits nonlinear transverse temperature and swelling distributions. The force vector derived from creep strains is obtained from the following integral:

$$\int_V \epsilon_{ij}^* C_{ijmn} \epsilon_{mn}^C dU.$$

To evaluate the integral it becomes necessary to know the creep strain variation across the flow duct. As shown later, an incremental analysis will be employed and the above integral to be evaluated becomes

$$\int_V \Delta \epsilon_{ij}^* C_{ijmn} \Delta \epsilon_{mn}^C dU.$$

Using the von Mises flow rule the triaxial creep strains can be related to uniaxial creep data

$$\Delta \epsilon_{mn}^C = \frac{3}{2} \frac{\Delta \epsilon_e^C}{\sigma_e} S_{ij}$$

where $S_{ij} = \sigma_{ij} - \frac{1}{3} \delta_{ij} \sigma_{kk}$.

Since the initial elastic stress distribution is known across the flow duct it is possible to calculate $\Delta \epsilon_{mn}^C$ for the first time interval at a fixed number of points uniformly distributed across the flow duct. Subsequent stress distributions at later time intervals can be determined by computing the change in stress at each of these points.

$$\Delta \sigma_{ij} = C_{ijmn} \left(\Delta \epsilon_{mn} - \Delta \epsilon_{mn}^C - \Delta \epsilon_{mn}^T - \Delta \epsilon_{mn}^S \right)$$

Knowledge of the stress distribution across the flow duct at each time interval, together with the specified creep law, permits the evaluation of the force vector derived from creep strains.

The general incremental procedure for the solution of creep problems proceeds in the following manner. At time equal to zero (solution point 1), the initial loads are applied and the elastic response ($\underline{K} \underline{x}_1 = \underline{R}_1$) calculated. This state of stress (initial elastic stress) is assumed to be constant during the following time interval (time interval 1). The creep strains occurring during the time interval can be evaluated at the end of the time interval and related to a force vector as previously discussed. Then for the second solution point new load vectors are computed including the contribution from the creep strains

$$\underline{K} \underline{x}_2 = \underline{R}_2 = \underline{R}_1^i + \Delta \underline{R}_{12}^i + \underline{R}_1^a + \Delta \underline{R}_{12}^a$$

where $\underline{R}_2$ is the total loads existing at solution point 2, $\Delta \underline{R}_{12}^i$ is the change in the initial strain "loads" including the creep during increment 1, and $\Delta \underline{R}_{12}^a$ is the change in the applied load vector. Observe that the applied loads may be modified at each time interval as well as the loads resulting

from initial strains. This feature is essential to permit the modeling of the loading due to stainless steel swelling in the flow ducts.

The solution procedure applies for incremental values of the displacements and loads and can be shown in the following manner.

Let $\mathbf{x}_2 = \mathbf{x}_1 + \Delta\mathbf{x}_{12}$ where $\Delta\mathbf{x}_{12}$ is the change in the displacements. This gives

$$\underline{\mathbf{K}} (\mathbf{x}_1 + \Delta\mathbf{x}_{12}) = \mathbf{R}_1^i + \mathbf{R}_1^a + \Delta\mathbf{R}_{12}^i + \Delta\mathbf{R}_{12}^a$$

$$\text{or } \underline{\mathbf{K}} \Delta\mathbf{x}_{12} = \Delta\mathbf{R}_{12}^i + \Delta\mathbf{R}_{12}^a, \text{ since } \underline{\mathbf{K}} \mathbf{x}_1 = \mathbf{R}_1^i + \mathbf{R}_1^a.$$

Knowledge of the change in displacements ($\Delta\mathbf{x}_{12}$) permits the calculation of the change in total strains ($\Delta\epsilon_{mn}$). Since the change in creep strains ($\Delta\epsilon_{mn}^c$), thermal strains ($\Delta\epsilon_{mn}^T$), and swelling strains are already known for the time interval, the change in stresses ($\Delta\sigma_{ij}$) can be computed as previously indicated. Adding the stresses ($\Delta\sigma_{ij}$) to the previous stress distribution gives the new stress distribution across the flow duct. This state of stress is assumed to be constant during the next time interval (time interval 2) and the incremental creep strains are calculated based upon the new stress distribution. This same procedure is repeated until the final time of interest is reached.

Summary

A general finite element procedure has been defined which will permit the creep analysis of flow ducts using the same structural idealization as used for the elastic analysis of flow ducts. Furthermore, any arbitrary creep law of the general functional form

$$\Delta\epsilon_e^c = F(\sigma_e, \epsilon_e^c, \phi, T, \Delta t)$$

may be used. Provision has been made in the formulation to accommodate the time dependent loading condition resulting from stainless steel swelling.

CODE DESIGN - BEAM BENDING CODE

V. B. Watwood and J. D. Rector

The analysis of beams using finite element techniques is a direct extension of the current technology. Since there is a commonness between all structural problems when these techniques are applied, the beam bending analysis code can be patterned after the existing structural analysis codes AXISOL and AXICRP. To achieve reliable results as soon as possible, all applicable sections of AXISOL-AXICRP will be incorporated within the new code. An advanced feature will be the capability of decomposing the stiffness matrix to provide rapid solutions.

A detailed description of the matrix decomposition scheme is given on the following pages.

Matrix Decomposition

The matrix equation $\frac{K}{N \times N} \frac{\gamma}{N \times 1} = \frac{R}{N \times 1}$ is usually solved by inverting K.

$\underline{K}$ can be decomposed to an equivalent set of matrices of special form.

$$\frac{K}{N \times N} = \frac{U^T}{N \times N} \frac{W}{N \times N} \frac{U}{N \times N} \quad \text{where:}$$

$$\underline{U} \text{ is of the form } \begin{bmatrix} 1 & x & x & . \\ & 1 & . & . \\ & & . & . \\ & & & 1 \end{bmatrix} \quad \text{i.e., an upper triangular matrix with 1's on the diagonal.}$$

$$W \text{ is of the diagonal form } \begin{bmatrix} W_{11} & & \\ & W_{22} & \\ & & \ddots \\ & & & W_{nn} \end{bmatrix}$$

$$\text{then } K \underline{x} = \underline{R}$$

$$\underline{U}^T \underline{W} \underline{U} \underline{Y} = \underline{R}$$

$$\underline{W} \underline{U} \underline{Y} = (\underline{U}^T)^{-1} \underline{R} = \underline{Y}$$

$$\underline{U} \underline{Y} = \underline{W}^{-1} \underline{Y} = \underline{Z}$$

$$\underline{Y} = \underline{U}^{-1} \underline{Z}.$$

$$\text{since } (\underline{U}^T)^{-1} \underline{R} = \underline{Y},$$

$$\underline{U}^T \underline{Y} = \underline{R}$$

$$\begin{bmatrix} 1 & & & & \\ x & 1 & & & \\ x & x & . & & \\ . & . & . & & \\ . & . & . & . & \\ x & x & . & . & x & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \\ Y_n \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \\ \\ R_n \end{bmatrix}$$

$\underline{Y}$ can be found by forward substitution, i.e., without inversion

$$\underline{W}^{-1} \underline{Y} = \underline{Z}$$

$$\begin{bmatrix} \frac{1}{W_{11}} & & & & \\ & \frac{1}{W_{22}} & & & \\ & & \ddots & & \\ & & & \frac{1}{W_{nn}} & \end{bmatrix} \begin{Bmatrix} Y_1 \\ Y_2 \\ . \\ . \\ Y_n \end{Bmatrix} = \begin{Bmatrix} Z_1 \\ Z_2 \\ . \\ . \\ Z_n \end{Bmatrix}$$

$\underline{Z}$ can be found directly

$$Z_i = \frac{Y_i}{W_{ii}}$$

the solution for $\underline{x}$ can be solved by backward substitution

$$\underline{Y} = \underline{U}^{-1} \underline{Z}$$

$$\underline{U} \underline{Y} = \underline{Z}$$

$$\begin{bmatrix} x & x & . & . & x \\ 1 & x & . & . & x \\ . & . & . & . & x \\ . & . & . & . & . \\ . & . & . & . & 1 \end{bmatrix} \begin{Bmatrix} \gamma_1 \\ \gamma_2 \\ . \\ . \\ \gamma_n \end{Bmatrix} = \begin{Bmatrix} z_1 \\ z_2 \\ . \\ . \\ z_n \end{Bmatrix}$$

When K is positive definite and symmetric, U and W can be constructed from K as follows:

Note

$$U_{ij} = 1 \text{ when } i = j$$

$$U_{ij} = 0 \text{ when } i > j$$

$$W_{ij} = 0 \text{ when } i \neq j$$

by definition

$$K_{ij} = \sum_{n=1}^N \sum_{m=1}^N U_{ni} W_{nm} U_{mj} \quad \text{N- size of } K \quad (1)$$

but $W_{nm} = 0$ when $n \neq m$ so

$$K_{ij} = \sum_{n=1}^N U_{ni} W_{nn} U_{nj} \quad (2)$$

Since K is symmetric, it suffices to look only at the cases where $i = j$ and $J > i$.

for $i = j$

$$K_{ii} = \sum_{n=1}^N U_{ni}^2 W_{nn} \quad (3)$$

but $U_{ni} = 0$ if $n > i$, so

$$K_{ii} = \sum_{n=1}^i U_{ni}^2 W_{nn} = \sum_{n=1}^{i-1} U_{ni}^2 W_{nn} + U_{ii}^2 W_{ii} \quad (4)$$

since $U_{ii} = 1$ by definition

$$W_{ii} = K_{ii} - \sum_{n=1}^{i-1} U_{ni}^2 W_{nn} \quad (5)$$

This equation for W_{ii} can be solved when values of U_{ni} and W_{nn} are known for $n < i$.

Now consider the second case, i.e., when $j > i$.

Again

$$K_{ij} = \sum_{n=1}^N U_{ni} W_{nn} U_{nj} \quad (6)$$

but since $j > i$ and $U_{ni} = 0$ when $n > i$,

$$K_{ij} = \sum_{n=1}^i U_{ni} W_{nn} U_{nj} = \sum_{n=1}^{i-1} U_{ni} W_{nn} U_{nj} + U_{ii} W_{ii} U_{ij} \quad (7)$$

and since $U_{ii} = 1$

$$U_{ij} = \frac{1}{W_{ii}} K_{ij} - \sum_{n=1}^{i-1} U_{ni} W_{nn} U_{nj} \quad (8)$$

This equation fails when $W_{ii} = 0$. However, it can be shown that the determinant of $\underline{U}$ is 1 and the determinant of $\underline{W}$ is

the product of the diagonal elements. Therefore since K is by definition a positive definite matrix, all diagonal elements of $\underline{W}$ must be nonzero.

$\frac{U}{N \times N}$ and $\frac{W}{N \times N}$ can be constructed from $\frac{K}{N \times N}$ using Equations (5) and (8).

Starting with Equation (5) and $i = 1$:

$$W_{11} = K_{11} - \sum_{n=1}^{i-1} U_{ni}^2 W_{nn} = K_{11} - 0 = K_{11} \quad (9)$$

substituting K_{11} into Equation (8) and using $i = 1$:

$$U_{ij} = \frac{1}{K_{11}} K_{ij} - \sum_{n=1}^{i-1} U_{ni} W_{nn} U_{nj} = \frac{K_{ij}}{K_{11}} \quad (10)$$

The W_{11} term and the U_{ij} terms are used in solving Equations (5) and (8) for $i = 2$. Stepping up in this manner completes $\underline{U}$ and $\underline{W}$.

Note

$$\begin{array}{cccccccc} 1 & U_{12} & U_{13} & U_{14} & \cdot & \cdot & \cdot & \cdot & U_{11} = \frac{K_{11}}{K_{11}} = 1 \\ & 1 & U_{23} & U_{24} & \cdot & \cdot & \cdot & \cdot & \\ & & 1 & U_{34} & U_{35} & U_{36} & & & \end{array}$$

Finally note that for every K_{ij} beyond the bandwidth of K , the corresponding $U_{ij} = 0$. Therefore n need not be considered when it exceeds the bandwidth of K .

IMPACT OF SWELLING MODEL ON FTR CORE DESIGN

R. J. Jackson

Introduction

Interim guidance was provided to FTR driver fuel designers in the form of equations describing the relationship among metal swelling, fast neutron fluence, and operating temperature. These equations are

for $\begin{cases} \text{AISI 304 stainless steel, solution treated} \\ \text{AISI 316 stainless steel, solution treated} \end{cases}$

$$\frac{\Delta V}{V} = 5.0 \times 10^{-36} (\phi t)^{1.66} \left[e^{-6800/RT} - (1.87 \times 10^4) e^{-27,000/RT} \right]$$

for AISI 316 stainless steel, cold worked

$$\frac{\Delta V}{V} = 9.4 \times 10^{-33} (\phi t)^{1.48} \left[e^{-5600/RT} - (1.78 \times 10^5) e^{-30,000/RT} \right]$$

to a volume change $\leq 1\%$

and at $T \leq 500^\circ\text{C}$.

Preliminary analyses were performed to determine the impact of the interim metal swelling equations, and the impact of current quantitative uncertainty of swelling on the FTR core design. The principally affected components are the fuel pin bundle with its associated spacer system, the subassembly flow ducts and the core radial restraint mechanism. The analyses considered duct packing, duct bending, and core packing. It is impractical to separate the effects of creep from analysis of duct bending so some creep effects are considered in that analysis. The results are summarized in Tables 4.2 and 4.3.

TABLE 4.2. *Summary of Component Bending Reflection*

Mechanism	Bending Source	Assumptions in Analysis	Restraint Condition	Burnup Limitation, 10 ³ MWd/MTM
Flow Duct Bending	Thermal camber	No creep	Fully restrained for operation	None
	Differential swelling	No creep		~40
	Thermal camber	100% stress relaxation		None
	Differential swelling	100% stress relaxation		Uncertain
	Thermal camber	No creep	Unlocked for refueling	None
	Differential swelling	No creep		Batch loading >10
	Thermal camber	100% stress relaxation		1/2 in. free bow deflection
	Differential swelling	100% stress relaxation		Uncertain
Radial Reflector Bending	Differential swelling	No creep	Unlocked for refueling	About 1 year life
	Differential swelling	100% stress relaxation		

TABLE 4.3. *Summary of Component Swelling in Radial Dimension*

Condition	Swelling Model	Burnup Limitation Criterion	Burnup Limitation, 10 ³ MWd/MTM
Core Packing	Mean value +95% conf. on mean -95% conf. on mean	0.100 in. between ducts	- 62 to 75 -
Duct-Pin Bundle Interaction. Wire Wrap	Arrhenius type equation Conversion to sol. treated Enhanced swelling	Contact between pin bundle and duct wall	- 33 30
Duct-Grid Spacer Interaction	Arrhenius type equation Conversion to sol. treated Enhanced swelling	Grid swelling to contact duct wall	- ~55 ~55

Summary

Differential swelling resulting in duct bending is the major fuel lifetime limiting item in the present design. More analysis of the duct bending problem is required. This must consider both the fuel shuffling schemes and a parametric study of the effects of creep, because no conservative assumption regarding creep rate is possible.

Potential limitations to fuel exposure include:

- (a) duct bending caused by thermal camber if significant creep occurs in-reactor
- (b) duct bending caused by differential volume increase if no creep occurs in-reactor
- (c) core packing (occupying of all available core space by swollen ducts)
- (d) duct packing (occupying of all available duct space by swollen fuel pins).

The uncertainty in extrapolating irradiation induced swelling of goal burnup is very large and can change the predicted burnup limitation within the range 30,000 to greater than 80,000 MWd/MTM, if duct packing becomes the limitation.

Flow Duct Bending

The flow ducts are exposed to radial fluence and temperature gradients. This causes more swelling to occur on the hotter, higher fluence side than on the opposite side. The gradients cause both thermal camber and bending from the differential swelling.

Calculations

The environment assumed in these calculations is tabulated below.

<u>Distance from Bottom of Core, in.</u>	<u>Hot Side</u>		<u>Cold Side</u>	
	<u>Fluence, 10²²(a)</u>	<u>Temp, °F</u>	<u>Fluence, 10²²(a)</u>	<u>Temp, °F</u>
	<u>Total</u>	<u>Total</u>	<u>Total</u>	<u>Total</u>
3	12.0	624	9.19	618
7	15.6	655	11.9	642
11	18.2	687	13.9	666
15	19.6	718	15.0	690
19	19.9	751	15.2	714
23	19.0	782	14.5	739
27	17.0	814	13.0	673
31	13.9	846	10.0	786
35	10.0	875	7.6	811

a. Corresponds to 83,000 MWd/MTM

Fluence values corresponding to other fuel burnup values are assumed linearly related to these values. The hot side of the duct is nearer the reactor center. Row 5 has the combination of temperature and fluence gradients which cause the most bending.

The Arrhenius type equation for type 304 solution treated stainless steel was used to calculate the volume increase for the various combinations of fluence and temperature along the length of the duct on both the hot and cold sides. No in-reactor creep was assumed, for conservatism. The difference in swelling of opposite sides of the duct cause a difference in axial dimension of the two opposite sides. In the absence of lateral restraint, this creates a bending deflection. The amount of this deflection can be calculated from geometric

considerations of the difference in axial extension on the two sides of the duct. However, for purposes of a computer calculation it is convenient to work in terms of a pseudo-temperature gradient across the duct which would cause the same difference in axial extension as the differential swelling. These pseudo-temperature gradients are compared with the actual temperature gradient in Figure 4.5. Note that for 80,000 MWd/MTM the pseudo-temperature gradient is more than an order of magnitude larger than the actual temperature gradient. The restrained duct displacements which result from the differential expansion are shown in Figure 4.6. This configuration corresponds to the normal operating condition. This analysis assumes the flow duct behaves as an elastic member and redistributes the applied moment. The unrestrained duct displacements resulting from differential swelling for a beam obeying elastic behavior are shown in Figure 4.7. This configuration is applicable to the condition at refueling when core restraint is removed.

FIGURE 4.5. Pseudotemperature Gradients Across Flow Ducts

FIGURE 4.6. Flow Duct Displacement Due to Swelling

FIGURE 4.7 Unrestrained Duct Displacement Due to Swelling

Cursory analysis of the core radial restraint mechanism indicates the reflector bar that is part of that mechanism is good for about one year based on similar analyses. The creep effects noted in the next section may change this limitation.

Creep Effect

If creep occurs during irradiation swelling, it will affect the resultant deflection curve. This effect is opposite for thermal camber and for bending from differential swelling, as noted on Figure 4.8. This figure compares the deflection curves for the restrained duct and unrestrained duct. Figure 4.8a shows that the thermal gradient, alone, causes the duct to bow outward during operation. If no creep occurs (and no plastic flow) the duct will return to a straight configuration when the restraints are released for refueling. Figure 4.8b indicates that if 100% stress relaxation occurs during operation, the duct will bow inward about 1/2 in. at the top when the restraints are released prior to refueling. Intermediate amounts of stress relaxation will cause a free bow condition intermediate between these two cases. Figure 4.8c indicates the duct will bow inward during operation if swelling alone occurs. This is because the innermost duct wall axial dimension increases more than the outermost duct wall. If no creep occurs this differential length increase in the core region will cause the deflection shape shown. When the restraints are removed the duct top will bow outward. Figure 4.8d represents the deflection curve resulting from differential expansion if in-reactor creep is very pronounced (100% stress relaxation). In the limiting case depicted here, the inward deflection of the duct is accomplished without distributing any moment to the adjacent spans. Without the self restraint of the adjacent spans, the inward deflection for this case is larger than when no creep occurs. When the restraints

a. THERMAL CAMBER
NO CREEP

b. THERMAL CAMBER
100% STRESS
RELAXATION

c. DIFFERENTIAL ΔV
NO CREEP

d. DIFFERENTIAL ΔV
100% STRESS
RELAXATION

FIGURE 4.8. Comparison of Creep Effects

are removed, the duct retains its same configuration because there are no residual stresses in the duct to cause the deflection curve to change.

It can be noted that creep is helpful in permitting higher burnup for a given differential swelling model in both the restrained and unrestrained cases. Conversely, creep is harmful in the thermal camber case because it causes more free bow. The substantial free bow deflection from thermal camber, if 100% stress relaxation occurred, would seriously affect the refueling operation. Because of these two opposing phenomena, either of which may be burnup-limiting, it is not possible to make a conservative assumption regarding creep rate. The design analysis must include a parametric study of effects of creep rate. Irradiation enhanced creep is, of course, known and expected, and yet to be measured under irradiation conditions prototypic of FTR.

Burnup Limitations

The restrained duct, if it were to bow inward into contact with an adjacent duct during operation, would exert a lateral force on the adjacent duct. If bridging or an intermittent slipping were to occur between contacting ducts, sudden reactivity changes might become possible. Further study is required to determine if this is a problem. Therefore, duct contact from bowing is tentatively defined as a burnup limitation. Conceptually, a nominal 0.100 in. clearance between ducts provides the limitation shown in Figure 4.9. This assumes the adjacent ducts remain straight. This provides only a first approximation but it is considered reasonable because Row 4 ducts will swell much less than those of Row 5. Without considering a specific fuel shuffling scheme, no other assumption appears as conservative.

FIGURE 4.9. Maximum Restrained Duct Deflection from Differential Swelling

If creep occurs to the extent of 100% stress relaxation during operation, the inward deflection from differential swelling will be greater than if no creep occurs. When the bent duct contacts the adjacent duct, the deflection curve will change shape rather than exert a force on the adjacent duct. Therefore, there is no corresponding duct touching limitation because of reactivity considerations for the case for which 100% stress relaxation occurs. During refueling it will be necessary to remove a deformed duct without damaging an adjacent duct (unless batch loading of the core is used). There is 0.100 in. allowed between ducts in the conceptual design, and releasing the radial restraint mechanism provides another 0.100 in. clearance at the duct top. Therefore, a free bow deflection of 0.200 in. has been assumed as a burnup limitation. This limitation does not consider the experimental work which will be done to determine the interaction of the

core components both during operation and refueling. The assumed limitation is listed in Table 4.2 and indicated on Figure 4.10 which shows the duct top deflection during refueling.

FIGURE 4.10. Maximum Unrestrained Duct Deflection Versus Burnup

Core Packing

For the purposes of this analysis core packing is defined as the condition existing when flow ducts have swelled into contact with their adjacent ducts. Because of the potential for duct bridging with subsequent sudden release (and accompanying reactivity increase), and the refueling problem with contacting ducts, this is identified as a burnup limitation. The effect of creep on changes in duct cross section has been found to be small by comparison with dimensional changes from swelling.

Calculations

The swelling model for type 304 solution treated stainless steel was used to predict the dimensional increase across the flats of the duct. The temperature and fluence environment used in the duct bending analysis was used for the initial calculations of swelling. Because of hot channel factor uncertainties, subsequent check calculations were based upon a maximum duct temperature of 950 °F. The swellings associated with Row 1 and with Row 5 ducts were calculated for the elevation 27 in. above the bottom of the core. This closely corresponds to the plane of maximum swelling. The calculated dimensional change and burnup for Row 5 was based upon an average dimensional increase or fluence for the hottest side and the cooler side of the duct.

The plot of dimensional change versus fuel burnup for Row 5 almost coincides with the corresponding plot for Row 1. Therefore, Figure 4.11 shows only the dimensional change for Row 1. This indicates core packing would not occur, based upon the mean value prediction. Core packing is predicted to occur, however, based on the upper 95% confidence limit of the mean at about 75,000 MWd/MTM. For the check case which considers about a 70 °F increase in maximum duct temperature, core packing is predicted at about 62,000 MWd/MTM based upon the 95% upper confidence limit on the mean. These burnup limitations are shown in Table 4.3.

Duct Packing

Duct packing is defined as the condition existing when the pin bundle swells such that its swollen dimension across the outside of the bundle equals the swollen dimension across the inside of the duct. The fuel pin bundle will increase in nominal dimension across the flats with increasing swelling. Similarly, the corresponding flow duct dimension would increase

FIGURE 4.22. *Duct Dimensional Change*

with burnup. The pin bundle cladding is hotter than the duct. For the temperatures involved, the swelling rate increases with increasing temperature. Therefore, a potential exists for the pin bundle to swell faster than the duct. This is defined as a burnup limitation because of the high radial restraint forces imposed on the bundle by the duct when packing occurs, which is predicted to lead to failure of the fuel pins.

Dimensional Changes

Metal swelling increases with both fluence and temperature (within the temperature range of concern). The combination of fluence and temperature which produces the maximum swelling occurs about 27 in. above the bottom of the core. Therefore,

analysis of duct packing was made at that elevation for a wire-wrapped bundle. Figure 4.12 shows the calculated nominal dimensional change across the flats of the pin bundle with burnup for the three swelling model extrapolations. This dimensional increase is based on an assumed bundle with 75% average pins and 25% peak pins. The useful limit of the Arrhenius equation model as shown by the solid line is about 28,000 MWd/MTM. At all higher burnup values the dimensional increase is shown as extrapolations by the three models being considered. A comparison of Figure 4.12 with Figure 4.11 indicates that duct packing is predicted at 30,000 and 33,000 MWd/MTM for "enhanced swelling" extrapolation and "conversion-to-solution-treated" extrapolation, respectively. It is worth noting these burnup limitations are extremely sensitive to the wire diameter size. If the spacer wire diameter were reduced 1 1/2 mils while retaining the present duct dimension, this burnup limitation would increase to about 55,000 MWd. The Arrhenius equation extrapolation predicts that duct packing will not occur. These burnup limitations are listed in Table 4.3.

A similar analysis of duct packing for a grid spaced system predicts that duct packing will occur at approximately 55,000 MWd/MTM for the two higher extrapolation models. However, there would be less space between swollen pins than in the wire wrapped case. The Arrhenius equation predicts that duct packing will not occur. These burnup limitations are also shown in Table 4.3.

FIGURE 4.12. Dimensional Change in Pin Bundle

5.0 CRITICAL MASS PHYSICS

CRITICAL PARAMETERS FOR 0 TO 7% PuO_2 - U(NAT)O_2 RODS IN WATER

C. L. Brown and D. R. Oden

The HAMMER system⁽¹⁾ has been found to calculate with reasonable accuracy material bucklings measured experimentally for 1.5 wt%, 2.0 wt%, 4.0 wt%, and 6.6 wt% $\text{PuO}_2/\text{PuO}_2\text{-UO}_2$ rods in water.⁽²⁻⁴⁾ For 36 experimental lattices, the average of the calculated $k_{\text{eff},s}$ is 1.015, and the range, 0.990 to 1.034 (Table 5.1). In all instances, calculated $k_{\text{eff},s}$ at the points of maximum buckling and minimum critical mass are equal to or greater than unity, which is conservative from the viewpoint of criticality safety.

Based on the above correlation, the HAMMER system was used to calculate minimum critical parameters for 0.3, 0.5, and 0.7 in. diameter rods in the enrichment range 0 to 7 wt% $\text{PuO}_2/\text{PuO}_2\text{-U(Nat)O}_2$ (Table 5.2). Minimum values, in Table 5.2 and Figures 5.1 through 5.6, were obtained by plotting each critical parameter for each rod diameter and enrichment as a function of water-to-fuel volume ratio. This information is needed to help establish reasonable criticality safety limits for handling, storage, and shipping of $\text{PuO}_2\text{-U(Nat)O}_2$ fuel rods and assemblies to be used in the Plutonium Recycle Program.

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TABLE 5.1. *Comparison of HAMMER System Calculations with Experimental Measurements*

Water-to-Fuel Volume Ratio	Measured Buckling, m ⁻²	Calculated Buckling, m ⁻²	Calculated k _{eff}
1.5 wt% PuO ₂ -UO ₂ Rods; 0.372 in. diam; 0.027 in. Zr Clad; 7.83 wt% ²⁴⁰ Pu ⁽²⁾			
1.123	48.0	48.28	1.001
1.582	65.1	68.19	1.010
2.730	78.5	85.59	1.021
3.813	74.9	82.31	1.021
5.167	60.9	68.43	1.022
5.605	55.2	62.51	1.021
2.0 wt% PuO ₂ -UO ₂ Rods; 0.508 in. diam; 0.030 in. Zr Clad; 7.65 wt% ²⁴⁰ Pu ⁽²⁾			
1.434	88.0	87.59	0.999
2.445	102.9	111.10	1.023
3.461	103.4	113.68	1.028
4.332	97.8	107.79	1.027
6.195	76.8	83.99	1.020
6.958	70.0	73.20	1.009
9.835	32.7	31.09	0.995
16.54 wt% ²⁴⁰ Pu			
2.445	88.4	94.56	1.018
3.461	89.0	97.27	1.023
4.332	81.9	91.52	1.027
6.195	61.8	68.32	1.018
6.958	55.5	57.79	1.006
23.50 wt% ²⁴⁰ Pu			
1.434	62.4	59.35	0.990
2.445	78.9	82.17	1.010
3.461	77.1	84.60	1.021
4.332	72.2	78.86	1.019
6.195	53.6	55.92	1.007
6.958	44.0	45.47	1.004
4.0 wt% PuO ₂ -UO ₂ Rods; 0.498 in. diam; 0.034 in. Zr Clad; 18.10 wt% ²⁴⁰ Pu ⁽³⁾			
1.931	94.7	94.56	1.000
2.566	107.9	111.17	1.009
3.624	108.7	121.40	1.034
4.533	107.9	120.81	1.034
7.271	88.4	96.34	1.021
10.874	59.5	61.14 *	1.004
11.580	41.1	42.97	1.005
6.6 wt% PuO ₂ -UO ₂ Rods; 0.337 in. diam; 0.023 in. Zr Clad; 8.57 wt% ²⁴⁰ Pu ⁽⁴⁾			
1.681	108.8	106.0	0.992
2.165	121.5(a)	126.7	1.012
4.699	159.6	169.2	1.022
5.673	159.3	170.8	1.026
10.754	128.4	140.5	1.029

a. Average of 2 Measurements

TABLE 5.2. *Minimum Critical Parameters for 0 to 7 wt% PuO₂-UO₂ Rods in Water*
(5% ²⁴⁰Pu; natural uranium; 95% theoretical density)

Enrichment, wt% PuO ₂	Mass, kg Oxide	Volume, ℓ	No. Rods ^(a)	∞ Cylinder Diameter, in.	∞ Slab Thickness, in.	Areal Density, ² kg Oxide/ft
<u>0.3 in. Diameter Rods</u>						
0.7	278.0	78.0	742	15.90	8.60	50.00
1.0	164.0	66.0	492	13.89	7.28	36.80
2.0	69.2	35.0	243	11.05	5.45	21.00
3.0	45.8	26.8	170	10.00	4.76	15.80
4.0	35.7	22.9	136	9.42	4.40	13.20
5.0	29.9	20.5	119	9.08	4.17	11.60
7.0	24.0	18.1	98	8.64	3.89	9.95
<u>0.5 in. Diameter Rods</u>						
0.7	300.0	98.5	290.0	16.01	8.64	55.2
1.0	185.0	66.5	200.0	13.90	7.31	42.1
2.0	85.5	37.0	108.0	11.28	5.59	26.3
3.0	60.8	29.0	81.5	10.32	4.95	21.0
4.0	49.8	25.3	68.8	9.78	4.59	18.4
5.0	43.8	22.9	61.8	9.43	4.39	16.9
7.0	37.1	20.4	54.2	9.02	4.12	15.1
<u>0.7 in. Diameter Rods</u>						
0.7	337.0	101.0	165.0	16.20	8.77	62.0
1.0	213.0	70.0	117.0	14.20	7.47	48.8
2.0	107.0	40.3	68.2	11.62	5.80	32.5
3.0	80.0	32.1	53.8	10.69	5.19	27.0
4.0	67.3	28.1	46.8	10.18	4.84	24.0
5.0	60.2	25.7	42.6	9.85	4.66	22.1
7.0	52.2	23.0	38.0	9.47	4.38	20.2

a. Rod infinite length; and in cylindrical geometry.

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FIGURE 5.2. Minimum Critical Mass as a Function of Fuel Enrichment

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FIGURE 5.2. Minimum Critical Volume as a Function of Fuel Enrichment

Neg 0694258-5

FIGURE 5.3. Minimum Critical Cylinder Diameter as a Function of Fuel Enrichment

Neg 0694258-4

FIGURE 5.4. Minimum Critical Number of Rods in Cylindrical Geometry as a Function of Fuel Enrichment

Neg 0694258-6

FIGURE 5.5. Minimum Critical Slab Thickness as a Function of Fuel Enrichment

Neg 069425817

FIGURE 5.6. Minimum Critical Areal Density as a Function of Fuel Enrichment

REFLECTOR SAVINGS FOR URANYL NITRATE OF LOW ENRICHMENT

C. A. Rogers, R. C. Lloyd, and S. R. Bierman

Calculations of reflector savings have been completed for spherical lattices of uranium rods in water surrounded by a uranyl nitrate reflector. Five computer programs were used to obtain the reported analytical results: HFN,⁽¹⁾ DTF,⁽²⁾ THERMØS,⁽³⁾ GAMTEC,⁽⁴⁾ EGGNIT.⁽⁵⁾ Each calculation required at least two of these programs: one to generate the cross sections and another to solve the diffusion equations. For the uranium rod-water lattices, EGGNIT, a modified version of GAMTEC, was used because it performs a better resonance integral calculation for lattices. Since EGGNIT uses only one thermal group, the thermal parameters were replaced by parameters from THERMØS, which uses 30 thermal groups. HFN and DTF were used to calculate critical dimensions.

The most reactive form of slightly enriched uranium is a lattice of uranium rods in water with the proper diameter and spacing. Table 5.3 gives a list of the parameters of the cores used in the reflector savings calculations. These configurations are very near to the most reactive configurations. Using each of the lattices given in Table 5.3, a set of curves that was generated using HFN gave the savings for reflectors of thicknesses of up to 50 cm and of uranium concentrations of up to 1000 g U/liter. In each calculation, the same ²³⁵U enrichment is used in both the core and reflector. Using 3.0 and 5.0 wt% ²³⁵U enriched uranyl nitrate, several DTF checkpoints were computed, and agreement with HFN is very good. With 20.3 cm (8 in.) reflectors, the two codes agree with each other to less than 0.3 cm for all concentrations of uranium in the reflectors.

Reflector savings of a sphere is defined as the difference between the radius of an unreflected critical sphere and the radius of a critical sphere of the same material surrounded by a reflector of uniform thickness.

TABLE 5.3. Core Configurations Used in Calculations

^{235}U Enrichment, wt %	Rod Diameter, cm	Center-to-Center Spacing, cm	Buckling, cm^{-2}	k_m
2.0	1.270	2.238	0.0110	1.29
3.0	1.016	2.144	0.0152	1.41
4.0	0.762	1.706	0.0182	1.48
5.0	0.762	1.706	0.0200	1.52

Figures 5.7 through 5.10 show the reflector savings as a function of reflector thickness for various enrichments of ^{235}U in the reflector. These curves depend greatly on the uranium concentration and enrichment. Ten centimeters of water has a reflector savings which is more than 95% of its maximum value. When enriched uranyl nitrate is added to the water, a greater thickness of reflector is required for the reflector savings to reach 95% of its maximum value. When k_∞ of the reflector is greater than unity, there is no longer an asymptotic approach to a constant value. Instead, an inflection point is reached on the reflector savings curve beyond which the slope of the curve increases with increasing thickness. At one particular thickness, the core is no longer needed and the reflector becomes critical by itself.

The similarity of curves of k_∞ plotted against reflector savings (with reflector thickness held constant) for different enrichments can be readily seen in Figure 5.11. A single curve could be used to approximate all of these curves to a fair degree of accuracy. Curves of lower enrichment lie slightly above those of higher enrichment, primarily as a result of differences in moderation.

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FIGURE 5.7. Reflector Savings for a 2.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere

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FIGURE 5.8. Reflector Savings for a 3.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere

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FIGURE 5.9. Reflector Savings for a 4.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere

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FIGURE 5.10. Reflector Savings for a 5.0 wt% ^{235}U Enriched Uranyl Nitrate Reflected Sphere

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FIGURE 5.11. Reflector Savings as a Function of Reflector K_{∞} for a 50.80 cm Thick Uranyl Nitrate Reflector Surrounding a Spherical Uranium Lattice

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