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REPORT

**PHYSICS RESEARCH QUARTERLY REPORT
JANUARY, FEBRUARY, MARCH, 1966**

By
THE STAFFS
of
REACTOR PHYSICS,
EXPERIMENTAL PHYSICS RESEARCH
and
CRITICAL MASS PHYSICS

APRIL 15, 1966

BATTELLE-NORTHWEST

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The Staffs of
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Experimental Physics Research
and
Critical Mass Physics

F. G. Dawson, Jr	Manager Reactor Physics
C. W. Lindenmeier	Manager Theoretical Physics
L. C. Schmid	Manager Light Moderator Reactor Physics
P. L. Hofmann	Manager Engineering Physics
H. L. Henry	Manager Experimental Reactors
R. E. Heineman	Manager High Temperature Reactor Physics
D. D. Lanning	Manager Heavy Moderator Reactor Physics
B. R. Leonard, Jr.	Manager Experimental Physics Research
E. D. Clayton	Manager Critical Mass Physics

Physics and Instruments Department

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MACHINE-ANALYSIS ABSTRACT--MAXIMUM LIKELIHOOD NONLINEAR VECTOR FIELDS--
 BATTELLE-NORTHWEST PROGRAM LIKELY--B. H. Duane, Theoretical Physics Unit,
 Nuclear Analysis Section

BNW Program LIKELY constructs maximum-likelihood multidimensional vector fields satisfying any statistical-or-deterministic linear-or-nonlinear vector field theory cast as a variational extremal subject to correlated-statistic or implicit-anholonomic constraints.

Analysis Logic

The physical theory (to which the vector measurements $\underline{y}_m \pm U_m^{1/2}$ at phase-space coordinates $\underline{x}_m$ are to be fitted by variation of parameters $\underline{P}$) is cast as a set of implicit (or explicit) equations:

Theory: $q(\underline{y}, \underline{x}, \underline{P}) = 0$.

Implicit-theory flexibility plays a key role here: both by direct acceptance of any physical conservation or balance law with no need for intricate (or unfeasible) transformation; and also by free acceptance of implicit anholonomic constraints, implied merely by providing fewer field equations q than there are field variables $\underline{y}$.

The statistically weighted and projectively constrained construction of the mean square misfit Q forms the foundation of the analysis logic:

Mean square misfit:

$$Q = \sum_m q_m \cdot \left[\sum_i \sum_j (\partial q_m / \partial y_m^i) U_m^{ij} (\partial q_m / \partial y_m^j) \right]^{-1} \cdot q_m.$$

This is the quantity to be minimized in achieving a best fit, and the optimality of its construction merits critical scrutiny in the context of each specific application.

The maximum-likelihood theory is constructed by solving the minimal-

misfit conditions:

Best theory: $\partial Q / \partial \underline{P} = 0$ and $\partial^2 Q / \partial \underline{P} \partial \underline{P}$ positive definite.

These nonlinear partial-differential equations are solved by iterating second-order analytic continuation to solid convergence:

Iterative algorithm:

$$\underline{P} = \underline{P} - (\partial Q / \partial \underline{P}) \cdot (\partial^2 Q / \partial \underline{P} \partial \underline{P})^{-1}.$$

Source

The source deck consists of a 2000-card 11-link overlay chain, formulated in the machine-independent dialect subset of IBM-Fortran-IV-12 and UNIVAC-Fortran-IV-CSC, and designed for both modular growth versatility and freedom from obsolescence.

Input

Input data, fed in either on free-format cards or via flexible input linkage, include: estimates and bounds for the theory parameters $\underline{P}$ (25-parameter limit); vector measurements $\underline{y}_m$ (100-vector limit, each with up to 5 components); corresponding phase-space coordinates $\underline{x}_m$ (if any); correlated measurement-uncertainty tensors U_m (100-tensor limit, each with up to 5 x 5 components); range and resolution of graphical output (up to 25 phase-space cross plots of

5-vector fields); and a theory subroutine for calculating the vector-misfit fields

$$[q(\underline{y}, \underline{x}, \underline{p}), \partial q(\underline{y}, \underline{x}, \underline{p})/\partial \underline{y}]$$

and their first and second parametric gradients

$$[\partial q/\partial \underline{p}, \partial^2 q/\partial \underline{y} \partial \underline{p}],$$

$$[\partial^2 q/\partial \underline{p} \partial \underline{p}, \partial^3 q/\partial \underline{y} \partial \underline{p} \partial \underline{p}] \quad .$$

Numerics

The computational numerics have been quite fully optimized with balanced emphasis upon reliability, versatility, and speed. Numeric techniques typifying the state of the art include:

- Full use of numeric rounding, to minimize accrual of truncation bias.
- No iterative peripheral transmission, to enhance both reliability and speed.
- Self-evaluative sequence of ill-conditioned-matrix inversion logic, based upon use of two entirely different matrix-inversion methods. The iterative loops use a biggest-residual-pivot one-sided single-sweep matrix-inversion method designed for both reliability and speed. The saddle-decision logic uses a more versatile matrix-inversion method designed for both reliability and firm saddle detection, involving biggest-residual-blade two-sided single-sweep saddle-preserving transformation to principal blades, followed by blade inversion and inverse dual transformation. Output includes an evaluative cross-check assessing the adequacy of both matrix-inversion methods.
- Two-stage recovery logic for failure of second-order analytic continua-

tion, branching first to cubic-expansion refinement of amplitude of last change, and then to reversing-and-halving of last change.

- Computation of Student and Fisher angular probability by angular Fourier transform.

Output

Output-processing versatility includes an exhaustive nonlinear statistical analysis of the residual misfit between measurements and theory, based upon nonlinear generalization of the small-sample angular-probability decision theory of Student and Fisher, and culminating with probabilistically-uncoupled assessments of both the intrinsic quality of the measurements and the intrinsic relevance of the theory, as well as the adequacy of the agreement. Output logic is in harmony with remote-transmission needs, with numeric tabulation normally limited to a few pages having high information density, and with all bulky detail normally compressed to graphical form.

Status

The IBM-7090 compilation of the source deck appears to be free of all flaws and has been placed on the computer library tape for exploratory evaluative application to reactor physics work. Working documentation of the theory and use of the IBM version is available upon request from the Theoretical Physics Unit, for use within BMI and AEC installations. The program is expected to be made available for unrestricted external distribution through the Argonne Code Center after completion of more comprehensive formal documentation.

SENSITIVITY OF THERMAL REACTOR PARAMETERS TO SCATTERING MODEL—J. R. Worden,
W. L. Purcell, and R. C. Liikala, Light Moderator Reactor Physics Unit,
Reactor Lattice Physics Section

*Abstract of a paper published in the Proceedings of Conference on
Neutron Cross Section Technology held at Washington, D.C., March
22-24, 1966.*

Comparative studies have been carried out to determine the effect of the choice of scattering model for the common moderating materials, H_2O , D_2O , and graphite, on calculated reactor parameters. Calculations of the thermal reproduction factor, $\bar{\eta}$, and the thermal utilization, f , have been made for uranium and/or plutonium for which measured values of parameters are available. The calculations were performed using the thermalization code, THERMOS,⁽¹⁾ in conjunction with kernels generated using various scattering models. Preliminary results for this study have been reported previously⁽²⁾ and this paper includes the results of and is an extension of the previous work.

The systems selected for study were chosen because measured values of the parameters, $\bar{\eta}$ and f , are available, and the neutron absorption in the systems is large because of either a large volume or high concentration of fuel in the cell. The criterion of

significant neutron absorption is necessary to assure a decent check on the scattering model, since all the models will predict the correct neutron spectrum in the limit of zero absorption.

For graphite moderator, the results given in the previous progress report were used for the paper to illustrate the effects due to the various scattering models for graphite (i.e., Parks kernel and gas model) and are shown in Table I.

The experiments using graphite moderator were also performed with H_2O coolant. Because of the excellent agreement between calculated and measured parameters for the dry cases, these same lattices were selected for study of scattering by light water.

Scattering kernels were generated from the Nelkin scattering model for protons bound in H_2O ⁽⁴⁾ and also from the Brown-St. John modified gas model.⁽⁵⁾ Initially, anisotropic scattering was neglected. The results of

TABLE I. Values of $\bar{\eta}$ and f for Uranium Fueled Graphite Lattices, Air Cooled

	2.5 in. Solid Rod		Tube-in-Tube	
	$\bar{\eta}$	f	$\bar{\eta}$	f
Experiment ⁽³⁾	1.289 ± 0.013	0.941 ± 0.002	1.304 ± 0.014	0.923 ± 0.003
Calculated:				
Parks	1.290	0.941	1.297	0.922
Gas (mass 12)	1.299	0.930	1.307	0.909

TABLE II. Values of $\bar{n}$ and f for Uranium Fueled Graphite Lattices, H_2O Cooled

	2.5 in. Solid Rod		Tube-in-Tube	
	$\bar{n}$	f	$\bar{n}$	f
Experiment ⁽³⁾	1.297 $\pm$ 0.015	0.871 $\pm$ 0.004	1.308 $\pm$ 0.015	0.831 $\pm$ 0.005
Calculated:				
Brown-St. John Gas	1.302	0.857	1.315	0.807
Nelkin ^a	1.300	0.858	1.316	0.807
Nelkin ^b	1.300	0.868	1.312	0.818

a. Neglecting anisotropy

b. Including an approximate correction for anisotropy

these calculations where anisotropic scattering was neglected were given in the previous progress report.⁽²⁾ A discrepancy between calculation and experiment of the order of 1 to 3% resulted assuming isotropic scattering. Because of the flux gradients in these lattices, it is necessary to make at least an approximate correction for anisotropic scattering in H_2O . This was accomplished by substituting the transport cross section for the scattering cross section in generating the scattering kernel, as suggested by Honeck.⁽⁶⁾ As can be seen in Table II, the calculated parameters for the solid rod agree with experiment if one includes a correction for anisotropic scattering. For the tube-in-tube geometry, where the effects due to anisotropy are even larger, the agreement between calculation and measurement is not good, but the error in f is reduced by a factor of 2 by including the approximate correction for anisotropic scattering.

For D_2O moderator, comparisons were made between calculations using the Brown-St. John⁽⁷⁾ modified gas model and an adaption of the Nelkin model to D_2O

by Honeck.⁽⁸⁾ The calculations were made for a lattice consisting of fuel elements in the form of 19 rod- UO_2 clusters containing D_2O coolant.⁽⁹⁾ The results of these calculations were given in the previous progress report. For this paper, the correction for anisotropic scattering were included in the calculations. The results are given in Table III.

TABLE III. Values of $\bar{n}$ and f for 8 in. D_2O Lattices Fueled with Natural UO_2 , 19-Rod Clusters

	$\bar{n}$	f
Experiment ⁽⁹⁾	1.322 $\pm$ 0.021	0.900 $\pm$ 0.005
Calculated:		
Brown-St. John	1.313	0.931
Nelkin ^a	1.309	0.931
Nelkin ^b	1.308	0.931

a. Neglecting anisotropy

b. Including approximation for anisotropy

As is seen in Table III, both models predict the same value of thermal utilization but the value, when compared with experiment, is 3 1/2% too large. Furthermore, making a correction for anisotropic scattering, does not result in a change in the calculated f .

The disagreement between calculation and experiment could perhaps be caused by errors introduced in constructing a cylindrical model of the cluster suitable for a 1-dimensional flux calculation. The adequacy of this model was tested, however, by comparison of the measured thermal utilization for a 19-rod cluster of UO_2 -0.9 wt% PuO_2 fuel in graphite with a calculation using the same cylindricized model of the cluster. The measured value of f for this lattice⁽¹⁰⁾ is $f = 0.968 \pm 0.002$ and the calculated values are $f = 0.965$ using the Parks kernel and $f = 0.959$ using the gas model. This tends to confirm the adequacy of the geometrical model of the cluster since it is assumed that scattering in graphite is adequately represented by the Parks model.

It is concluded that the Nelkin scattering model is adequate for H_2O . It is assumed that the discrepancy in the case of the tube-in-tube fuel geometry is due to improper treatment of anisotropic scattering in the calculational model. Further studies are planned that will determine the adequacy of the first moment transfers as calculated from the Nelkin scattering model.

It is tentatively concluded, from this limited study, that the models considered for scattering in D_2O need improvement. The statistics upon which this conclusion is reached are very poor, however, since only a single case was studied. Additional studies are planned to further investigate scattering in D_2O by the calculational methods described.

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COMPUTER ANALYSIS OF REACTOR NOISE DATA—L. C. Schmid and G. D. Seybold, Light
Moderator Reactor Physics Unit, Reactor Lattice Physics Section

Introduction

Computers have been utilized for the analysis of data from reactor noise experiments. The analysis is accomplished in two steps. First the amplitude data of the frequency spectrum of reactor noise is put in a form amenable to comparison to the theoretical function. In the second step, parameters are obtained from the experimental data by fitting the theoretical functions to the data using improved least squares fitting techniques.

Background

Noise analysis is a method whereby random variables are used to describe physical phenomena of a system. The random variable may be an externally applied disturbance or an internal fluctuation of the system. However to be useful the random variable must be describable by probability theory. The analysis is accomplished by determining what effect, as a function of frequency, the random variable has on the gain or power spectrum at the output of the system. As a result of the analysis the frequency response or transfer function of the system is obtained. In zero-power reactors the transfer function measurement is a means of evaluating the ratio β/λ of the effective neutron live time and delayed neutron fraction of a reactor loading. At higher power levels the transfer function measurement presents information on reactor stability as well as the ratio β/λ .

The frequency response of a reactor system to noise is the quotient (as a

function of frequency) of the amplitude of the reactor-power fluctuation and the amplitude of the perturbing function which causes the power fluctuation. The power-spectral density of a random noise having a poisson distribution is equal to a constant (power output in each unit bandwidth of the spectrum would be equal to a constant) while all other power spectral densities are a function of frequency. Such a random process, having a constant power spectral density is referred to as a "white noise." Thus if a system is excited by white noise either internally or externally the perturbing amplitudes are constant as a function of frequency and it is only necessary to analyze the random power fluctuations of the output to obtain the frequency dependence of the transfer function of the system.

In a reactor noise experiment an inherent fluctuation of the neutron population about some average value is used as the perturbing function to measure the transfer function of the reactor loading. The fluctuations occur because the delayed neutrons produced because of fission do not appear immediately and the prompt neutrons fail to sustain the neutron population by the fraction β . Since the statistical fluctuations of the neutron population have the characteristics of a poisson distribution it is only necessary to measure the reactor power fluctuations as a function of frequency to determine the transfer function.

The frequency dependence of the power fluctuations are determined using

a frequency analyzer which has a narrow band-pass filter which filters out all frequencies except those near the particular frequency at which the power fluctuation is being evaluated. Since the transfer function characterizes the frequency response of the system, it is related to the kinetic equations

$$\frac{dn}{dt} = \frac{\delta k}{\ell^*} n + \sum_{i=1}^j \lambda_i C_i \quad (1)$$

and

$$\frac{dC_i}{dt} = -\lambda_i C_i + \frac{\beta_i}{\ell^*} n \quad (2)$$

where C_i and λ_i are the delayed neutron precursor concentrations and decay constants. Equations (1) and (2) are a set of linear differential equations with constant coefficients when δk is constant. The use of a transfer function (since it is a measure of the frequency response) implies a $\delta k = f(t)$ and therefore the set of linear differential equations no longer have constant coefficients. However, it turns out that if sufficiently small excursions are assumed, the equations can be approximated by a set with constant coefficients which simplify the solution of the equations.

First Equations (1) and (2) can be combined into

$$\frac{dn}{dt} = \frac{\delta k}{\ell^*} n - \sum \frac{dC_i}{dt} \quad (3)$$

and if

$n = n_0 + \delta n$ and $C_i = C_{oi} + \delta C_i$ is assumed, then

$$\begin{aligned} \frac{dn}{dt} &= \frac{d\delta n}{dt} = \frac{\delta k}{\ell^*} n_0 + \frac{\delta k \delta n}{\ell^*} - \sum \frac{d\delta C_i}{dt} \\ &= \frac{\delta k}{\ell^*} n_0 - \sum \frac{d\delta C_i}{dt} \end{aligned} \quad (4)$$

and

$$\begin{aligned} \frac{dC_i}{dt} &= \frac{d\delta C_i}{dt} = \frac{\beta_i}{\ell^*} (n_0 + \delta n) \\ &\quad - \lambda_i (C_{oi} + \delta C_i) \\ &= \frac{\beta_i}{\ell^*} \delta n - \lambda_i \delta C_i \end{aligned} \quad (5)$$

since in steady state

$$\frac{dC_{oi}}{dt} = 0 = \frac{\beta_i}{\ell^*} n_0 - \lambda_i C_{oi} \quad (6)$$

Reducing Equations (4) and (5) to LaPlace-transform operational form gives

$$S\delta n(S) = \frac{n_0}{\ell^*} - \delta k(S) - S\sum \delta C_i(S) \quad (7)$$

and

$$S\delta C(S) = \frac{\beta_i}{\ell^*} \delta n(S) - \lambda_i \delta C_i(S) \quad (8)$$

Combining Equations (7) and (8) the transfer function,

$$TF = \frac{\delta n(S)}{\delta k(S)} = \frac{n_0}{\ell^*} \frac{1}{S \left[1 + \sum \frac{\beta_i}{\ell^* (S + \lambda_i)} \right]} \quad (9)$$

or

$$TF = \frac{\delta n(S)}{\delta k(S)} = \frac{n_0}{\ell^*} \frac{\prod_{i=1}^j (S + \lambda_i)}{S \prod_{i=1}^j (S + \alpha_i)} \quad (10)$$

where α_i are roots of j^{th} degree equation formed from expanding the denominator in Equation (9) and setting it equal to zero.

For the case of the approximation of one delayed group of neutrons the equivalent of Equation (9) is

$$\begin{aligned}
\frac{\delta n(S)}{\delta k(S)} &= \frac{n_o}{\ell^*} \frac{1}{S \left[1 + \frac{\beta}{\ell^* (S + \bar{\lambda})} \right]} \\
&= \frac{n_o}{S \ell^*} \frac{\ell^* (S + \bar{\lambda})}{\left[\ell^* (S + \bar{\lambda}) + \beta \right]} \\
&= \frac{n_o}{S \ell^*} \frac{(S + \bar{\lambda})}{S + \bar{\lambda} + \beta / \ell^*} \\
&\approx \frac{n_o}{S \ell^*} \frac{(S + \bar{\lambda})}{(S + \beta / \ell^*)} \quad (11)
\end{aligned}$$

The quantity which is usually compared to the experimental measurement is the power spectral density. This quantity is equal to the square of the absolute value of the transfer function

$$\begin{aligned}
|n(\omega)|^2 &= |TF|^2 \\
&= \left(\frac{n_o}{\ell^* \omega} \right)^2 \frac{\prod_{i=1}^j (\omega^2 + \lambda_i^2)}{\prod_{i=1}^j (\omega^2 + \alpha_i^2)} + N \\
&\approx \left(\frac{n_o}{\ell^* \omega} \right)^2 \frac{(\omega^2 + \bar{\lambda}^2)}{(\omega^2 + \alpha^2)} + N \quad (12)
\end{aligned}$$

$$S = j\omega = j2\pi f$$

$$\alpha = \bar{\beta} / \ell^* + \bar{\lambda} \approx \frac{\bar{\beta}}{\ell^*}$$

A term N which is independent of the frequency has been added to the right hand side of Equation (12). This term represents a quantity of white noise which is present in the experimental data because of the statistical nature in which the ion chamber and electronic equipment collects the information of the neutron fluctuations. This additional noise is white noise and hence

its amplitude is independent of frequency. Also, it is uncorrelated with the reactor noise and so it adds to the absolute value of the square of the transfer function, rather than the transfer function itself.

In order to acquire the reactor-noise data an ionization chamber is used to provide a current which is proportional to the neutron flux density. The current from the chamber is then suitably biased and amplified so only the fluctuating component of the signal is obtained. The fluctuating component is recorded with an FM magnetic-tape recorder for frequency analysis at a later time or passed through a filter system for direct analysis.

In the frequency analysis of the data the playback unit of the magnetic tape recorder is used to produce the signal and pass it through a DC amplifier to a band pass filter which is tuned to a frequency of interest. The output of the filter contains information from only a narrow band of frequency. This signal is amplified, rectified, and applied to an integrating circuit which is allowed to compute for a given time T . At the end of the fixed length of time the integrated amplitude V is recorded along with the frequency f and length of analysis time T . This process is repeated at other frequencies until the complete frequency range of interest is covered. In some cases multifilters are used to analyze more than one frequency at a time. Additional frequencies are analyzed by changing the speed of the recorder or the filter frequency. The band width of the filter is normally proportional to frequency and the amplitude output

from the integrating circuit must be divided by the square root of the frequency to correct for this effect. Thus the power output in decibels is

$$|n(f)|^2 = 10 \log \left[\frac{V(f, t)}{T} \right]^2 \frac{1}{f} = |TF|^2 \quad \dots(13)$$

The relative standard deviation of the determination of the power for a relatively narrow band pass filter is

$$\frac{\sigma_P}{P} = \left(\frac{1}{\Delta f T} \right)^{1/2} \quad (14)$$

if $\Delta f T \gg 1$ and where

$$P = \left(\frac{V(\Delta f, T)}{T} \right)^2 \frac{1}{f} .$$

Normally Equation (13) would not be a correct way for measuring the power spectrum of a general type of signal. The correct way for any signal is to determine the average power contained in a sample of the signal during time T . Thus, the signal from the ion chamber should be squared before it is averaged. However, the method of averaging the signal and then squaring as done in Equation (13) is identical to the correct way provided the distribution of the signal is Gaussian. The Gaussian property allows the average of the signal to be proportional to the square root of the average value of the square of the signal. In the measurement of the transfer function the shape is the important quantity and so the proportionality constant need not be determined.

Computer Analysis

Data analysis is accomplished with a Program NOISE written for the UNIVAC 1107 computer. Frequency analyzed data is converted to power data according to Equations (13) and (14). Then the parameters which best fit Equation (12) are obtained utilizing a new least squares fitting technique.

Program NOISE uses a generalized input routine, NAMELIST, to reduce input data requirements. The input data comprise a pseudo frequency f' , amplitude V , time T , and normalization constants $G1$ and $G2$. The parameter $G1$ converts the pseudo frequency f' to the true frequency of the analysis $f = G1 \cdot f'$. This correction is necessary when the playback speed of the recorder is different from the record speed. The parameter $G2$ corrects for possible differences in the gain of the systems which collect the time integrated signal V . The output data includes the input values, the true frequency f , the gain in absolute and in decibel units, and the uncertainty in per cent and in decibels.

For some time the parameters α_i , λ_i , n_0/ℓ^* , and N were obtained from Equation (12) by using the usual least squares fitting techniques which utilize first order differentials. Occasionally there have been problems with obtaining convergence.⁽¹⁻³⁾ Recently a general least squares approach⁽⁴⁾ incorporating nonlinear iterative logic and statistics which involve both first and second order differentials have been used successfully. In the fitting process Equation (12) is used and the data points are weighted inversely as the square of the standard deviation.

In addition to the parameters (and their errors) which result in the best fit to the data, the output consists of data points and the fitted curve plotted in decibels on semilog paper. Under certain conditions the program will stop after the data analysis and before the fitting process begins. In this case punched cards containing the data are furnished for use with Program LEARN.

The problems completed so far involve only the equation for one delayed group of neutrons. Based on experience obtained with the usual fitting techniques,⁽¹⁻³⁾ it is expected that the data which has been obtained in noise experiments is not accurate enough to define parameters for more than one average group of delayed neutrons. However, the general form of Equation (12) also describes frequency dependent amplification and attenuation effects of data acquisition or data analysis equipment and reactor kinetic effects⁽⁵⁾ introduced by reflectors. Therefore data which may have these effects included in addition to delayed neutron effects can be analyzed using the multigroup form of Equation (12).

Both Programs NOISE and the subroutine to be used with Program LEARN are available for use. A report is being written which will describe the two programs in detail.

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SENSITIVITY OF REACTOR MULTIPLICATION VALUES TO CROSS SECTION UNCERTAINTIES FOR
THERMAL SYSTEMS—R. C. Liikala, W. L. Purcell, and J. R. Worden, Light Moderator
Reactor Physics Units, Reactor Lattice Physics Section

*Abstract of a paper published in the Proceedings of the Conference on
Neutron Cross Section Technology held at Washington, D.C., March 22-24.*

This paper shows to what extent the theoretical prediction of reactor multiplications, k_{eff} is influenced by the uncertainty in the nuclear data involved. The study has been restricted to the nuclear data σ_a and $\nu\sigma_f$ for the fissile and fertile isotopes of the reactor systems analyzed.

The critical and/or subcritical experiments selected for analysis cover a wide range of fuel types and lattice spacings. The experiments are all simple, clean lattices, H_2O moderated, and include enriched $\text{UO}_2^{(1,2)}$, $\text{PuAl}^{(3-5)}$, and $\text{PuO}_2^{(6,7)}$ fuel. The fine structure of the lattice spectrum is determined from unit cell calculations. The unit cell calculations, theoretically based upon the Boltzmann transport equation, are performed using the computer codes HRG⁽⁸⁾, THERMOS⁽⁹⁾, and TEMPEST⁽¹⁰⁾. The multigroup constants for the homogenized lattice obtained from the fine structure calculations, along with multigroup constants for H_2O (computed using the HRG and TEMPEST codes), are utilized in computing lattice multiplication values, k_{eff} assuming one dimensional-multigroup diffusion theory using computer code HFN.⁽¹¹⁾ The isotopic multigroup cross sections are arbitrarily varied resulting in perturbed multiplication values. The end result of these calculations are multiplication sensitivity factors for each isotope and each energy group.

The multiplication, k_{eff} for all systems considered is most sensitive to changes in the thermal cross section of the principal fissile nuclide in the fuel. For any of the fission nuclides, the multiplication is more sensitive to a change in the product $\nu\sigma_f$ thermal than to the same percentage change in σ_a thermal. The multiplication sensitivities for changes in non-thermal group cross sections decrease with increased lattice spacing whereas the trend for thermal group cross sections is just the reverse.

The uncertainties in present values of cross section data are estimated and applied to the calculated multiplication sensitivities to arrive at uncertainties in reactor multiplications. Only the basic nuclear data for resonance and thermal neutron energies for U^{235} , U^{238} , Pu^{239} , and Pu^{240} are considered. The resonance data considered are the absorption and fission integrals for infinite dilution. The thermal data considered are the 2200 m/sec values.

The maximum uncertainty in k_{eff} due to the assumed cross section uncertainties is of the order of 1.0% (10 mk). The uncertainty in resonance cross section data leads to consistently larger uncertainties in k_{eff} than does the uncertainty in thermal cross sections. This is a reflection of the fact that major emphasis has been placed on the measurement of thermal cross sections

and the evaluation of consistent cross section sets for thermal values. The maximum uncertainty in k_{eff} due to uncertainties in resonance and thermal cross section data are respectively 1% (10 mk) and 0.5% (5 mk) for the systems analyzed. The uncertainties in k_{eff} due to thermal cross sections are comparable for uranium and plutonium fueled thermal systems.

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LUTETIUM ACTIVATIONS AND SPECTRAL INDICES IN A D_2O -MODERATED REACTOR WITH
Pu-Al 19-ROD CLUSTERS—V. O. Uotinen, Light Moderator Reactor Physics Unit,
Reactor Lattice Physics Section

Introduction

This report describes measurements in the PRCF of the energy and spatial distribution of neutrons in a cell consisting of a Pu-Al 19-rod cluster moderated with D_2O .⁽¹⁾ Calculations of the spatial distribution are also presented.

Spectral indices T and r , which characterize respectively the thermal component and the epithermal component of the neutron spectrum, have been measured. The indices have been deduced from gamma-ray activities of lutetium foils which were activated at ten radial positions in the reactor. Three of the foils were in the central cluster of Pu-Al rods. The remaining foils were in the moderator.

The spectral indices T and r have been compared with the results of similar measurements^(2,6) made in the PRTR. The spatial variations of lutetium activity and thermal flux have been compared with results of calculations performed with the transport theory code THERMOS.⁽³⁾

Epicadmium resonance integrals for Lu^{175} and Lu^{176} , which are used in the determination of the spectral indices,⁽⁴⁾ depend on the thickness of the cadmium covers that are used for the cadmium-covered activation. Curves have been generated which give the epicadmium resonance integrals as a function of cadmium thickness.

Description of Experiment

The PRCF was loaded with 19-rod clusters of 1.8 wt% Pu-Al fuel elements and

moderated with D_2O .⁽¹⁾ Each fuel rod is 88 in. long and 0.504 in. diam and is surrounded by a 0.03 in. thick Zircaloy jacket. The rods in the cluster are separated by a 0.072 in. wire wrap. Nominal values for the plutonium isotopic composition are 6.00 wt% Pu^{240} , 0.45 wt% Pu^{241} , <0.01 wt% Pu^{242} , and 93.55 wt% Pu^{239} . The reactor core was composed of seven 19-rod clusters in an 8 in. hexagonal lattice.

Foils 1.11 cm in diam and 0.12 cm thick which contain 10 mg/cm² lutetium,⁽⁵⁾ were used. Three of the foils were in specially fabricated separable fuel rods of the central fuel cluster, one in each of the three rings of fuel rods. The remaining seven foils were in the moderator held in position by a mechanical device (Figure 1) constructed from aluminum, which slid into the reactor through an access hole in the top template at a radius of 9.23 in. A foil holder on the bottom of this device was swung into a horizontal position, pointing toward the central fuel element. All ten foils were 2 in. below the vertical center line of the fuel elements.

Two irradiations were made, one with the foils enclosed in cadmium dishes 0.020 in. thick, the other with the foils enclosed in aluminum dishes of the same size. The activity of a foil was counted on an integral analyzer biased at 32 keV about 1 hr after the irradiation and again in four or five days. This allowed the Lu^{176m} and Lu^{177} activities to be separated. In addition,

FIGURE 1. Foil Positioner

the activities of a set of foils which had been irradiated in the external thermal column of the Thermal Test Reactor were counted for calibration purposes.

Analysis and Results

Spectral Indices

Spectral indices were deduced from the lutetium activities assuming that the thermal and slowing-down components of the spectrum were joined by means of four different functions. For each ring of the 19-rod PuAl cluster use was made of functions calculated⁽⁶⁾ specifically for these positions. These functions take into account the depletion of neutrons of energies near the Pu²³⁹ resonance at 0.3 eV. For positions in the moderator the "joining function" that was used was that measured for D₂O by Johansson et al.⁽⁷⁾

The parameter r was calculated for each lutetium isotope from the relation⁽⁸⁾

$$\frac{1}{r} = \sqrt{\frac{4T}{\pi T_0}} \left\{ \left[\frac{(CdR - \alpha) RI_{ec}}{g\sigma_0} \right]^{Lu} + \left(\frac{RI_{et}}{g\sigma_0} \right)^{1/v} \right\} \quad (1)$$

The CdR , RI_{ec} , α , g , and σ_0 are, respectively, the cadmium ratio, the epicadmium resonance integral, the ratio of the epithermal to the epicadmium resonance integral, the non- $1/v$ factor,⁽⁹⁾ and the 2200 m/sec absorption cross section. The $T_0 = 293.6$ °K and $(RI_{et}/g\sigma_0)^{1/v}$ is the ratio of the epithermal resonance integral to the product $g\sigma_0$ for a $1/v$ absorber. The quantities used for σ_0 were 13.26 and 2048.9 barns, respectively, for Lu¹⁷⁵ and Lu¹⁷⁶, and the value used for $(RI_{et}/g\sigma_0)^{1/v}$ was 0.969.⁽⁸⁾

The epicadmium and epithermal resonance integrals of Lu¹⁷⁵ were assumed to be 463.4 and 467 barns, respectively. The corresponding values for Lu¹⁷⁶ were assumed to be 1090 and 5507 barns. The values for the epicadmium resonance integrals were obtained by applying a correction to values that have been measured^(4,6) for 0.040 in. cadmium covers. A cadmium cutoff energy of 0.64 eV was used in the calculations. The measured epicadmium resonance integrals of Lu¹⁷⁵ and Lu¹⁷⁶ for 0.040 in. cadmium covers are 459 and 978 barns, respectively. Values for a range of cadmium thicknesses can be obtained from the curves in Figure 2. These curves were generated from cadmium ratios that

had been measured⁽¹⁰⁾ in the Thermal Test Reactor for various thicknesses of cadmium. In generating these curves it was assumed that the epicadmium activity is directly proportional to the epicadmium resonance integral.

FIGURE 2. Epicadmium Resonance Integral of Lu^{175} and Lu^{176} as Functions of Cadmium Thickness. Values of 459 and 978 Barns Were Assumed Valid for 0.040 in. Cadmium Covers

Cadmium ratios and values of T and r obtained at each position are shown in Table I. The errors reported for the

cadmium ratios include counting statistics and deviations in the measured activity due to instability of the counting equipment. The errors reported for the spectral index T are obtained using a formula derived in Reference 8. The errors reported for values of r include contributions from uncertainties in T , g , and the cadmium ratio.

In the moderator the cadmium ratio increases and the value of r decreases as the radius increases from 8.34 to 21.39 cm. This indicates that the ratio of the number of thermal neutrons to epithermal neutrons is increasing. This is to be expected, because the epithermal flux decreases rapidly as one approaches the edge of the core. The radius of the cylindricized core is 28 cm.

Spatial Distribution of Neutrons

The spatial distribution of thermal flux and lutetium activations were calculated with the one-dimensional transport theory code THERMOS.⁽³⁾ The flux and activations were integrated over the energy range from 0 to 0.415 eV. A cylindrical model of the cluster was used in which the fuel was in three concentric rings, separated by zones of

TABLE I. Spectral Indices for a PuAl Cell in the PRCF. The First Three Foils Were in a Central Rod, an Intermediate Rod and an Outer Rod of a 19-Rod Cluster. The Other Foils Were in the D_2O Moderator.

Position Radius, cm	Cadmium Ratio		Spectral Indices	
	Lu^{176m}	Lu^{177}	r	T $^{\circ}\text{K}(a)$
0	1.341 ± 0.003	41.4 ± 2.0	0.0656 ± 0.0006	321 ± 3
1.59	1.350 ± 0.002	47.4 ± 2.1	0.0633 ± 0.0003	328 ± 3
3.09	1.386 ± 0.002	43.7 ± 1.9	0.0588 ± 0.0004	319 ± 3
8.34	1.524 ± 0.009	62.9 ± 2.8	0.0457 ± 0.0006	307 ± 7
10.52	1.556 ± 0.006	65.8 ± 2.9	0.0435 ± 0.0005	302 ± 3
12.69	1.577 ± 0.004	63.4 ± 2.8	0.0422 ± 0.0003	299 ± 2
14.86	1.575 ± 0.007	65.7 ± 2.9	0.0418 ± 0.0005	307 ± 4
17.04	1.613 ± 0.006	65.1 ± 2.9	0.0399 ± 0.0004	298 ± 3
19.21	1.618 ± 0.004	62.6 ± 2.8	0.0391 ± 0.0003	305 ± 2
21.39	1.666 ± 0.009	66.8 ± 2.9	0.0365 ± 0.0006	302 ± 5

(a) The moderator temperature was 297 °K.

cladding and D_2O . Two separate calculations were made using two scattering models for D_2O , the Brown-St. John⁽¹¹⁾ modified gas model and Honeck's⁽¹²⁾ modified Nelkin model. The calculated thermal flux and lutetium activation have to be corrected for the overall buckling of the core before they can be compared with experiment. Correction factors were determined from a calculation⁽¹³⁾ using the diffusion theory code HFN.⁽¹⁴⁾

Measured activities of Lu^{176m} and Lu^{177} are plotted as a function of radius in Figures 3 and 4. Also shown are relative activities obtained from calculations using the two scattering models for D_2O . The results obtained using the gas model agree somewhat better with experiment, and both of the calculated curves overestimate the flux peaking in the moderator.

FIGURE 3. Measured and Calculated Lu^{176m} Activation. The Insert Shows the Position of the Foil Holder Relative to the Fuel Clusters. The Edge of the Cylindricized Core is at 28.2 cm.

The normalization of calculation and experiment at the center of the cell is somewhat arbitrary. A more meaningful comparison can perhaps be obtained by comparing ratios of the average activation in the moderator to average activation in the fuel rods. Such a comparison is shown in Table II. It is evident that there is a large discrepancy also between calculated and measured region average activations.

The THERMOS code was used to obtain lutetium cross sections as a function of radial position. These cross sections were divided out of the measured activities to convert the measured thermal activation to a thermal flux. This allows a comparison between "measured" and calculated region average fluxes (Table III).

FIGURE 4. Measured and Calculated Bare Minus Cadmium-Covered Activation of Lu^{176}

TABLE II. Region Average Thermal Activations in a PuAl D₂O Cell(a)

Region	Lu ¹⁷⁵			Lu ¹⁷⁶		
	THERMOS		Exp.	THERMOS		Exp.
	Nelkin	Gas		Nelkin	Gas	
Center	1.000	1.000	1.000	1.000	1.000	1.000
Intermediate rod	1.04	1.04	1.03	1.04	1.04	1.05
Outer rod	1.19	1.19	1.14	1.16	1.16	1.10
Moderator	1.87	1.81	1.52 ^b	1.70	1.66	1.49 ^b

(a) Normalized to 1.000 at center for each isotope

(b) This is an estimate. No experimental points were obtained near the cluster. The shape of the curve in this region was estimated.

The disagreement between calculated and measured activations and fluxes could be due to incorrect lutetium cross sections in the Battelle-Northwest Master Library. However, the activation of a 1/v detector (copper) was also calculated and found to be in excellent agreement with the calculated Lu¹⁷⁵ activation. This indicates that the Lu¹⁷⁵ thermal cross section that was used in the calculation is very nearly 1/v, as it should be. The Lu¹⁷⁶ cross sections cannot be easily checked because of a large resonance near thermal energies.

A more likely cause of the disagreement between calculated and measured activations and fluxes is an inadequate scattering model for D₂O. This conclu-

sion is supported by the recent work of Worden, et al., (15,16) which points out the disagreement in D₂O moderated systems between calculated and measured values of the disadvantage factor $\bar{\phi}$ MOD/ $\bar{\phi}$ FUEL, thermal utilization f , and thermal reproduction factor $\bar{\eta}$. The discrepancy could also be due to the choice of geometrical model for the cluster. However, it is concluded in References 15 and 16 that the geometrical model used to represent a 19-rod cluster (this model is similar to ours) is adequate. This conclusion is based on a comparison between a graphite-moderated 19-rod cluster of 2 wt% PuO₂-UO₂ and a D₂O-moderated cluster of natural UO₂. Excellent agreement is obtained between calculated and measured parameters for the PuO₂-UO₂ cluster using the Parks kernel for graphite, but for the D₂O case the agreement is poor. The same geometrical model was used in both cases, so the disagreement is attributed to the scattering model used for D₂O. A further indication of the adequacy of the geometrical model of the cluster is the good agreement between calculated and measured relative fluxes in the three fuel rods (Table III).

The THERMOS code solves the integral transport equation assuming isotropic scattering in the laboratory coordinate system. An approximate correction can be made for anisotropic scattering by substituting the transport cross section for the scattering cross section in generating the scattering kernel, as suggested by Honeck.⁽¹⁷⁾ Applying this anisotropy correction to deuterium had a small effect on the calculated parameters. The ratio of the average flux in the moderator to the average flux in

TABLE III. Region Average Thermal Fluxes in a PuAl D₂O Cell

	THERMOS		Experiment	
	Nelkin	Gas	Lu ¹⁷⁵	Lu ¹⁷⁶
Center rod	1.000	1.000	1.00	1.00
Intermediate rod	1.040	1.040	1.02	1.06
Outer rod	1.174	1.172	1.13	1.14
Moderator	1.761	1.723	1.44	1.55

the central fuel rod decreased by ~2%.

Comparison with PRTR Measurements

Spectrum measurements using lutetium foils have been made in the PRTR in a cell consisting of a PuAl 19-rod cluster moderated with D₂O. Spectral indices were deduced⁽²⁾ assuming that the joining function measured for D₂O by Johansson, et al.⁽⁷⁾ was valid at all experimental points. This function is not valid in the PuAl cluster.

The data were reanalyzed⁽⁶⁾ using joining functions calculated specifically for the PuAl cluster. However, in both the original analysis and the reanalysis, epicadmium resonance integrals were used which are valid for 0.040 in. cadmium covers, whereas in the experiment 0.030 in. cadmium covers were used. The correction for this reduces the values of the spectral index T by 2 to 4 °K.

The corrected results for the PRTR measurements and PRCF results that were made at corresponding positions are shown in Table IV. The values for the moderator in the PRCF are those determined at a radius of 8.34 cm.

Conclusions

The errors reported for values of r, T, and the cadmium ratios in the moderator do not allow for any errors which may result from repositioning the foil holder. In the analysis of the data it was assumed that the positions of the foils were identical during the two irradiations. Although this introduces an additional unknown uncertainty to the data, the results are consistent as a function of radial position.

The spectral indices in the cluster, as deduced from PRTR data and PRCF data, agree within their errors. The value of T deduced from the moderator in the PRTR is larger than the values of T in the cluster. This is contrary to what is expected; the spectrum is known to be harder in the cluster than in the moderator. This casts some doubt on the validity of T for the moderator as deduced from the PRTR data. The PRCF results, on the other hand, appear reasonable.

Calculations using the computer code THERMOS overestimate the thermal flux depression in the fuel. It is suggested

TABLE IV. Spectral Indices in a PuAl Cell as Measured in the PRCF and PRTR

Position of Foil	PRCF		PRTR	
	T, °K (a)	r	T, °K (b)	r
Center rod	321 ± 3	0.0656 ± 0.0006	336 ± 13	0.069 ± 0.004
Intermediate rod	328 ± 3	0.0633 ± 0.0003	335 ± 12	0.067 ± 0.004
Outer rod	319 ± 3	0.0588 ± 0.0004	313 ± 12	0.058 ± 0.004
Moderator	307 ± 7 ^(c)	0.046 ± 0.001	340 ± 12 ^(d)	0.052 ± 0.004

(a) Temperature of moderator was 297 °K.

(b) Temperature of moderator was 300 °K.

(c) At a radius of 8.34 cm

(d) As far as possible from any cluster

that this may be due to an inadequate scattering model for D_2O .

Acknowledgement

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REACTIVITY WORTH OF NEUTRON ABSORBING RODS IN PLUTONIUM-FUELED,
H₂O-MODERATED SYSTEMS—V. O. Uotinen and R. C. Liikala, Light
Moderator Reactor Physics Unit, Reactor Lattice Physics
Section

*Summary of a paper to be presented at the June, 1966
American Nuclear Society Meeting in Denver, Colorado.*

A series of experiments has been performed in the Critical Approach Facility (CAF) to measure reactivity worths of neutron absorbing rods. The absorbing rods consist of five rods containing various amounts of hafnium oxide in aluminum, four rods containing various amounts of boron in aluminum and an aluminum rod. Each rod is 0.368 in. in diameter, 36.0 in. long, and surrounded by a Zircaloy sleeve of 0.372 in. ID and 0.426 in. OD. The CAF was loaded with PuO₂-UO₂ or PuAl fuel rods in a hexagonal lattice. Various Pu²⁴⁰ concentrations and moderator-to-fuel ratios were used.

The measurements were made in the center of each loading. Multiplica-

tions were measured at several degrees of subcriticality with and without a poison rod present. Measurements were also made with a cadmium sleeve (0.435 in ID and 0.515 in. OD) surrounding each poison rod to obtain measurements of epicadmium worths. The critical number of fuel rods for each case was determined from inverse multiplication curves. The worth of each absorbing rod, with and without a cadmium cover, was found in terms of the number of additional peripheral fuel rods needed to offset the effect of the absorbing rod. The results are shown in Table I for the bare absorbing rods. The ratio of subcadmium worth to epicadmium worth is between 3 and 4 for the HfO₂ rods and

TABLE I. Worth for Removal of Neutron Absorbing Rods Relative to H₂O in Various Plutonium-Fueled, H₂O-Moderated Systems

Absorber	Concentration, g/cm ³	2 wt% PuO ₂ -UO ₂ 8 wt% Pu ²⁴⁰ V _m /V _f = 3.469		2 wt% PuO ₂ -UO ₂ 16 wt% Pu ²⁴⁰ V _m /V _f = 3.469		2 wt% PuO ₂ -UO ₂ 24 wt% Pu ²⁴⁰ V _m /V _f = 3.469		2 wt% PuO ₂ -UO ₂ 8 wt% Pu ²⁴⁰ V _m /V _f = 6.209		1.8 wt% Pu-Al 6 wt% Pu ²⁴⁰ V _m /V _f = 1.489	
		Reactivity Worth of Bare Absorbing Rods									
		ΔN ^(a)	mk ^(b)	ΔN	mk	ΔN	mk	ΔN	mk	ΔN	mk
Al	2.63	-0.1 ± 0.1	-0.1	0.0 ± 0.1	0.0	0.0 ± 0.2	0.0	0.0 ± 0.1	0.0	0.4 ± 0.2	1.7
HfO ₂	0.52	1.5 ± 0.1	2.0	2.4 ± 0.2	2.2	2.9 ± 0.2	2.0	1.7 ± 0.1	1.5	6.0 ± 0.2	2.57
HfO ₂	1.03	2.5 ± 0.2	3.3	3.3 ± 0.2	3.0	4.5 ± 0.2	3.0	2.5 ± 0.2	2.2	8.9 ± 0.1	3.79
HfO ₂	1.74	3.3 ± 0.2	4.3	4.4 ± 0.1	4.0	5.7 ± 0.2	3.8	3.3 ± 0.1	2.9	11.5 ± 0.3	4.90
HfO ₂	3.42	4.7 ± 0.1	6.1	6.0 ± 0.2	5.5	7.9 ± 0.2	5.2	4.7 ± 0.1	4.1	16.2 ± 0.2	6.85
HfO ₂	5.11	5.7 ± 0.2	7.4	6.8 ± 0.2	6.2	9.0 ± 0.2	5.9	5.3 ± 0.1	4.6	19.1 ± 0.2	8.04
B	0.00560	1.3 ± 0.1	1.7	1.9 ± 0.1	1.7	2.4 ± 0.3	1.6	1.6 ± 0.1	1.4	4.8 ± 0.2	2.05
B	0.0168	3.0 ± 0.2	3.9	3.7 ± 0.2	3.4	5.0 ± 0.1	3.3	3.4 ± 0.1	3.0	9.8 ± 0.3	4.17
B	0.0409	4.6 ± 0.1	6.0	5.8 ± 0.1	5.3	7.8 ± 0.1	5.2	4.9 ± 0.1	4.3	15.6 ± 0.5	6.60
B	0.0853	5.5 ± 0.3	7.1	7.0 ± 0.1	6.4	9.2 ± 0.2	6.0	5.6 ± 0.2	4.9	19.1 ± 0.4	8.04
Cd sleeve alone		7.6 ± 0.1	9.8	9.5 ± 0.2	8.6	12.2 ± 0.5	8.0	7.1 ± 0.2	6.1	25.7 ± 0.6	10.7
Cd sleeve containing H ₂ O		8.6 ± 0.4	11.0	10.6 ± 0.2	9.5	13.8 ± 0.4	9.0	7.6 ± 0.2	6.6	28.8 ± 0.8	12.0

(a) ΔN is the worth in terms of peripheral fuel rods.

(b) The values of ΔN were converted to mk by calculating Δk_{eff} due to small changes about the measured critical radius using the diffusion theory code HPN.

~ 10 for the boron rods. The worth as a function of Pu^{240} concentration in the fuel is shown in Figure 1 for a moderator-to-oxide volume ratio of 3.469.

Calculated worths in terms of peripheral rods are consistently smaller than measured worths. The methods used in the calculations are identical

to those described in another paper.* The computer codes HRG and THERMOS are used for cell computations and the one dimensional multigroup diffusion-theory code HFN for reactor calculations.

* R. C. Liikala and W. P. Stinson. "Experimental and Analytical Results for $\text{PuO}_2\text{-UO}_2\text{-H}_2\text{O}$ Lattices," *Trans. Am. Nucl. Soc.*, June, 1966.

FIGURE 1. Reactivity Worth of HfO_2 in a System of 2 wt% $\text{PuO}_2\text{-UO}_2$ Fuel Rods in H_2O ($V_m/V_f = 3.469$)

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