MASTER

CORRELATION LENGTHS, INCLUSIVE TRANSVERSE MOMENTUM DISTRIBUTIONS

AND

MULTIPERIPHERALISM[†]

Se-Yuen Mak and Chung-I Tan

Department of Physics

Brown University

Providence, Rhode Island 02912

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

[†]Supported in part by the U.S. Atomic Energy Commission (Report No. COO-3130TA-257).

Carl

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

ABSTRACT

Transverse momentum distributions in the pionization region of the single- and two-particle inclusive reactions are studied using both the general multi-peripheral considerations and a specific multi-Regge model with exponential damping in momentum transfers. The slope of the diffractive peak for elastic scattering is shown to set the scale for the inclusive transverse momentum distributions. The same reasonings also lead to a "broadening" of $\langle q_{\perp}^2 \rangle$ and to a peaking effect at $\cos\phi = -1$ for the single- and two-particle distributions respectively. The latter correlation effect is due to the existence of a J-plane branch point, thus having a correlation length $(\alpha_v - \alpha_c)^{-1}$. These general results are then verified by calculations based on an explicit multi-Regge model. Possible corrections to our results are also discussed.

I. Introduction

The basic ingredient of the multiperipheral model¹ (MPM) is the neglect of "long-range" correlation effects in particle productions at high energies. By incorporating direct channel unitarity, the integral equation approach to the MP dynamics allows us to demonstrate explicitly as its consequences the Regge behavior of exclusive processes¹⁻⁴ and the scaling properties of inclusive processes.^{5,6} These general properties are independent of the detail nature of the short-range correlations.

Experimentally, it is known that the distribution for the transverse momentum $q_{\perp}^2$ is roughly exponential. We concentrate in this paper on the consequences of this experimental input under the MP considerations. The "central" parameter of our discussion is the average $q_{\perp}^2$ for elastic scattering,⁷ Ω^{-1} , defined by

$$\Omega \equiv \left\langle \frac{d}{dt} \ln \left(\frac{d\sigma}{dt} \right)^{\text{elastic}} \right\rangle_{\text{average}} \quad (1.1)$$

In order to avoid the unnecessary complications of solving MP integral equations, we simply assume the solution has a reasonable spectrum in the J-plane -- a leading pole at $\alpha_V(0)$, and a branch point at $\alpha_c(0)$, $\alpha_c < \alpha_V$.

We first demonstrate that a MP consideration alone can lead to a transverse momentum cutoff for the single particle inclusive distribution in the pionization region of the form

$$f(q_{\perp}^2) \propto \exp \left\{ -\frac{\Omega}{2} q_{\perp}^2 \right\} \quad (1.2)$$

The same analysis is then used to obtain the large transverse momentum cutoff for the two-particle production. These results, obtained by general arguments, are then verified by an explicit calculation using an exponentially damped

multi-Regge model (MRM). We find, in particular, that the deviation from an uncorrelated distribution in the two-particle pionization region⁸ is

$$\Delta f_{12} \equiv f_{12} - f_1 f_2$$

$$\propto (\Sigma_{12})^{-(\alpha_v - \alpha_c)} \exp \left\{ -\Omega \left(\frac{2}{3} \kappa_1 + \frac{2}{3} \kappa_2 + [e^{-y} + \frac{2}{3} \cos \phi] \sqrt{\kappa_1 \kappa_2} \right) \right\}, \quad (1.3)$$

where $y \equiv y_1 - y_2 > 0$ is the relative rapidity, $\cos \phi = \hat{q}_1^\perp \cdot \hat{q}_2^\perp$, and

$$\kappa_1 \equiv q_1^{\perp 2} + \mu^2 \quad \text{and} \quad \kappa_2 \equiv q_2^{\perp 2} + \mu^2.$$

As $y \rightarrow 0$, Δf_{12} vanishes as $\exp\{-(\alpha_v - \alpha_c)y\}$, $(\Sigma_{12} \sim 2\sqrt{\kappa_1 \kappa_2} \cosh y)$, thus corresponding to a correlation length $(\alpha_v - \alpha_c)^{-1}$.

The concept of a limiting distribution (or scaling) for inclusive processes can best be described by using Mueller's Regge hypothesis.⁹ With the realization that inclusive distributions are directly related to discontinuities of elastic multiparticle scattering amplitudes,¹⁰ it is believed that Mueller's Regge hypothesis is probably on the same footing as the multi-Regge hypothesis¹¹ for the exclusive processes. In the case of the two-particle distribution,¹² the relevant multiparticle reaction is 4-to-4,

$$p_a + q_1' + q_2' + p_b \longrightarrow p_a' + q_1 + q_2 + p_b', \quad (1.4)$$

and the discontinuity is always taken in the crossed variable

$$M^2 = (p_a + p_b - q_1 - q_2)^2. \quad (1.5)$$

The discontinuity (21) X_2 , in the forward limit, i.e., $p_a = p_a'$, $q_1 = q_1'$, etc., is schematically represented by Fig. 1a, and it is related to the two-particle distribution f_{ab} ¹² for the process

$$p_a + p_b \longrightarrow q_1 + q_2 + \text{anything} \quad (1.6)$$

by

$$f_{ab}^{12} = (\sigma_{ab}^{tot})^{-1} \Delta^{-\frac{1}{2}}(s, m_a^2, m_b^2) \mathbb{X}_2. \quad (1.7)$$

The Mueller multi-Regge expansions for $\mathbb{X}_2$ in various kinematic regions¹² are illustrated by Figs. 1b - 1e. Each diagram corresponds to a limiting distribution for the region in question, and is controlled by the leading vacuum singularity α_v . The shape of the distribution is determined by "coupling functions" associated with vertices, which can only be obtained in models. The contributions from the lower singularities in the complex J-plane provide the corrections to the limiting distributions at "non-asymptotic" regions. In particular, these corrections in general do not possess factorization properties of the limiting distributions, thus leading to "short-range" correlation effects.

In the MPM, the discontinuity $\mathbb{X}_2$ is given by a sum of eight terms, A, B, ..., H, corresponding to different topologies where the two detected particles are emitted in the MP chain (Fig. 5). From the discussion of Section II, it is evident that not all eight MP diagrams contribute to the limiting distributions. By comparing Fig. 1 and Fig. 5, we find that the correspondence between the leading Mueller diagrams and the MP diagrams is:

Single Fragmentation of a: C, E, G, H

Single Fragmentation of b: D, F, G, H

Double Fragmentation: A, B, E, F, H

Correlated Pionization: G, H

Uncorrelated Pionization: H (1.8)

This correspondence can be established explicitly¹³ by a repeated use of the multiperipheral assumptions and by assuming the existence of an isolated leading vacuum singularity α_v . This, in turn, proves the scaling of the two-particle

distribution. In particular, it allows us to calculate the "coupling functions" in Mueller expansions in terms of parameters of the MPM.

Diagrams not listed in (1.8) correspond to non-asymptotic corrections. Particular attention is directed to the transition between the correlated and the uncorrelated pionization regions. Under our approximation the diagram H will be factorizable, so that the entire correlation effect in the pionization region is due to the diagram G. This short-range correlation can be shown to correspond to the existence of a J-plane branch point at α_c .⁸

Our program is to first calculate the gross transverse momenta behavior of the Mueller coupling functions by a general MP consideration. To avoid discussing the details of MP bootstrap, we shall assume the knowledge of the J-plane singularities at the forward limit and shall ignore the effects of Regge slopes. Under these conditions, we find from (1.1) that the elastic amplitude is parametrized by

$$T^{\text{elastic}}(s,t) \sim i \gamma(t)(s)^{\alpha_v} \gamma(t), \quad (1.9)$$

where

$$\gamma(t) = g e^{\frac{\Omega}{4} t}, \quad (1.10)$$

so that the total cross section is asymptotically given by

$$\sigma^{\text{tot}}(s) \sim g^2 s^{\alpha_v - 1}. \quad (1.11)$$

The cutoff behavior (1.2) and (1.3) can then be shown to follow from the "chain" structure of the MPM. In order to make our discussions self-contained, we review briefly the structure of the MPM and the scaling property of the two-particle distribution in Section II. Transverse momentum cutoffs for the single- and the two-particle distributions are derived in Section III. In Section IV, we obtain explicit representations for Σ_2 , using an exponentially

damped multi-Regge model, which not only verify results of Section III, but also provide us with distribution functions for other kinematic regions.

In a model with a single Regge pole, the branch cut α_c is located at $2\alpha(0) - 1$. It then follows that the "short-range" correlation assumption requires $\alpha_v(0) = \alpha(0) < 1$. Furthermore, the factor $\alpha_v - \alpha_c$ also reminds us of the triple-Regge behavior for the single particle production, where the distribution near the phase space boundary, $|x| \simeq 1$, behaves as

$$(1 - |x|)^{-(\alpha_v - \alpha_c) + 1},$$

x being the Feynman scaling variable.¹⁴ It can be shown that this behavior is related to the J-plane branch point contribution to the total cross section σ_{ab}^{tot} , so that it can again be identified with short-range correlation effects, provided that $\alpha(0) < 1$. These and other related questions are discussed in Section V.

II. Kinematics of Multiperipheral Model and Scaling

A. Single Particle Distribution in Multi-Regge Model

The single particle distribution function f_{ab}^1 is defined by

$$f_{ab}^1 \equiv (\sigma_{ab}^{\text{tot}})^{-1} (2E) \frac{d\sigma}{d^3q} = (\sigma_{ab}^{\text{tot}})^{-1} \Delta(s, m_a^2, m_b^2)^{-\frac{1}{2}} X_1, \quad (2.1)$$

where X_1 is the discontinuity of a 3-to-3 amplitude in $M^2 = (p_a + p_b - q)^2$, and Δ is the usual triangle function. By making a multi-Regge expansion for production amplitudes, X_1 is calculated by integrating over the final state phase space and summing over the number of produced particles. It is then given by a sum of three terms, corresponding to inequivalent production configurations in a MP chain. Each term involves one or two ladder sums, which can always be related to the solution of a MP integral equation (the CGL equation in the present case). In the pionization region, we find (Fig. 2a),

$$X_1(p_a; q; p_b) \simeq \frac{2}{(2\pi)^7} \int d^4Q_1 d^4Q_2 \delta^4(Q_1 + Q_2 + q) B_L(p_a; +Q_1, -Q_2) |\beta(t_1, \omega, t_2)|^2 \\ \times B_R(-Q_1, -Q_2; p_b) \quad (2.2)$$

The derivation of (2.2) can be found in I and II, and we follow closely notations introduced there: Σ 's always refer to subenergies between neighboring particles, t 's are momentum transfers squared inside a MP chain, ω 's are Toller angles, $\gamma(t)$ and $\beta(t, \omega, t')$ are the single- and the double-Regge couplings respectively. B_L and B_R , the auxiliary absorptive parts, are solutions of the CGL equation. The Regge factor $H(\Sigma, t)$, which enters in the build-up of B 's, is parametrized by $(\Sigma/\mu^2)^{\alpha(t)}$ where $\alpha(t)$ is the input Regge pole. (See Fig. 2b).

B. Two-Particle Distribution in Multi-Regge Model

Similarly, the two-particle distribution for the process (1.4) is related to the invariant cross section by

$$f_{ab}^{12} = (\sigma_{ab}^{tot})^{-1} (2E_1)(2E_2) \frac{d\sigma}{d^3q_1 d^3q_2} = (\sigma_{ab}^{tot})^{-1} \Delta(s, m_a^2, m_b^2)^{-1/2} X_2. \quad (2.3)$$

In the MPM, X_2 is given by a sum of eight terms,

$$X_2 = A + B + \dots + H, \quad (2.4)$$

corresponding to the MP configurations illustrated in Fig. 3. Again, using the MR expansion and performing the ladder sums, we find that each diagram in Fig. 3 can be expressed in terms of B's. For example, the diagram G is given by

$$G = \frac{2}{(2\pi)^{10}} \int d^4Q_1 d^4Q d^4Q_2 \delta^4(Q_1 + Q + q_1) \delta^4(Q_2 - Q + q_2) B_L(p_a; Q_1, -Q) \\ \times |\beta(t_1, \omega_1, t)|^2 |H(t, \Sigma_{12})|^2 |\beta(t, \omega_2, t_2)|^2 B_R(Q, Q_2; p_b), \quad (2.5)$$

where $t = Q^2$, $t_1 = Q_1^2$, $t_2 = Q_2^2$. The derivation of (2.5) and the representations for the rest of Fig. 3 are given in Appendix A.

f_{ab}^{12} , in general, depends on six independent variables,¹² which can be chosen to be: $s = (p_a + p_b)^2$, y_1, y_2 , $q_1^\perp, q_2^\perp$, $\cos\phi = \hat{q}_1^\perp \cdot \hat{q}_2^\perp$.

(y's are the rapidities. Our convention is $y_a \geq y_1 \geq y_2 \geq y_b$). However, for our subsequent discussions, it is often convenient to use invariants. They include

Energy variables: $s, \Sigma_{12} = (q_1 + q_2)^2, M^2 = (p_a + p_b - q_1 - q_2)^2, M_1^2 = (p_a + p_b - q_1)^2,$

$$M_2^2 = (p_a + p_b - q_2)^2, \quad \kappa_1 \equiv \mu^2 + q_1^{\perp 2}, \quad \kappa_2 \equiv \mu^2 + q_2^{\perp 2} \quad (2.6)$$

Momentum transfers: $u_1 = (p_a - q_1)^2, u_2 = (p_a - q_2)^2, v_1 = (p_b - q_1)^2,$

$$v_2 = (p_b - q_2)^2. \quad (2.7)$$

One useful relation among these variables is

$$\Sigma_{12} = 2\mu^2 + 2\sqrt{\kappa_1 \kappa_2} \cosh y - 2q_1^{\perp} q_2^{\perp} \cos\phi, \quad (y \equiv y_1 - y_2). \quad (2.8)$$

C. Scaling of the Two-Particle Distribution

The general multiperipheral assumption of strong dampings for all momentum transfers, together with the assumption of the Pomeranchuk dominance of the B's are sufficient to demonstrate the scaling of f_{ab}^{12} in a manner completely analogous to the treatment of f_{ab}^1 in II. The MP assumption allows us to show that phase space integrations for A, ..., H scale, and the dominant contributions of the integrands always come from regions where either B's already have the correct variable dependences or their behavior are known. Because of the "scaling" property¹⁻⁴ of the kernel of the CGL equation, it generally follows that B has Regge behavior (Fig. 4), i.e., as $s_1', s_1, s_2', s_2 \rightarrow \infty$,

$$B_L(p_a; -Q - q_1, -Q) \sim (s_1')^{\alpha_v} b_L(s_1/s_1', t_1) \quad (2.9)$$

$$B_R(Q, Q - q_2; p_b) \sim (s_2')^{\alpha_v} b_R(s_2/s_1', t_2) \quad (2.10)$$

where

$$s_1' = (-Q - q_1 + p_a)^2, \quad s_1 = (-Q + p_a)^2, \quad s_2' = (Q - q_2 + p_b)^2, \\ s_2 = (Q + p_b)^2, \quad (2.11)$$

and

$$t = Q^2, \quad t_1 = (Q + q_1)^2, \quad t_2 = (Q - q_2)^2.$$

It can then be shown that terms listed in (1.8) always have an asymptotic behavior, s^{α_v} , with coefficients which are functions of scaled variables only. Next we assume that α_v is the same vacuum singularity which controls the total cross section, (1.11), the scaling property of f_{ab}^{12} then follows. Since the demonstration of above

arguments is straight forward, we shall not present it here. However, certain technical details are given in Appendix B. To summarize: We replace each ladder sum by a vacuum Regge pole α_v , thus arriving at Fig. 5a through 5h, representing contributions A through H respectively.

III. Transverse Momentum Cutoff in the Pionization Region

The scaling property of the single particle distribution for the general MPM has been demonstrated in II, and an explicit form of the distribution has also been obtained in I for a MRM with exponential damping in momentum transfers. By combining these two approaches, transverse momentum cutoffs and correlations for the single- and the two-particle distributions can be obtained under a general MP consideration. We explain the procedure by first applying it to the single-particle case, where explicit results are known. We next treat the two-particle distribution in the pionization region.

A. Single-Particle Distribution

Separating $d\vec{Q}_1^\perp$, $d\vec{Q}_2^\perp$ from d^4Q_1 , d^4Q_2 , we find that X_1 in (2.2) can be written as (also use (2.9) and (2.10))

$$X_1 = \frac{(2)(\kappa)^{\alpha_v}(s)^{\alpha_v}}{(2\pi)^7} \int dx dz d^2\vec{Q}_1^\perp d^2\vec{Q}_2^\perp \delta^2(\vec{Q}_1^\perp + \vec{Q}_2^\perp + \vec{q}_\perp) \times b_L(x, t_1) |\beta(t_1, \omega, t_2)|^2 b_R(z, t_2), \quad (3.1)$$

where $x \equiv p_a \cdot Q_1 / p_a \cdot q$, $z \equiv p_b \cdot Q_2 / p_b \cdot q$, and

$$\kappa \equiv \frac{2(p_a \cdot q)(p_b \cdot q)}{p_a \cdot p_b} \frac{\mu^2}{q_\perp^2 + \mu^2}. \quad (3.2)$$

Using the MP hypothesis of strong damping in t_1 and t_2 , we find, (see II),

$$t_1 \simeq -Q_1^{\perp 2} - \kappa(z+1)x + O(1/s), \quad (3.3a)$$

$$t_2 \simeq -Q_2^{\perp 2} - \kappa(x+1)z + O(1/s), \quad (3.3b)$$

and the dominant contribution in (3.1) comes from $0 < x \leq O(1)$, $0 < y < O(1)$.

Any $Q_1^{\perp 2}$ and $Q_2^{\perp 2}$ dependence in (3.1) will only come from t_1 and t_2 via (3.3).

We first generalize the t cutoff for the single Regge coupling, (1.10), to the MP chain. By factorization, we expect a similar factor associated with each internal momentum transfer variable. For definiteness, we approximate

$$\beta(t, \omega, t_2) \sim \gamma(t_1) \tilde{\beta} \gamma(t_2) \quad (3.4)$$

where $\tilde{\beta}$ is assumed to be smooth,¹⁵ and later will be set to be $\gamma(\mu^2)^{-1}$. This rapid damping in t 's can be shown to survive the ladder summation, (see I), e.g.,

$$B_\ell(s_1, s_1', t_1) \sim (s_1')^{\alpha_v} \tilde{b}_\ell(x) \gamma^2(t_1),$$

and

$$\tilde{b}_\ell(x) \propto x^{2\bar{\alpha}} \quad (3.5)$$

where $\bar{\alpha}$ is taken to be the average $\alpha(t_1)$, ($\bar{\alpha} \sim \alpha(0)$). Independent of this specific assumption, we find

$$X_1 \sim s^{\alpha_v} \int dx dz d^2\vec{Q}_1^\perp d^2\vec{Q}_2^\perp \delta^2(\vec{Q}_1^\perp + \vec{Q}_2^\perp + \vec{q}_\perp) e^{\Omega(t_1 + t_2)} I(x, z, \dots) \quad (3.6)$$

where $I(x, z, \dots)$ is a smooth function.

The important observation to be made is that in order for $q_\perp^2$ to be large, either $Q_1^{\perp 2}$ or $Q_2^{\perp 2}$ (or both) has to increase, which then forces $|t_1|$, $|t_2|$ to grow. However, because of the presence of $\exp \{\Omega(t_1 + t_2)\}$, and the conditions (3.3a) and (3.3b), we find the large $q_\perp^2$ behavior gets most of its contribution from $0 < x, z \ll 1$. (That is, $0 < s_1'/s_1 \ll 1$, and $0 < s_2'/s_2 \ll 1$, yet s_1' and s_2' are still allowed to be large). We are then left with

$$X_1 \sim \int d^2\vec{Q}_1^\perp d^2\vec{Q}_2^\perp \delta^2(\vec{Q}_1^\perp + \vec{Q}_2^\perp + \vec{q}_\perp) \exp \{-\Omega(Q_1^{\perp 2} + Q_2^{\perp 2})\} \quad (3.7)$$

For $q_\perp^2 \rightarrow \infty$, asymptotic estimate yields

$$X_1 \propto \exp \left\{ -\Omega \left| Q_1^{\perp 2} + Q_2^{\perp 2} \right|_{\min} \right\} = \exp \left\{ -\frac{\Omega}{2} q_\perp^2 \right\}, \quad (3.8)$$

corresponding to the configuration

$$\vec{q}_1^\perp = \vec{q}_2^\perp = -\vec{q}_\perp/2 \quad . \quad (3.9)$$

Our crude calculation above agrees with the explicit calculation in I,¹⁶ and a similar result has also been obtained by Bali, Pignotti and Steele of Ref. 6. We believe that this "broadening" of $\langle q_\perp^2 \rangle$ is a general consequence of any MPM with rapid damping in t's. The origin of the broadening effect is the "sharing" of $\vec{q}_\perp$ by two adjacent t's in a MP chain. It is interesting to note that experimentally⁷ Ω ranges from 6 to 10(Gev/c)⁻², whereas the single particle distribution has an observed pionization cutoff¹⁷, (in (Gev/c)⁻²),

$$\exp \{-3.5 q_\perp^2\} \quad . \quad (3.10)$$

We would also like to point out that calculations¹⁸ based on the dual resonance model have yielded a cutoff behavior, $\exp \{-4\alpha' q_\perp^2\}$, $\alpha' \simeq 1$ (Gev/c)⁻¹, whereas the elastic large angle (e.g. $\theta_{cm} = \pi/2$) cutoff is $\exp \{-8\ln 2 \alpha' q_\perp^2\}$. However, we do not see any obvious connections between these two results.

B. Two-Particle Distribution

It follows from (1.8) that, for the pionization region, we only need to consider diagrams G and H. Under our approximation (2.9), (2.10), H is factorized (Appendix D), so that the cutoff in $\vec{q}_1^\perp$ and $\vec{q}_2^\perp$ is

$$H \sim \exp \left\{ -\frac{\Omega}{2} q_1^{\perp 2} \right\} \exp \left\{ -\frac{\Omega}{2} q_2^{\perp 2} \right\} = \exp \left\{ -\frac{\Omega}{2} (q_1^{\perp 2} + q_2^{\perp 2}) \right\}, \quad (3.11)$$

and the correlation effect is entirely contained in G. Following the procedure of Section III-A, we first introduce

$$x = \frac{p_a \cdot Q_1}{p_a \cdot (q_1 + q_2)}, \quad z = \frac{p_b \cdot Q_2}{p_a \cdot (q_1 + q_2)} \quad (3.12)$$

and find from (2.5) that the large $q_1^\perp$, $q_2^\perp$ contribution can be written as

$$G \sim \int dx \int dz \int d^2 \vec{Q}_\perp \exp \{ \Omega(t_1 + t + t_2) \} II(x, z, \dots) \quad (3.13)$$

where II is a smooth function, and

$$Q_1^{\perp 2} \sim -t_1 - (\kappa_1 + \kappa_2 + 2\sqrt{\kappa_1 \kappa_2} \cosh y)(z+1)x \quad (3.14a)$$

$$Q_1^2 \sim -t - \sqrt{\kappa_1 \kappa_2} e^{-y} [x(1+e^y) + 1][z(1+e^y) + 1], \quad (3.14b)$$

$$Q_2^{\perp 2} \sim -t_2 - (\kappa_1 + \kappa_2 + 2\sqrt{\kappa_1 \kappa_2} \cosh y)(x+1)z, \quad (3.14c)$$

$\kappa_i \equiv q_i^{\perp 2} + \mu^2$, $i = 1, 2$ and $\vec{Q}_1^\perp = -(\vec{Q}_\perp + \vec{q}_1^\perp)$, $Q_2 = \vec{Q}_\perp - \vec{q}_2^\perp$. In the limit κ_1, κ_2 large, $|t_1 + t + t_2|_{\min}$ occurs at $0 \sim y$, $z \ll 1$, so that we are left with

$$G \sim \exp \{ -\Omega(\kappa_1 + \kappa_2 + \sqrt{\kappa_1 \kappa_2} e^{-y}) \} \int d^2 \vec{Q}_\perp \exp \{ -\Omega[3Q_\perp^2 + 2\vec{Q}_\perp \cdot (\vec{q}_1^\perp - \vec{q}_2^\perp)] \} \quad (3.15)$$

A saddle point calculation indicates that the least damping occurs at

$$\vec{Q}_\perp = -\frac{1}{3}(\vec{q}_1^\perp - \vec{q}_2^\perp), \text{ and}$$

$$G \sim \exp \{ -\Omega[\frac{2}{3} q_1^{\perp 2} + \frac{2}{3} q_2^{\perp 2} + (e^{-y} + \frac{2}{3} \cos \phi) q_1^\perp q_2^\perp] \} \quad (3.16)$$

Since contributions from both G and H are positive, we find from (3.11) and (3.16) that the distribution $f_{12}(q_1^\perp, q_2^\perp, \cos \phi, y)$ is monotonic in $\cos \phi$ and has a maximum at $\phi = \pi$, for $y, q_1^\perp, q_2^\perp$ fixed. Furthermore, consider the case $e^{-y} < 2/3$, and say $q_1^\perp$ is large, it follows from (3.16) that the most probable distribution from Δf_{12} is $q_2^\perp = \frac{1}{2}(1 - \frac{3}{2} e^{-y}) q_1^\perp$ and $\phi = \pi$. That is, whenever a particle with a large transverse momentum is detected, it is most likely to find another particle also with large transverse momentum but pointing in the opposite direction (of course, we need to first subtract the uncorrelated distribution). We suggest that this is the signature of "multiperipheralism". (The present requirement $e^{-y} < 2/3$ is probably only an undesirable consequence of our approximation (3.4)). We stress that this phenomenon is not a consequence of the over-all

energy momentum conservations, but rather a reflection of the "short-range" correlation effect of the MPM.

As we increase the relative rapidity y , Δf^{12} in general will diminish. From Mueller's Regge expansion, we expect that the rate of decrease of the ratio G/H is controlled by the "gap" between α_v and the next dominant J-plane singularity. As we shall demonstrate in Section IV, the next singularity in the present model is a branch point at $\alpha_c = 2\alpha_0 - 1$.

IV. Results of an Exponentially-Damped Multi-Regge Model

The two-particle distribution is obtained using a concrete model which assumes Regge behavior in cluster energies and exponential damping in the MP momentum transfers. The model is basically the same as that first considered by Caneschi and Pignotti,¹⁹ and, as explained in I, we have been able to carry out analytic calculations without having to approximate phase space factors. Under the assumption of a rapid exponential damping in t : $\gamma(t) = g \exp(\frac{\Omega}{4} t)$, $\beta(t, \omega, t') = \gamma(\mu^2)^{-1} \gamma(t) \gamma(t')$, the CGL integral equation can be approximated by a separable kernel; and we obtain,

$$B_r(Q, Q_2; p_b) \simeq [(Q + p_b)^2 / (Q_2 + p_b)^2]^{2\alpha(t_2)} \frac{|\gamma(t_2)|^2}{|\gamma(\mu^2)|^2} [(Q_2 + p_b)^2]^{\alpha_v}, \quad (4.1)$$

in terms of which diagrams A, ..., H can be calculated. For instance, it follows from (4.1) and (2.6) that

$$G = \frac{2}{(2\pi)^{10}} g^4 e^{-2\Omega\mu^2} \int d^4Q e^{\Omega(t_1 + t + t_2)} (s_1')^{\alpha_v} (s_1/s_1')^{2\bar{\alpha}_1} (\Sigma_{12}/\mu^2)^{2\bar{\alpha}_0} (s^2/s_2')^{2\bar{\alpha}_2} (s_2')^{\alpha_v}, \quad (4.2)$$

where we have replaced α_1 by an average value $\bar{\alpha}_1 \simeq \alpha_1(0)$. This integral can actually be integrated to yield (Appendix C)

$$G = g^8 e^{-2\Omega\mu^2} (M^2)^{\alpha_v} (\Sigma_{12}/\mu^2)^{2\bar{\alpha}_0 - 1 - \alpha_v} \phi_{\alpha_1, \alpha_2}(c, a_1, a_2, b), \quad (4.3)$$

where $\phi_{\alpha_1, \alpha_2}(c, a_1, a_2, b)$ is given by Eq. (C.19).

Equation (4.1) also allows us to write down diagrams C and D directly and to show that H is precisely a product of two single particle distributions derived in I (Appendix D). Explicit expressions for E and F can be obtained similarly, whereas A is simply the non-forward elastic scattering cross section.

In what follows, we first describe the properties of the diagram G in various kinematic regions and then comment on the features of other diagrams.

A. Correlation is the Pionization Region

In this limit, we have, $M^2/s \rightarrow 1$, and

$$c - a_1 \sim -\Omega(2v_2 + v_1) \rightarrow \infty, \quad c - a_2 \sim -\Omega(2u_1 + u_2) \rightarrow \infty$$

$$a_1 + a_2 \sim 3\Omega M^2 \sim 2b \sim 2c \rightarrow \infty,$$

$$b - c \sim -\frac{\Omega}{3} [6\sqrt{\kappa_1 \kappa_2} e^{-y} + 3\sqrt{\kappa_1 \kappa_2} e^y + 2(\kappa_1 + \kappa_2) - 2(2q_1^\perp q_2^\perp \cos\phi + 2\mu^2)], \quad (4.4)$$

$$\lambda = \frac{(c - a_1)(c - a_2)}{a_1 + a_2} \simeq \frac{2\Omega}{3} (\kappa_1 + \kappa_2 + 2\sqrt{\kappa_1 \kappa_2} (e^{-y} + \frac{1}{4} e^y))$$

We can demonstrate that (a) $\phi_{\alpha_1, \alpha_2}$ reaches a limiting distribution, i.e., $\phi_{\alpha_1, \alpha_2} \rightarrow \phi_{\alpha_1, \alpha_2}(y, \kappa_1, \kappa_2, \cos\phi)$. (b) As $y \rightarrow \infty$, $\phi_{\alpha_1, \alpha_2}(\infty, \kappa_1, \kappa_2, \cos\phi) \neq 0$ and finite, so that

$$\Delta f^{12} \propto G/s^{\alpha_v} \sim (\Sigma_{12})^{-(\alpha_v - \alpha_c)} \sim \exp \{ -(\alpha_v - \alpha_c)y \}, \quad (4.5)$$

where $\alpha_c \equiv 2\bar{\alpha}_0 - 1$ is the leading vacuum branch point. Eq.(4.5) indicates that the branch cut contributes a short range correlation with a correlation length $(\alpha_v - \alpha_c)^{-1}$.

The analytic form of $\phi_{\alpha_1, \alpha_2}$ can be given in terms of elementary functions in the physically interesting cases of $\bar{\alpha}_1, \bar{\alpha}_2 = 0$ or $1/2$ through the use of Eq. (C.11). However, since we are primarily interested in the gross correlation effect, e.g., the large transverse momentum cutoff, we allow α_1, α_2 to be arbitrary and consider the limit

$$s \gg \kappa_1 \gg \mu^2, \quad s \gg \kappa_2 \gg \mu^2, \quad (4.6)$$

(More precisely, the limit $\kappa_i/s, \mu^2/\kappa_i \rightarrow 0, i = 1, 2$). In this case,

$$\lambda/\Omega\Sigma_{12} \simeq \left(\frac{1}{3}\right) \frac{\kappa_1 + \kappa_2 + 2\sqrt{\kappa_1 \kappa_2} (e^{-y} + \frac{1}{4} e^y)}{\sqrt{\kappa_1 \kappa_2} (\cosh y - \cos\phi)}$$

$$M^2/c \simeq \left(\frac{2}{3}\right) \left(\frac{1}{\Omega}\right)$$

$$\tau_1(c - a_1) \simeq \Omega [2\sqrt{\kappa_1 \kappa_2} e^{-y} + \kappa_1]$$

$$\tau_2(c - a_2) \simeq \Omega [2\sqrt{\kappa_1 \kappa_2} e^{-y} + \kappa_2] \quad (4.7)$$

and

$$-b + c + \Omega(\Sigma_{12}^{-2\mu^2}) \simeq -\Omega [2/3 (q_1^{\perp 2} + q_2^{\perp 2}) + (e^{-y} + 2/3 \cos\phi)\sqrt{\kappa_1 \kappa_2}] . \quad (4.8)$$

Using $\sigma_{\text{tot}} \simeq g^2 s^{\alpha_v - 1}$, we obtain from (C.21) and (1.7) that

$$\begin{aligned} \Delta f^{12} &\simeq \frac{g^6 e^{-2\Omega\mu^2}}{12(2\pi)^9 \Omega} \Gamma(\alpha_v - (2\bar{\alpha}_1 - 1)) \Gamma(\alpha_v - (2\bar{\alpha}_2 - 1)) \left[\frac{2\sqrt{\kappa_1 \kappa_2} (\cosh y - \cos\phi)}{\mu^2} \right]^{2\bar{\alpha}_0} \\ &\times [2\Omega^2(\kappa_1 + \kappa_2 + 2\sqrt{\kappa_1 \kappa_2} e^{-y} + \frac{\sqrt{\kappa_1 \kappa_2}}{2} e^y)]^{-\alpha_v - 1} [\Omega(2\sqrt{\kappa_1 \kappa_2} e^{-y} + \kappa_1)]^{2\bar{\alpha}_1} \\ &\times [\Omega(2\sqrt{\kappa_1 \kappa_2} e^{-y} + \kappa_2)]^{2\bar{\alpha}_2} \exp \{-\Omega[2/3 (q_1^{\perp 2} + q_2^{\perp 2} + q_1^{\perp} q_2^{\perp} \cos\phi) \\ &+ e^{-y} \sqrt{\kappa_1 \kappa_2}]\} . \end{aligned} \quad (4.9)$$

Comparing (4.9) with (3.16), we find that the large κ_1, κ_2 behavior is precisely that given by the general MP arguments. Our explicit calculation has furnished us with further informations on the correlation length, as well as a detailed correlation function.

B. Single Fragmentation of a:

The region can best be described by Feynman's scaled variables $x_1 \equiv q_1^{\parallel}/q_{\max}$, $x_2 \equiv q_2^{\parallel}/q_{\max} > 0$, in the C.M. of a, b. With our convention $q_1^{\parallel} > q_2^{\parallel}$, for $q_1^{\perp}$, $q_2^{\perp}$ finite, energy conservation requires $x_1 > x_2$, and $1 \geq x_1 \geq 0$, $1/2 \geq x_2 \geq 0$. In this case, we find that

$$c - a_2 \sim f(x_1, x_2, \kappa_1, \kappa_2) \quad (4.10)$$

is finite, and

$$\begin{aligned} c - a_1 &\sim \Omega s(2x_2 + x_1) \\ a_1 + a_2 &\sim 3\Omega s(1-x_1-x_2) \end{aligned} \quad (4.11)$$

so that

$$\Delta f_a^{12} \sim (\Sigma_{12}/\mu^2)^{-(\alpha_v - \alpha_c)} e^{-\Omega Y} \phi'_{\alpha_1, \alpha_2}(x_1, x_2, q_1^{\perp}, q_2^{\perp}, \phi) \quad (4.12)$$

where

$$Y = \frac{2x_1 + 3x_2 - x_2^2}{x_1(3 - 2x_1 - x_2)} \kappa_1 - \frac{2 - x_1}{3 - 2x_1 - x_2} \kappa_2 - \frac{2(1 + x_1)}{3 - 2x_1 - x_2} q_1^{\perp} q_2^{\perp} \cos \phi, \quad (4.13)$$

$$\Sigma_{12} \sim 2\mu^2 + \left(\frac{x_2}{x_1}\right) \kappa_1 + \left(\frac{x_1}{x_2}\right) \kappa_2 - 2q_1^{\perp} q_2^{\perp} \cos \phi. \quad (4.14)$$

Equation (4.12) indicates that Δf_a^{12} reaches a limiting distribution, and it vanishes when $x_1/x_2 \gg 1$, $(\Sigma/\mu^2 \rightarrow \infty)$. Equation (4.13) indicates that an exponential cutoff in κ_1 and κ_2 also exists where coefficients depend on x_1 , x_2 and $\cos \phi$. This is to be contrasted with the contribution from the diagram H, (Eq. (4.10) of I),

$$e^{-\Omega \left(\frac{1}{2-x_1} \kappa_1 + \frac{1}{2-x_2} \kappa_2 \right)} \quad (4.15)$$

Together we find that the damping is again most effective at $\phi = 0$, and the angle $\phi = \pi$ is preferred. In the limit $x_1, x_2 \rightarrow 0$, and

$$\frac{x_2}{x_1} \rightarrow (\kappa_2/\kappa_1)^{\frac{1}{2}} e^{-y} \quad (4.16)$$

Equation (4.12) reduces to Eq. (4.19), i.e., the fragmentation region smoothly approaches the pionization region. It also follows from (4.12) that the correlation length coming from G is again $(\alpha_v + \alpha_c)^{-1}$.

C. Deep Inelastic Region

This region is characterized by $|\vec{q}_1| = O(\sqrt{s})$, $|\vec{q}_2| = O(\sqrt{s})$ in the C.M. system. It has been shown in the case of the single particle production that the distribution has an exponential cutoff (Eq.(4.5) of I)

$$f_{ab} \sim \exp \left\{ -s\Omega \left[(1 - r/2) - \left[(1 - r/2)^2 - \frac{r^2}{4} \sin^2 \theta \right]^{\frac{1}{2}} \right] \right\}, \quad (\theta \neq 0), \quad (4.17)$$

where $r = q_0/q_{\max}$, $q_{\max} = \sqrt{s}/2$, $\sin \theta \sim q^\perp/q_0$. Similar cutoff has also been obtained by Huang and Segre recently using the DRM.²⁰ Strictly speaking, neither model is really applicable in this region where cross sections are extremely small. However, it is interesting to note that in both the DRM and the MPM, the behavior in this region and the cutoff in the pionization region are smoothly connected.

It can easily be shown that Eq.(C.21) is still applicable in this case. Introducing, in the C.M., ($q_{\max} = \sqrt{s}/2$)

$$r_1 = q_1^0/q_{\max}, \quad r_2 = q_2^0/q_{\max}, \quad (4.18)$$

$$z_1 = \cos \theta_1 = \hat{q}_1 \cdot \hat{z}, \quad z_2 = \cos \theta_2 = \hat{q}_2 \cdot \hat{z}, \quad z_{12} = \hat{q}_1 \cdot \hat{q}_2 \quad (4.19)$$

we find the dominant cutoff is of the form

$$\Delta f_{12} \sim \exp \{ -s\Omega J(r_1, r_2, z_1, z_2, z_{12}) \} \quad (4.20)$$

where

$$J = \left(\frac{1}{s}\right) \left[\frac{b-c}{\Omega} - \Sigma \right],$$

$$\frac{b}{s\Omega} = 1/2 \left[3 - \frac{3}{2} (r_1 + r_2) - \frac{1}{2} (r_1 z_1 - r_2 z_2) + \frac{3}{2} r_1 r_2 (1 - z_{12}) \right]$$

$$\frac{c}{s\Omega} = \left\{ \left(\frac{b}{s\Omega} \right)^2 - [2r_1(1 - z_1) + r_2(1 - z_2)][2r_2(1 + z_2) + r_1(1 + z_1)] \right.$$

$$\left. - 4r_1 r_2 (1 - z_{12}) \left[1 - r_1 - r_2 + \frac{r_1 r_2}{2} (1 - z_{12}) \right] \right\}^{1/2}$$

$$\frac{\Sigma}{s} = \frac{r_1 r_2}{2} (1 - z_{12}) . \quad (4.21)$$

Together with the fact that H is a product of two factors of the form (4.17), we see that f^{12} scales in the same manner as in the single particle case.

D. Other Diagrams

As we have already mentioned, the diagram H can be decomposed into a product of two single particle distribution, which, according to Eq. (1.8), contributes to all kinematic regions except at the boundary of the phase space. Near the boundary, end diagrams A, B, C, D, E, and F are important, leading to various types of triple-Regge behavior. For instance, at $x_1 + x_2 \sim 1$, $x_1 > x_2$, we find

$$C \sim g^6 e^{2\Omega\mu^2} s^{\alpha_v} \left(\frac{\Sigma}{\mu} \right)^{2\bar{\alpha}_0} (1 - x_1 - x_2)^{\alpha_v - 2\bar{\alpha}_2} (1 - x_1)^{2\bar{\alpha}_2} h, \quad (4.22)$$

$$h = \left(\frac{1}{2\pi} \right)^6 \exp \left\{ -\Omega \left((\kappa_1) \left[\frac{2 - x_2}{x_1} \right] + (\kappa_2) \left[\frac{1 - \kappa_1}{x_2} \right] + 2q_1^\perp q_2^\perp \cos\phi \right. \right. \\ \left. \left. - m^2 (2 - 2x_1 - x_2) \right) \right\} \quad (4.23)$$

Diagram D has a similar behavior at $x_1 + x_2 \sim 1$. In the di-triple-Regge limit where $x_1 \sim 1$, $x_2 \sim 1$, the diagram B gives a contribution

$$B \sim g^6 e^{-\Omega \mu^2} s^{\alpha_v} (1 - x_1)^{\alpha_v - 2\bar{\alpha}_1} (1 + x_2)^{\alpha_v - 2\bar{\alpha}_2} h', \quad (4.24)$$

$$h' = (2\pi)^{-6} e^{-\Omega[\frac{\kappa_1}{x_1} - \mu^2 - m^2(1 - x_1)]} e^{-\Omega[\frac{\kappa_2}{x_2} - \mu^2 - m^2(1 + x_2)]} \quad (4.25)$$

In this limit, the rest of the diagrams can all be shown to vanish relative to B, so that the distribution is independent of the angle ϕ -- another reflection of the short-range correlation of the MPM.²¹⁻²³ Diagram E and F are

factorizable, and they are given respectively by

$$E = \frac{g^2}{(2\pi)^3} e^{\frac{\Omega \mu^2}{2}} e^{-\Omega[\frac{\kappa_1}{x_1} - m^2(1 - x_1)]} (1 - x_1)^{\alpha_v - 2\bar{\alpha}} X_1(p_a - q_1; q_2; p_b)$$

$$F = \frac{g^2}{(2\pi)^3} e^{\frac{\Omega \mu^2}{2}} e^{-\Omega[-\frac{\kappa_2}{x_2} - m^2(1 + x_2)]} (1 + x_2)^{\alpha_v - 2\bar{\alpha}} X_1(p_a; q_1; p_b - q_2). \quad (4.26)$$

where X_1 is given by Eq. (2.2) .

V. Concluding Remarks

We have emphasized in this paper on two important features of two-particle correlation effects in the pionization region: (i) The transverse momenta are shown to be restricted to values of the order Ω^{-1} , and their distributions are in general correlated, reflecting the MP nature of the high energy production mechanism. (ii) These correlation effects are shown to diminish as the relative rapidity increases, with the correlation length given by the inverse of the separation between the location of the leading vacuum pole and that of the leading vacuum branch point. Although we have only demonstrated these features explicitly by using a simple and somewhat unrealistic MR model, we are confident of their generality because they can also be deduced heuristically by general MP considerations.

These are two sources of uncertainties which could possibly invalidate our conclusions. First, there is the complication of the secondary trajectories, some of which can lead to longer correlation lengths than the one we have considered. The generalization to include these singularities is a straight forward procedure; we associate a "correlation length" with each lower J-plane singularity contribution. However, since each "parent" trajectory yields a factorizable contribution to Δf^{12} , it will not lead to a $\cos\phi$ dependence that is characteristic of a MP mechanism: a term of the form $g(\kappa_1)(\Sigma_{12})^{-(\alpha_v - \alpha_M)} g(\kappa_2)$, α_M being the secondary trajectory, has a smooth $\cos\phi$ dependence and actually favors $\cos\phi = 1$. The peaking effect at $\cos\phi = -1$ can only come from non-factorizable singularities such as branch points and daughter trajectories. Our result represents a typical branch point contribution; and the effect of a daughter trajectory has already been studied in the context of the dual resonance model.¹² Remarkably, both results are qualitatively similar.

A more serious objection can be raised concerning the relative sign of the branch point contribution. Under quite general considerations, one can demonstrate that Δf^{12} should be predominantly negative,²⁴ whereas our model will always yield a positive contribution from the diagram G. It could be that our prescription for calculating the branch point contribution is incorrect; and a correct procedure will produce a "minus" sign, analogous to the case of the two-to-two absorptive correction calculation.²⁵ However, as we shall see shortly, this cannot be accomplished within a MP model with $\alpha_v(0) < 1$. Independent of this observation, we would like to argue that the physics of short-range correlations dictates this sign to be "plus". Since secondary trajectories cannot produce the $\cos\phi = -1$ peaking phenomenon, and since our arguments leading to the $\cos\phi$ dependence of G is independent of its overall sign, a negative branch point contribution will, therefore, lead to a peaking at $\cos\phi = 1$. If this were the case, we could no longer have "local" transverse momentum conservation, and it would, in turn, lead to a long-range transverse momentum correlation. We thus conclude that any negative contribution to Δf^{12} will come from secondary Regge trajectories. The notion that Regge poles themselves already contain absorptive effects is an interesting one. For instance, the possibility that Δf_{12} is negative would rule out a naive generalization of the exchange degeneracy to the present analysis. To summarize: A MP peaking effect at $\cos\phi = -1$ coming from a branch point will in general superimpose on top of a smooth and negative background in Δf^{12} .

Coming back to the question of the branch cut in MPM, we remark that the factorizable approximation for the diagram H is, strictly speaking, incorrect. This is because a ladder sum, aside from generating a Regge pole, also contains a branch point contribution itself. The term G roughly corresponds to the usual elastic AFS contribution,¹ and the complete branch cut effect is the sum of G and H.

For instance, it can be shown that Δf^{12} coming from G alone, when $\alpha' \neq 0$, has also an addition $(\ln \Sigma_{12})^{-1}$ factor, whereas the sum "softens"⁸ the branch cut, leading to a dependence

$$(\Sigma_{12})^{-(\alpha_v - \alpha_c)} (\ln \Sigma_{12})^{-1-\epsilon}$$

with $\epsilon > 0$. However, this will not change the sign of the net contribution when there is an isolated leading singularity $\alpha_v < 1$.²⁶ In view of the simple physical arguments presented in Section III, we believe that the rough transverse momentum correlation obtained earlier will not be altered by the modification from H. Mention should also be made on the effect of the possible dependence of the internal Reggeon-Reggeon coupling $|\beta|$ on the Toller angle.^{3,27} Consistent with the spirit of our approximation where polynomial variations are always ignored when compared with exponential cutoffs, we believe that any mild variation in the Toller angle dependence will not reflect itself strongly in the $\cos\phi$ dependence of the correlation function Δf^{12} .

Of course, all our present discussions depend on the assumption that $1 > \alpha_v > \alpha_c$. If $\alpha_v(0) = \alpha_c(0) = 2\alpha_v(0) - 1$, we find that the correlation length is ∞ , so that the correlation function only vanishes logarithmically: $\Delta f^{12} \sim (\ln \Sigma_{12})^{-1-\epsilon}$. It is generally believed that the Pomeranchon contribution has a small coefficient so that the double-Pomeranchon cut effect can probably be left out at our present experimental accuracy. Therefore, our results could represent the situation having a Pomeranchon α_v , an effective branch point $\alpha_c, \alpha_v > \alpha_c$, and together with another Regge pole yet to be specified. We strongly urge the experimental tests of our predictions.

Acknowledgement

We thank H. M. Fried and K. Kang for discussions on the concept of short-range correlations. Also one of us (C.-IT.) is happy to acknowledge fruitful discussions with D. Silverman during the early development of the ideas considered here.

Appendix A - Two-Particle Distribution and Multiperipheral Chain

We shall first derive Eq. (2.5), following essentially the same steps of the Appendix in I. Consider a particular MP chain with $N + 2$ final particles. Let the detected particle pair have momenta q_1 and q_2 , and let us specify the MR amplitude T_{N+2} by a set of four-momentum transfers to the left $\{p_i\}$, $1 \leq i \leq n$, and to the right $\{k_j\}$, $1 \leq j \leq m$, and in between q_1 and q_2 , Q . Whereas the contribution to the total cross-section due to this $N + 2$ -particle state is

$$\sigma^{N+2}(p_a, p_b) = \frac{1}{2^{\frac{1}{2}}(s, m_a^2, m_b^2)} \int |T_{N+2}|^2 d\phi^{N+2}, \quad (A.1)$$

the two-particle cross-section from this same configuration is

$$d\sigma^{N+2}(q_1, q_2; p_a, p_b) = \frac{d^4 q_1 \delta^+(q_1^2 - \mu^2) d^4 q_2 \delta^+(q_2^2 - \mu^2)}{2\Delta^{\frac{1}{2}}(s, m_1^2, m_2^2)} \sum_{n=1}^{N-1} \sum_{m=N-n}^{N-1} \times \int \frac{d^4 Q}{(2\pi)^{10}} d\phi^n d\phi^m |T_{N+2}|^2. \quad (A.2)$$

$d\phi^{N+2}$, $d\phi^n$, $d\phi^m$ are Lorentz invariant phase space factors given by Eq.(A.2), Eq.(A.5) of I. The summation over n is due to the freedom of the location of this pair (q_1, q_2) within the MP chain. Making a MP expansion for T_{N+2} , we find

$$|T_{N+2}(p_a, p_b; \{p_i\}, \{k_j\}, q_1, q_2)|^2 = \left| \frac{T_{n+1}(p_a, -Q; \{p_i\})}{\gamma(p_n^2)} \right|^2 \cdot |\beta(Q + q_1)^2, \omega_1, Q^2|^2 H(Q^2, \Sigma_{12}) \times |\beta(Q^2, \omega_2, (Q - q_2)^2)|^2 \left| \frac{T_{m+1}(Q, p_b; \{h_j\})}{\gamma(k_m^2)} \right|^2 \quad (A.3)$$

The two-particle distribution is obtained by summing Eq.(A.2) over N. Together with the restricted sum n, they can be converted into two unrestricted sums for n and m and performed because of the factorizable nature of T_{N+2} and $d\phi^{N+2}$. Recalling the definition of the CGL B-function,

$$B_\ell(p_a; -Q - q_1, -Q) = \frac{1}{2} \sum_{n=1}^{\infty} \int \left| \frac{T_{n+1}(p_a, -Q; \{p_1\})}{\gamma(p_n^2)} \right|^2 d\phi^n(p_a, -Q - q_1; \{p_1\}), \quad (A.4)$$

$$B_r(Q, Q - q_2; p_b) = \frac{1}{2} \sum_{m=1}^{\infty} \int \left| \frac{T_{m+1}(Q, p_b; \{k_1\})}{\gamma(k_m^2)} \right|^2 d\phi^m(Q - q_2, p_b; \{k_1\}), \quad (A.5)$$

Eq.(2.5) then follows.

The contributions from all other diagrams can be obtained in a similar manner. First, A is simply the non-forward 2-to-2 scattering cross section

$$A = \frac{1}{(2)(2\pi)^2} \delta^4(p_a + p_b - q_1 - q_2) |\gamma(t)|^2 (s/\mu^2)^{2\alpha(t)}. \quad (A.6)$$

Diagram B is given by what is essentially a CGL B function, but with both external lines being Regge poles.²³ Let it be B, and we find

$$B = \frac{1}{(2\pi)^6} |\gamma(t_1)|^2 B(p_a, p_a - q_1; p_b - q_2, p_b) |\gamma(t_2)|^2. \quad (A.7)$$

Diagrams C and D can both be expressed in terms of B

$$C = \frac{1}{(2\pi)^6} |\gamma(t) H(t, \Sigma_{12}) \beta(t_2, \omega_2, t_2)|^2 B_r(p_a - q_1 - q_2; p_b), \quad (A.8)$$

$$D = \frac{1}{(2\pi)^6} B_\ell(p_a; p_b - q_1 - q_2, p_b - q_2) |\beta(t_1, \omega_1, t) H(t, \Sigma_{12}) \gamma(t)|^2. \quad (A.9)$$

Diagrams E, F, and H involve both B_ℓ , B_r and B. They are respectively given by

$$E = \frac{2}{(2\pi)^6} |\gamma(t_1)|^2 \int \frac{d^4 Q}{(2\pi)^4} B(p_a, p_a - q_1; Q, Q + q_2) |\beta(t, \omega_2, t_2)|^2 B_r(-Q, -Q - q_2; p_b), \quad (A.10)$$

$$F = \frac{2}{(2\pi)^6} \int \frac{d^4 Q}{(2\pi)^4} B_\ell(p_a; -q_1 - Q, -Q) |\beta(t_1, \omega_1, t)|^2 B(Q + q_1, Q; p_b - q_2, p_b) |\gamma(t_2)|^2, \quad (A.11)$$

and

$$H = \frac{4}{(2\pi)^{14}} \int d^4 Q_1 d^4 Q_2 B_\ell(p_a; Q_1, -Q_3) |\beta(t_1, \omega_1, t_3)|^2 B(-Q_1, Q_3; Q_4, -Q_2) |\beta(t_4, \omega_2, t_2)|^2 B_r(-Q_4, Q_2; p_b). \quad (A.12)$$

Appendix B

For simplicity, we present here only the result for the diagram G in the correlated pionization region. We demonstrate that $G \sim (s)^{\alpha_v} f(y, q_1^{\perp 2}, q_2^{\perp 2}, \cos\phi)$ in the limit $s \rightarrow \infty$. We make use of the technique introduced in II: changing $p' \rightarrow p_a$, $p \rightarrow p_b$, $q' \rightarrow Q - q_2$, $q'' \rightarrow -Q - q_1$, $k \rightarrow q_1 + q_2$, the Jacobian of the transformation is

$$\begin{aligned} & \int d^4 Q_1 d^4 Q d^4 Q_2 \delta^4(Q_1 + Q + q_1) \delta^4(Q_2 - Q - q_2) \\ &= \int ds_1' ds_2' dt_1 dt_2 J \\ &= \int dx dz dt_1 dt_2 \frac{\theta(-\Delta((\vec{q}_1^{\perp} + \vec{q}_2^{\perp})^2, (\vec{Q}^{\perp} + \vec{q}_1^{\perp})^2, (\vec{Q}^{\perp} - \vec{q}_2^{\perp})^2))}{\{-\Delta((\vec{q}_1^{\perp} + \vec{q}_2^{\perp})^2, (\vec{Q}^{\perp} + \vec{q}_1^{\perp})^2, (\vec{Q}^{\perp} - \vec{q}_2^{\perp})^2)\}^{\frac{1}{2}}} [\kappa_1 + \kappa_2 \\ & \quad + 2\sqrt{\kappa_1 \kappa_2} \cosh y] \end{aligned} \quad (B.1)$$

where

$$\begin{aligned} x &= \frac{-p_a \cdot (Q + q_1)}{p_a \cdot (q_1 + q_2)} = \frac{s_1' - t_1 - m_1^2}{-u_1 - u_2 + 2(m_1^2 + \mu^2)} \\ z &= \frac{-p_b \cdot (q_2 - Q)}{p_b \cdot (q_1 + q_2)} = \frac{s_2' - t_2 - m_2^2}{-v_1 - v_2 + 2(m_2^2 + \mu^2)} \end{aligned} \quad (B.2)$$

and

$$\begin{aligned} (\vec{Q}^{\perp} - \vec{q}_2^{\perp})^2 &= -t_2 - [\kappa_1 + \kappa_2 + 2\sqrt{\kappa_1 \kappa_2} \cosh y] (x + 1)z, \\ (\vec{Q}^{\perp} + \vec{q}_1^{\perp})^2 &= -t_1 - [\kappa_1 + \kappa_2 + 2\sqrt{\kappa_1 \kappa_2} \cosh y] (z + 1)x. \end{aligned} \quad (B.3)$$

The assumption of Pomernanchuk-pole dominance for B_ℓ and B_r will lead to an overall factor s^{α_v} and leaving the integrand with factors which are functions of integration variables x, z, t_1, t_2 as well as scaling variables $y, q_1^{\perp 2}, q_2^{\perp 2}, \cos\phi$.

The only non-trivial step involved is to demonstrate that $Q^2 \equiv t$ is an implicit function of these variables. This can be proved by noting that, because of the MP assumption, we can show that

$$t \simeq -Q_1^2 + \sqrt{\kappa_1 \kappa_2} e^{-y} [x(1 + e^y) + 1] [z(1 + e^y) + 1] + O\left(\frac{1}{s}\right). \quad (B.4)$$

However, conservation of transverse momenta indicates that Q_1^2 is determined by $(t_1, t_2, q_1^\perp, q_2^\perp, x, z, \phi)$, which are scaled variables. The distribution can easily be shown to scale also in the single fragmentation region. We shall omit the proof since the calculation is lengthy though straight forward.

Appendix C - Evaluation of the Diagram G

The technique required to evaluate (4.2) is very similar to that used in I for the single particle distribution. We first approximate s_1, s_2 by $s_1 \simeq s_1' - u_1$, $s_2 \simeq s_2' - v_2$, and change integration variables to s_1', s_2', t and r , where $r \equiv P^2$, $P \equiv Q - q_2 + q_1$:

$$G = \frac{2}{(2\pi)^{10}} g^8 e^{-2\Omega\mu^2} \left(\frac{\Sigma_{12}}{\mu^2}\right)^{2\bar{\alpha}_0} e^{\Omega(\Sigma_{12} - 2\mu^2)} G'_{\alpha_1, \alpha_2}, \quad (C.1)$$

$$G'_{\alpha_1, \alpha_2} = \frac{1}{16} \int ds_1' ds_2' (s_1')^{\alpha_v} (s_2')^{\alpha_v} \left(\frac{s_1' - u_1}{s_1'}\right)^{2\bar{\alpha}_1} \left(\frac{s_2' - v_2}{s_2'}\right)^{2\bar{\alpha}_2} I(s_1', s_2') \quad (C.2)$$

$$I(s_1', s_2') = \int \frac{dt dr}{\sqrt{-\Delta_4}} \theta(-\Delta_4) e^{2\Omega t + \Omega r} \quad (C.3)$$

Δ_4 is the Gram-determinant of vectors $P, q_2 - q_1, p_a - q_1$, and $p_b - q_2$, and $I(s_1', s_2')$ is again an integral of the form performed in I (Eq. (3.12) of I). The identification is completed with the substitution $p' \rightarrow -P, q \rightarrow q_2 - q_1, p \rightarrow p_a - q_1$, and $k \rightarrow p_b - q_1$, so that $I(s_1', s_2')$ is given by

$$I(s_1', s_2') = 4\pi \exp[-b] \exp[a_1(s_1'/M^2) + a_2(s_2'/M^2)] \frac{\sinh[c\Delta^{\frac{1}{2}}(1, s_1'/M^2, s_2'/M^2)]}{c} \times \theta[\Delta^{\frac{1}{2}}(M^2, s_1', s_2')] \quad (C.4)$$

with

$$\begin{aligned} b &= \Omega(M^2 - v_2 - u_1) + \Omega/2 (M^2 - u_2 - v_1) \\ a_1 &= \Omega(M^2 + v_2 - u_1) + \Omega/2 (M^2 - u_2 + v_1) \\ a_2 &= \Omega(M^2 - v_2 + u_1) + \Omega/2 (M^2 + u_2 - v_1) \\ c &= \Omega\{\Delta(M^2, v_2, u_1) + \frac{1}{4}\Delta(M^2, u_2, v_1) + [M^2(M^2 - u_1 - u_2 - v_1 - v_2 + 2(q_2 - q_1)^2) \\ &\quad - (u_2 - v_1)(v_2 - u_1)]\}^{\frac{1}{2}} \end{aligned} \quad (C.5)$$

In arriving at (C.4) and (C.5), we have substituted into Eqs. (3.15) and (3.16) of I with

$$\Omega_\ell \rightarrow 2\Omega, \Omega_r \rightarrow \Omega, \mu^2 \rightarrow (q_2 - q_1)^2 = 4\mu^2 - \Sigma, M^2 \rightarrow M^2$$

$$t_\ell \rightarrow t, s_\ell' \rightarrow s_1', m_1^2 \rightarrow u_1, u_\ell \rightarrow u_2$$

$$t_r \rightarrow r, s_r' \rightarrow s_2', m_2^2 \rightarrow v_1, u_r \rightarrow v_2. \quad (C.6)$$

As in I, the s_1' and s_2' can next be performed by changing variables to

$$z(1-x) = s_1'/M^2, \quad x(1-z) = s_2'/M^2 \quad (C.7)$$

and Eq. (C.4) becomes

$$G'_{\alpha_1, \alpha_2} = \left(\frac{\pi}{8}\right) \frac{(M^2)^2 + 2\alpha_v}{c} \exp\{-b + c\} \int_0^1 \int_0^1 dx dz \theta(1-x-z) (1-x-z) \mathcal{J}_{\alpha_1, \alpha_2} \exp\{-(c-a_1)z + (c-a_2)x + (a_1+a_2)xz\}, \quad (C.8)$$

where

$$\mathcal{J}_{\alpha_1, \alpha_2}(x, z) = [z(1-x)]^{\alpha_v} [x(1-z)]^{\alpha_v} \left[1 + \frac{\tau_1}{z(1-x)}\right]^{2\bar{\alpha}_1} \left[1 + \frac{\tau_2}{x(1-z)}\right]^{2\bar{\alpha}_2}, \quad (C.9)$$

with

$$\tau_1 \equiv -u_1/M^2, \quad \tau_2 \equiv -v_2/M^2 \quad (C.10)$$

For the restricted case of $\alpha_1, \alpha_2 = 0, 1/2$ and $\alpha_v = 1$, we have

$\mathcal{J}_{\alpha_1, \alpha_2} = (M^2)^2 [z(1-x) + \tau_1 \delta_{\alpha_1, 1/2}] [x(1-z) + \tau_2 \delta_{\alpha_2, 1/2}]$, and these terms can be removed by formal differentiation. The result is

$$G'_{\alpha_1, \alpha_2} = \frac{\pi}{8} \frac{(M^2)^2 + 2\alpha_v}{c} \left(\frac{d}{da_1} + \tau_1 \delta_{\alpha_1, 1/2}\right) \left(\frac{d}{da_2} + \tau_2 \delta_{\alpha_2, 1/2}\right) \left(\frac{1}{a_1 + a_2}\right) \frac{d}{dc} \times \{\exp(-b+c) [\exp(\lambda_1) E_1(\lambda_1)] + \exp(-b-c) [\exp(\lambda_2) E_1(\lambda_2)] - \exp(-b+a_1) [\exp(\lambda_3) E_1(\lambda_3)] - \exp(-b+a_2) [\exp(\lambda_4) E_1(\lambda_4)]\}, \quad (C.11)$$

where

$$E_1(\lambda) = \int_\lambda^\infty dx x e^{-x}, \quad (C.12)$$

$$\begin{aligned}\lambda_1 &= \frac{(c - a_1)(c - a_2)}{a_1 + a_2}, & \lambda_2 &= \frac{(c + a_1)(c + a_2)}{a_1 + a_2} \\ \lambda_3 &= \frac{(c - a_1)(c + a_1)}{a_1 + a_2}, & \lambda_4 &= \frac{(c + a_2)(c - a_2)}{a_1 + a_2}\end{aligned}\quad (C.13)$$

The main feature of Eq.(C.11) is the exponential decrease of the terms as given by the exponents

$$\begin{aligned}-b + a_1 &\sim \Omega(2v_2 + v_1) \\ -b + a_2 &\sim \Omega(2u_1 + u_2) \\ -b \pm c &\sim (-b)(1 \mp \left\{1 - \left(\frac{\Omega^2}{4b^2}\right) [4(2u_1 + u_2)(2v_2 + v_1) + 8M^2 (q_2 - q_1)^2]\right\}^{\frac{1}{2}})\end{aligned}\quad (C.14)$$

In the regions of interests to us, we always have

$$0 < b - c \ll b - a_1, b - a_2, b + c \quad (C.15)$$

together with the fact that

$$\frac{1}{x+1} < e^x E_1(x) < \frac{1}{x}, \quad x > 0, \quad (C.16)$$

it follows that the first term in (C.11) always dominates over others.

More generally, we can scale z, x in (C.8) by $(c - a_1)$ and $(c - a_2)$

$$x' \equiv (c - a_2) x, \quad z' = (c - a_1) z, \quad (C.17)$$

and we find

$$G = g^8 e^{-2\Omega\mu^2} (M^2)^{\alpha_v} (\Sigma_{12}/\mu^2)^{2\bar{\alpha}_0 - 1 - \alpha_v} \phi_{\bar{\alpha}_1, \bar{\alpha}_2}(c, a_1, a_2, b) \quad (C.18)$$

$$\phi_{\bar{\alpha}_1, \bar{\alpha}_2} = \frac{1}{(8)(2\pi)^9} \left(\frac{M^2}{c}\right) [(3\Omega\lambda)\left(\frac{\mu^2}{\Sigma_{12}}\right)]^{-(\alpha_v+1)} e^{-b+c+\Omega(\Sigma_{12}-2\mu^2)} \times$$

$$\int_0^1 dx' dz' \theta\left(1 - \frac{x'}{c - a_2} - \frac{z'}{c - a_1}\right) \left(1 - \frac{x'}{c - a_2} - \frac{z'}{c - a_1}\right) [(c - a_1)(c - a_2)]^{\alpha_v}$$

$$g_{\alpha_1, \alpha_2} \left(\frac{x'}{c - a_2}, \frac{z'}{c - a_1}\right) \exp \left[- \left(x' + z' + \frac{x' z'}{\lambda} \right) \right] \quad (C.19)$$

In the limit

$$k_1 \gg \mu^2, \quad k_2 \gg \mu^2 \quad (C.20)$$

Eq. (C.19) yields

$$\begin{aligned} \phi_{\bar{a}_1, \bar{a}_2} \simeq & \frac{1}{(8)(2\pi)^9} \left(\frac{M^2}{c}\right) \left[\frac{3\Omega\lambda\mu^2}{\Sigma_{12}}\right]^{-(\alpha_v+1)} [\tau_1(c - a_1)]^{2\bar{a}_1} [\tau_2(c - a_2)]^{2\bar{a}_2} \\ & \times \Gamma(\alpha_v - (2\bar{a}_1 - 1)) \Gamma(\alpha_v - (2\bar{a}_2 - 1)) \exp \{-b + c + \Omega (\Sigma_{12}^{-2}\mu^2)\} \end{aligned} \quad (C.21)$$

Appendix D - Factorization of Diagram H

By solving a generalized CGL equation²³, one can show that B has a Regge behavior

$$B(-Q_1, Q_3; Q_4, -Q_2) \simeq \left[\frac{(Q_4 - Q_1)^2}{(Q_4 + Q_3)^2} \right]^{2\alpha(t_3)} A(Q_3; Q_4) \left[\frac{(Q_3 - Q_2)^2}{(Q_3 + Q_4)^2} \right]^{2\alpha(t_4)}, \quad (D.1)$$

where

$$A(Q_3; Q_4) \simeq g^2 \frac{|\gamma(t_3)|^2}{|\gamma(\mu^2)|^2} [(Q_3 + Q_4)]^{\alpha_v(0)} \frac{|\gamma(t_4)|^2}{|\gamma(\mu^2)|^2} \quad (D.2)$$

In the strong ordering approximation, we have

$$\left[\frac{(Q_4 - Q_1)^2}{(Q_4 + Q_3)^2} \right] \simeq \left[\frac{(p_b - Q_1)^2}{(p_b + Q_3)^2} \right], \quad \left[\frac{(Q_3 - Q_2)^2}{(Q_3 + Q_4)^2} \right] \simeq \left[\frac{(p_a - Q_2)^2}{(p_a + Q_4)^2} \right], \quad (D.3)$$

$$\text{and} \quad (Q_3 + Q_4)^2 \simeq \frac{(p_a + Q_4)^2 (p_b + Q_3)^2}{(p_a + p_b)^2}, \quad (D.4)$$

so that

$$\begin{aligned} B(-Q_1, Q_3; Q_4, -Q_2) &\simeq g^2 (s_{ab})^{-\alpha_v} \frac{|\gamma(t_3)|^2}{|\gamma(\mu^2)|^2} \left[\frac{(Q_1 - p_b)^2}{(Q_3 + p_b)^2} \right]^{2\alpha(t_3)} [(Q_3 + p_b)^2]^{\alpha_v} [(p_a + Q_4)^2]^{\alpha_v} \\ &\times \left[\frac{(p_a - Q_2)^2}{(p_a + Q_4)^2} \right]^{2\alpha(t_4)} \frac{|\gamma(t_4)|^2}{|\gamma(\mu^2)|^2} \end{aligned} \quad (D.5)$$

Substituting (D.5) into (A.12) and using (4.1), (2.2), we obtain

$$H = g^2(s_{ab})^{-\alpha_v} X_1(p_a; q_1; p_b) X_1(p_a; q_2; p_b). \quad (D.6)$$

It then follows from (1.11), (2.1) and (2.3) that the contribution to f_{ab}^{12} due to the diagram H is precisely

$$f_{ab}^{12}(H) = f^1(q_1) f^2(q_2) \quad (D.7)$$

Footnotes and Refereces

1. L. Bertocchi, S. Fubini, and M. Tonin, Nuovo Cimento 25, 626 (1962);
D. Amati, A. Stanghellini, and S. Fubini, Nuovo Cimento 26, 896 (1962).
2. G. F. Chew, M. L. Goldberger, and F. Low, Phys. Rev. Letters 22, 208 (1969);
G. F. Chew, and C. DeTar, Phys. Rev. 180, 1577 (1969); A. H. Mueller and
I. J. Muzinich, Ann. Phys. (N.Y.) 57, 20, 500 (1970); M. Ciafaloni, C. DeTar,
and M. N. Misheloff, Phys. Rev. 188, 2522 (1969).
3. M. L. Goldberger, C.-I Tan and J. M. Wang, Phys. Rev. 184, 1920 (1969);
D. Silverman and C.-I Tan, Phys. Rev. D1, 3479 (1970); S. Pinsky and
W. I. Weisberger, Phys. Rev. D2, 1640 (1970).
4. J. S. Ball and G. Marchesini, Phys. Rev. 188, 2209, 2508 (1969); G. F. Chew
and W. R. Frazer, Phys. Rev. 181, 1914 (1969); L. Caneschi and A. Pignotti,
Phys. Rev. 180, 1525 (1969); I. G. Halliday and L. M. Saunders, Nuovo Cimento
60A, 177 (1969).
5. D. Silverman and C.-I Tan, Nuovo Cimento 2A, 489 (1971), (hereafter referred
to as I).
6. D. Silverman and C.-I Tan, Phys. Rev. D3, 991 (1971), (hereafter referred to
as II); see also N. F. Bali, A. Pignotti and D. Steele, Phys. Rev. D3, 1167
(1971); C. DeTar, Phys. Rev. D3, 128 (1971); K. Wilson, Acta. Phys. Austr.
17, 37 (1963).
7. Experimental values of Ω can be found in Foley et al, Phys. Rev. Letters 11,
425, 503 (1963): (in the unit of $(\text{Gev}/c)^{-2}$) $\pi^+p \sim 8.9$, $\pi^-p \sim 9.7$, $\kappa^+p \sim 7$,
 $\kappa^-p \sim 10$, $pp \sim 10$, $\bar{p}p \sim 12$. A single exponential fit is adequate for
 $|t| \sim q_{\perp}^2 \lesssim 0.5(\text{Gev}/c)^2$, where the cross sections usually drop from $t = 0$
by several decades. It has been pointed out by Krisch (Phys. Rev. Letters
19, 1149, (1967)) that further structure exists in p-p reaction at larger

$|t|$ values, the slope parameter decreases from 10 to 3.45 for $1 \leq |t| \leq 4$. Since we are describing effects due to regions where cross sections are suppose to be "large", the relevant parameter is thus the former where more than 90% of the elastic events take place. As we shall discuss later, the corresponding region of interests to us for inclusive reactions is again $|q_1|^2 \leq 0.5(\text{Gev}/c)^2$.

8. Contribution coming from a branch point in general has an additional factor $(\ln \Sigma_{12})^{-1-\epsilon}$, where $\epsilon > 0$ due to the unitarity requirement (J. B. Bronzan and C. E. Jones, Phys. Rev. 160, 1494 (1971)). Under our present approximation where Regge slope effect is neglected, this logarithmic factor is absent.
9. A. H. Mueller, Phys. Rev. D2, 2963 (1970); for a generalization, see also H. Abarbanel, Phys. Letters 34B, 67 (1971).
10. H. P. Stapp, Phys. Rev. D3, 3177 (1971); C.-I Tan, Phys. Rev. D4, (1971); J. C. Polkinghorne (to be published).
11. N. F. Bali, G. F. Chew, and A. Pignotti, Phys. Rev. Letters 19, 614 (1967); see also T. W. Kibble, Phys. Rev. 131, 2282 (1963); K. A. Ter-Martirosyan, Sov. Phys. JEPT 17, 233 (1963); F. Zachariassen and G. Zweig, Phys. Rev. 160, 1322 (1967).
12. Two-particle distribution has been studied using the dual resonance model (DRM) by C. Jen, K. Kang, P. Shen and C.-I Tan, Phys. Rev. Letters 27, 458 (1971), and "Dual-Amplitude Analysis of Two-Particle Production. II", Brown Univ. Preprint: COO-3130TA-256 (submitted to Annals of Physics). Kinematics of two-particle distributions are discussed in detail in these articles. See also, A. Bassetto, M. Toller and L. Sertorio, CERN Th: 1326, May, 1971, for a general discussion.
13. For the single particle distribution, see, for instance, C.-I Tan, "High-Energy Inclusive Processes", Contribution 4, Springer Tracts in Modern Physics,

Vol. 60, (to be published).

14. R. Feynman, Phys. Rev. Letters 23, 1415 (1969); J. Benecke, T. T. Chou, C. N. Yang, and E. Yen, Phys. Rev. 188, 2159 (1969).
15. Dependence of β on Toller angle is also equivalent to an dependence on $\hat{Q}_1^\perp \cdot \hat{Q}_2^\perp \equiv \cos\phi_{12}$. However, as long as this dependence is smooth, our results on the large transverse momentum cutoff are expected to be valid.
16. In I, a factor 1/2 was left out in Eq.(3.16), associated with Ω_ℓ and Ω_r . Consequently, all equations in Section IV of I should have Ω_ℓ and Ω_r replaced by $\Omega_\ell/2$ and $\Omega_r/2$. After this replacement, our present result agrees with that of I if $\Omega = \Omega_\ell = \Omega_r$.
17. L. G. Ratner et al., Phys. Rev. Letters 27, 68 (1971); C. W. Akerlof et al., Phys. Rev. D3, 645 (1971).
18. C. E. DeTar, K. Kang, C.-I Tan, and J. H. Weis, Phys. Rev. D4, 425 (1971); see also D. Gordon and G. Veneziano, Phys. Rev. D1, 2116 (1971); M. A. Virasoro, Phys. Rev. D1, 2834 (1971); R. C. Brower and R. E. Waltz, Nuovo Cimento (to be published).
19. L. Caneschi and A. Pignotti, Phys. Rev. Letters 22, 1219 (1969); D. Silverman and C.-I Tan, Phys. Rev. D2, 233 (1970).
20. K. Huang, and G. Segre, Phys. Rev. Letters 27, 1095 (1971).
21. D. Freedman, E. Jones, F. Low and J. Young, Phys. Rev. Letters 26, 1197 (1971).
22. C. Jen, K. Kang, P. Shen and C.-I Tan, Phys. Rev. Letters 27, 754 (1971).
23. An integral equation for this B with both external lines being Regge poles has been discussed in more general terms in: "Construction of Multiperipheral Dynamical Equations and Inclusive Sum Rules" Chung-I Tan, Brown Univ. Preprint NYO-2262TA-254, submitted to Phys. Rev.

24. This point will be elaborated elsewhere. We note that this phenomenon has also been recognized by S.-H. H. Tye. (unpublished).
 25. See, for instance, L. Caneschi, Phys. Rev. Letters 23, 254 (1969).
 26. W. R. Frazer and C. H. Mehta, Phys. Rev. Letters 23, 258 (1969).
 27. C.-I Tan and J. M. Wang, Phys. Rev. 185, 1899 (1969).
-

Figure Captions

1. (a) Mueller diagram for a two-particle inclusive cross section.
(b) The limiting distribution for the fragmentation region of particle a.
(c) The double fragmentation limit.
(d) The pionization limit.
(e) The uncorrelated pionization limit.
2. (a) The pionization limit of a single-particle inclusive cross section.
(b) The multiperipheral summation for the CGL auxiliary function.
3. (a) - (h) Eight inequivalent production configurations in a MP chain.
4. Schematic representation of the Regge behavior of the CGL B function.
5. (a) - (h) Contributions to the Mueller expansion corresponding to the eight MP configurations depicted in Fig. 3(a) - (h).

(a)

(b)

(c)

(d)

(e)

FIGURE 1

(a)

$$B_l(p_a; Q_1, -Q_2) = \frac{1}{2} \sum_{n=1}^{\infty} \int d\Phi_n \left| \begin{array}{c} \text{Diagram (b)} \end{array} \right|^2$$

(b)

FIGURE 2

(a)

(b)

(c)

(d)

(e)

(f)

(g)

(h)

FIGURE 3

FIGURE 4

FIGURE 5