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HIGH ENERGY KAON-NUCLEON SCATTERING

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HIGH ENERGY KAON-NUCLEON SCATTERING

Ismail A. Sakmar
(Thesis).

May 1963

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A b s t r a c t

Backward and forward elastic $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$ scatterings at high energies are studied. The amplitudes of these processes as well as of the process $K\bar{K} \rightarrow N\bar{N}$ are expressed in terms of suitable partial wave amplitudes in crossed channels. These latter are analytically continued into the complex angular momentum plane and the scattering amplitudes approximated in terms of Regge poles. The quantum numbers of all three channels are discussed and a correspondence made between poles and known particles. In this respect there is a clear distinction between $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$ scattering. A variety of particles have been discovered with the quantum numbers of the antikaon-nucleon channel whereas in the kaon-nucleon channel there are no known particles. As a result different behaviors are expected in backward scattering. Since the particles controlling the backward scattering are fermions, fermion trajectories are discussed from the point of view of recent developments. The contributions of the Regge poles in the $K\bar{K} \rightarrow N\bar{N}$ process to forward $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$ scatterings are calculated. Differential cross section formulas for forward and backward scattering as well as total cross sections are given in terms of trajectories and residues.

I n t r o d u c t i o n

During the last few years two major developments have almost completely changed strong interactions physics. The first of these came about with the analytic continuation by Mandelstam¹ of the two-body-in two-body-out scattering amplitude in both the energy and the momentum transfer variables. Amplitudes describing three related processes were thereby interpreted as the boundary values of the same function. In the new S-matrix theory which followed this development, the analyticity in these two variables and the unitarity condition, which has to be simultaneously satisfied for three related processes, played the central role.^{2, 2}

The second development arose from Regge's discovery³ that the partial wave scattering amplitude might be analytically continued as a function of energy and angular momentum into the complex angular momentum (J) plane. In this way he was able to convert the partial wave expansion into an integral and define the scattering amplitude for unphysical values of the scattering angle. It followed that the poles of the partial wave scattering amplitudes in the J -plane (Regge poles) control the high energy behavior of the scattering amplitudes in crossed channels.

In this paper the high energy scattering of kaons from nucleons is investigated within the framework of these developments. The problem is similar to the pion-nucleon problem (which was investigated by Virendra Singh)⁴ as far as the spin is concerned. But the s and u -channels are not symmetric as in the π - N case, because the K -meson is not its own anti-particle. In general wherever the two problems are similar the same definitions and derivations are used.

In section II the Lorentz invariant variables are defined and the physical regions given. The analytical continuation of the partial wave amplitudes in J

both for the non-relativistic and relativistic cases are given in section III, introducing thereby also the J-parity. Section IV defines the amplitudes of the s and u-channels. The partial waves are analytically continued in J. The even and odd continuations are defined and the amplitudes are expressed in terms of these continuations. Finally the Sommerfeld-Watson transformation is applied. The natural variable of the amplitudes used is not the energy-squared but the energy. The amplitudes satisfy Mac-Dowell's⁵ reflection principle which leads to some important consequences about fermion trajectories. Recent developments and some new possibilities for fermion Regge poles are also discussed. The quantum numbers and the Regge poles of the s-and u-channels are discussed and possible trajectories suggested in accordance with what was said about the fermion Regge poles.

In section V backward scattering differential cross sections are obtained using the results of section IV. Since the crossing matrices are given for the invariant amplitudes A and B only and the B amplitude changes sign under $s \leftrightarrow u$ crossing, the expressions for the physical amplitudes are rather complicated. Moreover the fermion Regge poles become imaginary for negative energy-squared as a result of which the amplitudes oscillate. Singh in his paper⁶ UCRL-10416 had expressed the hope that for not too high energies in a certain region of the Mandelstam diagram one might use real Regge poles. From purely kinematical considerations it is shown that exactly in this region the Regge formalism can unfortunately not be used.

The t-channel amplitudes and the Regge poles in this channel are discussed in section VI. Finally the expressions for forward and total cross sections are derived and compared with experimental results.

K i n e m a t i c s

Mandelstam has shown that the two-body scattering amplitude describes three related physical processes when analytically continued both in energy and momentum transfer variables.

For kaon-nucleon scattering these three processes are:

$$\begin{aligned} s) & K + N \rightarrow K + N \\ t) & K + \bar{K} \rightarrow N + \bar{N} \\ u) & \bar{K} + K \rightarrow \bar{K} + N \end{aligned} \quad (2.1)$$

In Fig.1. kaon-nucleon scattering is schematically shown. As usual the invariant variables s, t and u are defined by:

$$\begin{aligned} s &= (p_1 + p_2)^2 \\ t &= (p_1 + p_4)^2 \\ u &= (p_1 + p_3)^2 \end{aligned} \quad (2.2)$$

where p_i is the 4-momentum of the incoming particle i .

These three variables are not independent since they satisfy the relation

$$s + t + u = 2(M^2 + m_K^2) \quad (2.3)$$

where M is the nucleon mass and m_K the kaon mass. The mass differences between different members of the same isotopic multiplet are ignored throughout this paper.

To preserve the symmetry all three variables are kept and used as tri-angular coordinates in the Mandelstam diagram Fig.2 when they are real. The metric we use is such that:

$$\begin{aligned} \text{if } p_i &= (E_i, \vec{p}_i) \quad p_j = (E_j, \vec{p}_j) \\ p_i p_j &= E_i E_j - \vec{p}_i \cdot \vec{p}_j \end{aligned} \quad (2.4)$$

We also take $\hbar = 1$; $c = 1$

From equations (2.2) the relation of s,t,u to the barycentric-system scattering angle and momentum may be found. For the s-channel they are

$$\begin{aligned} s &= M^2 + m_K^2 + 2q_s^2 + 2 \sqrt{(M^2 + q_s^2)(m_K^2 + q_s^2)} \\ t &= -2q_s^2(1 - \cos \theta_s) \\ u &= M^2 + m_K^2 - 2q_s^2 \cos \theta_s - 2 \sqrt{(M^2 + q_s^2)(m_K^2 + q_s^2)} \end{aligned} \quad (2.5)$$

where q_s is the magnitude of the c.o.m momentum and $\cos \theta_s$ the c.o.m scattering angle for the case the s-channel is physical. From (2.5) the following useful relations can be derived,

$$\begin{aligned} u &= \frac{1 - \cos \theta_s}{2} \frac{(M - m_K)^2}{s} - \frac{1 + \cos \theta_s}{2} (s - 2M^2 - 2m_K^2) \\ s \cdot u &= (M^2 - m_K^2)^2 - 2q_s^2(1 + \cos \theta_s)s \\ q_s^2 &= \frac{[s - (M + m_K)^2][s - (M - m_K)^2]}{4s} = \frac{[(W_s + M)^2 - m_K^2][(W_s - M)^2 - m_K^2]}{4W_s^2} \end{aligned} \quad (2.6)$$

where $W_s^2 = s$

Similarly the u-channel variables are found to be :

$$\begin{aligned} u &= M^2 + m_K^2 + 2q_u^2 + 2 \sqrt{(M^2 + q_u^2)(m_K^2 + q_u^2)} \\ s &= M^2 + m_K^2 - 2q_u^2 \cos \theta_u - 2 \sqrt{(M^2 + q_u^2)(m_K^2 + q_u^2)} \\ t &= -2q_u^2(1 - \cos \theta_u) \end{aligned} \quad (2.7)$$

while the t-channel variables are :

$$\begin{aligned} t &= 4(m_K^2 + q^2) = 4(M^2 + p^2) \\ s &= -q^2 - p^2 - 2qp \cos \theta_t \\ u &= -q^2 - p^2 + 2qp \cos \theta_t \end{aligned} \quad (2.8)$$

For the t-channel there are two momenta q and p ; q is the momentum of the kaon and p of the nucleon.

Now we can define the physical regions for each variable in the Mandelstam diagram Fig.(2). The physical regions for the three channels are given by :

$$-1 < \cos \theta < 1 \quad (2.9)$$

$$\text{and } (\text{c.o.m total energy})^2 \gg (m_1 + m_j)^2 \quad (2.10)$$

where m_1 and m_j are the two masses for the channel in question and the c.o.m. scattering angle in this channel.

With the help of (2.10) and (2.6) the physical regions are found to be:

$$\begin{aligned} \text{For the s-channel} \quad & t < 0 \\ & s > (M + m_K)^2 \\ & s.u < (M^2 - m_K^2)^2 \end{aligned} \quad (2.11)$$

$$\begin{aligned} \text{For the u-channel} \quad & t < 0 \\ & u > (M + m_K)^2 \\ & s.u < (M^2 - m_K^2)^2 \end{aligned} \quad (2.12)$$

$$\begin{aligned} \text{For the t-channel} \quad & t > (2M)^2 \\ & s.u > (M^2 - m_K^2)^2 \end{aligned} \quad (2.13)$$

Thus the s and u-channels are bounded by the line $t = 0$ (which represents forward scattering) and by one branch of the hyperbola

$$s.u = (M^2 - m_K^2)^2 \quad (2.14)$$

(which represents backward scattering).

The t-channel is the inside of the other branch of this hyperbola.

III

Regge Formalism

Analytic continuation of the partial wave amplitudes into the complex angular momentum plane and Regge poles: in the non-relativistic case:³

Before applying the Regge formalism to the kaon-nucleon problem it would be useful to look at the non-relativistic potential scattering of spinless particles which contains all the basic ideas we'll make use of later.

If one is given a superposition of Yukawa potentials

$$V(r) = \int_0^\infty \frac{e^{-\mu r}}{r} \rho_0(\mu^2) d\mu^2 \quad (3.1)$$

it is known that the partial wave expansion of the scattering amplitude

$$f(E, \cos \theta) = \sum_{l=0}^{\infty} (2l+1) f_l(k) P_l(\cos \theta) \quad (3.2)$$

converges inside an ellipse⁸ whose foci are at $\cos \theta = +1$ and $\cos \theta = -1$; its major axis is $2(1 + \frac{\mu_0^2}{2E})$, where μ_0 is the range of the Yukawa potential and $E = k^2$ the square of the momentum. Thus the amplitude is an analytic function inside this ellipse. To obtain the scattering amplitude outside the ellipse the above series can be converted into an integral using the so called Sommerfeld-Watson transform.⁹

$$f(E, \cos \theta) = \frac{1}{2i} \int_C \frac{(2l+1) f(l, k) P_l(-\cos \theta)}{\sin \pi l} dl \quad (3.3)$$

Here the contour C encloses only the poles coming from the denominator $\sin \pi l$ and shown in Fig.(3). The amplitudes $f(l, k)$ are the analytical continuations of the partial amplitudes $f_l(k)$ into the complex l plane.

One can easily verify that the residues at these poles give the original series:

$$2\pi i (\text{Residue at the pole } l=n) = 2\pi i \left[\frac{1}{2i} (2n+1) f(n, k) P_n(-\cos \theta) \lim_{l \rightarrow n} \frac{l-n}{\sin \pi l} \right]$$

$l \rightarrow n$

using l'Hôpital rule for the limit one finds

$$\begin{aligned}
 2\pi i \lim_{l \rightarrow n} (\text{Residue at the pole } l=n) &= \pi(2n+1) f_n(k) P_n(-\cos\theta) \frac{1}{\pi \cos \pi n} \\
 &= (2n+1) f_n(k) P_n(\cos\theta) (-1)^n \frac{1}{(-1)^n} \\
 &= (2n+1) f_n(k) P_n(\cos\theta)
 \end{aligned} \quad (3.4)$$

For the analytic continuation of the partial waves $f_l(k)$ into the complex l -plane Regge solved the Schroedinger equation for complex l . He also showed that $f(l, k)$ are meromorphic functions to the right of the line $\text{Re } l = -\frac{1}{2}$.

Now in addition to the poles coming from the denominator in (3.3) the amplitudes $f(l, k)$ may have poles (Regge poles). If one distorts the contour as shown in Fig. (4) as to include these poles but remaining to the right of $\text{Re } l = -\frac{1}{2}$ one picks up the contributions of these poles in addition to the poles coming from the denominator $\sin \pi l$. The integral along the infinite semi-circle vanishes provided $f(l, k)$ decreases fast enough in any direction $-\frac{\pi}{2} < \arg l < \frac{\pi}{2}$. It turns out that for real values of k or positive values of E the Regge poles are only in the first quadrant of the $J_{-\frac{1}{2}}$ plane. They move as the energy changes. Therefore they are sometimes also called moving poles.

Writing the Cauchy integral one finds

$$\begin{aligned}
 \frac{1}{2i} \oint_{C'} \frac{(2l+1) f(l, k) P_l(-\cos\theta)}{\sin \pi l} dl &= \frac{1}{2i} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{(2l+1) f(l, k) P_l(-\cos\theta)}{\sin \pi l} dl \\
 &+ \frac{1}{2i} \int_{\gamma} \frac{(2l+1) f(l, k) P_l(-\cos\theta)}{\sin \pi l} dl \\
 &= 2\pi i \sum_{\text{Regge poles}} \text{Residues} + 2\pi i \sum_{\substack{\text{Poles from} \\ \sin \pi l}} \text{Residues}
 \end{aligned} \quad (3.5)$$

as the contribution from the infinite semi circle vanishes we get

$$\frac{1}{2i} \int_{-\frac{1}{2} - i\infty}^{-\frac{1}{2} + i\infty} \frac{(2l+1)f(l,k)P_l(-\cos\theta)}{\sin \pi l} dl = 2\pi i \sum_{\text{Regge poles}} \frac{1}{2i} (2\alpha_i(E)+1) \frac{P_{\alpha_i(E)}(-\cos\theta)}{\sin \pi \alpha_i(E)} (\text{Residue of } f(l,k))_{l=\alpha_i(E)} + \sum_{l=0}^{\infty} (2l+1) f_l(k) P_l(\cos\theta) \quad (3.6)$$

where the last term is the scattering amplitude. Thus one obtains

$$f(E, \cos\theta) = \frac{1}{2i} \int_{-\frac{1}{2} - i\infty}^{-\frac{1}{2} + i\infty} \frac{(2l+1)f(l,k)P_l(-\cos\theta)}{\sin \pi l} dl + \sum_{\text{Regge poles}} (2\alpha_i(E)+1) \frac{P_{\alpha_i(E)}(-\cos\theta)}{\sin \pi \alpha_i(E)} \cdot \beta_i(k) \quad (3.7)$$

where $\beta_i(k) = (-\pi \text{Residue of } f(l,k))_{l=\alpha_i(E)}$

If one is not using the Schroedinger equation then one has to find another way for analytically continuing the partial wave amplitudes $f_l(k)$. For the Mandelstam representation this continuation was given by Froissart and Gribov. It is based on the known analytic continuation of the Legendre functions of the second kind. We'll first look at the case where there is no exchange potential because it introduces a complication called J-parity, which will be treated later. In this simple case the scattering amplitude is given by²

$$f(k^2, t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{\rho_0(t')}{t' - t} + \frac{1}{\pi^2} \int_0^{\infty} \int_{t_0}^{\infty} dk'^2 dt' \frac{\rho(k'^2, t')}{(k'^2 - k^2)(t' - t)} \quad (3.8)$$

where ρ_0 is the weight function of the Yukawa potentials already introduced in (3.1). ρ is the double spectral function and t is defined by $t = -2k^2(1 - \cos\theta)$

Or expressing in terms of the t discontinuity:

$$f_t(k^2, t) = \rho_0(t) + \frac{1}{\pi} \int dk'^2 \frac{\rho(k'^2, t)}{k'^2 - k^2}, \quad (3.9)$$

we have

$$f(k^2, t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{f_t(k^2, t')}{t' - t} \quad (3.10)$$

Projecting out the partial wave $f_1(k^2)$, one finds

$$\begin{aligned} f_1(k^2) &= \frac{1}{2} \int_{-1}^1 f(k^2, t) P_1(\cos \theta) d \cos \theta \\ &= \frac{1}{2\pi} \int_{-1}^1 \int_{t_0}^{\infty} \frac{f_t(k^2, t')}{t' - t} P_1(\cos \theta) d \cos \theta dt' \end{aligned} \quad (3.11)$$

Then, using Neumann's definition of the Legendre function of the second kind

$$Q_1(z) = \frac{1}{2} \int_{-1}^1 \frac{P_1(z')}{z - z'} dz' \quad (3.12)$$

and interchanging the order of integration, we have

$$\int_{-1}^1 \frac{P_1(\cos \theta)}{t' + 2k^2(1 - \cos \theta)} d \cos \theta = \frac{1}{2k^2} \int_{-1}^1 \frac{P_1(\cos \theta)}{\frac{t'}{2k^2} + 1 - \cos \theta} d \cos \theta = \frac{1}{k^2} Q_1\left(1 + \frac{t'}{2k^2}\right) \quad (3.13)$$

Hence

$$f_1(k^2) = \frac{1}{2\pi k^2} \int_{t_0}^{\infty} f_t(k^2, t') Q_1\left(1 + \frac{t'}{2k^2}\right) dt' \quad (3.14)$$

This is a suitable expression for the analytic continuation of $f_1(k^2)$ since the analytic continuation of Q_1 is known and its asymptotic behavior is like $1/z^{1+1}$. Thus if the absorptive part $f_t(k^2, t)$ behaves like a power of t for large t

$$f_t(k^2, t) \sim t^{N(k^2)} \quad t \rightarrow \infty \quad (3.15)$$

then the integral converges for

$$\operatorname{Re} l > \operatorname{Re} N \quad (3.16)$$

and defines $f(l, k^2)$.

For large $z = \cos \theta$ we see from eq.(3.7) that the integral along the line

$\operatorname{Re} l = -\frac{1}{2}$ vanishes, because

$$P_1(-z) \sim z^{\frac{1}{2}} = z^{\frac{1}{2} \operatorname{Re} l + i \operatorname{Im} l} = z^{-\frac{1}{2} + i \operatorname{Im} l} \rightarrow 0 \quad (3.17)$$

for $z \rightarrow \infty$

The Regge pole terms will behave like

$$\frac{(2\alpha_i(E) + 1)(-z)^{\alpha_i(E)}}{\sin \pi \alpha_i(E)} \beta_i(k) \quad (3.18)$$

$$\text{Since } t = -2k^2(1 - z) \\ t \sim z$$

Redefining $\beta_i(k)$ one finds for large t

$$f(E, \cos \theta) \approx (2\alpha(E) + 1) \frac{t^{\alpha(E)}}{\sin \pi \alpha(E)} \beta(E) \quad (3.19)$$

$t \rightarrow \infty$

where now $\alpha(E)$ is the Regge pole farthest to the right in the l -plane. From the expressions (3.14), (3.15), (3.16) and (3.19) one sees that the analytically continued partial wave amplitude is holomorphic to the right of the Regge pole with the largest real part. Obviously $f(l, k^2)$ coincides with $f_1(k^2)$ for integer values of l . Squires¹² has investigated the conditions for the uniqueness of the continuation. Equation (3.14) satisfies all these conditions.

Analytic continuation in the relativistic case and J-parity.

In potential scattering with an exchange potential or in the purely relativistic case the scattering amplitude can be written as

$$A(s, t, u) = \frac{1}{\pi} \int \frac{A_t(s, t')}{t' - t} dt' + \frac{1}{\pi} \int \frac{A_u(s, u')}{u' - u} du' \quad (3.20)$$

which is the fixed energy dispersion relation with A_t and A_u the absorptive parts in the t and u channels. If the scattered particles are of equal mass the variables are defined as:

$$\begin{aligned} s &= 4(q^2 + m^2) \\ t &= -2q^2(1 - \cos \theta) \\ u &= -2q^2(1 + \cos \theta) \end{aligned} \quad (3.21)$$

This gives

$$A(s, t, u) = \frac{1}{\pi} \int_{-m^2}^{\infty} \frac{A_t(s, t')}{t' + 2q^2(1 - \cos \theta)} dt' + \frac{1}{\pi} \int_{4m^2}^{\infty} \frac{A_u(s, u')}{u' + 2q^2(1 + \cos \theta)} du', \quad (3.22)$$

and projecting out the 1th partial wave,

$$A_1(s) = \frac{1}{2\pi q^2} \iint \frac{A_t(s, t') P_1(\cos \theta)}{(1 + \frac{t'}{2q^2}) - \cos \theta} d \cos \theta dt' + \frac{1}{2\pi q^2} \iint \frac{A_u(s, u') P_1(\cos \theta)}{(1 + \frac{u'}{2q^2}) + \cos \theta} d \cos \theta du' \quad (3.23)$$

interchanging the order of integration and using again Neuman's formula for the Legendre functions of the second kind (3.12) one gets

$$A_1(s) = \frac{1}{\pi q^2} \int A_t(s, t') Q_1(1 + \frac{t'}{2q^2}) dt' - \frac{1}{\pi q^2} \int A_u(s, u') Q_1(-1 - \frac{u'}{2q^2}) du' \\ = \frac{1}{\pi q^2} \int A_t(s, t') Q_1(1 + \frac{t'}{2q^2}) dt' - \frac{1}{\pi q^2} \int A_u(s, u') (-1)^{1+1} Q_1(1 + \frac{u'}{2q^2}) du' \quad (3.24)$$

or, changing the dummy variable u' to t' , one finds

$$A_1(s) = \frac{1}{\pi q^2} \int \left[A_t(s, t') + (-1)^1 A_u(s, t') \right] Q_1(1 + \frac{t'}{2q^2}) dt' \quad (3.25)$$

This expression would be suitable for an analytical continuation were it not for the factor $(-1)^1$ which diverges for a large negative imaginary part of l and makes the Sommerfeld-Watson transform impossible

$$(-1)^l = e^{i\pi(\text{Re } l + i \text{Im } l)} = e^{i\pi \text{Re } l} e^{-\pi \text{Im } l} \quad (3.26)$$

One way to get around this difficulty is to define two continuations one away from even values of l and the other away from odd values of l . Thus the even continuation agrees with the physical amplitude only at even integers whereas the odd continuation agrees with it only at odd integers. This is called J-parity;

in the first case the J-parity is said to be even and in the second case odd.

KN $\rightarrow$ KN and $\bar{K}N \rightarrow \bar{K}N$ channels

Amplitudes of the processes KN $\rightarrow$ KN and $\bar{K}N \rightarrow \bar{K}N$

The S-matrix for kaon - nucleon scattering is defined as in the case of pion - nucleon scattering:

$$S_{fi} = \delta_{fi} - i(2\pi)^4 \delta(q_i + p_i - q_f - p_f) \sqrt{\frac{M^2}{4\omega_i \omega_f E_i E_f}} \tau_{fi} \quad (4.1)$$

where M is the nucleon mass

ω_i = total energy of the initial kaon

ω_f = total energy of the final kaon

E_i = total energy of the initial nucleon

E_f = total energy of the final nucleon

τ_{fi} is the T matrix element between the initial and final nucleon Dirac spinors u_i and $\bar{u}_f$

$$\tau_{fi} = \bar{u}_f T u_i \quad (4.2)$$

If the kaon is a pseudoscalar particle T can be decomposed into two spin independent amplitudes in the following way

$$T = -AI + i\gamma \cdot Q\mathbf{B} \quad (4.3)$$

$$\text{where } Q = \frac{q_i + q_f}{2} \quad (4.4)$$

and γ = the Dirac matrix 4-vector

The differential cross section is given in the center-of-mass system as:

$$\frac{d\sigma}{d\Omega} = \sum \left| \langle \text{final} | f | \text{initial} \rangle \right|^2 \quad (4.5)$$

where the sum indicates an average over the initial and a sum over the final spin states.

Here f can be defined either as

$$f = f_1 + \frac{(\vec{\sigma} \cdot \vec{q}_f)(\vec{\sigma} \cdot \vec{q}_1)}{q_1 q_f} f_2 \quad (4.6)$$

or
$$f = f_{no} + i \vec{\sigma} \cdot \vec{n} f_{s.f} \sin \theta \quad (4.7)$$

where $\vec{n}$ is the unit vector perpendicular to the plane defined by the initial and final kaon momenta

$$\vec{n} = \frac{\vec{q}_1 \times \vec{q}_f}{q_1 q_f \sin \theta} \quad (4.8)$$

f_{no} = Non spin flip amplitude

$f_{s.f}$ = Spin flip amplitude

In terms of these amplitudes the differential cross section becomes

$$\frac{d\sigma}{d\Omega} = |f_1 + f_2|^2 - 2(1 - \cos \theta) \operatorname{Re}(f_1^* f_2) \quad (4.9)$$

or
$$\frac{d\sigma}{d\Omega} = |f_{no}|^2 + |f_{s.f}|^2 \sin^2 \theta \quad (4.10)$$

the relation between f_1 , f_2 and f_{no} , $f_{s.f}$ being

$$\begin{aligned} f_1 + \frac{(\vec{\sigma} \cdot \vec{q}_f)(\vec{\sigma} \cdot \vec{q}_1)}{q_1 q_f} f_2 &= f_1 + \frac{1}{q_1 q_f} (\vec{q}_f \cdot \vec{q}_1 + i \vec{\sigma} \cdot (\vec{q}_f \times \vec{q}_1)) f_2 \\ &= f_1 + \cos \theta f_2 + i \vec{\sigma} \cdot \vec{n} f_2 \sin \theta \end{aligned} \quad (4.11)$$

$$f_1 + \cos \theta f_2 = f_{no} \quad (4.12)$$

$$f_2 = f_{s.f} \quad (4.13)$$

The relations between f_1 , f_2 and the invariant amplitudes A, B are¹³

$$f_1 = \frac{E+M}{8\pi W} [A + (W-M)B] \quad (4.14)$$

$$f_2 = \frac{E-M}{8\pi W} [-A + (W+M)B] \quad (4.15)$$

or

$$f_1 = \frac{(W+M)^2 - m_K^2}{16\pi W^2} [A + (W-M)B] \quad (4.16)$$

$$f_2 = \frac{(W-M)^2 - m_K^2}{16\pi W^2} [-A + (W+M)B] \quad (4.17)$$

A and B in terms of f_1 and f_2 are

$$A = \frac{4\pi}{E^2 - M^2} [(W+M)(E-M)f_1 - (W-M)(E+M)f_2] \quad (4.18)$$

$$B = \frac{4\pi}{E^2 - M^2} [(E-M)f_1 + (E+M)f_2] \quad (4.19)$$

Here W is the total c.o.m energy and E the total c.o.m nucleon energy

$$E = \frac{W^2 + M^2 - m_K^2}{2W} \quad (4.20)$$

Since A and B are functions of s, t and u ; e.g of W^2 and not W one has the relation

$$f_1(-W) = -f_2(W)$$

which was first pointed out by Mac Dowell. We will see later that this reflection principle leads to very important consequences.

The decomposition of f_1 and f_2 into the partial wave amplitudes is given by

$$f_1 = \sum_J (f_{J-\frac{1}{2}, J-\frac{1}{2}}^J P_{J+\frac{1}{2}}' - f_{J+\frac{1}{2}, J+\frac{1}{2}}^J P_{J-\frac{1}{2}}') \quad (4.21a)$$

$$f_2 = \sum_J (f_{J+\frac{1}{2}, J+\frac{1}{2}}^J P_{J+\frac{1}{2}}' - f_{J-\frac{1}{2}, J-\frac{1}{2}}^J P_{J-\frac{1}{2}}') \quad (4.21b)$$

where J runs over $\frac{1}{2}, 3/2, 5/2, \dots$. Here $f_{J-\frac{1}{2}, J-\frac{1}{2}}^J$ is the transition amplitude from an initial state with total angular momentum J and orbital angular momentum $J-\frac{1}{2}$ to a final state with the same total angular momentum and orbital angular momentum. $f_{J+\frac{1}{2}, J+\frac{1}{2}}^J$ is the corresponding amplitude for the orbital angular momentum $J+\frac{1}{2}$. Since the total angular momentum and parity are conserved there are no transitions between the following orbital angular momentum states:

$$J-\frac{1}{2} \rightarrow J+\frac{1}{2}$$

$$J+\frac{1}{2} \rightarrow J-\frac{1}{2}$$

Therefore we'll abbreviate the amplitudes in the following manner

$$\begin{aligned} f_{J-\frac{1}{2}, J-\frac{1}{2}}^J &\equiv f_{J-\frac{1}{2}}^J \\ f_{J+\frac{1}{2}, J+\frac{1}{2}}^J &\equiv f_{J+\frac{1}{2}}^J \end{aligned} \quad (4.22)$$

The partial wave amplitudes can be projected out from eq.s(4.21) using the orthogonality of Legendre polynomials:

$$f_{J-\frac{1}{2}} = \frac{1}{2} \int_{-1}^1 (f_1 P_{J-\frac{1}{2}} + f_2 P_{J+\frac{1}{2}}) dz \quad (4.23a)$$

$$f_{J+\frac{1}{2}} = \frac{1}{2} \int_{-1}^1 (f_1 P_{J+\frac{1}{2}} + f_2 P_{J-\frac{1}{2}}) dz \quad (4.23b)$$

The partial waves of A and B are defined by

$$A_{J-\frac{1}{2}} = \int_{-1}^1 A(s, u, t) P_{J-\frac{1}{2}}(z) dz \quad (4.24a)$$

$$B_{J-\frac{1}{2}} = \int_{-1}^1 B(s, u, t) P_{J-\frac{1}{2}}(z) dz \quad (4.24b)$$

The relations between the partial waves of f_1, f_2 and A, B are:

$$f_{J-\frac{1}{2}} = \frac{E+M}{16\pi W} \left[A_{J-\frac{1}{2}} + (W-M)B_{J-\frac{1}{2}} \right] + \frac{E-M}{16\pi W} \left[-A_{J+\frac{1}{2}} + (W+M)B_{J+\frac{1}{2}} \right] \quad (4.25)$$

$$f_{J+\frac{1}{2}} = \frac{E+M}{16\pi W} \left[A_{J+\frac{1}{2}} + (W-M)B_{J+\frac{1}{2}} \right] + \frac{E-M}{16\pi W} \left[-A_{J-\frac{1}{2}} + (W+M)B_{J-\frac{1}{2}} \right] \quad (4.26)$$

From these formulas we see that one can analytically continue $f_{J-\frac{1}{2}}$ and $f_{J+\frac{1}{2}}$

if one knows how to continue $A_{J-\frac{1}{2}}$ and $B_{J-\frac{1}{2}}$. The amplitudes we are interested

in are essentially $f_{J-\frac{1}{2}}$ and $f_{J+\frac{1}{2}}$ (aside from their kinematical singularities which we will discuss later) because these are the partial wave amplitudes which give us f_1 and f_2 which in return give the cross sections. Thus the Sommerfeld-Watson transform will essentially be applied to $f_{J-\frac{1}{2}}$ and $f_{J+\frac{1}{2}}$.

The analytical continuation of the partial wave amplitudes for unequal masses:

For the kaon-nucleon problem the continuation process is similar to the one for the pion-nucleon case as treated by Singh^{4,6}. Except for the limits of integrals the derivation is the same and is included here for the sake of completeness. The scattering amplitude satisfies again the fixed energy dispersion relation:

$$A(s, t, u) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{A_t(s, t')}{t' - t} dt' + \frac{1}{\pi} \int_{(M+m_K)^2}^{\infty} \frac{A_u(s, u')}{u' - u} du' \quad (4.27)$$

where now s and t are given by (2.5). For u we use the relation (2.6). Substituting for t and u these expressions one finds

$$A(s, t, u) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} \frac{A_t(s, t')}{t' + 2q_s^2 (1 - \cos \theta)} dt' + \frac{1}{\pi} \int_{(M+m_K)^2}^{\infty} \frac{A_u(s, u')}{u' - \frac{(M^2 - m_K^2)^2}{s} + 2q_s^2 (1 + \cos \theta)} du' \quad (4.28)$$

Projecting out a partial wave, interchanging the order of integration and using again Neuman's formula for the Legendre polynomials of the second kind (3.12) one gets

$$A_{J+\frac{1}{2}}(s) = \frac{1}{\pi q_s^2} \int_{(m_K^2)^2}^{\infty} A_t(s, t') Q_{J+\frac{1}{2}} \left(1 + \frac{t'}{2q_s^2}\right) dt' - \frac{1}{\pi q_s^2} \int_{(M+m_K)^2}^{\infty} A_u(s, u') Q_{J+\frac{1}{2}} \left(-1 - \frac{u' - \frac{(M^2 - m_K^2)^2}{s}}{2q_s^2}\right) du' \quad (4.29)$$

Making a change of variable $u' = \frac{(M^2 - m_K^2)^2}{s} = t'$ (4.30)

where t' is a dummy variable and using again

$$Q_1(-z) = (-1)^{1+1} Q_1(z) \quad (4.31)$$

$$A_{J+\frac{1}{2}}(s) = \frac{1}{\pi q_s^2} \int \left[A_t(s, t') + (-1)^{J+\frac{1}{2}} A_u\left(s, t' + \frac{(M^2 - m_K^2)^2}{s}\right) \right] Q_{J+\frac{1}{2}} \left(1 + \frac{t'}{2q_s^2}\right) dt' \quad (4.32)$$

Again to avoid the divergence caused by $(-1)^{J+\frac{1}{2}}$ one defines an even and an odd continuation of the amplitudes $A_{J+\frac{1}{2}}$, even continuation agreeing with the physical amplitude for even values of $(J+\frac{1}{2})$ and the odd continuation for odd values of $(J+\frac{1}{2})$, e.g. in J plane we want the even continuations for both $J+\frac{1}{2}$ and $J-\frac{1}{2}$ amplitudes to be physical for $J = 1/2, 5/2, 9/2, \dots$ whereas the odd continuations should be physical for $J = 3/2, 7/2, 11/2, \dots$

$$A_{J-\frac{1}{2}}^{e,o}(s) = \frac{1}{\pi q_s^2} \int \left[A_t(s, t') \pm A_u\left(s, t' + \frac{(M^2 - m_K^2)^2}{s}\right) \right] Q_{J-\frac{1}{2}} \left(1 + \frac{t'}{2q_s^2}\right) dt' \quad (4.33)$$

$$A_{J+\frac{1}{2}}^{e,o}(s) = \frac{1}{\pi q_s^2} \int \left[A_t(s, t') \mp A_u\left(s, t' + \frac{(M^2 - m_K^2)^2}{s}\right) \right] Q_{J+\frac{1}{2}} \left(1 + \frac{t'}{2q_s^2}\right) dt' \quad (4.34)$$

where the upper sign refers to even and the lower sign to odd continuation. Now we know how to analytically continue $A_{J+\frac{1}{2}}$ and $B_{J+\frac{1}{2}}$. In fact we have

two continuations for each partial wave amplitude: $A_{J+\frac{1}{2}}^e$, $A_{J+\frac{1}{2}}^o$ and $B_{J+\frac{1}{2}}^e$, $B_{J+\frac{1}{2}}^o$.

Thus using the formulas (4.25) and (4.26) one also obtains the even and odd continuations of $f_{J+\frac{1}{2}}$ amplitudes. We now have to express f_1 and f_2 in terms

of f_1^e and f_1^o because those are the amplitudes we know how to continue. For this we write the series (4.21) as follows:

$$f_1 = f_1^e + f_1^o = \sum_{\substack{1/2, 3/2, \dots \\ J-1/2}} f_1^e P'_{J-1/2, J+1/2} - \sum_{\substack{1/2, 3/2, \dots \\ J+1/2}} f_1^e P'_{J+1/2, J-1/2} + \sum_{\substack{3/2, 5/2, \dots \\ J-1/2}} f_1^o P'_{J-1/2, J+1/2} - \sum_{\substack{3/2, 5/2, \dots \\ J+1/2}} f_1^o P'_{J+1/2, J-1/2} \quad (4.35)$$

defining the first two series as f_1^e and the last two as f_1^o (because by definition the even continuations are the ones which are physical for $J = 1/2, 5/2, 9/2, \dots$ whereas the odd continuations are physical for $J = 3/2, 7/2, 11/2, \dots$).

Using

$$P'_1(-z) = (-1)^{1+1} P'_1(z)$$

f_1^e and f_1^o can also be written as:

$$f_1^{e,o} = \sum_{\substack{1/2, 3/2, \dots \\ J-1/2}} f_1^{e,o} \left(\frac{P'_{J+1/2}(\cos\theta) \pm P'_{J+1/2}(-\cos\theta)}{2} \right) - \sum_{\substack{1/2, 3/2, \dots \\ J+1/2}} f_1^{e,o} \left(\frac{P'_{J-1/2}(\cos\theta) \mp P'_{J-1/2}(-\cos\theta)}{2} \right) \quad (4.36)$$

where the upper sign refers to f_1^e and the lower sign to f_1^o .

Similarly for f_2

$$f_2 = f_2^e + f_2^o = \sum_{\substack{1/2, 3/2, \dots \\ J+1/2}} f_2^e P'_{J+1/2, J+1/2} - \sum_{\substack{1/2, 3/2, \dots \\ J-1/2}} f_2^e P'_{J-1/2, J-1/2} + \sum_{\substack{3/2, 5/2, \dots \\ J+1/2}} f_2^o P'_{J+1/2, J+1/2} - \sum_{\substack{3/2, 5/2, \dots \\ J-1/2}} f_2^o P'_{J-1/2, J-1/2} \quad (4.37)$$

$$f_2^{e,o} = \sum_{\substack{1/2, 3/2, \dots \\ J+1/2}} f_2^{e,o} \left(\frac{P'_{J+1/2}(\cos\theta) \pm P'_{J+1/2}(-\cos\theta)}{2} \right) - \sum_{\substack{1/2, 3/2, \dots \\ J-1/2}} f_2^{e,o} \left(\frac{P'_{J-1/2}(\cos\theta) \mp P'_{J-1/2}(-\cos\theta)}{2} \right) \quad (4.38)$$

Since now f_1 and f_2 are expressed in terms of $f_{J+\frac{1}{2}}^e$ and $f_{J+\frac{1}{2}}^o$ the analytical continuations of which we already know we can apply the Sommerfeld-Watson transform and convert the series into integrals. But before doing this we have to settle a final question which is the singularity structure of $f_{J+\frac{1}{2}}^e$ and $f_{J+\frac{1}{2}}^o$ amplitudes. Keeping this point in mind let us write the Sommerfeld-Watson integral for f_1 and f_2 .

$$f_{1,2} = \frac{1}{2} \oint f_{J+\frac{1}{2}}^e \left(\frac{P'_{J+\frac{1}{2}}(\cos\theta) + P'_{J+\frac{1}{2}}(-\cos\theta)}{2} \right) \frac{dJ}{\cos\pi J} - \frac{1}{2} \oint f_{J+\frac{1}{2}}^e \left(\frac{P'_{J-\frac{1}{2}}(\cos\theta) - P'_{J-\frac{1}{2}}(-\cos\theta)}{2} \right) \frac{dJ}{\cos\pi J} \\ - \frac{1}{2} \oint f_{J+\frac{1}{2}}^o \left(\frac{P'_{J+\frac{1}{2}}(\cos\theta) - P'_{J+\frac{1}{2}}(-\cos\theta)}{2} \right) \frac{dJ}{\cos\pi J} + \frac{1}{2} \oint f_{J+\frac{1}{2}}^o \left(\frac{P'_{J-\frac{1}{2}}(\cos\theta) - P'_{J-\frac{1}{2}}(-\cos\theta)}{2} \right) \frac{dJ}{\cos\pi J} \quad (4.39)$$

where the upper signs refer to f_1 and the lower signs to f_2 .

Singularities of $f_{J+\frac{1}{2}}^{e,o}$ and $f_{J-\frac{1}{2}}^{e,o}$ amplitudes:

Since $f_{J+\frac{1}{2}}^{e,o}$ and $f_{J-\frac{1}{2}}^{e,o}$ are linear combinations of $A_{J+\frac{1}{2}}^{e,o}$, $A_{J-\frac{1}{2}}^{e,o}$ and $B_{J+\frac{1}{2}}^{e,o}$, $B_{J-\frac{1}{2}}^{e,o}$

they have the same Mandelstam singularities. But from the expressions (4.25) and (4.26) it can be seen that in addition to Mandelstam singularities $f_{J+\frac{1}{2}}^{e,o}$ and $f_{J-\frac{1}{2}}^{e,o}$ have also kinematical singularities in s (or u) plane arising from the factors containing W , e.g. $\sqrt{s}$ (or $\sqrt{u}$). To avoid these kinematical singularities (4.15) it is therefore more suitable to work in the W -plane rather than s (or u) plane. There is also a pole at $W = 0$ arising from vanishing of the denominator in (4.25 or (4.26). Finally let us look at the threshold behavior of the amplitudes $f_{J+\frac{1}{2}}^{e,o}$.

From the dispersion relation (4.32) for partial wave amplitudes $A_{J+\frac{1}{2}}^{e,o}$ and $B_{J+\frac{1}{2}}^{e,o}$ it can be seen that those amplitudes behave like $k^{2(J+\frac{1}{2})}$ at $k^2 = 0$ or $s = (M+m_K)^2$ which is the physical threshold. Thus if one wants to avoid the kinematical square root singularities in s-plane, the pole at $W = 0$ and if one wishes to take out the threshold behavior one defines the Frazer-Fulco amplitudes, which are functions of W instead of s (or u).^{4,15}

$$h_{J-\frac{1}{2}}^{e,o}(W) = \frac{16\pi W}{E+M} \frac{f_{J-\frac{1}{2}}^{e,o}}{(2k^2)^{J-\frac{1}{2}}} \quad (4.40a)$$

$$h_{J+\frac{1}{2}}^{e,o}(W) = \frac{16\pi W}{E-M} \frac{f_{J+\frac{1}{2}}^{e,o}}{(2k^2)^{J-\frac{1}{2}}} \quad (4.40b)$$

which in terms of the eight partial wave amplitudes $A_{J\pm\frac{1}{2}}^{e,o}$ and $B_{J\pm\frac{1}{2}}^{e,o}$ are given by the following expressions:

$$h_{J-\frac{1}{2}}^{e,o}(W) = \frac{A_{J-\frac{1}{2}}^{e,o}}{(2k^2)^{J-\frac{1}{2}}} + (W-M) \frac{B_{J-\frac{1}{2}}^{e,o}}{(2k^2)^{J-\frac{1}{2}}} + 2(E-M)^2 \left[-\frac{A_{J+\frac{1}{2}}^{e,o}}{(2k^2)^{J+\frac{1}{2}}} + (W+M) \frac{B_{J+\frac{1}{2}}^{e,o}}{(2k^2)^{J+\frac{1}{2}}} \right] \quad (4.41a)$$

$$h_{J+\frac{1}{2}}^{e,o}(W) = 2(E+M)^2 \left[\frac{A_{J+\frac{1}{2}}^{e,o}}{(2k^2)^{J+\frac{1}{2}}} + (W-M) \frac{B_{J+\frac{1}{2}}^{e,o}}{(2k^2)^{J+\frac{1}{2}}} \right] + \left[-\frac{A_{J-\frac{1}{2}}^{e,o}}{(2k^2)^{J-\frac{1}{2}}} + (W+M) \frac{B_{J-\frac{1}{2}}^{e,o}}{(2k^2)^{J-\frac{1}{2}}} \right] \quad (4.41b)$$

These amplitudes also satisfy Mac Dowell's⁵ reflection principle which turns out to be an important property of fermions:

$$h_{J-\frac{1}{2}}^{e,o}(W) = -h_{J+\frac{1}{2}}^{e,o}(-W) \quad (4.42)$$

This relation shows that if the amplitude $h_{J-\frac{1}{2}}^{e,o}(W)$ has the pole $J = \alpha(W)$ in the

complex J -plane then the amplitude $h_{\substack{e,0 \\ J+\frac{1}{2}}}(W)$ has the pole $J = \alpha(-W)$. There is

a similar relation between the residues of these amplitudes at the poles.

F e r m i o n R e g g e P o l e s

We were led by the existence of kinematical singularities in the physical amplitudes $f_{J-\frac{1}{2}}$ and $f_{J+\frac{1}{2}}$ as functions of energy-squared to define new amplitudes $h_{J-\frac{1}{2}}$ and $h_{J+\frac{1}{2}}$ which are functions of energy W . It was seen that those amplitudes also satisfy Mac Dowell's reflection principle, which in turn gave us the complex J -plane poles $\alpha_1(W)$ and $\alpha_2(W)$ of $h_{J-\frac{1}{2}}$ and $h_{J+\frac{1}{2}}$ as functions of W again. $\alpha_1(W)$ and $\alpha_2(W)$ were related by

$$\alpha_1(W) = \alpha_2(-W)$$

On the basis of real analyticity and the reflection principle more can be inferred about the poles of these amplitudes which differ only in space parity. The function $\alpha_1(W)$ has at an arbitrary point W in the complex W -plane the same value as the function $\alpha_2(W)$ at the point $(-W)$. Thus in the J -plane $\alpha_1(W)$ and $\alpha_2(W)$ trajectories coincide the only difference being in the sign of the parameter W at the same points. Since $\alpha_2(-W)$ is just the reflection of $\alpha_2(W)$ both these functions have the same value at $W = 0$. Consequently $\alpha_1(W)$ also coincides with $\alpha_2(W)$ at $W = 0$ as was first pointed out by Gribov.¹⁶

If W is pure imaginary the two operations "taking the complex conjugate" and "taking the negative" become the same so that the Mac Dowell principle and the real analyticity can further be combined to show that the poles of the two amplitudes become complex conjugates of each other.

Thus writing for $W = ib$ where b is a real number one has from

$$\begin{aligned} \alpha_1(ib) &= \alpha_2(-ib) && \text{(reflection principle)} \\ \text{and } \alpha_2(-ib) &= \alpha_2^*(ib) && \text{(real analyticity)} \\ \alpha_1(ib) &= \alpha_2^*(ib) \end{aligned}$$

This also was first shown by Gribov. Thus in addition to the right hand cut

in $u = W^2$ plane which starts at $u = (M+m_\pi)^2$, where M is the fermion mass and m_π the pion mass, there is also a left hand cut starting at $u = 0$.

Now let us say one of the trajectories, say $\alpha_1(W)$ makes a particle or a resonance. If the real part of $\alpha_1(W)$ keeps on decreasing for $W < 0$ this will mean that the real part of $\alpha_2(W)$ will decrease for $W > 0$ and won't make a particle. On the other hand if $\alpha_2(W)$ also makes a particle this will mean that the real part of $\alpha_1(W)$ has to increase again for $W < 0$. Thus in this case $\alpha_1(W)$ will have a minimum. Even if $\alpha_2(W)$ does not make a particle a minimum is still possible. Moreover α will have a left hand cut not only in u but also in W .

From all these considerations one would expect the fermion Regge trajectory to have a shape more or less like the one shown in Fig.(5). The loop for $W < 0$ is chosen in the lower half plane because if one moves from the positive real W axis in W -plane into the upper plane and approaches the positive imaginary axis the pole in J -plane should move from the positive W loop towards the positive imaginary W loop. One would not expect to meet the negative W loop on this way. One can also see this by writing a dispersion relation for a function $\psi(W)$ and looking at the relation derived from this by $W \rightarrow -W$ $\psi(W)$. It will be seen that the imaginary part has the opposite sign.

To summarize we can say that the same trajectory (both loops included) can be considered as the Regge trajectory of the pole of $h_{J+\frac{1}{2}}$ or of $h_{J-\frac{1}{2}}$. If it is taken to be $\alpha_1(W)$ the lower loop corresponds to negative energies and even if it reaches a physical J -value it won't correspond to a physical particle. On the other hand if we take the same trajectory to be $\alpha_2(W)$ which is the pole of $h_{J-\frac{1}{2}}$ then the lower loop will correspond to positive energies and if it makes a physical J -value it will correspond to a physical particle. The upper loop now will correspond to negative energies and when it crosses physical J -values

this won't correspond to physical particles. If we now plot $\text{Re } \alpha(W)$ versus W , the curve corresponding to the trajectories of Chew-Frautschi diagram will probably look as shown in Fig. (6). $\text{Re } \alpha_2(W)$ will be the reflection of this curve with respect to the $W = 0$ axis. Thus if $\text{Re } \alpha_1(W)$ goes through a physical value in the negative energy region this will mean there is a particle with this spin, with the mass equal to the absolute value of this energy but with opposite parity; e.g. one has to invert the energy and parity simultaneously. Of course we do not know if the two trajectories related by the reflection principle are the top trajectories. It may very well be that the top trajectory of one amplitude is related to a lower trajectory of the other amplitude. All the same it is interesting to investigate the consequences if this is not the case.

From what was previously said it follows that on the Chew-Frautschi diagram ¹⁷ two fermion Regge trajectories with the same quantum numbers but opposite parities meet each other on the $u = 0$ axis.

Now since

$$\frac{d \text{Re } \alpha(u)}{du} = \frac{1}{2W} \frac{d \text{Re } \alpha(W)}{dW}$$

if the minimum is not at $W = 0$ the derivative $d \text{Re } \alpha(W)/du$ will blow up at $u = 0$. If one of the tangents at $W = 0$ has the angle θ the other one has $(\pi - \theta)$ so that

$$\frac{d \text{Re } \alpha(u)}{du}$$

will go to the limit with different signs for each branch. That means that two trajectories with the same quantum numbers but different parities in Chew-Frautschi diagram not only meet each other on $u = 0$ axis but are actually a single trajectory touching this axis.

We'll see later that two possible candidates on Chew-Frautschi diagram which might have such a behavior are $(\Lambda_\alpha \Lambda_\beta)$ and $(\Sigma_\delta \Sigma_\gamma)$ pairs. Here we are using Rosenfeld's notation¹⁸

	Parity	Spin
α	+	1/2, 5/2, 9/2,
β	-	1/2, 5/2, 9/2,
γ	+	3/2, 7/2, 11/2,
δ	-	3/2, 7/2, 11/2,

Looking at the positions of these particles on Chew-Frautschi diagram one can draw one more conclusion if all the conjectures are correct. Since Σ_γ and Σ_δ trajectories are on different sides of the Λ_α trajectory this pair has to intersect $(\Lambda_\alpha \Lambda_\beta)$ pair. The possibility that these particles may belong to different trajectories which are not related by the reflection principle does of course exist, as mentioned previously. The other possibility is shown in Fig .(7).

Regge poles in the $\bar{K}N \rightarrow KN$ channel

The quantum numbers of this channel are

Baryon number $B = 1$
 Strangeness $S = -1$
 Isotopic spin $I = 0, 1$
 J-parity $= +, -$
 Parity $P = +, -$

In this channel, in addition to Λ and Σ there are a number of resonances (Y^* 's), which share the quantum numbers $B = 1$ and $S = -1$. Unfortunately the remaining quantum numbers of these resonances are in general not very well established. Their present situation is given in table (1)

Table 1 : $\bar{K}N \rightarrow \bar{K}N$ channel particles

$I = 0$

	J	J-parity	P	Mass (Mev)	Γ (Mev)	$I(J^P)$
Λ	$\frac{1}{2}$	+	+	1115		$0(\frac{1}{2}^+)$
Y_0^*	$\frac{1}{2}$	+	-	1405	50	$0(\frac{1}{2}^-)$
Y_0^*	$3/2$	-	-	1520	15	$0(3/2^-)$
Y_0^*	$\geq 5/2$	+	+	1815	120-300	$0(5/2^+)$

$I = 1$

	J	J-parity	P	Mass (Mev)	Γ (Mev)	$I(J^P)$
Σ	$\frac{1}{2}$	+	+	1190		$1(\frac{1}{2}^+)$
Y_1^*	$\geq 3/2$	-	+	1385	50	$1(3/2^+)$
Y_1^*	$3/2$	-	-	1660	60 ± 20	$1(3/2^-)$

Thus for $B = 1$ and $S = -1$ we have the following eight combinations:

- 1) $I = 0$, J-parity $= +$, $P = +$

One Regge pole with these quantum numbers is the Λ particle. Another candidate is the 1815 Mev Y_0^* resonance, if we believe that the spin assignment $5/2$ and parity assignment $+$ are correct.

- 2) $I = 0$, J-parity $= +$, $P = -$

One possible pole with these quantum numbers is the 1405 Mev Y_0^* .

- 3) $I = 0$, J-parity $= -$, $P = +$

At present there seems to be no candidate for this set of quantum numbers.

- 4) $I = 0$, J-parity $= -$, $P = -$

The only resonance which can be a candidate for this set is the 1520 Mev Y_0^* .

- 5) $I = 1$, J-parity $= +$, $P = +$

For this set we have the Σ particle pole.

- 6) $I = 1$, J-parity $= +$, $P = -$

There is no candidate for this set either.

- 7) $I = 1$, J-parity $= -$, $P = +$

One possible pole may be 1385 Mev Y_1^* .

- 8) $I = 1$, J-parity $= -$, $P = -$

The recently discovered 1660 Mev Y_1^* has this set of quantum numbers if the assignments are correct.

Let us see now in which amplitudes of the u-channel these poles appear:

1) $I = 0$

a) Even continuation amplitudes (See Fig. 5, 9):

I) $h_{J+\frac{1}{2}}^e(W_u)$ should be physical for $J = \frac{1}{2}$. This gives us

$$\text{Spin} = \frac{1}{2}$$

$$\text{Orbital angular momentum } l = 1$$

$$\text{Parity} = + \quad \text{if } \bar{K} \text{ parity is } (-)$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = +$$

The particle with these quantum numbers is Λ . Also ^{the} Λ_{1815} Mev Y_0^* is probably on the same trajectory.

II) Now let us consider the other amplitude which is related to $h_{J+\frac{1}{2}}^e$ by Mac

Dowell's reflection principle e.g. $h_{J-\frac{1}{2}}^e(W_u)$. For this amplitude we have

$$J = \frac{1}{2}$$

$$\text{Orbital angular momentum } l = 0$$

$$\text{Parity} = - \quad \text{if } \bar{K} \text{ parity is } (-)$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = +$$

1405 Mev Y_0^* seems to have these quantum numbers.

If now the trajectories of these particles are the ones related by the reflection principle they will look like as shown in Fig. (10). It is interesting to notice that the Regge pole has an imaginary part already at $W = M + m_K$.

b) Odd continuation amplitudes (See Fig. 11) :

I) $h_{J+\frac{1}{2}}^o(W_u)$. For the odd continuation to be physical J has to be $3/2, 7/2, \dots$

$$\text{Spin} = 3/2$$

$$\text{Orbital angular momentum } l = 2$$

$$\text{Parity} = - \quad \text{if } \bar{K} \text{ parity is } (-)$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = -$$

1520 Mev Y_0^* has the right quantum numbers.

$$\text{II) } h_{J-\frac{1}{2}}^0(W_u)$$

$$\text{Spin} = 3/2$$

$$\text{Orbital angular momentum } l = 1$$

$$\text{Parity} = + \quad \text{if } \bar{K} \text{ parity is } (-)$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = -$$

There is no particle with these quantum numbers. The possible trajectories of the Regge poles in the odd continuation amplitudes $h_{J+\frac{1}{2}}^0(W_u)$ and $h_{J-\frac{1}{2}}^0(W_u)$ are shown in Fig.(12)

2) $I = 1$ isotopic spin 1 case.

a) Even continuation amplitudes (see Fig. 13)

$$\text{I) } h_{J+\frac{1}{2}}^e(W_u)$$

$$\text{Spin} = \frac{1}{2}$$

$$\text{Orbital angular momentum } l = 1$$

$$\text{Parity} = +$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = +$$

The particle with these quantum numbers is Σ .

$$\text{II) } h_{J-\frac{1}{2}}^e(W_u)$$

$$\text{Spin} = \frac{1}{2}$$

$$\text{Orbital angular momentum } l = 0$$

$$\text{Parity} = -$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = +$$

At the present time there seems to be no particle with these quantum numbers. The possible trajectories of the Regge poles for $I = 1$ in the even continuation amplitudes $h_{J+\frac{1}{2}}^e$ and $h_{J-\frac{1}{2}}^e$ are shown in Fig.(14).

b) Odd continuation amplitudes (see Figs. 15, 16)

$$I) h_{J+\frac{1}{2}}^0(w_u)$$

$$\text{Spin} = 3/2$$

$$\text{Orbital angular momentum } l = 2$$

$$\text{Parity} = -$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = -$$

The newly discovered 1660 Mev Y_1^* has probably these quantum numbers.

$$II) h_{J-\frac{1}{2}}^0(w_u)$$

$$\text{Spin} = 3/2$$

$$\text{Orbital angular momentum } l = 1$$

$$\text{Parity} = +$$

$$J\text{-parity} = (-1)^{J-\frac{1}{2}} = -$$

One possible candidate for these quantum numbers was the 1385 Mev Y_1^* as we have previously seen. The possible trajectories of the Regge poles for $I = 1$ in the odd continuation amplitudes are shown in Fig.(17).

Regge Poles in the $KN \rightarrow KN$ channel

The quantum numbers of this channel are

Baryon number $B = 1$

Strangeness $S = 1$

Isotopic spin $I = 0, 1$

There are no known particles or resonances with these quantum numbers. Therefore the Regge pole trajectories (if they exist at all) of the even and odd continuation amplitudes do not move far enough to the right in the J -plane to make particles in this channel.

Because of this difference between the s - and u -channels, the Regge poles of which control the high energy backward cross sections of each other, these two channels are expected to behave differently in backward scattering.

High Energy Backward Scattering in the s - channel (KN $\rightarrow$ KN)

In the s-channel we have the following processes:

$$\begin{aligned}
 1) & K^+p \rightarrow K^+p, & K^0n \rightarrow K^0n & A_s^1 \\
 2) & K^+n \rightarrow K^+n, & K^0p \rightarrow K^0p & \frac{1}{2}(A_s^1 + A_s^0) \\
 3) & K^0p \rightarrow K^+n, & K^+n \rightarrow K^0p & \frac{1}{2}(A_s^1 - A_s^0) \quad \text{Charge exchange}
 \end{aligned} \tag{5.1}$$

where on the right side the amplitudes describing these processes are given.

Here the notation is such that the invariant amplitude describes a pure isotopic spin process given by the superscript (0,1) in the channel given by the subscript (not to be confused with the absorptive part). The pure isotopic spin invariant amplitudes in s- and u-channels are related by the following crossing matrices:

$$\begin{pmatrix} A_s^0 \\ A_s^1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A_u^0 \\ A_u^1 \end{pmatrix} \quad \begin{pmatrix} B_s^0 \\ B_s^1 \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} B_u^0 \\ B_u^1 \end{pmatrix} \tag{5.2}$$

The unpolarized differential cross section is given by

$$\frac{d\sigma}{d\Omega} = |f_1 + f_2|^2 - 2(1 - \cos\theta_s) \operatorname{Re}(f_1^* f_2) \tag{5.3}$$

When crossing the amplitudes f_1 and f_2 one has to be careful because the B amplitudes have an extra minus sign and the crossing relations for f_1 and f_2 are not the same as for A and B. To cross f_1 and f_2 we write the expressions (4.14) and (4.15) both for s- and u-channels. Here $f_{1,2}^s$ and $f_{1,2}^u$ are defined by (4.6) respectively in s- and u-channels. They are the amplitudes which when inserted in (4.9) give the differential cross section in their respective channels.

$$\begin{aligned}
 f_1^{s,I=0,1} &= \frac{E_s + M}{8\pi W_s} \left[A_s^{0,1} + (W_s - M) B_s^{0,1} \right] & f_1^{u,I=0,1} &= \frac{E_u + M}{8\pi W_u} \left[A_u^{0,1} + (W_u - M) B_u^{0,1} \right] \\
 f_2^{s,I=0,1} &= \frac{E_s - M}{8\pi W_s} \left[-A_s^{0,1} + (W_s + M) B_s^{0,1} \right] & f_2^{u,I=0,1} &= \frac{E_u - M}{8\pi W_u} \left[-A_u^{0,1} + (W_u + M) B_u^{0,1} \right]
 \end{aligned}
 \tag{5.4} \tag{5.5}$$

Let us take the first process as an example. The differential cross section for it is:

$$\frac{d\sigma}{d\Omega} = \left| f_1^{s,I=1} + f_2^{s,I=1} \right|^2 - 2(1 - \cos\theta_s) \operatorname{Re}(f_1^{s,I=1*} f_2^{s,I=1}) \tag{5.6}$$

To cross the amplitudes f_1 and f_2 one calculates $A_u^{0,1}$ and $B_u^{0,1}$ from (5.5) in terms of $f_1^{u,I=0,1}$ and $f_2^{u,I=0,1}$. Then with the help of the crossing matrices (5.2) $A_s^{0,1}$ and $B_s^{0,1}$ in (5.4) are expressed in terms of $A_u^{0,1}$ and $B_u^{0,1}$. Finally the expressions obtained from (5.5) for $A_u^{0,1}$, $B_u^{0,1}$ are inserted in (7.4). Thus $f_1^{s,I=0,1}$ and $f_2^{s,I=0,1}$ are expressed in terms of $f_1^{u,I=0,1}$ and $f_2^{u,I=0,1}$. When these results are inserted in (5.6) one obtains the differential cross section for the process 1).

Now let us look at the contributions of different poles in the u -channel to the amplitudes $f_{1,2}^{u,I=0,1}$. For this we make use of the expressions (4.39). We have already seen which poles contribute to which amplitudes. The contributions of the known particles and resonances are obtained easily from (4.39). Since the formulas are rather lengthy we will give only one example:

Contributions to $I = 0$, even J -parity f_1 amplitude:

The poles which contribute to these amplitudes are Λ , $Y_{0,1405}^*$, $Y_{0,1815}^*$. Since we assume that $Y_{0,1815}^*$ is on the same trajectory as Λ and $Y_{0,1405}^*$ is on the trajectory related to Λ_u trajectory by Mac Dowe'll's reflection principle, we are essentially dealing with one Regge pole. Hence using the expressions (4.40) we have the following contribution:

$$\begin{aligned}
 \left[f_1^{u, I=0} \right]_{\Lambda} &= (2\pi i)^{\frac{1}{2}} \frac{P'_{\alpha_{\Lambda}(-W)+\frac{1}{2}}(z) + P'_{\alpha_{\Lambda}(-W)+\frac{1}{2}}(-z)}{2} \frac{1}{\cos \pi \alpha_{\Lambda}(-W)} \frac{E+M}{16\pi W} (2k^2)^{\alpha_{\Lambda}(-W)-\frac{1}{2}} \\
 &\quad \cdot \lim_{J \rightarrow \alpha_{\Lambda}(-W)} \left[(J - \alpha_{\Lambda}(-W)) (-h_{J+\frac{1}{2}}^{\Theta}(-W)) \right] \\
 &= (2\pi i)^{\frac{1}{2}} \frac{P'_{\alpha_{\Lambda}(W)-\frac{1}{2}}(z) - P'_{\alpha_{\Lambda}(W)-\frac{1}{2}}(-z)}{2} \frac{1}{\cos \pi \alpha_{\Lambda}(W)} \frac{E-M}{16\pi W} (2k^2)^{\alpha_{\Lambda}(W)-\frac{1}{2}} \\
 &\quad \cdot \lim_{J \rightarrow \alpha_{\Lambda}(W)} \left[(J - \alpha_{\Lambda}(W)) h_{J+\frac{1}{2}}^{\Theta}(W) \right]
 \end{aligned} \tag{5.7}$$

Where z, W, E, k are understood to be the u -channel quantities. Also use has been made of the relations

$$\alpha_{Y^*}^{0,1405}(W) = \alpha_{\Lambda}(-W) \tag{5.8}$$

$$\text{and } h_{J+\frac{1}{2}}^{\Theta}(W) = -h_{J-\frac{1}{2}}^{\Theta}(-W) \tag{5.9}$$

with the provisions already discussed before.

The only other case where a relation like (5.8) might hold is for the poles of the $I = 1$, odd J -parity amplitudes:

$$\alpha_{Y^*}^{1,1660}(-W) = \alpha_{Y^*}^{1,1385}(W) \tag{5.10}$$

Finally we should like to point out that it is very difficult to guess where all these fermion trajectories will meet the $u = 0$ axis and which of them will dominate. One thing we can say is that the Regge pole terms in backward scattering will not be as dominant over the integral along the $\text{Re } J = -\frac{1}{2}$ line as in the case of forward scattering (see 3.7).

H i g h E n e r g y B a c k w a r d S c a t t e r i n g i n t h e
u - c h a n n e l ($\bar{K}N \rightarrow \bar{K}N$)

In the u-channel we have the following processes:

$$\begin{aligned}
 1) \quad & \bar{K}^+ p \rightarrow \bar{K}^+ p, \quad \bar{K}^0 n \rightarrow \bar{K}^0 n, \quad \frac{1}{2}(A_u^0 + A_u^1) \\
 2) \quad & \bar{K}^+ n \rightarrow \bar{K}^+ n, \quad \bar{K}^0 p \rightarrow \bar{K}^0 p, \quad A_u^1 \\
 3) \quad & \bar{K}^0 n \rightarrow \bar{K}^+ p, \quad \bar{K}^+ p \rightarrow \bar{K}^0 n, \quad \frac{1}{2}(A_u^0 - A_u^1)
 \end{aligned} \tag{5.11}$$

where on the right side the amplitudes describing these processes are given. The notation is the same as for (5.1). The crossing matrices for $s \rightarrow u$ or $u \rightarrow s$ crossing are identical and are given by the expressions (5.2).

The differential cross section formulas for the above processes 1), 2), 3) are the same formulas respectively for 2), 1), and 3) of $\bar{K}N \rightarrow \bar{K}N$ scattering. A sign difference for the processes 3) of $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \bar{K}N$ scattering amplitudes disappears in cross section formulas when the amplitudes are squared.

As was seen in connection with the poles of different channels the main difference between the $\bar{K}N \rightarrow \bar{K}N$ and $\bar{K}N \rightarrow \bar{K}N$ backward high energy scatterings is the absence of Regge particles in the crossed channel of the $\bar{K}N \rightarrow \bar{K}N$ scattering. One thus would expect to see a backward peak for $\bar{K}N \rightarrow \bar{K}N$ scattering whereas it shouldn't be observed for $\bar{K}N \rightarrow \bar{K}N$ scattering.

Difference between equal and unequal mass particle scatterings in the backward direction.

Before letting $\cos\theta_u$ go to infinity in the expressions like (5.7) which takes the amplitude into the crossed channel s for high energies, we will investigate this crossing for equal and unequal mass particle scatterings. The reason is that even though the above procedure is correct in the limit, one has to be careful for high but finite energies in the case of unequal mass particle scattering.

We first look at the equal mass case. Let the Regge pole be in ^{the} s -channel. The invariant variables are given for the s -channel by:

$$\begin{aligned} s &= 4(q_s^2 + 1) \\ t &= -2q_s^2(1 - \cos\theta_s) \\ u &= -2q_s^2(1 + \cos\theta_s) \end{aligned} \quad (5.12)$$

where the masses of the particles are taken to be unity.

Inserting

$$q_s^2 = \frac{s}{4} - 1 \quad (5.13)$$

in the expression for u one finds

$$u = (-\frac{1}{2}s + 2)(1 + \cos\theta_s) \quad (5.14)$$

Thus if $s \leq 0$ and $\cos\theta_s > 1$ we are in the physical region for the u -channel. Now in the u -channel the backward direction corresponds to $s = 0$ and the backward cone to

$$s = \text{small and negative} \quad .$$

Thus $\cos\theta_s \rightarrow \infty$ corresponds to $u \rightarrow \infty$.

It is also clear that u can not be positive if $\cos\theta_s$ is negative unless $s > 4$, which is not in the physical u -channel. The equal mass case is shown in the Mandelstam diagram Fig.(13)

We see that for $\cos\theta_s$ sufficiently large so that the Regge pole term dominates over the integral along the line $\text{Re } J = -\frac{1}{2}$ we will be in the physical region for u-channel if the energy is high enough.

Now let us turn to the unequal mass case. Again we take the Regge pole in s-channel. In this case the invariant variables are given by (2.5). We will also make use of the first formula of the eq.s (2.6). Again one can study in the Mandelstam diagram for the unequal mass case the behavior of the curves

$$\cos\theta_s = \text{a negative number}$$

$$\cos\theta_s = \text{a positive number}$$

The second term in the first formula of the eq.s (2.6) represents a straight line in (u, s) plane given by

$$u = -\frac{1}{2}(1 + \cos\theta_s)(s - 2(M^2 + m^2)) \quad (5.15)$$

When $\cos\theta_s$ varies this line turns around the point

$$s = 2(M^2 + m^2) \quad ; \quad u = 0$$

The curve (2.6) is asymptotic to this line and to the line $s = 0$. Except at $s = 0$ it is a single valued function of s and has two branches. In figures (19) and (20) two curves are shown, one with $\cos\theta_s$ negative the other positive and both with large absolute values. All the curves

$$\cos\theta_s = \text{constant}$$

intersect each other at two fixed points

$$A) \quad s = (M+m)^2 \quad ; \quad u = (M-m)^2$$

$$B) \quad s = (M-m)^2 \quad ; \quad u = (M+m)^2$$

since these two points automatically satisfy the eq.(2.6).

From Figs (19), (20) one sees that if

$$\cos \theta_s \rightarrow -\infty$$

one can never enter into the backward cone of the physical u-channel.

If on the other hand

$$\cos \theta_s \rightarrow +\infty$$

one can enter the backward cone of the physical u-channel.

But in either case the small region between the

$$\text{line} \quad s = 0$$

$$\text{and the hyperbola} \quad u.s = (M^2 - m^2)^2$$

is excluded. Thus if one is doing an experiment between these two curves the Regge poles in the crossed channel can not give any information for this region.

Hence the Regge formalism about high energy scattering can not be applied to a finite region centered around the backward direction. Outside of this region where $s < 0$, it can be applied. But we have seen that for $s < 0$ the Regge pole becomes complex (two complex conjugate poles) and the amplitude oscillates. These kinematical considerations show that the hope expressed by Singh that one could avoid these oscillations if the experiment is done in the region discussed above is not realized. On the other hand even though the amplitudes oscillate Gribov has shown that the backward differential cross section does not, due to cancellations.

Regge Poles in the $K\bar{K} \rightarrow N\bar{N}$ channel

The quantum numbers of this channel are:

Baryon number $B = 0$

Strangeness $S = 0$

Isotopic spin $I = 0, 1$

J-parity $= +, -$

Parity $P = +, -$

G-parity $G = +, -$

$$\begin{cases} P = (-1)^{L+1} & \text{for } N\bar{N} \text{ system} \\ P = (-1)^L & \text{for } K\bar{K} \text{ system} \end{cases}$$

$$\begin{cases} G = (-1)^{L+S+I} & \text{for } N\bar{N} \text{ system} \\ G = (-1)^{L+I} & \text{for } K\bar{K} \text{ system} \end{cases}$$

where L = orbital angular momentum

and s = spin state of the system.

There are a number of particles with the quantum numbers

$$B = 0$$

$$S = 0$$

$$I = 0, 1$$

some with well established assignments, others with less certainty about their quantum numbers. A list of those particles is given in table (2).¹⁸

Table 2 : $K\bar{K} \rightarrow N\bar{N}$ channel particles.

$$I = 0$$

	J	J-parity	P	G	Mass (Mev)	Γ (Mev)	$I(J^{P,G})$
Pom. (P)	2	+	+	+			$0(2^{++})$
ABC	0	+	+	+			$0(0^{++})$
η	0	+	-	+	550	< 10	$0(0^{-+})$
ω	1	-	-	-	780	< 15	$0(1^{--})$

I = 1

	J	J-parity	P	G	Mass (Mev)	Γ (Mev)	$I(J^{P,G})$
π	0	+	-	-	140		$1(0^{--})$
ρ	1	-	-	+	750	100	$1(1^{-+})$

There are 16 different combinations for I, J-parity, P and G which are shown below:

I = 0

I = 1

	J-par.	P	G		J-par.	P	G	
1)	+	+	+	Pom., P', ABC	9)	+	+	+
2)	+	-	+	η	10)	+	-	+
3)	-	+	+		11)	-	+	+
4)	-	-	+		12)	-	-	+
5)	+	+	-		13)	+	+	-
6)	+	-	-		14)	+	-	-
7)	-	+	-		15)	-	+	-
8)	-	-	-	ω	16)	-	-	-

Of these combinations eight are eliminated by the condition

$$G = (-1)^{L+I} \quad \text{for } K\bar{K} \text{ system.}$$

Since the K meson has 0 spin the angular momentum of the $K\bar{K}$ system L has to be equal to the spin of the exchanged particle, if the angular momentum is to be conserved.

Thus

$$G = (-1)^{J+I}$$

This excludes the combinations 3), 4), 5), 6) and 9), 10), 15), 16). Actually there are no particles observed for these eight sets of quantum numbers.

On the other hand parity conservation, angular momentum conservation ($J=L$) and the relation $P = (-1)^L$ for the $K\bar{K}$ system eliminates the combinations 2), 7), 11) and 14).

The $\bar{N}N$ system requires a more detailed analysis. The G parity of $\bar{N}N$ system is given by

$$G = (-1)^{L+S+I}$$

The intrinsic parity of the $\bar{N}N$ system is (-).

Thus we have:

1) If $s = 0$ (singlet state) $G = (-1)^{L+I}$

a) $I = 0$ case $G = (-1)^L$

I) If $L = \text{even}$

then $G = \text{even}$, $P = \text{odd}$

and we have $I = 0$
 $J = +$
 $P = -$
 $G = +$

II) If $L = \text{odd}$

then $G = \text{odd}$, $P = \text{even}$

and we have $I = 0$
 $J = -$
 $P = +$
 $G = -$

b) $I = 1$ case $G = (-1)^{L+1}$

I) If $L = \text{even}$

then $G = \text{odd}$, $P = \text{odd}$

and we have $I = 1$
 $J = +$
 $P = -$
 $G = -$

II) If $L = \text{odd}$

then $G = \text{even}$, $P = \text{even}$

and we have $I = 1$
 $J = -$
 $P = +$
 $G = +$

2) If $s = 1$ (triplet state) $G = (-1)^{L+I+1}$

a) $I = 0$ case $G = (-1)^{L+1}$

I) If $L = \text{even}$

then $G = \text{odd}$, $P = \text{odd}$

Remembering that for the triplet $\bar{N}N$ spin state s and L can be combined to give either even or odd J we have

$I = 0$
 $J = +$
 $P = -$
 $G = -$

II) If $L = \text{odd}$

then $G = \text{even}$, $P = \text{even}$

With the same argument as under I) one has

$I = 0$
 $J = -$
 $P = +$
 $G = +$

b) $I = 1$ case $G = (-1)^{L+2}$

I) If $L = \text{even}$

then $G = \text{even}$, $P = \text{odd}$

and we have

$$\begin{aligned} I &= 1 \\ J &= \mp \\ P &= - \\ G &= + \end{aligned}$$

II) If $L = \text{odd}$

then $G = \text{odd}$, $P = \text{even}$

and we have

$$\begin{aligned} I &= 1 \\ J &= \mp \\ P &= + \\ G &= - \end{aligned}$$

This analysis shows us that in no case is it possible to obtain the combinations 4), 5), 9), and 16). Therefore these combinations are doubly ruled out. Also from the same analysis we see in which state the $\bar{N}N$ system has to be in order to exchange in this channel a particle with certain quantum numbers.

If we now consider the initial and final states simultaneously we have:

$$G_{\bar{N}N} = (-1)^{L_{\bar{N}N} + I + s}$$

where $L_{\bar{N}N}$ is the orbital angular momentum of the $\bar{N}N$ system and s its spin state.

$$G_{\bar{K}K} = (-1)^{L_{\bar{K}K} + I}$$

where $L_{\bar{K}K}$ is the orbital angular momentum of the $\bar{K}K$ system.

From G-parity conservation one sees that if N and $\bar{N}$ are in a singlet state ($s = 0$) then $L_{\bar{N}N}$ and $L_{\bar{K}K}$ are simultaneously odd or even. But the intrinsic parity of the $\bar{N}N$ system is $(-)$ whereas the intrinsic parity of the $\bar{K}K$ system is $(+)$. Hence the parity conservation requires $L_{\bar{N}N}$ and $L_{\bar{K}K}$ to have opposite evenness or oddness. This contradiction tells us that the $\bar{N}N$ system can not be in a singlet state ($s = 0$) regardless of its isotopic spin.

$s = 0$ gives us the following quantum number combinations:

$$\begin{aligned} 1) \quad I = 0 \quad \text{if } L_{\bar{N}N} = \text{even} \quad J\text{-parity} &= (-1)^J = + \quad \text{since } J = L_{\bar{N}N} \quad \text{because } s = 0 \\ P &= (\text{Intrinsic parity of } \bar{N}N) \cdot (\text{Evenness or oddness of } L_{\bar{N}N}) = - \\ G &= (-1)^{L_{\bar{N}N} + I + s} = + \end{aligned}$$

This is the combination 2) with the quantum numbers of 1)

$$\text{if } L_{\overline{N}\overline{N}} = \text{odd} \quad J\text{-parity} = (-1)^J = -$$

$$P = +$$

$$G = -$$

This is the combination 7).

$$2) \quad I = 1 \quad \text{if } L_{\overline{N}\overline{N}} = \text{even} \quad J\text{-parity} = (-1)^J = +$$

$$P = -$$

$$G = -$$

This is the combination 14) with the quantum numbers of 11).

$$\text{if } L_{\overline{N}\overline{N}} = \text{odd} \quad J\text{-parity} = (-1)^J = -$$

$$P = +$$

$$G = +$$

This is the combination 11).

Thus we conclude that the combinations 2), 7), 14), and 11) are ^{also} ruled out if one considers $K\overline{K}$ and $\overline{N}\overline{N}$ systems simultaneously and finds that the $\overline{N}\overline{N}$ system can not be in the $s = 0$ state.

To sum up we can say that of the possible 16 quantum number combinations for $B = 0$, $S = 0$, $I = 0, 1$ in the $K\overline{K} - \overline{N}\overline{N}$ channel 12 are excluded. For the 4 remaining combinations the only particles which are more or less known and which can be exchanged in the t -channel are:

$$\rho_{01}, P', A_2, \omega, f_2.$$

Amplitudes of the t-channel ($K\bar{K} \rightarrow N\bar{N}$)

In this channel we use the amplitudes χ_+ and χ_- defined by

$$\begin{aligned}\chi_+ &= (M^2 - \frac{1}{2}t)A + \frac{1}{4}N(u-s)B \\ \chi_- &= E\end{aligned}\tag{6.1}$$

Also in the s-channel we will use the same amplitudes because the differential cross section formula for s-channel takes a very simple form in terms of these amplitudes. Moreover crossing from t to s-channel becomes particularly simple.

In the t-channel the definitions (6.1) are equivalent to the following definitions in terms of the helicity amplitudes of Frazer and Fulco:

$$\begin{aligned}\chi_+ &= 8\pi E p F_{++} \\ \chi_- &= \frac{8\pi}{q \sin\theta_t} F_{+-}\end{aligned}\tag{6.2}$$

where E is the nucleon total energy in the c.o.m system, p the nucleon momentum and q the kaon momentum. θ_t is the scattering angle in t-channel. With the help of formulas (3.6) and (3.7) of Frazer and Fulco and the above relations χ_+ and χ_- can easily be expanded in terms of the partial helicity amplitudes:

$$\begin{aligned}\chi_+ &= -8\pi i E \sqrt{p/q} \sum_J (J+\frac{1}{2}) S_+^J P_J(\cos\theta_t) \\ \chi_- &= -\frac{8\pi i}{q\sqrt{qp}} \sum_J \frac{J+\frac{1}{2}}{J(J+\frac{1}{2})} S_-^J P_J(\cos\theta_t)\end{aligned}\tag{6.3}$$

Here S_+^J and S_-^J are the S-matrix elements for scattering into a state with total angular momentum J and in which N and $\bar{N}$ have the same (+) or the opposite (-) helicities. The kaon mass is taken to be unity. The sum is over all inte-

ger values of J in contrast to the $\pi\pi - N\bar{N}$ case where one sums only over even values of J for $I = 0$ and over odd values of J for $I = 1$. The reason for this is the following : Two pions are always in an even G-parity state because the G-parity of n pions is given as a function of the numbers of pions alone. Thus the G-parity of the $N\bar{N}$ system too has to be even. This gives:

$$G_{N\bar{N}} = (-1)^{L_{N\bar{N}} + I + s} = \text{even}$$

$$L_{N\bar{N}} + I + s = \text{even}$$

$$\text{if } s = 0 \quad L_{N\bar{N}} + I = \text{even}$$

$$\text{for } I = 0 \quad L_{N\bar{N}} = \text{even} \rightarrow L_{\pi\pi} = \text{even} \rightarrow J = \text{even}$$

$$\text{for } I = 1 \quad L_{N\bar{N}} = \text{odd} \rightarrow L_{\pi\pi} = \text{odd} \rightarrow J = \text{odd}$$

$$\text{if } s = 1 \quad L_{N\bar{N}} + I + 1 = \text{even}$$

$$L_{N\bar{N}} + I = \text{odd}$$

$$\text{for } I = 0 \rightarrow L_{N\bar{N}} = \text{odd} \rightarrow \text{Parity} = (N\bar{N} \text{ intrinsic parity}) \cdot (\text{odd}) = \text{even}$$

$$\rightarrow L_{\pi\pi} = \text{even} \rightarrow J = \text{even}$$

$$\text{for } I = 1 \rightarrow L_{N\bar{N}} = \text{even} \rightarrow \text{Parity} = (N\bar{N} \text{ intrinsic parity}) \cdot (\text{even}) = \text{odd}$$

$$\rightarrow L_{\pi\pi} = \text{odd} \rightarrow J = \text{odd}$$

Hence one always has $J = \text{even}$ for $I = 0$

$J = \text{odd}$ for $I = 1$

For $K\bar{K} \rightarrow N\bar{N}$ such is not the case. The G-parity of the $K\bar{K}$ system depends on the orbital angular momentum and isotopic spin of the system. This point has already been discussed.

To calculate the forward scattering in s-channel and the total cross section we have to find the contribution of Regge poles in t-channel to the amplitudes $\chi_{+}^{0,1}$, $\chi_{-}^{0,1}$. For this we have to apply the Sommerfeld-Watson transformation to the formulas (6.3). But these formulas are not suitable yet for the transformation because of the factors $\sqrt{J(J+1)}$ in the denominators which would introduce singularities at $J = 0$ and $J = -1$. Therefore we incorporate the denominator into a new amplitude defined by:

$$(J+\frac{1}{2}) \frac{S_{-}^{0,1 J}}{\sqrt{J(J+1)}} = \Lambda_{-}^{0,1 J} \quad (6.4)$$

The other amplitude $S_{+}^{0,1 J}$ gives no difficulty.

Analytical continuation of S_{+}^J and A_{-}^J amplitudes into the complex J-plane.

Now we want to analytically continue $A_{-}^{0,1J}$ and $S_{+}^{0,1J}$ into the complex J-plane. The procedure is again the same, e.g. we try to express $A_{-}^{0,1J}$ and $S_{+}^{0,1J}$ in terms of functions the analytical continuations of which are already known. For this we write first S_{+}^J in terms of A and B using (3.3), (3.8) and (3.11) of Frazer and Fulco:

$$S_{+}^J = -\frac{M}{8\pi E} \sqrt{q/p} \int_{-1}^1 d \cos \theta_t P_J(\cos \theta_t) \left(-\frac{p^2}{M} A + q p \cos \theta_t B \right) \quad (6.5)$$

The first term gives us the partial wave A_J . As before to express A_J in terms of the discontinuities of the amplitude A and Q functions we write the Mandelstam representation for A:

$$A = \frac{1}{\pi} \int_{(M+m_K)^2}^{\infty} \frac{A_s(t, s')}{s' - s} ds' + \frac{1}{\pi} \int_{(m_Y+m_{\pi})^2}^{\infty} \frac{A_u(t, u')}{u' - u} du' \quad (6.6)$$

Projecting out the partial wave and using the t-channel variables (2.8) one finds:

$$A_J = \int_{-1}^1 A P_J(\cos \theta_t) d \cos \theta_t \quad (6.7)$$

$$A_J = \frac{1}{\pi} \int_{-1}^1 \int_{(M+m_K)^2}^{\infty} \frac{A_s(t, s')}{s' - (-q^2 - p^2 - 2qp \cos \theta_t)} P_J(\cos \theta_t) d \cos \theta_t ds' \quad (6.8)$$

$$+ \frac{1}{\pi} \int_{-1}^1 \int_{(m_Y+m_{\pi})^2}^{\infty} \frac{A_u(t, u')}{u' - (-q^2 - p^2 - 2qp \cos \theta_t)} P_J(\cos \theta_t) d \cos \theta_t du'$$

Interchanging the order of integration and using Neumann's definition for Q function (3.12) we obtain:

$$A_J = -\frac{1}{\pi q p} \int_{-\infty}^{\infty} \frac{A_s(t, s') Q_J(-\frac{s' + q^2 + p^2}{2qp}) ds'}{(M + m_K)^2} + \frac{1}{\pi q p} \int_{-\infty}^{\infty} \frac{A_u(t, u') Q_J(\frac{u' + q^2 + p^2}{2qp}) du'}{(m_Y + m_\pi)^2} \quad (6.9)$$

Using the same dummy variable for both integrals and inserting

$$Q_J(-z) = (-1)^{J+1} Q_J(z) \quad (6.10)$$

we finally get

$$A_J = \frac{1}{\pi q p} \int_{-\infty}^{\infty} \left[A_u(t, s') - (-1)^{J+1} A_s(t, s') \right] Q_J\left(\frac{s' + q^2 + p^2}{2qp}\right) ds' \quad (6.11)$$

The second term in the expression for S_+ is of the form

$$\int_{-1}^1 B \cos \theta_t P_J(\cos \theta_t) d \cos \theta_t \quad (6.12)$$

We again write the Mandelstam representation for B

$$B = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{B_s(t, s')}{s' - s} ds' + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{B_u(t, u')}{u' - u} du' \quad (6.13)$$

$(M + m_K)^2$
 $(m_Y + m_\pi)^2$

$$B = -\frac{1}{2qp\pi} \int_{-\infty}^{\infty} \frac{B_s(t, s')}{-\left(\frac{s' + q^2 + p^2}{2qp}\right) - \cos \theta_t} ds' + \frac{1}{2qp\pi} \int_{-\infty}^{\infty} \frac{B_u(t, u')}{\frac{u' + q^2 + p^2}{2qp} - \cos \theta_t} du' \quad (6.14)$$

$(M + m_K)^2$
 $(m_Y + m_\pi)^2$

Making use of the relation

$$t Q_J(t) = \frac{1}{2} \int_{-1}^1 \frac{z P_J(z)}{t - z} dz \quad (6.15)$$

the expression (6.12) can be evaluated:

$$\int_{-1}^1 B \cos \theta_t P_J(\cos \theta_t) d \cos \theta_t = -\frac{1}{q p \pi} \int_{-\infty}^{\infty} \frac{B_s(t, s') \left(-\frac{s' + q^2 + p^2}{2qp}\right) Q_J\left(-\frac{s' + q^2 + p^2}{2qp}\right) ds'}{(M + m_K)^2} + \frac{1}{q p \pi} \int_{-\infty}^{\infty} \frac{B_u(t, u') \left(\frac{u' + q^2 + p^2}{2qp}\right) Q_J\left(\frac{u' + q^2 + p^2}{2qp}\right) du'}{(m_Y + m_\pi)^2} \quad (6.16)$$

Again we use the same dummy variable and obtain

$$\int B \cos \theta_t P_J(\cos \theta_t) d \cos \theta_t = \frac{1}{q p \Pi} \int \left[B_u(t, s') + (-1)^J B_s(t, s') \right] \left(\frac{s' + q^2 + p^2}{2qp} \right) Q_J \left(\frac{s' + q^2 + p^2}{2qp} \right) ds' \quad (6.17)$$

Inserting the results we obtained for the first and second terms of the expression for S_+^J (6.5) we finally obtain:

$$S_+^J = \frac{2}{8\pi^2 i \sqrt{t}} \sqrt{q/p} \int \left[A_u(t, s') + (-1)^J A_s(t, s') \right] Q_J \left(\frac{s' + q^2 + p^2}{2qp} \right) ds' - \frac{2M}{8\pi^2 i \sqrt{t}} \sqrt{q/p} \int \left[B_u(t, s') - (-1)^J B_s(t, s') \right] \left(\frac{s' + q^2 + p^2}{2qp} \right) Q_J \left(\frac{s' + q^2 + p^2}{2qp} \right) ds' \quad (6.18)$$

The expression for Λ_-^J is obtained from the definition of Λ_-^J in terms of S_-^J and (3.4), (3.8) and (3.12) in Frazer Fulco.²¹

$$\Lambda_-^J = - \frac{q^2}{32\pi i} \sqrt{p/q} \frac{J+1}{J} \int_{-1}^1 B \left[\cos \theta_t P_J(\cos \theta_t) - P_{J+1}(\cos \theta_t) \right] d \cos \theta_t \quad (6.19)$$

$$\text{since } z P_J(z) - P_{J+1}(z) = \frac{J}{2J+1} (P_{J-1}(z) - P_{J+1}(z)) \quad (6.20)$$

the expression for Λ_-^J simplifies to

$$\Lambda_-^J = - \frac{q^2}{16\pi i} \sqrt{p/q} \int_{-1}^1 B \left[P_{J-1}(\cos \theta_t) - P_{J+1}(\cos \theta_t) \right] d \cos \theta_t \quad (6.21)$$

Writing the Mandelstam representation for B one finds :

$$\int_{-1}^1 B P_{J-1}(\cos \theta_t) d \cos \theta_t = B_{J-1} = \frac{1}{\pi q p} \int \left[B_u(t, s') - (-1)^J B_s(t, s') \right] Q_{J-1} \left(\frac{s' + q^2 + p^2}{2qp} \right) ds' \quad (6.22a)$$

$$\int_{-1}^1 B P_{J+1}(\cos \theta_t) d \cos \theta_t = B_{J+1} = \frac{1}{\pi q p} \int \left[B_u(t, s') - (-1)^J B_s(t, s') \right] Q_{J+1} \left(\frac{s' + q^2 + p^2}{2qp} \right) ds'$$

Inserting these in the expression for Λ_-^J we finally find:

$$(6.22b)$$

$$\Lambda_-^J = -\frac{1}{16\pi^2} \sqrt{q/p} \int \left[B_u(t, s') - (-1)^J B_s(t, s') \right] \left[Q_{J-1} \left(\frac{s'+q^2+p^2}{2qp} \right) - Q_{J+1} \left(\frac{s'+q^2+p^2}{2qp} \right) \right] ds' \quad (6.23)$$

Now the expressions (6.18) and (6.23) are suitable for the analytic continuation of the amplitudes S_+^J and Λ_-^J into the complex J -plane since the analytic continuation of the Q functions are known. As before the only difficulty arises from the factor $(-1)^J$. Therefore we again define two continuations for each of the amplitudes: Even continuation for $J = \text{even}$ and odd continuation for $J = \text{odd}$. Thus we obtain the amplitudes $S_+^{0,1 J, e}$ and $\Lambda_-^{0,1 J, e}$.

Now that we know the analytic continuation of S_+^J and Λ_-^J we can apply the Sommerfeld-Watson transformation to the series (6.3) and distort the contour as done before. When $\cos \theta_t \rightarrow \infty$ again the contribution from the integral along the line $\text{Re } J = -\frac{1}{2}$ will vanish and the main contribution will come from the Regge pole terms.

Sommerfeld-Watson transformation on χ_+ and χ_- amplitudes:

As before we separate χ_+ and χ_- into two parts by summing over even and odd integers respectively. Thus:

$$\begin{aligned} \chi_+ = \chi_+^e + \chi_+^o &= -8\pi i E \sqrt{p/q} \sum_{J=0,2,4,\dots} (J+\frac{1}{2}) S_+^J P_J(\cos\theta_t) \\ &\quad - 8\pi i E \sqrt{p/q} \sum_{J=1,3,5,\dots} (J+\frac{1}{2}) S_+^J P_J(\cos\theta_t) \end{aligned} \quad (6.24)$$

$$\chi_- = \chi_-^e + \chi_-^o = -\frac{8\pi i}{q\sqrt{qp}} \sum_{J=0,2,4,\dots} \Lambda_-^J P_J'(\cos\theta_t) - \frac{8\pi i}{q\sqrt{qp}} \sum_{J=1,3,5,\dots} \Lambda_-^J P_J'(\cos\theta_t)$$

$$\text{Using } P_J'(-z) = (-1)^{J+1} P_J'(z) \quad (6.25)$$

$$\text{and } P_J(-z) = (-1)^J P_J(z) \quad (6.26)$$

one can also write $\chi_+^{e,o}$ and $\chi_-^{e,o}$ as:

$$\chi_+^{e,o} = -8\pi i E \sqrt{p/q} \sum_{J=0,1,2,\dots} (J+\frac{1}{2}) S_+^{J,o} \frac{P_J(\cos\theta_t) \mp P_J(-\cos\theta_t)}{2} \quad (6.27)$$

$$\chi_-^{e,o} = -\frac{8\pi i}{q\sqrt{qp}} \sum_{J=0,1,2,\dots} \Lambda_-^{J,o} \frac{P_J'(\cos\theta_t) \mp P_J'(-\cos\theta_t)}{2} \quad (6.28)$$

where the upper signs refer to e and the lower signs to o.

χ_+ and χ_- can now be written applying the Sommerfeld-Watson transformation:

$$\begin{aligned}
 \chi_{+}^{0,1} &= (1/2)8\pi iE \sqrt{p/q} \int (J+\frac{1}{2}) \binom{0,1 J, e}{S_{+}} \frac{P_J(\cos\theta_t) + P_J(-\cos\theta_t)}{2} \frac{dJ}{\sin\pi J} \\
 &- (1/2)8\pi iE \sqrt{p/q} \int (J+\frac{1}{2}) \binom{0,1 J, o}{S_{+}} \frac{P_J(\cos\theta_t) - P_J(-\cos\theta_t)}{2} \frac{dJ}{\sin\pi J} \\
 &= (\chi_{+}^{0,1})_{+} + (\chi_{+}^{0,1})_{-}
 \end{aligned} \tag{6.29a}$$

$$\begin{aligned}
 \chi_{-}^{0,1} &= (1/2) \frac{8\pi i}{q\sqrt{qp}} \int \binom{0,1 J, e}{\Lambda_{-}} \frac{P'_J(\cos\theta_t) - P'_J(-\cos\theta_t)}{2} \frac{dJ}{\sin\pi J} \\
 &- (1/2) \frac{8\pi i}{q\sqrt{qp}} \int \binom{0,1 J, o}{\Lambda_{-}} \frac{P'_J(\cos\theta_t) + P'_J(-\cos\theta_t)}{2} \frac{dJ}{\sin\pi J} \\
 &= (\chi_{-}^{0,1})_{+} + (\chi_{-}^{0,1})_{-}
 \end{aligned} \tag{6.29b}$$

Regge pole contributions to χ_{+} and χ_{-} amplitudes:

Finally the contributions of Regge poles to the χ_{+} and χ_{-} amplitudes will be:

$$\left[\chi_{+}^{0,1} \right]_{+} = \pm 2\pi i (1/2)8\pi iE \sqrt{p/q} (J+\frac{1}{2}) \frac{P'_{\alpha}(\cos\theta_t) \mp P'_{\alpha}(-\cos\theta_t)}{2 \sin \pi \alpha} \lim_{J \rightarrow \alpha} \left[(J-\alpha) \binom{0,1 J, e}{S_{+}} \right] \tag{6.30a}$$

$$\left[\chi_{-}^{0,1} \right]_{+} = \pm 2\pi i (i/2) \frac{8\pi i}{q\sqrt{qp}} \frac{P'_{\alpha}(\cos\theta_t) \mp P'_{\alpha}(-\cos\theta_t)}{2 \sin \pi \alpha} \lim_{J \rightarrow \alpha} \left[(J-\alpha) \binom{0,1 J, o}{\Lambda_{-}} \right]$$

(6.30b)

where the upper signs refer to the even J-parity Regge poles and the lower signs to odd J-parity Regge poles.

As $\cos \theta_t \rightarrow \infty$ there is no complications arising as in the case of backward scattering. The t-channel variables are given by eq.s (2.8). Finding q^2 and p^2 from the first equation and inserting into the second gives

$$s = -\frac{1}{2}t + M^2 + m^2 - \frac{1}{2} \sqrt{(t-4M^2)(t-4m^2)} \cos \theta_t \quad (6.31)$$

Whatever the value of $\cos \theta_t$ may be, for $t = 4M^2$ and $t = 4m^2$ the curve has the same values which are the appexes of the hyperbola

$$u.s = (M^2 - m^2)^2 \quad (6.32)$$

Thus all curves $\cos \theta_t = \text{constant}$ pass through these two points. They approach the lines $t = 4M^2$ and $t = 4m^2$ as $\cos \theta_t \rightarrow \infty$.

From Fig.(21) it is seen that there is no difficulty in entering the physical regions s or u since the curve $\cos \theta_t = \text{const.}$ intersects the line $t = 0$. Following Frautschi, Gell-Mann and Zachariasen²⁴ one can redefine the residues in order to avoid coefficients with variable dimensions.

For $\cos \theta_t \rightarrow \infty$ $P_\alpha(\cos \theta_t) \rightarrow s^\alpha$ and $P'_\alpha(\cos \theta_t) \rightarrow \alpha s^{\alpha-1}$

and the above formulas simplify to

$$\left[\chi_{+}^{0,1} \right]_{\pm} = \pm \left(\frac{s}{2M} \right)^{\alpha_{\pm}(t)} \left[\frac{1 \pm e^{-i\pi \alpha_{\pm}(t)}}{2 \sin \pi \alpha_{\pm}(t)} \right] C_{\chi_{+}, \pm}^{0,1} \quad (6.33)$$

$$\left[\chi_{-}^{0,1} \right]_{\pm} = \pm \left(\frac{s}{2M} \right)^{\alpha_{\pm}(t) - 1} \left[\frac{1 \pm e^{-i\pi \alpha_{\pm}(t)}}{2 \sin \pi \alpha_{\pm}(t)} \right] C_{\chi_{-}, \pm}^{0,1}$$

The definition of the coefficients in these last formulas can be obtained in terms of the residues of λ_-^J and S_+^J :

$$\text{Let us call } \lim_{J \rightarrow \alpha} \left[(J-\alpha) \begin{pmatrix} 0,1 J,0^e \\ S_+ \end{pmatrix} \right] = \begin{pmatrix} 0,1 e \\ b_+^0 \end{pmatrix}$$

$$\lim_{J \rightarrow \alpha} \left[(J-\alpha) \begin{pmatrix} 0,1 J,0^e \\ \lambda_- \end{pmatrix} \right] = \begin{pmatrix} 0,1 e \\ b_-^0 \end{pmatrix}$$

From the second equation of the relations (2.8) one finds

$$\cos \theta_t \rightarrow -\frac{s}{2qp}$$

For large $\cos \theta_t$, $P_\alpha(\cos \theta_t)$ and $P'_\alpha(\cos \theta_t)$ behave like

$$P_\alpha \rightarrow \frac{2^\alpha \Gamma(\alpha + \frac{1}{2})}{\sqrt{\pi} \Gamma(\alpha + 1)} \cos \theta_t^\alpha$$

$\cos \theta_t \rightarrow \infty$

$$P'_\alpha \rightarrow \frac{2^\alpha \Gamma(\alpha + \frac{1}{2})}{\sqrt{\pi} \Gamma(\alpha)} (\cos \theta_t)^{\alpha-1}$$

$\cos \theta_t \rightarrow \infty$

The behavior of $P_\alpha(-\cos \theta_t)$ and $P'_\alpha(-\cos \theta_t)$ is found from the relations:

$$P_\alpha(-\cos \theta_t) = e^{i\pi\alpha} P_\alpha(\cos \theta_t) - \frac{2}{\pi} \sin \pi\alpha Q_\alpha(\cos \theta_t),$$

and

$$Q_\alpha(\cos \theta_t) \rightarrow \frac{\sqrt{\pi} \Gamma(\alpha + 1)}{2^{\alpha+1} \Gamma(\alpha + \frac{3}{2})}$$

$$\cos \theta_t \rightarrow \infty$$

Thus one obtains

$$C_{\chi_{+,-}^{0,1}} = \pm (-8\pi^2 i E) \sqrt{p/q} (\alpha_{\pm} + \frac{1}{2}) \frac{\Gamma(\alpha_{\pm} + \frac{1}{2})}{\sqrt{\pi} \Gamma(\alpha_{\pm} + 1)} (-1)^{\alpha_{\pm}} \left(\frac{2M}{qp}\right)^{\alpha_{\pm}} \begin{pmatrix} 0,1 & 8 \\ & b_{+} \end{pmatrix}$$

$$C_{\chi_{-,-}^{0,1}} = \pm \left(\frac{-16\pi^2 i}{q\sqrt{qp}} \right) \frac{\Gamma(\alpha_{\pm} + \frac{1}{2})}{\sqrt{\pi} \Gamma(\alpha_{\pm})} (-1)^{\alpha_{\pm}-1} \begin{pmatrix} 0,1 & 8 \\ & b_{-} \end{pmatrix}$$

where the kaon mass is taken to be unity.

VII

High energy $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$ forward elastic scattering

We again have the processes (5.1). The differential cross section is given by:

$$\frac{d\sigma}{d\Omega} = |f_1|^2 + |f_2|^2 + \cos\theta_s (f_1 f_2^* + f_2 f_1^*) \quad (7.1)$$

Using the expressions for f_1 and f_2 in terms of A and B (4.14) and (4.15) and inserting $\cos\theta_s = 1 + \frac{t}{2k_s^2}$ the differential cross section becomes:

$$\begin{aligned} \frac{d\sigma}{d\Omega} = & \frac{2}{(8\pi W)^2} \left[2M^2 - \frac{t}{2k_s^2} (E^2 - M^2) \right] |A|^2 \\ & + \frac{2M}{(8\pi W)^2} \left[2\omega M + \frac{t}{2k_s^2} (E^2 - M^2) \right] (AB^* + BA^*) \\ & + \frac{2}{(8\pi W)^2} \left[2M^2 \omega^2 + \frac{t}{2k_s^2} (E^2 - M^2)(W^2 - M^2) \right] |B|^2 \end{aligned} \quad (7.2)$$

where $W^2 = s$ is the total c.o.m. energy-squared

$$E = \frac{W^2 + M^2 - 1}{2W} \quad \text{the total c.o.m. nucleon energy} \quad (7.3)$$

$$\omega = \frac{s - M^2 - 1}{2M} \quad \text{the laboratory energy of the incident kaon } (m_k=1).$$

The expression (7.2) can also be written as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} (M^2 - \frac{t}{1}) |A|^2 + \frac{2M}{64\pi^2 s} (s - M^2 - 1 + \frac{1}{2}t) (AB^* + BA^*) + \frac{1}{64\pi^2 s} \left[(s - M^2 - 1)^2 + t(s - M^2) \right] |B|^2$$

In the expression (7.2) we again have to make the following substitutions for the corresponding KN $\rightarrow$ KN processes:

$$\begin{aligned}
 1) \quad A &\rightarrow A_s^1 \rightarrow \frac{1}{2}(A_t^0 + A_t^1) & B &\rightarrow B_s^1 \rightarrow \frac{1}{2}(B_t^0 + B_t^1) \\
 2) \quad A &\rightarrow \frac{1}{2}(A_s^0 + A_s^1) \rightarrow \frac{1}{2}(A_t^0 - A_t^1) & B &\rightarrow \frac{1}{2}(B_s^0 + B_s^1) \rightarrow \frac{1}{2}(B_t^0 - B_t^1) \quad (7.5) \\
 3) \quad A &\rightarrow \frac{1}{2}(A_s^1 - A_s^0) \rightarrow A_t^1 & B &\rightarrow \frac{1}{2}(B_s^1 - B_s^0) \rightarrow B_t^1
 \end{aligned}$$

Here we expressed the processes 1 through 3 of (5.1) as combinations of the pure isotopic spin amplitudes in the s-channel and then used the following crossing matrix to express them in terms of the pure isotopic spin amplitudes in the t-channel:

$$\begin{pmatrix} A_s^0 \\ A_s^1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & -3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} A_t^0 \\ A_t^1 \end{pmatrix}$$

Now let us again introduce the χ_+ and χ_- amplitudes defined by (6.1). In terms of these amplitudes the s-channel differential cross section becomes:

$$\frac{d\sigma}{d\Omega} = \frac{1}{16\pi^2 s(M^2 - \frac{1}{4}t)} \left\{ |\chi_+|^2 + \frac{t}{16} [su - (M^2 - 1)^2] |\chi_-|^2 \right\} \quad (7.6)$$

We see that the cross term has dropped out. Moreover since the eq.(7.6) is obtained from eq.(7.4) by inserting for A and B

$$A = \frac{1}{M^2 - \frac{1}{4}t} (\chi_+ - M \frac{s-u}{4} \chi_-) \quad (7.7)$$

$$B = \chi_-$$

which are in s-channel pure isotopic spin amplitudes, when the substitutions (7.5) are made we will end up with the corresponding substitutions for the χ_+ and χ_- amplitudes. That is one has to replace χ_+ and χ_- in (7.6) by

$$1) \quad \chi_+ \rightarrow \chi_{+s}^1 \rightarrow \frac{1}{2}(\chi_{+t}^0 + \chi_{+t}^1) \quad \chi_- \rightarrow \chi_{-s}^1 \rightarrow \frac{1}{2}(\chi_{-t}^0 + \chi_{-t}^1)$$

$$2) \quad \chi_+ \rightarrow \frac{1}{2}(\chi_{+s}^0 + \chi_{+s}^1) \rightarrow \frac{1}{2}(\chi_{+t}^0 - \chi_{+t}^1)$$

$$\chi_- \rightarrow \frac{1}{2}(\chi_{-s}^0 + \chi_{-s}^1) \rightarrow \frac{1}{2}(\chi_{-t}^0 - \chi_{-t}^1) \quad (7.9)$$

$$3) \quad \chi_+ \rightarrow \frac{1}{2}(\chi_{+s}^1 - \chi_{+s}^0) \rightarrow \chi_{+t}^1 \quad \chi_- \rightarrow \frac{1}{2}(\chi_{-s}^1 - \chi_{-s}^0) \rightarrow \chi_{-t}^1$$

Since

$$\frac{d\sigma}{dt} = \frac{\pi}{s} \frac{d\sigma}{dn} \quad (7.10)$$

we obtain after inserting the expressions (6.33) in this formula

$$\frac{d\sigma}{dt} \rightarrow \frac{1}{16\pi(M^2 - \frac{1}{4}t)} \left(\frac{s}{2M}\right)^{2\alpha(t)-2} \left| \frac{1 \pm e^{-i\pi\alpha(t)}}{2 \sin \pi\alpha(t)} \right|^2 \left\{ |c_{\chi_+}|^2 - \frac{t}{16} |c_{\chi_-}|^2 \right\} \quad (7.11)$$

where $\alpha(t)$ is the Regge pole with the largest real part. Here it should be emphasized that there are two independent residues at each pole.

We have already seen what particles can be exchanged in the t-channel. The $I = 0$ particles in the t-channel have the same contribution to s- and u-channel pure isotopic spin amplitudes (at the same point in Mandelstam diagram) whereas the $I = 1$ particles contribute with opposite signs. To find the contribution of each pole let us write again the t-channel amplitudes as sum of even and odd J-

parity parts as given by the eq.s (6.29) and (6.30).

$$\chi_{+t}^0 = (\chi_{+t}^0)_+ + (\chi_{+t}^0)_- \quad \chi_{-t}^0 = (\chi_{-t}^0)_+ + (\chi_{-t}^0)_- \quad (7.12)$$

$$\chi_{+t}^1 = (\chi_{+t}^1)_+ + (\chi_{+t}^1)_- \quad \chi_{-t}^1 = (\chi_{-t}^1)_+ + (\chi_{-t}^1)_-$$

Thus the vacuum poles P, P' and the ABC particle will contribute only to $(\chi_{+t}^0)_+$ and $(\chi_{-t}^0)_+$

ω will contribute only to $(\chi_{+t}^0)_-$ and $(\chi_{-t}^0)_-$

ρ will contribute only to $(\chi_{+t}^1)_-$ and $(\chi_{-t}^1)_-$

From the definition of the variables s, t , and u by the eq.s (2.5) and (2.7) for each case where the corresponding channel is physical we see that one has to interchange s and u if one wants the u -channel to be physical. But interchanging s and u means

$$\cos \theta_t \rightarrow -\cos \theta_t.$$

From (6.29) and (6.30) we see that under such an interchange the χ_+ amplitudes for example become:

$$\begin{aligned} [\chi_{+t}(\cos \theta_t)]_+ &= [\chi_{+t}(-\cos \theta_t)]_+ , \\ [\chi_{+t}(\cos \theta_t)]_- &= -[\chi_{+t}(-\cos \theta_t)]_- \end{aligned}$$

As the result of these considerations one sees that the

vacuum poles P, P' and ABC have the same contributions to the physical processes

$$KN \rightarrow KN \text{ and } \bar{K}N \rightarrow \bar{K}N$$

ω has opposite contributions to the physical processes $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$.

ρ has the same contributions to the physical processes $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$.

Thus for the different processes in s-channel we have to make the following substitutions:

$$\begin{aligned}
 1) \quad \chi_{+s}^1 &= \frac{1}{2}(\chi_{+t}^0 + \chi_{+t}^1) = \frac{1}{2}([\chi_{+t}^0]_{+} + [\chi_{+t}^0]_{-} + [\chi_{+t}^1]_{+} + [\chi_{+t}^1]_{-}) \\
 &= \frac{1}{2} \left\{ \left(\frac{s}{2M} \right)^{\alpha_p} \frac{1+e^{-i\pi\alpha_p}}{2 \sin \pi\alpha_p} C_{\chi_{+P}}^0 + \left(\frac{s}{2M} \right)^{\alpha_{p'}} \frac{1+e^{-i\pi\alpha_{p'}}}{2 \sin \pi\alpha_{p'}} C_{\chi_{+P'}}^0 + \left(\frac{s}{2M} \right)^{\alpha_{ABC}} \frac{1+e^{-i\pi\alpha_{ABC}}}{2 \sin \pi\alpha_{ABC}} C_{\chi_{+ABC}}^0 \right. \\
 &\quad \left. - \left(\frac{s}{2M} \right)^{\alpha_{\omega}} \frac{1-e^{-i\pi\alpha_{\omega}}}{2 \sin \pi\alpha_{\omega}} C_{\chi_{+\omega}}^0 - \left(\frac{s}{2M} \right)^{\alpha_{\rho}} \frac{1-e^{-i\pi\alpha_{\rho}}}{2 \sin \pi\alpha_{\rho}} C_{\chi_{+\rho}}^1 \right\} \quad (7.13a)
 \end{aligned}$$

Let us write this formula as:

$$\chi_{+s}^1 = \frac{1}{2} \left\{ (P) + (P') + (ABC) - (\omega) - (\rho) \right\}$$

where each term is defined by the corresponding term in eq. (7.13a)

$$\begin{aligned}
 2) \quad \chi_{+}^1 &= \frac{1}{2}(\chi_{+s}^0 + \chi_{+s}^1) = \frac{1}{2}(\chi_{+t}^0 - \chi_{+t}^1) = \frac{1}{2}([\chi_{+t}^0]_{+} + [\chi_{+t}^0]_{-} - [\chi_{+t}^1]_{+} - [\chi_{+t}^1]_{-}) \\
 &= \frac{1}{2} \left\{ (P) + (P') + (ABC) - (\omega) + (\rho) \right\} \quad (7.13b)
 \end{aligned}$$

$$\begin{aligned}
 3) \quad \chi_{+}^1 &\rightarrow \frac{1}{2}(\chi_{+s}^1 - \chi_{+s}^0) = \chi_{+t}^1 = [\chi_{+t}^1]_{+} + [\chi_{+t}^1]_{-} \\
 &= -(\rho) \quad (7.13b)
 \end{aligned}$$

For the χ_{-} amplitudes we will have similar expressions where $\left(\frac{s}{2M}\right)$ has one power less and instead of the coefficients $C_{\chi_{+}}$ we have the coefficients $C_{\chi_{-}}$. In these formulas the kaon-mass is taken to be unity.

For the physical processes in the u-channel the substitutions become using the crossing matrix:¹³

$$\begin{pmatrix} 0 \\ A \\ u \\ 1 \\ A \\ u \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ A \\ t \\ 1 \\ A \\ t \end{pmatrix}$$

$$1) \quad \frac{1}{2}(\chi_{+u}^0 + \chi_{+u}^1) = \frac{1}{2}(\chi_{+t}^0 + \chi_{+t}^1) = \frac{1}{2} \left\{ \left[\chi_{+t}^0(s,t,u) \right]_+ + \left[\chi_{+t}^0(s,t,u) \right]_- + \left[\chi_{+t}^1(s,t,u) \right]_+ + \left[\chi_{+t}^1(s,t,u) \right]_- \right\}$$

and interchanging s and u to make the u-channel physical

$$\begin{aligned} \chi_{+} &\rightarrow \frac{1}{2} \left\{ \left[\chi_{+t}^0(u,t,s) \right]_+ + \left[\chi_{+t}^0(u,t,s) \right]_- + \left[\chi_{+t}^1(u,t,s) \right]_+ + \left[\chi_{+t}^1(u,t,s) \right]_- \right\} \\ &= \frac{1}{2} \left\{ \left[\chi_{+t}^0(s,t,u) \right]_+ - \left[\chi_{+t}^0(s,t,u) \right]_- + \left[\chi_{+t}^1(s,t,u) \right]_+ - \left[\chi_{+t}^1(s,t,u) \right]_- \right\} \\ &= \frac{1}{2} \left[(P) + (P') + (ABC) + (\omega) + (\rho) \right] \end{aligned} \quad (7.14a)$$

$$\begin{aligned} 2) \quad \chi_{+u}^1 &= \frac{1}{2}(\chi_{+t}^0 - \chi_{+t}^1) = \frac{1}{2} \left\{ \left[\chi_{+t}^0(s,t,u) \right]_+ + \left[\chi_{+t}^0(s,t,u) \right]_- - \left[\chi_{+t}^1(s,t,u) \right]_+ - \left[\chi_{+t}^1(s,t,u) \right]_- \right\} \\ \chi_{+} &\rightarrow \frac{1}{2} \left\{ \left[\chi_{+t}^0(u,t,s) \right]_+ + \left[\chi_{+t}^0(u,t,s) \right]_- - \left[\chi_{+t}^1(u,t,s) \right]_+ - \left[\chi_{+t}^1(u,t,s) \right]_- \right\} \\ &= \frac{1}{2} \left\{ \left[\chi_{+t}^0(s,t,u) \right]_+ - \left[\chi_{+t}^0(s,t,u) \right]_- - \left[\chi_{+t}^1(s,t,u) \right]_+ + \left[\chi_{+t}^1(s,t,u) \right]_- \right\} \\ &= \frac{1}{2} \left[(P) + (P') + (ABC) + (\omega) - (\rho) \right] \end{aligned} \quad (7.14b)$$

$$\begin{aligned} 3) \quad \frac{1}{2}(\chi_{+u}^0 - \chi_{+u}^1) &= \chi_{+t}^1 = \left[\chi_{+t}^1(s,t,u) \right]_+ + \left[\chi_{+t}^1(s,t,u) \right]_- \\ A &\rightarrow \left[\chi_{+t}^1(u,t,s) \right]_+ + \left[\chi_{+t}^1(u,t,s) \right]_- \\ &= \left[\chi_{+t}^1(s,t,u) \right]_+ - \left[\chi_{+t}^1(s,t,u) \right]_- \\ &= (\rho) \end{aligned} \quad (7.14c)$$

Again with similar formulas for the χ_- amplitudes.

These expressions substituted into the formula (7.6) give us the differential cross sections for the processes 1 through 3 of (5.1) and (5.11) both for $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$ channels.

VIII.

Total cross sections.

The total cross section is given by the formula

$$\sigma^{\text{tot}} = \frac{4\pi}{q} \text{Im}(f_1 + f_2)_{t=0} \quad (8.1)$$

Expressing f_1 and f_2 in terms of A, B and using the eq.s (4.14) and (4.15)

$$f_1 + f_2 = \frac{M}{4\pi W} (A + \omega B) \quad (8.2)$$

where $\omega = \frac{EW - M^2}{M} = \frac{s - M^2 - 1}{2M}$ is the kaon lab. energy

$$E = \frac{W^2 + M^2 - 1}{2W} \quad \text{nucleon total energy} \quad (8.3)$$

$$W^2 = s \quad \text{c.o.m total energy-squared}$$

σ^{tot} can also be written as:

$$\sigma^{\text{tot}} = \frac{M}{qW} \text{Im}(A + \omega B)_{t=0} \quad (8.4)$$

For high energies $\frac{M}{qW} \rightarrow \frac{2M}{s}$ (8.5)

$$\omega \rightarrow \frac{s}{2M}$$

and we obtain $\sigma^{\text{tot}} = \frac{2M}{s} \text{Im}(A + \frac{s}{2M} B)_{t=0}$ (8.6)

It is also known that the c.o.m energy-squared is proportional to the laboratory

total energy of the incident particle. Making use of this relation and using the high energy expressions (7.13) and (7.14) for the physical processes $KN \rightarrow KN$ and $\bar{K}N \rightarrow \bar{K}N$ respectively one can form different combinations between the total cross sections. To avoid confusion with the ω particle we use for the kaon laboratory total energy E_K now. (The residues are redefined)

$$\begin{cases}
 \sigma_{\bar{K}p - K^+p} = C_{\omega} E_K^{\alpha_{\omega}(0)-1} + C_E E_K^{\alpha_f(0)-1} \\
 \sigma_{\bar{K}p + K^+p} = C_P + C_{P'} E_K^{\alpha_{P'}(0)-1} + C_{ABC} E_K^{\alpha_{ABC}(0)-1} \\
 \sigma_{\bar{K}n - K^+n} = C_{\omega} E_K^{\alpha_{\omega}(0)-1} - C_f E_K^{\alpha_f(0)-1} \\
 \sigma_{\bar{K}n + K^+n} = C_P + C_{P'} E_K^{\alpha_{P'}(0)-1} + C_{ABC} E_K^{\alpha_{ABC}(0)-1} \\
 \sigma_{K^+p - K^+n} = -C_f E_K^{\alpha_f(0)-1} \\
 \sigma_{K^+p - K^+n} = C_P + C_{P'} E_K^{\alpha_{P'}(0)-1} + C_{ABC} E_K^{\alpha_{ABC}(0)-1} - C_{\omega} E_K^{\alpha_{\omega}(0)-1} \\
 \sigma_{\bar{K}p - K^+n} = C_{\omega} E_K^{\alpha_{\omega}(0)-1} \\
 \sigma_{\bar{K}p + K^+n} = C_P + C_{P'} E_K^{\alpha_{P'}(0)-1} + C_{ABC} E_K^{\alpha_{ABC}(0)-1} + C_f E_K^{\alpha_f(0)-1} \\
 \sigma_{\bar{K}n - K^+p} = C_{\omega} E_K^{\alpha_{\omega}(0)-1} \\
 \sigma_{\bar{K}n + K^+p} = C_P + C_{P'} E_K^{\alpha_{P'}(0)-1} + C_{ABC} E_K^{\alpha_{ABC}(0)-1} - C_f E_K^{\alpha_f(0)-1} \\
 \sigma_{\bar{K}p - K^+n} = C_f E_K^{\alpha_f(0)-1} \\
 \sigma_{\bar{K}p + K^+n} = C_P + C_{P'} E_K^{\alpha_{P'}(0)-1} + C_{ABC} E_K^{\alpha_{ABC}(0)-1} + C_{\omega} E_K^{\alpha_{\omega}(0)-1}
 \end{cases}$$

(10.7)

It is known from the recent high energy experiments that the $K^+ p$ total cross section is ^{roughly} constant at 18.4 mb, whereas the $K^- p$ total cross section decreases from 28.2 mb for a kaon momentum of 4 Bev/c to 21.6 mb for a kaon momentum of 19.1 Bev/c. One can take

$$\sigma^{K^+ p} = C_p$$

making use of the equality of the particle and anti-particle cross sections according to the Pomeranchuk theorem. If the ABC term is negligible at these energies one can also estimate $C_{P'}^{and \alpha_{P'}(0)}$ by fitting the data to the first two terms as was done by W.F.Baker, R.L.Cool, E.W.Jenkins, T.F.Kycia, R.H.Phillips and A.L.Read²⁶ who found

$$C_{P'} = 21.7 \pm 6.2$$

$$\alpha_{P'}(0) = 0.36 \pm 0.13$$

which is not very different from that found by Igi.²⁷ Thus the sign and the magnitude of the residues of P and P' are fairly known. On the other hand to be able to say anything about the sign and the magnitude of the residues of ω and ρ one has to know the high energy behavior of the $K^+ n$ and $K^- n$ cross sections.

C o n c l u s i o n

High energy kaon-nucleon scattering has been considered on the basis of Regge particles in the various channels. Special attention has been given to the fermion trajectories in the $\bar{K}N \rightarrow \bar{K}N$ channel and some possible relations between the associated particles investigated. Contributions of Regge poles to the amplitudes have been calculated and expressions for forward, backward and total cross sections derived.

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Fig.1 Kaon-nucleon scattering diagram.

Fig.2 Mandelstam diagram for kaon-nucleon scattering.

Fig.3 Contour of the Sommerfeld-Watson integral.

Fig.4 Distorted contour of the Sommerfeld-Watson integral

Fig.5 A possible fermion Regge trajectory.

Fig.6 Real part of the pole as function of the energy.

Fig.7 A possible connection between Λ_α and Λ_β or Σ_δ and Σ_δ trajectories if these are the trajectories related by Mac Dowell's reflection principle.

Fig.8 Poles of $I=0$, even parity, even continuation amplitude $h^e_{J+\frac{1}{2}}(W_u)$

Fig.9 Pole of $I=0$, odd parity, even continuation amplitude $h^e_{J-\frac{1}{2}}(W_u)$

Fig.10 Regge trajectory of $I = 0$ even continuation amplitudes if Λ_u and Λ_p are the trajectories related by Mac Dowell's reflection principle.

Fig.11 Pole of $I = 0$ odd parity, odd continuation amplitude $h^0_{J+\frac{1}{2}}(w_u)$

Fig.12 Regge trajectory of $I = 0$ odd continuation amplitudes (Λ_y)

Fig.13 Pole of the $I = 1$ even parity even continuation amplitude $h_{J+\frac{1}{2}, u}^e(w)$

Fig.14 Regge trajectory of the $I = 1$ even continuation amplitudes.

Fig.15 Pole of the $I = 1$ odd parity
odd continuation amplitude

$$h_{J+\frac{1}{2}}^o(w_u)$$

Fig.16 Pole of the $I = 1$ even parity
odd continuation amplitude

$$h_{J-\frac{1}{2}}^o(w_u)$$

Fig.17 Regge trajectory of the $I = 1$ odd continuation amplitudes if Σ_d and Σ_y
are the trajectories related by Mac Dowell's reflection principle.

Fig.18 Crossing from s-channel to u-channel by letting $\cos\theta_s$ go to infinity (equal mass case).

Fig.19 Crossing from s-channel to u-channel by letting $\cos\theta_s$ go to $-\infty$ (unequal mass case).

Fig.20 Crossing from s-channel to u-channel by letting $\cos\theta_s$ go to $+\infty$ (unequal mass case).

Fig.21 Crossing from t-channel to s (or u) channel by letting $\cos \theta_t$ go to infinity.

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