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THE ECONOMICS OF FUEL DEPLETION IN FAST BREEDER REACTOR BLANKETS

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S.T. Brewer, E.A. Mason and M.J. Driscoll

November, 1972

Department of Nuclear Engineering
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

AEC Research and Development Report
UC-34 Physics
Contract AT(11-1)-3060
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF NUCLEAR ENGINEERING

Cambridge, Massachusetts

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U.S. Atomic Energy Commission, Headquarters
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Dr. Paul Greebler, Manager
Nuclear Energy Division
Breeder Reactor Department
General Electric Company
310 DeGuigne Drive
Sunnyvale, California 94086

(1 copy)

Dr. Harry Morewitz, Manager
LMFBR Physics and Safety Projects
Atomics International
P.O. Box 309
Canoga Park, California 91305

(1 copy)

Mr. Malcolm Dyos, Manager
Nuclear Development, LMFBR Project
Westinghouse Electric Corporation
Advanced Reactors Division
Waltz Mill Site
P.O. Box 158
Madison, Pennsylvania 15663

(1 copy)

Dr. Robert Avery, Director
Reactor Physics Division
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60539

(1 copy)

Dr. Charles A. Preskitt, Jr.
Mgr., Atomic and Nuclear Department
Gulf Radiation Technology
P.O. Box 608
San Diego, California 92112

(1 copy)

ABSTRACT

A fast breeder reactor fuel depletion-economics model was developed and applied to a number of 1000 MWe LMFBR case studies, involving radial blanket-radial reflector design, radial blanket fuel management, and sensitivity of energy costs to changes in the economic environment.

Choice of fuel cost accounting philosophy, e.g. whether or not to tax plutonium revenue, was found to have significant effect on absolute values of energy costs, without, however, distorting design rankings, comparative results, and irradiation time optimization.

A single multigroup physics computation, to obtain the flux shape and local spectra for depletion calculations, was found to be sufficient for preliminary design and sensitivity studies. The major source of error in blanket depletion results was found to be the assumption of a fixed flux shape over an irradiation cycle; spectrum hardening in the radial blanket with irradiation is of minor importance.

The simple depletion-economics model was applied to several 1000 MWe LMFBR case studies. Advantages of a moderating reflector were found to increase as blanket thickness was reduced. For a 45 cm radial blanket, a beryllium metal reflector offered little improvement, in blanket fuel economics, over sodium; for a 15 cm blanket, beryllium increased net blanket revenue by about 60%. An improvement of about 30% in net blanket revenue resulted when each radial blanket annular region was assumed to be exposed to its own local optimum irradiation time. Optimum radial blanket irradiation time and the net blanket revenue (mills/kWhe) at this optimum were found to be approximately linear in the unit fuel cycle costs, i.e. fabrication and reprocessing costs (\$/kgHM) and fissile market value (\$/kg).

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Computer calculations were performed at the MIT Information Processing Center.

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CHAPTER 1

INTRODUCTION AND SUMMARY

1.1 INTRODUCTION

A Fast Breeder Reactor blanket performs several functions: fertile-to-fissile converter, reflector, shield. In addition, it produces some power, thereby relieving, slightly, the power burden on the core. Of these functions, the fissile breeding objective is considered paramount. For current 1000 MWe designs, a fast reactor without blankets is not a breeder; although most of the conversion is accomplished in the core (internal breeding ratio ~ 0.8), a fertile blanket is required to achieve overall breeding ratios above unity.

An AEC sponsored program is underway at MIT using the Fast Reactor Blanket Test Facility (BTF) to investigate blanket neutronics for the LMFBR effort (60, 61). To guide the selection of blanket mock-up experiments, comparative studies have been made of the fuel economics of several LMFBR blanket-reflector configurations.

Objectives of the work reported here were twofold: (1) to develop a simple depletion-economics calculational tool for survey evaluations of LMFBR blanket configurations; and (2) to perform several comparative studies around a 1000 MWe reference LMFBR configuration. The 1000 MWe case studies involve choice of radial reflector material (Be-metal vs. sodium), radial blanket thickness, advantages of local fuel management in the radial blanket, and the sensitivity of LMFBR fuel energy costs to changes in the economic environment.

1.2 OUTLINE OF THE REPORT

Calculational methods for FBR fuel depletion economics are developed in Chapters 2 and 3. Chapter 2 deals with the accounting details involved in determining energy costs by reactor region (core, axial blanket, radial blanket). The depletion method is developed in Chapter 3.

In Chapter 4, the energy cost and depletion methods are combined to form a computational tool for evaluating the fuel economic performance - in units of mills/KWhe or \$/kg HM/year - of regions under either batch or scatter fuel management schemes. (Appendix C describes a computer program, SPPIA, developed to perform the depletion-economics computations.) A reference LMFBF configuration is selected, and the integrated depletion-economics model is applied to this reactor, in a reference economic environment.

In Chapter 5, the fuel depletion-economics model is applied to a series of case studies in which the radial blanket thickness, radial reflector material, fuel management scheme, and economic environment are varied around the reference.

Chapter 6 summarizes major conclusions of the study and lists several recommendations for future efforts.

Appendix B describes a preliminary scoping study examining the economic viability of FBR blankets as reactor unit size increases.

1.3 QUALITATIVE DISCUSSION OF FBR BLANKET DESIGN CONSIDERATIONS AND LITERATURE SURVEY

The major economic objective¹ of FBR blanket design is to maximize the

¹ Another objective frequently adopted is the maximization of blanket breeding ratio. This objective, which is usually not consistent with the net revenue (or minimum power cost) objective, is macro-economic in nature, and is keyed to national fuel resource conservation considerations.

net blanket fissile revenue, that is, to maximize the fissile credit less fabrication, reprocessing, and carrying charges. At the same time thermal-hydraulic engineering design seeks to minimize the effects of the blanket power swing over a refueling cycle interval and to minimize the power gradient across the blanket. Other engineering considerations are the shielding role of the blanket, and possible material constraints on blanket exposure.

The blanket designer has several design variables and options to work with in meeting these objectives while satisfying the constraints. Some of the major variables and options are discussed qualitatively below. Studies which have addressed these considerations are referenced.

Blanket Thickness

Selection of blanket thickness involves a tradeoff between the fissile plutonium production rate and fuel cycle costs - fabrication, reprocessing, and associated carrying charges. An incremental increase in blanket thickness imposes additional fabrication and reprocessing costs while providing some additional fissile production. The incremental increase in fissile production decreases with blanket thickness because of flux attenuation. An incremental increase in thickness beyond some point is unprofitable - the added fissile revenue is not sufficient to offset the added fabrication and reprocessing costs.

The "optimum" thickness depends on the economic environment - fissile value ($\$/\text{kg Pu}_f$), fabrication cost ($\$/\text{kg HM}$) and reprocessing cost ($\$/\text{kgHM}$). Thick blankets are indicated when fissile value is high and/or fabrication and reprocessing costs are low. Thicker blankets may also be in order when leakage flux to the blanket is increased due to changes in core design.

The Westinghouse LMFR Follow-On Studies (73), Task I, have shown that the optimum radial blanket thickness is not sharp, that is, the

blanket profit is a weak function of blanket thickness. This conclusion is borne out in the present study. The Westinghouse optimum thickness is between 25 and 30 cm, again consistent with the present study.

Blanket Irradiation Time

Below some irradiation time, T_1 , the bred fissile inventory in the blanket is not sufficient to offset the blanket fabrication, reprocessing, and carrying charges. At T_1 , the "breakeven point", the revenue from bred fissile is just equal to fabrication, reprocessing, and carrying charges. Beyond T_1 , the blanket produces a net profit. As irradiation time, T , is further increased, Pu239 is produced at a decreasing rate, because of the burnup of both fertile U238 and fissile Pu239, and the fissile credit averaged over irradiation time, T , decreases. Also, as irradiation time T increases, carrying charges increase, and direct fabrication and reprocessing charges decrease. Taken together, these opposing effects result in an optimum irradiation time, T_{opt} , at which the net revenue in \$/kg HM/year (or in mills/KWhe) is a maximum.

Local optimum irradiation time decreases, and local net revenue at the optimum increases, with increased local flux. Thus regions near the blanket-core interface reach their optima sooner and produce more revenue than regions deeper in the blanket. For pancaked cores, the axial blanket optimum irradiation time is less than that of the radial blanket. Thinner blankets enjoy shorter optimum irradiation times.

Several studies have assessed optimum blanket irradiation times for particular designs (1, 4, 12, 70). Typical local optima range from about two to about eight years across the radial blanket.

Engineering considerations such as burnup, power swing, corrosion, and irradiation damage of cladding may tend to limit feasible irradiation time.

Blanket Fuel Management Scheme

Axial blanket fuel management is constrained to that of the core since axial blanket fuel assemblies are merely extensions of core assemblies in present LMFBR designs. The core-axial blanket fuel management scheme adopted in the 1000 MWe LMFBR Follow-on Studies (69, 70, 71, 72, 73) can be described as a region-scatter scheme. In this scheme, the core-axial blanket is divided into annular regions. At each refueling event, fractions $g_1, g_2, \dots$ of regions 1, 2, $\dots$ are discharged and replaced with fresh fuel. Fuel sees only one position in the reactor. The discharge fractions $g_1, g_2, \dots$ decrease with distance from the core centerline, implying that irradiation times increase with distance from the core centerline. This procedure enhances flux flattening and discharge burnup uniformity.

Radial blanket fuel management is independent of that of the core-axial blanket, with the restriction, of course, that blanket refueling dates coincide with those of the core-axial blanket, to minimize reactor shutdowns for refueling. With the exception of Westinghouse (73) the scheme selected in the 1000 MWe Follow-on Studies is region-scatter. Again, irradiation time increases and discharge fraction decreases with distance of the region from the core-blanket interface, thus implementing flux flattening across the blanket. Batch management is the special case of scatter management in which the discharge fractions are set equal to unity, i.e. at each refueling event for a given region, 100% of the fuel is discharged and replaced with fresh fuel.

Other schemes proposed for the radial blanket are out-in, in-out, and fuel assembly rotation. The Westinghouse Follow-on design (73) specifies in-out. In this scheme, fresh fuel is loaded in the innermost blanket

region, and is moved outward in subsequent refuelings, remaining in each annular region for one or more cycles. Fuel is discharged, finally, from the outermost region. Advantages (10) of the in-out management are power flattening, reduction of local power swing, and burnup uniformity. An earlier study (4) argued qualitatively that in-out management would be uneconomic due to the prolonged holdup of bred fissile. This was not demonstrated quantitatively.

In the out-in scheme, fresh fuel is loaded in the outermost region, moved inward, and discharged from the innermost region. The scheme has the advantage of achieving uniform burnup, and would tend to reduce the power swing over an irradiation cycle. However, out-in would tend to aggravate the power tilt across the blanket. Out-in management was compared (4) to fixed element management (batch or scatter) and was found to have only a few percent profit advantage.

A recent study (17) has investigated the optimum out-in throughput for a 1000 MWe LMFBR radial blanket. The study determined the effect of throughput on 10-year fuel cycle costs. Halving of the radial blanket out-in throughput increased fuel cycle costs (from optimal) by less than 5%. Increasing the throughput by a factor of about 1.5 increased the 10 year fuel cycle cost by about 1%.

The optimum throughput analysis reported in this (17) study was used as an illustration of a computational method for selecting optimal FBR fuel management strategies in a changing economic environment. The method permits changing fuel management during plant life (in response to changes in the economic environment) in order to minimize fuel costs during the remainder of plant life. In the radial blanket illustration cited, remaining plant life is 10 years.

Fuel element rotation has been studied by Westinghouse (10).

Rotation may be considered a sub-fuel management scheme in that it may be used in conjunction with the other schemes. During a refueling, fuel assemblies are simply rotated in place, thus moving fuel with high fissile content deeper into the blanket. Advantages of rotation are power-flattening and reduction of local power swing over an irradiation cycle. Westinghouse has shown that the maximum (with time) rod peaking factor for a radial blanket rod adjacent to the core can be reduced by about 20% by rotation. The reduction in power peaking across the blanket was not reported. Also, the effect of rotation on breeding economics was not reported.

Inner Radial Moderator

Insertion of a layer of moderating material between core and blanket would offer the advantage of softening the leakage flux entering the blanket, improving the fertile capture rate per incident neutron. On the other hand, the incident flux (entering the blanket) would be diminished due to absorption and reflection by the moderating layer. Thus the net effect of inner radial moderator configuration on blanket breeding is not qualitatively clear. Furthermore, one might expect the moderating layer to return more neutrons to the core and to degrade the returning spectrum. The net effect (on critical mass and internal breeding ratio) of the improved reflection plus degraded core spectrum is also not intuitively evident.

Perks and Lord (5) have performed survey calculations on the inner radial moderator concept, using a variety of moderating materials and thicknesses. Candidate materials were graphite (82% graphite), graphite-steel (41% graphite, 51% stainless steel) and sodium (100% sodium). The inner radial moderator configuration consistently resulted in a small reduction in critical mass, an increase in internal breeding ratio, a

reduction in blanket breeding ratio, and a net reduction in total breeding ratio. Their (5) cost results show that the core fissile inventory reduction does not offset the breeding revenue reduction; thus, the inner radial moderator concept does not appear economically attractive.

Moderated Blankets

Replacing some blanket fuel with moderator material would tend to soften the blanket spectrum, enhancing the conversion rate per unit of fuel. Opposing this effect is the lessened gross breeding occasioned by the diminished fuel content. Some candidate moderating materials are graphite, ZrH_2 , and BeO .

Two studies (4, 12) have investigated the breeding economics of moderated blankets. Hasnain (4) considered graphite in an LMFBR radial blanket, while Mayer (12) considered graphite, ZrH_2 , and BeO in a steam-cooled fast reactor (SCFR) radial blanket. In all cases, the inclusion of moderating materials (at the expense of fuel volume) led to a reduction in breeding ratio. Core parameters (k_{eff} , critical mass) were only slightly affected. Both studies concluded that moderated blankets offered no significant economic advantages.

Another study (17) has shown that seeding a typical LMFBR radial blanket with carbon leads to a slight improvement in the breeding performance of the inner radial blanket: about 10% increase in inner radial blanket fissile concentration. The outer radial blanket was found to be practically unaffected.

Radial Reflector

Functions of the radial reflector are: (1) to enhance radial blanket performance by flattening blanket flux, and, possibly, by softening the return spectrum; and (2) to provide a neutron shield for structural materials outside the reactor. Two major design decisions are choice of radial reflector composition and choice of radial reflector thickness.

In the Westinghouse LMFBR Follow-On work (73), Fe, C, Ni, and Na (reference case) reflectors were compared for a 10.5 inch thick radial blanket. Maximum improvement (over the Na reflected case) in radial blanket fuel economic performance was only 0.008 mills/KWHe (the 12 inch graphite reflector). A 3 inch Fe reflector provided minimum improvement (0.002 mills/KWHe). A 3 inch Ni reflector resulted in 0.007 mills/KWHe savings. Choice of radial reflector material and thickness was found to have little effect on power ratios across the blanket. Nickel provided a significant improvement in flux attenuation and was selected as the preferred reflector material.

Using the BR-1 reactor, Russian experimenters (6) have studied the effect of reflector composition on radial blanket breeding. Be, C, Ni, Fe, Cu, 1 Kh 18NgT steel, water, and extended blanket material were compared. The thicknesses of these reflectors were chosen such that any further increase in thickness resulted in negligible increase in blanket U238 (n, γ) captures. "Reflector efficiency" was defined as

$$B_i = A_i / A_{xB}$$

where

A_i = additional U238 (n, γ) captures resulting from addition of reflector of material i.

A_{xB} = additional U238 (n, γ) captures resulting from extending the blanket.

The base radial blanket thickness was not given, nor could it be inferred. Two types of blankets - uranium carbide and metallic uranium - were used.

Table 1.1 summarizes the results. The reflector efficiency for the extended blanket case was unity, by definition. All other efficiencies were less than unity, indicating that an extended blanket is preferable if fabrication and reprocessing costs are ignored. The results show that

TABLE 1.1
EFFECT OF RADIAL REFLECTOR ON RADIAL BLANKET BREEDING,
RUSSIAN EXPERIMENTAL RESULTS (6)

Reflector Material	Reflector Thickness (cm)	Bi	
		Uranium Carbide Blanket	Metallic Uranium Blanket
Be	140	0.54	0.86
C	600	0.50	-
Ni	192	0.47	0.51
Fe	184	0.42	0.28
Steel	160	0.33	0.40
Cu	184	0.24	0.41
Water	144	0.23	0.49
UC		1.00	-
U-met.		-	1.00

moderating reflectors (Be, water) are significantly more effective for metallic blankets than for carbide blankets, owing to the harder spectrum in metallic blankets and the potential for improved U238 (n, γ) capture. For both carbide and metallic blankets, Be is the preferred reflector.

The study included no analysis of the fissile revenue-fuel cycle cost tradeoff in extending the blanket. Thus from their results, Table 1.1, it is not possible to reach a firm economic judgement vis-a-vis replacement of blanket material with reflector.

In an analytic study at MIT (61) it was found that for an 18 inch blanket, no improvement in blanket breeding was accomplished by increasing the reflector (Fe) thickness beyond 18 inches. Similarly, no improvement was noted in extending an unreflected 18 inch blanket by more than an additional 18 inches, i.e. beyond a total unreflected thickness of 36 inches. Thus an 18 inch iron reflector and a 36 inch radial blanket are effectively infinite.

A German study (12) has evaluated radial reflector materials for steam cooled FBRs. Candidate materials were steam, water, ZrH_2 , BeO, graphite, steel, UO_2 (extended blanket), and U metal. The radial blanket in all cases, was 35 cm thick, and composed of 56 v/o UO_2 and 18 v/o structural material. Reflectors, in all cases, were 80 v/o reflector material, 10 v/o steel, and 10 v/o coolant.

The reflector materials were first ranked by their effect on "breeding rate" (undefined). Optimum reflector thickness was selected such that further increase in thickness increased the breeding rate by less than 1%. Table 1.2 summarizes the results of the breeding rate ranking.

The moderating reflectors are ZrH_2 , BeO, and graphite. Of these, ZrH_2 has the strongest moderating effect, but it is also the strongest absorber and thus the weakest net reflector. It has the least beneficial effect on blanket breeding. The less-thermalizing and less-absorbing BeO and graphite return more neutrons, albeit at higher energies, and result in higher blanket breeding.

TABLE 1.2

EFFECT OF RADIAL REFLECTOR ON RADIAL BLANKET BREEDING,
GERMAN STUDY (12)

Reflector Material	Optimum Reflector Thickness (cm)	B ¹
BeO	12-16	0.023
graphite	12-16	0.021
steel	6-8	0.015
UO ₂	6-8	0.013
U-metal	6-8	0.013
ZrH ₂	4	0.011

1/ B = radial breeding rate - radial breeding rate with no reflector

The shielding effectiveness of the materials was also considered. In these studies, reflector thickness was held constant at 8 cm. Flux values (in arbitrary units) at the outer edge of the reflectors are shown in Table 1.3. If the objective is to minimize high energy flux, ZrH_2 would be the preferred reflector. The other moderating reflectors, BeO and graphite, are somewhat poorer attenuators.

The breeding rate and shielding effectiveness surveys described above were based on "snapshot" multigroup physics computations. In a further study, the same author (12) evaluated the blanket revenues, with the various reflectors, at optimum irradiation times. Fabrication costs of the blanket were ignored entirely. Also portions of the blanket which would not yield a net profit (after reprocessing) were not counted. That is, these unprofitable regions did not burden the blanket with any cost whatever; they were simply not considered to be reprocessed. Table 1.4 summarizes the percent revenue improvements (over the case with no reflector) resulting from the addition of the various reflectors. The oversimplified economic assumptions apparently account for the inconsistency in reflector rankings between Tables 1.2 and 1.4.

Metallic vs. Oxide Blankets

The economics of metallic and oxide blankets have been compared by Klickman (1). Core design was held fixed. Optimum thickness for the metallic blanket (~ 20 cm) was about one half that of the oxide blanket (~ 40 cm). For these thicknesses, the two blankets had approximately the same breeding ratio, uranium content, and flux attenuation characteristics. Burnup limitations were assumed to be 5000 MWD/MT for the metallic blanket and 25,000 MWD/MT for the oxide blanket. The study showed that the low burnup limitation severely disadvantages the metallic blanket - its regional optimum irradiation times cannot be achieved. The oxide blanket's ir-

TABLE 1.3
SHIELDING PERFORMANCE OF REFLECTORS,
GERMAN STUDIES (12)

Reflector Material	<u>Flux at Outer-Edge of an 8 cm Reflector</u>	
	Fast Flux 0.8-10.5 Mev (arbitrary units)	Total Flux 0-10.5 Mev (arbitrary units)
BeO	1.63	49.66
graphite	2.68	53.30
steel	2.77	39.14
UO ₂	2.41	33.28
U-metal	2.00	25.46
ZrH ₂	1.09	33.24

TABLE 1.4

EFFECT OF RADIAL REFLECTOR ON BLANKET REVENUE,
GERMAN STUDIES (12)

Reflector Material	Blanket Revenue Improvement with respect to reference ¹
BeO	11.6%
graphite	12.9
steel	6.8
UO ₂	4.0
U-metal	3.2
ZrH ₂	9.8

1/ Reference = no reflector

radiation time was not so-limited. Even without the burnup limitations, the oxide blanket was found to be economically preferable.

1.4 SUMMARY

1.4.1 Objectives

The objectives of this work were twofold:

- (1) to develop a simple depletion-economics calculational tool for survey evaluations of LMFBR blanket configurations; and
- (2) to perform several comparative studies around a 1000 MWe reference LMFBR configuration.

The 1000 MWe case studies (2), to which model (1) was applied, dealt with (a) effect of choice of radial reflector material (Be-metal vs. Na) and radial blanket thickness on radial blanket fuel economics, (b) the advantage of operating each radial blanket region on its own local optimum irradiation schedule, and (c) the sensitivity of LMFBR fuel energy costs to the economic environment.

A preliminary study examined the economic viability of FBR blankets as reactor size is increased. The reactor size-blanket economics study used only the economics equations developed in task (1) above. Depletion information was obtained from simple, one energy group, spherical geometry breeding ratio expressions. Three cases were compared over a range of core sizes: (a) a spherical core surrounded by a breeding blanket, with no fissile burnup in the blanket; (b) a spherical core surrounded by a sodium reflector (no blanket); and (c) a spherical core surrounded by a breeding blanket, with blanket burnup (power) accounted for.

1.4.2 The Depletion-Economics Model (Chapters 2,3)

The depletion-economics model has two parts: (a) the cost analysis

model which yields the fuel components of energy cost, given unit fabrication and reprocessing costs ($\$/\text{kgHM}$), plutonium market values ($\$/\text{kgPu}_f$), money costs (discount and tax rates), and the nuclide balance data; and (b) the physics-depletion model, which yields the nuclide balance data - load and discharge masses of fertile and fissile materials - used in the cost analysis model. The depletion economics model is programmed in the computer code SPPIA, described in Appendix C. Given local physics data (local flux and flux-averaged cross sections) from a single multigroup physics computation, and given the economic parameters, the code yields fuel costs locally (or for an annular region) in $\$/\text{kgHM}/\text{year}$, and energy costs by major region (core, axial blanket, radial blanket) in mills/KWhe.

1.4.2.1 Cost Analysis Model

Despite attempts to standardize nuclear fuel cost accounting methodology (21,22,23,24), a casual review of methods actually used in design evaluations and tradeoff studies reveals substantial inconsistencies. Furthermore, FBR blankets impose several unique accounting problems: blanket fuel appreciates with irradiation, raising certain tax questions; and the long irradiation times in the radial blanket make the treatment of blanket carrying charges important. For these reasons, a cash flow method (CFM) was adopted in the present work.

A general CFM expression for the levelized cost of electricity (mills/KWhe) was derived and applied to FBR fuel costs. When applied to a region (core, axial blanket, or radial blanket) or subregion under fixed-element (batch or scatter) management, the equations reduce to forms giving local fuel economic performance, e.g. in an annular zone, or at a "point", in mills/KWhe or $\$/\text{kgHM}/\text{year}$:

$$\bar{e} = \frac{1000}{E} M_{HM}^0 \left[\frac{C_{fiss} \epsilon_0 F^{mp}(T)}{T} + \frac{C_{fab} F^{fab}(T)}{T} + \frac{C_{repr} F^{repr}(T)}{T} - \frac{C_{fiss} \epsilon(T) F^{mc}(T)}{T} \right] \quad (1-1)$$

material purchase
fabrication
reprocessing
material credit

where $\bar{e}$ is the local levelized fuel component of the energy cost (mills/KWhe), E is the electrical energy produced by the reactor in one year (kWhe/yr), T is the local irradiation time (yr.), C_{fab} and C_{repr} are the unit fabrication and reprocessing costs (\$/kgHM), C_{fiss} is the fissile plutonium price (\$/kg), ϵ_0 is the initial enrichment, $\epsilon(T)$ is the discharge enrichment (kg fissile discharged per kg of heavy metal loaded), $F^q(T)$ is the carrying charge factor for cost component q , and M_{HM}^0 is the mass of heavy metal loaded. The term in brackets [] may be regarded as a figure of merit representing local fuel economic performance, having units of dollars per year per local kilogram of heavy metal loaded.

The carrying charge factors, $F^q(T)$, are given by

$$F^q(T) = \frac{1}{1-\tau} \left[\frac{1}{(1+x)^{Tq}} - \tau \right] \quad \text{for capitalized costs or revenues}$$

$$= \frac{1}{(1+x)^{Tq}} \quad \text{for non-capitalized costs or revenues (expensed cost or taxed revenue) (1-2)}$$

where

$$x = (1 - \tau)r_b f_b + r_s f_s = \text{"discount rate"} \quad (1-3)$$

and where τ is the income tax rate, f_b and f_s are the debt and equity fractions, r_b and r_s are the debt and equity rates of return, and T^q is the time between the cash flow transaction q and the irradiation midpoint.

The "front end" components, fabrication and material purchase, are normally capitalized. The "backend" components, reprocessing and material credit, may be capitalized or not, according to tax interpretation. If they are not capitalized, then revenue from the sale of plutonium is taxed as ordinary income, along with electricity revenue, and reprocessing charges are treated as tax deductible expenses in the year in which they occur. The two methods, capitalizing and not capitalizing backend transactions, were compared and were found to have a significant effect on absolute values of energy costs. However, choice of method does not distort comparative or incremental results, e.g. design rankings, optimum blanket irradiation time, sensitivity studies. In the case studies to which the depletion-economics model was applied, material credit was consistently taxed and reprocessing charges were consistently expensed.

The CFM treatment of carrying charges is embodied in Equations (1-2) above. Two approximate methods, here labeled "Simple Interest Method" (SIM) and "Compound Interest Method" (CIM), were identified in the literature:

$$F^q = 1 + y_q T^q \quad (\text{SIM}) \quad (1-4)$$

and

$$F^q = (1 + y_q)^{T^q} \quad (\text{CIM}) \quad (1-5)$$

where

$$\begin{aligned}
 y_q &= x/(1-\tau) && \text{for capitalized costs or revenues} \\
 &= x && \text{for non-capitalized costs or} \\
 & && \text{revenues (expensed costs or} \\
 & && \text{taxed revenues)}
 \end{aligned} \tag{1-6}$$

The CFM expressions were shown, through series expansions, to reduce to SIM and CIM for small $T^q y_q$. SIM underpredicts, while CIM overpredicts, the carrying charge factor. Because radial blanket irradiation times are typically long, the CFM method was selected for use in the case studies of this report.

1.4.2.2 Physics-Depletion Model

The function of the physics-depletion model is to furnish discharge fuel composition, $\epsilon(T)$, to the cost analysis model for use in computing material credit.

In the method developed for this work, the "Semi-Analytic Method" (SAM), local physics data (fluxes and spectrum-weighted cross sections) from a single multigroup calculation are used in the analytic solutions of the reaction rate equations to obtain discharge fissile content:

$$\epsilon = \frac{M_{49} + M_{41}}{M_{HM}^0} \tag{1-7}$$

$$M_{49} = N_{49} V \frac{\tilde{M}_{49}}{N_{av}}, \quad M_{41} = N_{41} V \frac{\tilde{M}_{41}}{N_{av}} \tag{1-8}$$

$$\begin{aligned}
 N_{49} = N_{28}^0 A \exp(-\sigma_a^{28} \theta) [1 - \exp(-(\sigma_a^{49} - \sigma_a^{28}) \theta)] \\
 + N_{49}^0 \exp(-\sigma_a^{49} \theta)
 \end{aligned} \tag{1-9}$$

$$N_{41} = N_{28}^0 AB_1 C_1 \exp(-\sigma_a^{28} \theta) - N_{28}^0 AB_2 C_2 \exp(-\sigma_a^{49} \theta) \quad (1-10)$$

$$+ N_{49}^0 B_2 C_2 \exp(-\sigma_a^{49} \theta) + \beta_1 C_3 \exp(-\sigma_a^{40} \theta) + \beta_2 \exp(-\sigma_a^{41} \theta)$$

$$A = \sigma_c^{28} / (\sigma_a^{49} - \sigma_a^{28})$$

$$B_1 = \sigma_c^{49} / (\sigma_a^{40} - \sigma_a^{28})$$

$$B_2 = \sigma_c^{49} / (\sigma_a^{40} - \sigma_a^{49})$$

$$C_1 = \sigma_a^{40} / (\sigma_a^{41} - \sigma_a^{28}) \quad C_2 = \sigma_c^{40} / (\sigma_a^{41} - \sigma_a^{49}) \quad C_3 = \sigma_c^{40} / (\sigma_a^{41} - \sigma_a^{40})$$

$$\beta_1 = N_{40}^0 - (N_{28}^0 AB_1 - N_{28}^0 AB_2 + N_{49}^0 B_2)$$

$$\beta_2 = N_{41}^0 - (N_{28}^0 AB_1 C_1 - N_{28}^0 AB_2 C_2 + N_{49}^0 B_2 C_2 + \beta_1 C_3) \quad (1-11)$$

$$\theta = \int^T \phi(T') dT' = \text{local flux time} \quad (1-12)$$

M_{49}, M_{41} = discharge masses of Pu239, Pu241 respectively

N_{49}, N_{41} = discharge atom density of Pu239, Pu241 respectively

$\tilde{M}_{49}, \tilde{M}_{41}$ = atomic masses of Pu239, Pu241 respectively

N_{av} = Avogadro's Number

V = volume of the zone (1-13)

Local flux and local spectrum-weighted cross sections are taken from a single multigroup physics computation, and are assumed constant over a fueling cycle.

Several effects complicate the physics-depletion characteristics of FBR blankets: (1) spectrum softening with distance from the core-blanket interface; (2) spectrum hardening with irradiation time, due to the

relatively large buildup of fissile plutonium in the blanket; (3) flux shift, i.e. increase in blanket flux with irradiation time, due to buildup of fissile plutonium in the blanket; and (4) heterogeneity effects occasioned by the soft blanket spectrum, and aggravated, in the case of radial blankets, by larger pin diameters.

Effect (1) requires that cross sections be input to the depletion calculation with sufficient spatial detail, i.e. a separate cross section set, properly flux weighted, for each of many blanket regions. Since the accurate spatial description of blanket physics is a prime concern in the Blanket Test Facility work, no attempt was made to determine potential savings in computational effort through reduced spatial detail. Instead, attention was concentrated on effects (2) and (3).

Effects (2) and (3) suggest that static physics calculations be performed sufficiently often, during a depletion calculation, to correct the local fluxes and cross sections. Since most of the computational effort is absorbed by the multigroup calculations, computer expense can be significantly reduced by minimizing their frequency, that is by maximizing the irradiation time intervals over which flux shape and local spectra are assumed constant. For this reason, studies were performed to assess the effects of item (2), spectrum hardening, and item (3) flux shift, on depletion calculation results. Qualitatively, the two effects operate in opposite directions, spectrum hardening tending to decrease blanket discharge fissile inventory, flux shift tending to increase blanket discharge fissile inventory.

Three parallel depletion calculations were performed for a reference 1000 MWe LMFBR:

(a) a 26 energy group time step depletion calculation (26G-TSD), which accounted for both spectrum changes and flux shift;

(b) a 1 energy group time step depletion calculation (1G-TSD), which accounted only for flux shift; and

(c) a "semi-analytic method" (SAM) calculation, which accounts for neither spectrum change nor flux shift with irradiation.

The two approximate methods, (b) and (c), used local spectrum-weighted cross sections from the initial (time zero) method (a) solution. In addition, method (c) used local fluxes from the initial method (a) solution. The computer program 2DB (26) was used for calculations (a) and (b). Method (a) used the Bondarenko 26 group cross section set (48), heterogeneity-corrected by the program 1DX (27).

The calculations assumed batch management of both core (plus axial blanket) and radial blanket. Core and axial blanket fuel was assumed replaced after two years irradiation, corresponding to an average burnup of 100,000 MWD/MT. Radial blanket fuel was assumed irradiated to four years. The use of batch management in these calculations imposes a severe test of the constant flux, constant spectrum assumptions. For the same irradiation time, the variations of composition, flux shape, and spectra over a cycle interval are greater for batch management than for scatter management.

Principal findings of the methods study described above are listed below.

(1) For the core, the discharge fissile inventories from the three calculations were practically in exact agreement (errors less than 0.1%).

(2) For the axial blanket, 1G-TSD overpredicted¹ discharge fissile inventory by less than 4%, while SAM underpredicted¹ by less than 4%.

1. Compared to the 26G-TSD calculation.

(3) For the radial blanket, 1G-TSD overpredicted¹ discharge fissile inventory by about 10%, due to its soft cross sections. SAM underpredicted¹ discharge fissile inventory by around 10%, in spite of its soft cross sections, because of its low flux values.

(4) Of the two effects examined in this exercise, spectrum hardening and flux shift, the latter was found to be dominant.

The SAM calculation, performed by the program SPPIA, resulted in computer time savings (over the 26G-TSD, performed by 2DB) of on the order of 90%, while the 1G-TSD (2DB) led to about 60% time savings. In addition to depletion results, the SPPIA computation obtained fuel costs by region, as functions of irradiation time.

The effect of heterogeneity corrections (i.e. U238 resonance, spatial self-shielding) on radial blanket depletion results was examined. Heterogeneity influences blanket fissile production in two opposing ways: (a) the lower effective U238 microscopic capture cross section, σ_c^{28} , depresses the conversion rate, tending to decrease bred fissile inventory; (b) viewing blanket neutronics as an attenuation process, the lower σ_c^{28} results in higher blanket fluxes, tending to increase the conversion rate and bred fissile inventory. Of these two opposing effects, (a) dominates and heterogeneity leads to a net adverse effect on blanket breeding.

Two multigroup physics computations were performed using, respectively, 26 group infinitely dilute cross sections and 26 group heterogeneity - corrected cross sections in the blanket. Local fluxes and one group cross sections from these two computations were then input to SAM to obtain depletion results with and without heterogeneity corrections. Comparison of the two SAM results showed that blanket heterogeneity reduced fissile discharge inventory by about 10% for irradiation times of interest (2-7 years). A similar study (30) showed that heterogeneity corrections for a
1. Compared to the 26G-TSD calculation.

typical LMFBR axial blanket diminished calculated axial blanket Pu239 discharge mass by as much as 3%.

1.4.3 1000 MWe LMFBR Case Studies (Chapter 5)

The depletion-economics model established above was applied to case studies involving radial blanket thickness, choice of radial reflector material, radial blanket fuel management, and the sensitivity of LMFBR fuel energy costs to the economic environment.

1.4.3.1 Radial Blanket Thickness and Radial Reflector Material

Combinations of three radial blanket thicknesses (15, 30, 45 cm) and two radial reflector materials (sodium, beryllium metal) were evaluated. The total radial dimension (blanket plus reflector) was held fixed at 95 cm, since even the thinnest (50 cm) reflector is effectively infinite (6,61). The core and axial blanket configuration was also held fixed. Core volume was 4908 liters, core height-to-diameter ratio was 0.4 and the axial blanket was 40 cm thick. Core and axial blanket fuel economics were found to be insensitive to radial blanket/reflector design changes. A solid beryllium metal reflector (no coolant, no structural material) was selected as a limiting case, i.e. as the reflector apt to provide maximum improvement in radial blanket fuel economics.

Figure 1.1 and Table 1.5 summarize the results of the blanket thickness-reflector material survey. "Reference" and "more favorable" economic environments, for radial blankets, are defined in Table 1.6. Principal findings are listed below.

1. The relative advantage of the moderating reflector, Be-metal, increases as the reflector is moved nearer the high flux zones of the blanket, that is, as the blanket thickness decreases. For a thick

FIG. 1.1 EFFECT OF RADIAL BLANKET THICKNESS AND RADIAL REFLECTOR MATERIAL ON RADIAL BLANKET FUEL ECONOMICS

TABLE 1.5 EFFECT OF RADIAL BLANKET THICKNESS AND RADIAL
REFLECTOR MATERIAL ON RADIAL BLANKET FUEL ECONOMICS

Configur- ation #	Radial Blanket Thickness (cm)	Radial Reflector Material	M_{49}/T @ T=2yr. (kg/yr.)	Reference Economic Environment			More Favorable Economic Environment		
				T_{opt} (yr)	$\bar{e}_{RB}$ @ T_{opt} (mills/KWHe)	M_{49} @ T_{opt} (kg)	T_{opt} (yr)	$\bar{e}_{RB}$ @ T_{opt} (mills/ KWHe)	M_{49} @ T_{opt} (kg)
1	45	Na	158	6-1/2	-0.037	825	3-1/2	-0.237	512
2	45	Be-metal	160	6-1/2	-0.040	845	3-1/2	-0.243	521
1A	30	Na	141	4-3/4	-0.058	596	2-1/2	-0.242	342
2A	30	Be-metal	157	4-1/2	-0.072	610	2-1/2	-0.279	380
1B	15	Na	97	3-1/2	-0.055	304	2	-0.188	194
2B	15	Be-metal	130	2-3/4	-0.087	308	1-1/2	-0.276	205

TABLE 1.6

RADIAL BLANKET ECONOMIC ENVIRONMENT

	Reference	More Favorable
Fabrication, \$/kgHM	69	40
Reprocessing, \$/kgHM	31	31
Fissile Market Value, \$/kg	10,000	20,000
Discount Rate, %	8	8

(45 cm) blanket, the effect of radial reflector material choice is only slight.

2. For either reflector, reducing the blanket thickness always reduces the bred plutonium inventory of the blanket, that is, the plutonium forfeited in the region eliminated is greater than the additional plutonium bred in the remaining region as a result of improvement of its breeding performance (σ_c^{238}).

3. Optimum irradiation time decreases as the radial blanket thickness decreases and as the economic environment improves. The effect of the choice of radial reflector material on optimum irradiation time is more pronounced the thinner the blanket.

4. Radial blanket thickness optimization is weak, that is, net blanket revenue does not display a sharp peak as radial blanket thickness is reduced from 3 rows to 2 rows to 1 row (15 cm per row). Thick blankets are indicated when fabrication and reprocessing costs decrease and/or fissile market value increases.

1.4.3.2 Advantage of Local Fuel Management

Fuel management schemes addressed in this study are characterized as "fixed fuel" schemes, i.e. fuel sees only one position in the reactor. During a refueling event a fraction, g , of a region's fuel is discharged and replaced with fresh fuel ("scatter" management). If all of the region's fuel ($g=1.0$) is replaced, the region is said to be "batch" managed.

The entire radial blanket may be batch or scatter managed, in which case all fuel experiences the same irradiation time. Alternatively, the blanket may be divided into annular regions (rows), with each irradiated to its own local optimum irradiation time, again in a batch or scatter

management scheme. The advantage of operating each radial blanket annular region on its own local optimum irradiation schedule was estimated for the reference LMFBR configuration (45 cm blanket, Na radial reflector). Net radial blanket revenue in mills/KWhe was found to be about 30% higher when local management was assumed. The local optimum irradiation time ranged from 2.5 years (at the core blanket interface) to about 12 years (at the blanket-reflector interface), while the optimum irradiation time for the blanket as a whole was 6.5 years.

Another advantage of local fuel management, not quantified in the present studies, is the power flattening effect.

1.4.3.3 Sensitivity of LMFBR Fuel Energy Costs to the Economic Environment

Costs generated throughout the fuel cycle are ultimately transferred to the utility company and borne, along with the utility company's carrying charges, by the electricity consumer via the fuel component of the levelized cost (price) of electricity in mills/KWhe. Economic environment is defined here as the unit costs for fabrication¹ and reprocessing (\$/kgHM), the fissile Pu market value (\$/kgPu fissile) and the utility company discount rate(%). The sensitivity of reference LMFBR fuel energy costs (mills/KWhe) to components of the economic environment was examined by varying each parameter around the reference values given in parentheses in Table 1.7. Sensitivity of region "s" fuel cost ($\bar{e}_s$) to cost component "q", about reference environment "o", is represented by the "sensitivity coefficient",

$(A_{q,s})_o$, defined by

$$(A_{q,s})_o = (C_{q,s} / \bar{e}_s)_o \left(\partial \bar{e}_s / \partial C_{q,s} \right) \quad (1-14)$$

1. Carrying charges of the fuel cycle industries are included in their unit costs (\$/kgHM). Carrying charge components of energy costs refer to utility company carrying charges.

TABLE 1.7

RANGES OF ECONOMIC ENVIRONMENT PARAMETERS

Unit Processing Costs [\$/kgHM]

Fabrication

Core	150-(314)-330
Axial Blanket	20-(80)-314
Radial Blanket	20-(69)-100

Reprocessing

Core	15-(31)- 60
Axial Blanket	15-(31)- 60
Radial Blanket	15-(31)- 60

Nuclide Market Values (\$/kg)

Fertile (C_{28} , C_{40})	0
Fissile (C_{49} , C_{41})	5000-(10,000)-25,000

Utility Company Financial Parameters

Income Tax Rate (τ)	(0.5)
Discount Rate (x)	0.06-(0.08)-0.10

() indicates reference value

Table 1.8 summarizes the sensitivity coefficients for the reference core, axial blanket, and radial blanket. Fabrication and reprocessing components include their respective carrying charges. The material component is the net direct fissile material cost (fissile material purchase less fissile material credit) plus the material carrying charges (inventory). For all three regions, the energy costs for fuel are seen to be most sensitive to unit fissile value and least sensitive to unit reprocessing cost.

For the core and axial blanket, irradiation time is set by the burnup limit of the core. Thus, for these regions, Equation (1-1) reduces to simple linear relations of the unit costs:

$$\bar{e}_s = a_{fab,s} C_{fab,s} + a_{repr,s} C_{repr,s} + a_{mat,s} C_{fiss} \quad (1-15)$$

where

$$a_{q,s} = \frac{1000}{ET} M_{HM}^0 g(T) = \text{constant}.$$

Hence, for these regions, sensitivity coefficients simply represent the fractions of the regional cost, $\bar{e}_s$, contributed by the respective components:

$$(A_{q,s})_0 = \left(\frac{\bar{e}_{q,s}}{\bar{e}_s} \right)_0 \quad (1-16)$$

where

$$\bar{e}_{q,s} = a_{q,s} C_{q,s}$$

The radial blanket energy cost of interest is the fuel cost at the optimum irradiation time, $(\bar{e}_{RB})_{Topt}$. Since the optimum irradiation time is an implicit function of the economic environment parameters, the Equation (1-1) for the radial blanket does not reduce exactly to a simple linear form. However, sensitivity results from the SPPIA program,

TABLE 1.8

SENSITIVITY COEFFICIENTS, $(A_{q,s})_o^*$, FOR REFERENCE LMFBR
CORE, AXIAL BLANKET, AND RADIAL BLANKET

q s	Core	Axial Blanket	Radial Blanket
Fabrication	0.357	-0.495**	-2.15**
Reprocessing	0.025	-0.140**	-0.44**
Material	<u>0.628</u>	<u>1.635</u>	<u>+3.59</u>
	1.000	1.000	1.00

$$* \quad (A_{q,s})_o = \frac{\Delta \bar{e}_s / (\bar{e}_s)_o}{\Delta C_{q,s} / (C_{q,s})_o}$$

** These terms are negative because the $(\bar{e}_s)_o$ for the blankets are negative.

Figures 1.2, 1.3 and 1.4, showed that $(\bar{e}_{RB})_{Topt}$ is practically linear in C_{fab} , C_{repr} , and C_{fiss} over the expected ranges of these parameters. Thus, Equations (1-15) and (1-16) are applicable to the radial blanket near reference economic conditions.

Figures 1.2, 1.3, and 1.4 also show that $Topt$ is approximately linear in C_{fab} , C_{repr} , and C_{fiss} , and that $Topt$ decreases with improvement in the radial blanket's economic environment.

Figure 1.5 shows the regional (core, axial blanket, and radial blanket) and total fuel costs as functions of fissile plutonium value. Several features are noted:

- (a) Due to the core fissile inventory component, the total reactor fuel energy cost, $\bar{e}_{reactor}$, increases with C_{fiss} despite the fact that the reactor produces more fissile plutonium than it consumes.
- (b) The axial blanket is more profitable than the radial blanket, because the axial blanket sees more neutrons in this particular, but typical, design ($H/D = 0.4$).
- (c) The axial blanket breakeven point occurs at about 3.9 \$/gm.
- (d) The radial blanket breakeven point occurs at about 7.25 \$/gm.
- (e) As fissile price increases, the blankets become more viable, substantially offsetting the higher core inventory costs.

It is unlikely that the disparity between axial blanket profit and radial blanket profit would be diminished significantly by reasonable changes in the thickness or composition of either blanket. The axial blanket advantage is largely inherent: the axial blanket enjoys a higher flux, and higher fissile generation rate per unit of heavy metal loaded, and a short optimum irradiation time close to that set by the core burnup limit, (2 years). Hence axial blanket fissile credit is not threatened by overwhelming processing and material carrying charges.

FIG. 1.2 SENSITIVITY OF OPTIMUM RADIAL BLANKET FUEL ENERGY COST TO UNIT FABRICATION COST

FIG. 1.3 SENSITIVITY OF OPTIMUM RADIAL BLANKET FUEL ENERGY COST TO UNIT REPROCESSING COST

FIG. 1.4 SENSITIVITY OF OPTIMUM RADIAL BLANKET FUEL ENERGY COST TO FISSILE PLUTONIUM PRICE

FIG. 1.5 EFFECT OF FISSILE PU PRICE ON TOTAL REACTOR FUEL ENERGY COST

1.4.4 Reactor Size and Blanket Fuel Economics

A semiquantitative scoping study was performed to examine the effect of reactor unit rating on the economic viability of blankets. As core size increases (holding core shape fixed), core fuel economics improve due to the decreased critical enrichment and increased internal breeding ratio. At the same time, core surface-to-volume ratio and external breeding ratio diminish, and blanket fuel economics degenerate.

All of the major assumptions in this preliminary study penalized the blanket. A spherical core was assumed throughout the range of core size, that is, core geometry spoiling to maintain negative sodium void coefficients was not accounted for. A one-zone core was assumed, whereas a graded enrichment scheme would have enhanced blanket economics. The increased control requirements, and associated costs, involved in increasing the internal breeding ratio much above unity, were ignored.

Figure 1.6 shows that in spite of these (and other) penalties, the blanket concept is economically preferable to a non-breeding reflector (Na) for reactor ratings well over 1000 MWe. Beyond the "indifference point", the advantage of the "no-blanket" configuration is only very slight. Thus, it is likely that blankets will remain an important part of LMFBR design for the foreseeable future.

1.5 CONCLUSIONS AND RECOMMENDATIONS

The most significant findings and recommendations are summarized in the following paragraphs.

Choice of fuel cost accounting method has a significant effect on absolute values of energy costs (mills/KWhe), but does not distort comparative and incremental results, design rankings, optimization of fuel residence times, etc. Choice of taxing method can, however, affect the

FIG. 1.6 REACTOR FUEL ENERGY COSTS WITH AND WITHOUT A BREEDING BLANKET

optimized thickness of blankets.

A single multigroup physics computation, to obtain the flux shape and local spectra for depletion calculations, is sufficient for evaluating blanket/reflector design changes and for scoping and sensitivity studies. The major source of error in depletion results is the assumption of constant local flux over an irradiation cycle.

Choice of radial reflector material is important for radial blankets of one or two rows of subassemblies (15-30 cm). The relative advantage of a moderating reflector increases as the reflector is moved nearer the high flux zones of the blanket, that is, as the blanket thickness decreases from three (45 cm) rows to two (30 cm) rows to one (15 cm) row of subassemblies.

Radial blanket thickness optimization is weak, i.e. net blanket revenue does not display a sharp peak as radial blanket thickness is reduced from three rows to two rows to one row. Significant improvement (~30% increase in net blanket revenue) results from irradiating each radial blanket region to its own, local optimum irradiation time.

Both the optimum radial blanket irradiation time and the corresponding radial blanket net revenue are approximately linear functions of the unit costs in dollars per kilogram for fabrication, reprocessing, and fissile material. For increased fissile costs, both blankets (axial and radial) become more important in offsetting the increased core fissile inventory costs.

Based on a simple examination of reactor size versus blanket fuel economics, blankets are expected to remain an important part of LMFBR design for the foreseeable future.

Fast breeder reactor blanket design and fuel management has not received attention, in the open literature, commensurate with its importance. Design and fuel management study results tend to be highly specialized and fragmentary, making normalizations and comparisons difficult. A comparative evaluation of scatter, batch, out-in, and in-out equilibrium radial blanket fuel management schemes, for a fixed reactor configuration, is recommended.

The flexibility of radial blanket fuel management, after the reactor is in operation, presents the opportunity of optimizing reload strategies in accordance with the current and projected economic environments. Further effort in this area is recommended.

Interactions between engineering design and fuel management parameters should be examined with the aim of better understanding and characterizing the blanket. Radial blanket fuel management directly influences the degree of power flattening across the blanket, the power swing over an irradiation cycle, and the core-blanket power split. The associated economic trade offs are not well understood. In particular, an analysis of the benefits and penalties of blanket fissile seeding is recommended.

In brief, the most important recommendation is that, whatever aspects of blanket fuel management are subjected to further scrutiny, this be done on a more global basis, at the minimum taking into consideration the strong interaction of management schemes and the flow orificing pattern adopted.

Since unit sizes are projected to increase to 2000 MWe and beyond after the year 2000, a more thorough parametric study of blanket performance versus reactor rating is recommended.

CHAPTER 2

FUEL COST ANALYSIS METHOD

2.1 INTRODUCTION

2.1.1 Objectives of the Chapter

The purpose of this chapter is to establish a fuel cost analysis method, for fast breeder reactors, capable of

- (a) ranking major technological alternatives, e.g. choice of blanket and reflector materials,
- (b) optimizing certain design and fuel management variables, e.g. optimum blanket thickness, optimum blanket irradiation time(s), and
- (c) determining the sensitivity of fuel cost to changes in the economic environment, e.g. changes in unit fabrication and reprocessing costs, sale value of fissile nuclides, and cost of money parameters.

A secondary purpose of the chapter is to compare alternative fuel cost accounting methods. While alternative methods frequently yield significantly different absolute values of power costs in mills/KWhe, choice of accounting method should not distort the ranking of design alternatives, the values of optimized parameters nor the results of the sensitivity studies.

2.1.2 Background: Utility Company Economics

In non-regulated industries, the objective function used in measuring performance is profit (to be maximized), i.e. the stockholders' return on investment. By contrast, the objective function in utility company economics is usually the cost of electricity to the customer (to be minimized), or price, since a ceiling is imposed on investors' rate of return by regulatory

agencies. Hence, in utility economic analyses, investors' rate of return is treated as an expense to the customer, is fixed in the calculations, and the cost (price) of electricity to the customer must be such that the company's revenue from the sale of electricity balances all of its costs in generating and delivering that electricity, including payment of investors' principal and return, taxes, and direct expenses.

For purposes of selecting generating plant type, design studies, etc., the electricity generated by a given plant is normally burdened with the costs associated with that plant alone. For purposes of preparing financial statements and setting electricity rates, however, the costs of different plants are, of course, mixed.

Generation costs are normally divided into three categories: plant capitalization, fuel, and operating and maintenance costs. Only LMFBR fuel costs are considered in this report. Costs generated throughout the nuclear fuel cycle are ultimately transferred to the utility company, burdened to the production of electricity, and borne by the customer, along with the company's costs of capital associated with the fuel, in the form of a levelized price (cost) of electricity (mills/KWHe).

2.1.3 Scope of the Cost Analysis Model

Figure 2.1 illustrates the scope of the cost analysis model established for the present work. The model has the features and restrictions listed below.

1. The model is restricted to fuel costs. It excludes the other major cost categories: plant capitalization, and operating and maintenance.
2. The model treats costs associated with an individual plant or class of plant, i.e. LMFBR, in a given economic environment. Other concerns such as the mix of plant types (capacity planning), coupling effects,

generating schedules (dispatching) are not treated.

3. The "economic environment" is set, in parametric fashion, outside the cost analysis, and is assumed to be constant throughout plant life. The "economic environment" is defined here as the market value of fuel materials, prices of fuel cycle processing services, and the utility company's cost of money parameters. Such macroeconomic concerns as supply-demand effects in the market place (value of fissile isotopes), allocation of national resources, and effects of total processing industry throughput on processing prices, are not treated by the model.

4. As shown in Figure 2.1, the cost analysis model requires irradiation-depletion data from a fuel depletion model. (Fuel depletion models for LMFBR's are discussed in Chapter 3.) Output of the cost analysis model includes the following:

- * fuel component of electricity costs, in mills per kilowatt hour (electric), by major region, i.e. core, axial blanket, radial blanket; and

- * local annual fuel costs, in units of dollars per year per kilogram of heavy metal loaded, by major region, subregion (an annulus of fuel), or at a point.

2.1.4 Outline of the Chapter

A general expression for the levelized cost of electricity is derived in Section 2.2. This formulation, labeled here as the "cash flow method" (CFM), is applicable to all categories of costs: plant capitalization, fuel, and operating and maintenance costs. The derivation presented in Section 2.2 follows that of Vondy (19), with the following exception: an option is included for taxing revenue from sources other than the sale of electricity.

The CFM expression is applied to fast breeder reactor fuel costs in Section 2.4. Two methods for the tax treatment of post-irradiation transactions are derived.

In Section 2.4, the CFM expressions are specialized for fixed-fuel management schemes, in which fuel assemblies "see" only one irradiation position. A figure of merit representing local fuel economic performance (\$/year/local kg heavy metal) is derived.

In Section 2.5, several fuel cost accounting methods are compared. The effect of post-irradiation tax assumptions is shown. The cash flow method (CFM) is related to and numerically compared with two other methods: the simple-interest method (SIM) and the compound interest method (CIM).

A sample CFM calculation is present in Section 2.6. Major features of the blanket fuel cost vs. irradiation time characteristic are shown, i.e. the breakeven point and the optimum irradiation time.

2.2 DERIVATION OF A GENERAL EXPRESSION FOR THE LEVELIZED COST OF ELECTRICITY (CASH FLOW METHOD)

A general expression for the levelized cost of electricity, $\bar{e}$ (mills/KWhe) - applicable to plant capitalization costs, fuel costs and operating and maintenance costs - is derived in this section. The general expression is applied to FBR fuel costs in Section 2.3.

An electric utility company interacts financially with

- * customers (electricity revenue, $\bar{e} E_j$);
- * capital equipment suppliers (capitalized costs, Z_j);
- * fuel suppliers, fabricators, reprocessors (capitalized costs Z_j , expensed costs, O_j)
- * federal, state and local governments (income tax, T_j , property tax, π_j)

- * investors, including stockholders (owners) and bondholders (Y_j)
- * and others.

Nomenclature is given in Appendix A.

Figure 2.2 illustrates the detailed accounting treatment of utility company cash flows in year j .

Book value (liability to investors) at the end of year j is the book value, Y_j , in effect during year j plus any new capitalization which occurred during year j . Residual revenue for year j (revenue after taxes, current expenses, and return to investors have been subtracted), R_j , is applied against end-of-year book value to obtain the book value in effect for the following year, $j + 1$, i.e.

$$Y_{j+1} = Y_j - R_j + Z_j \quad (2-1)$$

Residual revenue, R_j , is a function of the levelized price (cost) of electricity, $\bar{e}$. Application of the boundary conditions

$$Y_1 = Z_0 \quad (2-2)$$

and

$$Y_{N+1} = 0 \quad (2-3)$$

to equation (2-1) yields an expression for $\bar{e}$, as is shown below.

Residual revenue, R_j , is given by

$$\begin{aligned} R_j &= (\text{taxable revenue} + \text{non-taxable revenue}) \\ &\quad - (\text{current expenses}) \\ &\quad - (\text{investors return}) \\ &\quad - (\text{income tax}) \\ &= \frac{\bar{e}E_j}{1000} + V_j + V_j' \\ &\quad - O_j - (r_b f_b Y_j + r_s f_s Y_j) - T_j \end{aligned} \quad (2-4)$$

FIG. 2.2 UTILITY COMPANY CASH FLOW ACCOUNTING

Income tax, T_j , is computed by applying an income tax rate (τ) to the net taxable revenues, i.e.

$$\begin{aligned} T_j &= \tau [\text{net taxable revenues, } j] \\ &= \tau [(\text{taxable revenue})_j - (\text{tax deductions})_j] \end{aligned} \quad (2-5)$$

Taxable revenue is given by

$$(\text{taxable revenue})_j = \frac{\bar{e} E_j}{1000} + V_j \quad (2-6)$$

The second term V_j (taxable revenue from sources other than the sale of electricity) is included for flexibility and generality.

Allowed tax deductions are:

- current (non-capitalized) costs, O_j ;
- return on debt capital (bonds), $r_b f_b Y_j$; and
- depreciation of capitalized costs, D_j :

$$(\text{tax deductible expenses})_j = O_j + r_b f_b Y_j + D_j \quad (2-7)$$

Thus income tax for year j is given by

$$T_j = \tau \left[\frac{\bar{e} E_j}{1000} + V_j - O_j - r_b f_b Y_j - D_j \right] \quad (2-8)$$

Substituting Equation (2-8) into Equation (2-4) for T_j , one obtains

$$\begin{aligned} R_j &= \frac{\bar{e} E_j}{1000} + V_j + V'_j - O_j - r_b f_b Y_j - r_s f_s Y_j \\ &\quad - \tau \left[\frac{\bar{e} E_j}{1000} + V_j - O_j - r_b f_b Y_j - D_j \right] \\ &= (1 - \tau) \left[\frac{\bar{e} E_j}{1000} + V_j - O_j \right] + \tau D_j \\ &\quad - [(1 - \tau) r_b f_b + r_s f_s] Y_j + V'_j \end{aligned} \quad (2-9)$$

The residual revenue R_j is available to pay back the bondholders' and stockholders' principal, as shown in (2-1), and may be thought of as a book depreciation payment. Substituting Equation (2-9) into Equation (2-1) for R_j ,

$$Y_{j+1} = Y_j + Z_j - (1-\tau) \left[\frac{\bar{e} E_j}{1000} + V_j - O_j \right] + \tau D_j + V'_j - [(1-\tau) r_b f_b + r_s f_s] Y_j \quad (2-10)$$

Defining

$$A_j = Z_j - (1-\tau) \left[\frac{\bar{e} E_j}{1000} + V_j - O_j \right] - \tau D_j + V'_j \quad (2-11)$$

and collecting terms on Y_j , one obtains

$$Y_{j+1} = [1 + (1-\tau) r_b f_b + r_s f_s] Y_j + A_j \quad (2-12)$$

Defining

$$x \equiv (1-\tau) r_b f_b + r_s f_s, \quad (2-13)$$

Equation (2-12) becomes

$$Y_{j+1} = (1+x) Y_j + A_j. \quad (2-14)$$

The quantity x defined in Equation (2-13) will presently be identified as the appropriate discount rate for present value calculations.

The recursive relation (2-14), together with the boundary conditions (2-2), (2-3) and the definition of A_j , Equation (2-11), leads to a closed form expression for levelized price of electricity, E . Equation (2-14) is applied repeatedly, as follows, subject to the boundary conditions:

$$Y_1 = Z_0 = A_0 \quad (2-2)$$

$$Y_2 = (1+x)Y_1 + A_1 = (1+x)A_0 + A_1$$

$$Y_3 = (1+x)^2 A_0 + (1+x)A_1 + A_2$$

.

.

$$Y_i = \sum_{j=0}^{i-1} (1+x)^{i-1-j} A_j$$

.

.

.

$$Y_{N+1} = \sum_{j=0}^N (1+x)^{N-j} A_j = 0 \quad (2-3)$$

$$= (1+x)^N \sum_{j=0}^N (1+x)^{-j} A_j = 0 \quad (2-15)$$

Substituting Equation (2-11) for A_j into Equation (2-15), and solving for $\bar{e}$,

$$\bar{e} = 1000 \frac{\sum_{j=0}^N (1+x)^{-j} \left[\frac{Z_j}{1-\tau} - \frac{V_j}{1-\tau} - \frac{\tau}{1-\tau} D_j - V_j + O_j \right]}{\sum_{j=0}^N (1+x)^{-j} E_j} \quad (2-16)$$

The factor $(1+x)^{-j}$ is seen to be the "discount factor", or "present value factor",

$$w(j) = (1+x)^{-j} \quad (2-17)$$

and x , defined by

$$x = (1-\tau)r_b f_b + r_s f_s \quad (2-13)$$

is identified as the "discount rate".

The derivation above assumes that the capital structure (f_b , f_s) and rates of return (r_b , r_s) are fixed.

The cash flow approach used in deriving Equation (2-16) has many advantages. First, Equation (2-16) is exact; that is, it avoids the tax iteration of methods which treat income taxes as costs to be present-valued separately. In the cash flow method derived above, allowance for income taxes is implicit in the coefficients of the present values ($1/1-\tau$, $\tau/1-\tau$, 1) and in the discount rate.

Another advantage is that vagaries in the interpretation of the discount rate are circumvented. The discount rate is defined within the cash flow derivation above. It may be calculated once the tax rate (τ), capital structure (f_b , f_s), and debt and equity rates of return (r_b , r_s) are set.

Further, the cash flow method (CFM) is flexible. A broad range of financial conventions may be accommodated within the CFM structure. Whether to "capitalize" (Z , V') or "expense" (0 , V) a given transaction is the user's decision, subject to IRS and state regulations. Equation (2-16) specifies how these transactions are treated. Different classes of transactions must be treated in different ways, when taxes are present, and the CFM specifies how.

Finally, in other methods (20) two depreciation schedules sometimes appear (depreciation for tax purposes, and depreciation for computing investors' return), leading to confusion. In the CFM described above, depreciation enters the calculation only in the tax computation. The depreciation of investors principal is set by the assumption that residual revenue R_j is applied toward paying back this principal. This restriction imposes a standard convention, of sorts, but permits alternative designs to be evaluated on a standard basis, free of accounting artifices.

2.3 APPLICATION TO FBR FUEL COSTS

A general expression for the levelized cost of electricity, Equation (2-16), was developed in Section 2.3. In the present section, that formulation is specialized to treat FBR fuel costs.

2.3.1 Separation of Costs

For convenience and flexibility, one wishes the costs to be as separable and additive as possible. They are made additive, in this report, by burdening the total reactor energy with each cost item individually, as opposed to charging the energy released by a single fuel stream with costs associated with that stream.

The reactor fuel costs are separated, or classified, as follows:

(a) by fuel stream. A fuel stream is a distinct sequence of fuel lots which sees one (batch or scatter management) or more (e.g. in-out, out-in management, etc.) positions in the reactor. A fuel stream can normally be identified with a major region (core, axial blanket, or radial blanket), and may be characterized by features such as pin diameter, initial isotope composition, etc.

(b) by component, for each stream. Each fuel lot of each stream incurs certain "component" costs as it proceeds through the fuel cycle, i.e. material purchase, fabrication, reprocessing, and material credit.

(c) as direct costs or carrying charges. Each component of each stream includes direct and carrying charge subcomponents. Thus there are eight (8) cost items per fuel stream.

If S is the number of fuel streams, there are $S \times 8$ separate cost items in mills/KWhe which may be added directly to obtain total reactor fuel cost. These cost items may be regrouped as desired to assist technological economic insight, to make results comparable with other studies, etc.

For example, direct material purchase and direct material credit may be combined to form "burnup" cost. Their carrying charges may be combined to form "inventory" cost.

2.3.2 Application to FBR Fuel Costs

The cost expressions associated with an arbitrary fuel stream are developed below. Total reactor fuel cost may be found by summing the costs of the individual streams, provided the denominator of Equation (2-16), discounted total plant electricity, is retained in each of the stream equations. For the moment, attention is focused on the numerator.

For an arbitrary fuel stream, the numerator of Equation (2-16) may be expressed as a sum over the fuel lots (m) rather than over years (j):

$$\begin{aligned}
 (\text{numerator}) &= \sum_{j=0}^N w(j) \left[\frac{Z_j}{1-\tau} - \frac{V_j}{1-\tau} - \frac{\tau}{1-\tau} D_j - V_j - O_j \right] \\
 &= \sum_{m=1}^n \left\{ \frac{Z_m}{1-\tau} w(t_m^Z) - \frac{V'_m}{1-\tau} w(t_{m,k}^{V'}) \right. \\
 &\quad - \frac{\tau}{1-\tau} \sum_k (D_m)_k w(t_{m,k}^D) \\
 &\quad \left. - V_m w(t_m^V) + O_m w(t_m^O) \right\} \quad (2-18)
 \end{aligned}$$

where the t's are the times, in years, from beginning of plant life to the indicated transactions.

For each fuel lot m , four direct cash flows are identified, as shown in Figure 2.3:

material purchase, z_m^{mp} ;

fabrication cost, z_m^{fab} ; [\$/lot]

reprocessing cost, z_m^{repr} ; and

material credit, $-z_m^{mc}$.

Each of these transactions is to be assigned to one of the classes of transactions appearing in Equation (2-16) and Equation (2-18).

These classes of transactions and two methods of assigning the lot's costs to them are summarized in Table 2.1. In both methods, the pre-irradiation transactions (material purchase and fabrication) are capitalized. Method A treats post-irradiation transactions as non-capitalized; that is, revenue from the sale of valuable isotopes is taxed as ordinary income, along with revenue from the sale of electricity, and reprocessing cost is written off as a tax deductible expense in the year that it occurs. Method B capitalizes the post-irradiation transactions.

Method B is strictly applicable, or at least conceptually correct, in cases of depreciating fuel, such as thermal reactor fuel, which experiences no net fissile gain during irradiation. For depreciating fuel,

$$D = (z_m^{fab} + z_m^{mp}) - (z_m^{mc} - z_m^{repr}) > 0 \quad (2-19)$$

and the interpretation of the tax depreciation credit, $\tau D/(1-\tau)$, is clear and correct. On the other hand, if fuel appreciates, as in a fast breeder reactor blanket,

$$D < 0, \quad (2-20)$$

FIG. 2.3 TIMING OF CASH FLOWS ASSOCIATED WITH A FUEL LOT

TABLE 2.1 TAX TREATMENT OF FUEL TRANSACTIONS

<u>Trans- action Type</u>	<u>Description</u>	<u>Tax Factor</u>	<u>Method A</u>	<u>Method B</u>
Z	capitalized cost	$1/1-\tau$	material purchase; fabrication	material purchase; fabrication; reprocessing
O	expensed cost	1	reprocessing	
V'	capitalized (non-taxable) revenue from sources other than sale of electricity	$1/1-\tau$		material credit
V	non-capitalized (taxable) revenue from sources other than sale of electricity	1	material credit	
D	depreciation for tax purposes; fictitious; for each Z-type cost, there must appear a depreciation term	$\tau/1-\tau$	material purchase and fabrication	(material purchase and fabrication) - (material credit - reprocessing)

and the tax depreciation credit reverses sign. This would imply a "tax appreciation debit", an entity not recognized in accounting nor allowed by taxing authorities. There is no symmetry with regard to tax depreciation: appreciation is not the opposite, or mirror image, of depreciation, for tax purposes.

Tax regulations require that ordinary income - that from the sale of the firm's ordinary product - be taxed at the corporate rate, τ , at the time that the income is realized (product exchanged for cash). Other regulations and rates cover extraordinary income, capital gains, losses, etc. The "ordinary" product of electric utility companies is electricity. It is conceivable that fissile material could become another ordinary product for utilities operating breeders, and that this new income will be taxed in the same manner as that from the sale of electricity.

How or whether this income is to be taxed, and the details of the fuel account structure, are beyond the scope of this report, and must await resolution by tax and regulatory agencies, or by common usage. Methods A and B are compared in Section 2.5. Choice of method has a significant effect on absolute values of levelized fuel cost, particularly in the radial blanket, but does not distort the economic ranking of design alternatives, nor does it significantly affect optimized parameters such as exposure. Method A was selected and used consistently in the case studies of Chapter 5.

2.3.2.1 Method A

Under Method A, Equation (2-18) becomes

$$\begin{aligned}
 (\text{numerator}) = & \sum_m^n \left\{ \frac{z_m^{\text{mp}}}{1-\tau} w(t_m^{\text{mp}}) + \frac{z_m^{\text{fab}}}{1-\tau} w(t_m^{\text{fab}}) \right. \\
 & + z_m^{\text{repr}} w(t_m^{\text{repr}}) - z_m^{\text{mc}} w(t_m^{\text{mc}}) \\
 & \left. - \frac{\tau}{1-\tau} (z_m^{\text{mp}} + z_m^{\text{fab}}) \sum_k^K g_k w(t_m^k) \right\}, \quad (2-21)
 \end{aligned}$$

where the inner summation is taken over the tax depreciation credits, k , for lot m , and the g_k are fractional coefficients determined by the tax depreciation schedule used.

Capitalized costs associated with the fuel lot m are amortized (depreciated for tax purposes) over the time the lot is in productive use, i.e. the time it spends in the reactor. Except for tax purposes, the depreciation schedule is immaterial; only the initial and discharge values of the fuel are relevant. Tax depreciation need not be consistent with the actual change of fuel value during irradiation since the IRS permits certain fictitious tax depreciation schedules. For consistency, simplicity, and because actual fuel value changes are practically linear, straight line depreciation of fuel is frequently used in utility accounting⁽⁸⁰⁾.

With straight line tax depreciation, the tax depreciation term for lot m becomes

$$\begin{aligned}
 & \frac{\tau}{1-\tau} (z_m^{\text{mp}} + z_m^{\text{fab}}) \sum_k^K g_k w(t_m^k) \\
 = & \frac{\tau}{1-\tau} \frac{(z_m^{\text{mp}} + z_m^{\text{fab}})}{K} \sum_k^K w(t_m^k) \\
 = & \frac{\tau}{1-\tau} \frac{(z_m^{\text{mp}} + z_m^{\text{fab}})}{K} w(t_m) \sum_k^K w(t_m^k), \quad (2-22)
 \end{aligned}$$

where

$t_m \equiv$ time from beginning of plant life to midpoint of irradiation of lot m ,

$T_m^k \equiv t_m^k - t_m$

$t_m^k \equiv$ time of depreciation credit k .

Little accuracy is lost if only one tax depreciation credit is assumed and it occurs at the irradiation midpoint. Table 2.2 illustrates the effect of this assumption for an irradiation time of five years. With this assumption, Equation (2-21) becomes

$$\begin{aligned}
 (\text{numerator}) = & \sum_m^n \left\{ \frac{z_m^{\text{mp}}}{1-\tau} w(t_m^{\text{mp}}) + \frac{z_m^{\text{fab}}}{1-\tau} w(t_m^{\text{fab}}) \right. \\
 & + z_m^{\text{repr}} w(t_m^{\text{repr}}) - z_m^{\text{mc}} w(t_m^{\text{mc}}) \\
 & \left. - \frac{\tau}{1-\tau} (z_m^{\text{mp}} + z_m^{\text{fab}}) w(t_m) \right\} \quad (2-23)
 \end{aligned}$$

Grouping the terms by component (mp, fab, repr, mc) and recalling Equation (2-16), the contribution of the fuel stream ($\bar{e}$) to the total reactor fuel cost is given by

$$\begin{aligned}
 \bar{e} &= \bar{e}_{\text{mp}} + \bar{e}_{\text{fab}} + \bar{e}_{\text{repr}} + \bar{e}_{\text{mc}} \\
 \bar{e}_{\text{mp}} &= 1000 \frac{\sum_m^n z_m^{\text{mp}} \left[\frac{w(t_m^{\text{mp}})}{1-\tau} - \frac{\tau}{1-\tau} w(t_m) \right]}{\sum_j^N w(j) E_j}
 \end{aligned}$$

TABLE 2.2 EFFECT OF ASSUMING A SINGLE TAX DEPRECIATION CREDIT

K = 5 "exact"			K = 1 "approx."		
k	T_m^k	$w(T_m^k)$	k	T_m^k (yr)	$w(T_m^k)$
1	-2	1.1664			
2	-1	1.08000			
3	0	1.00000	1	0	1.00000
4	+1	0.92593			
5	+2	0.85754			
<hr/>			<hr/>		
$\frac{1}{K} \sum_k^K w(T_m^k) =$					
			1.00594		
			1.00000		

$$\begin{aligned}
\bar{e}_{fab} &= 1000 \frac{\sum_m^n z_m^{fab} \left[\frac{w(t_m^{fab})}{1-\tau} - \frac{\tau}{1-\tau} w(t_m) \right]}{\sum_j^N w(j) E_j} \\
\bar{e}_{repr} &= 1000 \frac{\sum_m^n z_m^{repr} w(t_m^{repr})}{\sum_j^N w(j) E_j} \\
\bar{e}_{mc} &= -1000 \frac{\sum_m^n z_m^{mc} w(t_m^{mc})}{\sum_j^N w(j) E_j}
\end{aligned} \tag{2-24}$$

Defining

$$\begin{aligned}
T_m^{mp} &\equiv t_m - t_m^{mp} = T_m'^{mp} + 1/2 T_m \\
T_m^{fab} &\equiv t_m - t_m^{fab} = T_m'^{fab} + 1/2 T_m \\
T_m^{repr} &\equiv t_m - t_m^{repr} = -(T_m'^{repr} + 1/2 T_m) \\
T_m^{mc} &\equiv t_m - t_m^{mc} = -(T_m'^{mc} + 1/2 T_m),
\end{aligned} \tag{2-25}$$

observing that

$$w(a) w(b) = w(a+b), \tag{2-26}$$

one obtains, for the fuel stream in question,

$$\bar{e}_{mp} = 1000 \frac{\sum_m^n w(t_m) z_m^{mp} \left[\frac{w(-T_m^{mp})}{1-\tau} - \frac{\tau}{1-\tau} \right]}{\sum_j^n w(j) E_j}$$

$$\bar{e}_{fab} = 1000 \frac{\sum_m^n w(t_m) z_m^{fab} \left[\frac{w(-T_m^{fab})}{1-\tau} - \frac{\tau}{1-\tau} \right]}{\sum_j^N w(j) E_j}$$

$$\bar{e}_{repr} = 1000 \frac{\sum_m^n w(t_m) z_m^{repr} [w(-T_m^{repr})]}{\sum_j^N w(j) E_j}$$

$$\bar{e}_{mc} = -1000 \frac{\sum_m^n w(t_m) z_m^{mc} [w(-T_m^{mc})]}{\sum_j^N w(j) E_j} \quad (2-27)$$

2.3.2.2 Direct and Carrying Charge Contributions

Each fuel stream component can further be separated into direct and carrying charge subcomponents. For component q (mp, fab, repr, or mc), of lot m , the direct dollar cost is z_m^q . Let $(z_m^q)^{**}$ be the carrying charge component. Then the total dollar cost, $(z_m^q)^*$, is given by

$$\begin{aligned}
(z_m^q)^* &\equiv (\text{direct})_m^q + (\text{Ca.Chg.})_m^q \\
&= z_m^q + (z_m^q)^{**} \\
&= z_m^q + f_m^q z_m^q \\
&= z_m^q F_m^q
\end{aligned} \tag{2-28}$$

where

$$F_m^q \equiv 1 + f_m^q$$

The carrying charge factors (F_m^q) are readily identified as the bracketed [] terms in Equation (2-27), and are summarized in Table 2.3.

Levelized cost of electricity associated with the fuel stream's component q is therefore

$$\begin{aligned}
\bar{e}_q &= (\pm) 1000 \frac{\sum_m^n w(t_m) z_m^q F_m^q}{\sum_j^N w(j) E_j} \\
&= 1000 \frac{\sum_m^n w(t_m) z_m^q}{\sum_j^N w(j) E_j} + 1000 \frac{\sum_m^n w(t_m) z_m^q f_m^q}{\sum_j^N w(j) E_j} \\
&= \bar{e}_{q,\text{direct}} + \bar{e}_{q,\text{CaChg}}
\end{aligned} \tag{2-29}$$

2.3.2.3 Method B

In Method B, the reprocessing and material credit transactions are capitalized. Only the reprocessing and material credit carrying charge factors differ between Methods A and B.

TABLE 2.3 SUMMARY OF EXPRESSIONS FOR CARRYING CHARGE FACTORS, F_m^q , BY CASH FLOW METHOD

Component <u>q</u>	Method <u>A</u>	Method <u>B</u>
material purchase	$\frac{1}{1-\tau} [w(-T_m^{\text{mp}}) - \tau]$	$\frac{1}{1-\tau} [w(-T_m^{\text{mp}}) - \tau]$
fabrication	$\frac{1}{1-\tau} [w(-T_m^{\text{fab}}) - \tau]$	$\frac{1}{1-\tau} [w(-T_m^{\text{fab}}) - \tau]$
reprocessing	$w(-T_m^{\text{repr}})$	$\frac{1}{1-\tau} [w(-T_m^{\text{repr}}) - \tau]$
material credit	$w(-T_m^{\text{mc}})$	$\frac{1}{1-\tau} [w(-T_m^{\text{mc}}) - \tau]$

Following a derivation parallel to Equation (2-21) through Equation (2-29), the Method B carrying charge factors shown in Table 2.3 are found. Equations (2-29) are common to both methods.

2.3.2.4 Direct Dollar Costs Per Lot

A fuel lot's direct costs are given in terms of unit costs (\$/kg) as follows:

$$\begin{aligned}
 z_m^{\text{mp}} &= C_{28} M_{28}^0 + C_{49} M_{49}^0 + C_{40} M_{40}^0 + C_{41} M_{41}^0 + C_{42} M_{42}^0 \\
 z_m^{\text{fab}} &= C_{\text{fab}} M_{\text{HM}}^0 \\
 z_m^{\text{repr}} &= C_{\text{repr}} M_{\text{HM}}^0 \\
 z_m^{\text{mc}} &= C_{28} M_{28}(\text{T}) + C_{49} M_{49}(\text{T}) + C_{40} M_{40}(\text{T}) + C_{41} M_{41}(\text{T}) + C_{42} M_{42}(\text{T})
 \end{aligned} \tag{2-30}$$

where

$$M_{\text{HM}}^0 = M_{28}^0 + M_{49}^0 + M_{40}^0 + M_{41}^0 + M_{42}^0$$

The M's are masses of the isotopes U238, Pu239, Pu240, Pu241, Pu242 contained in the lot m in kilograms. Isotope values (C_{28} , C_{49} , ...) have units of dollars per kilogram of the isotope in question. Processing unit costs (C_{fab} , C_{repr}) have units of dollars per kilogram of heavy metal (U, Pu) loaded, and depend, of course, on the region for which the lot is intended (core, axial blanket, radial blanket).

The unit costs (C_{fab} , C_{repr} , C_{28} , C_{49} , ...) together with the cost of money parameters (τ , f_b , r_b , f_s , r_s) are regarded in this report as forming the "economic environment," and are set parametrically, external to the present fuel costing model. Economic environment is varied in the sensitivity studies of Chapter 5.

2.4 SIMPLIFICATIONS FOR BATCH AND SCATTER FUEL MANAGEMENT SCHEMES; LOCAL FUEL ECONOMIC PERFORMANCE

2.4.1 Batch Fuel Management

"Batch" fuel management of a region (core, axial blanket or radial blanket) is defined here as a scheme in which the region's fuel is discharged and replaced as a whole, and the fuel associated with the region or fuel stream sees only one position in the reactor.

If plant load factor remains approximately constant throughout plant life, and if physics-depletion characteristics of the stream are insensitive to the fuel management of the remainder of the reactor, then the fuel lots in the stream are identical: the load and discharge compositions of all lots are the same, the irradiation times for the lots are the same, and for the fuel stream in question, there are no beginning-of-plant-life and end-of-plant-life transients. For such a fuel stream, Equation (2-29) becomes

$$\bar{e}_q = 1000 \frac{z^q F^q \sum_{m=1}^n w(t_m)}{E \sum_{j=1}^N w(j)} \quad (2-31)$$

where $\bar{e}_q$ is the levelized energy cost (mills/KW-hr) for cost component q , F^q is the carrying charge factor for component q , E is the annual electricity produced in KW-hr, n is the number of fuel lots in the fuel stream throughout plant life, N is the number of years of plant life, and z^q is the direct cost in dollars for component q associated with a fuel lot, that is, associated with the total fuel volume of the batch-managed region.

The expression may be further simplified by noting that

$$\frac{\sum_{m=1}^n w(t_m)}{\sum_{j=1}^N w(j)} \approx \frac{n}{N} = \frac{1}{T} \quad (2-32)$$

where T is the irradiation time of a fuel lot in years.

Equation (2-32) is exact for $n = N$ ($T = 1$ year). Error in the approximation (2-32) increases with decrease in n (increase in T). Irradiation times of interest are about six years (radial blanket fuel) or less. Table 2.4 shows that the error, for a six year irradiation time and an 8% discount rate, is less than 1%.

The factor $1/T$ may be thought of as the fuel throughput in lots per year, for the fuel stream in question.

Equation (2-31) becomes

$$\bar{e}_q = \frac{1000}{ET} \sum_q z^q F^q \quad (2-33)$$

and the total cost associated with the stream is

$$\bar{e} = \frac{1000}{ET} \sum_q z^q F^q \quad (2-34)$$

2.4.2 Scatter Fuel Management

"Scatter" fuel management of a region or subregion is a scheme in which a fraction g of the region's fuel is discharged and replaced during a refueling event. As in batch management, the fuel sees only one position in the reactor.

Because a fraction $1-g$ of the initial load is incompletely irradiated, the region experiences a beginning of plant life fuel transient. Once in

TABLE 2.4 EFFECT OF THE APPROXIMATION

$$\frac{\sum_m^n w(t_m)}{\sum_j^N w(j)} \approx \frac{1}{T}$$

T(yr)	n	N	$\frac{\sum_m^n w(t_m)}{\sum_j^N w(j)}$	$\frac{n}{N}$	Error
1	30	30	1.00000	1.00000	0
6	5	30	0.1653066	0.166666	less than 1%

equilibrium, the fuel lots' irradiation times and load and discharge compositions are identical, and assuming plant load factor remains constant, Equation (2-31) modified for equilibrium scatter fueling is:

$$(\bar{e}_q)_{EQ} = \frac{1000 \sum_m^n w(t_m)}{E \sum_j^N w(j)} z^q F^q, \quad (2-31A)$$

where the fuel lot size is one-gth of the total fuel volume of the region.

The expression corresponding to Equation (2-32) for equilibrium scatter management is:

$$\frac{\sum_m^n w(t_m)}{\sum_j^N w(j)} \approx \frac{n}{N} = \frac{1}{gT} \quad (2-32A)$$

Thus,

$$(\bar{e}_q)_{EQ} = \frac{1000}{ET} z^q F^q \quad (2-33A)$$

and the total equilibrium fuel cost associated with the stream is

$$(\bar{e})_{EQ} = \frac{1000}{ET} \sum_q z^q F^q \quad (2-34A)$$

Comparison of Equations (2-33), (2-34), (2-33A) and (2-34A) shows that the functional forms of the batch and equilibrium scatter cost equations are identical. Indeed, the processing costs - fabrication, reprocessing - are numerically equal as well, for the same irradiation time. Only the material costs

$$\begin{aligned} z^{mp} &= C_{28} M_{28}^0 + C_{49} M_{49}^0 + \dots \\ z^{mc} &= C_{28} M_{28}(T) + C_{49} M_{49}(T) + \dots \end{aligned} \quad (2-30)$$

may differ, owing to the possibly-differing physics-depletion characteristics of the two management schemes.

For the same irradiation time (T), the time averaged composition, e.g. fissile plutonium content, of a region managed by scatter refueling is qualitatively the same as if it were batch managed. In scatter management, however, the composition varies within a smaller range, since its cycle time is shorter - one gth that of the batch managed region.

2.4.3 Local Fuel Economic Performance

It is useful to display the fuel economic performance as a function of position in one of the major regions (core, axial blanket, radial blanket), under batch or scatter management. For example, one may wish to know the performance of the third row of fuel assemblies in the radial blanket, and how it compares to that of the second row, etc. For purposes of comparing these subregions, a figure of merit independent of their volumes is desired.

From Equation (2-34), the fuel costs associated with a stream (region or subregion) is

$$\begin{aligned} \bar{c} &= \bar{c}_{mp} + \bar{c}_{fab} + \bar{c}_{repr} + \bar{c}_{mc} \\ &= \frac{1000}{E} \left[\frac{z^{mp} F^{mp}(T)}{T} + \frac{z^{fab} F^{fab}(T)}{T} \right. \\ &\quad \left. + \frac{z^{repr} F^{repr}(T)}{T} - \frac{z^{mc}(T) F^{mc}(T)}{T} \right] \end{aligned} \quad (2-35)$$

where the carrying charge factors and direct material credit have been expressed as functions of irradiation time, T .

Using Equations (2-30) to express the direct dollar costs in terms of unit costs, one obtains

$$\bar{e} = \frac{1000}{E} \left[\frac{(C_{28}^{M_{28}^0} + C_{49}^{M_{49}^0} + \dots) F^{mp}(T)}{T} + \frac{C_{fab}^{M_{HM}^0} F^{fab}(T)}{T} + \frac{C_{repr}^{M_{HM}^0} F^{repr}(T)}{T} - \frac{(C_{28}^{M_{28}}(T) + C_{49}^{M_{49}}(T) + \dots) F^{mc}(T)}{T} \right] \quad (2-36)$$

The contribution of fertile material (U238, Pu240) to the value of fuel is insignificant. For core fuel

$$C_{fert}^{M_{fert}^0} < C_{fiss}^{M_{fiss}^0}, C_{fab}^{M_{HM}^0}, C_{repr}^{M_{HM}^0}$$

$$C_{fert}^{M_{fert}}(T) < C_{fiss}^{M_{fiss}}(T), C_{fab}^{M_{HM}^0}, C_{repr}^{M_{HM}^0} \quad (2-37)$$

For blanket fuel,

$$C_{fert}^{M_{fert}^0} < C_{fiss}^{M_{fiss}}(T), C_{fab}^{M_{HM}^0}, C_{repr}^{M_{HM}^0}$$

$$C_{fert}^{M_{fert}}(T) < C_{fiss}^{M_{fiss}}(T) \quad (2-38)$$

In either case, the assumption

$$a) \quad C_{fert} \approx 0 \quad (2-39)$$

has a negligible effect on results. In addition, the following assumption is made:

$$b) \quad C_{fiss} = C_{49} = C_{41} \quad (2-40)$$

With assumptions a) and b), Equation (2-36) reduces to

$$\begin{aligned} \bar{e} = & \frac{1000}{E} \left[\frac{C_{fiss} (M_{49}^0 + M_{41}^0) F^{mp}(T)}{T} \right. \\ & + \frac{C_{fab} M_{HM}^0 F^{fab}(T)}{T} \\ & + \frac{C_{repr} M_{HM}^0 F^{repr}(T)}{T} \\ & \left. - \frac{C_{fiss} (M_{49}(T) + M_{41}(T)) F^{mc}(T)}{T} \right] \quad (2-41) \end{aligned}$$

Multiplying and dividing by M_{HM}^0 ,

$$\begin{aligned} \bar{e} = & \frac{1000}{E} M_{HM}^0 \left[\frac{C_{fiss} \epsilon_0 F^{mp}(T)}{T} \right. \\ & + \frac{C_{fab} F^{fab}(T)}{T} \\ & + \frac{C_{repr} F^{repr}(T)}{T} \\ & \left. - \frac{C_{fiss} \epsilon(T) F^{mc}(T)}{T} \right] \quad (2-42) \end{aligned}$$

where

$$\begin{aligned} \epsilon_0 &= (M_{49}^0 + M_{41}^0) / M_{HM}^0 \\ \epsilon(T) &= (M_{49}(T) + M_{41}(T)) / M_{HM}^0 \quad (2-43) \end{aligned}$$

The term in brackets may be regarded as a figure of merit representing local fuel performance, having units of dollars per year per local kilogram of heavy metal. Normalizing to kilograms of initial heavy metal in this manner removes the volume effect in comparing regions or subregions. Except through the discharge composition, $\epsilon(T)$, in the material credit term, the local performance is independent of reactor power level, or load factor.

2.5 COMPARISON OF FUEL COST ACCOUNTING METHODS

2.5.1 Effect of Tax Assumptions in the Cash Flow Method

In Section 2.3, two tax interpretations were applied to the treatment of post-irradiation transactions:

Method A: The post-irradiation transactions are not capitalized i.e. reprocessing cost is written off as a tax deductible expense in the year that it occurs, and material credit is taxed as ordinary income, along with the sale of electricity.

Method B: The post-irradiation transactions are capitalized. Choice of Method A or B affects only the carrying charges associated with reprocessing and material credit. All direct costs and the carrying charges on pre-irradiation costs remain the same.

For a tax rate of 50%, the carrying charges on material credit and reprocessing under Method B are double those from Method A, i. e.

$$\frac{(\bar{e}^{\text{repr}})_B^{**}}{(\bar{e}^{\text{repr}})_A^{**}} = \frac{(F^{\text{repr}})_B - 1}{(F^{\text{repr}})_A - 1}$$

$$= \frac{\frac{1}{1-\tau} \left[w(-T^{\text{repr}}) - \tau \right] - 1}{w(-T^{\text{repr}}) - 1} = 2 ; \quad (2-44)$$

and similarly,

$$\frac{(\bar{e}^{mc})_B^{**}}{(\bar{e}^{mc})_A^{**}} = 2 \quad (2-45)$$

Thus,

$$\frac{(\bar{e}^{mc})_B^{**} + (\bar{e}^{repr})_B^{**}}{(\bar{e}^{mc})_A^{**} + (\bar{e}^{repr})_A^{**}} = 2 \quad (2-46)$$

The carrying charges on reprocessing and material credit are but two of the eight subcomponents making up the total cost of the fuel lot. Also, the carrying charges on reprocessing and material credit are of opposite sign, tending to cancel.

The discrepancy between total costs of the lot, as calculated by Methods A and B, increases with irradiation time as carrying charges in general become more important, and as the material credit (plutonium buildup) from blanket fuel further overshadows blanket reprocessing costs.

Figures 2.4, 2.5 and 2.6 compare the two methods as applied to core, axial blanket and radial blanket fuel costs, respectively, of the reference LMFBR (Chapter 4). Method A is seen to result in lower total fuel costs in all three regions. The discrepancy in core fuel costs is about 10% at the design residence time of two years. In the axial blanket, the discrepancy is about 20% at two years. For the radial blanket, the discrepancy is quite severe, although the two methods yield comparable optimum residence times - 6 years in Method B, 6.5 years in Method A.

2.5.2 Relationship of the Cash Flow Method to Two Other Accounting Methods

For cost component q , lot m , the carrying charge in dollars, $(z_m^q)^{**}$, and the total cost in dollars, $(z_m^q)^*$, are given by

FIG. 2.4 EFFECT OF POST-IRRADIATION TAX ASSUMPTION ON LEVELIZED CORE FUEL ENERGY COST

FIG. 2.5 EFFECT OF POST-IRRADIATION TAX ASSUMPTION ON LEVELIZED AXIAL BLANKET FUEL ENERGY COST.

FIG. 2.6 EFFECT OF POST-IRRADIATION TAX ASSUMPTION ON
LEVELIZED RADIAL BLANKET FUEL ENERGY COST

$$(z_m^q)^{**} = z_m^q f_m^q$$

and

$$(z_m^q)^* = z_m^q F_m^q$$

where

$$F_m^q \equiv 1 + f_m^q \quad (2-28)$$

2.5.2.1 Cash Flow Method (CFM)

The cash flow method (CFM) of Sections 2.2 and 2.3 showed that for capitalized costs or capitalized revenues the carrying charge factor F_m^q takes the form

$$(F_m^q)_{CFM} = \frac{1}{1-\tau} [w(-T_m^q) - \tau] \quad (\text{capitalized}) \quad (2-47)$$

while for expensed costs and taxed revenues the factor is

$$(F_m^q)_{CFM} = w(-T_m^q) \quad (\text{expensed costs; taxed revenues}) \quad (2-48)$$

where

$$w(-T_m^q) \equiv \frac{1}{(1+x)^{-T_m^q}} = (1+x)^{T_m^q} \equiv \text{discount factor} \quad (2-49)$$

$$x \equiv (1-\tau) r_b f_b + r_s f_s \equiv \text{discount rate} \quad (2-50)$$

Two other methods of treating nuclear fuel carrying charges are commonly used:

- (a) Simple Interest Method (SIM), (20, 22, 23)
- (b) Compound Interest Method (CIM), (83)

The cash flow method (CFM) is related to these "approximate" methods below.

2.5.2.2 Simple Interest Method (SIM)

The SIM associates carrying charges with areas under a fuel lot's value histogram. Figure 2.7 shows the general appearance of such histograms for (a) depreciating core fuel and (b) appreciating blanket fuel, and how they may be constructed from the cash flows. Figures 2.8 and 2.9 are value histograms for the reference reactor core and radial blanket, respectively, defined in Chapter 4. Pre-irradiation transactions (material purchase and fabrication are assumed to occur simultaneously, as are the post-irradiation transactions (reprocessing and material credit).

In the SIM, the carrying charge and total dollar costs for component q of lot m are given by

$$(z_m^q)^{**}_{SIM} = (z_m^q) T_m^q y_q \quad (2-51)$$

and

$$(z_m^q)^{*}_{SIM} = (z_m^q) (1 + T_m^q y_q) \quad (2-52)$$

respectively, where

$$\begin{aligned} T_m^q &= T_m'^q + 1/2 T_m \quad (\text{pre-irradiation cash flows}) \\ &= -(T_m^q + 1/2 T_m) \quad (\text{post-irradiation cash flows}) \end{aligned} \quad (2-53)$$

$y_q \equiv$ carrying charge rate applicable to component q .

Thus the carrying charge factors F_m^q for the SIM are given by

$$(F_m^q)^{SIM} = 1 + T_m^q y_q. \quad (2-54)$$

The factors $(F_m^q)^{CFM}$, given by Equations (2-47) and (2-48), reduce to $(F_m^q)^{SIM}$ for small $T_m^q y_q$. Expanding the discount factor in Equation (2-47) in a binomial series, one finds, for capitalized costs or capitalized revenues,

(a) FBR CORE FUEL

(b) FBR BLANKET FUEL

FIG. 2.7 CONSTRUCTION OF VALUE-TIME PLOTS FROM FUEL LOT CASH FLOWS

FIG. 2.8 VALUE-TIME PLOTS FOR A CORE FUEL BATCH

FIG. 2.9 VALUE-TIME PLOTS FOR A RADIAL BLANKET FUEL BATCH

$$\begin{aligned}
(F_m^q)_{CFM} &= \frac{1}{1-\tau} [w(-T_m^q) - \tau] \\
&= \frac{1}{1-\tau} [(1+x)T_m^q - \tau] \\
&= \frac{1}{1-\tau} \left[1-\tau + T_m^q x + \frac{T_m^q (T_m^q - 1)x^2}{2!} + \dots \right] \\
&= 1 + T_m^q \frac{x}{1-\tau} + \frac{T_m^q (T_m^q - 1)}{2!} \frac{x^2}{1-\tau} + \dots
\end{aligned} \tag{2-55}$$

For small $T_m^q x / 1-\tau$ Equation (2-55) reduces to the approximate form

$$(F_m^q)_{CFM} \approx 1 + T_m^q \frac{x}{1-\tau} = 1 + T_m^q y_q = (F_m^q)_{SIM} \tag{2-56}$$

Thus for capitalized costs and capitalized revenues (not taxed), the correct carrying charge rate y_q for the SIM is

$$y_q = \frac{x}{1-\tau} \quad (\text{capitalized costs or capitalized revenues}) \tag{2-57}$$

where the discount rate (x) is given by (2-50).

Similarly, for expensed costs and taxed revenues,

$$\begin{aligned}
(F_m^q)_{CFM} &= w(-T_m^q) = (1+x)T_m^q \\
&= 1 + T_m^q x + \frac{T_m^q (T_m^q - 1) x^2}{2!} + \dots
\end{aligned} \tag{2-58}$$

For small $T_m^q x$,

$$(F_m^q)_{CFM} \approx 1 + T_m^q x = 1 + T_m^q y_q = (F_m^q)_{SIM} \tag{2-59}$$

Thus for expensed costs and taxed revenues the correct carrying charge

rate y_q in the SIM is

$$y_q = x \quad \begin{array}{c} \text{(expensed costs or} \\ \text{taxed revenues)} \end{array} \quad (2-60)$$

where, as before, the discount rate (x) is given by Equation (2-50).

2.5.2.3 Compound Interest Method (CIM)

The restriction on SIM Equations (2-56) and (2-57) that $T_m^q y_q$ be small suggests that SIM may be improved by continuing (2-56) in a series of powers of y_q :

$$\begin{aligned} (F_m^q)_{\text{CIM}} &= 1 + T_m^q y_q + \frac{T_m^q (T_m^q - 1)}{2!} y_q^2 + \dots \\ &= (1 + y_q)^{T_m^q} \end{aligned} \quad (2-61)$$

Thus for the compound interest method (CIM),

$$(z_m^q)^{**}_{\text{CIM}} = (z_m^q) [(1 + y_q)^{T_m^q} - 1] \quad (2-62)$$

and

$$(z_m^q)^* = (z_m^q) (1 + y_q)^{T_m^q} \quad (2-63)$$

where, again,

$$y_q = \frac{x}{1 - \tau} \quad \begin{array}{c} \text{(capitalized costs or} \\ \text{capitalized revenues)} \end{array} \quad (2-57)$$

$$= x \quad \begin{array}{c} \text{(expensed costs or} \\ \text{taxed revenues)} \end{array} \quad (2-60)$$

The $(F_m^q)_{\text{SIM}}$ factors reduce, of course, to the $(F_m^q)_{\text{CIM}}$. Comparing Equations (2-61) and (2-60) with Equation (2-48), one observes that the $(F_m^q)_{\text{CIM}}$ and $(F_m^q)_{\text{CFM}}$ are identical for expensed costs and taxed revenues.

However, the $(F_m^q)_{\text{CIM}}$ and $(F_m^q)_{\text{CFM}}$ are not identical for capitalized costs and capitalized revenues, as can be seen by comparing the expansions (2-61) and (2-55): their first order terms are identical, but a CIM term of order $k > 1$ is in error by a factor of $(1 - \tau)^{k-1}$. The CIM overpredicts the true (CFM) factor F_m^q , for capitalized transactions.

2.5.2.4 Summary

Table 2.5 summarizes the distinction between the three methods of computing carrying charge factors, F_m^q .

Figure 2.10 illustrates the discrepancies between the three methods of computing F_m^q for a capitalized transaction. A discount rate of 8% and a tax rate of 50% are assumed in Figure 2.10. The SIM underpredicts F_m^q , while CIM overpredicts F_m^q . In both cases, errors are significant only for large T_m^q , or large y_q , or both. Since radial blanket fuel may be irradiated as long as ten years, the CFM derived in Sections 2.2 and 2.3 was selected.

2.6 SAMPLE CALCULATION: BEHAVIOR OF BLANKET FUEL COSTS WITH IRRADIATION TIME

Results using the fuel cost model derived in this Chapter exhibit features one would expect qualitatively. Figures 2.5 and 2.6 show the levelized fuel costs of the reference reactor's axial and radial blankets, respectively, as functions of irradiation time. Two major characteristics are noted.

(1) A "breakeven" exposure point exists, at which material credit just balances the material purchase, fabrication, and reprocessing costs:

$$m.c. = m.p. + \text{fab.} + \text{repr.} \quad (2-64)$$

For blanket irradiation times below this point, fissile material produced

TABLE 2.5 SUMMARY OF EXPRESSIONS FOR CARRYING CHARGE FACTOR (F_m^q) BY CFM, CIM AND SIM

Transaction Type	Expression for Carrying Charge Factor, F_m^q		
	CFM	CIM	SIM
Capitalized Costs (Z) and Revenues (V')	$\frac{1}{1-\tau} [(1+x) T_m^q - \tau]$	$(1+y_q) T_m^q$ $y_q = \frac{x}{1-\tau}$	$1+y_q T_m^q$ $y_q = \frac{x}{1-\tau}$
Non-Capitalized Transactions: Expensed Costs (O) Taxed Revenues (V)	$(1+x) T_m^q$	$(1+y_q) T_m^q$ $y_q = x$	$1+y_q T_m^q$ $y_q = x$

Discount Rate: $x = (1-\tau) r_b f_b + r_s f_s$
Discount Intervals:

$$T_m^q \equiv T_m'^q + 1/2 T_m \text{ (pre-irradiation transactions)}$$

$$\equiv -(T_m'^q + 1/2 T_m) \text{ (post irradiation transactions)}$$

$$(F_m^q)_{SIM} < (F_m^q)_{CFM} < (F_m^q)_{CIM}$$

For $T_m^q y_q < < 1$,

$$(F_m^q)_{CFM} \rightarrow (F_m^q)_{SIM}$$

$$(F_m^q)_{CIM} \rightarrow (F_m^q)_{SIM}$$

FIG. 2.10 COMPARISON OF CARRYING CHARGE FACTORS FOR CAPITALIZED TRANSACTIONS COMPUTED BY CFM, CIM, AND SIM

is not sufficient to yield a net revenue.

(2) An optimum exposure point exists, at which the cost is a minimum (most negative).

Figures 2.11 and 2.12 display radial blanket cost components as functions of irradiation time, under Method A. Material purchase (U238) has been ignored in both plots since its contribution is an order of magnitude below other components. Direct material credit is the only negative component; positive costs are shown netted against material credit in both figures. Carrying charges on fabrication and reprocessing components have been combined in "processing carrying charges".

In Figure 2.11 levelized costs are in units of dollars per kilogram of heavy metal (U + Pu) loaded. Total reactor energy released during irradiation of the radial blanket, is shown at the top of the figure. Direct material credit, $|C_{fiss} \epsilon(T)|$, is seen to increase (become more negative) as irradiation time increases, but at a decreasing rate. (The depletion model used in generating $\epsilon(T)$ is described in Chapter 3.) Inventory cost, $C_{fiss} \epsilon(T) [F^{mc}(T) - 1]$, is seen to increase with irradiation time. Direct processing costs, C_{fab} , C_{repr} are, of course, constant, while their carrying charge, $C_{fab} [F^{fab}(T) - 1] + C_{repr} [F^{repr}(T) - 1]$, increases with irradiation time.

The costs of Figure 2.11, in dollars per kilogram of heavy metal, are to be assigned to the total reactor energy released during radial blanket irradiation, $E(\text{KWH}/\text{year})T(\text{year})$. Since E is constant, one may divide the costs of Figure 2.11 by blanket irradiation time T to obtain the radial blanket fuel costs in units of dollars per year per kilogram of heavy metal, Figure 2.12. This is the term in brackets in Equation (2.42), applied to the entire radial blanket. Figure 2.12 also includes a scale indicating the radial blanket levelized power costs in mills/KWHr.

FIG. 2.11 COMPONENTS OF RADIAL BLANKET LEVELIZED COSTS
AS FUNCTIONS OF IRRADIATION TIME

FIG. 2.12 COMPONENTS OF RADIAL BLANKET FUEL LEVELIZED ANNUAL COSTS AND ENERGY COSTS AS FUNCTIONS OF IRRADIATION TIME

From Figure 2.12, an optimum radial blanket irradiation time is seen to exist because of the opposing behavior of material credit and processing costs with irradiation time. Material credit decreases (becomes less negative) while processing cost decreases (become less positive).

2.7 SUMMARY

A general expression for levelized cost (price) of electricity is derived in Section 2.2. This formulation, labeled the "Cash Flow Method" (CFM) is applied to FBR fuel costs in Section 2.3. When applied to a region (core, axial blanket, or radial blanket) under batch or scatter fuel management, Section 2.4, the equations reduce to forms giving local fuel economic performance, e.g. in an annular fuel region or at a point. Table 2.6 below summarizes the CFM fuel cost equations.

Effects of the choice of accounting methods are examined in Section 2.5. Within the CFM, two options for treating post-irradiation transactions are identified:

Method A. Tax revenue from the sale of fissile material (material credit) as ordinary income, along with electricity revenue; treat reprocessing costs as tax deductible expenses.

Method B. Capitalize the revenue from the sale of fissile material; capitalize the reprocessing cost.

Choice between these two tax interpretations has a significant effect on absolute values of power costs (mills/KWhr), but does not distort comparative and incremental results, e.g. design rankings, optimization of radial blanket residence time, sensitivity studies.

The CFM is related to and compared with two approximate methods of treating carrying charges in Section 2.5: The "Compound Interest Method" (CIM); and the "Simple Interest Method" (SIM). For both capitalized and

TABLE 2.6 SUMMARY OF FBR FUEL COST ANALYSIS EQUATIONS (CASH FLOW METHOD)

1. Total Reactor Fuel Cost: $\bar{e}(\text{reactor}) = \sum_s (\bar{e})_s \dots \frac{\text{mills}}{\text{KWHe}}$ $s \equiv \text{fuel stream index}$

2. Fuel Stream Cost: $(\bar{e})_s = \sum_q (\bar{e}_q)_s \dots \frac{\text{mills}}{\text{KWHe}}$ $q \equiv \text{cost component index}$
 $= \text{mp, fab, repr, mc}$

3. Component Costs of a Stream: $(\bar{e}_q)_s = (\bar{e}_{q,\text{direct}})_s + (\bar{e}_{q,\text{CaChg}})_s \dots \frac{\text{mills}}{\text{KWHe}}$

$$= 1000 \left(\frac{\sum_m^n w(t_m) z_m^q}{\sum_j^N w(j) E_j} + \frac{\sum_m^n w(t_m) z_m^q f_m^q}{\sum_j^N w(j) E_j} \right)_s$$

$$= 1000 \left(\frac{\sum_m^n w(t) z_m^q F_m^q}{\sum_j^N w(j) E_j} \right)_s$$

$m \equiv \text{fuel lot index, stream } s$

TABLE 2.6 - continued

4. Carrying Charge Factors:

$$F_m^q \equiv 1 + f_m^q \equiv \text{carrying charge factor, component } q, \text{ fuel lot } m \text{ of a given fuel stream}$$

$$= \frac{1}{1-\tau} [w(-T_m^q) - \tau] \dots \text{ for } q \text{ capitalized}$$

$$= w(-T_m^q) \dots \text{ for } q \text{ not capitalized (expensed cost or tax revenues)}$$

where

$$w(t) \equiv (1+x)^{-t} \equiv \text{discount factor}$$

$$x \equiv (1-\tau)r_b f_b + r_s f_s \equiv \text{discount rate}$$

T_m^q as defined in Figure 2.3.

$\tau \equiv$ tax rate

$f_b \equiv$ bond fraction

$f_s \equiv$ stock fraction

$r_b \equiv$ rate of return to bondholders

$r_s \equiv$ rate of return to stockholders

5. Tax Assumptions

<u>Component, q</u>	<u>Method A</u>	<u>Method B</u>
material purchase	capitalized	capitalized

TABLE 2.6 - continued

<u>Component, q</u>	<u>Method A</u>	<u>Method B</u>
fabrication	capitalized	capitalized
reprocessing	not capitalized (expensed)	capitalized
material credit	not capitalized (taxed)	capitalized

6. Direct Dollar Costs (per lot, per stream)

$$\text{material purchase} \dots z_m^{\text{mp}} = C_{28}^0 M_{28}^0 + C_{49}^0 M_{49}^0 + C_{40}^0 M_{40}^0 + C_{41}^0 M_{41}^0 + C_{42}^0 M_{42}^0 \dots \text{\$/lot}$$

$$\text{fabrication} \dots z_m^{\text{fab}} = C_{\text{fab}}^0 M_{\text{HM}}^0$$

$$\text{reprocessing} \dots z_m^{\text{repr}} = C_{\text{repr}}^0 M_{\text{HM}}^0$$

$$\text{material credit} \dots z_m^{\text{mc}} = C_{28}^0 M_{28}(T) + C_{49}^0 M_{49}(T) + C_{40}^0 M_{40}(T) + C_{41}^0 M_{41}(T) + C_{42}^0 M_{42}(T)$$

7. For batch or scatter fuel management of fuel stream s:

$$(\bar{e})_s = \frac{1000}{E} M_{\text{HM}}^0 \left[\frac{C_{\text{fissile}}^0 F_o^{\text{mp}}(T)}{T} + \frac{C_{\text{fab}}^0 F^{\text{fab}}(T)}{T} + \frac{C_{\text{repr}}^0 F^{\text{repr}}(T)}{T} - \frac{C_{\text{fissile}}^0 \epsilon(T) F^{\text{mc}}(T)}{T} \right] \frac{\text{mills}}{\text{KWHe}}$$

[] = figure of merit, local fuel performance . . . \\$/yr.

kg HM loaded

noncapitalized costs, the carrying charge factors of CIM and CFM are identical. Errors in carrying charge factors, introduced by use of CIM or SIM, are slight for typical core and axial blanket irradiation times - about one percent for an irradiation time of two years. At six years, representative of radial blanket batch irradiation, the errors are about +10% for the CIM, and -10% for the SIM.

The CFM with tax treatment method A is used consistently in the case studies of Chapter 5.

Chapter 3

PHYSICS - DEPLETION MODEL

3.1 INTRODUCTION

Fuel cost analysis requires that fuel discharge composition be known as a function of irradiation time. The objective of the present chapter is to establish a fast breeder reactor fuel depletion model suitable for fuel economic scoping, survey, ranking and sensitivity studies, but sufficiently simple to permit its use, in the form of a computer code, without incurring excessive computer time or capacity requirements.

Effects of spectral, spatial and time detail on FBR physics-depletion calculations have been the subject of several recent studies (28), (29), (30), (37), (44). In particular, Little et.al. (28) have found that little detail is required for accurate core depletion calculations. This is due to the spatial uniformity of spectrum in the core, and the slow variations of flux magnitude and spectrum with irradiation time.

Blanket physics-depletion is considerably more complex, due to the following:

- (1) spectrum softening with distance from the core-blanket interface;
- (2) spectrum hardening with irradiation time due to the relatively large buildup of fissile plutonium in the blanket;
- (3) flux shift, i.e. increase in blanket flux with irradiation time, due to the buildup of fissile plutonium in the blanket; and
- (4) spatial self-shielding (heterogeneity) effects occasioned by the softer blanket spectrum, and aggravated, in the case of radial

blankets, by larger fuel pin diameters.

Effect (1) requires that cross-sections be input to the calculation with sufficient spatial detail, i.e. a separate cross-section set, properly flux weighted, for each of many blanket regions. Effects (2) and (3) suggest that physics calculations be performed sufficiently often, during a time step depletion calculation, to correct the local cross-sections and fluxes.

Parametric and survey studies commonly require the evaluation of numerous cases involving different configurations and compositions, and if models are unnecessarily complex, the computation costs may be prohibitive. In physics-depletion calculations, practically all of the computer time is absorbed by the static physics computations (neutron balances) which yield flux magnitudes and spectra at discrete points in irradiation time and space. The calculations of composition changes between these points in time, using the nuclide depletion equations, require negligible effort in terms of computer time. Hence, computer expense can be significantly reduced by decreasing the number of static physics calculations, i.e. maximizing the length of the irradiation time intervals over which flux magnitudes and spectra are treated as being constant. For this reason, studies were performed, Section 3.4, to assess the effects of item (2), spectrum hardening, and item (3), flux shift, on depletion calculation results.

Conventional multigroup time step depletion (MG-TSD) methods (26), against which simpler methods are to be judged, permit local fluxes and spectra to vary with irradiation. One group time step depletion calculations (1G-TSD) permit local fluxes (flux shape) but not spectra, to vary with irradiation. Both MG-TSD and 1G-TSD models yield reactivity behavior with irradiation, or, alternatively, may involve performing

criticality searches on control poison concentrations. Time step depletion methods are described in Section 3.2.

A simpler (and less expensive) depletion model, the "Semi-Analytic Depletion Method" (SAM), has been developed in this study, and is introduced in Section 3.3. This method is based on the assumptions that both flux (local) and spectrum (local) are constant with irradiation time (or exposure), thus requiring only one multigroup neutron balance for a given reactor configuration. The assumptions of constant local flux and local spectrum, while suitable for the core, may be questionable for the blankets, due to the rapid buildup of fissile plutonium in the blankets. Effects of these assumptions on core, axial blanket, and radial blanket depletion results are evaluated in Section 3.4 by comparing results of three parallel depletion calculations (26G-TSD, 1G-TSD, and SAM).

In SAM, only one multigroup physics calculation is performed. Thus SAM does not provide the k_{eff} behavior with irradiation. This limitation is discussed in Section 3.5.

Effects of spatial self-shielding on blanket depletion results are estimated in Section 3.7.

3.2. DESCRIPTION OF TIME STEP DEPLETION (TSD) CALCULATIONS

Depletion calculations are central to several reactor design functions, yielding space-time-dependent power densities for thermal design, reactivity and criticality information for control system design and absorber management, and fuel discharge compositions for fuel management.

Conventional time step depletion (TSD) calculations may be characterized as one or more neutron balance (in space and energy) computations ("physics"), separated in the irradiation time domain by depletion ("burnup") computations

describing the changes in nuclide populations at each point or region. This process is shown schematically in Figure 3.1. Preparation for the TSD calculation includes the selection of a standard multigroup cross-section set, e.g. the Russian Set (48), the Yiftah-Okrent-Moldauer Set (49), the Hansen-Roach Set (50), or cross-section file, e.g. ENDF/B. More refined calculations may require corrections to this parent cross-section data. Such corrections may be performed by a "cross-section generating program", such as LDX (27), MC² (39), or TDOWN (40). For example, given a reactor configuration (geometry, composition) and the parent cross-section data, LDX performs resonance and spatial shielding corrections by region, and collapses to regional few group sets if desired. The resulting multigroup set is then used as input to the TSD calculation, which may be performed by a computer program such as 2DB (26), PHENIX (31), PYRE (34), or CITATION (47).

With reactor geometry and initial compositions, $\{N_i(\vec{r}, 0)\}$, as input, the TSD begins with a physics calculation at time zero. This calculation yields initial flux shapes and position-dependent spectra, $\phi(\vec{r}, E, 0)$, and the initial effective multiplication constant $k_{\text{eff}}(0)$. (Some programs provide the option of performing a "criticality search" on compositions or dimensions to obtain $k_{\text{eff}} = 1$ at each physics calculation.) The flux profile is normalized to the reactor power specified.

Flux shape and spectrum, $\phi(\vec{r}, E, 0)$, serve as input to the depletion calculation and are assumed constant over the next time interval, that is, until the next physics calculation. Depletion calculations are performed individually for each specified zone. Each zone normally consists of several spatial mesh points; zone fluxes are obtained by appropriate spatial averaging, and nuclide composition is assumed uniform over the zone. The zone depletion equation for nuclide j is

FIG. 3.1 MULTIGROUP TIME STEP DEPLETION CALCULATION

$$\begin{aligned}
\frac{dN_j}{dt} = & -\lambda_j N_j && \text{(decay loss)} \\
& - N_j \sum_{k=1}^n \phi_k \sigma_{k,a}^j && \text{(absorption loss)} \\
& + \lambda_i N_i && \text{(decay source)} \\
& + \sum_m N_m \sum_{k=1}^n \phi_k \sigma_{k,c}^m && \text{(capture source)} \\
& + \sum_q N_q y_{q,j} \sum_{k=1}^n \phi_k \sigma_{k,f}^q && \text{(fission source)}
\end{aligned}
\tag{3-1}$$

where

$j \equiv$ nuclide index

$k \equiv$ energy group index

$m \equiv$ capture parent index

$i \equiv$ decay parent index

$q \equiv$ fission parent index

$N_j \equiv$ atom density, nuclide j , in the zone (atoms/cm³)

$\phi_k \equiv$ group k neutron flux in the zone (n/cm²sec)

$\sigma_{k,e}^j \equiv$ microscopic cross-section for event e , group k , nuclide j (cm²)

$e = a$, absorption

$= c$, capture

$= f$, fission

$\lambda_j \equiv$ decay constant, nuclide j (sec⁻¹)

$t \equiv$ time (sec)

$y_{q,j} \equiv$ yield of nuclide j per fission of nuclide q

Rates of neutron-induced reactions may be expressed in terms of spectrum-weighted cross sections (collapsed to one group) by noting that, in the zone in question,

$$\sum_{k=1}^n \phi_k \sigma_{k,x}^y = \frac{\left(\sum_{k=1}^n \phi_k \right) \left(\sum_{k=1}^n \phi_k \sigma_{k,x}^y \right)}{\left(\sum_{k=1}^n \phi_k \right)} \quad (3-2)$$

$$= \phi \sigma_x^y$$

where

$$\phi \equiv \text{total flux} \equiv \sum_{k=1}^n \phi_k \quad (3-3)$$

$$\sigma_x^y \equiv \begin{array}{l} \text{spectrum averaged one} \\ \text{group cross-section,} \\ \text{nuclide y, event x} \end{array} = \frac{\sum_{k=1}^n \phi_k \sigma_{k,x}^y}{\sum_{k=1}^n \phi_k} \quad (3-4)$$

Coefficients of the nuclide depletion equation are thus seen to be one group microscopic cross-sections, collapsed in the zone spectrum, and Equation (3-1) takes the form

$$\frac{dN_j}{dt} = -\lambda_j N_j - N_j \phi \sigma_a^j + \lambda_i N_i + \sum_m N_m \phi \sigma_c^m + \sum_q N_q y_{q,i} \phi \sigma_f^q \quad (3-5)$$

The collapsing implied in going from Equation (3-2) to Equation (3-5) does not represent any further approximation or loss of generality.

With the flux information $\phi(\bar{r}, E, 0)$ from the initial physics calculations, zone fluxes and cross-sections are computed with Equations (3-3), (3-4) and used in equations of the form of Equation (3-5) to determine zone

compositions at the end of the time interval Δt , assuming the ϕ and σ are constant over Δt . Equations (3-5) may be solved analytically, if the important nuclide chains are simple and known, or in a finite difference fashion, if more flexibility in programming is desired.

The reactor composition thus computed, $\{N_j(\bar{r}, t)\}$ at $t = \Delta t$, is used in the next physics calculation to obtain $\phi(\bar{r}, t, E)$ and k_{eff} at $t = \Delta t$. Again, the flux is normalized to the specified constant reactor power. Zone fluxes and cross-sections collapsed in zone spectra are then used in the depletion calculation for the second time step, yielding $\{N_j(\bar{r}, t)\}$ at $t = 2\Delta t$. This cycle of "physics-collapse-deplete" is repeated until the specified end of the calculation is reached.

The procedure just described accounts for variations in flux shape with irradiation since physics calculations are performed periodically. If these physics calculations are of two or more energy groups, variations in spectra (with irradiation) are also accounted for. The TSD computation may be made more refined by decreasing the irradiation time interval between physics calculations and increasing the number of energy groups. One group TSD calculations will assess the flux shape variation, but not spectrum variations.

3.3 SEMI-ANALYTIC DEPLETION METHOD (SAM)

3.3.1 Introduction

If local flux and local spectrum are assumed constant over the entire irradiation time, then only one multigroup physics calculation is necessary. The local ϕ 's and σ 's from this "snapshot" may be used in the analytic solutions of the coupled depletion equations of the form (3-5) to obtain the local fuel composition as a function of irradiation time. Such a procedure will be labeled the "Semi-Analytic Depletion Method (SAM)".

3.3.2 Analytic Solution of Depletion Equations

For a uranium-plutonium fueled FBR, the important nuclides (for fuel economics) are U238, Pu239, Pu240, Pu241, Pu242. Their coupled depletion equations are:

$$\text{U238: } \frac{dN_{28}}{dt} = -N_{28} \phi \sigma_a^{28} \quad (a)$$

$$\text{Pu239: } \frac{dN_{49}}{dt} = N_{28} \phi \sigma_c^{28} - N_{49} \phi \sigma_a^{49} \quad (b)$$

$$\text{Pu240: } \frac{dN_{40}}{dt} = N_{49} \phi \sigma_c^{49} - N_{40} \phi \sigma_a^{40} \quad (c)$$

(3-6)

$$\text{Pu241: } \frac{dN_{41}}{dt} = N_{40} \phi \sigma_c^{40} - N_{41} \phi \sigma_a^{41} - \lambda_{41} N_{41} \quad (d)$$

$$\text{Pu242: } \frac{dN_{42}}{dt} = N_{41} \phi \sigma_c^{41} - N_{42} \phi \sigma_a^{42} \quad (e)$$

where

$$\phi \sigma_x^y = \sum_{k=1}^n \phi_k \cdot \frac{\sum_{k=1}^n \phi_k \sigma_{k,x}^y}{\sum_{k=1}^n \phi_k}$$

Without decay terms, nuclide depletion equations and their solutions may be expressed in more convenient form with flux time (θ),

$$\begin{aligned} \theta(t) &\equiv \int_0^t \phi(t') dt' \\ d\theta(t) &\equiv \phi(t) dt \end{aligned} \quad (3-7)$$

as the independent variable. Of the nuclides involved, only Pu241 suffers

a significant decay loss (half life 14 years). Assuming for the moment that Pu241 decay may be neglected, Equations (3-6) take the following form:

$$\begin{aligned}
 \text{U238: } \frac{dN_{28}}{d\theta} + \sigma_a^{28} N_{28} &= 0 \\
 \text{Pu239: } \frac{dN_{49}}{d\theta} + \sigma_a^{49} N_{49} &= \sigma_c^{28} N_{28} \\
 \text{Pu240: } \frac{dN_{40}}{d\theta} + \sigma_a^{40} N_{40} &= \sigma_c^{49} N_{49} \\
 \text{Pu241: } \frac{dN_{41}}{d\theta} + \sigma_a^{41} N_{41} &= \sigma_c^{40} N_{40} \\
 \text{Pu242: } \frac{dN_{42}}{d\theta} + \sigma_a^{42} N_{42} &= \sigma_c^{41} N_{41}
 \end{aligned} \tag{3-8}$$

Assuming the cross-sections $\{\sigma\}$ are independent of flux time (θ), the solutions of the set (3-8), to be applied locally, are

$$\text{U238: } N_{28} = N_{28}^0 \exp(-\sigma_a^{28} \theta) \tag{a}$$

$$\text{Pu239: } N_{49} = N_{28}^0 A \exp(-\sigma_a^{28} \theta) [1 - \exp(-(\sigma_a^{49} - \sigma_a^{28}) \theta)] + N_{49}^0 \exp(-\sigma_a^{49} \theta) \tag{b}$$

$$\begin{aligned}
 \text{Pu240: } N_{40} &= N_{28}^0 AB \exp(-\sigma_a^{28} \theta) - N_{28}^0 AB_2 \exp(-\sigma_a^{49} \theta) \\
 &+ N_{49}^0 B_2 \exp(-\sigma_a^{49} \theta) + \beta_1 \exp(-\sigma_a^{40} \theta)
 \end{aligned} \tag{c}$$

$$\begin{aligned}
 \text{Pu241: } N_{41} &= N_{28}^0 AB_1 C_1 \exp(-\sigma_a^{28} \theta) - N_{28}^0 AB_2 C_2 \exp(-\sigma_a^{49} \theta) \\
 &+ N_{49}^0 B_2 C_2 \exp(-\sigma_a^{49} \theta) + \beta_1 C_3 \exp(-\sigma_a^{40} \theta) \\
 &+ \beta_2 \exp(-\sigma_a^{41} \theta)
 \end{aligned} \tag{d}$$

$$\begin{aligned}
 \text{Pu242: } N_{42} = & N_{28}^{AB} C_1 D_1 \exp(-\sigma_a^{49} \theta) - N_{28}^{AB} C_2 D_2 \exp(-\sigma_a^{49} \theta) \\
 & + N_{49}^0 B_2 C_2 D_2 \exp(-\sigma_a^{49} \theta) + \beta_1 C_1 D_3 \exp(-\sigma_a^{40} \theta) \\
 & + \beta_2 D_4 \exp(-\sigma_a^{41} \theta) + \beta_3 \exp(-\sigma_a^{42} \theta)
 \end{aligned} \quad (e)$$

where

$$A \equiv \frac{\sigma_c^{28}}{\sigma_a^{49} - \sigma_a^{28}}$$

$$B_1 \equiv \frac{\sigma_c^{49}}{\sigma_a^{40} - \sigma_a^{28}}$$

$$B_2 \equiv \frac{\sigma_c^{49}}{\sigma_a^{40} - \sigma_a^{49}}$$

$$C_1 \equiv \frac{\sigma_c^{40}}{\sigma_a^{41} - \sigma_a^{28}}$$

$$C_2 \equiv \frac{\sigma_c^{40}}{\sigma_a^{41} - \sigma_a^{49}}$$

$$C_3 \equiv \frac{\sigma_c^{40}}{\sigma_a^{41} - \sigma_a^{40}}$$

$$D_1 \equiv \frac{\sigma_c^{41}}{\sigma_a^{42} - \sigma_a^{28}}$$

$$D_2 \equiv \frac{\sigma_c^{41}}{\sigma_a^{42} - \sigma_a^{49}}$$

$$D_3 \equiv \frac{\sigma_c^{41}}{\sigma_a^{42} - \sigma_a^{40}}$$

$$D_4 \equiv \frac{\sigma_c^{41}}{\sigma_a^{42} - \sigma_a^{41}}$$

$$\beta_1 \equiv N_{40}^0 - (N_{28}^{AB} C_1 - N_{28}^{AB} C_2 + N_{49}^0 B_2)$$

$$\beta_2 \equiv N_{41}^0 - (N_{28}^{AB} C_1 - N_{28}^{AB} C_2 + N_{49}^0 B_2 C_2 + \beta_1 C_3)$$

$$\beta_3 \equiv N_{42}^0 - (N_{28}^{AB} C_1 D_1 - N_{28}^{AB} C_2 D_2 + N_{49}^0 B_2 C_2 D_2 + \beta_1 C_1 D_3 + \beta_2 D_4)$$

In expressing Equation (3-6d) in terms of flux time, Equation (3-8d), Pu241 decay was neglected. Without this assumption, the details of the flux history and out of pile processing and cooling schedules would be required

for estimates of Pu241 and Pu242 populations.

Using the representative FBR blanket reaction rate

$$\sigma_a^{41} \phi \simeq 1.8 \times 10^{-8} \text{ sec}^{-1}, \quad (3-11)$$

the Pu 241 decay rate is seen to be about 10% of its absorption loss rate:

$$\frac{\lambda_{41}}{\sigma_a^{41} \phi} \simeq \frac{1.57 \times 10^{-9} \text{ sec}^{-1}}{1.8 \times 10^{-8} \text{ sec}^{-1}} = 0.087 \simeq 10\% \quad (3-12)$$

The solution of Equation (3-6d) assuming constant N_{40} , ϕ , and σ 's is

$$N_{41} = \frac{N_{40}^0 \sigma_c^{40}}{\sigma_a^{41} \left(\frac{\lambda_{41}}{(1 + \sigma_a^{41} \phi)} \right)} \left[1 - \exp \left(- \sigma_a^{41} \phi \left(1 + \frac{\lambda_{41}}{\sigma_a^{41} \phi} \right) t \right) \right] + N_{41}^0 \exp \left(- \sigma_a^{41} \phi \left(1 + \frac{\lambda_{41}}{\sigma_a^{41} \phi} \right) t \right) \quad (3-13)$$

Letting N_{41}^* denote the Pu241 concentration calculated by ignoring decay, and assuming no initial Pu241, one finds that for an irradiation time of 10 years,

$$\frac{N_{41}}{N_{41}^*} \simeq 0.97 \quad (3-14)$$

Under these conditions, the assumption of no Pu241 decay results in an over-prediction of about 3% in N_{41} .

3.3.3 Summary

The semi-analytic depletion method (SAM) assumes that local spectra and local flux magnitudes do not change with irradiation time. The assump-

tion of constant local fluxes contains another assumption: that the reactor flux shape does not change with irradiation time.

SAM differs from the time step depletion method in that only one multi-group physics computation is performed. Local fluxes and local spectrum-weighted cross-sections from this single physics computation are used as input to Equation (3-7) and Equations (3-9) to obtain local fuel compositions as functions of irradiation time.

3.4 EFFECTS OF THE ASSUMPTIONS OF CONSTANT LOCAL FLUX AND SPECTRUM

3.4.1 Description

The semi-analytic method (SAM) uses neutronic data (local σ 's and ϕ 's) from a single multigroup neutron balance to obtain local fuel composition as a function of irradiation time. These local σ 's and ϕ 's are assumed constant throughout the irradiation life of the fuel. The constant ϕ and constant σ assumptions produce opposing errors in calculations of blanket discharge fissile inventory.

The effects of these assumptions were estimated by comparing results of three parallel depletion calculations (Methods a, b and c) described in Table 3.1 and Figure 3.2. Two sets of comparisons were made:

(1) Reactor #1: Reference Reactor (sodium radial reflector); no fission products included in the TSD methods; comparison of methods a, b, and c.

(2) Reactor #2: Identical in all respects to reactor #1, except that Be metal is used as the radial reflector; fission products included in the TSD method; comparison of methods a and c.

The reference reactor (#1) configuration is described in Figure 3.3. Selection of this reactor as a reference is discussed in Chapter 4. The Be-reflected reactor (#2) is obtained by replacing the Na of zone 24 with

TABLE 3.1 DEPLETION CALCULATIONAL METHODS TO DETERMINE EFFECTS OF CONSTANT FLUX, CONSTANT SPECTRUM

<u>Method</u>	<u>Description</u>	<u>Program</u>	<u>ASSUMPTIONS</u>		
			<u>Input Data Source</u>	<u>Allowed to Vary</u>	
				<u>Local ϕ's</u>	<u>Local σ's</u>
a	26G-TSD	2DB*	σ (E) 's from LDX	yes	yes
b	1G-TSD	2DB*	σ 's from a at t=0	yes	no
c	SAM	SPPIA**	ϕ 's and σ 's from a at t=0	no	no

* Ref. (26)

** SPPIA is a program written to perform SAM calculations, and is described in Appendix C.

1 Ref. 48

2 Ref. 81

3 Ref. 27

4 Ref. 26

5 SPPIA is a program written to perform SAM calculations, and is described in Appendix C.

FIG. 3.2 PROCEDURE FOR METHODS COMPARISONS: 26G-TSD vs. 1G-TSD vs. SAM

VOLUME FRACTIONS

REGION	CORE	RADIAL BLANKET	AXIAL BLANKET	RADIAL REFL.	AXIAL REFL.	AX. REFL. FOR RAD. BKT.
ZONE	1-4	5-19	20-23	24	25	26
SODIUM	0.50	0.30	0.50	1.00	0.50	0.30
OXIDE	0.30*	0.50	0.30	0	0	0
SS CR-FE-NI	0.20	0.20	0.20	0	0.50	0.70

* INITIAL CORE ENRICHMENT = 14%

FIG. 3.3 REFERENCE LMFBR CONFIGURATION (REACTOR #1).

Be metal.

Zone definitions are also shown in Figure 3.3. In all three methods, each burnable zone (1, 2, ..., 23) was depleted individually, that is, a zone was the basic volume unit over which number densities were "smeared" at each time step. The same zone definitions were used for inputting the space-dependent one group cross-sections for methods b(1G-TSD) and c(SAM) and the flux shape for method c (SAM).

For purposes of displaying results, and for use in fuel economic analysis (but not in the physics-depletion calculations) zone compositions were combined to form "region" compositions. For example, the inner-most radial blanket region ("V") is the sum of zones 5, 6 and 7, and

$$M_{49}^V(t) = M_{49}^5(t) + M_{49}^6(t) + M_{49}^7(t) ,$$

$$\epsilon_{49}^V(t) = M_{49}^V(t) / M_{HM}^V(0) \quad (3-15)$$

The TSD calculations (a, b) allowed k_{eff} to vary with burnup; that is, criticality searches were not performed. Effects of this simplification (neglect of flux shape and spectral perturbations introduced by control poison) are considered negligible (29).

In the TSD runs (a, b), physics computations were performed at 150 equivalent full power day (EFPD) intervals. Each of these physics computations yielded the current k_{eff} , the current flux shape, and in the case of the multigroup run (a), the local spectra. After each physics computation, the flux was normalized to a total reactor power of 2576 MW_t, corresponding to 1000 MW_e and a plant thermal efficiency of 39%.

In the TSD calculations, the core and axial blanket were assumed discharged at 600 EFPD, corresponding to an average core burnup of 100,000

MWD_t/MT. At a plant load factor of 82%, this is consistent with a core residence time of two years. The radial blanket remained in situ until the end of the depletion calculation at 1200 EFPD (4 years). Batch fuel management of the radial blanket and the core-axial blanket combination was assumed, which imposes a more severe test of constant flux, constant spectrum assumptions than scatter management.

3.4.2 Results (with Time as the Independent Variable)

The results of the methods-comparisons are reported below by major region (core, axial blanket, radial blanket).

Core

Tables 3.2 and 3.3 show the core depletion results (fissile plutonium population) for reactors #1 and #2 respectively. Excellent agreement between methods a, b and c is noted: at two years irradiation (100,000 MWD_t/MT), discrepancies in fissile masses are about 0.03% for Pu239 and about 1% for Pu241. Zone spectra remain practically constant, accounting for the success of method b. This, together with the result that zone fluxes were fairly constant, explains the close agreement of method c with the other two methods.

The second core load (600 to 1200 EFPD) shows a very slight increase in discharge Pu239 and decrease in discharge Pu241, compared to the first core load. This results from the flux shift to the radial blanket, which is not replaced at two years. As fissile Pu builds up in the radial blanket, the radial blanket provides a greater fraction of total reactor power, decreasing somewhat the powerload, and flux, in the core.

Axial Blanket

Axial blanket depletion results for reactors #1 and #2 are shown in

TABLE 3.2

COMPARISON OF CORE DEPLETION RESULTS FOR

REACTOR #1 (REFERENCE REACTOR)

Equiv. Reactor Full Power Time (D)	Core Res. Time (Yr.)		Method (a)	Method (b)	Method (c)
			<u>26G-TSD</u>	<u>1G-TSD</u>	<u>SAM</u>
0	0	ϵ_{49}	14.05	14.05	14.05
		ϵ_{41}	0	0	0
300	1	ϵ_{49}	12.89	12.89	12.89
		ϵ_{41}	0.062	0.062	0.062
600	2	ϵ_{49}	11.90	11.90	11.90
		ϵ_{41}	0.175	0.174	0.175
600+	0	ϵ_{49}	14.05	14.05	14.05
		ϵ_{41}	0	0	0
900	1	ϵ_{49}	12.93	12.93	12.89
		ϵ_{41}	0.058	0.058	0.062
1200	2	ϵ_{49}	11.97	11.98	11.90
		ϵ_{41}	0.168	0.165	0.175

$$\epsilon_{49} = 100 \times M_{49} / M_{HM}^0 \quad (\%)$$

$$M_{49} = \text{Pu239 mass}$$

$$\epsilon_{41} = 100 \times M_{41} / M_{HM}^0 \quad (\%)$$

$$M_{41} = \text{Pu241 mass}$$

$$M_{HM}^0 = \text{mass of heavy metal loaded (kg)} = 12623 \text{ kg}$$

TABLE 3.3

COMPARISON OF CORE DEPLETION RESULTS FOR
REACTOR #2 (Be-RADIAL REFLECTOR)

<u>Equiv. Reactor Full Power Time (D)</u>	<u>Core Res. Time (Yr.)</u>		<u>Method (a) 26G-TSD</u>	<u>Method (c) SAM</u>
0	0	ϵ_{49}	14.05	14.05
		ϵ_{41}	0	0
300	1	ϵ_{49}	12.89	12.90
		ϵ_{41}	0.062	0.062
600	2	ϵ_{49}	11.94	11.91
		ϵ_{41}	0.168	0.175
600+	0	ϵ_{49}	14.05	14.05
		ϵ_{41}	0	0
900	1	ϵ_{49}	12.93	12.90
		ϵ_{41}	0.058	0.062
1200	2	ϵ_{49}	12.02	11.91
		ϵ_{41}	0.159	0.175

$$\epsilon_{49} = 100 \times M_{49}^0 / M_{HM}^0 (\%)$$

$$M_{49} = \text{Pu239 mass}$$

$$\epsilon_{41} = 100 \times M_{41}^0 / M_{HM}^0 (\%)$$

$$M_{41} = \text{Pu241 mass}$$

$$M_{HM}^0 = \text{mass of heavy metal loaded (kg)} = 12623 \text{ kg}$$

Tables 3.4 and 3.5 respectively.

The 1G-TSD method b, which assumes constant spectra but accounts for flux shape changes, overpredicts the discharge Pu239 content at two years by about 4%, relative to method a. This occurs because the cross-sections input to method b, obtained by collapsing the spectra from method a's solution at time zero, are too soft. (Blanket spectra harden with irradiation due to the fissile buildup.) Method c, SAM, underpredicts Pu239 content by about 4%, in spite of its soft cross-section set, because its input zone fluxes, from the method a time zero solution, are too low. (Blanket flux increases with irradiation due to fissile buildup in the blanket.)

The second axial blanket batch (600 + to 1200 EFPD) suffers a slight decrease (~3%) in discharge fissile content, compared to the first batch. The second axial blanket batch experiences a lower flux than the first, owing to the buildup of flux and power in the radial blanket.

Fission product buildup in the blanket tends to diminish plutonium breeding, by competing with U238 resonance capture and by generally hardening the spectrum. Comparison of 26G-TSD axial blanket results for Reactor #1 (no fission products) and Reactor #2 (with fission products) shows that this is not a discernable effect.

Radial Blanket

Figures 3.4 and 3.5 display the radial blanket depletion method comparisons for reactors #1 and #2 respectively. Results for the innermost annular region (zones 5,6,7) and outermost annular region (zones 17,18,19), as well as for the entire radial blanket, are shown.

As in the axial blanket, the 1G-TSD method consistently overpredicts Pu239 content, while SAM consistently underpredicts Pu239 content. The cross-sections input to 1G-TSD and SAM were obtained by collapsing in the

TABLE 3.4

COMPARISON OF AXIAL BLANKET DEPLETION RESULTS

FOR REACTOR #1 (REFERENCE LMFBR)

Equiv. Reactor Full Power Time (D)	Axial Blanket Res. Time (Yr.)		Method (a)	Method (b)	Method (c)
			26G-TSD	1G-TSD	SAM
0	0	ϵ_{49}	0	0	0
		ϵ_{41}	0	0	0
300	1	ϵ_{49}	2.09	2.15	2.08
		ϵ_{41}	0.0019	0.0019	0.0019
600	2	ϵ_{49}	3.85	4.00	3.71
		ϵ_{41}	0.0113	0.0148	0.0119
600+	0	ϵ_{49}	0	0	0
		ϵ_{41}	0	0	0
900	1	ϵ_{49}	2.03	2.08	2.08
		ϵ_{41}	0.0017	0.0018	0.0019
1200	2	ϵ_{49}	3.74	3.89	3.71
		ϵ_{41}	0.01006	0.01312	0.0119

$$\epsilon_{49} = 100 \times M_{49} / M_{HM}^0 (\%)$$

$$M_{49} = \text{Pu239 mass}$$

$$\epsilon_{41} = 100 \times M_{41} / M_{HM}^0 (\%)$$

$$M_{41} = \text{Pu241 mass}$$

$$M_{HM}^0$$

$$= \text{mass of heavy metal loaded (kg)} = 10093 \text{ kg}$$

TABLE 3.5

COMPARISON OF AXIAL BLANKET DEPLETION RESULTS
FOR REACTOR #2 (Be-RADIAL REFLECTOR)

<u>Equiv. Reactor Full Power Time (D)</u>	<u>Axial Blanket Res. Time (yr)</u>		<u>Method (a)</u> <u>26G-TSD</u>	<u>Method (c)</u> <u>SAM</u>
0	0	ϵ_{49}	0	0
		ϵ_{41}	0	0
300	1	ϵ_{49}	2.0884	2.0794
		ϵ_{41}	0.0019	0.0019
600	2	ϵ_{49}	3.8836	3.6887
		ϵ_{41}	0.0112	0.0121
600+	0	ϵ_{49}	0	0
		ϵ_{41}	0	0
900	1	ϵ_{49}	2.0278	2.0794
		ϵ_{41}	0.00167	0.0019
1200	2	ϵ_{49}	3.7702	3.6887
		ϵ_{41}	0.0098	0.0121

$$\epsilon_{49} = 100 \times M_{49} / M_{HM}^0 \quad (\%)$$

$$M_{49} = \text{Pu239 mass}$$

$$\epsilon_{41} = 100 \times M_{41} / M_{HM}^0 \quad (\%)$$

$$M_{41} = \text{Pu241 mass}$$

$$M_{HM}^0 = \text{mass of heavy metal loaded (kg)} = 10093 \text{ kg}$$

FIG. 3.4 COMPARISON OF RADIAL BLANKET DEPLETION RESULTS FOR REACTOR #1 (REFERENCE LMFBR)

FIG. 3.5 COMPARISON OF RADIAL BLANKET DEPLETION RESULTS FOR REACTOR #2 (BE-RADIAL REFLECTOR)

26G-TSD time zero local spectra. Blanket spectra harden with irradiation due to fissile buildup. This accounts for the 1G-TSD overprediction. The zone fluxes input to SAM were taken from the 26G-TSD time zero solution. Blanket flux increases with irradiation, also because of fissile buildup. This explains SAM's underprediction, despite its soft cross-sections. Evidently the flux shift effect overrides the spectrum hardening effect.

At four years (1200 EFPD), SAM underpredicts the Pu239 content in the innermost region by about 5%, and that in the outermost region by about 20%. Since regions deep in the blanket produce relatively little plutonium, the relatively large error there has little effect on overall radial blanket Pu239 content, which SAM underpredicts by about 10% at 4 years.

The core's nearly constant flux and spectra has greatest influence on blanket fuel near the core-blanket interface, tending to validate SAM's assumptions there. SAM's error increases with distance from the core.

Fission product buildup in the blanket tends to diminish plutonium breeding rate, by competing with U238 resonance capture and by hardening the spectrum. Comparison of 26G-TSD radial blanket results for Reactor #1 (no fission products) and Reactor #2 (with fission products) shows that this is not a discernable effect.

Use of initial fluxes and spectrum-weighted cross-sections as input to SAM resulted in SAM's underprediction of blanket Pu239 inventories. Better agreement between SAM and 26G-TSD results would be expected if the fluxes and cross-sections for SAM were generated by a multigroup physics calculation in which a "representative" amount of fissile material were included in the blanket. Choice of the "representative" amount of fissile material would involve a guess, and perhaps an iteration, for each irradiation time to be evaluated. For example, for a 4 year irradiation, the "representative" amount might be the fissile inventory at 2 years; for an

8 year irradiation, it might be the fissile inventory at 4 years, etc.

To test these ideas, a SAM calculation (SAM_4) was performed using radial blanket fluxes and cross-sections from the 26G physics solution at 4 years. Results of the SAM_4 calculation were compared to those of SAM_0 and the 26G-TSD "truth" calculation, as shown schematically in Figure 3.6. Figure 3.7 displays this comparison.

As expected, SAM_4 overpredicts $\epsilon_{48}(T)$ while SAM_0 underpredicts $\epsilon_{49}(T)$. Table 3.6 shows that the discrepancy between SAM_4 and SAM_0 is about 24% (of SAM_4), independent of irradiation time. Comparing the two SAM calculations with the 26G-TSD "truth", one notes that SAM_0 error increase with irradiation time, while SAM_4 error decreases. Although the two SAM calculations disagreed in ϵ_{49} , and therefore radial blanket material credit (mills/KW-hr) by about 24%, they both yielded an optimum blanket irradiation time of about 6 years.

In the case studies of Chapter 5, clean (initial) conditions are used consistently in SAM, tending to underestimate the economic value of all blanket cases considered.

3.4.3 Results (With Burnup as the Independent Variable)

Depletion method comparisons in Section 3.4.2 were presented with time as the independent variable. With time as the independent variable, SAM requires that one assume constant local flux (and therefore flux shape) throughout the fuel irradiation, or that one use a time-averaged local flux. Table 3.4 and Figure 3.4 of Section 3.4.2 show that the constant flux assumption is the dominant source of error in SAM blanket depletion

FIG. 3.6 PROCEDURE FOR COMPARING SAM RESULTS USING CLEAN AND IRRADIATED FUEL NEUTRONIC DATA

FIG. 3.7 COMPARISON OF SAM RADIAL BLANKET RESULTS USING CLEAN AND IRRADIATED FUEL NEUTRONIC DATA

TABLE 3.6

COMPARISON OF SAM_0 , SAM_4 , and 26G-TSD
RADIAL BLANKET DEPLETION RESULTS

		Irradiation Time (years)		
		2	4	8
Error ϵ_{49} (%)	(1) SAM_0	0.975	1.764	2.942
	(2) SAM_4	1.295	2.329	3.838
	(3) 26G-TSD	1.030	1.972	-
	(3) - (1) / (3)	0.05	0.10	-
	(3) - (2) / (3)	0.25	0.18	-
	(2) - (1) / (2)	0.24	0.24	0.24

calculations¹. This assumption may be relaxed if, instead of time, a time integral property at a point or zone is selected as the independent variable representing degree of exposure, eg. flux-time (θ),

$$A \equiv \int \phi \, dt \quad (3-16)$$

or burnup (B),

$$B = \frac{CV}{M_{HM}^0} \int_{\theta} \{ N_F(\theta') \, \sigma_f \} \, d\theta' \quad (3-17)$$

where the brackets $\{ \}$ represent a summation over fissionable materials, V is the zone volume, C is energy released per fission, and M_{HM}^0 is the mass of heavy metals (U + Pu) loaded in the zone.

This section presents a comparison of 26G-TSD and SAM results for reactor #2 (Be-reflected), with burnup as the independent variable. Having "normalized out" the local flux variation, any discrepancy between 26G-TSD and SAM results should be attributable to SAM's assumption of constant local spectra. Further, part of the spectral variation is ameliorated since burnup ($\sim \sigma_f \int d\theta \, N_F(\theta)$), as well as composition $N_F(\theta)$, is affected by the spectrum, and in the same direction.

Zone burnup in the 26G-TSD was calculated from fission product inventories. In SAM, the $N(\theta)$ equations (3-9) were integrated and solved directly for burnup.

Figures 3.4 and 3.5 showed the largest discrepancy between 26G-TSD and SAM depletion results (vs. time) to be in the outermost region of the

1. The constant flux and constant spectrum assumptions produce opposing errors in $\epsilon_{\text{fissile}}$. Refer to Table 3.4 and Figure 3.4. The effect of the constant spectrum assumption is seen by comparing the 1G-TSD and 26G-TSD results, while comparison of 1G-TSD and SAM results shows the influence of the constant flux assumption.

radial blanket. This comparison, together with the burnup-normalized results are shown in Figure 3.8. With burnup as the independent variable, the two methods yield practically identical results. Similar agreement was found in all radial blanket, axial blanket, and core regions and zones.

Figure 3.9 displays overall radial blanket depletion results by the two methods, with both time and overall radial blanket burnup as independent variables. Again, excellent agreement is noted when burnup is the independent variable.

The excellent agreement in depletion results, when burnup is the independent variable, reinforces the conclusion of Section 3.2.2 that flux shift is more important than spectral variations with irradiation time. Unfortunately, fuel economic analysis requires time as the independent variable in order to assess carrying charges. With time as the independent variable, SAM requires that constant local fluxes and fixed flux shape be assumed. Future work in developing FBR blanket calculational tools should be aimed at predicting the flux shift to the radial blanket, by simple, inexpensive methods.

Composition - Burnup Characteristics

Figure 3.10 shows the ϵ_{49} - burnup characteristics for the innermost and outermost annular regions of the radial blanket. Characteristic curves for the interior regions lie in the shaded area. The curve for the outermost region lies above the others because this region enjoys the softest spectrum in the radial blanket. Data for Figure 3.10 was generated by SAM.

Figure 3.11 compares the depletion characteristics of radial blanket, axial blanket, and core. The axial blanket curve lies above that of the radial blanket because of its softer spectrum (more Na). Core Pu239 fraction decreases with burnup, e.g. the internal breeding ratio is less than unity.

FIG. 3.8 DEPLETION RESULTS FOR OUTER-MOST RADIAL BLANKET REGION:
TIME VERSUS BURNUP AS THE INDEPENDENT VARIABLE

FIG. 3.9 DEPLETION RESULTS FOR THE ENTIRE RADIAL BLANKET:
TIME VERSUS BURNUP AS THE INDEPENDENT VARIABLE

○ 26G-TSD

▽ SAM

FIG. 3.10 BURNUP-FISSILE CONCENTRATION CHARACTERISTICS FOR INNER-MOST AND OUTER-MOST RADIAL BLANKET REGIONS

FIG. 3.11 COMPARISON OF BURNUP-FISSILE CONCENTRATION CHARACTERISTICS OF CORE, AXIAL BLANKET, AND RADIAL BLANKET

3.5 CRITICALITY AND REACTIVITY

In fuel economic analysis, two major functions of a physics-depletion model are (1) to provide fuel composition as a function of irradiation exposure, and (2) to provide criticality and reactivity information. The first is required for computing direct material costs ("burnup costs"), for core and blanket fuel. The second is closely related to core material carrying charges ("inventory cost"), i.e. carrying charges associated with the excess fissile material required to maintain criticality throughout fuel life.

Effects of simplifying assumptions (constant local fluxes and constant local spectra) on irradiated fuel composition results were estimated in Section 3.4 by comparing results of three depletion methods (26G-TSD, 1G-TSD and SAM). The present section concerns criticality and reactivity information (or lack of it) from the two simplified methods (1G-TSD, SAM).

3.5.1 SAM

The SAM procedure as outlined in Section 3.3 includes only one neutron balance computation — to obtain the flux shape, spectra and k_{eff} . Thus the lifetime behavior of k_{eff} is not determined in SAM. However, this is not considered a serious restriction in economic sensitivity and scoping studies.

A rigorous model would include the constraint that enough excess reactivity be loaded to ensure criticality at the desired discharge burnup. Computationally, this may require several complete depletion iterations (at various load enrichments, ϵ_0), as suggested in Figure 3.12, to determine the correct ϵ_0 .

FIG. 3.12 ILLUSTRATION OF DEPLETION ITERATION TO SELECT INITIAL CORE ENRICHMENT

Load enrichment ϵ_0 affects FBR fuel costs through the material components: direct material purchase, direct material credit, and their associated carrying charges (inventory charges).

Effect of ϵ_0 on Direct Material Costs (Core)

Direct material cost ("burnup cost") represents the difference between direct material purchase costs and direct material credits. The direct material purchase component of FBR core fuel cost is directly proportional to load enrichment, ϵ_0 . (By contrast, the same component in light water reactors increases in a parabolic fashion with ϵ_0 , because of the separative work cost.) With flux and cross-sections held constant, Equation (3-9) shows that material credit is also proportional to ϵ_0 , though with a lower constant of proportionality than direct material purchase because higher ϵ_0 implies less fertile material available for breeding. Thus changes in direct material purchase cost and material credit, due to adjustments in ϵ_0 , tend to cancel.

Effect of ϵ_0 on Material Inventory Costs (Core)

The effect of ϵ_0 on core inventory costs is illustrated in Fig. 3.13. Two cases are shown: (1) reactor critical at beginning of fuel life; and (2) reactor critical at end of fuel life. Assuming simple interest, inventory cost is proportional to the area under the fissile fraction (ϵ) plot. The difference in inventory costs between the two cases is represented by the shaded area, $iT \Delta\epsilon$. Both plots are practically linear, and have approximately the same slope, i.e. about 2% in two years, representative of 100,000 MWD_t/MT core burnup. The discrepancy in inventory costs for this simple example is thus

$$\frac{(\text{inv})_2 - (\text{inv})_1}{(\text{inv})_2} = \frac{iT \Delta\epsilon}{iT\epsilon_c + 1/2 iT \Delta\epsilon} \approx 12\% \quad (3-18)$$

The discussion above ignores the slight perturbation of core spectrum

FIG. 3.13 ILLUSTRATION OF THE EFFECT OF CORE INITIAL ENRICHMENT ON MATERIAL INVENTORY COST

and flux level (for the same power) due to the changes in ϵ_0 . Results of 26G-TSD calculations, Section 3.4, show that core spectrum and flux were practically constant over a two year irradiation which resulted in a fissile fraction decrease of $\Delta \epsilon \approx 2\%$. This suggests that ϵ_0 adjustments near criticality, to obtain the desired BOL excess reactivity, would have little effect on core spectrum, flux, and therefore, on discharge compositions.

The discussion above also ignores the perturbation of core spectrum and flux shape due to control poisons. Other studies (29) have concluded that the inclusion of control poisons in TSD calculations (criticality search option) has little effect on depletion results.

The SAM procedure is used in Chapter 5 to compare the fuel economics of various radial blanket sizes (45, 30, and 15 cm) and radial reflector materials. It was found that changing the radial blanket-radial reflector configuration had negligible effect on critical enrichment.

To summarize, the inability of SAM to provide the lifetime behavior of reactivity is not a serious limitation for survey-type, comparative economic studies, provided, of course, that the one snapshot physics calculation is performed at conditions near criticality.

3.5.2 1G-TSD

Comparison of 1G-TSD versus 26G-TSD reactivity information is shown in Table 3.7. Using the expression¹

$$\frac{\delta K}{K} \approx 0.2 \frac{\delta \epsilon}{\epsilon},$$

the 1G-TSD method would result in an "error" of about 15% in critical mass (or inventory cost). The discrepancy in reactivity swing over one core lifetime is negligible.

1. From the 26G-TSD calculation for Reactor #1 (no fission products).

TABLE 3.7

COMPARISON OF MULTIPLICATION CONSTANT VALUES
FROM 26G-TSD AND 1G-TSD CALCULATIONS

Method	k_{eff}^0	Δk_{eff} (per core life)
(a) 26G-TSD	1.0004	0.0245
(b) 1G-TSD	<u>0.9631</u>	<u>0.0248</u>

$$\delta K_{\text{eff}}^0 = 0.0363 \quad \delta(\Delta K_{\text{eff}}) = -0.0003$$

$$\delta \equiv (a) - (b)$$

The 1G-TSD method requires one multigroup physics computation, anyway, to generate its position-dependent cross-sections, and the k_{eff} from this computation is presumably available. This, together with the ΔK -lifetime data from the 1G-TSD calculation, may be used to obtain excess reactivity requirements, fissile loading, and inventory costs.

3.6 COMPARISON OF COMPUTER TIME REQUIREMENTS FOR 26G-TSD, 1G-TSD AND SAM

Table 3.8 compares the MIT IBM 360/65 computer time requirements for the 26G-TSD, 1G-TSD, and SAM calculations discussed in Sections 3.4 and 3.5. All physics and TSD calculations were performed by 2DB.

The reactor configuration was the reference LMFBR described in Figure 3.3, with 1232 mesh points (44 x 28) and 23 active burnup zones. Depletion calculations spanned 1200 EFPD with 150 EFPD time steps for the TSD methods.

3.7 EFFECT OF HETEROGENEITY ON BLANKET DEPLETION RESULTS

Among the effects making irradiated blanket compositions difficult to predict is spatial self-shielding of resonance capture (heterogeneity effects) resulting from the relatively soft blanket spectrum and, for radial blankets, aggravated by the large pin diameters (30).

Figure 3.14 shows the heterogeneity-corrected core and blanket U238 absorption cross-sections as functions of energy group index. These cross-sections were generated by the 1DX code (27) using the Russian 26 group set (48) as input. The 1DX run (a) changed the cross-section format from Russian format to DTF format, (b) corrected group definitions (lethargy interval) from the Russian definitions to uniform lethargy widths of 0.5, commencing with the upper boundary of group 1 at 10.5 Mev, and (c) performed heterogeneity corrections for a typical FBR composition and geometry.

TABLE 3.8COMPARISON OF COMPUTER TIME* REQUIREMENTSFOR 26G-TSD, 1G-TSD, and SAM26G-TSD Method

ten 26G static physics calculations (@ 30 min.per)	300 min.
eight depletion step calculations	<u>negligible</u>
Total 26G-TSD method	300 min.

1 G-TSD Method

one 26G static physics calculation	30 min.
ten 1G static physics calculations (@ 25 min.per)	25 min.
eight depletion step calculations	<u>negligible</u>
Total, attributable to 1G-TSD method	55 min.

SAM

one 26G static physics calculation	30 min.
one depletion step calculation	<u>negligible</u>
Total attributable to SAM	30 min.

* IBM 360/65

FIG. 3.14 26-GROUP U238 ABSORPTION CROSS SECTIONS FOR CORE AND BLANKET AS GENERATED BY 1DX PROGRAM FROM THE RUSSIAN SET

Because of the relatively hard core spectrum, the core heterogeneity corrections are slight, and the "core" cross-section sets are not greatly different from the infinitely-dilute Russian set. In this section, the blanket heterogeneity effects will be measured relative to core conditions, i.e. the core cross-sections are considered as infinitely dilute. To avoid confusion, the following notation is used in this section:

$\sigma(j,k)$	a local cross-section found by collapsing the multigroup parent set j over the local spectrum from a multigroup physics computation in which multigroup set k was used as input for the blanket cross-sections.
j or k	= He . . . heterogeneity-corrected blanket parent multigroup set
	= Ho . . . homogeneous (infinitely dilute) core parent multigroup set
$\phi(k)$	local neutron flux, from a multigroup physics computation in which multigroup cross-section set k was used as input for the blanket cross-sections.

Figure 3.15 displays $\sigma_a^{28}(\text{Ho}, \text{He})$ and $\sigma_a^{28}(\text{He}, \text{He})$ as functions of radial position along the midplane of the reference reactor(#1) at time zero. The blanket heterogeneity effect, $\sigma_a^{28}(\text{Ho}, \text{He}) - \sigma_a^{28}(\text{He}, \text{He})$ is seen to increase with depth into the blanket, due to spectrum softening with depth.

Heterogeneity affects blanket fissile production in two opposing ways:

- (a) the lower effective U238 microscopic capture cross-section, σ_c^{28} , depresses the U238 to Pu239 conversion rate, leading to lower bred fissile Pu inventories;
- (b) viewing blanket neutronics as an attenuation process, in a macroscopic sense, the lower effective microscopic absorption cross-sections (the most significant being U238) results in higher blanket macroscopic fluxes, tending to increase the U238 capture rate and bred fissile inventories.

FIG. 3.15 SPECTRUM-WEIGHTED ONE-GROUP U238 ABSORPTION CROSS SECTIONS AS FUNCTIONS OF RADIAL POSITION

Of the two opposing effects, (a) dominates and heterogeneity influences blanket breeding adversely.

To separate and quantify these effects, three parallel SAM radial blanket calculations were performed, using as local input

- (1) σ (Ho, He) , ϕ (He)
- (2) σ (He, He) , ϕ (He)
- (3) σ (Ho, Ho) , ϕ (Ho) .

Comparison of (1) and (2) provides a measure of effect (a), the depressed U238 capture cross-section. The overall net effect, on fissile breeding, is shown by comparing (2) and (3). Two separate multigroup physics computations, with the 2DB program, generated the local $\{\sigma, \phi\}$ data - one using "He" in the radial blanket, and the other using "Ho" in the radial blanket. The reference reactor configuration, Figure 3.3, was assumed in both computations. Spatial detail for depletion purposes and for purposes of inputting cross-sections and fluxes to SAM, was the same as in the studies of Section 3.4. Table 3.9 shows the local U238 capture rate data in zone #8 (see Figure 3.3) for the three SAM calculations.

The overall radial blanket depletion results, by the three SAM calculations, are compared in Figure 3.16. Comparing (1) and (2) isolates effect (a), that of the depressed σ_c^{28} . The discrepancy in discharge Pu239 inventory is about 20%, resulting in a discrepancy of about 20% in material credit. Effect (a) is offset significantly by effect (b), as can be seen by comparing (1) and (3). The overall self-shielding effect on Pu239 discharge - the combined effects (a) and (b) - is seen by comparing (2) and (3). Self-shielding reduces discharge Pu239 content by about 10%.

TABLE 3.9U238 CAPTURE DATA ILLUSTRATING RADIAL BLANKET HETEROGENEITY EFFECT

	σ_c^{28}	ϕ	$\sigma_c^{28} \phi$
(1) $\sigma_c^{28}(\text{Ho}, \text{He})$			
$\phi(\text{He})$	0.447	1.879	0.840
(2) $\sigma_c^{28}(\text{He}, \text{He})$			
$\phi(\text{He})$	0.374	1.879	0.703
(3) $\sigma_c^{28}(\text{Ho}, \text{Ho})$			
$\phi(\text{Ho})$	0.4168	1.778	0.740

FIG. 3.16 EFFECT OF HETEROGENEITY CORRECTION ON RADIAL BLANKET DEPLETION RESULTS

3.8 SUMMARY

A fast breeder reactor fuel depletion method, suitable for survey, ranking, and sensitivity studies, is established and tested in this Chapter. This method, labeled the Semi-Analytic Method (SAM), assumes that local spectra and fluxes do not vary with irradiation time. These assumptions are tested by comparing SAM results with those of multigroup (26 group) and one group time step depletion calculations. These assumptions are found to have negligible effect on core depletion results (error in discharge composition is less than 0.1%). Error in blanket depletion results is tolerable for purposes of comparative studies: less than 4% in axial blanket results, and about 10% in radial blanket results. Batch fuel management is assumed in the test calculations, thus placing maximum strain on the constant flux, constant spectrum assumptions. Of these two assumptions (which result in opposing errors), the constant flux assumption is found to be the most significant.

SAM results in a computer time savings of about 90%, and is selected for application to the case studies of Chapter 5. As applied in this report, SAM is restricted to fixed fuel schemes, i.e. batch or scatter fuel management.

A further limitation of SAM is that it does not yield reactivity history, since only one physics (multigroup) computation is performed per configuration per fuel lifetime. This is not considered a serious restriction in the sensitivity and comparative studies for which it is intended.

Effects of heterogeneity corrections on blanket depletion results are also examined in this chapter. Blanket heterogeneity is found to reduce fissile discharge inventory by about 10% for irradiation times of interest (2-7 years).

CHAPTER 4

INTEGRATED DEPLETION-ECONOMICS MODEL, SELECTION OF REFERENCE LMFBR, REFERENCE LMFBR FUEL ECONOMICS

4.1 INTRODUCTION

Fast breeder reactor fuel cost analysis and fuel depletion methods were discussed separately in Chapters 2 and 3, respectively. The "cash flow method (CFM)" was selected for cost analysis; the "semi-analytic method (SAM)" using neutronic data from a single multigroup calculation (per reactor configuration) was selected for the depletion model.

The purposes of the present chapter are:

1. to combine these two models into a procedure (Section 4.2) useful for scoping, survey, and sensitivity studies;
2. to compare the selected LMFBR reference configuration to other 1000 MWe LMFBR reference designs (Section 4.3); and
3. to apply the integrated depletion-economics model to the reference economic environment (Section 4.4).

Section 4.2 summarizes the calculational procedure used in the remainder of the report. Section 4.4 serves as an illustration of the procedure, while establishing the base case around which the case studies and sensitivity studies of Chapter 5 are performed. Also, the effect of core enrichment zoning on blanket fuel economics is estimated in Section 4.4.

4.2 INTEGRATED DEPLETION-ECONOMICS MODEL

Figure 4.1 is a schematic of the integrated depletion-economics model.

The step-by-step procedure is described below.

1. The reactor configuration is set in accordance with normal

FIG. 4.1 INTEGRATED DEPLETION-ECONOMICS MODEL

design procedures which need not be the subject of detailed discussion here.

(a) Geometry (shape) and dimensions are established.

(b) Initial compositions of the major regions - core, axial blanket, radial blanket, reflectors - are selected. Reactor geometry and composition are of course chosen to be compatible in terms of achieving criticality, and beginning-of-life (BOL) excess reactivity needed to achieve the rated burnup.

(c) Reactor power rating is specified, satisfying maximum local power density constraints and the desired power shape criteria at the point in life considered, typically BOL.

Depletion Calculation

2. Each major fuel-bearing region is subdivided into depletion zones. Depletion zones should be sufficiently numerous to give the detailed space-dependent depletion information desired and to account for large flux gradients and spectrum variations.

3. A single "snapshot" physics computation is performed to obtain $\phi(r, E)$. A standard multigroup program such as ANISN (46) or 2DB (26) may be used for this purpose. The program 2DB was used throughout the present study. In the present study, BOL nuclide inventories are used in the "snapshot" calculation. Flux shape and local spectra are used to obtain zone flux magnitudes, $\tilde{\phi}$, normalized to rated reactor power, and zone one-group cross sections:

$$\sum_{\text{zones}} (V \tilde{\phi} \Sigma_f)_{\text{zone}} = \text{Rated Reactor Power}$$

$$\sigma = \frac{\sum_g \tilde{\phi}_g \sigma_g}{\tilde{\phi}}$$

$$\tilde{\phi} = \sum_g \tilde{\phi}_g \quad (4-1)$$

where $\tilde{\phi}$ is the zone flux (total, integrated over energy and zone volume) at rated reactor power, V is the zone volume, Σ_f is the total macroscopic fission cross section for the zone, $\tilde{\phi}_g$ is the zone flux in energy group g .

Input to the multigroup physics computation includes, in addition to reactor configuration information, a multigroup cross section set. This multigroup set could be obtained from a cross section processing program such as 1DX (27) or MC² (39), making appropriate corrections, regionally, on a standard set or file, for self-shielding. Multigroup cross section data used in this report consisted of the 26 group Russian set (48), 1DX-corrected (27, 81) for the core and blanket.

4. Zone fluxes and zone cross sections, found in step 3, and plant load factor (L) are input to the "semi-analytic depletion method (SAM)" calculation, Equations (3-9), to obtain zone compositions as a function of flux time, burnup, and irradiation time. The load factor is used to scale the fluxes for Equations (3-7) and (3-9):

$$\phi = L \tilde{\phi} \quad (4-2)$$

Equations (3-9) yield the zone number densities at irradiation time T : $N_{28}(T)$, $N_{49}(T)$, ..., $N_{42}(T)$. The nuclide number densities are converted to masses by equations of the form

$$M_i(T) = N_i(T) V \frac{\tilde{M}_i}{N_{av}} \quad (4-3)$$

where

$M_i(T)$ = mass of nuclide i in the depletion zone at irradiation time T , kg;

V = zone volume, liters;

$\tilde{M}_i$ = atomic mass of nuclide i ;

N_{av} = Avogadro 's number

Since many economics parameters are normalized to mass of heavy metals (U, Pu) loaded, it is convenient to define the nuclide fractions

$$\epsilon_i(T) = M_i(T) / M_{HM}^0 \quad (4-4)$$

where

$$\begin{aligned} M_{HM}^0 &= \text{mass of heavy metals (U, Pu) loaded in zone} \\ &= M_{28}^0 + M_{49}^0 + \dots + M_{42}^0 \end{aligned}$$

5. Zone masses may be summed over an annular region which is batch or scatter-managed to obtain the masses discharged from fuel lots irradiated in that region:

$$\begin{aligned} \left[M_i(T) \right]_{\text{annular region}} &= \sum_k \left[M_i(T) \right]_{\text{zone } k} \\ \left[\epsilon_i(T) \right]_{\text{annular region}} &= \frac{\left[M_i(T) \right]_{\text{annular region}}}{\left[M_{HM}^0 \right]_{\text{annular region}}} \end{aligned} \quad (4-5)$$

Cost Calculations

6. The following economics parameters are set:

. Unit fuel processing costs, dollars per kg of heavy metal,

$$C_{fab}, C_{repr} \quad [\$/\text{kg HM}]$$

. Isotope market values, dollars per kg of isotope,

$$C_{28}, C_{49}, \dots, C_{42} \quad [\$/\text{kg}]$$

. Financial parameters

Tax rate (τ)

Capital structure (f_b, f_s)

Utility rates of return (r_b, r_s)

7. Using the economics parameters of step 6 and the nuclide masses from step 4 (local, depletion zone) or step 5 (annular region), fuel costs are calculated from the CFM cost equations developed in Chapter 2.

The levelized cost of electricity (mills/KWH) associated with a depletion zone or annular region is given by

$$\bar{e} = \frac{1000}{E} M_{HM}^0 \left[\begin{array}{l} \frac{(C_{28} \epsilon_{28}^0 + C_{49} \epsilon_{49}^0 + \dots)}{T} F^{mp}(T) \quad \text{(material purchase)} \\ + \frac{C_{fab} F^{fab}(T)}{T} \quad \text{(fabrication)} \\ + \frac{C_{repr} F^{repr}(T)}{T} \quad \text{(reprocessing)} \\ - \frac{(C_{28} \epsilon_{28}(T) + C_{49} \epsilon_{49}(T) + \dots)}{T} F^{mc}(T) \quad \text{(material credit)} \end{array} \right] \quad (4.6)$$

where $M_{HM}^0, \epsilon_{28}^0, \epsilon_{49}^0, \dots, \epsilon_{28}(T), \epsilon_{49}(T), \dots$ are either depletion zone quantities (step 4) or annular region quantities (step 5).

Other terms in Equation (4-6) are defined below:

E = electrical energy produced by the plant per year, $\frac{kwh}{yr}$

1000 = conversion factor, mills/dollar

$F^q(T)$ = carrying charge factor for cost component q , defined such that

Total cost (q) = direct cost (q) $\times F^q(T)$
 carrying charge (q) = direct cost (q) $\times [F^q(T) - 1]$.

The cost components of Equation (4-6) above correspond to chronological events in the fuel cycle, e.g. material purchase → fabrication → (irradiation) → reprocessing → material credit. Costs may be restructured as desired, e.g.

$$\begin{aligned}\text{material costs} &= \text{material, direct} + \text{material, carrying charge} \\ &= \text{"burnup cost"} + \text{"inventory cost"}\end{aligned}$$

$$\begin{aligned}\text{processing costs} &= \text{fabrication} + \text{reprocessing} \\ &= \text{processing, direct} + \text{processing, carrying charge}\end{aligned}$$

The bracketed term in Equation (4-6) may be regarded as a figure of merit representing local fuel economic performance, having units of dollars per year per local kilogram of heavy metal loaded.

8. Steps 4 through 7 generate local (by depletion zone), annular region, and major regional (core, axial blanket, radial blanket) depletion and economics results as a function of irradiation time.

The process may be repeated for different economic environments (step 6), using a single set of neutronics data from step 3, that is, the multi-group physics computation need be performed only once per reactor configuration.

A computer program, SPPIA, was developed to perform steps 4, 5, and 7, e.g. to perform SAM depletion calculations and CFM cost calculations, using, as input, the zone neutronic information from step 3 and the economic parameters set in step 6. This program is described in Appendix C.

4.3 REFERENCE LMFBR CONFIGURATION

In order to select a representative LMFBR configuration for blanket case studies, a brief survey was made of U.S. reactor manufacturers' 1000 MWe LMFBR 1964 (65, 64, 67, 68) and 1968 Follow-on (67, 70, 71, 74, 75, 76) designs, the Karlsruhe (77, 78) designs, the reference reactors used in

design calculations for the MIT Blanket Test Facility (60), and reference reactors used in other blanket analytical studies (1, 4, 5). Only uranium-plutonium ceramic fueled reactors were considered. Modular and annular geometries were excluded. Key variables surveyed were those base design parameters which affect blanket fuel economics directly and prominently: core height-to-diameter ratio, core power density, blanket size, and core and blanket compositions. Table 4.1 summarizes this data for the reactors considered, together with the LMFBR reference design adopted for the present studies. The reference LMFBR arrangement is shown in Figure 4.2.

The reference LMFBR adopted is identical to that used in the depletion method studies of Chapter 3. It closely resembles the Karlsruhe designs, and the reference LMFBR used for the MIT Blanket Test Facility design calculations. The two subcases (1, 1') of the reference LMFBR are identical except that 1' has two core enrichment zones while 1 has a single uniform load enrichment.

4.4 REFERENCE LMFBR FUEL ECONOMICS

One-Zone Core

In this section, the procedure outlined in Section 4.2 is applied to the reference LMFBR described in Section 4.3.

The reference economic environment assumed for these calculations is summarized in Table 4.2. This data is representative of base data used in LMFBR design studies (62, 63, 64, 65, 66, 69, 70, 71, 72, 73) and falls within ranges projected for the mature U.S. nuclear fuel cycle economy (18). Unit fuel cycle costs are sensitive to national nuclear fuel cycle throughput and capacity, and are closely coupled to reactor design and operation (fuel pin design, discharge enrichment). Since these concerns lie beyond the purview of this study, the economic environment is treated parametrically

TABLE 4.1 SURVEY OF LMFBR DESIGNS

1964 1000 MWe LMFBR Designs

		GE Ref. (65,67,68)	CE Ref. (64,67,68)
Core Height	(cm)	60.96	76.2
Diameter	(cm)	356.62	219.46
Height/Diameter		0.1709	0.347
Volume	(liters)	6030	2895
Axial Blanket Thickness x2	(cm)	91.4	91.4
Radial Blanket Thickness	(cm)	38.1	42.6
Core Power Density	(kw _t /liter)	365	695
Core & Axial Blanket Composition			
Fuel	(v/o)	34.8	25.6
Structural Metal	(v/o)	18.8	7.9
Sodium	(v/o)	46.4	66.5
Radial Blanket Composition			
Fuel	(v/o)	50.7	45
Structural Metal	(v/o)	17.2	12
Sodium	(v/o)	32.1	43
Fuel Enrichment Zoning			
No. of Core Enrichment Zones		3	2
Enrichment Zone #		I II III	I II
Load Enrichment	(%)	15.67	10.68
Enrichment Zone Volume	(liters)	2010 2010 2010	1447.5 1447.5

TABLE 4.1 - Continuation

1968 1000 MWe LMFBR Follow-on Designs

		AI Ref. (70,74)	CE Ref. (71,76)	B&W Ref. (69,75)
Core Height	(cm)	127	60.96	88.73
Diameter	(cm)	235	270.36	306.68
Height/Diameter		0.54	0.226	0.29
Volume	(liter)	5506	3498	6516
Axial Blanket Thickness x 2	(cm)	60.96	91.44	71.12
Radial Blanket Thickness	(cm)		30.8	30.8
Core Power Density	(kw _t /liter)	408	608	346
Core&Axial Blanket Composition				
Fuel	(v/o)	31.0	38.63	44.2
Structural Metal	(v/o)	18.9	13.10	16.9
Sodium	(v/o)	50.1	39.98	35.4
Radial Blanket Composition				
Fuel	(v/o)	57.7	52.66	64.3
Structural Metal	(v/o)	13.1	15.01	16.3
Sodium	(v/o)	29.2	32.33	19.5
Fuel Enrichment Zoning				
No. of Core Enrichment Zones		2	2	3
Enrichment Zone #		I II	I II	I II III
Load Enrichment	(%)	11.6 16.0	10.13 12.05	
Enrichment Zone Volume	(liters)			

TABLE 4.1 Continuation

		Karlsruhe Ref. (77)	Karlsruhe (Jansen) Ref. (78)	Forbes Ref. (60)
Core Height	(cm)	95.5	94	95
Diameter	(cm)	286	282	200
Height/Diameter		0.333	0.333	0.475
Volume	(liters)	6132	5868	2983
Axial Blanket Thickness x 2	(cm)	80	80	80
Radial Blanket Thickness	(cm)	45.3	46	45
Core Power Density	(kw _t /liter)	393		
Core & Axial Blanket Composition				
Fuel	(v / o)	30.4	30	35
Structural Metal	(v / o)	19.6	20	15
Sodium	(v / o)	50.0	50	50
Radial Blanket Composition				
Fuel	(v / o)	48.3	50	50
Structural Metal	(v / o)	21.9	20	20
Sodium	(v / o)	29.8	30	30
Fuel Enrichment Zoning				
No. of Core Enrichment Zones		2	2	1 2
Enrichment Zone #		I II	I II	I I II
Load Enrichment (%)		10.66 13.93		12.43 11.54 15.06
Enrichment Zone Volume (liters)		3066 3066		2983 1462 1521

TABLE 4.1 Continuation

		Klickman Ref. (1)	Hasnain Ref. (4)	Perks Ref. (5)
Core Height	(cm)	105	93	119.38
Diameter	(cm)	131	104	193.04
Height/Diameter		0.801	0.9	0.62
Volume	(liters)	1414	800	3400
Axial Blanket Thickness x 2	(cm)	92	80	122
Radial Blanket Thickness	(cm)	40	45	61
Core Power Density	(kw./ liter)	443	1000	329
Core & Axial Blanket Composition				
Fuel	(v/o)	50	30	30.8
Structural Metal	(v/o)	14	20	25.0
Sodium	(v/o)	36	50	36.4
Radial Blanket Composition				
Fuel	(v/o)		60	
Structural Metal	(v/o)		18	varied
Sodium	(v/o)		22	
Fuel Enrichment Zoning				
No. of Core Enrichment Zones		1	1	2
Enrichment Zone #		I	I	I II
Load Enrichment	(%)	9.8		17.5 12.93
Enrichment Zone Volume	(liter)	1414		
Plant Rated Thermal Power	(MW _t)	620	800	1200
Plant Rated Electrical Power	(MW _e)	230		500

TABLE 4.1 Continuation

		Reference LMFBR for Present Studies		
Core Height	(cm)	100		
Diameter	(cm)	250		
Height/Diameter		0.4		
Volume	(liters)	4906		
Axial Blanket Thickness x 2	(cm)	80		
Radial Blanket Thickness	(cm)	45		
Core Power Density	(kw _t /liter)	500		
Core & Axial Blanket Composition				
Fuel	(v/o)	30		
Structural Metal	(v/o)	20		
Sodium	(v/o)	50		
Radial Blanket Composition				
Fuel	(v/o)	50		
Structural Metal	(v/o)	20		
Sodium	(v/o)	30		
Fuel Enrichment Zoning				
No. of Core Enrichment Zones		1*	2**	
Enrichment Zone #		I	I	II
Load Enrichment	(%)	14	11	17
Enrichment Zone Volume	(liters)	4906	2540	2366
Plant Rated Thermal Power	(MW _t)		2560	
Plant Rated Electrical Power	(MW _e)		1000	

* Reactor #1

** Reactor #1'

VOLUME FRACTIONS

REGION	CORE	RADIAL BLANKET	AXIAL BLANKET	RADIAL REFL.	AXIAL REFL.	AX.REFL. FOR RAD.BKT.
ZONE	1-4	5-19	20-23	24	25	26
SODIUM	0.50	0.30	0.50	1.00	0.50	0.30
OXIDE	0.30*	0.50	0.30	0	0	0
SS CR-FE-NI	0.20	0.20	0.20	0	0.50	0.70

* INITIAL CORE ENRICHMENT = 14%

FIG. 4.2 REFERENCE LMFBF CONFIGURATION (REACTOR #1)

TABLE 4.2REFERENCE ECONOMIC ENVIRONMENT

<u>Unit Fuel Processing Costs</u>	<u>\$/kg HM</u>		
	<u>Core</u>	<u>Axial Blanket</u>	<u>Radial Blanket</u>
Fabrication (C_{fab})	314	80	69
Reprocessing (C_{repr})	31.5	31.5	31.5

<u>Isotope Market Values</u>	<u>\$/kg</u>
U238 (C_{28})	0
Pu239 (C_{49})	10,000
Pu240 (C_{40})	0
Pu241 (C_{41})	10,000
Pu242 (C_{42})	0

Financial Parameters

Income Tax Rate (τ)	0.5
Capital Structure	
Bond(Debt) Fraction (f_b)	0.5
Stock (Equity) Fraction (f_s)	0.5
Rates of Return	
Bonds (r_b)	0.07
Stocks (r_s)	0.125
Discount Rate *	(x) 0.08

$$* x = (1 - \tau)r_b f_b + r_s f_s$$

in Chapter 5, where the sensitivity of LMFBR fuel costs (mills/KWHe) to economic environment is estimated.

Table 4.3 lists pre-irradiation, irradiation, and post-irradiation times assumed for these reference calculations. Table 4.4 summarizes the reference plant power data.

Figures 4.3, 4.4, and 4.5 show fuel costs (mills/KWHe) versus irradiation time for the reference LMFBR (#1) core, axial blanket, and radial blanket. Batch fuel management of each of these regions is assumed. The reference one-zone core irradiation time of two years corresponds to an average core burnup of 102,000 MWD_t/MT. Core fuel cost at this exposure is seen to be 0.97 mills/KWHe. It is assumed, for practical reasons, that the axial blanket must be operated on the same fueling schedule as the core, i.e. two year irradiation time. The optimum irradiation time for the axial blanket, under the reference economic environment, is about two years (Figure 4.4). At two years, the axial blanket fuel cost is -0.141 mills/KWHe, a net revenue.

Unlike the axial blanket, the irradiation time for the radial blanket may be fixed independently of the core. Figure 4.5 shows its "breakeven" irradiation time to be about three years. Its optimum irradiation time is about six years, corresponding to a fuel cost of -0.036 mills/KWHe, (-8.1 \$/yr./kg HM), a net revenue.

Comparing the fuel economics of the axial and radial blankets, one notes: (a) the relatively low net revenue provided by the radial blanket at its optimum irradiation time, i.e. 8.1 \$/yr./kgHM for the radial blanket vs. 100 \$/yr./kgHM for the axial blanket; and (b) the relatively "sluggish" behavior of the radial blanket, i.e. the radial blanket is slow in reaching its maximum net revenue. The comparatively poor radial blanket performance is the result of its relatively

TABLE 4.3 REFERENCE FUEL CYCLE TIMING

<u>Region</u>	<u>Scheme</u>	<u>Pre-Irradiation</u>		<u>Irradiation</u>	<u>Post-Irradiation</u>	
		<u>Times (yr.)</u>		<u>Time (yr.)</u>	<u>Times (yr.)</u>	
		<u>Material</u> <u>Purchase</u>	<u>Fabri-</u> <u>cation</u>		<u>Repro-</u> <u>cessing</u>	<u>material</u> <u>credit</u>
core	batch	0.5	0.5	2	0.5	0.5
axial blanket	batch	0.5	0.5	2	0.5	0.5
radial blanket	batch	0.5	0.5	variable	0.5	0.5

TABLE 4.4REFERENCE PLANT POWER PARAMETERS

Plant Rated Power, electrical	1000 MWe
Plant Thermal Efficiency (η)	39%
Plant Load Factor (L)	83%

FIG. 4.3 REFERENCE LMFBR CORE FUEL ENERGY COST AS A FUNCTION OF EXPOSURE

FIG. 4.4 REFERENCE LMFBR AXIAL BLANKET FUEL ENERGY COST AS A FUNCTION OF EXPOSURE

FIG. 4.5 REFERENCE LMFBR RADIAL BLANKET FUEL COST
AS A FUNCTION OF EXPOSURE

low average neutron flux (axial leakage predominates) and its somewhat harder spectrum (the radial blanket contains less sodium per unit volume).

Figures 4.6, 4.7 and 4.8 display the reference LMFBR (#1) radial blanket neutronic and fuel economic spatial characteristics. Figure 4.6 shows ϕ , σ_c^{28} , and $\phi \sigma_c^{28}$ as functions of radial position along the mid-plane ($Z=0$) determined by the 2DB computation, step 3, Section 4.2. Figure 4.7 shows the resulting local fuel economic performance (\$/yr/kgHM) versus irradiation time, for the annular regions labeled V, VIII, XI, XIV, and XVII in Figure 4.2. Breakeven and optimum irradiation times increase with distance from the core-blanket interfact because the fissile plutonium buildup is most rapid in the high flux regions near the core. Regions near the core show relatively sharp optima, compared to the regions XIV, XVII deep in the blanket. The optimum irradiation time (6.5 years) for the batch-managed radial blanket is indicated by the dotted line. At the optimum, annular region XVII (outermost) incurs a net cost, rather than a net revenue, indicating that the radial blanket, under the economic environment assumed, is too thick.

Figure 4.8 displays the variation of local fuel performance (\$/yr/kgHM) with distance from the core, at irradiation times of 2, 4, 6, and 8 years. Net revenue from the entire radial blanket may be identified with the net area under (+) and above (-) the curve. For a two year irradiation, about two-thirds of the 45 cm radial blanket incurs a net cost, i.e. the fissile produced is not enough to offset fabrication, reprocessing, and carrying charges. At 6.5 years, the optimum, the outer one-third (outer row of fuel assemblies) is unprofitable. The slope of the plot (\$/yr-kgHM-cm) decreases with irradiation time, as the inner regions pass their optimum irradiation times.

FIG. 4.6 LOCAL NEUTRONICS OF THE REFERENCE LMFBR RADIAL BLANKET

FIG. 4.7 FUEL ECONOMIC PERFORMANCE OF ANNULAR REGIONS IN THE REFERENCE LMFBR RADIAL BLANKET

FIG. 4.8 FUEL ECONOMIC PERFORMANCE AS A FUNCTION OF RADIAL POSITION IN THE REFERENCE LMFBR RADIAL BLANKET

One Versus Two Core Enrichment Zones

In addition to its advantages to core economics (53), core enrichment zoning enhances blanket breeding by increasing blanket flux. In order to estimate the effect of core enrichment zoning on blanket depletion economics, a two-zone core reactor (Reactor #1') was compared to the one-zone core reactor (Reactor #1) evaluated above. In this comparison, core fissile inventory loaded, rated reactor power, and core size were fixed.

Since both rated power and core size are held fixed, the effect of power flattening on core fuel depletion economics (either more allowable power, or a smaller core and lower critical mass are possible) are not accounted for. Also unaccounted for is the inventory cost of the additional fissile mass required for k_{eff} -equivalence with the one zone core reactor. These two unaccounted for effects tend to cancel. Since primary interest here is the blanket fuel economics, the adjustments in core conditions are not deemed necessary, although the net improvement in core fuel economics may, because core fuel costs (mills/KWHe) dominate, be more significant than the increased blanket revenue.

The comparison is shown in Table 4.5. When the core is zoned as prescribed, radial blanket revenue increases by about 150%, the axial blanket revenue by about 6%. Taken together, the incremental improvement in blanket revenue is about 0.07 mills/KWHe, a savings of the order of 5 to 10% in total reactor fuel cost.

4.5 SUMMARY

The fuel economics and depletion methods established in Chapters 2 and 3, respectively, are combined in this Chapter to form a simple step-by-step procedure. (Appendix C describes the computer program, SPPIA, developed to perform the fuel economics-depletion computations.)

TABLE 4.5EFFECT OF CORE ENRICHMENT ZONING ON BLANKET FUEL ECONOMICS

	Reactor #1 (1-zone core)	Reactor #2 (2-zone core)
Core		
Enrichment Zone #	-	I II
Zone Enrichment, %	14	11 17
Zone Volume, liters	4906	2540 2366
Average Enrichment, %	14	14
Radial Blanket		
Fuel Cost, mills/KWhe	-0.036	-0.093
Optimum Irradiation Time, years	6.5	4.5
$\Phi(0,0)/\Phi(125,0)$	6.294	2.831
Axial Blanket		
Fuel Cost, mills/KWhe	-0.141	-0.150
$\Phi(0,0)/\Phi(0,50)$	2.526	2.507
$\Phi(0,0)/\Phi(110,50)$	7.297	3.759

A reference 1000 MWe LMFBR configuration, representative of those used in current design studies, is specified; the depletion-economics model is applied to this reference design. Major characteristics of FBR depletion-economics are noted. The beneficial effect of core enrichment zoning on blanket fuel economics is demonstrated.

CHAPTER 5

1000 MWe LMFBR CASE STUDIES

5.1 INTRODUCTION

In this chapter, the calculational procedure outlined in Section 4.1 is applied to a number of case studies. Objectives of these case studies are:

- (1) to determine the effects, on reactor fuel economic performance, of radial blanket thickness and radial reflector material;
- (2) to determine the economic advantage of operating each radial blanket region (annular) on its own local optimum irradiation schedule; and
- (3) to examine the sensitivity of core, axial blanket, and radial blanket fuel energy costs (mills/KWHe) to the economic environment.

Radial blanket thickness and radial reflector are varied, as shown in Table 5.1. All other design parameters - core and axial blanket geometry and compositions, and radial blanket composition - are held fixed at reference reactor values given in Section 4.3, i.e. Reactor #1. Core and axial blanket fuel power costs (mills/KWHe) are found to be quite insensitive to radial blanket thickness and choice of radial reflector material (Section 5.2), and are thus ignored in evaluating the radial blanket configurations. Results of the radial blanket thickness - radial reflector material case studies are presented in Section 5.3.

The assumption of a one-zone core in these studies penalizes radial blanket economics in all cases considered. Thus, the absolute values of net radial blanket revenue (in mills/KWHe) of individual configurations should not be taken as representative or typical. However, comparative

TABLE 5.1 CASE DEFINITIONS

Reactor Configuration #	Radial Blanket Thickness (cm)	Radial Reflector Material
1 (reference)	45	Na
1A	30	Na
1B	15	Na
2	45	Be metal
2A	30	Be metal
2B	15	Be metal

conclusions and trends demonstrated by these studies are unaffected by the one-zone assumption.

Two radial blanket fuel management schemes are compared in Section 5.4: (a) "whole blanket" management, in which all blanket fuel sees the same irradiation time, the optimum for the blanket as a whole; and (b) "regional" management, in which each annular region is exposed to its own local optimum irradiation time. In either case, fuel sees only one position in the reactor.

The sensitivity of core, axial blanket, and radial blanket fuel power costs to changes in the economic environment is evaluated in Section 5.5. Also, simple linear forms of the cost equations are obtained in Section 5.5.

5.2 EFFECTS OF RADIAL BLANKET THICKNESS AND RADIAL REFLECTOR MATERIAL ON CORE AND AXIAL BLANKET FUEL DEPLETION ECONOMICS

Assessment of the economic (energy costs) effects of radial configuration design changes may be simplified considerably by ignoring their influence on core and axial blanket fuel depletion economics, that is by considering only the radial blanket depletion economics. For the reference geometry selected ($H/D = 0.4$), one would expect such changes in core and axial blanket fuel economic performance to be small because of the relatively small radial leakage from the core. That these changes are smaller, by orders of magnitude, than the simultaneous changes in radial blanket fuel depletion economics, is demonstrated in this section.

Changes in radial blanket thickness and/or radial reflector material can affect core and axial blanket fuel economics in two ways: (1) by affecting the core fissile inventory required for criticality, and thereby affecting core inventory cost; and (2) by perturbing the flux magnitude and spectra in the core and axial blanket, causing changes in

X depletion and material credit results.

(1) Core inventory cost is closely proportional to $yT\epsilon$, where y is the carrying charge rate (per annum), T is the time the fissile material is in the possession of the utility company, and ϵ is the critical enrichment. Thus the fractional change in core inventory cost, Inv. (mills/KWHe) , caused by a change in critical enrichment is given by

$$\frac{\Delta \text{Inv.}}{\text{Inv.}} = \frac{\Delta \epsilon}{\epsilon}$$

or

$$\frac{\Delta \text{Inv.}}{\text{Inv.}} \approx 5 \frac{\Delta K}{K}$$

where the expression¹

$$\frac{\Delta K}{K} \approx 0.2 \frac{\Delta \epsilon}{\epsilon}$$

has been used.

To illustrate the insensitivity of core inventory cost to radial configuration changes, Cases 1 (45 cm radial blanket with Na radial reflector) and 2B (15 cm radial blanket with Be radial reflector) are compared. From a reactivity point of view, these are the two most disparate cases. The multigroup physics computations for the two cases showed that for the same core fissile content, their values of K_{eff} differ by less than 0.0002.

Thus

$$\frac{\Delta \text{Inv.}}{\text{Inv.}} = 5 \times \frac{0.0002}{1.0} = 0.001.$$

The core inventory cost (Inv.) for Case 1 is 0.4147 mills/KWHe. Thus

1. From the 26G-TSD calculation for Reactor #1 (no fission products).

$$\Delta \text{Inv.} = \text{Inv.} \times \frac{\Delta \text{Inv.}}{\text{Inv.}} \approx 0.4147 \times 0.001$$

$$\approx 0.00042 \text{ mills/KWHe}$$

The difference in optimum radial blanket fuel costs between the two cases is 0.05 mills/KWHe¹, which dwarfs the difference in core inventory costs. Of the six Na and Be reflected cases considered (Table 5.1), the minimum difference between radial blanket fuel costs is found to be 0.004 mills/KWHe (Cases 1 and 2)¹, which is an order of magnitude greater than the maximum difference in core inventory costs.

(2) To illustrate the insensitivity of core and axial blanket fuel costs to the changes in flux shape and spectra occasioned by a radial configuration change, Table 5.2 shows the fuel costs of core, axial blanket and radial blanket for Cases 1 and 2B. The core results do not include the core inventory correction (0.0004 mills/KWHe) estimated above.

To summarize, core and axial blanket fuel depletion-economics is quite insensitive to choice of radial reflector material and radial blanket thickness:

$$(1) \quad \Delta \bar{\epsilon}_{\text{RB}} \gg \Delta \text{Inv.}_{\text{core}}$$

$$(2) \quad \Delta \bar{\epsilon}_{\text{RB}} \gg \Delta \bar{\epsilon}_{\text{core}}, \Delta \bar{\epsilon}_{\text{AB}}$$

For this reason, only the radial blanket fuel economic performance was considered in ranking the reactor configurations of Table 5.1.

5.3 RADIAL BLANKET THICKNESS AND RADIAL REFLECTOR MATERIAL

Reducing the radial blanket thickness has several effects on the radial blanket fuel economic performance:

1. Section 5.3.

4

TABLE 5.2

EFFECTS OF RADIAL CONFIGURATION CHANGES
ON CORE AND AXIAL BLANKET FUEL COSTS

Reactor Configuration #	Radial Blanket Thickness (cm)	Radial Reflector Material	$\bar{e}_{RB}$	$\bar{e}_{core}$	$\bar{e}_{AB}$
1	45	Na	-0.03635	0.96975	-0.14130
2B	15	Be	<u>-0.08687</u>	<u>0.96981</u>	<u>-0.14176</u>
	(2B-1)	=	-0.05052	0.00006*	-0.00046

* Does not include core inventory correction.

(a) fabrication and reprocessing costs for the region eliminated are saved;

(b) the plutonium which would have been bred in the region eliminated is forfeited;

(c) the breeding performance ($\overline{\sigma_c^{28} \phi}$) of the remaining radial blanket is improved, by bringing the high-albedo and moderating reflector nearer to the high flux regions of the blanket; and

(d) coolant pumping power requirements for the radial blanket are reduced. These effects suggest that an economic optimum thickness may exist.

Radial reflector composition influences radial blanket fuel economic performance through

(e) improved albedo, i.e. a beneficial effect on neutron economy through flux enhancement in the blanket, as well as overall flux flattening; and

(f) improved moderating ratio, softening radial blanket spectra and enhancing capture by U238.

Other economic considerations associated with choice of reflector material and reflector design are:

(g) radial reflector coolant pumping requirements;

(h) radial reflector material purchase and fabrication costs and exposure limits; and

(i) shielding performance of the radial reflector.

The economic consequences of changes in (d) radial blanket coolant pumping requirements, (g) radial reflector coolant pumping requirements, (h) radial reflector material and fabrication costs, and exposure limits, and (i) radial reflector shielding performance, are currently being investigated by others at MIT (13). The studies reported here embrace only

the depletion-economics considerations listed above, (a), (b), (c), (e), and (f). Further, the depletion-economics comparisons are biased toward the exotic moderating reflector: a solid reflector (100 % Be metal) is assumed. A real reflector would probably consist of clad BeO, with as much as 20 % required for coolant. Also, the costs of the Be reflector are not included in the tradeoff study.

Figure 5.1 shows the radial blanket fuel costs (mills/KWhe) as functions of irradiation time for the cases 1, 2, 1A, 2A, 1B, and 2B defined in Table 5.1. The reference economic environment, Tables 4.2 and 5.4, was assumed. Table 5.3 summarizes the optimum irradiation times, fuel costs, fissile plutonium breeding rates, and discharge fissile plutonium inventories in both reference and more favorable economic environments defined in Table 5.4. Figure 5.2 summarizes the radial blanket fuel costs (at optimum irradiation time) for the six combinations of radial blanket thickness and radial reflector material, under the two economic environments.

Several features are noted in the results presented in Figure 5.1 and Table 5.3.

(1) The importance of reflector material choice, with respect to blanket fuel economics, decreases with increased blanket thickness. Choice of beryllium results in an improvement in blanket revenue of about 60% over sodium, for a 15 cm radial blanket. For a 45 cm blanket, the improvement is only about 8%.

(2) For either sodium or beryllium reflectors, reducing the blanket thickness always reduces the plutonium bred by the blanket, i.e. effect (c) mentioned above is not sufficient to offset effect (b).

(3) Optimum irradiation time decreases with decreased radial blanket thickness.

FIG. 5.1 EFFECT OF RADIAL BLANKET THICKNESS AND RADIAL REFLECTOR MATERIAL ON RADIAL BLANKET FUEL ECONOMICS

TABLE 5.3

EFFECT OF RADIAL BLANKET THICKNESS AND RADIAL REFLECTOR MATERIAL ON RADIAL BLANKET FUEL ECONOMICS

Config- uration #	Radial Blanket Thickness (cm)	Radial Reflector Material	M_{49}/T @ T=2yr (kg/yr)	Reference Economic Environment			More Favorable Economic Environment		
				T_{opt} (yr)	$\bar{e}_{RB}$ @ T_{opt} (mills/KWHe)	M_{49} @ T_{opt} (kg)	T_{opt} (yr)	$\bar{e}_{RB}$ @ T_{opt} (mills/ KWHe)	M_{49} @ T_{opt} (kg)
1	45	Na	158	6-1/2	-0.037	825	3-1/2	-0.237	512
2	45	Be-metal	160	6-1/2	-0.040	845	3-1/2	-0.243	521
1A	30	Na	141	4-3/4	-0.058	596	2-1/2	-0.242	342
2A	30	Be-metal	157	4-1/2	-0.072	610	2-1/2	-0.279	380
1B	15	Na	97	3-1/2	-0.055	304	2	-0.188	194
2B	15	Be-metal	130	2-3/4	-0.087	308	1-1/2	-0.276	205

TABLE 5.4REFERENCE AND MORE FAVORABLE
ECONOMIC ENVIRONMENTS

	<u>Reference</u>	<u>More Favorable</u>
Radial Blanket Fabrication Cost, \$/kgHM	69	40
Radial Blanket Reprocessing Cost, \$/kgHM	31.50	31.50
Fissile Market Value, \$/kg	10,000	20,000
Discount Rate, %	8	8

FIG. 5.2 EFFECT OF ECONOMIC ENVIRONMENT ON OPTIMUM RADIAL BLANKET THICKNESS

(4) The effect of the choice of radial reflector material on the optimum irradiation time is more pronounced the thinner the blanket.

(5) Optimum irradiation time decreases as the economic environment improves.

(6) In the reference economic environment, the sodium reflected radial blanket displays a weak optimum thickness between 15 and 30 cm. For the beryllium reflected radial blanket, the optimum, if it exists, is between 0 and 15 cm, that is, the 15 cm beryllium reflected blanket is superior to the 30 and 45 cm blankets, owing to effect (c) above.

The increment of 15 cm is a representative thickness for a row of fuel subassemblies. LMFBR operators may have the option of adding or subtracting radial blanket rows in accordance with current economic conditions - fissile market value, fabrication and reprocessing costs, etc. Thicker radial blankets are indicated when (i) fabrication and reprocessing costs decreases, and/or (ii) fissile market value increases, as seen in Figure 5.2.

Figures 5.3 and 5.4 show the local neutronics obtained from the multi-group "snapshot" physics computations (2DB). In Figure 5.3 the beginning of fuel life capture reaction rate per U238 atom, $\sigma_c^{28} \phi$, is plotted along the radial blanket midplane ($z=0$). Breeding performance, for a particular case, may be associated qualitatively with the area under that case's $\sigma_c^{28} \phi$ vs. r curve. The improvement of the breeding performance of inner regions by reducing the blanket thickness, effect (c) above, is noted. This effect is quite weak for the sodium reflected blanket, e.g. $\sigma_c^{28} \phi$ in the inner 15 cm of radial blanket is insensitive to the location of the radial reflector. However, the improvement is quite pronounced for the Be reflected blanket, and the advantage of the Be reflector (over the Na reflector) is seen to increase as blanket thickness decreases.

FIG. 5.3 LOCAL U238 CAPTURE REACTION RATES ($\sigma_c^{28}\phi$) FOR VARIOUS RADIAL BLANKET THICKNESSES AND RADIAL REFLECTOR MATERIALS

FIG. 5.4 LOCAL NEUTRONICS IN THE RADIAL BLANKET

Figure 5.4 shows the resolution of the U238 capture rate into its components, σ_c^{28} and ϕ . Both reflective (blanket ϕ) and moderating (blanket σ_c^{28}) properties of Be are superior to those of Na.

5.4 RADIAL BLANKET FUEL MANAGEMENT SCHEMES

Case studies discussed elsewhere in this report assume that all radial blanket fuel sees the same irradiation time, the optimum for the blanket as a whole. From Figure 4.7, the optimum irradiation times of the annular regions range from about three years for the innermost region to about ten years for the outermost. At the "whole blanket" irradiation time of 6.5 years, the inner regions are past their optimum, while outer regions are under-exposed.

Clearly, an improvement in radial blanket performance would result if each annular region were irradiated to its own local optimum exposure. This scheme is labeled "regional" management in this report. Table 5.5 compares the radial blanket fuel energy costs (mills/KWHe) under whole blanket and regional management schemes. The regional scheme improves blanket profit by about 30%. Other advantages of the regional scheme, not implicit in Table 5.5, are power flattening and reduction of power buildup in the inner regions over an irradiation cycle.

Other fuel management schemes proposed for FBR radial blankets are:

- out-in (4)
- in-out (10, 73), and
- fuel assembly rotation (10)

The out-in scheme has been evaluated by Hasnain and Okrent (4) for a somewhat smaller reactor (800 liter core) than the reference reactor

TABLE 5.5WHOLE BLANKET VS. REGIONALFUEL MANAGEMENT SCHEMES

<u>Reactor Configuration</u>	<u>Radial Blanket Fuel Energy Costs (mills/KWHe)</u>	
	<u>Whole Blanket Management</u>	<u>Regional Management</u>
#1 (reference): 45 cm radial blanket; Na radial reflector	-0.0364	-0.0494
#2: 45 cm radial blanket; Be radial reflector	-0.0394	-0.0519

assumed in the present study. In this scheme, fresh fuel is loaded in the outermost blanket region, then moved inward in the following consecutive cycles, and discharged, finally, from the innermost region. The fuel economic advantage of out-in management, relative to batch and scatter schemes, is that uniformity of discharge composition may be achieved. However, Hasnain and Okrent found little fuel economic advantage in this scheme. A major engineering advantage is that local power density change with time is lessened, thereby reducing orificing control requirements. The out-in scheme would not, however, be expected to assist in power flattening.

Although it was not demonstrated quantitatively in their study, Hasnain and Okrent (4) argued that in-out management would be uneconomic, due to the holdup of bred plutonium. Froelich (10) has shown that in-out management has a strong power flattening effect, as well as reducing the power swing of radial blanket fuel over an irradiation cycle.

Froelich (10) has also shown that rotation of fuel assemblies implements power flattening and decreases power swing.

Advantages and disadvantages of various radial blanket fuel management options are summarized in Table 5.6.

5.5 SENSITIVITY OF FUEL ENERGY COSTS TO THE ECONOMIC ENVIRONMENT

5.5.1 Introduction

The purpose of this section is to examine the sensitivity of the reference LMFBR (Reactor #1) fuel energy costs (mills/KWhe) to changes in economic environment.

Unit costs (\$/kgHM, \$/kg fissile), credits (\$/kg fissile), and carrying charges throughout the nuclear fuel cycle are ultimately transferred to the utility company, burdened to the production of electricity, and, together

TABLE 5.6 RADIAL BLANKET FUEL MANAGEMENT SCHEMES

<u>Fuel Management Scheme</u>	<u>References</u>	<u>Advantages</u>	<u>Disadvantages</u>
1. Fixed-Fuel Schemes			
A. Batch and Scatter			
(1) Whole Blanket Management	(this report)	.simplicity	.non-uniform burnup .power tilt across blanket
(2) Regional Management	(69,70,71, 72,73), (this report)	.power flattening .more uniform burnup than whole blanket management; local optima achieved	. complexity
B. Batch	(this report)	.simplicity	.more severe power swing than scatter management
C. Scatter	(69,70,71, 72,73), (this report)	. reduces power swing	
2. Moving-Fuel Schemes			
A. Out-in	(4)	.uniform burnup .reduction in power swing	.aggravates power tilt across blanket
B. In-out	(10,73)	.uniform burnup .reduction in power swing .power flattening	

with utility company carrying charges, borne by the electricity consumer via a levelized price (cost) of electricity in mills/KWhe. These unit costs, credits, and carrying charges are determined largely by factors outside the scope of this report, e.g. supply-demand effects in the market place, technical-economic characteristics of fuel cycle processes, fuel processing capacities and throughputs, availability and structure of capital, fuel element design, etc. Thus, in this report, the following variables are regarded as comprising the "economic environment" and are treated in parametric fashion, as input to the depletion-economics calculations:

unit costs for fabrication and reprocessing, C_{fab} and C_{repr} , in dollars per kilogram of heavy metal (U + Pu);

nuclide market values, C_{28} , C_{49} , ..., C_{42} in dollars per kilogram of the respective nuclide;

utility company financial parameters i.e. tax rate (τ), capital structure (f_b , f_s) and rates of return (r_b , r_s) from which the discount rate (x) may be determined, $x = (1 - \tau)r_b f_b + r_s f_s$.

In the sensitivity studies reported below, the economic parameters were varied over the ranges shown in Table 5.7. Reference values are shown in parentheses. Reference values of C_{fab} and C_{repr} are typical of those projected for oxide-fueled LMFBR's by the USAEC Fuel Recycle Task Force (18), and of those assumed in the several 1000 MWe LMFBR design studies. Reference values of the utility company financial parameters, leading to a reference discount rate of 8%, are typical of those of the Commonwealth Edison Company in the late 1960's (82).

The fuel energy costs (mills/KWhe) associated with the major regions (core, axial blanket, radial blanket) were computed by the procedure

TABLE 5.7

RANGES OF ECONOMIC ENVIRONMENT PARAMETERS *

Unit Processing Costs [\$/kgHM]

Fabrication (C_{fab})

Core	150-(314)-330
Axial Blanket	20-(80)-314
Radial Blanket	20-(69)-100

Reprocessing (C_{repr})

Core	15-(31)-60
Axial Blanket	15-(31)-60
Radial Blanket	15-(31)-60

Nuclide Market Values [\$/kg]

Fertile (C_{28}, C_{40})	0
Fissile (C_{49}, C_{41})	5000-(10,000)-25,000

Utility Company Financial Parameters

Income Tax Rate (τ)	(0.5)
Discount Rate (x)	0.06-(0.08)-0.10

* Reference values are given in parentheses ().

outlined in Section 4.1. Each economic parameter was varied individually, holding all other parameters fixed at their reference values.

5.5.2 Core and Axial Blanket

Figures 5.5 through 5.12 display the behavior of reference reactor core and axial blanket fuel energy costs as the economic environment is varied around the reference environment.

Since the core and axial blanket irradiation times are fixed by the core burnup limit, core and axial blanket fuel energy cost equations can be reduced to convenient linear forms. From Equation (4-6), the fuel energy cost associated with region s (core, axial blanket, or radial blanket) is given by

$$\begin{aligned}\bar{e}_s &= \bar{e}_{fab,s} + \bar{e}_{repr,s} + \bar{e}_{mat'l,s} \\ &= \frac{1000 M_{HM}^0}{ET} [C_{fab,s} F_{fab}(T) + C_{repr,s} F_{repr}(T) \\ &\quad + C_{fiss} (\epsilon_s^0 F_{mp}(T) - \epsilon_s(T) F_{mc}(T))] \quad (5-1)\end{aligned}$$

With irradiation time (T) fixed, Equation (5-1) reduces to a linear expression in the unit costs $C_{fab,s}$, $C_{repr,s}$, C_{fiss} :

$$\bar{e}_s = a_{fab,s} C_{fab,s} + a_{repr,s} C_{repr,s} + a_{mat'l,s} C_{fiss} \quad (5-2)$$

where the constants $\{a_{q,s}\}$ are given by

$$a_{q,s} = \left(\frac{\bar{e}_{q,s}}{C_{q,s}} \right)_x = \left(\frac{\bar{e}_{q,s}}{C_{q,s}} \right)_0 = \frac{1000}{ET} M_{HM}^0 g(T) .$$

FIG. 5.5 EFFECT OF UNIT FABRICATION COST ON CORE FUEL ENERGY COST

FIG. 5.6 EFFECT OF UNIT REPROCESSING COST ON CORE FUEL ENERGY COST

FIG. 5.7 EFFECT OF FISSILE PLUTONIUM PRICE ON CORE FUEL ENERGY COST

FIG. 5.8 EFFECT OF DISCOUNT RATE ON CORE FUEL ENERGY COST

FIG. 5.9 EFFECT OF UNIT FABRICATION COST ON AXIAL BLANKET FUEL ENERGY COST

FIG. 5.10 EFFECT OF UNIT REPROCESSING COST ON AXIAL BLANKET FUEL ENERGY COST

FIG. 5.11 EFFECT OF FISSILE PLUTONIUM PRICE ON AXIAL BLANKET FUEL ENERGY COST

FIG. 5.12 EFFECT OF DISCOUNT RATE ON AXIAL BLANKET FUEL ENERGY COST

The subscript q denotes the cost component (fabrication, reprocessing, material); the subscript 0 denotes the reference economic condition, while x refers to an arbitrary economic condition.

Core and axial blanket irradiation time is set by the burnup limit of the core, in this case, 2 years, corresponding to an average core burn-up of 102,000 MWD_t/MT. Equation (5-2) for the core is

$$\begin{aligned}\bar{e}_{\text{core}} = & 1.09 \times 10^{-3} C_{\text{fab,core}} + 0.784 \times 10^{-3} C_{\text{repr,core}} \\ & + 0.06027 \times 10^{-3} C_{\text{fiss}}\end{aligned}\quad (5-3)$$

For the axial blanket,

$$\begin{aligned}\bar{e}_{\text{AB}} = & 0.872 \times 10^{-3} C_{\text{fab,AB}} + 0.625 \times 10^{-3} C_{\text{repr,AB}} \\ & - 0.02307 \times 10^{-3} C_{\text{fiss}}\end{aligned}\quad (5-4)$$

A "sensitivity coefficient", $(A_{q,s})_0$, is defined, to measure the sensitivity of $\bar{e}_s$ to changes in parameter $C_{q,s}$ from its reference value, all other parameters remaining fixed at their reference values:

$$\begin{aligned}(A_{q,s})_0 &= \left(\frac{C_{q,s}}{\bar{e}_s} \right)_0 \frac{\partial \bar{e}_s}{\partial C_{q,s}} \\ &= \frac{\Delta \bar{e}_s / (\bar{e}_s)_0}{\Delta C_{q,s} / (C_{q,s})_0} \\ &= \left(\frac{C_{q,s}}{\bar{e}_s} \right)_{a_{q,s}} = \left(\frac{\bar{e}_{q,s}}{\bar{e}_s} \right)_0\end{aligned}\quad (5-5)$$

Values of $(A_{q,s})_0$ for the core and axial blanket are summarized in Table 5.8. For both the core and axial blanket, the material (fissile) component dominates, i.e. energy costs are most sensitive to C_{fiss} . Fabrication is the next most important component, followed by reprocessing.

Changes in $\bar{e}_s$ due to simultaneous changes in several economic parameters may be computed by linear superposition:

$$\Delta \bar{e}_s = a_{fab,s} \Delta C_{fab,s} + a_{repr,s} \Delta C_{repr,s} + a_{matl,s} \Delta C_{fiss}$$

or

(5-6)

$$\frac{\Delta \bar{e}_s}{\bar{e}_s} = A_{fab,s} \frac{\Delta C_{fab,s}}{C_{fab,s}} + A_{repr,s} \frac{\Delta C_{repr,s}}{C_{repr,s}} + A_{fiss} \frac{\Delta C_{fiss}}{C_{fiss}}$$

5.5.3 Radial Blanket

Figures 5.13 through 5.16 display the behavior of radial blanket fuel power costs as the economic environment is varied around the reference environment. The optimum irradiation time (T_{opt}) is seen to decrease as C_{fiss} increases, as $C_{fab,RB}$ decreases, as $C_{repr,RB}$ decreases, or as the discount rate (x) increases.

Unlike the axial blanket, the radial blanket may be fuel-managed independently of the core. The radial blanket may thus be irradiated to its optimum exposure, which occurs somewhat beyond the two year core burnup limit. One is concerned, then, with the sensitivity of the optimum radial blanket fuel energy cost, $(\bar{e}_{RB})_{T_{opt}}$, to the economic environment.

The optimum irradiation time, T_{opt} , is an implicit function of the economic environment. Hence, the fuel energy cost expression (5-1) does not reduce exactly to a linear form (5-2) in the unit costs C_{fab}, C_{repr} ,

TABLE 5.8

CORE AND AXIAL BLANKET SENSITIVITY COEFFICIENTS, $(A_{q,s})_0^*$

$q \downarrow \quad s \rightarrow$	<u>Core</u>	<u>Axial Blanket</u>
Fabrication	0.357	-0.495 **
Reprocessing	0.025	-0.140 **
Material	<u>0.628</u>	<u>1.635</u>
	1.000	1.000

$$* (A_{q,s})_0 = \frac{\bar{e}_s / (\bar{e}_s)_0}{C_{q,s} / (C_{q,s})_0}$$

** These terms are negative because $(\bar{e}_{AB})_0$ is negative.

FIG. 5.13 EFFECT OF UNIT FABRICATION COST ON RADIAL BLANKET FUEL ENERGY COST

FIG. 5.14 EFFECT OF UNIT REPROCESSING COST ON RADIAL BLANKET FUEL ENERGY COST

FIG. 5.15 EFFECT OF FISSILE PLUTONIUM PRICE ON RADIAL BLANKET FUEL ENERGY COST

FIG. 5.16 EFFECT OF DISCOUNT RATE ON RADIAL BLANKET FUEL ENERGY COST

C_{fiss} as is the case for core and axial blanket. However, plots of $(\bar{e}_{RB})_{Topt}$ versus the unit costs, Figures 5.17, 5.18 and 5.19 show that $(\bar{e}_{RB})_{Topt}$ is nearly linear in C_{fab} , C_{repr} , and C_{fiss} .

For small perturbations in the economic environment (ΔC) the assumption of linearity is especially good. This is shown by the broken lines in Figures 5.17, 5.18 and 5.19, which represent linearizations at the reference conditions, e.g.

$$(\bar{e}_{RB})_{Topt} = \left(\frac{\bar{e}_{fab,RB}}{C_{fab,RB}} \right)_{0,Topt} C_{fab,RB} + (\bar{e}_{repr,RB})_{0,Topt} + (\bar{e}_{matl,RB})_{0,Topt} \quad (5-7)$$

The constants of proportionality, $\left(\frac{\bar{e}_{q,RB}}{C_{fab,RB}} \right)_{0,Topt}$, in units of 10^{-3} kg/KWhe are

$$\left(\frac{\bar{e}_{fab,RB}}{C_{fab,RB}} \right)_{0,Topt} = 1.154 \times 10^{-3}$$

$$\left(\frac{\bar{e}_{repr,RB}}{C_{repr,RB}} \right)_{0,Topt} = 0.519 \times 10^{-3}$$

$$\left(\frac{\bar{e}_{matl,RB}}{C_{fiss,RB}} \right)_{0,Topt} = -0.0132 \times 10^{-3}$$

A composite linear form, analogous to Equation (5-2), valid near reference economic conditions, is

FIG. 5.17 SENSITIVITY OF OPTIMUM RADIAL BLANKET FUEL ENERGY COST TO UNIT FABRICATION COST

FIG. 5.18 SENSITIVITY OF OPTIMUM RADIAL BLANKET FUEL ENERGY COST TO UNIT REPROCESSING COST

FIG.5.19 SENSITIVITY OF OPTIMUM RADIAL BLANKET FUEL ENERGY COST TO FISSILE PLUTONIUM PRICE

$$\begin{aligned}
(\bar{e}_{RB})_{Topt} &= a_{fab,RB} C_{fab,RB} + a_{repr,RB} C_{repr,RB} + a_{matl,RB} C_{fiss} \\
&= \left(\frac{\bar{e}_{fab,RB}}{C_{fab,RB}} \right)_{0,Topt} C_{fab,RB} + \left(\frac{\bar{e}_{repr,RB}}{C_{repr,RB}} \right)_{0,Topt} C_{repr,RB} \\
&\quad + \left(\frac{\bar{e}_{matl,RB}}{C_{fiss}} \right)_{0,Topt} C_{fiss} \\
&= 1.154 \times 10^{-3} C_{fab,RB} + 0.519 \times 10^{-3} C_{repr,RB} \\
&\quad - 0.0132 \times 10^{-3} C_{fiss} . \tag{5-8}
\end{aligned}$$

Values of the radial blanket sensitivity coefficients, defined as in Section 5.5.2, are given in Table 5.9. Optimum radial blanket fuel energy cost is seen to be most sensitive to fissile price, and least sensitive to unit reprocessing cost.

5.5.4 Fissile Market Price and the Economic Potential of LMFBR Blankets

In Sections 5.5.2 and 5.5.3, core, axial blanket, and radial blanket fuel energy costs were found to be most sensitive to fissile market price (C_{fiss}). It was also shown that $\bar{e}_{core}$ and $\bar{e}_{AB}$, assuming a fixed irradiation time, are linear in the unit costs $\{C_{q,s}\}$ while $(\bar{e}_{RB})_{Topt}$ is approximately linear in the $\{C_{q,RB}\}$.

Figure 5.20 shows the reactor fuel energy costs as a function of C_{fiss} . The total reactor fuel energy cost is the sum of the energy costs associated with each region:

$$\bar{e}_{reactor} = \bar{e}_{core} + \bar{e}_{AB} + (\bar{e}_{RB})_{Topt} \quad \frac{\text{mills}}{\text{KWh}}$$

TABLE 5.9

RADIAL BLANKET SENSITIVITY COEFFICIENTS*, $(A_{q, RB})_{0, Topt}$

q	$(A_{q, RB})_{0, Topt}$
Fabrication	-2.15 **
Reprocessing	-0.44 **
Material (fissile)	<u>+3.59</u>
	1.00

$$* (A_{q, RB})_{0, Topt} = \frac{(\Delta \bar{e}_{RB})_{Topt} / (\bar{e}_{RB})_{0, Topt}}{\Delta C_{q, RB} / (C_{q, RB})_{0, Topt}}$$

** These terms are negative because $(\bar{e}_{RB})_{0, Topt}$ is negative.

FIG. 5.20 EFFECT OF FISSILE PU PRICE ON TOTAL REACTOR FUEL ENERGY COST

The values of $(\bar{e}_{RB})_{Topt}$ used in constructing Figure 5.20 were taken from actual computed results, Figures 5.19 and 5.15, rather than the linear approximation, Equation (5-8).

Figure 5.20 displays several features:

- (a) $\bar{e}_{reactor}$ increases with C_{fiss} despite the fact that the reactor produces more fissile material than it consumes. This is due to the high core fissile inventory cost. An increase in C_{fiss} results in an increase in net direct material revenue, but this advantage is overcome by the increased core inventory costs.
- (b) The axial blanket is more profitable than the radial blanket, because the axial blanket sees more neutrons ($H/D=0.4$). A two enrichment zone core configuration (Reactor 1', Section 4.4), of course, upgrades radial blanket performance by enhancing radial leakage.
- (c) Below $C_{fiss} \approx 8$ \$/gm, the blankets are of marginal importance. As the fissile price increases, the blankets become more viable, substantially offsetting the high core inventory cost.
- (d) The axial blanket breakeven point, from Figure 5.20, occurs at 3.88 \$/gm. This is confirmed by solving Equation (5-4) for C_{fiss} with $\bar{e}_{AB} = 0$.
- (e) The radial blanket breakeven point, from Figure 5.20, occurs at 7.25 \$/gm. This agrees with the solution of the linear approximation, Equation (5-8), with $(\bar{e}_{RB})_{Topt} = 0$.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

In the preceding chapters, a variety of specific conclusions have been reached in regard to a broad spectrum of FBR fuel economic questions. An FBR fuel depletion-economics model was developed and applied to a number of 1000 MWe LFBR case studies. In this chapter, the specific conclusions, in both the methods development and application phases, are reviewed as a prelude to a discussion of broader issues and recommendations as to the scope and direction of future work.

6.1 CONCLUSIONS

The major conclusions of this study are summarized below.

Depletion-Economics Methods

1. Blankets impose several unique accounting problems: blanket fuel appreciates with irradiation, thus raising certain income tax questions; and the long irradiation times in the radial blanket make the accounting treatment of blanket carrying charges important. Two methods of treating post-irradiation transactions were compared:

Method A. Tax the revenue from sale of fissile material (material credit) as ordinary income, along with electricity revenue; treat reprocessing costs as tax deductible expenses in the year in which they occur;

Method B. Capitalize fissile revenue and reprocessing costs.

Method A results in significantly lower values of levelized fuel costs (mills/KWhe). While the choice between fuel cost accounting methods

has a significant effect on absolute values of energy costs, it does not distort comparative and incremental results, e.g. design rankings, optimization of radial blanket residence times, etc. However, choice of method B would lead to the selection of thinner blankets, since under method B more of the radial blanket is unprofitable.

The question of whether or not to tax fissile revenue must ultimately be resolved by taxing authorities or by common usage. Fast breeder reactors have two major products: electricity and fissile material. Thus it would appear that fissile revenue should properly be taxed as ordinary income, along with the revenue from the sale of electricity. For this reason, method A is recommended for future studies.

2. Several effects complicate the physics-depletion of FBR blankets: spectrum softening with distance from the core-blanket interface; spectrum hardening and flux shift with irradiation; and heterogeneity effects. The two exposure dependent effects - spectrum hardening and flux shift - influence the blanket breeding rate in opposing directions. The flux shift effect dominates. A single multigroup physics computation, to obtain the flux shape and local spectra for depletion calculations, is sufficient for evaluating blanket/reflector design changes, scoping studies, and sensitivity studies. The major source of error in computing fissile buildup by this procedure is the assumption of constant flux over an irradiation cycle.

Corrections (to the U238 capture cross section) for blanket heterogeneity effects influence computed fissile production in two opposing ways: (a) reduction in σ_c^{28} , tending to reduce the bred fissile inventory; and (b) reduction in neutron attenuation, tending to increase blanket fluxes and increase breeding rate. Of these effects, (a) dominates, and heterogeneity leads to a net adverse effect on blanket breeding. For a

typical radial blanket, heterogeneity corrections lead to a reduction in discharge fissile inventories of about 10%.

It was also found that core and axial blanket fuel costs are insensitive to radial blanket/reflector configuration changes.

1000 MWe LMFBR Case Studies

3. Substitution of a moderating reflector (e.g. Be) for the outer radial blanket row/s can improve overall radial blanket economic performance significantly. Choice of radial reflector material, e.g. Be vs. Na has little effect on the fuel economics of thick (~ 45 cm) radial blankets. The relative advantage of a moderating reflector increases as the reflector is moved nearer the high flux zones of the blanket, that is, as the blanket thickness decreases.

4. Reducing blanket thickness (by replacing outer blanket regions with reflector) reduces the bred fissile inventory of the blanket, that is, the plutonium forfeited in the region eliminated is greater than the additional plutonium bred in the remaining region due to its improved breeding performance ($\sigma_c^{28}\phi$). This loss of plutonium revenue is opposed by savings in fabrication and reprocessing costs of the blanket elements eliminated. Radial blanket thickness optimization is weak, i.e. net blanket revenue does not display a sharp peak as radial blanket thickness is reduced from 3 rows to 2 rows to 1 row.

5. Core enrichment zoning, in addition to its advantages to core fuel economics, significantly enhances breeding in the radial blanket, and can increase net radial blanket revenue by a factor of two or more.

6. Optimum radial blanket fuel residence times increase with distance from the core, ranging from approximately 2 years for fuel assemblies nearest the core to about 10 years for fuel in the outer row of a 45 cm

blanket. Thus, if the blanket is irradiated to a single irradiation time (whole blanket management), the optimum irradiation time for the blanket as a whole, then inner blanket fuel is overexposed and the outer blanket fuel is underexposed. Significant improvement ($\sim 30\%$ increase in net blanket revenue) results from irradiating each annular region to its own, local optimum irradiation time.

7. For a fixed irradiation time set by the burnup limit of core fuel, core and axial blanket fuel costs (mills/KWHe) are simple linear functions of the unit costs for fabrication, reprocessing ($\$/\text{kgHM}$) and fissile material ($\$/\text{kg fissile}$). Core and axial blanket fuel costs are most sensitive to fissile value. Fabrication is the next most important component.

Unlike the axial blanket, the radial blanket may be fuel-managed independently of the core, and thus may be irradiated to its optimum exposure which occurs somewhat beyond core residence times. The radial blanket fuel cost (or net revenue) at its optimum irradiation time is an implicit function of the unit costs for fabrication, reprocessing and fissile material, as is the optimum irradiation time. Thus the optimum radial blanket fuel cost is not exactly linear in the unit costs, as is the case for core and axial blanket. However, both the optimum irradiation time and the corresponding radial blanket fuel cost (or net revenue) are approximately linear functions of the unit costs. Optimum radial blanket fuel cost (mills/KWHe) is most sensitive to fissile price.

For increased fissile prices, both blankets (axial and radial) become more important in offsetting the increased core fissile inventory costs.

6.2 RECOMMENDATIONS

1. Blanket Power and Blanket Seeding

The availability of neutrons for fertile-to-fissile conversion is a key factor in blanket economics. Deep in the blanket (far from the core), where neutrons are relatively scarce, the conversion rate may be too low to overcome the rates of increase of fabrication and fissile carrying charges with residence time, resulting in net positive costs for fuel assemblies there. Thus design or fuel management options which tend to increase blanket flux have the potential of enhancing blanket economics.

Overall reactor fuel economic performance, represented by the net total fuel costs of core and blankets (mills/KWHe), is improved by increasing blanket power. This statement assumes that the maximum local power density in the core is held fixed.

A scheme which increases both blanket multiplication (and hence blanket flux) and blanket power is the inclusion of fissile material in blanket load fuel, or blanket seeding. Depending on the orificing scheme adopted, other advantages of blanket seeding are increased coolant exit temperatures (reducing the mixed mean temperature degradation) and reduced blanket power-swing over an irradiation cycle. Another incentive for seeding, or some equivalent accommodation, is the desirability of modifying a given blanket so that the theoretically optimum blanket thickness matches the discrete thickness allowed by fixed subassembly dimensions. Blanket seeding has a number of disadvantages, including increased inventory cost, increased pumping power requirements, decreased fertile material load available for conversion, and, in the case of uranium seeding, additional processing charges, for enrichment.

Table 6.1 summarizes the advantages and disadvantages of blanket seeding. A parametric study to examine these tradeoffs in a more quantitative manner is recommended.

Other investigators (12) have shown that reducing the load fissile enrichment from that of natural uranium (0.7% U235) to zero has negligible effect on blanket breeding. Thus substitution of natural uranium for enrichment plant tails ($\sim 0.2\%$ U235) would offer no improvement in blanket breeding.

Figure 3.11 shows that the local breeding ratio decreases from greater than unity to below unity as load enrichment is increased from zero (blankets) to about 14% (core), because of the enhanced competition for neutrons by the fissile species. At some intermediate point, the local blanket breeding ratio will be unity. Offsetting the diminished local blanket breeding ratio is the increased blanket flux, which tends to increase net blanket revenue at the local optimum irradiation time and to decrease the local optimum irradiation time.

2. Radial Blanket Fuel Management

The literature search, Appendix D, failed to render a comprehensive, comparative evaluation of alternate fuel management strategies for a fixed reactor configuration. This is perhaps due to the complexity of fuel management calculations and the lack, until recently, of computer programs flexible enough to survey all feasible options with ease. Several recently developed programs promise to be useful for FBR fuel management studies: PHENIX (31), REBUS (32), FUMBLE (45), and ZDECOST (17). A comparative evaluation of scatter, batch, out-in, and in-out equilibrium radial blanket fuel management schemes is recommended. Table 6.2 gives qualitative advantages and disadvantages of these options.

TABLE 6.1 ADVANTAGES AND DISADVANTAGES OF BLANKET SEEDING

ADVANTAGES

- . Increased blanket multiplication (more neutrons available for fertile-to-fissile conversion)
- . "Bonus power". For a fixed local power density limit in the core, increased reactor power.
- . Reduced mixed mean temperature degradation from blanket coolant.
- . Decreased power swing over an irradiation cycle.

DISADVANTAGES

- . For the same fuel volume fraction, slightly less fertile material available for conversion.
- . Increased blanket coolant pumping requirements.
- . Increased fissile inventory costs.
- . In the case of U235, a possible increase in processing costs (enrichment).

TABLE 6.2 RADIAL BLANKET FUEL MANAGEMENT SCHEMES

<u>Fuel Management Scheme</u>	<u>References</u>	<u>Advantages</u>	<u>Disadvantages</u>
1. Fixed-Fuel Schemes			
A. Batch and Scatter			
(1) Whole Blanket Management	(this report)	.simplicity	.non-uniform burnup .power tilt across blanket
(2) Regional Management	(69), (70), (71), (72), (73), (this report)	. power flattening .more uniform burnup than whole blanket management; local optima achieved.	.complexity
B. Batch	(this report)	.simplicity	.more severe power swing than scatter management.
C. Scatter	(69), (70), (71), (72), (73), (this report)	.reduces power swing	.complexity
2. Moving-Fuel Schemes			
A. Out-in	(4)	.uniform burnup .reduction in power swing	.aggravates power tilt across blanket .large amount of handling per fuel element

TABLE 6.2 (continued)

<u>Fuel Management Schemes</u>	<u>References</u>	<u>Advantages</u>	<u>Disadvantages</u>
B. In-out	(10), (73)	<ul style="list-style-type: none">. uniform burnup. reduction in power swing. power flattening	<ul style="list-style-type: none">. large amount of handling per fuel element

Once the reactor is in operation, the radial blanket offers the major remaining area of fuel management flexibility. The operator may tailor its size (or number of rows of subassemblies), load composition (enrichment in fissile plutonium or uranium, or possibly the use of fertile thorium), and fuel management scheme to current and projected economic environments. Thus, for example, if higher plutonium prices and/or lower fabrication and reprocessing costs are forecast, he may decide to add a row of subassemblies. Under other conditions, he may elect to alter the radial reflector design, include moderating material in the radial blanket, etc. A recent study (17) has addressed the general problem of optimizing FBR fuel management options in a variable economic environment. Continued effort in this area is recommended, with the aim of parameterizing and simplifying fuel management decision-making.

3. FBR Fuel Management and Design

In-reactor fuel management, particularly that of the blanket, is commonly treated as an after-thought in reactor design. The reactor is designed first; then fuel management is optimized, subject to the configuration selected and its engineering limitations and constraints. Ideally, the tasks of design and of setting the fuel management scheme should be intimately coupled, with the goal of reaching a more "global" optimum.

For example, the power swing (and hence orificing requirements) over an irradiation cycle in a radial blanket region, i , may be diminished by decreasing the fraction, g_i , of fuel replaced in that region per refueling event. This, in turn decreases the region throughput, increases the irradiation time, and increases the fissile inventory in the region, which may affect the net blanket fuel revenue adversely. Similarly, a degree of power flattening across the blanket can be achieved

by dividing the radial blanket into annular regions and assigning them refueling fractions (g_1 , g_2 , ...) which decrease with distance from the core. The design benefits of power shaping should be included explicitly in the analysis of fuel management strategies.

Whatever aspects of blanket fuel management are subjected to further scrutiny should be approached on a more global basis, at the minimum taking into consideration the strong interaction of management schemes and the flow orificing pattern adopted. x

Finally, since unit sizes are projected to increase to 2000 MWe and beyond after the year 2000, a more thorough parametric study of blanket performance versus reactor rating is recommended. The reactor size-blanket fuel economics preliminary study in Appendix B identifies some of the design-fuel management issues which should be addressed by such an effort.

APPENDIX A - NOMENCLATURE

Chapter 2 - Nomenclature

Subscripts, Superscripts and Abbreviations.

q	cost component index; mp, fab, repr or mc
mp	material purchase
fab	fabrication
repr	reprocessing
mc	material credit
28	U238
49	Pu239
40	Pu240
41	Pu241
42	Pu242
HM	heavy metal (U + Pu)
CFM	cash flow method
CIM	compound interest method
SIM	simple interest method

Levelized Cost (Price) of Electricity

$\bar{e}$	levelized cost (price) of electricity associated with fuel.	$\frac{\text{mills}}{\text{KWH}_e}$
	Depending on context, the symbol $\bar{e}$ denotes:	
	*total reactor levelized fuel cost (sum over all fuel streams)	
	*levelized fuel cost associated with a given fuel stream (sum over the cost components, q, of the fuel stream).	

Unit Costs

C_{fab}	unit fabrication cost for a given type fuel	$\frac{\$}{\text{kgHM}}$
------------------	---	--------------------------

C_{repr}	unit reprocessing cost for a given type fuel	$\frac{\$}{\text{kgHM}}$
C_j	market value of isotope j	$\frac{\$}{\text{kg}}$

Annual Quantities for CFM Derivation

E_j	electrical energy generated by the plant, year j	KWHe
V_j	taxable revenue from sources other than the sale of electricity, year j	\$
O_j	tax deductible cost, year j	\$
D_j	depreciation for tax purposes, year j	\$
V'_j	non-taxable revenue, year j	\$
Z_j	capitalized cost (new capitalization), year j	\$
Y_j	book value (liability to investors) in effect during year j	\$
T_j	income tax, year j	\$
$w(j)$	discount factor, $(1+x)^{-j}$	

Costs Associated with a Fuel Lot

z_m^q	direct cost, cost component q, fuel lot m	\$
$(z_m^q)^{**}$	carrying charge associated with cost component q, fuel lot m	\$
$(z_m^q)^*$	total cost associated with component q, fuel lot m;	\$
$= z_m^q + (z_m^q)^{**}$		

Time

t_m^q	time from beginning of plant life to transaction q, fuel lot m	yr.
t_m	time from beginning of plant life to irradiation midpoint of fuel lot m	yr.
T_m	irradiation time of fuel lot m	yr.
T_m^q	pre-irradiation time, component q, fuel lot m; time between transaction q and start of irradiation	yr.
$T_m'^q$	post-irradiation time, component q, fuel lot m; time between end of irradiation and transaction q	yr.
T_m^q	time between transaction q of lot m, and midpoint of irradiation	yr.

Financial Parameters

x	discount rate
f_b	fraction of capital from bondholders
f_s	fraction of capital from stockholders
r_b	bondholders' rate of return
r_s	stockholders' rate of return
r	income tax rate
F_m^q	carrying charge factor associated with cost component q, fuel lot m; $(z_m^q) * / z_m^q$
y_q	carrying charge rate associated with cost component q

Fuel Composition

$M_{28}(T),$ $M_{49}(T)...$	Mass of indicated nuclide after irradiation	kg
$M_{28}^0,$ $M_{49}^0,...$	Initial mass of indicated nuclide	kg

$\epsilon(T)$ Fissile concentration after irradiation time T,

$$M_{49}(T) + M_{41}(T)/M_{HM}^0$$

ϵ_0 Initial fissile concentration (enrichment),

$$M_{49}^0 + M_{41}^0 / M_{HM}^0$$

Chapter 3 - Nomenclature

$\sigma_{k,e}^j$ microscopic cross-section for event e,
energy group k, nuclide j barns

σ_e^j spectrum averaged one group cross-section,
event e, nuclide j barns

$$\sum_{k=1}^n \phi_k \sigma_{k,e}^j / \sum_{k=1}^n \phi_k$$

ϕ_k neutron flux in energy group k $n/cm^2 \text{ sec}$

ϕ total flux, $n/cm^2 \text{ sec}$

$$\sum_{k=1}^n \phi_k$$

N_j atom density, nuclide j atoms/cm^3

N_j^0 initial atom density, nuclide j atoms/cm^3

θ flux time
 $\equiv \int^t \phi(t') dt'$

Appendix B - Nomenclature

R_c core radius cm

V_c core volume liter

B_c^2 critical core buckling cm^{-2}

ϵ_c critical core enrichment

b_i	internal breeding ratio, $\frac{\text{fissile production rate in core}}{\text{fissile destruction rate in core}}$	
b_x	external breeding ratio, $\frac{\text{fissile production rate in blanket}}{\text{fissile destruction rate in core}}$	
$\theta_{c,ox}$	oxide volume fraction in core	
$\theta_{b,ox}$	oxide volume fraction in blanket	
f	fraction of reactor thermal power produced in blanket	
$\Delta M_{c,49}(T_c)$	change in core Pu239 inventory during core irradiation time T_c	kg
$M_{c,49}^0$	initial core Pu239 inventory	kg
$\Delta M_{b,49}(T_b)$	change in blanket Pu239 inventory during blanket irradiation time T_b	kg
$\bar{e}_{c,proc}$	levelized core processing (fabrication + reprocessing) cost, including carrying charges	<u>mills</u> KWH _e
$\bar{e}_{b,proc}$	levelized blanket processing (fabrication + reprocessing) cost, including carrying charges	<u>mills</u> KWH _e
$\bar{e}_{c,BU}$	levelized core burnup (direct material) cost, including material purchase and material credit	<u>mills</u> KWH _e
$\bar{e}_{c,Inv.}$	levelized core inventory cost (material carrying charge)	<u>mills</u> KWH _e
$\bar{e}_{b,mat}$	levelized blanket material cost (revenue), including carrying charge	<u>mills</u> KWH _e
$\bar{e}_c$	total levelized core fuel cost, $\bar{e}_{c,proc} + \bar{e}_{c,BU} + \bar{e}_{c,Inv.}$	<u>mills</u> KWH _e
$\bar{e}_b$	total levelized blanket fuel cost $\bar{e}_{b,proc} + \bar{e}_{b,mat}$	<u>mills</u> KWH _e
$\bar{e}$	total levelized reactor fuel cost $\bar{e}_c + \bar{e}_b$	<u>mills</u> KWH _e

APPENDIX B

REACTOR SIZE AND BLANKET FUEL ECONOMICS

B.1 INTRODUCTION

Very large reactors (> 1000 MWe), advantaged by economics of scale, have been predicted. For example, the EEI "Fast Breeder Reactor Report" (58) suggests that light water reactors and fast breeder reactors may have unit ratings in excess of 2000 MWe by the year 1990. The purpose of this appendix is to examine relationships between FBR core size (or power rating) and FBR blanket fuel economics.

As core size is increased, (holding shape fixed), core neutron leakage (per core fission) and external breeding ratio decrease. Net blanket revenue (plutonium credit less fabrication and reprocessing costs), per unit of power, decreases. At the same time, core critical enrichment decreases, resulting in lower core fissile inventory costs. With a higher fertile concentration in the core, the internal breeding ratio is enhanced, diminishing the burnup (direct material) component of core fuel energy cost. Indeed, for a sufficiently large core, the internal breeding ratio exceeds unity and the burnup component becomes a revenue.

For these reasons, the economic importance of the blanket tends to decrease with reactor size. In fact, it may be worthwhile, for a sufficiently large core, to substitute a non-breeding reflector for the breeding blanket. This would eliminate fabrication and reprocessing costs, although this advantage may be offset by the cost of the added reflector. In addition, the core neutron economy may be improved, provided the reflector has superior neutronic properties compared to the breeding blanket,¹

1. Design studies for the FFTF (84) have shown that Ni is superior to a breeding blanket, as a core reflector.

resulting in even lower critical enrichment (lower core inventory cost) and higher internal breeding ratio (lower burnup cost). If the reflector is merely sodium coolant, it need not be fabricated or cooled in situ and blanket fabrication costs and pumping requirements are eliminated. Against these advantages of a non-breeding reflector must be weighed its obvious disadvantage - the loss of blanket plutonium revenue.

The arguments above are largely academic. First, the core shape was assumed to be held fixed as core size increases. In reality, high leakage geometries (e.g. pancake, annular) may be required for large cores to enhance the negative component of the sodium void coefficient. Another reason for spoiling large core geometry is to hold the internal breeding rate near unity, thus minimizing reactivity control requirements, control systems costs, and parasitic loss of neutrons available otherwise for breeding.

The purpose of this appendix is to examine some of these qualitative arguments in a semi-quantitative way. Using simple, one energy group, spherical geometry neutronics equations, the fuel economics of reactors with and without blankets were compared as core size (power rating) is increased. Three cases are considered:

Case A: Spherical core with a breeding blanket, assuming no Pu239 burnup in the blanket¹;

Case A*: Spherical core with a breeding blanket, corrected for Pu239 burnup in the blanket, blanket power fraction = 0.1¹;

Case B: Spherical core with a sodium reflector (no breeding blanket).

1. Case A*, a refinement of Case A, is included to examine the effect of blanket burnup on overall energy costs (mills/KWhe).

The spherical core size limit for a negative sodium void coefficient (50% sodium loss) have been determined in an extensive parametric study by Terasawa, et.al. (55). This limit is indicated in the results of Section B.3.

B.2 EQUATIONS

B.2.1 Summary

The neutron balance, depletion, and economics working equations used in this study are summarized in Table B.1. Major assumptions are listed in Table B.2. Table B.3 gives the region compositions and one group physics data. Table B.4 presents the economic data. Residence times and power-related parameters are given in Table B.5. The equations of Table B.1 are used as follows:

1. For a given core volume (V_c), the core critical buckling (B_c^2) is found by solving the transcendental critical Equation (B-4A) or (B-4B).
2. With this value of critical buckling, critical core enrichment (ϵ_c) is computed, using Equation (B-15). Critical mass ($M_{c,49}^0$) may also be computed at this time, from ϵ_c and V_c , using Equation (B-17).
3. The internal breeding ratio (b_i), a function of ϵ_c , is determined from Equation (B-20). The external breeding ratio (b_x), a function of both ϵ_c and B_c , is found from Equation (B-24).
4. Masses of Pu239 discharged from core and blanket ($M_{c,49}$, $M_{b,49}$) are found from Equations (B-36) and (B-37) respectively, with the known values of V_c , ϵ_c , b_i and b_x . This step may be by-passed since the economics equations incorporate (B-36) and (B-37).
5. Core and blanket levelized fuel energy costs are determined from Equations (B-48) through (B-78), with the values of ϵ_c , b_i and b_x

TABLE B.1 - SUMMARY OF WORKING EQUATIONS

Neutronic and Depletion Equations

(a) Critical Core Buckling:

$$6.24 V_C^{1/3} B_C \cot 6.24 V_C^{1/3} B_C = -0.3 V_C^{1/3} - 0.1915 \dots \text{(Case A, A*)} \dots \text{(B-4A)}$$

$$= -0.04815 V_C^{1/3} - 1.4561 \dots \text{(Case B)} \dots \text{(B-4B)}$$

(b) Critical Core Enrichment:

$$\epsilon_c = 0.0651 + 62.76 B_C^2 \dots \text{(Case A, A*, B)} \dots \text{(B-15)}$$

(c) Critical Core Mass:

$$M_{c,49}^0 = 2.64 V_C \epsilon_c \dots \text{(Case A, A*, B)} \dots \text{(kg)} \dots \text{(B-17)}$$

(d) Breeding Ratios:

$$b_i = 0.1255 \left(\frac{1 - \epsilon_c}{\epsilon_c} \right) \dots \text{(Case A, A*, B)} \dots \text{(B-20)}$$

$$b_x = 76.19 B_C^2 / \epsilon_c \dots \text{(Case A)} \dots \text{(B-24)}$$

(e) Discharge Pu239 Mass

$$M_{c,49} = 2.64 V_C \epsilon_c + 0.3250 (b_i - 1) V_C \dots \text{(Case A, A*, B)} \dots \text{(kg)} \dots \text{(B-36)}$$

$$M_{b,49} = 0.975 b_x V_C \dots \text{(Case A)} \dots \text{(B-37)}$$

Cost Equations

(f) Core

$$\text{Burnup, } \bar{\epsilon}_{c,BU} = -1.16 (b_i - 1) \dots \text{(Case A, B)} \dots \text{(mills/KWHe)} \dots \text{(B-48)}$$

Inventory, $\bar{e}_{c,inv} = 3.335 \bar{e}_c + 0.1250 (b_i - 1) \dots \dots \dots$ (Case A,B) . . (mills/KWHe). (B-49)

Processing, $\bar{e}_{c,proc} = 0.3964 \dots \dots \dots$ (Case A,B) . . (mills/KWHe). (B-50)

(g) Blanket

Material, $\bar{e}_{b,mat} = -0.886 b_x \dots \dots \dots$ (Case A) . . . (mills/KWHe). (B-60)

Processing, $\bar{e}_{b,proc} = 0.2555 \dots \dots \dots$ (Case A) . . . (mills/KWHe). (B-61)

Corrections for Blanket Pu239 Burnup (Case A*)

(e)* Discharge Pu239 Mass (Blanket):

$M_{b,49}^* = 0.8 M_{b,49} = 0.78 b_x V_c \dots \dots \dots$ (Case A*) . . . (kg) (B-69)

(f)* Core Cost Equations:

$\bar{e}_{c,BU}^* = 0.9 \bar{e}_{c,BU} \dots \dots \dots$ (Case A*) . . . (mills/KWHe). (B-75)

$\bar{e}_{c,inv}^* = 0.9 \bar{e}_{c,inv} \dots \dots \dots$ (Case A*) . . . (mills/KWHe). (B-76)

$\bar{e}_{c,proc}^* = 0.9 \bar{e}_{c,proc} \dots \dots \dots$ (Case A*) . . . (mills/KWHe). (B-77)

(g)* Blanket Cost Equations:

$\bar{e}_{b,mat}^* = 0.72 \bar{e}_{b,mat} \dots \dots \dots$ (Case A*) . . . (mills/KWHe). (B-79)

$\bar{e}_{b,proc}^* = 0.9 \bar{e}_{b,proc} \dots \dots \dots$ (Case A*) . . . (mills/KWHe). (B-78)

Total Fuel Energy Cost Equations

Core: $\bar{e}_c = \bar{e}_{c,BU} + \bar{e}_{c,inv} + \bar{e}_{c,proc} \dots \dots \dots \text{(Case A,B)} \dots \text{(mills/KWHe)}$

$\bar{e}_c^* = \bar{e}_{c,BU}^* + \bar{e}_{c,inv}^* + \bar{e}_{c,proc}^* \dots \dots \dots \text{(Case A*)} \dots \text{(mills/KWHe)}$

Blanket: $\bar{e}_b = \bar{e}_{b,mat} + \bar{e}_{b,proc} \dots \dots \dots \text{(Case A)} \dots \text{(mills/KWHe)}$

$\bar{e}_b^* = \bar{e}_{b,mat}^* + \bar{e}_{b,proc}^* \dots \dots \dots \text{(Case A*)} \dots \text{(mills/KWHe)}$

Reactor: $\bar{e} = \bar{e}_c \dots \dots \dots \text{(Case B)} \dots \text{(mills/KWHe)}$

$\bar{e} = \bar{e}_c + \bar{e}_b \dots \dots \dots \text{(Case A)} \dots \text{(mills/KWHe)}$

$\bar{e}^* = \bar{e}_c^* + \bar{e}_b^* \dots \dots \dots \text{(Case A*)} \dots \text{(mills/KWHe)}$

Plant Power Rating

$Pe = 0.2 V_c \dots \dots \dots \text{(Case A,B)} \dots \text{(MWe)} \dots \dots \dots \text{(B-64)}$

$Pe^* = 0.222 V_c \dots \dots \dots \text{(Case A*)} \dots \text{(MWe)} \dots \dots \dots \text{(B-65)}$

(V_c in liters)

TABLE B.2 - ASSUMPTIONS

1. Geometry

- . Spherical core
- . Infinite outer region (for criticality calculation)
- . Blanket Volume = 3 x core volume (for blanket processing cost calculation)

2. For Case A, there is no Pu239 burnup in the blanket (blanket power fraction is zero).*

For Case A*, Pu 239 burnup in the blanket is accounted for (blanket power fraction is 0.1).*

3. Core rated power density is independent of core size.

4. No fissile material is loaded in the blanket.

5. Core and blanket have the same one group cross sections (no spectral effects).

6. No higher isotopes of Plutonium are considered.

7. Core enrichment is uniform (no zoned enrichment scheme).

8. Increased control requirements for internal breeding ratios substantially above unity are ignored.

9. There are no sodium void coefficient restrictions on the size of a spherical core.

* For blanket irradiation times near six years, U238 fission provides about 15% of blanket energy.

TABLE B.3 - REGION COMPOSITIONS AND ONE-GROUP DATA

<u>Region</u>	<u>Nuclide</u>	<u>Volume Fraction</u>	<u>Theoretical Atom Density ($\frac{\text{atom}}{\text{barn-cm}}$)</u>	<u>σ_a (barns)</u>	<u>$\nu \sigma_f$ (barns)</u>	<u>σ_t (barns)</u>	<u>η</u>	<u>Composite Parameters</u>
Core	U238(oxide)	0.3	0.024	0.35	0.14	7.65	0.4	$D_c = \text{core diff.}$ $C \text{ coeff} \approx 1.65 \text{ cm}$
	Pu239(oxide)		0.025	2.36	5.66	8.0	2.4	$\Sigma_{a,p} = \text{non-fuel}$
	Na	0.55	0.025	0.0016		3.29		absorption
	Fe	0.15	0.085	0.010		2.50		$\approx 0.0002 \text{ cm}^{-1}$
Blanket (Case A, A*)	U238(oxide)	0.6	0.024	0.35	0.14	7.65	0.4	$D_b = \text{blanket diff.}$ $b \text{ coeff.} \approx 1.96 \text{ cm}$
	Na	0.2	0.025	0.0016		3.29		
	Fe	0.2	0.085	0.010		2.50		$X_b = 0.04 \text{ cm}^{-2}$
Na Reflec- tor (case B)	Na	1.0	0.024	0.0016		3.29		$D_b = 4.05 \text{ cm}$ $X_b = 0.00314 \text{ cm}^{-2}$

TABLE B.5 - PLANT POWER RELATED PARAMETERS
AND BATCH FUEL TIMING

Rated Core Power Density	(q''')	500 kw_t/liter
Plant Load Factor	(L)	0.8
Net Thermal Efficiency	(η)	0.4
Fuel Irradiation Times		
core	(T_c)	2 years
blanket	(T_b)	6 years
Pre and Post-Irradiation Times (core and Blanket)		
material purchase	(T'_{mp})	0.5 years
fabrication	(T_{fab})	0.5
reprocessing	(T''_{repr})	0.5
material credit	(T''_{mc})	0.5

determined above.

B.2.2 Derivation of Equations

B.2.2.1 Neutronic and Depletion Equations

Critical Core Buckling

The one-group criticality equation for a spherical core surrounded by an infinite outer region is

$$B_c R_c \cot B_c R_c = 1 - \frac{D_b}{D_c} (X_b R_c + 1) \quad (\text{B-1})$$

where

$$B_c^2 = (\text{critical core buckling})$$

$$= \frac{k_{\infty} - 1}{L_c^2} = \frac{1}{D_c} [v \Sigma_{f,c} - \Sigma_{a,c}] ,$$

$$X_b^2 = \frac{1 - k_{\infty b}}{L_b^2} = -B_b^2 = \frac{1}{D_b} [\Sigma_{a,b} - v \Sigma_{f,b}] ,$$

$R_c \equiv$ core radius

$D_i \equiv$ diffusion coefficient, region i

$\Sigma_{f,i} \equiv$ macroscopic fission cross section, region i

$\Sigma_{a,i} \equiv$ macroscopic absorption cross section, region i

$v \equiv$ neutron yield per fission

subscript i = c (core)

= b (outer region)

The "outer region (b)" is either a breeding blanket (Case A) or a sodium reflector (Case B). In the latter case, the outer region is not a multiplying medium, and $X_b^2 = 1/L_b^2$, as usual.

The core radius expressed in terms of core volume (V_c) is given by

$$\begin{aligned} R_c &= (3/4 \pi V_c)^{1/3} \\ &= 6.24 V_c^{1/3} \end{aligned} \quad (B-2)$$

where V_c is in liters, and R_c is in centimeters. Equation (B-1) thus becomes

$$B_c (6.24 V_c^{1/3}) \cot B_c (6.24 V_c^{1/3}) = - \frac{D_b}{D_c} [X_b (6.24 V_c^{1/3}) + 1] \quad (B-3)$$

Using the data of Table 6.3, Equation (B-3) becomes

$$6.24 V_c^{1/3} B_c \cot(6.24 V_c^{1/3} B_c) = -0.3 V_c^{1/3} - 0.1915 \dots \text{Case A} \quad (B-4A)$$

$$= -0.04815 V_c^{1/3} - 1.4561 \dots \text{Case B} \quad (B-4B)$$

Critical Core Enrichment

For a given core volume, the transcendental Equation (B-4A) or (B-4B) is solved for critical core buckling, by trial and error, or graphically. An expression relating critical core enrichment (ϵ_c) and critical core buckling is developed below.

The core diffusion equation, for criticality, is

$$D_c \nabla^2 \phi - \Sigma_{a,c} \phi + \nu \Sigma_{f,c} \phi = 0 \quad (B-5)$$

or

$$\nabla^2 \phi + B_c^2 \phi = 0$$

where

$$B_c^2 = \frac{\nu \Sigma_{f,c} - \Sigma_{a,c}}{D_c}$$

or

$$-D_c B_c^2 - \Sigma_{a,c} + \nu \Sigma_{f,c} = 0 \quad (B-6)$$

But

$$\begin{aligned} \nu \Sigma_{f,c} &= \nu_{28} \Sigma_{f,c,28} + \nu_{49} \Sigma_{f,c,49} \\ &= \eta_{28} \Sigma_{a,c,28} + \eta_{49} \Sigma_{a,c,49} \end{aligned} \quad (B-7)$$

where the subscripts 28 and 49 denote U238 and Pu239 respectively. Substituting (B-7) into (B-6),

$$\eta_{28} \Sigma_{a,c,28} + \eta_{49} \Sigma_{a,c,49} = \Sigma_{a,c} + D_c B_c^2 \quad (B-8)$$

Letting

$$\Sigma_{a,c,p} \equiv \text{non-fuel, macroscopic absorption cross section in the core,}$$

the total core macroscopic cross section is given by

$$\Sigma_{a,c} = \Sigma_{a,c,28} + \Sigma_{a,c,49} + \Sigma_{a,c,p} \quad (B-9)$$

Substituting (B-9) into (B-8) one obtains

$$\begin{aligned} &(\eta_{28} - 1) \Sigma_{a,c,28} + (\eta_{49} - 1) \Sigma_{a,c,49} \\ &= \Sigma_{a,c,p} + D_c B_c^2 \end{aligned} \quad (B-10)$$

The heavy metals uranium and plutonium have approximately the same theoretical number densities in the form of oxides (Table B.3). Thus their macroscopic cross sections may be expressed in terms of enrichment, as follows:

$$\Sigma_{a,c,49} = N_{49}^* \epsilon_c \theta_{c,ox} \sigma_{a,49} \quad (B-11)$$

$$\Sigma_{a,c,28} = N_{28}^* (1 - \epsilon_c) \theta_{c,ox} \sigma_{a,28} \quad (B-12)$$

where

N_{49}^* , N_{28}^* $\equiv$ theoretical number densities of Pu239 and U238 in the form of oxides, atoms/barn-cm

$\epsilon_c \equiv$ critical core enrichment
 $\equiv$ Pu239 number density / (Pu239 number density + U238 number density)

$\theta_{c,ox} \equiv$ volume fraction of oxide in core (B-13)

$\sigma_{a,49} \equiv$ Pu239 microscopic absorption cross section, barns

$\sigma_{a,28} \equiv$ U238 microscopic absorption cross section, barns.

Substituting (B-11) and (B-12) into (B-10) and solving for critical core enrichment,

$$\epsilon_c = \frac{\Sigma_{a,c,p} + D_c B_c^2 - (\eta_{28} - 1) N_{28}^* \theta_{c,ox} \sigma_{a,28}}{(\eta_{49} - 1) N_{49}^* \theta_{c,ox} \sigma_{a,49} - (\eta_{28} - 1) N_{28}^* \theta_{c,ox} \sigma_{a,28}} \quad (B-14)$$

Assuming the representative values

$$\Sigma_{a,c,p} = 0.0002 \text{ cm}^{-1}$$

$$\theta_{c,ox} = 0.3$$

and using the one group data of Table B.3, Equation (B-14) becomes

$$\epsilon_c = 0.0651 + 62.76 B_c^2 \quad (B-15)$$

Critical Core Mass (Pu239)

Equation (B-15) is used to determine the critical core enrichment, using the critical core buckling found from Equation (B-4A) or (B-4B). Critical core mass (Pu239), $M_{c,49}^0$, may be computed from enrichment as follows:

$$M_{c,49}^0 = \epsilon_c M_{c,HM} = \epsilon_c V_c \theta_{c,ox} \phi_{ox} C \quad (B-16)$$

where

M_{49}^0 $\equiv$ critical core fissile mass (kg)

ϕ_{ox} $\equiv$ oxide density ≈ 10 (kg HM oxide/liter HM oxide)

C $\equiv 0.88$ (kg IM/kg IM oxide)

$M_{c,HM}$ = mass of heavy metal (U + Pu) in the core (kg).

For an oxide volume fraction of 0.3,

$$\begin{aligned} M_{c,49} &= 2.64 V_c \epsilon_c \\ M_{c,HM} &= 2.64 V_c \end{aligned} \quad (B-17)$$

Breeding Ratios

The Pu239 inventories in discharged core and blanket fuel are to be expressed in terms of internal (core) and external (blanket) breeding ratios, respectively. Internal breeding ratio (b_i) is defined as

$$b_i \equiv \frac{\text{Pu239 production rate in the core}}{\text{Pu239 consumption rate in the core}} \quad (B-18)$$

Thus

$$b_i = \frac{N_{c,28} \sigma_{c,28}}{N_{c,49} \sigma_{a,49}} = \frac{1 - \epsilon_c}{\epsilon_c} \frac{\sigma_{c,28}}{\sigma_{a,49}} \quad (B-19)$$

where

$\sigma_{c,28}$ $\equiv$ U238 microscopic capture cross section

$\sigma_{a,49}$ $\equiv$ Pu239 microscopic absorption cross section.

Using the cross section data of Table B.3,

$$b_i = 0.1255 \frac{1 - \epsilon_c}{\epsilon_c} \quad (B-20)$$

The external breeding ratio (b_x) is defined as follows:

$$b_x \equiv \frac{\text{Pu239 production rate in the blanket}}{\text{Pu239 consumption rate in the core}} \quad (\text{B-21})$$

Thus,

$$b_x = \left(\frac{\text{core neutron leakage rate}}{\text{Pu239 consumption rate in core}} \right) \cdot \left(\frac{\text{U238 capture rate in the blanket}}{\text{core neutron leakage rate}} \right) \quad (\text{B-22})$$

Assuming no leakage from the outer face of the blanket, and no blanket multiplication,

$$b_x = \left(\frac{D_c B_c^2}{N_{49}^* \epsilon_c \theta_{c,ox} \sigma_{a,49}} \right) \cdot \left(\frac{\Sigma_{c,b,28}}{\Sigma_{a,b}} \right) \quad (\text{B-23})$$

where

$\Sigma_{c,b,28}$ = U238 macroscopic capture cross section in the blanket

$\Sigma_{a,b}$ $\equiv$ total macroscopic absorption cross section in the blanket

Using the data of Table B.3,

$$b_x = 76.19 \frac{B_c^2}{\epsilon_c} \quad (\text{B-24})$$

Discharge Pu239 Inventories

The change in core Pu239 inventory over the fuel lifetime in the core is given by

$$\Delta M_{c,49}(T_c) \equiv M_{c,49}(T_c) - M_{c,49}^0$$

$$\begin{aligned}
&= (\text{Pu239 atom production rate} - \\
&\quad \text{Pu239 atom consumption rate})_{\text{core}} \times T_c G \\
&= \left(\frac{\text{Pu239 atom production rate}}{\text{Pu239 atom consumption rate}} \right. \\
&\quad \times \frac{\text{Pu239 atom consumption rate}}{\text{total fission rate}} \\
&\quad \times \text{total fission rate} \Big)_{\text{core}} \times T_c G \\
&\quad - \left(\frac{\text{Pu239 atom consumption rate}}{\text{total fission rate}} \times \text{total fission rate} \right)_{\text{core}} \times T_c G \\
&= b_i \left(\frac{1 + \alpha_{49}}{1 + \delta_c} \right) \frac{(1-f) LQ}{EPF} T_c G \\
&\quad - \left(\frac{1 + \alpha_{49}}{1 + \delta_c} \right) \frac{(1-f) LQ}{EPF} T_c G \\
&= \left(\frac{1 + \alpha_{49}}{1 + \delta_c} \right) \frac{(1-f) LQ}{EPF} T_c G (b_i - 1), \quad \text{B-25) }
\end{aligned}$$

where

$\Delta M_{c,49}(T_c)$ $\equiv$ change in core Pu239 inventory over fuel life-time, kg

$M_{c,49}(T_c)$ $\equiv$ discharge Pu239 inventory, core, kg.

$\alpha_{49} \equiv \sigma_{c,49} / \sigma_{f,49}$

$\delta_c \equiv N_{c,28} \sigma_{f,28} / N_{c,49} \sigma_{f,49}$

$f \equiv$ fraction of reactor power produced in the blanket

$Q \equiv$ rated reactor thermal power, kw_t

$T_c \equiv$ fuel residence time in core, years

$L \equiv$ plant load factor

$G \equiv 39.67 \times 10^{-26}$ kg Pu239/atom Pu239

$EPF \equiv$ energy per fission $= 1.016 \times 10^{-21}$ kw-yr/fission

Evaluating the constant, G/EPF , and substituting

$$\Delta M_{c,49}(T_c) = \frac{1 + \alpha_{49}}{1 + \delta_c} (1-f) LQ T_c (b_i - 1) \times 3.9045 \times 10^{-4} \quad (B-26)$$

Letting

$$\frac{1 + \alpha_{49}}{1 + \delta_c} \simeq 1.04 \quad (B-27)$$

and assuming negligible blanket power,

$$f \simeq 0, \quad (B-28)$$

equation (B-26) becomes

$$\Delta M_{c,49}(T_c) = LQT_c (b_i - 1) \times 4.06 \times 10^{-4} \quad (B-29)$$

A similar development for the blanket yields

$$\Delta M_{b,49}(T_b) \equiv M_{b,49}(T_b) - M_{b,49}^0 \quad (B-30)$$

$$= \left[b_x(1-f) \left(\frac{1 + \alpha_{49}}{1 + \delta_c} \right) - f \left(\frac{1 + \alpha_{49}}{1 + \delta_b} \right) \right] T_b LQ \times 3.9045 \times 10^{-4}$$

where

$\Delta M_{b,49}(T_b)$ $\equiv$ change in blanket Pu239 inventory over fuel lifetime, kg

$M_{b,49}^0$ $\equiv$ load Pu239 inventory in blanket kg

$M_{b,49}(T_b)$ $\equiv$ discharge Pu239 inventory, blanket kg

$\delta_b \equiv N_{b,28} \sigma_{f,28} / N_{b,49} \sigma_{f,49}$

$T_b \equiv$ fuel residence time in blanket kg

Using Equations (B-27) and B-28), Equation (B-30) reduces to

$$\Delta M_{b,49}(T_b) = LQT_b b_x \times 4.06 \times 10^{-4} \quad (B-31)$$

Rated thermal power (Q) is given by

$$Q = q_c''' V_c$$

where q_c''' is the rated core thermal power density in kw_t/liter . Equations (B-29) and (B-31) become

$$\Delta M_{c,49}(T_c) = Lq_c''' V_c T_c (b_i - 1) \times 4.06 \times 10^{-4} \quad (B-32)$$

and

$$\Delta M_{b,49}(T_b) = Lq_c''' V_c T_b b_x \times 4.06 \times 10^{-4} \quad (B-33)$$

Using the values for q_c''' , L, T_c and T_b from Table B.5, Equations (B-32) and (B-33) become

$$\Delta M_{c,49} = 0.325 (b_i - 1) V_c \quad (B-34)$$

$$\Delta M_{b,49} = 0.975 b_x V_c \quad (B-35)$$

The Pu239 inventory changes during irradiation were defined above as

$$\Delta M_{c,49}(T_c) = M_{c,49}(T_c) - M_{c,49}^0$$

$$\Delta M_{b,49}(T_b) = M_{b,49}(T_b) - M_{b,49}^0$$

Combining these definitions with (B-34) and (B-35), using Equation (B-17) for $M_{c,49}^0$, and assuming that no Pu239 is loaded in the blanket, one obtains

$$M_{c,49} = 2.64 V_c \epsilon_c + 0.325 (b_i - 1) V_c \quad (B-36)$$

and

$$M_{b,49} = 0.975 b_x V_c \quad (B-37)$$

B.2.2.2 Economics Equations

The expressions for levelized unit energy costs (mills/KWhe) associated with regions under batch management are derived in Chapter 2. The following cost components are identified with each region: material purchase, fabrication, reprocessing, and material credit. Each of these components are further subdivided into direct and carrying charge sub-components, such that for region "s", component "q",

$$(\text{cost})_{s,q} = (\text{direct cost})_{s,q} F_q(T_s) \quad (B-38)$$

$$\begin{aligned} (\text{carrying charge})_{s,q} &= (\text{direct cost})_{s,q} (F_q(T_s) - 1) \\ &= (\text{direct cost})_{s,q} f_q(T_s) \end{aligned} \quad (B-39)$$

where $F_q(T_s)$ is a carrying charge factor emerging from the levelizing process and T_s is the residence time of region "s" fuel. Expressions for the carrying charge factors are derived in terms of utility company financial parameters in Chapter 2.

For the purposes of this study, the cost components and subcomponents are re-aggregated as follows:

(1) core direct material components (direct material purchase and direct material credit) are combined to form the core "burnup" or "depletion" component, $\bar{e}_{c,BU}$ (mills/KWhe);

(2) carrying charges associated with the core material components (material purchase and material credit) are combined to form the core "inventory" component, $\bar{e}_{c,inv}$ (mills/KWhe);

(3) core fabrication and reprocessing components (including their direct and carrying charges) are combined to form the core "processing" component, $\bar{e}_{c,proc}$ (mills/KWhe);

(4) blanket material components (including their direct and carrying charges) are combined to form the blanket material component, $\bar{e}_{b,mat}$ (mills/KWhe); and

(5) blanket fabrication and reprocessing components (including their direct and carrying charges) are combined to form the blanket "processing" component, $\bar{e}_{b,proc}$ (mills/KWhe).

Core Fuel Energy Costs

From the cost equations derived in Chapter 2, the core fuel energy costs defined above are given by

$$\bar{e}_{c,BU} = \frac{1000}{ET_C} C_{49} \Delta M_{c,49}(T_C) = \frac{1000}{ET_C} C_{49} [M_{c,49}^0 - M_{c,49}(T_C)] \quad (B-40)$$

$$\bar{e}_{c,inv} = \frac{1000}{ET_C} C_{49} [M_{c,49}^0 f_{mp}(T_C) - M_{c,49}(T_C) f_{mc}(T_C)] \quad (B-41)$$

$$\bar{e}_{c,proc} = \frac{1000}{ET_C} M_{c,HM} [C_{c,fab} F_{fab}(T_C) + C_{c,repr} F_{repr}(T_C)] \quad (B-42)$$

where

$E \equiv$ electric energy produced per year, kwhe/year

$$= 8760 \eta L q_C^m V_C \quad (B-43)$$

$\eta \equiv$ net thermal efficiency, kw_e/kw_t

Chapter 2 gives expressions for the carrying charge factors, $F(=1+f)$, in terms of utility company financial parameters:

$$\begin{aligned} f_{mp}(T_c) &= F_{mp}(T_c) - 1 \\ &= \frac{1}{1-\tau} \left[(1+x)^{T'_{mp}+1/2T_c} - \tau \right] - 1 \end{aligned} \quad (B-44)$$

$$\begin{aligned} f_{mc}(T_c) &= F_{mc}(T_c) - 1 \\ &= (1+x)^{-(T''_{mc}+1/2T_c)} - 1 \end{aligned} \quad (B-45)$$

$$F_{fab}(T_c) = \frac{1}{1-\tau} \left[(1+x)^{T'_{fab}+1/2T_c} - \tau \right] \quad (B-46)$$

$$F_{repr}(T_c) = (1+x)^{-(T''_{repr}+1/2T_c)} \quad (B-47)$$

Using the data of Tables B.4 and B.5,

$$f_{mp}(T_c=2) = 0.2445$$

$$f_{mc}(T_c=2) = -0.1089$$

$$F_{fab}(T_c=2) = 1.2445$$

$$F_{repr}(T_c=2) = 0.8911$$

Further, using Equations (B-36), (B-17) and the data of Tables B.4 and B.5, Equations (B-40), (B-41), and (B-42) become

$$\bar{e}_{c,BU} = -1.16 (b_i - 1) \quad (B-48)$$

$$\bar{e}_{c,inv} = 3.335 \epsilon_c + 0.1260 (b_i - 1) \quad (B-49)$$

$$\bar{e}_{c,proc} = 0.3964 \quad (B-50)$$

The total fuel energy cost associated with the core, $\bar{e}_c$, is given by

$$\begin{aligned}\bar{e}_c &= \bar{e}_{c,BU} + \bar{e}_{c,inv} + \bar{e}_{c,proc} \\ &= 3.335 \text{ €}_c - 1.034 (b_i - 1) + 0.3964\end{aligned}\quad (B-51)$$

Blanket Fuel Energy Costs

From the cost equations derived in Chapter 2, the blanket fuel energy costs defined above are

$$\bar{e}_{b,mat} = \frac{1000}{ET_b} C_{49} [M_{b,49}^0 F_{mp}(T_b) - M_{b,49}(T_b) F_{mc}(T_b)] \quad (B-52)$$

$$\bar{e}_{b,proc} = \frac{1000}{ET_b} M_{b,HM} [C_{b,fab} F_{fab}(T_b) + C_{b,repr} F_{repr}(T_b)] \quad (B-53)$$

where E is given by Equation (B-43).

From Chapter 2,

$$F_{mp}(T_b) = \frac{1}{1-\tau} [(1+x)^{T'_{mp}+1/2T_b} - \tau] \quad (B-54)$$

$$F_{mc}(T_b) = (1+x)^{-(T''_{mc} + 1/2T_b)} \quad (B-55)$$

$$F_{fab}(T_b) = \frac{1}{1-\tau} [(1+x)^{T'_{fab}+1/2T_b} - \tau] \quad (B-56)$$

$$F_{repr}(T_b) = (1+x)^{-(T''_{repr}+1/2T_b)} \quad (B-57)$$

Using the data of Tables B.4 and B.5,

$$F_{mp}(T_b=6) = 1.6175$$

$$F_{mc}(T_b=6) = 0.7641$$

$$F_{fab}(T_b=6) = 1.6175$$

$$F_{repr}(T_b=6) = 0.7641$$

For the purposes of estimating blanket processing costs, the blanket volume is assumed to be three times the core volume,

$$V_b = 3 V_c ; \quad (B-58)$$

and

$$M_{b, HM} = 3 V_c \theta_{b, ox} \varphi_{ox} C$$

where

$\theta_{b, ox} \equiv$ oxide volume fraction, blanket

$\varphi_{ox} \equiv$ oxide density

$C \equiv 0.88$ (kgHM/ kgHM oxide)

Further it is assumed that no Pu239 is loaded in the blanket,

$$M_{b, 49}^0 = 0 \quad (B-59)$$

Using Equations (B-54), (B-55), (B-43), (B-59), the material carrying charges factor above, and data from Tables B.4 and B.5, Equation (B-52) becomes

$$\bar{e}_{b, mat} = -0.886 b_x \quad (B-60)$$

Using Equations (B-56), (B-57), (B-43), the processing carrying charge factors above, and data from Tables B.4 and B.5, Equation (B-53) becomes

$$\bar{e}_{b, proc} = 0.2555 \quad (B-61)$$

The total fuel energy cost associated with the blanket, $\bar{e}_b$, is given by

$$\begin{aligned} \bar{e}_b &= \bar{e}_{b, mat} + \bar{e}_{b, proc} \\ &= -0.886 b_x + 0.2555 \end{aligned} \quad (B-62)$$

B.2.2.3 Rated Power and Core Volume

Core rated thermal power is given by

$$Q = q_c''' V_c \quad \text{kw}_t$$

where q_c^m is the rated core power density in kw_t/liter and V_c is the core volume in liters. If the blanket is assumed to produce negligible power (Case A), then plant rated electrical power, P_e , in MWe is

$$P_e = \eta Q \times 10^{-3} = \eta q_c''' V_c \times 10^{-3} \quad \text{MWe} \quad (\text{B-63})$$

where η is the net thermal efficiency. For rated core power density of $500 \text{ kw}_t/\text{liter}$ and a net thermal efficiency of 40%,

$$P_e = 0.2 V_c \quad \text{MWe} \quad (\text{B-64})$$

If the blanket is assumed to produce 10% of the total thermal power (Case A*), then plant rated electrical power, P_e^* , is

$$P_e^* = 0.22 V_c \quad \text{MWe} \quad (\text{B-65})$$

B.2.3.4 Corrections for the Blanket Pu239 Burnup Assumption: Case A*

In deriving the equations above, it was assumed that there was no burnup of Pu239 in the blanket, i.e. that blanket power contribution was approximately zero, and that all of the bred Pu239 was available at end of blanket fuel life.

The assumption has several effects on fuel energy costs. For the same core power rating, the cost components $\bar{e}_{c,BU}$, $\bar{e}_{c,inv}$, $\bar{e}_{c,proc}$, and $\bar{e}_{b,proc}$ are over estimated, since the total delivered energy E , in the denominators of the cost equations, is underestimated. The assumption effects the blanket material component, $\bar{e}_{b,mat}$, a revenue, in two ways, both tending toward an overestimation of this revenue: the total delivered

energy E is underestimated (denominator) and the discharge Pu239 inventory $M_{b,49}$ (numerator) is overestimated.

Corrections for the assumption are developed in this section. Numerical comparisons of Cases A and A* are given in Section B.4.

Correction Factors for Blanket Discharge Pu239 Inventory

The assumption in question causes the blanket Pu239 discharge inventory to be overestimated. To compensate for this, a correction factor (C F) is defined

$$CF \equiv M_{b,49}^* (T_b) / M_{b,49} (T_b) \quad (B-66)$$

where

$M_{b,49} (T_b) \equiv$ blanket Pu239 discharge inventory assuming no Pu239 burnup (Case A).

$M_{b,49}^* (T_b) \equiv$ blanket Pu239 discharge inventory, allowing for Pu239 burnup (Case A*)

The correction factor CF is estimated by two methods below.

Method (1)

The "true" discharge inventory, $M_{b,49}^* (T_b)$, is given by Equation (B-30) while the assumed discharge inventory $M_{b,49} (T_b)$ is found by setting blanket power fraction (f) equal to zero in Equation (B-30). Using Equation (B-30) and Equation (B-66),

$$CF = 1 - f - (f/b_x) \left[(1 + \delta_c) / (1 + \delta_b) \right] \quad (B-67)$$

The parameters δ_c and δ_b are given by

$$\delta_c = \frac{N_{c,28}}{N_{c,49}} \frac{\sigma_{f,28}}{\sigma_{f,49}} = \frac{1 - \epsilon_c}{\epsilon_c} \frac{\sigma_{f,28}}{\sigma_{f,49}} \quad (B-68)$$

and

$$\delta_b = \frac{N_{b,28}}{N_{b,49}} \frac{\sigma_{f,28}}{\sigma_{f,49}} = \frac{1 - \epsilon_b}{\epsilon_b} \frac{\sigma_{f,28}}{\sigma_{f,49}}$$

where ϵ_b is some representative Pu239 fraction in the blanket, i.e. at midpoint of an equilibrium cycle. Letting

$$\epsilon_c \approx 0.15$$

$$\epsilon_b \approx 0.02$$

and using the cross section data of Table B.3, one finds

$$\frac{1 + \delta_c}{1 + \delta_b} = 0.5$$

Assuming an external breeding ratio (b_x) of ~ 0.5 and a blanket power fraction (f) of $\sim 10\%$,

$$CF \approx 0.8 \quad (B-69)$$

Method (2)

The U238, Pu239 population equations, in terms of flux time (θ), are

$$\frac{dN_{b,28}}{d\theta} + \sigma_{c,28} N_{b,28} = 0$$

$$\frac{dN_{b,49}}{d\theta} + \sigma_{a,49} N_{b,49} = \sigma_{c,28} N_{b,28}$$

Assuming $N_{b,49}^0 = 0$, and that $\sigma_{a,49} N_{b,49} \neq 0$, the solution for $N_{b,49}$ is

$$N_{b,49}^* = N_{b,28}^0 \frac{\sigma_{c,28}}{\sigma_{a,49} - \sigma_{a,28}} \exp(-\sigma_{a,28}\theta) [1 - \exp(-[\sigma_{a,49} - \sigma_{a,28}]\theta)] \quad \text{(Case A*)} \quad (B-70)$$

The solution, assuming that $N_{b,49}^0 = 0$ and that $\sigma_{a,49} N_{b,49} = 0$, is

$$N_{b,49} = N_{b,28}^0 \frac{\sigma_{c,28}}{\sigma_{a,28}} [1 - \exp(-\sigma_{a,28} \theta)] \quad (\text{B-71})$$

(Case A)

Thus,

$$CF = \frac{N_{b,49}^*}{N_{b,49}} = \frac{e^{-\sigma_{a,28} \theta} - e^{-\sigma_{a,49} \theta}}{1 - e^{-\sigma_{a,28} \theta}} \quad (\text{B-72})$$

For a blanket flux of 10^{15} n/cm² sec and irradiation time of six years,

$$\theta \simeq 1.9 \times 10^{23} \text{ n/cm}^2$$

Using the cross section data of Table B.3,

$$CF \simeq 0.76 \quad (\text{B-73})$$

which is in rough agreement with the CF computed by the first method.

Corrections to the Economic Equations

A blanket power fraction of 10% and blanket discharge Pu239 correction factor (CF) of 0.80 are assumed (Method 1).

Quantities with a superscript asterisk (*) denote Case A* (with blanket burnup) values; those without asterisks denote Case A (without blanket Pu239 burnup).

With $f=10\%$,

$$E^* = E/0.9 \quad (\text{B-74})$$

and

$$\bar{e}_{c,BU}^* = 0.9 \bar{e}_{c,BU} \quad (\text{B-75})$$

$$\bar{e}_{c,inv}^* = 0.9 \bar{e}_{c,inv} \quad (\text{B-76})$$

$$\bar{e}_{c,proc}^* = 0.9 \bar{e}_{c,proc} \quad (B-77)$$

$$\bar{e}_{b,proc}^* = 0.9 \bar{e}_{b,proc} \quad (B-78)$$

The blanket material component is corrected both for E and $M_{b,49}$

$$\bar{e}_{b,mat}^* = (0.9) (0.8) \bar{e}_{b,mat} = 0.72 \bar{e}_{b,mat} \quad (B-79)$$

B.3 SAMPLE CALCULATION

Table B.6 presents sample calculations for Cases A, A* and B, for a core of 4000 liters.

B.4 RESULTS

Neutronics

Principal neutronics results are given in Figures B.1 and B.2. Core critical enrichments for Cases A, A* (breeding blanket) and Case B (sodium reflector) are plotted versus core volume in Figure B.1. Breeding ratios are shown as a function of core volume in Figure B.2. Enrichments and breeding ratios are independent of the blanket burnup assumption, with core volume as the independent variable. Another scale is shown to relate core volume to plant power rating.

As expected, critical enrichment and external breeding ratio decrease with core size, while internal breeding ratio increases. Case B (sodium reflector) enjoys slightly lower critical enrichment than Cases A, A* (breeding blanket). The lower critical enrichment of Case B improves its internal breeding ratio, but not enough to offset the sacrifice of external breeding.

TABLE B.6 - SAMPLE CALCULATIONS

	<u>CASE⁽¹⁾</u>		
	<u>A</u>	<u>A*</u>	<u>B</u>
Core Volume, V_c (liters)	4000	4000	4000
Reactor Power Rating, P_e (MWe)	800	890	800
Critical Buckling, B_c^2 (cm ⁻²)	0.0007156	0.0007156	0.000552
Critical Enrichment, ϵ_c	0.1096	0.1096	0.0994
Critical Mass, $M_{c,49}^0$ (kg)	1155	1155	1050
Internal Breeding Ratio, b_i	1.0198	1.0198	1.1367
External Breeding Ratio, b_x	0.4976	0.4976	-
Total Breeding Ratio, b	1.5174	1.5174	1.1367
Discharge Pu239			
Core ($T_c=2$ yr), $M_{c,49}$ (kg)	1181	1181	1228
Blanket ($T_b=6$ yr), $M_{b,49}$ (kg)	1940	1550	-
Fuel Energy Costs (mills/KWhe):			
Core Burnup, $\bar{\epsilon}_{c,BU}$	-0.0229	-0.0203	-0.1585
Core Inventory, $\bar{\epsilon}_{c,inv}$	0.3675	0.3300	0.3490
Core Processing, $\bar{\epsilon}_{c,proc}$	0.3964	0.357	0.3964
Core Total, $\bar{\epsilon}_c$	0.7410	0.666	0.5869
Blanket Material, $\bar{\epsilon}_{b,mat}$	-0.441	-0.318	-
Blanket Processing, $\bar{\epsilon}_{b,proc}$	0.2555	0.230	-
Blanket Total, $\bar{\epsilon}_b$	-0.1855	-0.088	-
Reactor Total, $\bar{\epsilon}$	0.5555	0.4805	0.5869

- (1) Case A: Spherical core with breeding blanket (No Pu239 burnup in blanket)
Case A*: Spherical core with breeding blanket (with Pu239 burnup in blanket)
Case B: Spherical core with sodium reflector

FIG. B.1 CRITICAL CORE ENRICHMENT AS A FUNCTION OF CORE VOLUME

FIG. B.2 BREEDING RATIOS AS FUNCTION OF CORE VOLUME

Case A versus Case A*:

Effect of Blanket Pu239 Burnup Assumption

The "no blanket power" (no blanket burnup) assumption affects energy costs (mills/KWhe) in several ways. For the same core power rating,

- (a) the core fuel cost components (burnup, inventory and processing) are overestimated since the total energy delivered, in the denominators of the cost equations, is underestimated;
- (b) the blanket processing component is similarly overestimated;
- (c) the blanket material component, a revenue, is overestimated because the discharge Pu239 inventory is overestimated and the total energy delivered is underestimated.

Figures B.3 and B.4 show the effects (on energy costs) of the "no blanket power" assumption. In Figure B.3, the assumption is seen to lead to an overprediction, $\bar{e}_{TOT} > \bar{e}_{TOT}^*$, of order 0.05 mills/KWhe. This is not insignificant compared to the effects of the radial design and management changes demonstrated in Chapter 5. Figure B.4 compares the blanket costs (net revenues), with and without the "no blanket power" assumption. The assumption is seen to favor blanket revenue significantly, $\bar{e}_b < \bar{e}_b^*$. This bias is, however, offset by the overestimation of core fuel costs.

Cases A, A* versus Case B:

A Breeding Blanket versus A Sodium Reflector

Figure B.5 shows the total reactor fuel energy costs as functions of plant power rating, for cases A, A* and B. The breeding blanket is seen to be advantageous to about 1350 MWe (Case A), or to about 1600 MWe (Case A*). The indifference points are not sharp and definitive, owing to the similarity of the slopes of the curves; a slight change in the economic environment could have a large effect on the indifference point. Beyond

FIG. B.3 EFFECT OF BLANKET BURNUP (POWER FRACTION)
ASSUMPTION ON REACTOR FUEL ENERGY COSTS

FIG. B.4 EFFECT OF BLANKET BURNUP (POWER FRACTION)
ASSUMPTION ON BLANKET FUEL ENERGY COSTS

FIG. B.5 REACTOR FUEL ENERGY COSTS WITH AND WITHOUT
A BREEDING BLANKET

the indifference points, the advantage of the reflector-only configuration is very slight.

The fuel-economic advantages and disadvantages of substituting a non-breeding reflector for a breeding blanket, in large LMFBRs, are summarized qualitatively in Table B.7, and are shown quantitatively, component-by-component, in Figures B.6 and B.7.

Figure B.6 shows the total core fuel energy costs for Cases A and B. The difference $\Delta \bar{e}_c$ is the savings in core fuel energy cost occasioned by the substitution of a sodium reflector for the breeding blanket. The savings is seen to increase with core size. Figure B.7 disaggregates the core fuel savings, $\Delta \bar{e}_c$, into its components $\Delta \bar{e}_{c,inv}$ and $\Delta \bar{e}_{c,BU}$. The improvement is seen to result largely from the burnup component savings, $\Delta \bar{e}_{c,BU}$ (higher internal breeding ratio), the inventory savings $\Delta \bar{e}_{c,Inv}$ being practically negligible.

Offsetting the core savings $\Delta \bar{e}_c$ is the loss of net blanket revenue, $\Delta \bar{e}_b$, as shown in Figure B.7.

Core Size and the Sodium Void Coefficient

Terasawa et.al. (55) have performed parametric studies similar to the study described above. As in the present study, spherical geometry was adopted. The Terasawa study was confined to LMFBR neutronics and involved sodium void and Doppler coefficients, in addition to the variables of the present study, e.g. core volume, critical enrichment, breeding ratios, etc.

Of particular interest are their results concerning enrichment (core size), sodium void coefficient, and internal breeding ratio. The studies reveal that for a sodium volume fraction above about 10% (structure $\approx 15\%$, fuel $\leq 75\%$), it is impossible to have both (a) an internal breeding ratio above unity and (b) a negative sodium void coefficient (50% sodium loss). For a fuel volume fraction of 30% (assumed in present studies),

TABLE B.7 - ADVANTAGES AND DISADVANTAGES OF SUBSTITUTING A
SODIUM REFLECTOR FOR A BREEDING BLANKET

AdvantagesDisadvantagesCORE

1. Lower core inventory cost due to lower critical enrichment;

$$\Delta \bar{e}_C, \text{ inv}$$

2. Higher internal breeding performance (lower burnup cost) due to lower critical enrichment;

$$\Delta \bar{e}_{C, \text{BU}}$$

OUTER REGION

3. No processing costs $\longleftrightarrow \Delta \bar{e}_b \longrightarrow$ 4. No external breeding

FIG. B.6 CORE FUEL ENERGY COSTS FOR REACTORS WITH AND WITHOUT BREEDING BLANKETS

FIG. B.7 FUEL ENERGY COST COMPONENTS FOR REACTORS WITH AND WITHOUT BREEDING BLANKETS

$$(a) \quad b_i \geq 1 \text{ for } \epsilon_c \leq 0.107$$

$$(b) \quad (\Delta k)_{Na} \leq 0 \text{ for } \epsilon_c \geq 0.12$$

where Terawawa's atom ratio, E'_c ($= \text{Pu239/U238}$), has been converted to enrichment ϵ_c ($= \text{Pu239/Pu239+U238}$). The threshold (a) for $b_i=1$ is confirmed by the present study, Figures B.1 and B.2.

From Figure B.1, the enrichment (b) of 12% occurs at a core volume of 2700 liters, or reactor power rating of 540 MWe (case A) or 600 MWe (Case A*). Thus, from Figure B.5, the breeding blanket cases are preferable to the sodium reflector case, for conservatively permissible values of sodium void coefficient (≤ 0). For core volumes greater than 2700 liters, core geometry may be altered to increase leakage, to maintain a negative sodium void coefficient. This would favor external breeding.

Discussion

Several major assumptions were adopted in this study - not only to simplify the analysis, but also to provide a net bias against the blanket concept:

(1) Minimum leakage geometry (spherical) was assumed throughout the range of core volumes considered. In actual design, the core geometry may have to be "spoiled" to enhance the negative component of the sodium void coefficient in large FBRs.

(2) It was assumed that blanket volume was three times core volume. Thus while the leakage decreases as core size increases, the blanket continues to be charged with constant processing charge per unit energy delivered. In actual design practice, the blanket size would be optimized, and one would expect the blanket-to-core volume ratio to decrease with core size.

(3) In Case A, blanket power (blanket Pu239 burnup) was neglected. This assumption was seen to disfavor the blanket concept.

(4) Blanket cross sections were assumed to be the same as for the core. In reality, blanket spectra will be softer, favoring external breeding.

(5) A one zone core was assumed. An optimal material loading pattern would call for some multi-zone enrichment scheme, with outer core zones having higher load enrichments, (52, 53), tending to increase blanket flux.

(6) The study did not account for increased control costs and control absorption neutronic penalties associated with increasing internal breeding ratio substantially above unity.

In spite of these penalties, the blanket concept is seen to be economically desirable (over the no-blanket configuration) to an "indifference point" somewhat over 1000 MWe. Beyond this "indifference point", the advantage of the sodium reflector (no blanket) configuration is very slight.

APPENDIX C

SPPIA, A DEPLETION-ECONOMICS PROGRAM FOR FAST BREEDER REACTORS

C.1 DESCRIPTION OF PROGRAM

The program SPPIA performs depletion-economics calculations for fast breeder reactors under fixed-element fuel management schemes, i.e. batch or scatter management. Input includes fuel cycle cost information and the neutronics data (local fluxes and spectrum-weighted cross sections) from a single physics computation. The program uses the "Semi-Analytic Depletion Method (SAM)" described in Chapter 3 to obtain local nuclide concentrations as functions of irradiation time. The cash flow equations developed in Chapter 2 are used to compute local and aggregate fuel costs in mills/KWhe, \$/yr/kgHM, and \$/kgHM as functions of irradiation time. Chapter 4 gives a step-by-step procedure for using the FBR fuel depletion-economics model embodied in SPPIA.

Local fluxes and cross sections may be taken from a multigroup physics program run for the reactor configuration assumed. Using these local ϕ 's and σ 's, SPPIA solves for time-dependent concentrations in Equations (3-9). The cost equations (2-30), (2-33), and (2-34) are then used to obtain local (by depletion zone), annular-regional, and major regional (i.e. core, axial blanket, radial blanket) fuel costs as functions of irradiation time.

C.2 INPUT INSTRUCTIONS

Table C.1 describes the input data for SPPIA. Control constants and economic data for a single computer run are given on cards 1 through 9. Neutronic data for each depletion zone are provided by cards 10 through 14, each zone having an individual set 10-14.

TABLE C.1
SPPIA INPUT

<u>Variable</u>	<u>Columns</u>	<u>Format</u>	<u>Description</u>
<u>CARD 1</u>			
ID(11)	1-66	11A6	Identification card.
<u>CARD 2</u>			
NCASE	1-6	I6	The number of depletion zones.
NVR	7-12	I6	The number of contiguous depletion zones per region, e.g. annular region. Although each zone is depleted individually, discharge compositions and economics are computed by regions which may consist of more than one zone.
NPRINT	13-18	I6	If NPRINT=0, print out of zone depletion and economics results is omitted. Only region results are printed. If NPRINT=1, both zone and region results are printed.
<u>CARD 3</u>			
NTS	1-6	I6	The number of time steps. Depletion and economics results are printed out after each time step.
<u>CARD 4</u>			
DT	1-12	F12.8	Duration of a time step, full power days.
<u>CARD 5</u>			
EFF	1-12	F12.8	Net thermal efficiency.
<u>CARD 6</u>			
F\$KG/M	1-10	F10.2	Unit fabrication cost, \$/KG/M
R\$KG/M	11-20	F10.2	Unit reprocessing cost, \$/KG/M
S\$KG49	21-30	F10.2	Price of Pu239, \$/KG Pu239

<u>Variable</u>	<u>Columns</u>	<u>Format</u>	<u>Description</u>
<u>CARD 6 (continued)</u>			
S\$KG28	31-40	F10.2	Price of U238, \$/KG U238
S\$KG40	41-50	F10.2	Price of Pu240, \$/KG Pu240
S\$KG41	51-60	F10.2	Price of Pu241, \$/KG Pu241
S\$KG42	61-70	F10.2	Price of Pu242, \$/KG Pu242
<u>CARD 7</u>			
TAX	1-12	F12.8	Income tax rate
BDRTE	13-24	F12.8	Bondholders' rate of return
BDFRN	25-36	F12.8	Bond Fraction
SKRTE	37-48	F12.8	Stockholders' rate of return
SKFRN	49-60	F12.8	Stock fraction
<u>CARD 8</u>			
SLF	1-12	F12.8	Plant load factor
CAPMWE	13-24	F12.8	Plant rated capacity, MWe
<u>CARD 9</u>			
TFPRE	1-12	F12.8	Time prior to beginning of irradiation that fabrication cash flow occurs, years.
TMPPRE	13-24	F12.8	Time prior to beginning of irradiation that material purchase cash flow occurs, years.
TRPPST	25-36	F12.8	Time after end of irradiation that reprocessing cash flow occurs, years.
TMCPST	37-48	F12.8	Time after end of irradiation that material credit occurs, years.

Note: There are NCASE sets of Cards 10-14, each set corresponding to a single depletion zone. Zone neutronic data is provided on Cards 10-14.

CARD 10

SIGA28	1-12	F12.8	U238 microscopic absorption cross section, barns.
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<u>Variable</u>	<u>Columns</u>	<u>Format</u>	<u>Description</u>
<u>CARD 10 (continued)</u>			
SIGF28	13-24	F12.8	U238 microscopic fission cross section, barns.
SIGA49	25-36	F12.8	Pu239 microscopic absorption cross section, barns
SIGF49	37-48	F12.8	Pu239 microscopic fission cross section, barns
<u>CARD 11</u>			
SIGA40	1-12	F12.8	Pu240 microscopic absorption cross section, barns
SIGF40	13-24	F12.8	Pu240 microscopic fission cross section, barns
SIGA41	25-36	F12.8	Pu241 microscopic absorption cross section, barns
SIGF41	37-48	F12.8	Pu241 microscopic fission cross section, barns
SIGA42	49-60	F12.8	Pu242 microscopic absorption cross section, barns
SIGF42	61-72	F12.8	Pu242 microscopic fission cross section, barns
<u>CARD 12</u>			
FLUX	1-12	F12.8	Neutron flux, 10^{15} n/cm ² -sec
<u>CARD 13</u>			
U28NO	1-12	F12.8	Initial U238 atom density, atoms/barn-cm
P49NO	13-24	F12.8	Initial Pu239 atom density, atoms/barn-cm
P40NO	25-36	F12.8	Initial Pu240 atom density, atoms/barn-cm
P41NO	37-48	F12.8	Initial Pu241 atom density, atoms/barn-cm
P42NO	49-60	F12.8	Initial Pu242 atom density, atoms/barn-cm
<u>CARD 14</u>			
VOL	1-12	F12.8	Zone volume, liters

C.3 SAMPLE PROBLEM

This section describes an SPPLA computer run for the reference LMFBR radial blanket (Figure 4.2).

The radial blanket is divided into fifteen depletion zones. These depletion zones were combined to form five annular regions, three depletion zones per annular region. Figure C.1 shows the input data card deck. The reference economic environment, Table 4.2, is assumed. Depletion zone neutronic data is taken from a 26 energy group multigroup computation using the program 2DE. Figure C.2 is SPPLA printed output giving composition and economics results for the innermost annular region (V) at an irradiation time of 3 years. The same computer run yields similar printed output for each of the NTS irradiation intervals, for each of the annular regions. Depletion-economics of each depletion zone can be obtained as well, by setting the control constant NPRINT equal to or greater than unity. Table C.2 interprets the variable names and table headings appearing in printed output.

FIG. C.1 SPPIA SAMPLE PROBLEM INPUT DECK

SPPIA SAMPLE PROBLEM: REFERENCE RAD BLANKET (SPPIA#1.03)

```

15      3      0
16
150.00
0.388198      10.00
69.00      31.50      10000.0      0.0      0.0      0.0      0.0
0.5      0.07      0.5      0.125      0.5
0.823      1000.0
0.5      0.5      0.5      0.5
0.3669      0.02363      3.016      2.246
1.151      0.2557      4.613      3.844      0.8939      0.1784
2.5735
0.0109 E 00
200.28
0.3752      0.0258      3.153      2.325
1.223      0.2483      4.881      4.057      0.9499      0.1714
1.778
0.0109 E 00
200.28
0.4479      0.009459      4.512      3.112
1.961      0.167      7.653      6.298      1.508      0.09461
0.5483
0.0109 E 00
320.44
0.3905      0.01610      3.400      2.463
1.343      0.2131      5.379      4.453      1.036      0.1378
1.87895
0.0109 E 00

```

FIG. C.1 SPPIA SAMPLE PROBLEM INPUT DECK - CONTINUATION

208.14

0.3991 0.01531 3.558 2.555

1.431 0.2070 5.687 4.700 1.103 0.1321

1.30035

0.0109 E 00

208.14

0.4669 0.007158 4.868 3.321

2.174 0.1512 8.381 6.891 1.662 0.07968

0.40861

0.0109 E 00

333.00

0.4251 0.009883 3.980 2.797

1.657 0.1731 6.530 5.375 1.271 0.1001

1.13375

0.0109 E 00

439.82

0.4339 0.009347 4.159 2.903

1.762 0.1686 6.884 5.662 1.350 0.09584

0.7874

0.0109 E 00

439.82

0.4954 0.004854 5.429 3.651

2.510 0.1336 9.535 7.836 1.911 0.06311

0.255475

0.0109 E 00

703.72

0.4734 0.005180 4.875 3.321

2.170 0.1382 8.310 6.815 1.658 0.06721

FIG. C.1 SPPIA SAMPLE PROBLEM INPUT DECK - CONTINUATION

0.53545

0.0109 E 00

471.24

0.4822	0.004876	5.082	3.445		
2.298	0.1351	8.730	7.160	1.754	0.06436

0.37464

0.0109 E 00

471.24

0.5371	0.002824	6.343	4.193		
3.069	0.1157	11.44	9.409	2.326	0.04632

0.127945

0.0109 E 00

753.98

0.5328	0.002663	6.248	4.138		
2.985	0.1146	11.11	9.104	2.279	0.04518

0.197415

0.0109 E 00

765.76

0.5436	0.002481	6.548	4.317		
3.176	0.1122	11.75	9.635	2.419	0.04301

0.141005

0.0109 E 00

765.76

0.5990	0.001527	8.023	5.197		
4.101	0.1002	15.09	12.44	3.097	0.03223

0.05346

0.0109 E 00

1225.22

FIG. C.2 SPPIA SAMPLE PROBLEM PRINTED OUTPUT (PARTIAL)

				RTIMEY	TIMED				
				2.99605656	900.0000				
AFT	AEFTOT	ABNUP	APFRN						
0.11294E 00	0.23992E 05	0.77238E 04	0.10348E-01						
AU28M	AP49M	AP40M	AP41M						
0.29754E 04	0.99274E 02	0.60680E 01	0.27195E 00	AP42M					
A28YR	A49MR	A40MR	A41MR	0.97431E-02					
0.95788616	0.03195970	0.00195349	0.00008755	A42MR	EPS				
				0.00000314	0.03204725				
CCFMP	CCFF	CCFRP	CCFMC						
0.3324451	0.3324451	-0.1425308	-0.1425308						
CCFMP	CCFF	CCFRPB	CCFMCB						
0.3324451	0.3324451	-0.2850618	-0.2850618						
POWER COSTS (MILLS/KWH)									
MATPUR	FAB	REPR(A)	MATCRE(A)	TOTALS(A)	REPR(B)	MATCRE(B)	TOTALS(B)		
0.0	0.00992267	0.00452992	-0.04596025	-0.03150767	0.00452992	-0.04596025	-0.03150767	DIRECT	
0.0	0.00329874	-0.00064565	0.00655075	0.00920384	-0.00129131	0.01310151	0.01510894	CARCHG	
0.0	0.01322141	0.00388426	-0.03940950	-0.02230383	0.00323861	-0.03285874	-0.01639873	TOTALS	
MATERIAL	PROCESSING			TOTALS	METHOD(A)				
DIR(RU)	-0.04596025	DIR	0.01445259	-0.03150767	DIRECT				
CARCHG(INV)	0.00655075	CARCHG	0.00265309	0.00470384	CARCHG				
TOTMAT	-0.03940951	TOTPROC	0.01710568	-0.02230383	TOTALS				
MATERIAL	PROCESSING			TOTALS	METHOD(B)				
DIR(RU)	-0.04596025	DIR	0.01445259	-0.03150767	DIRECT				
CARCHG(INV)	0.01310151	CARCHG	0.00200744	0.01510894	CARCHG				
TOTMAT	-0.03285875	TOTPROC	0.01646002	-0.01639872	TOTALS				
POWER COSTS (\$/YR/KGHHLOADED)									
MATPUR	FAB	REPR(A)	MATCRE(A)	TOTALS(A)	REPR(B)	MATCRE(B)	TOTALS(B)		
0.0	23.03	10.51	-106.67	-73.13	10.51	-106.67	-73.13	DIRECT	
0.0	7.66	-1.50	15.20	21.36	-3.00	30.41	35.07	CARCHG	
0.0	30.69	9.02	-91.47	-51.77	7.52	-76.26	-38.06	TOTALS	
MATERIAL	PROCESSING			TOTALS	METHOD(A)				
DIR(RU)	-106.67	DIR	33.54	-73.13	DIRECT				
CARCHG(INV)	15.20	CARCHG	6.16	21.36	CARCHG				
TOTMAT	-91.47	TOTPROC	39.70	-51.77	TOTALS				
MATERIAL	PROCESSING			TOTALS	METHOD(B)				
DIR(RU)	-106.67	DIR	33.54	-73.13	DIRECT				
CARCHG(INV)	30.41	CARCHG	4.66	35.07	CARCHG				
TOTMAT	-76.26	TOTPROC	38.20	-38.06	TOTALS				
POWER COSTS (\$/KGHHLOADED)									
MATPUR	FAB	REPR(A)	MATCRE(A)	TOTALS(A)	REPR(B)	MATCRE(B)	TOTALS(B)		
0.0	69.00	31.50	-319.60	-219.10	31.50	-319.60	-219.10	DIRECT	
0.0	22.94	-4.49	45.55	64.00	-8.98	91.10	105.06	CARCHG	
0.0	91.94	27.01	-274.04	-155.10	22.52	-228.49	-114.03	TOTALS	
MATERIAL	PROCESSING			TOTALS	METHOD(A)				
DIP(RU)	-319.60	DIR	100.50	-219.10	DIRECT				
CARCHG(INV)	45.55	CARCHG	18.45	64.00	CARCHG				
TOTMAT	-274.04	TOTPROC	118.95	-155.10	TOTALS				
MATERIAL	PROCESSING			TOTALS	METHOD(B)				
DIR(RU)	-319.60	DIR	100.50	-219.10	DIRECT				
CARCHG(INV)	91.10	CARCHG	13.96	105.06	CARCHG				
TOTMAT	-228.49	TOTPROC	114.46	-114.03	TOTALS				

MATCRE/MATPUR+FAB+REPR

TABLE C.2
INTERPRETATION OF SPPLA PRINTED OUTPUT

<u>Variable</u>	<u>Description</u>	<u>Units</u>
RTIMEY	Actual irradiation time	years
TIMED	Irradiation time in full power days	days
ABNUP	Burnup in the annular region	MWD/MTHM
APFRN	Fraction of total reactor power supplied by the annular region	

Nuclide Masses:

AU28M	Mass of U238 in annular region after irradiation	kg
AP49M	Mass of Pu239 in annular region after irradiation	kg
AP40M AP41M AP42M	Masses of Pu239,241,242 in annular region after irradiation	kg
AHMKGL	Initial mass of heavy metal (U+Pu)	kg

Nuclide Fractions:

A28MR	AU28M/AHMKGL
A49MR	AP49M/AHMKGL
A40MR	AP49M/AHMKGL
A41MR	AP41M/AHMKGL
A42MR	AP42MR/AHMKGL
EPS	Fissile Mass/Initial Mass of Heavy Metal

<u>Label or Heading</u>	<u>Description</u>
MATPUR	Material Purchase Component
FAB	Fabrication Component
REPR(A)	Reprocessing Component (Tax Method A)

<u>Label or Heading</u>	<u>Description</u>
REPR(B)	Reprocessing Component (Tax Method B)
MATCRE(A)	Material Credit Component (Tax Method A)
MATCRE(B)	Material Credit Component (Tax Method B)
DIR	Direct Component
CACHG	Carrying Charge Component
TOTMAT	Total Material Component (Burnup+Inventory)
TOTPROC	Total Processing Component (fabrication+reprocessing, including their carrying charges)

C.4 FORTRAN LISTING

The SPPLA FORTRAN listing is given on the following pages.

C	SPP1-A 'SPECIAL PURPOSE PROGRAM 1-A'	CARD0001
C		CARD0002
C	FAST BREEDER REACTOR FUEL DEPLETION AND ECONOMICS CALCULATIONS	CARD0003
C	FBR BLANKET TEST FACILITY, MASSACHUSETTS INSTITUTE OF TECHNOLOGY	CARD0004
C	S.T. BREWER, M.J. DRISCOLL, E.A. MASON	CARD0005
C	* * * * *	CARD0006
C	GIVEN ECONOMIC PARAMETERS AND LOCAL PHYSICS DATA THIS PROGRAM	CARD0007
C	COMPUTES LOCAL, ANNULAR-REGIONAL, AND REGIONAL (CORE,	CARD0008
C	AXIAL BLANKET,RADIAL BLANKET) FUEL COSTS AS FUNCTIONS OF	CARD0009
C	IRRADIATION TIME UNDER SCATTER OR BATCH FUEL MANAGEMENT SCHEMES.	CARD0010
C	* * * * *	CARD0011
		CARD0012
	DIMENSION AU28M(30),AP49M(30),AP40M(30),AP41M(30),AP42M(30)	CARD0013
	DIMENSION AMPD(30),AFBD(30),ARPD(30),AMCD(30),ADT(30),	CARD0014
1	AMPCC(30),AFBCC(30),ARPC(30),AMCCC(30),ACCT(30),	CARD0015
2	AMPTT(30),AFBTT(30),ARPTT(30),AMCTT(30),AT(30)	CARD0016
	DIMENSION FAMPD(30),FAFBD(30),FARPD(30),FAMCD(30),FADT(30),	CARD0017
1	FAMPCC(30),FAFBCC(30),FARPC(30),FAMCCC(30),FACCT(30),	CARD0018
2	FAMPTT(30),FAFBTT(30),FARPTT(30),FAMCTT(30),FAT(30)	CARD0019
	DIMENSION AEFTOT(30),ABNUP(30),SFTV(30),AFT(30)	CARD0020
		CARD0021
	DIMENSION ARPCCB(30),ARPTTB(30),AMCCCB(30),AMCTTB(30),	CARD0022
1	ACCTB(30),ATB(30)	CARD0023
	DIMENSION FARPCB(30),FAMCCB(30),FACCTB(30),FARPTB(30),	CARD0024
1	FAMCTB(30),FATB(30)	CARD0025
	DIMENSION A28MR(30),A49MR(30),A40MR(30),A41MR(30),A42MR(30),	CARD0026
1	EPS(30)	CARD0027
	DIMENSION ABU(30),AINV(30),AINVB(30),AMAT(30),AMATB(30),APRSD(30)	CARD0028
1	,APRSC(30),APRST(30),APRSCB(30),AAT(30),AATB(30),AADT(30),	CARD0029
2	AACCT(30),AACCTB(30)	CARD0030
	DIMENSION APRSTB(30)	CARD0031
	DIMENSION ID(11)	CARD0032
	DOUBLE PRECISION ID	CARD0033
		CARD0034
		CARD0035
C	FORMATS FOR READ CARD INPUT	CARD0036

```

1  FORMAT(3I6)
2  FORMAT(4F12.8)
22 FORMAT(6F12.8)
3  FORMAT(F12.8)
4  FORMAT(5F12.8)
5  FORMAT(F12.8)
6  FORMAT(I6)
7  FORMAT(F12.8)
21 FORMAT(2F12.8)
31 FORMAT(7F10.2)
41 FORMAT(5F12.8)
51 FORMAT(2F12.8)
61 FORMAT(4F12.8)

```

C

```

      PRINT 4442
4442 FORMAT(///99H PROGRAM "SPPLA" * * * * FBR BLANKET TEST FACILITY, M
      1IT * * * * S.T.BREWER, M.J.DRISCOLL, E.A.MASON)
      PRINT 4443
4443 FORMAT(///18X,63H FAST BREEDER REACTOR FUEL DEPLETION AND ECONOMICS
      1 CALCULATIONS)
      READ 4444,(ID(I),I=1,11)
4444 FORMAT(11A6)
      PRINT 4445,(ID(I),I=1,11)
      PRINT 400
4445 FORMAT(/////24H PROBLEM IDENTIFICATION:, 11A6)

      PRINT 411

```

C

C

```

      READ AND PRINT CARD INPUT DATA:
      CONTROL CONSTANTS; ECONOMIC AND POWER PARAMETERS
      READ 1, NCASE,NVR,NPRINT
      PRINT 2010
2010 FORMAT(1X,6H NCASE,3X,4H NVR,1X,7H NPRINT)
      PRINT 2011, NCASE,NVR,NPRINT

```

```

CARD0037
CARD0038
CARD0039
CARD0040
CARD0041
CARD0042
CARD0043
CARD0044
CARD0045
CARD0046
CARD0047
CARD0048
CARD0049
CARD0050
CARD0051
CARD0052
CARD0053
CARD0054
CARD0055
CARD0056
CARD0057
CARD0058
CARD0059
CARD0060
CARD0061
CARD0062
CARD0063
CARD0064
CARD0065
CARD0066
CARD0067
CARD0068
CARD0069
CARD0070
CARD0071
CARD0072

```

2011 FORMAT(1X,2I6,1X,I6)

```
      READ 6, NTS
      READ 7, DT
      READ 21, EFF, DENOX
      READ 31, F$KGHM, R$KGHM, S$KG49, S$KG28, S$KG40, S$KG41, S$KG42
      READ 41, TAX, BDRTE, BDFRN, SKRTE, SKFRN
      READ 51, SLF, CAPMWE
      READ 61,   TFPRE, TMPPRE, TRPPST, TMCPST
      PRINT 406
406  FORMAT(3X,4H NTS)
      PRINT 416, NTS
416  FORMAT(1X,I6)
      PRINT 407
407  FORMAT(3X,3H DT)
      PRINT 417,DT
417  FORMAT(1X,F12.8)
      PRINT 500
500  FORMAT(1X,19H COST ANALYSIS DATA)
      PRINT 598
598  FORMAT(4X,4H EFF,4X,6X,6H DENOX)
      PRINT 599, EFF, DENOX
599  FORMAT(1X,2F12.6)
      PRINT 531
531  FORMAT(3X,7H F$KGHM,2X,3X,7H R$KGHM,2X,3X,7H S$KG49,
1     2X,3X,7H S$KG28,2X,3X,7H S$KG40,2X,3X,7H S$KG41,2X,3X,7H S$KG42)
      PRINT 532, F$KGHM, R$KGHM, S$KG49, S$KG28, S$KG40, S$KG41, S$KG42
532  FORMAT(1X,2F12.8,5F12.2)
      PRINT 541
541  FORMAT(3X,4H TAX,5X,3X,6H BDRTE,3X,3X,6H BDFRN,3X,3X,6H SKRTE,3X,
1     3X,6H SKFRN)
      PRINT 542, TAX, BDRTE, BDFRN, SKRTE, SKFRN
542  FORMAT(1X,5F12.8)
      PRINT 551
551  FORMAT(3X,4H SLF,5X,3X,7H CAPMWE)
      PRINT 552, SLF, CAPMWE
```

CARD0073
CARD0074
CARD0075
CARD0076
CARD0077
CARD0078
CARD0079
CARD0080
CARD0081
CARD0082
CARD0083
CARD0084
CARD0085
CARD0086
CARD0087
CARD0088
CARD0089
CARD0090
CARD0091
CARD0092
CARD0093
CARD0094
CARD0095
CARD0096
CARD0097
CARD0098
CARD0099
CARD0100
CARD0101
CARD0102
CARD0103
CARD0104
CARD0105
CARD0106
CARD0107
CARD0108

```

552 FORMAT(1X,F12.8,F12.1)
    PRINT 561
561 FORMAT(3X,6H TFPRE,3X,2X,7H TMPPRE,3X,2X,7H TRPPST,3X,2X,
1 7H TMCPST)
    PRINT 562,TFPRE,TMPPRE,TRPPST,TMCPST
562 FORMAT(1X,4F12.8)

```

```

C      INITIALIZE
      DO 10001 J = 1,30
      AU28M(J) = 0.0
      AP49M(J) = 0.0
      AP40M(J) = 0.0
      AP41M(J) = 0.0
      AP42M(J) = 0.0
      AMPD(J) = 0.0
      FAMPD(J) = 0.0
      AFBD(J) = 0.0
      FAFBD(J) = 0.0
      ARPD(J) = 0.0
      FARPD(J) = 0.0
      AMCD(J) = 0.0
      FAMCD(J) = 0.0
      ADT(J) = 0.0
      FADT(J) = 0.0
      AMPCC(J) = 0.0
      FAMPCC(J) = 0.0
      AFBCC(J) = 0.0
      FAFBCC(J) = 0.0
      ARPCC(J) = 0.0
      FARPCC(J) = 0.0
      AMCCC(J) = 0.0
      FAMCCC(J) = 0.0
      ACCT(J) = 0.0
      FACCT(J) = 0.0
      AMPTT(J) = 0.0
      FAMPTT(J) = 0.0

```

```

CARD0109
CARD0110
CARD0111
CARD0112
CARD0113
CARD0114
CARD0115
CARD0116
CARD0117
CARD0118
CARD0119
CARD0120
CARD0121
CARD0122
CARD0123
CARD0124
CARD0125
CARD0126
CARD0127
CARD0128
CARD0129
CARD0130
CARD0131
CARD0132
CARD0133
CARD0134
CARD0135
CARD0136
CARD0137
CARD0138
CARD0139
CARD0140
CARD0141
CARD0142
CARD0143
CARD0144

```

AFBTT(J) = 0.0
 FAFBTT(J) = 0.0
 ARP TT(J) = 0.0
 FARPTT(J) = 0.0
 AMCTT(J) = 0.0
 FAMCTT(J) = 0.0
 AT(J) = 0.0
 FAT(J) = 0.0
 ARPCCB(J) = 0.0
 ARP TT B(J) = 0.0
 AMCCCB(J) = 0.0
 AMCTTB(J) = 0.0
 ACCTB(J) = 0.0
 ATB(J) = 0.0
 FARPCB(J) = 0.0
 FAMCCB(J) = 0.0
 FACCTB(J) = 0.0
 FARPTB(J) = 0.0
 FAMCTB(J) = 0.0
 FATB(J) = 0.0
 A28MR(J) = 0.0
 A49MR(J) = 0.0
 A40MR(J) = 0.0
 A41MR(J) = 0.0
 A42MR(J) = 0.0
 EPS(J) = 0.0

 ABU(J) = 0.0
 AINV(J) = 0.0
 AINVB(J) = 0.0
 AMAT(J) = 0.0
 AMATB(J) = 0.0
 APRSD(J) = 0.0
 APRSC(J) = 0.0
 APRST(J) = 0.0
 APRSCB(J) = 0.0

CARD0145
 CARD0146
 CARD0147
 CARD0148
 CARD0149
 CARD0150
 CARD0151
 CARD0152
 CARD0153
 CARD0154
 CARD0155
 CARD0156
 CARD0157
 CARD0158
 CARD0159
 CARD0160
 CARD0161
 CARD0162
 CARD0163
 CARD0164
 CARD0165
 CARD0166
 CARD0167
 CARD0168
 CARD0169
 CARD0170
 CARD0171
 CARD0172
 CARD0173
 CARD0174
 CARD0175
 CARD0176
 CARD0177
 CARD0178
 CARD0179
 CARD0180

```

APRSTB(J) = 0.0
AAT(J) = 0.0
AATB(J) = 0.0
AADT(J) = 0.0
AACCT(J) = 0.0
AACCTB(J) = 0.0
AEFTOT(J) = 0.0
ABNUP(J) = 0.0
SFTV(J) = 0.0
AFT(J) = 0.0

```

10001 CONTINUE

```

NCOUNT = 0.0
AHMKGL = 0.0
ATHMLD = 0.0
AVOL = 0.0

```

```

AU28MO = 0.0
AP49MO = 0.0
AP40MO = 0.0
AP41MO = 0.0
AP42MO = 0.0
IAR = 0

```

```

C
C CASE LOOP
C EACH CASE IS A ZONE OR A LOT OF FUEL
DO 10 I = 1, NCASE
  NCOUNT = NCOUNT + 1
C READ CARD INPUT FOR CASE # I : PHYSICS DATA
  READ 2, SIGA28, SIGF28, SIGA49, SIGF49
  READ 22, SIGA40, SIGF40, SIGA41, SIGF41, SIGA42, SIGF42
  READ 3, FLUX
  READ 4, U28ND, P49ND, P40ND, P41ND, P42ND
  READ 5, VOL

```

```

CARD0181
CARD0182
CARD0183
CARD0184
CARD0185
CARD0186
CARD0187
CARD0188
CARD0189
CARD0190
CARD0191
CARD0192
CARD0193
CARD0194
CARD0195
CARD0196
CARD0197
CARD0198
CARD0199
CARD0200
CARD0201
CARD0202
CARD0203
CARD0204
CARD0205
CARD0206
CARD0207
CARD0208
CARD0209
CARD0210
CARD0211
CARD0212
CARD0213
CARD0214
CARD0215
CARD0216

```

```

    PRINT 400
400  FORMAT(1H1)
    PRINT 401
401  FORMAT(8H CASE NO)
    PRINT 101,I
101  FORMAT(16///)
C   PRINT OUT INPUT DECK IMAGE
    PRINT 411
411  FORMAT(12X,17H INPUT DECK IMAGE)
    PRINT 4111
4111 FORMAT(1X,23H PHYSICS-DEPLETION DATA)
    PRINT 420
420  FORMAT(3X,7H SIGA28,2X,3X,7H SIGF28,2X,3X,7H SIGA49,3X,7H SIGF49)
    PRINT 20, SIGA28,SIGF28,SIGA49,SIGF49
    20  FORMAT(1X,4F12.8)
    PRINT 367
367  FORMAT(3X,7H SIGA40,2X,3X,7H SIGF40,2X,
1     3X,7H SIGA41,2X,3X,7H SIGF41,2X,
2     3X,7H SIGA42,2X,3X,7H SIGF42)
    PRINT 368, SIGA40,SIGF40,SIGA41,SIGF41,SIGA42,SIGF42
368  FORMAT(1X,6F12.8)
    PRINT 413
413  FORMAT(3X,5H FLUX)
    PRINT 403, FLUX
403  FORMAT(1X,F12.8)
    PRINT 404
404  FORMAT(3X,6H U28NO,2X,3X,6H P49NO,2X,3X,6H P40NO,2X,3X,6H P41NO,
1     2X,3X,6H P42NO)
    PRINT 414, U28NO,P49NO,P40NO,P41NO,P42NO
414  FORMAT(1X,5F12.8)
    PRINT 405
405  FORMAT(3X,4H VOL)
    PRINT 415,VOL
415  FORMAT(1X,F12.8)

```

C

```

CARD0217
CARD0218
CARD0219
CARD0220
CARD0221
CARD0222
CARD0223
CARD0224
CARD0225
CARD0226
CARD0227
CARD0228
CARD0229
CARD0230
CARD0231
CARD0232
CARD0233
CARD0234
CARD0235
CARD0236
CARD0237
CARD0238
CARD0239
CARD0240
CARD0241
CARD0242
CARD0243
CARD0244
CARD0245
CARD0246
CARD0247
CARD0248
CARD0249
CARD0250
CARD0251
CARD0252

```

C	CALCULATE CONSTANT PARAMETERS (OUTSIDE THE TIME LOOP)	CARD0253
	CONSTA = (SIGA28-SIGF28)/(SIGA49-SIGA28)	CARD0254
	CONSTB = U28NO*CONSTA	CARD0255
	CB1 = (SIGA49-SIGF49)/(SIGA40-SIGA28)	CARD0256
	CB2 = (SIGA49-SIGF49)/(SIGA40-SIGA49)	CARD0257
	BETA1 = P40NO-(U28NO*CONSTA*CB1-U28NO*CONSTA*CB2+P49NO*CB2)	CARD0258
	CC1 = (SIGA40-SIGF40)/(SIGA41-SIGA28)	CARD0259
	CC2 = (SIGA40-SIGF40)/(SIGA41-SIGA49)	CARD0260
	CC3 = (SIGA40-SIGF40)/(SIGA41-SIGA40)	CARD0261
	BETA2 = P41NO-(U28NO*CONSTA*CB1*CC1-U28NO*CONSTA*CB2*CC2	CARD0262
1	+P49NO*CB2*CC2+BETA1*CC3)	CARD0263
	CD1 = (SIGA41-SIGF41)/(SIGA42-SIGA28)	CARD0264
	CD2 = (SIGA41-SIGF41)/(SIGA42-SIGA49)	CARD0265
	CD3 = (SIGA41-SIGF41)/(SIGA42-SIGA40)	CARD0266
	CD4 = (SIGA41-SIGF41)/(SIGA42-SIGA41)	CARD0267
	BETA3 = P42NO-(U28NO*CONSTA*CB1*CC1*CD1-U28NO*CONSTA*CB2*CC2*CD2	CARD0268
1	+P49NO*CB2*CC2*CD2 + BETA1*CC3*CD3 + BETA2*CD4)	CARD0269
		CARD0270
	PRINT 600	CARD0271
600	FORMAT(1H1)	CARD0272
	PRINT 601	CARD0273
601	FORMAT(1X,15H OUTPUT FOLLOWS///)	CARD0274
	PRINT 602	CARD0275
602	FORMAT(3X,7H CONSTA)	CARD0276
	PRINT 102,CONSTA	CARD0277
102	FORMAT(1X,E15.5)	CARD0278
	PRINT 1801	CARD0279
1801	FORMAT(6X,4H CB1,5X,6X,4H CB2,5X,6X,6H BETA1)	CARD0280
	PRINT 802, CB1,CB2,BETA1	CARD0281
802	FORMAT(1X,3E15.5)	CARD0282
	PRINT 803	CARD0283
803	FORMAT(6X,4H CC1,5X,	CARD0284
1	6X,4H CC2,5X,	CARD0285
2	6X,4H CC3,5X,	CARD0286
3	6X,6H BETA2)	CARD0287
	PRINT 1804, CC1,CC2,CC3,BETA2	CARD0288


```

1804 FORMAT(1X,4E15.5)
      PRINT 805
805  FORMAT(6X,4H CD1,5X,6X,4H CD2,5X,
1      6X,4H CD3,5X,6X,4H CD4,5X,6X,6H BETA3)
      PRINT 806,CD1,CD2,CD3,CD4,BETA3
806  FORMAT(1X,5E15.5)

```

```

C      DOLLAR COST PER LOT
      U28MO=U28ND*VOL*395.25
      P49MO=P49ND*VOL*396.92
      P40MO = P40ND*VOL*398.62
      P41MO = P41ND*VOL*400.28
      P42MO = P42ND*VOL*401.96
      HMKGLD = U28MO + P49MO + P40MO + P41MO + P42MO
      FAB$L = HMKGLD*F$KGHM
      REPR$L = HMKGLD*R$KGHM
      SMP$L = P49MO*S$KG49 + U28MO*S$KG28 + P40MO*S$KG40 +
1      P41MO*S$KG41 + P42MO*S$KG42

      PRINT 920
920  FORMAT(4X,6H U28MO,5X,4X,6H P49MO,5X,4X,6H P40MO,5X,4X,6H P41MO,5X
1      ,4X,6H P42MO,5X,5X,7H HMKGLD)
      PRINT 921, U28MO,P49MO,P40MO,P41MO,P42MO,HMKGLD
921  FORMAT(1X,6F15.5)

      PRINT 922
922  FORMAT(3X,6H SMP$L,3X,3X,6H FAB$L,3X,3X,7H REPR$L)
      PRINT 923, SMP$L,FAB$L,REPR$L
923  FORMAT(1X,3F12.2)

      VFOX = HMKGLD/(VOL*DENDX*0.88)
      PRINT 901
901  FORMAT(1X,5H VFOX)
      PRINT 902, VFOX
902  FORMAT(1X,F12.8)

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CARD0289
CARD0290
CARD0291
CARD0292
CARD0293
CARD0294
CARD0295
CARD0296
CARD0297
CARD0298
CARD0299
CARD0300
CARD0301
CARD0302
CARD0303
CARD0304
CARD0305
CARD0306
CARD0307
CARD0308
CARD0309
CARD0310
CARD0311
CARD0312
CARD0313
CARD0314
CARD0315
CARD0316
CARD0317
CARD0318
CARD0319
CARD0320
CARD0321
CARD0322
CARD0323
CARD0324

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C   LEVELIZING PARAMETERS
    TAXF1 = 1.0/(1.0-TAX)
    TAXF2 = TAX/(1.0-TAX)
    DISRTE = (1.0-TAX)*BDRTE*BDFRN + SKRTE*SKFRN
C   ACTUAL ANNUAL ENERGY PRODUCED BY REACTOR PLANT(KWH/YEAR)
    AARPE = SLF*CAPMWE*1000.0*8760.0

    AARPT = AARPE/EFF
    PRINT 611
611  FORMAT(/ /3X,7H DISRTE)
    PRINT 612, DISRTE
612  FORMAT(1X,F12.8)
    PRINT 621
621  FORMAT(/ /3X,6H AARPE,6X,3X,6H AARPT)
    PRINT 622, AARPE,AARPT
622  FORMAT(1X,2E15.5)
    PRINT 400

C   TIME LOOP
C   N = NTS
    DO 100 J=1,N
    TJ = J
C   EQUIVALENT FULL POWER TIME
    TIMED = TJ*DT
    TIME = TIMED*86400.0
C   CALCULATE U238, PU239, PU240, PU241, PU242
C   ZONE ATOM DENSITIES (ATOMS/BARN-CM), MASSES (KG)
    FT = FLUX*TIME*(1.00E-09)
    TMFN1 = (EXP(-FT*SIGA28))*(1.0-EXP(-FT*(SIGA49-SIGA28)))
    P49N = (CONST8*TMFN1)+(P49ND*EXP(-SIGA49*FT))
    P49M = P49N*VOL*396.92
    U28N = U28ND*EXP(-SIGA28*FT)
    U28M = U28N*VOL*395.25
    P40N = U28ND*CONSTA*CB1*EXP(-SIGA28*FT)
1   -U28ND*CONSTA*CB2*EXP(-SIGA49*FT)

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CARD0325
CARD0326
CARD0327
CARD0328
CARD0329
CARD0330
CARD0331
CARD0332
CARD0333
CARD0334
CARD0335
CARD0336
CARD0337
CARD0338
CARD0339
CARD0340
CARD0341
CARD0342
CARD0343
CARD0344
CARD0345
CARD0346
CARD0347
CARD0348
CARD0349
CARD0350
CARD0351
CARD0352
CARD0353
CARD0354
CARD0355
CARD0356
CARD0357
CARD0358
CARD0359
CARD0360

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2	+P49ND*CB2*EXP(-SIGA49*FT)	CARD0361
3	+BETA1*EXP(-SIGA40*FT)	CARD0362
	P40M = P40N*VOL*398.62	CARD0363
	P41N = U28ND*CONSTA*CP1*CC1*EXP(-SIGA28*FT)	CARD0364
1	-U28ND*CONSTA*CB2*CC2*EXP(-SIGA49*FT)	CARD0365
2	+P49ND*CB2*CC2*EXP(-SIGA49*FT)	CARD0366
3	+BETA1*CC3*EXP(-SIGA40*FT)	CARD0367
4	+BETA2*EXP(-SIGA41*FT)	CARD0368
	P41M = P41N*VOL*400.28	CARD0369
	P42N = U28ND*CONSTA*CB1*CC1*CD1*EXP(-SIGA28*FT)	CARD0370
1	-U28ND*CONSTA*CB2*CC2*CD2*EXP(-SIGA49*FT)	CARD0371
2	+P49ND*CB2*CC2*CD2*EXP(-SIGA49*FT)	CARD0372
3	+BETA1*CC3*CD3*EXP(-SIGA40*FT)	CARD0373
4	+BETA2*CD4*EXP(-SIGA41*FT)	CARD0374
5	+BETA3*EXP(-SIGA42*FT)	CARD0375
	P42M = P42N*VOL*401.96	CARD0376
C	ACTUAL (REAL) TIME (YEARS)	CARD0377
	RTIMEY = TIMED/(365.0*SLF)	CARD0378
	IF(NPRINT.LT.1) GO TC 1401	CARD0379
	PRINT 400	CARD0380
	PRINT 713	CARD0381
713	FORMAT(50X,7H RTIMEY)	CARD0382
	PRINT 703, RTIMEY	CARD0383
703	FORMAT(50X,F12.8)	CARD0384
1401	CONTINUE	CARD0385
		CARD0386
		CARD0387
C	PARAMETERS RELATED TO ZONE ENERGY AND POWER	CARD0388
C	1. ATOMS FISSIONED IN REGION(LOT)	CARD0389
	AFIS49= SIGF49*VOL*((EXP(-SIGA49*FT)-1.0)*((CONSTB-P49ND)/SIGA49)	CARD0390
1	-(EXP(-SIGA28*FT)-1.0)*(CONSTB/SIGA28))*(1.00E+27)	CARD0391
	AFIS28= SIGF28*VOL*(U28ND/SIGA28)*(1.0-EXP(-SIGA28*FT))*(1.00E+27)	CARD0392
	AFISTT=AFIS49+AFIS28	CARD0393
C	2. FISSION ENERGY RELEASED IN REGION(LOT) (MWDT)	CARD0394
	EF49=(3.715)*(1.00E-22)*AFIS49	CARD0395
	EF28=(3.715)*(1.00E-22)*AFIS28	CARD0396

	EFTOT=EF49+EF28	CARD0397
C	3. NUMBER OF PU239 ATOMS CONSUMED IN REGION(LOT)	CARD0398
	ACON49=AFIS49*(SIGA49/SIGF49)	CARD0399
C	4. NUMBER OF PU239 ATOMS BRED IN REGION(LOT).	CARD0400
	ABRD49=AFIS28*((SIGA28-SIGF28)/SIGF28)	CARD0401
C	5. REGION(LOT) TIME INTEGRAL BREEDING RATIO...	CARD0402
C	PU239 BRED IN REGION/PU239 CONSUMED IN REGION	CARD0403
	BR1=ABRD49/ACON49	CARD0404
C	6. ACTUAL THERMAL ENERGY RELEASED BY REACTOR DURING IRRADIATION	CARD0405
C	OF THIS LOT(REGION) (MWDT)	CARD0406
	TERTR=AARPT*RTIMEY/(24.0*1000.0)	CARD0407
C	7. FRACTION OF REACTOR THERMAL ENERGY RELEASED IN THIS REGION(LOT)	CARD0408
	PERNL=EFTOT/TERTR	CARD0409
C	8. BURNUP IN REGION (LOT) (MWDT/MT OF HM LOADED)	CARD0410
	THMLD = HMKGLD/1000.0	CARD0411
	BURNUP = EFTOT/THMLD	CARD0412
C	9. REGION (LOT) THERMAL POWER AT TIME OF DISCHARGE (RTIMEY)	CARD0413
C	AT RATED PLANT CAPACITY (MWT)	CARD0414
	PDMW49=3.715*P49N*SIGF49*FLUX*VOL*(86400.0)*(1.00E-04)	CARD0415
	PDMW28=3.715*U28N*SIGF28*FLUX*VOL*(86400.0)*(1.00E-04)	CARD0416
	PDTOT=PDMW49+PDMW28	CARD0417
C	10. REGION (LOT) THERMAL POWER DENSITY AT TIME OF DISCHARGE	CARD0418
C	(RTIMEY) AT RATED PLANT CAPACITY (KWT/LITER)	CARD0419
	PDENS=PDTOT*1000.0/VCL	CARD0420
C	11. ACTUAL (AT LOAD FACTOR) REGION (LOT) TIME AVERAGED	CARD0421
C	POWER DENSITY (KWT/LITER)	CARD0422
	PDENSA=(EFTOT*SLF*1000.0)/(VOL*TIMED)	CARD0423
		CARD0424
	IF(NPRINT.LT.1) GO TO 1402	CARD0425
	PRINT 603	CARD0426
603	FORMAT(///5X,6H TIMED,4X,	CARD0427
1	6X,3H FT,6X,	CARD0428
2	4X,6H TMFN1,5X,	CARD0429
3	5X,5H U28N,5X,	CARD0430
4	5X,5H U28M,5X,	CARD0431
5	5X,5H P49N,5X,	CARD0432

6	5X,5H P49M)	CARD0433
	PRINT 613, TIMED,FT,1MFN1,U28N,U28M,P49N,P49M	CARD0434
613	FORMAT(1X,7E15.5)	CARD0435
	PRINT 467	CARD0436
467	FORMAT(5X,5H P40N,5X,5X,5H P41N,5X,5X,5H P42N)	CARD0437
	PRINT 468,P40N,P41N,P42N	CARD0438
468	FORMAT(1X,3E15.5)	CARD0439
	PRINT 469	CARD0440
469	FORMAT(5X,5H P40M,5X,5X,5H P41M,5X,5X,5H P42M)	CARD0441
	PRINT 470, P40M,P41M,P42M	CARD0442
470	FORMAT(1X,3E15.5)	CARD0443
		CARD0444
	PRINT 697	CARD0445
697	FORMAT(4X,7H AFIS49,4X,4X,7H AFIS28,4X,4X,7H AFISTT)	CARD0446
	PRINT 698,AFIS49,AFIS28,AFISTT	CARD0447
698	FORMAT(1X,3E15.5)	CARD0448
	PRINT 688	CARD0449
688	FORMAT(4X,5H EF49,6X,4X,5H EF28,6X,4X,6H EFTOT)	CARD0450
	PRINT 689, EF49,EF28,EFTOT	CARD0451
689	FORMAT(1X,3E15.5)	CARD0452
	PRINT 677	CARD0453
677	FORMAT(4X,7H ACON49,4X,4X,7H ABRD49,4X,11X,4H BR1)	CARD0454
	PRINT 678,ACON49,ABRD49,BR1	CARD0455
678	FORMAT(1X,3E15.5)	CARD0456
	PRINT 671	CARD0457
671	FORMAT(4X,6H TERTR,5X,4X,6H PFRNL,5X,4X,7H BURNUP)	CARD0458
	PRINT 672, TERTR, PFRNL,BURNUP	CARD0459
672	FORMAT(1X,3E15.5)	CARD0460
	PRINT 673	CARD0461
673	FORMAT(4X,7H PDMW49,4X,4X,7H PDMW28,4X,4X,6H PDTOT)	CARD0462
	PRINT 674, PDMW49,PDMW28,PDTOT	CARD0463
674	FORMAT(1X,3E15.5)	CARD0464
	PRINT 675	CARD0465
675	FORMAT(4X,7H PDENSD,4X,4X,7H PDENSA)	CARD0466
	PRINT 676,PDENSD,PDENSA	CARD0467
676	FORMAT(1X,2E15.5)	CARD0468

1402 CONTINUE

C CALCULATE POWER COSTS (MILLS/KWHE)
C POWER COSTS, DIRECT (MILLS/KWH)
TPPE = 1000.0/(AARPE*RTIMEY)
C MATERIAL PURCHASE
PCMPD = SMP\$L*TPPE
C FAB
PCFBD = FAB\$L*TPPE
C REPROCESSING
PCRPD = REPR\$L*TPPE
C MATERIAL CREDIT
SMC\$L = P49M*\$S\$KG49 + U28M*\$S\$KG28 + P40M*\$S\$KG40 + P41M*\$S\$KG41
1 + P42M*\$S\$KG42

IF(NPRINT.LT.1) GO TO 1403

PRINT 301

301 FORMAT(3X,6H SMC\$L)

PRINT 302,SMC\$L

302 FORMAT(1X,F12.2)

PRINT 2222

1403 CONTINUE

PCMCD = -SMC\$L*TPPE

C POWER COSTS,CARRYING CHARGES (MILLS/KWH)

C MATERIAL PURCHASE

TMP = 0.5*RTIMEY + TMPPRE

DISFMP = 1.0/((1.0+DISRTE)**(-TMP))

CCFMP = TAXF1*DISFMP - TAXF2 - 1.0

PCMPCC = (PCMPD)*(CCFMP)

PCMPTT = PCMPD + PCMPCC

C FAB

TFB = 0.5*RTIMEY + TFPRE

DISFF = 1.0/((1.0+DISRTE)**(-TFB))

CCFF = TAXF1*DISFF - TAXF2 - 1.0

PCFBCC = (PCFBD)*(CCFF)

CARD0469
CARD0470
CARD0471
CARD0472
CARD0473
CARD0474
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CARD0493
CARD0494
CARD0495
CARD0496
CARD0497
CARD0498
CARD0499
CARD0500
CARD0501
CARD0502
CARD0503
CARD0504

	PCFBTT = PCFBD + PCFBCC	CARD0505
C	REPROCESSING	CARD0506
	TRP = -(0.5*RTIMEY + TRPPST)	CARD0507
	DISFRP = 1.0/((1.0+DISRTE)**(-TRP))	CARD0508
	CCFRP = DISFRP - 1.0	CARD0509
	PCRPPC = (PCRPD)*(CCFRP)	CARD0510
	PCRPTT = PCRPD + PCRPPC	CARD0511
	CCFRPB = TAXF1*DISFRP - TAXF2 - 1.0	CARD0512
	PCRPCB = (PCRPD)*(CCFRPB)	CARD0513
	PCRPTB = PCRPD + PCRPCB	CARD0514
C	MATERIAL CREDIT	CARD0515
	TMC = -(0.5*RTIMEY + TMCPST)	CARD0516
	DISFMC = 1.0/((1.0+DISRTE)**(-TMC))	CARD0517
	CCFMC = DISFMC - 1.0	CARD0518
	PCMCCC = (PCMCD)*(CCFMC)	CARD0519
	PCMCTT = PCMCD + PCMCCC	CARD0520
	CCFMCB = TAXF1*DISFMC - TAXF2 - 1.0	CARD0521
	PCMCCB = (PCMCD)*(CCFMCB)	CARD0522
	PCMCTB = PCMCD + PCMCCB	CARD0523
		CARD0524
C	TOTALS	CARD0525
C	DIRECT	CARD0526
	PCDT=PCMPD+PCFBD+PCRPPC+PCMCD	CARD0527
C	CARRYING CHARGE	CARD0528
	PCCCT=PCMPCC+PCFBCC+PCRPPC+PCMCCC	CARD0529
	PCCCTB = PCMPCC + PCFBCC + PCRPCB + PCMCCB	CARD0530
C	GRAND TOTAL	CARD0531
	PCT=PCDT+PCCCT	CARD0532
	PCTB = PCDT + PCCCTB	CARD0533
		CARD0534
	IF(NPRINT.LT.1) GO TO 1404	CARD0535
	PRINT 303	CARD0536
303	FORMAT(3X,6H CCFMP,3X,3X,5H CCFF,4X,3X,6H CCFRP,3X,3X,6H CCFMC)	CARD0537
	PRINT 304,CCFMP,CCFF,CCFRP,CCFMC	CARD0538
304	FORMAT(1X,4F12.7)	CARD0539
	PRINT 30310	CARD0540

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30310 FORMAT(3X,6H CCFMP,3X,3X,5H CCFF,4X,3X,7H CCFRPB,2X,3X,7H CCFMCB)
      PRINT 3041, CCFMP,CCFF,CCFRPB,CCFMCB
3041  FORMAT(1X,4F12.7)
      PRINT 800
      800 FORMAT(//24H POWER COSTS (MILLS/KWH))
      PRINT 804
      PRINT 814,PCMPD,PCFBD,PCRPD,PCMCD,PCDT,PCRPD,PCMCD,PCDT
      PRINT 815,PCMPCC,PCFBCC,PCRPC,PCMCCC,PCCCT,PCRPCB,PCMCCB,PCCCTB
      PRINT 816,PCMPPT,PCFET,PCRPT,PCMCT,PCT,PCRPTB,PCMCTB,PCTB

```

1404 CONTINUE

C

C SUM OR AVERAGE OVER ZONES TO OBTAIN ANNULAR REGION RESULTS

```

AU28M(J) = AU28M(J) + U28M
AP49M(J) = AP49M(J) + P49M
AP40M(J) = AP40M(J) + P40M
AP41M(J) = AP41M(J) + P41M
AP42M(J) = AP42M(J) + P42M

AMPD(J) = AMPD(J) + PCMPD
AMPCC(J) = AMPCC(J) + PCMPCC
AMPTT(J) = AMPTT(J) + PCMPTT
AFBD(J) = AFBD(J) + PCFBD
AFBCC(J) = AFBCC(J) + PCFBCC
AFBTT(J) = AFBTT(J) + PCFBTT
ARPD(J) = ARPD(J) + PCRPD
ARPCC(J) = ARPCC(J) + PCRPC
ARPTT(J) = ARPTT(J) + PCRPT
AMCD(J) = AMCD(J) + PCMCD
AMCCC(J) = AMCCC(J) + PCMCCC
AMCTT(J) = AMCTT(J) + PCMCTT
ADT(J) = ADT(J) + PCDT
ACCT(J) = ACCT(J) + PCCCT
AT(J) = AT(J) + PCT

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CARD0541
 CARD0542
 CARD0543
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 CARD0574
 CARD0575
 CARD0576

ARPCCB(J) = ARPCCB(J) + PCRPCB
 ARPTTB(J) = ARPTTB(J) + PCRPTB
 AMCCCB(J) = AMCCCB(J) + PCMCCB
 AMCTTB(J) = AMCTTB(J) + PCMCTB
 ACCTB(J) = ACCTB(J) + PCCCTB
 ATB(J) = ATB(J) + PCTB

SFTV(J) = SFTV(J) + FT*VOL

AEFTOT(J) = AEFTOT(J) + EFTOT

100 CONTINUE

C

AHMKGL = AHMKGL + HMKGLD

ATHMLD = AHMKGL/ 1000.0

AVOL = AVOL + VOL

CONV1 = 1.0/AHMKGL

AU28MO = AU28MO + U28MO

AP49MO = AP49MO + P49MO

AP40MO = AP40MO + P40MO

AP41MO = AP41MO + P41MO

AP42MO = AP42MO + P42MO

A28MRO = AU28MO*CONV1

A49MRO = AP49MO*CONV1

A40MRO = AP40MO*CONV1

A41MRO = AP41MO*CONV1

A42MRO = AP42MO*CONV1

EPSD = A49MRO + A41MRO

IF(NCOUNT.LT.NVR) GO TO 10

IAR = IAR + 1

PRINT 400

PRINT 1001

1001 FORMAT(3X,23H ANNULAR REGION RESULTS)

CARD0577
 CARD0578
 CARD0579
 CARD0580
 CARD0581
 CARD0582
 CARD0583
 CARD0584
 CARD0585
 CARD0586
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 CARD0602
 CARD0603
 CARD0604
 CARD0605
 CARD0606
 CARD0607
 CARD0608
 CARD0609
 CARD0610
 CARD0611
 CARD0612

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PRINT 1002
1002 FORMAT(/ /6X,4H IAR)
PRINT 1003, IAR
1003 FORMAT(6X,I2)
PRINT 2001
2001 FORMAT(/ /5X,5H AVOL,5X,4X,7H AHMKGL)
PRINT 2002, AVOL, AHMKGL
2002 FORMAT(1X,2E15.5)
PRINT 2222
PRINT 1004
RTIMEY = 0.0
TIMED = 0.0
PRINT 1005, RTIMEY, TIMED
PRINT 1006
PRINT 1007, AU28MO, AP49MO, AP40MO, AP41MO, AP42MO
PRINT 10071
PRINT 1107, A28MRO, A49MRO, A40MRO, A41MRO, A42MRO, EPSO

CONV = AARPE/(1000.0*AHMKGL)

DO 10000 J=1,NTS
FAMPD(J) = AMPD(J)*CONV
FAMPCC(J) = AMPCC(J)*CONV
FAMPTT(J) = AMPTT(J)*CONV
FAFBD(J) = AFBBD(J)*CONV
FAFBCC(J) = AFBCC(J)*CONV
FAFBTT(J) = AFBTT(J)*CONV
FARPD(J) = ARPD(J)*CONV
FARPCCT(J) = ARPCCT(J)*CONV
FARPTT(J) = ARPTT(J)*CONV
FAMCD(J) = AMCD(J)*CONV
FAMCCC(J) = AMCCC(J)*CONV
FAMCTT(J) = AMCTT(J)*CONV
FADT(J) = ADT(J)*CONV
FACCT(J) = ACCT(J)*CONV
FAT(J) = AT(J)*CONV

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CARD0613
CARD0614
CARD0615
CARD0616
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CARD0632
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CARD0634
CARD0635
CARD0636
CARD0637
CARD0638
CARD0639
CARD0640
CARD0641
CARD0642
CARD0643
CARD0644
CARD0645
CARD0646
CARD0647
CARD0648

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FARPCB(J) = ARPCCB(J)*CONV
 FAMCCB(J) = AMCCCB(J)*CONV
 FACCTB(J) = ACCTB(J)*CONV
 FATB(J) = ATB(J)*CONV
 FARPTB(J) = ARPTTB(J)*CONV
 FAMCTB(J) = AMCTTB(J)*CONV

C

OTHER GROUPINGS OF COSTS

ABU(J) = AMPD(J) + AMCD(J)
 AINV(J) = AMPCC(J) + AMCCC(J)
 AINVB(J) = AMPCC(J) + AMCCCB(J)
 AMAT(J) = ABU(J) + AINV(J)
 AMATB(J) = ABU(J) + AINVB(J)
 APRSD(J) = AFB(D)(J) + ARPD(J)
 APRSC(J) = AFBCC(J) + ARPCC(J)
 APRST(J) = APRSD(J) + APRSC(J)
 APRSCB(J) = AFBCC(J) + ARPCCB(J)
 APRSTB(J) = APRSD(J) + APRSCB(J)
 AAT(J) = AMAT(J) + APRST(J)
 AATB(J) = AMATB(J) + APRSTB(J)
 AADT(J) = ABU(J) + APRSD(J)
 AACCT(J) = AINV(J) + APRSC(J)
 AACCTB(J) = AINVB(J) + APRSCB(J)

AFT(J) = SFTV(J)/AVOL

ABNUP(J) = AEFTOT(J)/ATHMLD

A28MR(J) = AU28M(J)*CONV1
 A49MR(J) = AP49M(J)*CCNV1
 A40MR(J) = AP40M(J)*CONV1
 A41MR(J) = AP41M(J)*CONV1
 A42MR(J) = AP42M(J)*CCNV1
 EPS(J) = A49MR(J) + A41MR(J)

CARD0649
 CARD0650
 CARD0651
 CARD0652
 CARD0653
 CARD0654
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 CARD0660
 CARD0661
 CARD0662
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 CARD0664
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 CARD0670
 CARD0671
 CARD0672
 CARD0673
 CARD0674
 CARD0675
 CARD0676
 CARD0677
 CARD0678
 CARD0679
 CARD0680
 CARD0681
 CARD0682
 CARD0683
 CARD0684

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TJ = J
TIMED = TJ*DT
RTIMEY = TIMED/(365.0*SLF)
TIME = TIMED*86400.0
PRINT 400
PRINT 1004
1004 FORMAT(45X,7H RTIMEY,6X,6H TIMED)
PRINT 1005, RTIMEY,TIMED
1005 FORMAT(44X,F12.8,3X,F12.4)

TERTR = AARPT*RTIMEY/(24.0*1000.0)
APFRN = AEFTOT(J)/TERTR
PRINT 2003
2003 FORMAT(5X,4H AFT,6X,4X,7H AEFTOT,4X,4X,6H ABNUP,5X,
1 4X,6H APFRN)
PRINT 2004, AFT(J),AEFTOT(J),ABNUP(J),APFRN
2004 FORMAT(1X,4E15.5)

PRINT 1006
1006 FORMAT(5X,6H AU28M,4X,5X,6H AP49M,4X,
1 5X,6H AP40M,4X,5X,6H AP41M,4X,5X,6H AP42M)
PRINT 1007, AU28M(J),AP49M(J),AP40M(J),AP41M(J),AP42M(J)
1007 FORMAT(1X,5E15.5)

PRINT 10071
10071 FORMAT(4X,6H A28MR,5X,4X,6H A49MR,5X,4X,6H A40MR,5X,
1 4X,6H A41MR,5X,4X,6H A42MR,5X,5X,6X,4H EPS)
PRINT 1107, A28MR(J),A49MR(J),A40MR(J),A41MR(J),A42MR(J),EPS(J)
1107 FORMAT(1X,5F15.8,5X,F15.8)

TMP = 0.5*RTIMEY + TMPPRE
DISFMP = 1.0/((1.0+DISRTE)**(-TMP))
CCFMP = TAXF1*DISFMP - TAXF2 - 1.0
TFB = 0.5*RTIMEY + TFPRE
DISFF = 1.0/((1.0+DISRTE)**(-TFB))

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CARD0685
CARD0686
CARD0687
CARD0688
CARD0689
CARD0690
CARD0691
CARD0692
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CARD0700
CARD0701
CARD0702
CARD0703
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CARD0705
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CARD0709
CARD0710
CARD0711
CARD0712
CARD0713
CARD0714
CARD0715
CARD0716
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CARD0718
CARD0719
CARD0720

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CCFF = TAXF1*DISFF - TAXF2 - 1.0
TRP = -(0.5*RTIMEY + TRPPST)
DISFRP = 1.0/((1.0+CISRTE)**(-TRP))
CCFRP = DISFRP - 1.0
CCFRPB = TAXF1*DISFRP - TAXF2 - 1.0
TMC = -(0.5*RTIMEY + TMCPST)
DISFMC = 1.0/((1.0+DISRTE)**(-TMC))
CCFMC = DISFMC - 1.0
CCFMCB = TAXF1*DISFMC - TAXF2 - 1.0
PRINT 303
PRINT 304,CCFMP,CCFF,CCFRP,CCFMC
PRINT 30310
PRINT 3041,CCFMP,CCFF,CCFRPB,CCFMCB

PRINT 1008
1008 FORMAT(/ /24H POWER COSTS (MILLS/KWH))
PRINT 804
PRINT 814,AMPD(J),AFBD(J),ARPD(J),AMCD(J),ADT(J),ARPD(J),AMCD(J),
1 ADT(J)
PRINT 815,AMPCC(J),AFBCC(J),ARPC(C),AMCCC(J),ACCT(J),ARPCCB(J),
1 AMCCCB(J),ACCTB(J)
PRINT 816,AMPTT(J),AFBTT(J),ARPTT(J),AMCTT(J),AT(J),ARPTTB(J),
1 AMCTTB(J),ATB(J)

PRINT 3030
PRINT 3032,ABU(J),APRSD(J),AADT(J)
PRINT 3033,AINV(J),APRSC(J),AACCT(J)
PRINT 3034,AMAT(J),APRST(J),AAT(J)
PRINT 3031
PRINT 3032,ABU(J),APRSD(J),AADT(J)
PRINT 3033,AINVB(J),APRSCB(J),AACCTB(J)
PRINT 3034,AMATB(J),APRSTB(J),AATB(J)

PRINT 1013
1013 FORMAT(/ /30H POWER COSTS ($/YR/KGHMLADED))

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CARD0721
CARD0722
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CARD0746
CARD0747
CARD0748
CARD0749
CARD0750
CARD0751
CARD0752
CARD0753
CARD0754
CARD0755
CARD0756

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PRINT 804
PRINT 1014, FAMPD(J),FAFBD(J),FARPD(J),FAMCD(J),FADT(J),
1 FARPD(J),FAMCD(J),FADT(J)
PRINT 1015, FAMPCC(J),FAFBCC(J),FARPC(J),FAMCCC(J),FACCT(J),
1 FARPCB(J),FAMCCB(J),FACCTB(J)
PRINT 1016, FAMPTT(J),FAFBTT(J),FARPTT(J),FAMCTT(J),FAT(J),
1 FARPTB(J),FAMCTB(J),FATB(J)

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ABU(J) = ABU(J)*CONV
AINV(J) = AINV(J)*CONV
AINVB(J) = AINVB(J)*CCNV
AMAT(J) = AMAT(J)*CONV
AMATB(J) = AMATB(J)*CONV
APRSD(J) = APRSD(J)*CCNV
APRSC(J) = APRSC(J)*CONV
APRST(J) = APRST(J)*CCNV
APRSCB(J) = APRSCB(J)*CONV
APRSTB(J) = APRSTB(J)*CONV
AAT(J) = AAT(J)*CONV
AATB(J) = AATB(J)*CONV
AADT(J) = AADT(J)*CONV
AACCT(J) = AACCT(J)*CCNV
AACCTB(J) = AACCTB(J)*CONV

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PRINT 3030
PRINT 3042, ABU(J),APRSD(J),AADT(J)
PRINT 3043, AINV(J),APRSC(J),AACCT(J)
PRINT 3044, AMAT(J),APRST(J),AAT(J)
PRINT 3031
PRINT 3042, ABU(J),APRSD(J),AADT(J)
PRINT 3043, AINVB(J),APRSCB(J),AACCTB(J)
PRINT 3044, AMATB(J),APRSTB(J),AATB(J)

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CARD0757
 CARD0758
 CARD0759
 CARD0760
 CARD0761
 CARD0762
 CARD0763
 CARD0764
 CARD0765
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 CARD0768
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 CARD0771
 CARD0772
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 CARD0783
 CARD0784
 CARD0785
 CARD0786
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 CARD0790
 CARD0791
 CARD0792

2222	FORMAT(//5X,80H METHOD(A): (MATCHRE-REPR) IS TAXED. METHOD(B):	CARD0793
	1(MATCHRE-REPR) IS CAPITALIZED.)	CARD0794
804	FORMAT(2X,7H MATPUR,3X,4X,4H FAB,4X,3X,8H REPR(A),1X,2X,	CARD0795
	1 10 H MATCHRE(A),3X,4X,10H TOTALS(A),7X,8H REPR(B),1X,2X,	CARD0796
	2 10H MATCHRE(B),3X,4X,10H TOTALS(B))	CARD0797
814	FORMAT(1X,4F12.8,4X,F12.8,5X,2F12.8,4X,F12.8,5X,7H DIRECT)	CARD0798
815	FORMAT(1X,4F12.8,4X,F12.8,5X,2F12.8,4X,F12.8,5X,7H CARCHG)	CARD0799
816	FORMAT(1X,4F12.8,4X,F12.8,5X,2F12.8,4X,F12.8,5X,7H TOTALS)	CARD0800
1014	FORMAT(1X,4F12.2,4X,F12.2,5X,2F12.2,4X,F12.2,5X,7H DIRECT)	CARD0801
1015	FORMAT(1X,4F12.2,4X,F12.2,5X,2F12.2,4X,F12.2,5X,7H CARCHG)	CARD0802
1016	FORMAT(1X,4F12.2,4X,F12.2,5X,2F12.2,4X,F12.2,5X,7H TOTALS)	CARD0803
3030	FORMAT(3X,9H MATERIAL,18X,3X,11H PROCESSING,16X,3X,7H TOTALS,20X,	CARD0804
	1 10H METHOD(A))	CARD0805
3031	FORMAT(3X,9H MATERIAL,18X,3X,11H PROCESSING,16X,3X,7H TOTALS,20X,	CARD0806
	1 10H METHOD(B))	CARD0807
3032	FORMAT(6X,8H DIR(BU),4X,F12.8,6X,4H DIR,8X,F12.8,6X,F12.8,	CARD0808
	1 7H DIRECT)	CARD0809
3033	FORMAT(6X,12H CARCHG(INV),F12.8,6X,7H CARCHG,5X,F12.8,6X,F12.8,	CARD0810
	1 7H CARCHG)	CARD0811
3034	FORMAT(6X,7H TOTMAT,5X,F12.8,6X,8H TOTPROC,4X,F12.8,6X,F12.8,	CARD0812
	1 7H TOTALS)	CARD0813
3042	FORMAT(6X,8H DIR(BU),4X,F12.2,6X,4H DIR,8X,F12.2,6X,F12.2,	CARD0814
	1 7H DIRECT)	CARD0815
3043	FORMAT(6X,12H CARCHG(INV),F12.2,6X,7H CARCHG,5X,F12.2,6X,F12.2,	CARD0816
	1 7H CARCHG)	CARD0817
3044	FORMAT(6X,7H TOTMAT,5X,F12.2,6X,8H TOTPROC,4X,F12.2,6X,F12.2,	CARD0818
	1 7H TOTALS)	CARD0819
		CARD0820
		CARD0821
		CARD0822
		CARD0823
		CARD0824
4013	PRINT 4013	CARD0825
4013	FORMAT(//27H POWER COSTS (\$/KGHMLoaded))	CARD0826
	FAMPD(J) = FAMPD(J) * RTIMEY	CARD0827
	FAFBD(J) = FAFBD(J) * RTIMEY	CARD0828
	FARPD(J) = FARPD(J) * RTIMEY	

FAMCD(J) = FAMCD(J)	* RTIMEY	CARD0829
FADT(J) = FADT(J)	* RTIMEY	CARD0830
FAMPCC(J) = FAMPCC(J)	* RTIMEY	CARD0831
FAFBCC(J) = FAFBCC(J)	* RTIMEY	CARD0832
FARPCC(J) = FARPCC(J)	* RTIMEY	CARD0833
FAMCCC(J) = FAMCCC(J)	* RTIMEY	CARD0834
FACCT(J) = FACCT(J)	* RTIMEY	CARD0835
FARPCB(J)=FARPCB(J)	* RTIMEY	CARD0836
FAMCCB(J) = FAMCCB(J)	* RTIMEY	CARD0837
FACCTB(J) = FACCTB(J)	* RTIMEY	CARD0838
FAMPTT(J) = FAMPTT(J)	* RTIMEY	CARD0839
FAFBTT(J) = FAFBTT(J)	* RTIMEY	CARD0840
FARPTT(J) = FARPTT(J)	* RTIMEY	CARD0841
FAMCTT(J) = FAMCTT(J)	* RTIMEY	CARD0842
FAT(J) = FAT(J)	* RTIMEY	CARD0843
FARPTB(J) = FARPTB(J)	* RTIMEY	CARD0844
FAMCTB(J) = FAMCTB(J)	* RTIMEY	CARD0845
FATB(J) = FATB(J)	* RTIMEY	CARD0846
ABU(J) = ABU(J)	* RTIMEY	CARD0847
AINV(J) = AINV(J)*RTIMEY		CARD0848
AINVB(J) = AINVB(J)	* RTIMEY	CARD0849
AMAT(J) = AMAT(J)	* RTIMEY	CARD0850
AMATB(J) = AMATB(J)	* RTIMEY	CARD0851
APRSD(J) = APRSD(J)	* RTIMEY	CARD0852
APRSC(J) = APRSC(J)	* RTIMEY	CARD0853
APRST(J) = APRST(J)	* RTIMEY	CARD0854
APRSCB(J) = APRSCB(J)	* RTIMEY	CARD0855
APRSTB(J) = APRSTB(J)	* RTIMEY	CARD0856
AAT(J) = AAT(J)	* RTIMEY	CARD0857
AATB(J) = AATB(J)	* RTIMEY	CARD0858
AADT(J) = AADT(J)	* RTIMEY	CARD0859
AACCT(J) = AACCT(J)	* RTIMEY	CARD0860
AACCTB(J) = AACCTB(J)	* RTIMEY	CARD0861
PRINT 804		CARD0862
PRINT 1014, FAMPD(J), FAFBD(J), FARPD(J), FAMCD(J), FADT(J),		CARD0863
1 FARPD(J), FAMCD(J), FADT(J)		CARD0864

PRINT 1015, FAMPCB(J),FAFBCC(J),FARPCB(J),FAMCCC(J),FACCT(J),	CARD0865
1 FARPCB(J),FAMCCB(J),FACCTB(J)	CARD0866
PRINT 1016, FAMPTT(J),FAFBTT(J),FARPTT(J),FAMCTT(J),FAT(J),	CARD0867
1 FARPTB(J),FAMCTB(J),FATB(J)	CARD0868
PRINT 3030	CARD0869
PRINT 3042, ABU(J),APRSD(J),AADT(J)	CARD0870
PRINT 3043, AINV(J),APRSC(J),AACCT(J)	CARD0871
PRINT 3044, AMAT(J),APRST(J),AAT(J)	CARD0872
PRINT 3031	CARD0873
PRINT 3042, ABU(J),APRSD(J),AADT(J)	CARD0874
PRINT 3043, AINVB(J),APRSCB(J),AACCTB(J)	CARD0875
PRINT 3044, AMATB(J),APRSTB(J),AATB(J)	CARD0876
	CARD0877
RMCD = FAMCD(J)/(FAMPD(J)+FAFBD(J)+FARPD(J))	CARD0878
RMCT = FAMCTT(J)/(FAMPTT(J)+FAFBTT(J)+FARPTT(J))	CARD0879
RMCTB = FAMCTB(J)/(FAMPTT(J)+FAFBTT(J)+FARPTB(J))	CARD0880
PRINT 8210	CARD0881
8210 FORMAT(/5X,23H MATCRE/MATPUR+FAB+REPR)	CARD0882
PRINT 8211	CARD0883
8211 FORMAT(3X,7H DIRECT,2X,3X,7H TOT(A),2X,3X,7H TGT(B))	CARD0884
PRINT 8212, RMCD, RMCT, RMCTB	CARD0885
8212 FORMAT(1X,3F12.6)	CARD0886
	CARD0887
AU28M(J) = 0.0	CARD0888
AP49M(J) = 0.0	CARD0889
AP40M(J) = 0.0	CARD0890
AP41M(J) = 0.0	CARD0891
AP42M(J) = 0.0	CARD0892
AMPD(J) = 0.0	CARD0893
FAMPD(J) = 0.0	CARD0894
AFBD(J) = 0.0	CARD0895
FAFBD(J) = 0.0	CARD0896
ARPD(J) = 0.0	CARD0897
FARPD(J) = 0.0	CARD0898
AMCD(J) = 0.0	CARD0899
FAMCD(J) = 0.0	CARD0900

ADT(J) = 0.0
FADT(J) = 0.0
AMPCC(J) = 0.0
FAMPCC(J) = 0.0
AFBCC(J) = 0.0
FAFBCC(J) = 0.0
ARPCC(J) = 0.0
FARPCC(J) = 0.0
AMCCC(J) = 0.0
FAMCCC(J) = 0.0
ACCT(J) = 0.0
FACCT(J) = 0.0
AMP TT(J) = 0.0
FAMP TT(J) = 0.0
AFB TT(J) = 0.0
FAFB TT(J) = 0.0
ARPTT(J) = 0.0
FARPTT(J) = 0.0
AMCTT(J) = 0.0
FAMCTT(J) = 0.0
AT(J) = 0.0
FAT(J) = 0.0

AEFTOT(J) = 0.0
ABNUP(J) = 0.0
SFTV(J) = 0.0
AFT(J) = 0.0

ARPCCB(J) = 0.0
ARPTTB(J) = 0.0
AMCCCB(J) = 0.0
AMCTTB(J) = 0.0
ACCTB(J) = 0.0
ATB(J) = 0.0
FARPCB(J) = 0.0
FAMCCB(J) = 0.0

CARD0901
CARD0902
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CARD0928
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CARD0934
CARD0935
CARD0936

FACCTB(J) = 0.0
 FARPTB(J) = 0.0
 FAMCTB(J) = 0.0
 FATB(J) = 0.0
 A28MR(J) = 0.0
 A49MR(J) = 0.0
 A41MR(J) = 0.0
 A42MR(J) = 0.0
 EPS(J) = 0.0
 ABU(J) = 0.0
 AINV(J) = 0.0
 AINVB(J) = 0.0
 AMAT(J) = 0.0
 AMATB(J) = 0.0
 APRSD(J) = 0.0
 APRSC(J) = 0.0
 APRST(J) = 0.0
 APRSCB(J) = 0.0
 APRSTB(J) = 0.0
 AAT(J) = 0.0
 AATB(J) = 0.0
 AADT(J) = 0.0
 AACCT(J) = 0.0
 AACCTB(J) = 0.0

10000 CONTINUE

NCOUNT = 0.0
 AHMKGL = 0.0
 ATHMLD = 0.0
 AVOL = 0.0
 AU28MD = 0.0
 AP49MD = 0.0
 AP40MD = 0.0
 AP41MD = 0.0

CARD0937
 CARD0938
 CARD0939
 CARD0940
 CARD0941
 CARD0942
 CARD0943
 CARD0944
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