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NUCLEAR PIPING DESIGN

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NUCLEAR PIPING DESIGN

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OAK RIDGE NATIONAL LABORATORY
Oak Ridge, Tennessee 37830
operated by
UNION CARBIDE CORPORATION
Nuclear Division
for the
U.S. ATOMIC ENERGY COMMISSION

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FOREWORD

This document was originally prepared by Teledyne Materials Research, a Teledyne Company, under Subcontract Number 3059 with Union Carbide Corporation, Nuclear Division, as an activity of the RDT Standards Program at Oak Ridge National Laboratory. The information was compiled by D. F. Landers of Teledyne Materials Research under the direction of W. R. Gall of Oak Ridge National Laboratory. This document is intended as a reference to guide those procuring nuclear piping to be used in water-cooled nuclear reactor systems under the purview of the United States Atomic Energy Commission Division of Reactor Development and Technology. It may also be of use to others as a guide to the application of the rules of the ANSI Standard Code for Pressure Piping B31.7, Nuclear Power Piping, and Section III, Nuclear Power Plant Components, of the ASME Boiler and Pressure Vessel Code in the design of nuclear power piping.

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NUCLEAR PIPING DESIGN

Abstract

The design of piping systems to comply with the rules for Class-1 piping stipulated in the ANSI Standard Code for Pressure Piping B31.7, Nuclear Power Piping, and in Section III, Nuclear Power Plant Components, of the ASME Boiler and Pressure Vessel Code is discussed in this manual. The rules are explained where clarification is needed, and methods of analysis for pressure, thermal, cyclic, and earthquake loads are presented in detail. Guidance in the preparation of the design specification is also presented.

1. INTRODUCTION

Piping systems purchased for nuclear power plants prior to July 1, 1971, are required to meet the rules set forth in the ANSI Standard Code for Pressure Piping B31.7, Nuclear Power Piping.¹ Those purchased after July 1, 1971, are required to meet the rules for piping set forth in Section III, Nuclear Power Plant Components, of the ASME Boiler and Pressure Vessel Code.² The rules of ANSI B31.7 have been incorporated in Subarticle NB-3600 of the 1971 edition of Section III of the ASME Boiler and Pressure Vessel Code, hereafter referred to as "Section III". The objective of these codes is to require comprehensive analysis of all parts of nuclear piping systems to attain a high level of confidence in the capability of these systems to sustain all possible loads without failure throughout their intended lifetime. Techniques and methods of analysis that meet the design requirements established in the Code are presented herein to provide guidance in interpreting these requirements. The requirements and the technical justification for them are discussed in some cases involving departures from past practice to assist the designer in understanding the effect of the new approaches on the design and analysis of the system.

In general, the requirements of ANSI B31.7 parallel those for vessels in Section III. However, the amount of effort that would be required to perform complete and detailed analyses of all points in the piping systems of a nuclear plant would far surpass that required for all the vessels in the plant. Consequently, a simplified approach is permitted that enables the designer to establish the worst possible combination of stresses that could exist in a piping system and to compare this condition with the specified requirements. For those points that by this simplified analysis do not meet the requirements, the designer may choose to either redesign to reduce stresses or make detailed analyses of the overstressed points to determine whether the requirements are met.

Although criteria for combining stresses and the limits to be met by various combinations are given in the Code, methods of analysis for determining all stresses are not given. Methods of analysis that may be used to meet the criteria are given in this manual. However, it is not to be inferred that these are the only acceptable methods or that they are suitable for all applications. The designer may use any method that he can substantiate as being adequate for the problem at hand.

The information presented in this manual is applicable to all reactor coolant and associated system piping with operating temperature limitations of 700°F and below for ferritic steel piping and 800°F and below for austenitic stainless steel and nonferrous piping and to all external structures attached to the piping such as those required for support, operation, servicing, and testing. This information is presented in the order in which the piping designer might perform his work. Since this manner of presentation may not agree with the order of presentation established in the ANSI Standard Code B31.7 or in Section III of the ASME Boiler and Pressure Vessel Code, reference to the applicable B31.7 paragraph is made for convenience and the corresponding paragraph in Section III is given within parentheses.

2. PRESSURE DESIGN

The initial task confronting the piping designer is determination of the wall thicknesses required for a particular piping system. The procedures used are not new, and few new rules to meet the basic wall thickness requirements are presented in ANSI B31.7 and Section III. Some aspects of these procedures are discussed in Refs. 3 through 10.

2.1 Basic Wall Thickness

The flow requirements and nominal pipe sizes for a piping system must be given in the design specification. With this information and the design pressure, the minimum required wall thickness (t_m) can be determined by using Eq. 1 in Division 1-704 of ANSI B31.7 (NB-3641).

$$t_m = \frac{PD_o}{2(S_m + yP)} + a \quad (1)$$

where

P = internal design pressure, psi,

D_o = outside diameter of pipe, in.,

S_m = maximum allowable stress in material caused by internal pressure at the design temperature, psi,

y = 0.4, and

a = additional thickness, in.

One important departure from past practice should be recognized when determining the minimum required wall thickness of piping. Subdivision 1-705.1 of ANSI B31.7 (NB-3652) stipulates in Eq. 9 that the internal pressure stress, seismic stress (single amplitude), and weight stresses be less than $1.5S_m$.

$$B_1 \left(\frac{PD_o}{2t} \right) + B_2 \left(\frac{D_o}{2I} \right) M_1 \leq 1.5S_m \quad (9)$$

The designer will be dealing with austenitic stainless steel or other nonferrous piping materials in many applications. Because of the cost per pound of these materials, the tendency is to provide very little

margin on the calculated and purchased minimum wall thicknesses. This tendency can lead to problems relative to the provision of adequate reinforcement in branch connections that could result in high primary plus secondary stress [Eq. 10 in Subdivision 1-705.2 of ANSI B31.7 (NB-3653.1)]. Such problems can be solved by providing increased wall thickness in the region of the branch connection only. However, while providing this increased thickness, the designer must also be aware of the effect of thickness on thermal stresses.

In the case of low-pressure systems, Eq. 9 may govern determination of the minimum required wall thickness. It can be seen that increasing the thickness will reduce the calculated stress of Eq. 9 by directly increasing the thickness (t) in the pressure load part of the equation and by increasing the moment of inertia (I) in the weight and seismic load part of the equation. The achievement of an acceptable thickness by using Eq. 9 becomes an iterative process. That is, the thickness from Eq. 1 is used, and if the calculated stress is greater than $1.5S_m$, another thickness must be selected and a new weight analysis must be made. This new thickness and weight analysis will require reconsideration of the support locations and loadings. This process must be repeated until an acceptable stress level is calculated.

The designer can with experience resolve the problem of the minimum wall thickness in low-pressure systems being controlled by weight and seismic considerations rather than internal pressure by applying a generous "a" factor to Eq. 1. In addition to threading and corrosion requirements, the "a" factor provides for structural strength of the pipe during erection.

2.2 Standard Fittings

No minimum thickness analysis is required for fittings purchased and used in accordance with the approved standards and pressure ratings given in Table 1-726.1 of ANSI B31.7 (Table NB-3691-1). However, the designer must provide assurance that short-radius elbows manufactured in accordance

with ANSI B16.28¹¹ shall have a minimum thickness in the crotch region 20% greater than required by Eq. 1 in Division 1-704 of ANSI B31.7 (NB-3641). The properties of short-radius welding elbows and tube bends are discussed in Refs. 12 through 15.

2.3 Pipe Bends

The wall thickness of pipe bends after bending must meet the minimum wall thickness requirements calculated by using Eq. 1 in Division 1-704 of ANSI B31.7 (NB-3641). A list of suggested minimum thicknesses prior to bending is given in Table 1-704.2.1 of ANSI B31.7 (Table NB-3642.1-1). These values are based on experience and good shop practice, but they do not assure satisfaction of the minimum wall thickness requirements or freedom from wrinkling in the crotch during bending. The designer is cautioned to discuss this with the fabricator to insure that these requirements are met. Failure to meet them could result in the designer having to perform a detailed analysis of the pipe in accordance with the rules of Appendix F of ANSI B31.7 (NB-3200) or develop stress indices for the bend to be used in the applicable equations of Division 1-705 of ANSI B31.7 (NB-3650). Various aspects of pipe bends are discussed in Refs. 16 through 34.

2.4 Intersections

Intersections that are not purchased in accordance with the applicable standards given in Table 1-726.1 of ANSI B31.7 (NB-3691-1) must be analyzed in accordance with the rules given in Subdivision 1-704.3 of ANSI B31.7 (NB-3643). The rules of this subdivision are based on the area replacement technique. That is, if a portion of pipe material that is subject to membrane stress is removed, that material must be replaced in close proximity to the area of removal. The amount of metal to be replaced and the location or limits within which it must be provided are described in detail in ANSI B31.7 (NB-3600) for both welded branch

connections and extruded outlets. Methods for the analysis of nozzles and intersections are discussed in Refs. 35 through 61.

2.5 Mitered Joints, Miscellaneous Fittings, and Flanges

Limitations are stipulated in ANSI B31.7 (NB-3600) for the use of mitered joints, and these limitations pertain to minimum thickness and geometry. No stress indices are available, and the use of mitered joints would require the development of these indices by the designer. The design of mitered joints is covered in Chapter 7 of Ref. 4.

Special rules are provided for attachments, blanks, reducers, and flanges. One item requiring careful attention by the designer is the flange. The rules in ANSI B31.7 and Section III (NB-3600) for flange design differ from those of past practice, particularly with respect to bolting. At the time ANSI B31.7 was being written, the opinion of the committee was that flanged joints are not generally used in Class-1 piping systems. Therefore, little guidance in bolting analysis or indices is provided in ANSI B31.7 or Section III. However, nuclear reactor plants that are experimental in nature could require the use of flanged joints.

The loadings that must be considered when flange bolting is analyzed are preload, pressure, differential thermal expansion of the mating flanges and the bolts, and expansion moments and forces. Some of these loads are discussed in Refs. 62 through 68. The methods of analysis used to determine these loads are quite detailed, and they are given in Appendix A of this manual. The resulting stress analysis using the loads determined in Appendix A is covered in detail in the applicable sections of this manual. At this point in the design, the designer need only be aware that the final selection of bolt size and material must be based on analyses to be provided later and that these analyses are quite different from those of past practice.

2.6 Wall Transitions and Pipe Ovality

The analysis of joints that involve a change in wall thickness, as illustrated in Fig. 1-727.3.1 of ANSI B31.7 (NB-4233-1), as well as joints between pipe and socket-welding fittings is discussed in Chapter 10 of Ref. 4 and in Ref. 69.

The determination of stresses in straight pipe that is out of round and is subject to internal pressure is discussed in Chapter 6 of Ref. 4 and in Refs. 70, 71, and 72.

3. INITIAL FLEXIBILITY ANALYSIS

The case of weight loading was discussed briefly in Subsection 2.1 in relation to Eq. 9 given in Subdivision 1-705.1 of ANSI B31.7 (NB-3652). One problem concerning the possibility of iteration for low-pressure systems was pointed out in this discussion. A more basic problem arises from the situation where the standard procedure of the design agency has been to rely on a hanger designer and supplier to provide adequate pipe support. The support for the piping system provided by the hanger designer was such that the maximum bending stress would not exceed a given value. For example, meeting the requirements of the hanger spacing table provided in ANSI B31.1⁷³ assured a bending stress in a straight pipe of approximately 1500 psi. One might assume that reliance on a hanger designer and supplier to provide adequate pipe support is no longer acceptable because of the need to include weight effects in the moment (M_1) term of Eq. 9 early in the design stage. This assumption is incorrect.

The requirements of Eq. 9 in ANSI B31.7 and NB-3600 can be satisfied by following past procedures with one exception. The hanger designer must be given the maximum allowable weight moment that can be carried by the system. The value of this moment can be readily determined by deciding how much of the allowable stress ($1.5S_m$) should be set aside for weight loading and calculating the moment required to produce that stress. The ANSI Code B31.7 and Section III require that consideration of all restraints, including hangers, be included in the analysis of the piping system for expansion moments. The common practice of relying on a hanger supplier for the design of hangers, supports, and anchors does not relieve the piping designer of his responsibility for these items.

When the piping designer provides his own supports, an initial flexibility analysis is required. This analysis is performed to provide a preliminary check on the flexibility stresses and to provide the deflection information necessary to properly design the support system. Design of the support system follows this analysis. The initial flexibility analysis is performed on the layout of the piping system, including all known attachments to the piping such as branch lines, equipment, etc. In

addition, this analysis shall include all equipment deflections and rotations and account for the maximum range of service temperatures anticipated. No other considerations need be included at this time since the purpose of the analysis is to determine the maximum deflections, thereby permitting the design of a proper and adequate support system. Methods of performing flexibility analyses of piping systems are discussed in Refs. 74 through 79.

3.1 System Imbalance

There are conditions which may bring about major unbalanced effects in piping systems that are not accounted for in ANSI B31.7 or Section III. Some of these are pointed out here since they affect stress levels.

Strains calculated on an elastic basis are sufficiently accurate for systems in which there are no severe plastic strain concentrations. However, elastic-based calculations fail to reflect the actual strain distribution in unbalanced systems where only a small length of the piping undergoes plastic strain while the major portion of the length remains essentially elastic. In these cases, the weaker or higher stressed portions will be subjected to plastic strain concentrations because of the elastic follow-up of the stiffer or lower stressed portions of the piping. That is, the imposed deflection on the piping system will be absorbed almost entirely in the weaker portion of the system and the remainder of the system will remain unchanged, as is illustrated in Fig. 3.1.

Fig. 3.1. Absorption of Loading by Weaker Portion of Unbalanced Piping System.

This imbalance of a piping system can be produced by

1. the use of small pipe runs in series with larger or stiffer pipe with the small pipe relatively lightly stressed,
2. local reduction in size of a cross section or local use of a weaker material, and
3. by the use in a system of uniform size of a configuration in which most of the piping lies near a straight line drawn between the anchors or terminals with only a small portion of the piping projected away from this line that absorbs most of the expansion strain.

Conditions such as these should be avoided, particularly where materials of relatively low ductility are used. Piping design to minimize creep concentrations is discussed in Ref. 80.

3.2 Longitudinal Loads

Direct stresses resulting from longitudinal loads are not calculated in Division 1-705 of ANSI B31.7 (NB-3650) because the moment loading in a normal configuration that is caused by the longitudinal load produces bending stresses so large that in comparison the direct stresses become insignificant. Although unlikely, a case could develop where the direct stress resulting from the longitudinal load is significant. For example, a straight run of pipe between two anchor points that is heated will produce zero bending moments but very high longitudinal compressive loads. The designer is therefore cautioned to check end loadings to determine the magnitude and effect of the calculated loads.

4. SUPPORT AND VIBRATION CONTROL

There are no specific methods provided in ANSI B31.7 or Section III for support and/or vibration control of a piping system. There is a requirement that all stresses be calculated and compared with allowable values. This requirement presents no problems relative to weight stresses since many of the presently available flexibility computer programs can be used to calculate weight stresses.

A procedure for performing the design of a system of support is presented here. The initial flexibility analysis provides the designer with deflections of the piping system at any point, and this information is used to determine whether a spring or rigid type of hanger will be used when the location of the support is determined. The spacing, location, and selection of hangers are discussed in Refs. 73, 74, 75, 77, and 81.

4.1 Hanger Spacing

The data tabulated below were taken from ANSI B31.1⁷³ to provide the designer with a conservative guideline for the selection of the initial hanger spacing for the piping system. The data given are for straight pipe without concentrated loads.

Nominal Pipe Size (in.)	Suggested Maximum Span in Feet Between Supports for Pipe Containing	
	<u>Water</u>	<u>Steam, Gas, or Air</u>
1	7	9
2	10	13
3	12	15
4	14	17
6	17	21
8	19	24
12	23	30
16	27	35
20	30	39
24	32	42

The data tabulated on the preceding page are based on the equation

$$S = 1.2 \frac{w\ell^2}{z},$$

where

S = maximum bending stress, psi,

w = total unit weight, lb/ft,

ℓ = length of pipe span between supports, ft, and

z = section modulus, in.³

The tabulated values are based on the assumption that the value of S = 1500 psi, but this value may be varied by the designer. However, he is cautioned not to choose a high value of S since it must be included with the values for pressure and other effects in meeting the allowable stresses of Eq. 9 in the Code.

4.2 Hanger Location Considerations

By using the data given in Subsection 4.1, the designer has arrived at an initial hanger spacing. However, the location of the actual supports involves some additional considerations relative to the piping itself, adjacent structures, calculated deflections, and accessibility. Six major considerations pertinent to the location of supports are as follows.

1. The supports should be located as close as possible to heavy concentrated loads imposed on the piping system by valves, flanges, minor vessels, etc. However, the designer must avoid attaching the support directly to a component (valve, strainer, etc.) that structurally is not his responsibility without first checking with the component supplier to determine the acceptability of the support.

2. Piping supports should be located on straight runs rather than on bends, elbows, or tees. These latter components are usually the most highly stressed portions of the system, and any additional restraint at these locations should be avoided. The localized restraint of welded attachments to elbows and bends will reduce the flexibility of these components and require experimental determination of the ANSI B31.7 stress index and flexibility factor.

3. The structure used as a foundation for the hanger must be capable of carrying the load imposed by the piping. The ideal situation is to attach supports to large structural members such as columns or trusses. It may be necessary to provide additional intermediate steel reinforcement to have an adequate foundation for hangers. This additional steel reinforcement should be provided judiciously to eliminate interferences with other piping, equipment, electrical cables, etc.; and the steel should be checked by the designer to insure its adequacy for carrying the imposed loading and to determine its deflection at the point of support attachment. The deflection of the intermediate steel reinforcement is required as input to the flexibility analysis. The designer responsible for the building structure should approve all support attachments and/or additional foundations.

4. Hangers which require examination after installation must be accessible for the required examination. Supports should not be located on sections of piping that require periodic removal. Avoiding such locations will eliminate the need for temporary support of the adjacent piping during removal.

5. Whenever possible, unidirectional supports should be located at points of zero or minimum deflection in the direction of the support. This will eliminate the need for springs and permit the use of rigid supports. Obviously, some decisions must be made concerning this consideration in relation to the first consideration, the most critical being the concentrated load situation of the first consideration. The restraint of rigid supports on the deflection of the piping system in the other two directions (normal to the plane of the support) must be considered. Short rod hangers can provide considerable restraint in these other directions, as is illustrated in Fig. 4.1. The free deflection of point A in Fig. 4.1 is along the X axis. By applying a rigid support that is short, the designer can introduce an additional restraint that will result in the displacement Z' . This displacement should be a part of the final flexibility analysis.

Fig. 4.1. Restraint of Short Rod Hangers on Deflection of Piping in Directions Normal to the Plane of the Support.

4.3 Hanger Selection

Once the locations of the hangers have been determined, the types of hangers to be used in these locations must be selected. There are two general types of hangers: rigid and spring.

4.3.1 Rigid Hangers

The word "rigid" is used to describe the essential stiffness of the hanger in the direction of the support. A rod hanger with eyebolt attachments to the building structure and the pipe provides freedom in two directions (X and Z) and is essentially rigid in the direction of the rod (Y), as is illustrated in Fig. 4.2. There are many other types of rigid hanger, and the degree of rigidity provided can vary from complete restraint in all three directions, as is provided by a structural anchor, to restraint in only one direction, as is provided by the rod hanger illustrated in Fig. 4.2.

Fig. 4.2. Typical Rod Type of Rigid Pipe Hanger.

4.3.2 Spring Hangers

The word "spring" is used to describe any hanger that is not considered rigid in the direction of the support. Such hangers are generally helical spring hangers, but any fabricated support that acts as a spring is considered to be a spring hanger.

A spring hanger provides complete or partial freedom in three directions (X, Y, and Z), as is illustrated in Fig. 4.3. The helical spring

Fig. 4.3. Typical Helical Spring Pipe Hanger.

hanger illustrated in Fig. 4.3 provides some restraint in the direction of support, but when compared with the rigid type of hanger, the helical spring hanger is relatively free. The spring constant, which acts as a restraint in the direction of the support, of the spring hanger must be included in the flexibility analysis. This spring constant is readily available for standard types of spring hangers on the market, but the constant must be calculated or determined experimentally for special hangers.

There is one special type of spring hanger that provides a supporting load which does not vary with deflection. That is, the spring constant is zero and the hanger provides no restraint in the direction of the support. This type of spring hanger is commonly referred to as a constant support hanger. Constant compression of the spring is maintained by providing a linkage to permit free deflection. The constant support hanger is usually located at points of high deflection (above 2 in.) or at equipment connections where the calculated expansion load must be maintained.

4.3.3 Selection Considerations

The selection of the best type of hanger is dependent on a number of conditions of which the most important is the effect of the hanger on vibration control and on the flexibility of the system. The use of rigid hangers is the most effective technique in controlling vibration, and this consideration must be weighed against the effect of rigid restraints on the stresses in the piping system. Some guidelines for making hanger selections are as follows.

1. Rigid supports should be located at
 - (a) points of zero deflection in the direction of the support load,
 - (b) points of negligible deflection in the direction of the support load in relation to the remainder of the system, and
 - (c) points where the deflection in the direction of the support load is not negligible but the stresses are so low that an additional restraint probably will not overstress the system.

2. Spring supports should be located at
- (a) points of large* vertical deflection and
 - (b) near equipment connections where additional restraint could overload the equipment.

4.4 Hydraulic Snubbers

Hydraulic snubbing devices which increase damping may be used to control vibration of the piping system. These devices offer high resistance to rapid displacements caused by dynamic loads while permitting essentially free movement of piping undergoing the very gradual displacements caused by normal (non-dynamic) loads such as those imposed by thermal expansion. The hydraulic snubber is designed to lock when subjected to an impulse (dynamic load), thereby not permitting relatively rapid motion of the piping. The results of tests performed on some hydraulic snubbers indicate that they essentially perform this intended function with varying degrees of lockup efficiency dependent upon the direction of the load (compression or tension on the snubber) and the velocity of the load. Relationships required to determine the maximum force the snubber must transmit and the degree of damping required to reduce the amplitude of vibration to a tolerable magnitude are given on page 270 of Ref. 77.

Hydraulic snubbers may also be used at intermediate points in a piping system as a means of increasing the response frequency of the system to seismic motion.

4.5 Role of Design and Analysis in Installation

Previous experience with nuclear reactor plants indicates that the coordination of design, analysis, and installation activities related to piping supports has been a weak area in the development of these plants.

*The absolute value of the vertical deflection can vary from system to system before being considered large, but it averages about 1/4 in.

The following problems relative to piping supports have been among the most evident.

1. Failure of the design agency to provide the hanger designer and supplier with the ranges of piping deflection computed in an initial flexibility analysis has resulted in insufficient travel in the spring hangers supplied.

2. The lack of consideration of system operating loads has resulted in the imposition of much larger loads on the support system than those that would be imposed by thermal expansion. Some of these operating loads are imposed by (1) sudden pressurization of the system, (2) sudden stoppage of flow, (3) sudden changes in pump speed, and (4) water slugs.

3. Inadequate deflection values can be obtained when the initial analysis of the piping system accounts for an intermediate anchor and all that is installed is a restraint against deflection and not rotation. This analysis procedure generally results in conservative values with respect to the moments on the system, but the deflection characteristics of the piping can change considerably, making the support system inadequate. For example, consider the piping system illustrated in Fig. 4.4. Points A, B, and C are analyzed as anchors, but in the support design and

Fig. 4.4. Diagram Illustrating Possible Differences Between Analytical and Design and Installation Approaches for Piping System Supports.

installation, points A and C are built as force and moment absorbing structures (anchors) while point B is built as a force absorbing structure only (a pin point). In the analysis where points A, B, and C are considered to be anchors, point 1 will deflect in a negative Y direction (down). During actual operation of the piping system with point B installed as a pin point, point 1 will deflect in a positive Y direction (up). If insufficient travel is available in the support system near point 1, the system as installed could be inadequate.

4. Other typical sources of support problems in piping systems arise from improper installation of restraints such as installation in the wrong direction, inadequate support structure for restraints, and insufficient clearances on limit stops.

The designer is responsible for insuring that all hangers, supports, restraints, and anchors are installed as designed. The locations and directions of restraints should be checked, and this check should also provide assurance that no additional restraints are present. Cold loads on all hangers should be checked to insure proper installation. The available travel on hydraulic snubber pistons and spring hangers should be compared with the predicted thermal deflections at the snubber and hanger locations to insure that the available travel is adequate.

5. DETERMINATION OF EXTERNAL LOADS

The equations in Division 1-705 of ANSI B31.7 (NB-3650) require that the resultant moment (M_1) be known. The various loading conditions that must be considered when calculating the external loads that make up M_1 are discussed here. The actual techniques used to combine the moments resulting from various loading conditions to arrive at M_1 for Eqs. 9 and 10 of the Code are given in Sections 6 and 7, respectively, of this manual. To determine the resultant moment, analyses must be performed to determine the loads resulting from

1. thermal expansion and equipment displacements,
2. weight effects, and
3. seismic effects.

5.1 Thermal Expansion Loads and Equipment Displacements

The analysis to determine the loads resulting from thermal expansion requires consideration of the actual operating conditions rather than the design conditions. For example, where the temperature of one line in a multi-branch system is held essentially constant while the temperature of the other lines fluctuates, the loads for these fluctuations must be calculated for each specific transient. An example of such a system is illustrated schematically in Fig. 5.1. Assume that Line 1 is continually operated at a temperature of 500°F, Line 2 is operated at temperatures between 285 and 513°F, and that Line 3 is operated at temperatures between ambient and 500°F. This results in three different maximum temperature conditions, which are tabulated below.

Condition Number	Temperature in Line		
	1 (°F)	2 (°F)	3 (°F)
1	500	285	85
2	500	513	85
3	500	500	500

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Fig. 5.1. Example of a Multi-Branch System Where Temperature of One Line is Held Constant While Temperature of Other Lines Fluctuates.

The designer is required to analyze the three conditions for the piping arrangement tabulated on the preceding page, and not one of these conditions is the same as would be developed by using the maximum or design temperature for each line at the same time.

Any equipment displacements from causes other than thermal expansion must also be considered when performing the final flexibility analysis. The effects of motion of all branch lines, equipment, restraints, and supports as well as their resistance to motion of the pipe must be included. Where variable support spring hangers and spring sway braces are used, the spring constant of the hanger or brace must be included in the analysis. As a minimum, the forces and moments acting on the system should be calculated at

1. the end of all welded fittings (tees, elbows, reducers, etc.);
2. the midpoint of all bends and elbows;
3. a distance of one diameter on the run and branch of all branch connections not covered in item 1;
4. all equipment connections (vessels, pumps, valves, etc.), anchors, and guides; and
5. the mating surface of all flanged joints.

Flexibility factors are given in the Code for standard components only. It is conservative to assume that a component is rigid (provides no flexibility), and this is quite common practice for flanges and valves. The designer may develop a flexibility factor for a given component by using the rules outlined in Appendix E-300 of ANSI B31.7 (NB-3680).

5.2 Weight Loading

Weight loading has often been neglected in the past for two reasons; one being justifiable and the other questionable. The justifiable approach has been to provide supports in a manner outlined in Section 4, thereby reducing the stresses imposed on the system by weight effects to insignificant values. The questionable approach has been to assume that weight loading is not a critical problem. At the present time, the calculation of stresses resulting from weight loading is required by the Code regardless of whether or not piping supports have been provided.

Most flexibility computer programs in use today calculate weight stresses by using one of two generally acceptable techniques. The first technique is to provide uniform loading to the piping system as a function of the weight of the pipe and contained fluid. The second technique is to provide concentrated loads on the piping system at specified intervals. The concentrated loads represent the weight of the pipe and contents for a specific length of piping. The second technique will give results that are almost identical to those of the first technique if the intervals or spacings of the concentrated loads are reasonable.

5.3 Seismic Loads

Seismic loads must be known to calculate the resultant moment (M_1) used in Eqs. 9, 10, and 11 of the Code. The design specification should stipulate the plant operating condition (full load, etc.) under which the specified earthquake is assumed to occur. Two sets of seismic moments are required to perform a Code analysis. The first set includes only

the moments resulting from inertia effects (seismic weight), and these moments are used in the resultant moment (M_i) value for Eq. 9. The second set includes the moments resulting from inertia effects plus the moments resulting from seismic motion of attachments (vessels, anchors, supports, etc.) to the piping, and these moments are used in the M_i value for Eqs. 10 and 11. The calculation of seismic loads is discussed in detail in Section 9 of this manual.

5.4 Expansion Tests

An expansion test has been included as a part of the pre-operational testing of some of the newer nuclear reactor plants. During pre-operational testing, actual deflections at pre-established locations are measured and compared with the predicted deflections for these locations. At this point, two important questions arise. They are (1) What is an acceptable deviation between the predicted and measured values of deflection? and (2) What should be done when an acceptable deviation is exceeded?

Examining the second question first, one must consider just what information went into the predicted values. Normally, a flexibility analysis for a given temperature in which assumed values are assigned for the deflections of attached vessels is used to predict system deflections at established locations. During actual operation, temperature variations in these vessels, vessel supports, and in the piping system can affect deflections drastically. It is therefore imperative that all possible information relative to actual operating temperatures, piping geometry, and support structures be known before the deflections are predicted.

The information used in the deflection predictions should be checked when actual deflections are measured. When all concerned parties agree that the conditions for the predicted and measured deflections are the same, an allowable deviation from the predicted values can be applied. If this deviation is exceeded, a detailed site inspection of the piping system should be made to establish that no unaccounted for interferences exist. If none are present, the flexibility analysis should be rerun

using the measured deflection values. If the results of this new analysis indicate acceptable stress and moment levels, a procedure for in-service measurement of deflections for the piping system should be instituted. If these measured values show repeatability, the system should be considered acceptable.

To establish an acceptable deviation from the predicted values of deflection at pre-established locations in the piping system, the stress or moment levels in the system should be considered. When performing the type of analysis prescribed in ANSI B31.1 and Section III, an allowable deviation could be calculated by using the expression

$$\Delta = 1 - \frac{S_{\text{actual}}}{S_{\text{allowable}}}, \quad (5.1)$$

where

Δ = allowable deviation,

S_{actual} = calculated expansion stress, and

$S_{\text{allowable}}$ = allowable stress.

When performing the type of analysis prescribed in the Code, an allowable deviation could be calculated by substituting moments for stress in Eq. 5.1.

$$\Delta = 1 - \frac{M_{i \text{ actual}}}{M_{i \text{ allowable}}} \quad (5.2)$$

where

$M_{i \text{ actual}}$ = calculated resultant moment, and

$M_{i \text{ allowable}}$ = maximum allowable resultant moment (M_1) from Eq. 10 of the Code when all loads and operating conditions are considered.

6. PROTECTION AGAINST MEMBRANE FAILURE

Some discussion of the philosophy behind the equations in Division 1-705 of ANSI B31.7 (NB-3650) and the applicable stress indices is appropriate. It should be noted that this discussion is applicable to all equations and is not limited to just membrane protection only.

6.1 Philosophy Behind Design Rules of the Code

During formulation of the design rules of ANSI B31.7, two questions had to be answered. These questions were (1) How can the design philosophy of The Criteria of Section III of the ASME Boiler and Pressure Vessel Code⁸² be utilized without completely confusing the average piping designer who has never been required by previous piping codes to analyze discontinuity and thermal stresses? and (2) How can assurance be provided that the stresses in a specific fitting or pipe are adequate without requiring an expensive analysis for a relatively inexpensive piece of equipment? Both of these questions reflect the need for a simplified solution that will cover all variables.

The simplified solution provided in ANSI B31.7 and Section III covers all variables of loading and geometry in a conservative manner by

1. considering the stresses resulting from all loading conditions to be additive in all cases and by
2. assuming that the stress indices (B, C, and K factors) have the maximum possible value for a given fitting subjected to a given load.

Actually, the maximum stresses resulting from two separate loading conditions could be 90° apart in a given fitting. For example, the maximum stress in an elbow that results from in-plane bending occurs at the side of the elbow and the maximum stress resulting from pressure occurs at the crotch. The equations in Division 1-705 of ANSI B31.7 (NB-3650) assume that these stresses occur at the same location in the fitting and that they are additive.

The theoretical solution of stresses in an elbow subjected to one loading condition is an arduous task. When this effort is multiplied by the number of applied loads and the number of elbows in the system, the effort required to determine the stresses becomes staggering. Simplification of analysis is provided in the Code by the use of stress indices, where a stress index is defined as the ratio of a particular stress to a nominal stress. The nominal stress in the Code is that for a straight pipe with the same diameter and wall thickness of the specific piping component being analyzed. The stress index corrects this stress to the maximum value of stress known to exist for a given piping component subjected to a given load. When multiplied by the nominal stress, the stress index will give a conservative estimate of the stress in the piping component. Thus, the theoretical solution must be performed only to develop the stress index. Some aspects of theoretical analysis are discussed in Refs. 83, 84, and 85.

A design analysis performed in accordance with the rules of the Code provides protection from two separate types of failure. These are (1) membrane or catastrophic failure (Eq. 9 of the Code) and (2) fatigue or leak failures (Eqs. 10 and 11 of the Code). Since the subject of discussion here is protection against membrane failure, the Code equation of concern is Eq. 9.

$$B_1 \left(\frac{PD_o}{2t} \right) + B_2 \left(\frac{D_o}{2I} \right) M_i \leq 1.5S_m, \quad (9)$$

where

B_1, B_2 = primary stress indices for the specific piping component being investigated;

P = design pressure;

D_o = outside diameter of pipe, in;

t = nominal wall thickness of piping component, in.;

I = moment of inertia, in.⁴;

M_i = resultant moment loading resulting from loads caused by (1) weight, (2) earthquake, and (3) other mechanical loads; and

S_m = allowable stress value, psi.

Protection against membrane failure must necessarily include the consideration of the appropriate loading conditions. In accordance with Eq. 9, protection against membrane failure is provided by limiting the total stress resulting from primary or non-self-limiting loads to $1.5S_m$. Primary or non-self-limiting loads include those resulting from pressure, weight, and seismic weights. Only one-half the range or single-amplitude seismic loads are considered in Eq. 9. Those loads associated with restraint of deformation are excluded. For example, internal pressure produces a deformation, and restraint of that deformation reduces the resultant stress but not the load. Consider a cylinder with a thin diaphragm. As one side of the diaphragm is pressurized, deformation occurs and stresses result. If the thickness of the diaphragm is increased, the deformations and stresses will decrease but the load will remain constant.

6.2 Loadings

The resultant moment (M_i) caused by weight, seismic, and other mechanical loads sustained by the piping component must be determined for use in Eq. 9 of the Code. When determining the value of M_i in accordance with the Code, the in-plane bending, out-of-plane (perpendicular) bending, and torsion components (M_1 , M_2 , and M_3) of the loading moments should be calculated algebraically by using the signs of the components of the seismic loading in combination with those of the weight loads and other mechanical loads. An example of this computation is as follows.

<u>Moment Components</u>	<u>Weight Load</u>	<u>Seismic Load</u>	<u>Total</u>	
M_1	+13,000	$\pm 8,500$	+21,500	+ 4,500
M_2	-14,000	$\pm 2,000$	-12,000	-16,000
M_3	- 1,000	± 350	- 650	- 1,350

If the method of seismic analysis is such that only magnitudes without relative algebraic signs are determined, the most disadvantageous combination of loading components must be used to determine the value of M_i . The loads for each moment component are added, and the relative

algebraic sign of the weight loading component is assigned to the total for each moment component. An example of this computation is as follows.

<u>Moment Component</u>	<u>Weight Load</u>	<u>Seismic Load</u>	<u>Total</u>
M_1	+13,000	8,500	+21,500
M_2	-14,000	2,000	-16,000
M_3	- 1,000	350	- 1,350

The resulting totals for the components of the loading moments are used to determine the value of M_1 for the piping component under consideration as explained in footnote 5 to Table D-201 in Appendix D of ANSI B31.7 (NB-3683.2-1). For straight-through piping components, curved pipe, and welding elbows; the value of $M_1 = (M_1^2 + M_2^2 + M_3^2)^{1/2}$. For the example computation where the relative algebraic signs of the components of the seismic load were known, the value of $M_1 = 24,630$ should be used in Eq. 9. For the example computation where the relative algebraic signs of the components of the seismic load were not known, the value of $M_1 = 26,800$ must be used in Eq. 9.

In footnote 5 to Table D-201 in Appendix D of ANSI B31.7 (NB-3683.2-1), two coordinate systems are used to designate components of moment loadings. For straight-through piping components, curved pipe, and welding elbows; the components of moment loading are designated as M_1 , M_2 , and M_3 . For branch connections and tees, the subscripts x, y, and z are used to designate components of moment loading whereas the subscripted numerals 1, 2, and 3 designate locations at which the loads are applied. The procedures used to determine the resultant moment (M_1) for branch connections or tees to be used in Eq. 9 are similar to those for straight-through piping components, curved pipe, and welding elbows; and these procedures are also given in footnote 5 to Table D-201 in Appendix D of ANSI B31.7 (NB-3683.2-1).

6.3 Stresses

Basic protection for internal pressure is provided as described in Section 2 of this manual. However, since protection against membrane failure requires the consideration of all non-self-limiting loads, the solution of Eq. 9 in the Code requires the analyst to have determined the weight and seismic loads, as described in Section 5. When examining Eq. 9, one finds two terms: pressure and bending moment. It is only necessary to calculate the stress in an equivalent straight pipe ($PD_o/2t$ and $M_i D_o/2I$) and multiply the results by the applicable B_1 and B_2 stress index factors. The stress indices for specific piping components are given in Appendix D of ANSI B31.7 (NB-3680).

If the requirements for "Pressure Design of Components" in Division 1-704 (NB-3640) and for "Satisfaction of Primary Stress Intensity" in Subdivision 1-705.1 of ANSI B31.7 (NB-3652) have been met by following the procedures outlined in Sections 2 and 6 of this manual, the design is acceptable for membrane protection and catastrophic failure is not to be expected.

7. PROTECTION AGAINST FATIGUE FAILURE

The ANSI Code B31.7 provides for protection against two types of fatigue failure. These are (1) fatigue failure in which the gross structure is subjected to elastic cycling and (2) fatigue failure in which the gross structure is subjected to plastic cycling. The determination by the designer of whether or not the structure cycles elastically is discussed in this section. This determination is accomplished by meeting the requirements of Eq. 10 of the Code. The "shakedown" criterion, as embodied in Eq. 10, states that the maximum primary plus secondary stress intensity range exclusive of stress concentration effects shall not exceed $3S_m$. Briefly, this criterion requires that after a few cycles of load application, the maximum stress will cycle within the range of tensile yield strength and compressive yield strength and will therefore be subjected to cycling within the elastic range. Satisfaction of this criterion will permit the calculation of stresses based completely on elastic behavior and will insure that incremental distortion, other plastic cycling, or ratcheting will not occur.

Equation 10 of the Code is as follows.

$$S_n = C_1 \left(\frac{P_o D_o}{2t} \right) + C_2 \left(\frac{D_o}{2I} \right) M_i + \frac{1}{2(1-\nu)} E \alpha \left| \Delta T_1 \right| + C_3 E_{ab} \left| \alpha_a T_a - \alpha_b T_b \right|, \quad (10)$$

where

$S_n \leq P_L + P_b + P_e + Q$ [as defined in Table F-104 of Appendix F in ANSI B31.7 (Fig. NB-3222-1)];

C_1, C_2, C_3 = secondary stress indices for the specific piping component being investigated;

P_o = range of operating pressure, psi;

D_o = outside diameter of pipe, in.;

t = nominal wall thickness of piping component, in.;

I = moment of inertia, in.⁴;

M_i = range of moment loading resulting from thermal expansion, anchor movements from any cause, seismic effects, and other mechanical loads;

ν = Poisson ratio = 0.3;

$E\alpha$ = modulus of elasticity (E) times the mean coefficient of thermal expansion (α), psi/°F;

$|\Delta T_1|$ = range of absolute value (without regard to sign) of the temperature difference between the temperature of the outside surface (T_o) and the temperature of the inside surface (T_i) of the piping component, assuming moment-generating equivalent linear temperature distribution;

E_{ab} = average modulus of elasticity of the two parts of the gross discontinuity, psi;

α_a = mean coefficient of expansion on side "a" of a gross discontinuity such as a branch-to-run, flange-to-pipe, or socket-fitting-to-pipe gross discontinuity, in./in. per °F;

T_a = range of average temperature minus the room temperature on side "a" of a gross discontinuity, °F;

α_b = mean coefficient of expansion on side "b" of a gross discontinuity, in./in. per °F; and

T_b = range of average temperature minus the room temperature on side "b" of a gross discontinuity, °F.

The first two terms of Eq. 10 are familiar inasmuch as they are similar to the terms in Eq. 9, the differences being that the "B" stress indices of Eq. 9 are replaced by "C" indices in Eq. 10 and the value of M_i is different. In Eq. 10, all terms predict the maximum primary and secondary stress resulting from the specific load associated with each term (P , pressure; M_i , bending moment; and ΔT , temperature difference). Since primary and secondary stresses are calculated in Eq. 10, all self-limiting and non-self-limiting loads that cycle must be considered. Non-self-limiting loads were discussed in Section 6 of this manual. Therefore, only the self-limiting loads resulting from pressure, external bending moments, and temperature or thermal gradients are discussed here. Some fatigue tests on piping components are discussed in Ref. 86; elasticity relative to pipe and piping components is discussed in Refs. 9, 87, and 88; and the principles and effects of temperature are discussed in Refs. 89 through 92.

7.1 Pressure

The loads resulting from pressure that are self-limiting and are considered in the "C" stress indices of Eq. 10 are those which occur at a structural discontinuity. For example, consider a piece of straight pipe joined to another pipe of dissimilar cross section, as is shown in Fig. 7.1. The assembly will be subjected to discontinuity stresses at the junction because the free radial deflection of one type of cylinder under pressure is different from that of the other, and this results in

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Fig. 7.1. Discontinuity Loads Produced at Junction of Two Pipes With Dissimilar Cross Sections.

a restraint on free deformation. This restraint produces forces and moments at the discontinuity that are necessary to provide compatibility of the structure. The loads H_1 , H_2 , M_1 , and M_2 illustrated in Fig. 7.1(c) are the discontinuity reactions required to maintain the compatibility of point A at the junction of the two pipes of dissimilar cross section. The discontinuity loads are self-limiting in nature since they are a function of the relative restraint of adjacent members rather than the imposed loading which produces the deflections.

The redundant loads are computed as follows. The radial deflection is determined from the expression

$$\mu = \frac{H}{2D\lambda^3} + \frac{M}{2D\lambda^2} + \frac{PR^2}{Et} \left(1 - \frac{\nu}{2}\right), \quad (7.1)$$

where

H = shear load, lb/in.,

$$D = \frac{Et^3}{12(1 - \nu^2)},$$

$$\lambda = \left[\frac{3(1 - \nu^2)}{R_m^2 t^2} \right]^{1/4},$$

M = moment, in.-lb/in.,

P = internal pressure, psi,

R = inside radius of pipe, in.,

R_m = mean radius, in.,

E = modulus of elasticity, psi,

t = nominal thickness of pipe wall, in., and

ν = Poisson's ratio = 0.3.

The rotation is determined from the expression

$$\beta = \frac{H}{2D\lambda^2} + \frac{M}{D\lambda}. \quad (7.2)$$

Equations 7.1 and 7.2 are used for both pipes with dissimilar cross sections since the deflections and rotations must be equal.

$$\begin{aligned} \mu \text{ of Pipe 1} &= \mu \text{ of Pipe 2} \\ \beta \text{ of Pipe 1} &= \beta \text{ of Pipe 2} \end{aligned} \quad (7.3)$$

The two simultaneous equations of the form given in Eq. 7.3 must be solved to determine the two unknowns H and M . However, this type of

solution is not required for standard piping components because the stress indices given in the Code account for this type of loading.

It is interesting to note that the influence of discontinuity loads (and all self-limiting loads) is local within the region of the discontinuity and fades away along the pipe. At a distance from the discontinuity where the radial deflection of the pipe is the same as that for a semi-infinite (free) cylinder, as is illustrated in Fig. 7.1(d), the influence of the discontinuity reactions is negligible.

7.2 Bending Moments

The definition of M_i for Eq. 10 of the Code differs from that for Eq. 9 inasmuch as M_i is defined for Eq. 10 as the range of moment loading resulting from (1) thermal expansion, (2) anchor movements from any cause, (3) seismic effects, and from (4) other mechanical loads. The seismic loading is considered in conjunction with the operating conditions in determining the value of M_i . The moments resulting from an earthquake may alternate in sign as the piping system vibrates, and these signs are used to algebraically calculate the sums of the moments. The range of moments resulting from seismic motion [Column 3 in the example under the definition of M_i for Eq. 10 given in Subdivision 1-705.2 of ANSI B31.7 (NB-3653.1)] is the algebraic sum of the maximum moments in each direction. These values represent the maximum range of moments for the cold condition (no thermal expansion). However, the hot condition must be considered, and the moments resulting from the earthquake for both vibratory conditions must be combined algebraically with the moments resulting from thermal expansion and other mechanical loads. These combinations of values are given in the earthquake range columns (1) and (2) of the Code example. The value of M_i for Eq. 10 is the largest resultant moment of earthquake range (Col. 3), Col. (1), or Col. (2) calculated in accordance with the rules given in Appendix D of ANSI B31.7 (NB-3680).

The value of M_i in Eq. 10 includes all cyclic non-self-limiting and self-limiting external moments. Weight effects need not be considered in the range loading since they are non-cyclic in nature. To illustrate the

differences between the types of moments generated by non-self-limiting and self-limiting loads, consider the moments generated by the weight of the piping system and its contents. These loads are always present and can only be accommodated by the amount of material provided in the pipe (the strain energy). If there is insufficient material, the loads will continue to produce deformation of the structure until failure occurs. Changing the amount of material or the design does not change the applied load, but it does change the effect of the load on the structure. However, since weight loading is a constant (non-cyclic) load, it is not necessary to consider the effects of weight loading on shakedown and fatigue. Weight stress affects the fatigue life of a structure when the mean stress about which the alternating component of stress cycles is changed from the zero value. Changing the mean stress to some value other than zero has a deleterious effect on the fatigue life. However, the fatigue curves given in the Code are adjusted for the maximum effect of mean stress. That is, the fatigue curves are based on the assumption that the maximum mean stress possible is present regardless of whether or not the piping component under consideration is subjected to a mean stress. There is no way the structure can reduce or change the value of the applied load if the load is non-self-limiting.

Self-limiting loads are quite different and are actually a secondary effect, as in the case of loads at a discontinuity that result from pressure loading. Consider a piping system that is limited to some temperature above ambient. In an unrestrained state, the system will undergo free expansion. However, since the system is anchored or restrained, loads are generated in the system to keep it in the desired position. These loads are a function of the amount of restraint on the free deflection, and the system must accommodate restraint of deflection rather than applied moments.

A simple comparison that might be made involves a bar fixed at one end and subjected to an applied load in one case and an applied deflection in a second case. If the load is large enough to produce yielding, progressive elongation will occur with each load application and failure will eventually result. If the load is large enough to produce stresses well in excess of the yield strength, failure will occur in the first

load application. If the applied deflection is equal to that produced by the first application of load, elongation will occur on the first load cycle but no elongation will occur on subsequent cycles since the deflection is controlled. The load which results from the deflection case is less than that which results from the applied load case after the first loading cycle.

7.3 Temperature

The two temperature terms of Eq. 10 are stresses that result from self-limiting loads produced by thermal gradients. The first temperature term in Eq. 10,

$$\frac{1}{2(1 - \nu)} E \alpha |\Delta T_1| ,$$

describes the stress resulting from a linear radial thermal gradient. However, the shape of the thermal gradient will usually be found to be exponential rather than linear, as is illustrated in the figure under Eq. 11 in Paragraph 1-705.3.1 of ANSI B31.7 (NB-3653.2) reproduced here in Fig. 7.2.

Fig. 7.2. Graphic Presentation of Non-Linear and Linear Thermal Gradient Through Wall Thickness Given in the Code.

The dashed line in Fig. 7.2 is a graphic representation of the linear gradient that produces a bending moment through the wall equal to

that moment produced by the actual gradient, represented by the solid line in Fig. 7.2. The ΔT_2 portion of the actual thermal gradient is only considered in fatigue analysis and is discussed in Section 8 of this manual. A derivation of the ΔT_1 and ΔT_2 terms used in ANSI B31.7 is presented in Ref. 9.

The second temperature term in Eq. 10,

$$C_3 E_{ab} \left| \alpha_a T_a - \alpha_b T_b \right| ,$$

describes the stress resulting from temperature differences occurring in adjacent parts or from a difference in the coefficient of thermal expansion (α) of adjacent parts or from both. An example of this phenomenon is the junction of two pipes or cylinders with dissimilar cross sections discussed in Subsection 7.1. At some time during the process of heating the piping system, the mean temperature of the thinner cylinder may be 480°F while the mean temperature of the thicker cylinder is 300°F. Discontinuity thermal stresses will therefore occur. The thinner cylinder will attempt to expand in the radial direction some quantity $R\alpha\Delta T$. The thicker cylinder will also attempt to expand but not as much since it is 180°F cooler. The thicker cylinder imposes a restraint on the thinner cylinder, generating internal forces and moments to provide compatibility of the structure just as in the pressure case. The same circumstances occur at a junction of two different metals even when both are at the same temperature because the different mean coefficients of thermal expansion (α) of the different metals result in different values of radial deflection. The radial deflection (μ) resulting from thermal expansion is computed in the same manner used for pressure, discussed in Subsection 7.1, except that the

$$\frac{PR^2}{Et} \left(1 - \frac{\nu}{2} \right)$$

term in Eq. 7.1 is replaced with the thermal deflection term $R\alpha\Delta T$.

The C_3 factor of Eq. 10 for axial geometric discontinuities given in the Code is 1.8. This value is derived from the case of a cylinder built into a wall. In reality, there are no standard piping components that introduce this type of restraint on a pipe. Variation of the C_3 factor as a function of the thickness ratio of adjacent parts is illustrated in Fig. 7.3. Using C_3 values for various thickness ratios

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Fig. 7.3. Variation of the C_3 Secondary Stress Index Factor as a Function of the Thickness Ratio of Adjacent Parts.

plotted in Fig. 7.3, the maximum thermal discontinuity stress can be determined by using the expression

$$\sigma_{\max} = C_3 E (\alpha_a T_a - \alpha_b T_b) , \quad (7.4)$$

where

E = modulus of elasticity (assumed to be the same for both thin- and thick-walled cylinders);

α_a = coefficient of thermal expansion for thin cylinder, as shown in Fig. 7.3; and

α_b = coefficient of thermal expansion for thicker cylinder.

7.4 Combination of Loading Conditions as a Function of Time

In the preceding discussion (Subsection 7.3), the need to consider loads as a function of time is introduced. This is a difficult consideration, and a simple analysis for Eq. 10 of the Code is presented here to introduce this concept to the designer. This analysis is of the loading values resulting from three operating conditions. Condition 1 is a transient which occurs very rapidly, and the maximum values of loads occur at substantially different times. Because of this, it is difficult to determine the maximum value when combined with other times in the transient, and multiple times are investigated for condition 1 in this example analysis. Another method of handling condition 1 is discussed at the end of this analysis. Condition 2 is the normal operating condition, and condition 3 is a power change condition. The loading values resulting from these operating conditions are tabulated below.

<u>Condi-</u> <u>tion</u>	<u>P_o</u> <u>(psi)</u>	<u>M₁</u> <u>(ft-lb)</u>	<u>M₂</u> <u>(ft-lb)</u>	<u>M₃</u> <u>(ft-lb)</u>	<u>ΔT₁</u> <u>(°F)</u>	<u>T_a-T_b</u> <u>(°F)</u>	<u>Cycles</u>
0	0	0	0	0	0	0	250
1a	+2000	0	0	0	+ 10	+ 5	25
1b	+2000	+ 2000	- 500	- 200	+130	+10	25
1c	+2000	+ 6000	- 3000	- 1000	+ 80	+30	25
1d	+2000	+10000	- 5000	- 1600	+ 15	+ 7	25
2	+1500	- 3000	-12000	+12000	- 10	-12	250
3	+2600	+ 8700	+ 2100	+15000	+ 22	+ 5	1000

The statement that "The range of stress as defined in Equation (10) consists of the difference in stress resulting from the consideration of two sets of loading conditions." in Subdivision 1-705.2 of ANSI B31.7 (NB-3653.1) makes a detailed investigation of the combinations of loading conditions necessary. Since the maximum range of stress must be calculated, each and every combination of loading conditions must be considered. The sets of conditions and loading values that must be used in Eq. 10 for the three operating conditions of this example case are given in Table 7.1.

Table 7.1. Sets of Loading Conditions To Be Used in Eq. 10 For Example Analysis

Sets of Condi- tions	P_o (psi)	M_1 (ft-lb)	M_2 (ft-lb)	M_3 (ft-lb)	M_i (ft-lb)	$ \Delta T_1 $ (°F)	$ T_a - T_b $ (°F)
0-1a	2000	0	0	0	0	10	5
0-1b	2000	2000	500	200	2070	130	10
0-1c	2000	6000	3000	1000	6800	80	30
0-1d	2000	10000	5000	1600	11300	15	7
0-2	1500	3000	12000	12000	17200	10	12
0-3	2600	8700	2100	15000	17500	22	5
1a-2	500	3000	12000	12000	17200	20	17
1b-2	500	5000	11500	12200	17500	140	22
1c-2	500	9000	9000	13000	18200	90	42
1d-2	500	13000	7000	13600	20000	25	19
1a-3	600	8700	2100	15000	17500	12	0
1b-3	600	6700	2600	15200	16800	108	5
1c-3	600	2700	5100	16000	17000	58	25
1d-3	600	2700	7100	16600	18300	7	2
2-3	1100	11700	14100	7000	19600	32	17

The values of P_o , M_i , ΔT_1 , and $T_a - T_b$ given in Table 7.1 must be used in Eq. 10 with the proper C indices, and the maximum value calculated must be compared with the value of $3S_m$ at the temperature of the material being considered. If the maximum value calculated from the above loads is less than $3S_m$, the designer can proceed to Eq. 11 of the Code since the structure cycles elastically for all conditions. If the maximum value of S_n calculated by using Eq. 10 exceeds $3S_m$, the set of loading conditions must be treated in a fatigue evaluation by using Eqs. 12 and 13 of ANSI B31.7 (Eqs. 12 and 14 of Section III). The designer must then load at the second highest value of S_n calculated and compare it to the value of $3S_m$. If the second value is less than $3S_m$, the designer can proceed to Eq. 11 for all of the other sets of loading conditions.

To illustrate the procedure, the following analysis is made by assuming that the loads given in Table 7.1 were calculated for a type-304 20-in. stainless steel long-radius elbow with a wall thickness of 1.500 in. that is welded (with a flush weld) to a 20-in. pipe with a wall thickness of 1.000 in. The transition between the two is tapered in accordance with Fig. 1-727.3.1 of ANSI B31.7 (NB-4233-1). The corresponding stress indices taken from Table D-201 in Appendix D of ANSI B31.7 (NB-3683.2-1)

are tabulated below. The C stress indices for the elbow are applicable to the center of the elbow, but they have been applied here to the end of the elbow. This is a conservative practice since the ovality effect decreases with length around the elbow. However, it would not be conservative to use $C = 1.0$ because there is some ovality all around the elbow.

Stress Index	Elbow	Transition	Flush Girth Weld	Value Used in Eq. 10
C_1	1.22	1.4	1.0	1.71
C_2	3.00	1.2	1.0	3.6
C_3	1.0	1.0	1.0	1.0

When the calculated C indices and known geometries are used, Eq. 10 is reduced to the form

$$S_n = \frac{1.71(20)}{2(1.5)}P_o + \frac{3.6(20)}{2(3755)}M_i + \frac{1(277)}{2(1 - 0.3)}|\Delta T_1| + 1.0(277)|T_a - T_b|$$

$$= 11.4P_o + 0.01M_i + 197.9|\Delta T_1| + 277|T_a - T_b|$$

The loads given in Table 7.1 are used in this equation to produce the values given in Table 7.2.

Table 7.2 Calculated Values of Eq. 10 Terms for Different Loading Conditions of Example Analysis

Sets of Condi- tions	$11.4P_o$	$0.01 \times$ $12M_i$	$197.9 \Delta T_1 $	$277 T_a - T_b $	S_n
0-1a	22800	0	1980	1390	26200
0-1b	22800	250	25700	2770	51500
0-1c	22800	800	15800	8310	47400
0-1d	22800	1360	3000	1940	29100
0-2	17100	2064	1980	3320	24500
0-3	29640	2100	4400	1390	37500
1a-2	5700	2064	4000	4700	16500
1b-2	5700	2100	27700	6100	41600
1c-2	5700	2180	17800	11600	37300
1d-2	5700	2400	4900	5300	18300
1a-3	6840	2100	2400	0	11300
1b-3	6840	2020	21400	1390	31600
1c-3	6840	2040	11500	6900	27300
1d-3	6840	2200	1400	550	11000
2-3	12540	2350	6300	4710	25900

The designer can now select the worst combination of conditions, remembering that the values for condition 1 have been taken at four different times. Once a value for condition 1 is taken in a set, all other values in that set of conditions are eliminated. For example, the worst set of conditions in which condition 1 is included is set 0-1b. This selection eliminates sets 0-1a, 0-1c, and 0-1d as well as 0 and 1b in combinations with any other conditions from future consideration of the worst combinations of conditions.

The allowable value of $3S_m$ is taken from Table A.1 in Appendix A of ANSI B31.7 (I-1.1, I-1.2, and I-1.3). Assuming that the temperature of the metal at the point being considered was 600°F at one time during each and every set of conditions, the allowable value of $3S_m = 46,800$ psi. The value of S_n for condition set 0-1b = 51,500 psi, which is greater than $3S_m$. The values for all other sets of conditions are less than $3S_m$. Thus, the condition set 0-1b must be treated in the fatigue evaluation by using Eqs. 12 and 13 of ANSI B31.7 (Eqs. 12 and 14 of Section III) since the stress does not cycle elastically. This treatment is discussed in Section 8 of this manual. All other sets of loading conditions are treated by using Eq. 11. This treatment is also discussed in Section 8.

The second method of handling the loads resulting from the rapid transient of condition 1 previously referred to is a simpler and more conservative way of treating such transient conditions. This method involves compression of the maximum values for each load in the four sub-conditions of condition 1 (1a, 1b, 1c, and 1d) into one overall condition. For these four sub-conditions of condition 1, the application of this method would result in the values tabulated below.

Condi- tion	P_o (psi)	M_1 (ft-lb)	M_2 (ft-lb)	M_3 (ft-lb)	ΔT_1 (°F)	$T_a - T_b$ (°F)	Cycles
1	+2000	+10000	-5000	-1600	+130	+30	25

The worst combination of values involving the sub-conditions of condition 1 given in Table 7.2 is condition set 0-1b. The same condition-0 values are combined with the new condition-1 values, and Eq. 10 is solved for this set of conditions.

$$S_n = 22,800 + 1360 + 25,700 + 8310 = 58,170 \text{ psi} .$$

This value of $S_n = 58,170$ psi calculated by using the second method for handling transient conditions is larger than that (51,500 psi) calculated by using the more precise method first discussed, but the second method provides the designer with a simpler and more conservative manner of solving Eq. 10 by reducing the number of combinations of conditions requiring computation. In this example analysis, the number of condition sets in Table 7.2 was reduced from 15 to 6 by using this second method for handling transient conditions.

7.5 Temperature Distribution

Under the provisions of ANSI B31.7 and Section III, the designers of piping systems must evaluate the stresses resulting from temperature changes in the materials from which the components of these systems are made. If the geometry is complicated, the evaluation will impose great difficulties. However, there are usually no essential difficulties in the evaluation of a cylindrical component such as straight pipe. Even so, this evaluation is not a trivial procedure, and the calculations involved are discussed in Ref. 93.

The temperature changes in the material of piping systems that are usually of concern are those that result from changes in the temperature of the fluids which flow through or are contained within the components of the system. In most cases, the wall-thickness-to-diameter ratio of the pipe is small enough to justify consideration of the pipe wall as a slab with zero curvature rather than a tube with finite curvature. It is also usually reasonable to consider the exterior surface of the pipe to be perfectly insulated. However, this assumption may not be conservative in the unusual case where the outer surface of the pipe is exposed to ambient air. In all such applications, the designer should check the results by other methods.

Thus, the problem usually involves a slab of metal of uniform, finite thickness with one perfectly insulated face and the other face convectively exposed to a fluid whose temperature varies in a known way with respect to time. The problem can be further simplified by assuming that the metal

has a uniform initial temperature (T_o). It can also be assumed that the physical properties of the pipe material are constant and that the surface heat transfer constant (for transfer of heat between the interior surface of the pipe and the fluid flowing in it) is constant.

7.5.1 Step and Linear Fluid Temperature Changes

Two cases of fluid temperature change are most important. One is a "step" change in which the temperature of the fluid is suddenly changed by a finite amount. The second case is a "linear" change in which the temperature of the fluid changes at a constant rate of so many degrees per minute or hour. With solutions available for these two cases, any case of fluid temperature variation can be covered by approximating it with a piece-wise linear variation (with jumps) if necessary. Obviously, the more complicated the temperature variation, the more complicated the solution will be. However, as simple a representation as possible should be used to approximate the actual fluid temperature variation.

Analytic equations for the temperature of the metal as a function of time and of position within the wall look more complicated than they really are. These equations involve infinite series, but for all values of time except those immediately after the start of the transient (the step or linear change in the temperature of the fluid), only a few terms need to be included in the sums since the subsequent terms are too small to make much difference. The equation for a step change in the temperature of the fluid is

$$T(x, \theta) = T_o + (T_f - T_o) \left[1 - \sum_{n=1}^{\infty} c_n \cos\left(\frac{m_n x}{t}\right) \exp(-m_n^2 N_F) \right], \quad (7.5)$$

where

x = distance into slab measured from insulated face, ft,

θ = time from start of transient, hours,

T_o = initial uniform temperature, °F,

T_f = temperature of fluid at time θ (a constant value: the temperature of the fluid after the step change), °F,

$c_n = (4 \sin m_n) / (2m_n + \sin 2m_n)$, dimensionless,
 m_n = nth root of the equation $m \tan m = N_B$, dimensionless,
 t = thickness of slab, ft, and
 N_F = Fourier number, dimensionless.

The Biot number

$$N_B = \frac{ht}{k}, \quad (7.6)$$

where

h = surface film heat transfer coefficient, Btu/hr·ft²·°F,
 t = thickness of slab, ft, and
 k = thermal conductivity of pipe material, Btu/hr·ft·°F;
 and the Fourier number

$$N_F = \frac{\epsilon \theta}{t^2}, \quad (7.7)$$

where

ϵ = thermal diffusivity of pipe material, ft²/hr,
 θ = time from start of transient, hr, and
 t = thickness of slab, ft.

The equation for a linear change in the temperature of the fluid is

$$T(x, \theta) = T_o + T_f - T_o \left[1 - \sum_{n=1}^{\infty} c_n \cos \left(\frac{m_n x}{t} \right) \frac{1 - \exp(-m_n^2 N_F)}{m_n^2 N_F} \right], \quad (7.8)$$

where the temperature of the fluid (T_f) changes linearly with time (θ) and the other terms are as defined for Eq. 7.5. Since the solution of Eqs. 7.5 and 7.8 involves a lot of calculation, the most important consequences of these equations were evaluated by using a digital computer with "double precision" arithmetic and employing 30 terms in the sums. The results of these evaluations were plotted in terms of the coefficients for the average (A) temperature through the thickness of the slab, the temperature of the heated (H) surface (surface in contact with the fluid), and the temperature of the insulated (I) surface as functions of the common logarithm of the Fourier number for a varying Biot number for both step and linear changes in the temperature of the fluid. Case 1, the step change, is indicated by the subscript 1 (A_1 , H_1 , and I_1); and

case 2, the linear change, is denoted by the subscript 2 (A_2 , H_2 , and I_2). These plots are shown in Figs. 7.4 through 7.9.

Fig. 7.4. Coefficient A_1 for a Step Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.5. Coefficient H_1 for a Step Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.6. Coefficient I_1 for a Step Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.7. Coefficient A_2 for a Linear Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.8. Coefficient H_2 for a Linear Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.9. Coefficient I_2 for a Linear Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

The following simple equations are to be used in conjunction with the data plotted in Figs. 7.4 through 7.9.

$$T_A = T_O + (T_f - T_O)A, \quad (7.9)$$

$$T_H = T_O + (T_f - T_O)H, \quad (7.10)$$

and
$$T_I = T_O + (T_f - T_O)I; \quad (7.11)$$

where

T_A = average temperature through the thickness of the metal slab, °F,

T_O = initial uniform temperature of metal slab, °F,

T_f = temperature of fluid after the change, °F,

A = coefficient for average temperature for either step temperature change (A_1) or linear temperature change (A_2),

T_H = temperature of the surface in contact with the fluid, °F,

H = coefficient for heat surface for either a step temperature change (H_1) or a linear temperature change (H_2),

T_I = temperature of insulated surface, °F, and

I = coefficient for insulated surface for either a step temperature change (I_1) or a linear temperature change (I_2).

The value of T_A , T_H , and T_I for any specified value of θ can be determined by using the data plots as follows. The Biot number (N_B) is first calculated to determine which curve in the plot is applicable to the particular situation. The Fourier number (N_F) corresponding to the desired value of time (θ) is then calculated and its common (base 10) logarithm is found. This gives the abscissa value at which the chart is entered and read upward to intersect the proper N_B curve. At this point, the value of the coefficient being sought is read at the left. Interpolation will probably be necessary, and it must be carefully done to obtain accurate results. Use of the data plots is further illustrated in the following example problems.

(a) Example Problem for Step Temperature Change in Fluid. The conditions of this problem are as follows. Water at a pressure of 2000 psia and a temperature of 300°F flows at a rate of 20 ft/sec through a carbon steel pipe with an outside diameter of 12.750 in. and a wall thickness of 0.406 in. The flow has been steady for some time when suddenly the

temperature of the water changes to 350°F, the other quantities remaining unchanged. Determine the values of T_A , T_H , and T_I one second after the change.

To solve the problem, values of the Reynolds number and the Prandtl number were first determined. The Reynolds number

$$N_R = \frac{VD_i\rho}{\mu} = 10.8 \times 10^6, \quad (7.12)$$

where

V = mean velocity of flow = 72,000 ft/hr,

D_i = inside diameter of pipe = 0.9948 ft,

ρ = density of water = 55.62 lb/ft³, and

μ = viscosity of water = 0.369 lb/hr·ft;

and the Prandtl number

$$N_P = \frac{C_p\mu}{k} = 0.987, \quad (7.13)$$

where

C_p = specific heat = 1.049 Btu/lb·°F and

k = thermal conductivity = 0.392 Btu/hr·ft²·°F per ft.

These values were used in the equation for the Nusselt number

$$N_N = 0.023(N_R)^{0.8}(N_P)^{0.4} = \frac{hD}{k} \quad (7.14)$$

to determine that the film heat transfer coefficient $h = 3820$ Btu/hr·ft²·°F for the 350°F temperature of the water after the step change, maintained constant during this problem. Since the thickness of the metal slab $t = 0.406$ in. = 0.034 ft and the thermal conductivity of the pipe material $k = 29.5$ Btu/hr·ft²·°F per ft, the Biot number

$$N_B = \frac{3820(0.034)}{29.5} = 4.403. \quad (7.6)$$

The diffusivity (ϵ) of carbon steel is given as 0.02 in.²/sec = 0.5 ft²/hr and the Fourier number

$$N_F = \frac{0.5}{3600(0.034)^2} = 0.12. \quad (7.7)$$

The common logarithm (base 10) of 0.12 = -0.921. This value and the Biot number were used with the charts shown in Figs. 7.4, 7.5, and 7.6 and the values of the coefficients were read as $A_1 = 0.23$, $H_1 = 0.69$, and

$I_1 = 0.04$. These values of the coefficients were then used in Eqs. 7.9, 7.10, and 7.11 to determine the value of T_A , T_H , and T_I one second after the temperature change in the fluid.

$$T_A = 300 + (50)(0.23) = 312^\circ\text{F} , \quad (7.9)$$

$$T_H = 300 + (50)(0.69) = 334^\circ\text{F} , \quad (7.10)$$

and
$$T_I = 300 + (50)(0.04) = 302^\circ\text{F} . \quad (7.11)$$

(b) Example Problem for Step and Linear Temperature Change in Fluid.

For fluid flowing through a pipe with a wall thickness of 0.1 ft and a diffusivity of $0.5 \text{ ft}^2/\text{hr}$, the flow conditions are such that the Biot number $N_B = 3.0$. The pipe and its contents were originally at a temperature of 120°F . The temperature of the fluid suddenly jumps to 150°F and then increases linearly, reaching 200°F in 90 seconds, after which it remains constant at 200°F . Determine T_A as a function of time until it has nearly reached 200°F .

In the solution of this problem, the Fourier number (N_F) is used as the variable, and two decades (a range of 100 to 1) are covered. Note that

$$N_F = \frac{0.5\theta}{3600(0.01)} = \frac{\theta}{72} . \quad (7.15)$$

Thus, when $\theta = 90$ seconds, N_F has the value of 1.25. This value is denoted as N_F^* . The actual temperature variation may be thought of as the superposition of three parts. These are

1. a step change of 30° starting at $\theta = 0$ and $N_F = 0$,
2. a linear change of 50° in 90 seconds denoted as $T_2 = 400$ starting at $N_F = 0$, and
3. a linear change at the same rate but in the negative sense starting at $\theta = 90$ seconds, denoted as $T_2^* = -400^*$ starting at $N_F^* = N_F - 1.25$.

What appeared to be a reasonable number of conveniently spaced values of $\log N_F$ were selected to cover the time interval of interest. $\log N_F$ was used as the basic parameter for ease of interpolation, and an engineers scale and a slide rule were used to attain as accurate an interpolation as possible. The results of these calculations are given in Table 7.3.

Table 7.3. Calculations for Example Problem Involving Both Step and Linear Temperature Change in Fluid

Log N_F	-1.0	-0.5	-0.4	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
N_F	0.1	0.316	0.398	0.501	0.631	0.794	1.000	1.259	1.585	1.995	2.512	3.162	3.981	5.012	6.310	7.943	10.000
θ (sec)	7.2	22.8	28.6	36.1	45.4	57.2	72.0	90.6	114.1	143.6	180.8	227.6	286.6	360.8	452.3	571.9	719.9
$N_F^* = N_F - \bar{N}_F$								0.009	0.335	0.745	1.262	1.912	2.731	3.762	5.060	6.693	8.750
Log N_F^*								-2.046	-0.475	-0.127	0.101	0.281	0.436	0.575	0.704	0.826	0.942
$f_1 = A_1 N_F$	0.170	0.402	0.465	0.537	0.615	0.700	0.778	0.840	0.903	0.950	0.975	0.990	0.997	1.0	1.0	1.0	1.0
$f_2 = A_2 N_F$	0.098	0.233	0.277	0.327	0.377	0.443	0.499	0.563	0.632	0.700	0.750	0.796	0.833	0.865	0.899	0.917	0.933
$f_2^* = A_2 N_F^*$								0.010	0.238	0.421	0.564	0.682	0.762	0.820	0.867	0.880	0.925
T_1	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30	30
T_2	4	12.64	15.92	20.04	25.24	31.76	40	50.36	63.40	79.80	100.48	126.48	159.24	200.48	252.4	317.72	400
T^*								-0.36	-13.40	-29.80	-50.48	-76.48	-109.24	-150.48	-202.4	-267.72	-350
$f_1 T_1$	5.1	12.1	14.0	16.1	18.4	21.0	23.3	25.2	27.1	28.5	29.2	29.7	29.9	30	30	30	30
$f_2 T_2$	0.4	2.9	4.4	6.6	9.5	14.1	20.0	28.4	40.1	55.9	75.4	100.7	132.6	173.4	226.9	291.3	373.2
$f_1 T_1^*$								0.0	-3.2	-12.5	-28.5	-52.2	-83.2	-123.4	-175.5	-235.6	-323.8
$f_2 T_2^* + f_1 T_1^*$	0.4	2.9	4.4	6.6	9.5	14.1	20.0	28.4	36.9	43.4	46.9	48.5	49.4	50.0	(51.4)	(55.7)	(49.4)
SUM	5.5	15.0	18.4	22.7	27.9	35.1	43.3	53.6	64.0	71.9	76.1	78.2	79.3	80	80	80	80
T_A	125.5	135.0	138.4	142.7	147.9	155.1	163.3	173.6	184.0	191.9	196.1	198.2	199.3	200.0	200.0	200.0	200.0

The calculations given in Table 7.3 are self explanatory except possibly those in line 15. This line shows what was done with the positive and negative linear changes T_2 and T_2^* , both of which eventually become quite large although their difference becomes and remains 50° . The large values of T_2 and T_2^* are multiplied by the factors $f_2 = A_2 N_F$ and $f_2^* = A_2 N_F^*$, which cannot be precisely read from the charts. Some error is therefore introduced. However, since it is known that $f_2 T_2 + f_2^* T_2^*$ must approach 50° from below as N_F gets larger, the accuracy of the calculations can be monitored and corrections can be made. Accordingly, the last three entries in line 15 (each enclosed in parentheses) should be corrected to the value of 50° . Thus, we get $f_1 A_1 + f_2 A_2 + f_2^* A_2^*$ (in line 16), which is the change in T_A , and the original $T_A = 120^\circ$ is added to this to obtain the final result given in line 17.

It is noted that fewer values of $\log N_F$ could probably have been used since the entire transient is substantially over insofar as T_A is concerned by the time $\log N_F = 0.5$. This probably would not be the case if the value of interest were T_I . On the other hand, the value of T_H would have settled down to 200°F far earlier than this. Sufficient information is given in Table 7.3 to plot T_A (line 17) as a function of θ (line 3) so that the originally stated problem is solved.

7.5.2 Thermal Gradient in Pipe Wall

Charts were also developed⁹³ for coefficients to be used in determining the quantities $|\Delta T_1|$ and $|\Delta T_2|$ in Eqs. 10 and 11 of ANSI B31.7 and Section III. The quantity $|\Delta T_1|$ in Eq. 10 is defined as the range of absolute value of the temperature difference between the temperature of the outside surface and the temperature of the inside surface of the piping component where moment generating equivalent linear temperature distribution is assumed. This range of temperature difference is used in the first temperature term of Eq. 10 to describe the stress resulting from a linear radial thermal gradient. However, the shape of the thermal gradient will usually be found to be exponential rather than linear, as is illustrated in Fig. 7.2 (page 36). The quantity $|\Delta T_2|$ used in Eq. 11

of the Code is defined as the range of absolute value for that portion of the nonlinear thermal gradient through the wall thickness not included in $|\Delta T_1|$ of Eq. 10 (as shown in Fig. 7.2).

In this discussion of these quantities, we will let

t = thickness of pipe wall,

y = radial position in the wall measured positive outward from the mid-thickness position (y lies in the range $\pm t/2$),

$T(y)$ = temperature as a function of radial position,

T_H = temperature at inside or heated surface,

T_I = temperature at outside or insulated surface, and

T_A = average temperature through the thickness of the metal.

As indicated in Fig. 7.10, the actual temperature distribution may be thought of as being divided into three parts. The first of these parts is the average temperature which has a constant value.

$$T_A = \left(\frac{1}{t} \right) \int_{-t/2}^{t/2} T(y) dy . \quad (7.16)$$

The second part is a linear variation of temperature through the pipe wall, of which the total variation

$$v = \left(\frac{12}{t^2} \right) \int_{-t/2}^{t/2} y T(y) dy . \quad (7.17)$$

Fig. 7.10. Decomposition of Temperature Distribution Range.

The third part is simply what is left. The average values of the second and third part are each zero, and the "first moment" of the third part is zero. The resulting definitions are then

$$\Delta T_1 = v, \quad (7.18)$$

and
$$\Delta T_2 = \text{Max} \left[|T_A - T_I| - \frac{|v|}{2}, |T_H - T_A| - \frac{|v|}{2}, 0 \right]. \quad (7.19)$$

The value of ΔT_2 is the algebraically largest of the three quantities within the brackets in Eq. 7.19, where the vertical lines indicate absolute value. If the first two quantities are negative, the result is zero. The rationale of the equation for ΔT_2 is certainly not obvious, but it is reasonable and has been accepted by the Code committees.

For the two cases of interest here, ΔT_1 and ΔT_2 are given by the following simple equations.

$$\Delta T_1 = (T_f - T_o)L \quad (7.20)$$

and
$$\Delta T_2 = (T_f - T_o)N, \quad (7.21)$$

where

T_f = temperature of the fluid after the change, °F,

T_o = initial uniform temperature of metal slab (pipe wall), °F,

L = coefficient for linear variation of temperature through pipe wall which may be either a step variation (L_1) or a ramp variation (L_2), and

N = coefficient for nonlinear variation of temperature through pipe wall which may be either a step variation (N_1) or a ramp variation (N_2).

The values of L and N are plotted as functions of the common logarithm of the Fourier number for a varying Biot number in Figs. 7.11 through 7.14. The use of the data in these plots in conjunction with Eqs. 7.20 and 7.21 is illustrated in the following example problem.

For fluid flowing through a pipe with a wall thickness of 0.1 ft and a diffusivity of 0.5 ft²/hr, the flow conditions are such that the Biot number $N_B = 3.0$. The pipe and its contents were originally at a temperature of 120°F when a slug of fluid at a temperature of 200°F flowed past a point, taking 15 seconds to get past. Thereupon, the temperature of

Fig. 7.11. Coefficient L_1 for a Step Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.12. Coefficient N_1 for a Step Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.13. Coefficient L_2 for a Linear Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

Fig. 7.14. Coefficient N_2 for a Linear Change in Fluid Temperature as a Function of the Common Logarithm of the Fourier Number for a Varying Biot Number (from Ref. 94).

the fluid suddenly resumed its former steady-state value of 120°F. Find the range of ΔT_1 resulting from this transient.

In the solution of this problem, the Fourier number (N_F) is used as the variable, and

$$N_F = \frac{0.5\theta}{3600(0.01)} = \frac{\theta}{72} , \quad (7.15)$$

where θ is in seconds. The calculations for this problem are given in Table 7.4, and the results for ΔT_1 are given in the last line.

Table 7.4. Calculations for Example Problem for Linear Thermal Gradient Through Pipe Wall

θ	0	5	10	15	20	25	30	35	40
θ^*				0	5	10	15	20	25
$\text{Log } N_F$	$-\infty$	-1.16	-0.86	-0.68	-0.56	-0.46	-0.38	-0.31	-0.26
$\text{Log } N_F^*$				$-\infty$	-1.16	-0.86	-0.68	-0.56	-0.46
$f_1 = L_1 N_F$	0	0.451	0.549	0.541	0.513	0.469	0.431	0.380	0.344
$f_2 = L_1 N_F^*$				0	0.451	0.549	0.541	0.513	0.469
$80(f_1 - f_2)$	0	36.1	43.9	43.3	5.0	-6.4	-8.8	-10.6	-10.0

These results indicate a rapid rise to a positive value of about 44°F followed by a rapid decrease to a negative value of -10.6°F and a long slow increase back to zero. The maximum positive value can be pinpointed a little more closely. At $\log N_F = -0.79$, $L_1 = 0.551$, which is the maximum value possible for $N_B = 3.0$. Correspondingly, the maximum value of ΔT_1 is 44.1°F at $\theta = 11.7$ seconds. The most negative value is approximately -10.6°F at $\theta = 35$ seconds. Thus, the range of ΔT_1 is from one extreme to the other, namely

$$|\Delta T_1| = 44.1 + 10.6 = 54.7^\circ\text{F} . \quad (7.22)$$

The same curves and procedure can be used for calculating the temperatures of adjacent points on a piping system to put into the T_a and T_b terms of Eq. 10 of the Code. The designer need only work with the T_A curves in this case since T_a and T_b represent the average temperature of specific locations.

8. FATIGUE EVALUATIONS

Fatigue failure occurs when the maximum stress from all loads and displacements is concentrated at a point and continued cycling of the stress produces a crack which propagates through the material and results in a leak. The basic philosophy of protection against fatigue failure and the requirements that must be met to assure shakedown or cycling within the elastic range were discussed in Section 7 of this manual. The actual fatigue evaluations that must be performed when shakedown is and is not assured are discussed in this section. These evaluations include the elastic fatigue analysis and the elastic-plastic fatigue analysis. Some aspects of the first are discussed in Refs. 9, 87, and 95 through 99; and plastic analysis is discussed in Ref. 100.

8.1 Elastic Fatigue Analysis

Equation 11 of the Code provides the designer with a tool for calculating the peak stress intensity. The peak stress intensity at a point is merely the maximum stress intensity calculated at that point (from Eq. 10 of the Code), including any local structural discontinuity (notch) effects plus any local thermal stresses that are present.

$$S_p = K_1 C_1 \left(\frac{P_o D_o}{2t} \right) + K_2 C_2 \left(\frac{D_o}{2I} \right) M_1 + \frac{1}{2(1-\nu)} K_3 E \alpha |\Delta T_1| + K_3 C_3 E_{ab} |\alpha_a T_a - \alpha_b T_b| + \frac{1}{1-\nu} E \alpha_1 |\Delta T_2|, \quad (11)$$

where

$S_p \leq P_L + P_b + P_e + Q + F$ as defined in Table F-104 of Appendix F in ANSI B31.7 (Fig. NB-3222-1),

K_1, K_2, K_3 = local stress indices for the specific piping component being investigated [Appendix D of ANSI B31.7 (NB-3680)], and

$|\Delta T_2|$ = range of absolute value (without regard to sign) for that portion of the nonlinear thermal gradient through the wall thickness not included in $|\Delta T_1|$ of Eq. 10, °F.

The definition of $|\Delta T_2|$ is illustrated in Fig. 7.2 (page 36) and discussed in Subsection 7.5.2, and all other terms in Eq. 11 are as defined for Eq. 10 of the Code (page 30 in this manual).

Examination of Eq. 11 indicates that "K" values have been added to the terms that appear in Eq. 10 to represent notch effects and that a new temperature term has been added to the equation. The "K" values are stress indices which predict the notch effects, and they are generally elastic stress concentration factors. The local thermal stress $|\Delta T_2|$ is a function of the nonlinear portion of the thermal gradient through the wall thickness.

8.2 Elastic-Plastic Fatigue Analysis

When the value of S_n calculated from Eq. 10 of the Code exceeds the value of $3S_m$, the fatigue evaluation must include the effects of plastic cycling. Since stresses are allowed to exceed the shakedown limits of Eq. 10, a limitation must be placed on the magnitude of the expansion stresses to prevent the development of a hinge moment and possible collapse of the system. This is done in ANSI B31.7 by requiring that the total number of cycles in which $3S_m$ is exceeded be less than 1000 and that the stress limits of Eqs. 9 and 12 of the Code be met.

$$S_e = C_2 \left(\frac{D_o}{2I} \right) M_i \leq 3S_m, \quad (12)$$

where

S_e = expansion stress, psi,

C_2 = secondary stress index for specific piping component being investigated [Appendix D of ANSI B31.7 (NB-3680)],

D_o = outside diameter of pipe, in.,

I = moment of inertia, in.⁴, and

M_i = range of moment loading resulting from thermal expansion and anchor movements.

Equation 12 requires that the moment loading (M_i) resulting from thermal expansion and anchor movements will not produce a primary plus secondary stress in excess of $3S_m$.

However, the requirement in ANSI B31.7 that the total number of cycles in which $3S_m$ is exceeded be less than 1000 is replaced in Section III by a requirement that Eq. 13 be met, where Eq. 13 of Section III is as follows.

$$C_1 \frac{P_o D_o}{2t} + C_2 \frac{D_o M_1}{2I} + C_3' E_{ab} |\alpha_a T_a - \alpha_b T_b| \leq 3S_m ,$$

where

P_o = range of operating pressure,

C_1, C_2 = secondary stress indices for the specific component under investigation (NB-3680), and

C_3' = stress index value given in Code Case No. 1486 and in revised Table NB-3683.2-1 (Winter 1971 Addenda to Section III).

If the hinge moment requirements of Eq. 12 are satisfied, the peak stress intensity for each transient condition is calculated by using Eq. 11. The alternating stress intensity is calculated for those transients which do not meet the requirements of Eq. 10 by using Eq. 13 of ANSI B31.7 (Eq. 14 of Section III). This equation provides for the decay of fatigue life for cycling in the plastic range.

$$S_{alt} = \frac{1}{2} K_e S_p , \quad \text{ANSI B31.7 (13)}$$

Section III (14)

where

S_{alt} = alternating stress intensity,

$$K_e = 1.0 + \frac{1-n}{n(m-1)} \left[\frac{S_n}{3S_m} - 1 \right] \quad \text{for } 1.0 < \frac{S_n}{3S_m} < m ,$$

$$= \frac{1}{n} \quad \text{for } \frac{S_n}{3S_m} > m ,$$

m, n = material parameters used to compensate for reduction in cycle life in plastic cycling,

S_p = peak stress intensity value calculated from Eq. 11, and

S_n = primary plus secondary stress intensity value calculated from Eq. 10.

Values of the material parameters m and n are given in the Code for the various classes of code materials. The values given in the following

tabulation were taken from Paragraph 1-705.4.1 of ANSI B31.7 (NB-3653.6).

<u>Material</u>	<u>m</u>	<u>n</u>
Low-alloy steel	2.0	0.2
Martenistic stainless steel	2.0	0.2
Carbon steel	3.0	0.2
Austenitic stainless steel	1.7	0.5
Nickel-chrome-iron	1.7	0.33

When S_{alt} values for each transient condition have been determined by using Eq. 13 of ANSI B31.7 (Eq. 14 of Section III), the cumulative usage factor must be calculated. The cumulative usage factor (U) shall not exceed 1.0, and the rules for its determination are given in Paragraph 1-705.3.4 of ANSI B31.7 [NB-3222.4(e)(5)].

8.3 Example Fatigue Evaluation

The following example evaluation is provided to demonstrate the solution of Eqs. 11, 12, and 13 (14) of the Code for a simple case. The three operating conditions discussed in Subsection 7.4 of this manual (page 39) are used for this case to provide continuity. In performing an analysis in accordance with the rules of the Code, the designer would complete the solution to Eq. 10, as outlined in Subsection 7.4 of this manual, and proceed with the analysis as follows.

The loads for the three conditions tabulated on page 39 with the values for ΔT_2 tabulated below must be used when investigating all of the possible loading combinations resulting from these three conditions.

<u>Condi- tion</u>	<u>ΔT_2 (°F)</u>
0	0
1a	+35
1b	+22
1c	+12
1d	+ 3
2	- 3
3	+ 6

The sets of conditions and loading values that must be used in the solution of Eq. 11 are given in Table 7.1 (page 40 of this manual) and the values of ΔT_2 that must be included for these sets of conditions are tabulated below.

Sets of Condi- tions	$ \Delta T_2 $ (°F)	Sets of Condi- tions	$ \Delta T_2 $ (°F)
0-1a	35	1a-2	38
0-1b	22	1b-2	25
0-1c	12	1c-2	15
0-1d	3	1d-2	6
0-2	3	1a-3	29
0-3	6	1b-3	16
		1c-3	6
		1d-3	3
		2-3	3

Equation 10 was reduced by applying the calculated C indices and known geometries and the loads given in Table 7.1 were used to produce values of the basic terms in Eq. 10 for each set of loading conditions. These values are given in Table 7.2 (page 41) of this manual. Therefore, the only unknowns remaining in Eq. 11 for the specific sets of loading conditions are

$$K_1, K_2, K_3, \text{ and } \frac{1}{1 - \nu} E \alpha |\Delta T_2| .$$

Values for the K indices are given in Table D-201 in Appendix D of ANSI B31.7 (NB-3680). As stated in Subsection 7.4, this analysis is being performed for a type 304 stainless steel radius elbow 20 in. long with a wall thickness of 1.500 in. that is welded (with a flush weld) to a 20-in. pipe with a wall thickness of 1.000 in.; and the transition between the two is tapered in accordance with Fig. 1.727.3.1 of ANSI B31.7 (NB-4233-1). The applicable K indices values are tabulated below.

Stress Index	Elbow	Tapered Transition	Flush Girth Weld	Value Used in Eq. 11
K_1	1.0	1.5	1.1	1.65
K_2	1.0	1.8	1.1	1.98
K_3	1.0	1.5	1.1	1.65

By using the solution to Eq. 10 given in Subsection 7.4 and applying the K indices tabulated on the preceding page, Eq. 11 can be reduced to the form

$$\begin{aligned}
 S_p &= 1.65(11.4P_o) + 1.98(0.01M_i) + 1.65(197.9|\Delta T_1|) \\
 &\quad + 1.65(277|T_a - T_b|) + \frac{1}{1 - 0.3}(277)|\Delta T_2| \\
 &= 18.81P_o + 0.0198M_i + 326.54|\Delta T_1| + 457.05|T_a - T_b| + 395.7|\Delta T_2|
 \end{aligned}$$

The loads given in Table 7.1 (page 41) and tabulated on page 68 for $|\Delta T_2|$ are used in this form of Eq. 11 to produce the values given in Table 8.1.

Table 8.1. Calculated Values of Eq. 11 Terms for Different Loading Conditions of Example Analysis

Sets of Condi- tions	18.81P _o	0.0198	326.54	457.05	395.7	S _p
		× 12M _i	× ΔT ₁	× T _a - T _b	× ΔT ₂	
0-1a	37600	0	3300	2300	13800	57000
0-1b	37600	490	42500	4600	8700	93800
0-1c	37600	1600	26100	13700	4700	83700
0-1d	37600	2700	4900	3200	1200	49600
0-2	28200	4100	3200	5500	1200	42200
0-3	48900	4200	7200	2300	2400	65000
1a-2	9400	4100	6500	7800	15000	42700
1b-2	9400	4200	45700	10000	9900	79200
1c-2	9400	4300	29400	19100	5900	68100
1d-2	9400	4800	8200	8750	2400	33600
1a-3	11300	4200	3900	0	11500	30900
1b-3	11300	4000	35300	2300	6300	59200
1c-3	11300	4040	18900	11400	2400	48000
1d-3	11300	4400	2300	900	1200	20150
2-3	20700	4650	10400	7800	1200	44750

The calculated values of S_n and S_p for each set of conditions are given in Table 8.2. The calculated values of S_{alt} are also given. The values of S_{alt} = 1/2(S_p) except in those cases where the value of S_n is greater than the value of 3S_m. For such cases, the value of S_{alt} must be calculated by using Eq. 13 of ANSI B31.7 (Eq. 14 of Section III). This is the case for condition set 0-1b of this example analysis, but before

Table 8.2. Calculated Stress Values of Eqs. 10 and 11 for Different Loading Conditions of Example Analysis Compared With Allowable Stress Values

Sets of Condi- tions	S_n (psi)	$3S_m$ (psi)	S_p (psi)	S_{alt} (psi)
0-1a	26,200	46,800	57,000	28,000
0-1b	51,500	46,800	93,800	a
0-1c	47,400	46,800	83,700	
0-1d	29,100	46,800	49,600	24,800
0-2	24,500	46,800	42,200	21,100
0-3	37,500	46,800	65,000	32,500
1a-2	16,500	46,800	42,700	21,350
1b-2	41,600	46,800	79,200	39,600
1c-2	37,300	46,800	68,100	34,050
1d-2	18,300	46,800	33,600	16,800
1a-3	11,300	46,800	30,900	15,450
1b-3	31,600	46,800	59,200	29,600
1c-3	27,300	46,800	48,000	24,000
1d-3	11,000	46,800	20,150	10,075
2-3	25,900	46,800	44,750	22,375

^aCondition set 0-1b results in maximum value of S_n in excess of $3S_m$ for combinations of condition 0 and 1, thereby eliminating all other sets involving condition 1 from consideration; and the value of S_{alt} must be calculated by using Eq. 13 (14) of the Code.

Eq. 13 (14) can be used to calculate the value of S_{alt} for this set, the designer must demonstrate that the set under consideration does not produce a hinge moment by meeting the requirements of Eq. 12.

$$S_e = C_2 \left(\frac{D_o}{2I} \right) M_i \leq 3S_m \quad (12)$$

The value of $C_2 (D_o/2I)M_i$ for condition set 0-1b is obtained from Table 7.2 (page 41 of this manual). Thus, for set 0-1b

$$S_e = 250 \text{ psi},$$

which is less than $3S_m$ (46,800 psi) for set 0-1b. The value of S_{alt} for set 0-1b can now be calculated by using Eq. 13 (14) of the Code.

$$S_{alt} = \frac{1}{2} K_e S_p \quad (13)$$

$$(14)$$

where

$$S_n/3S_m = 51,500/46,800 = 1.10,$$

$$m = 1.7,$$

$$n = 0.5, \text{ and}$$

$$K_e = 1.0 + \frac{1 - 0.5}{0.5(1.7 - 1)}(1.10 - 1) = 1.14 .$$

$$S_{alt} = \frac{1}{2}(1.14)(93,800) = 53,500 \text{ psi} .$$

When there are two or more types of stress cycles that produce significant stresses, their cumulative effect must be evaluated in accordance with the rules of Paragraph 1-705.3.4 of ANSI B31.7 [NB-3222.4(e)(5)]. Performance of this portion of the fatigue evaluation may be facilitated by plotting the values of S_p (calculated by using Eq. 11 of the Code) for the loading conditions as a function of time, as is illustrated in Fig. 8.1.

Fig. 8.1. Values of S_p Plotted as a Function of Time for Loading Conditions of Example Analysis.

The condition sets to be considered in this portion of the fatigue evaluation are tabulated below.

<u>Set</u>	<u>Cycles</u>	<u>Comments</u>
0-1b	25	Eliminates 1b, reduces 0 to 225 cycles
0-3	225	Eliminates 0, reduces 3 to 775 cycles
2-3	775	

It should be noted that in going from condition 0 to condition 1b, condition 2 is automatically considered since the only reversal of stress occurs in going up to 1b and then back down to 0. The usage factor (U) is calculated for each of these three sets of conditions from the expression

$$U = n/N ,$$

where

n = specified number of times the stress cycle will be repeated during the life of the system and

N = maximum number of stress cycle repetitions that could be allowed if this type of cycle were the only one acting on the system.

The calculated value of S_{alt} for the condition set under consideration is used in the applicable design fatigue curve shown in Figs. 1-705.3.3 (a), (b), or (c) of ANSI B31.7 (I-9-1, I-9-2, or I-9-4) to determine the allowable number of cycles (N). The resulting values for the three sets of conditions considered in this example evaluation are tabulated below.

Condition Set	Specified Number of Cycles (n)	S_{alt} (psi)	Allowable Number of Cycles (N)	Usage Factor (U)
0-1b	25	53,500	17,000	0.001
0-3	225	32,500	280,000	0.001
2-3	775	22,375	∞	0.000

cumulative usage factor = 0.002

In this case, the values for N for the corresponding values of S_{alt} were taken from Fig. 1-705.3.3(b) in ANSI B31.7 (I-9-2). Since the cumulative usage factor is less than 1.0, the design is acceptable with respect to fatigue.

9. SEISMIC MOTION ANALYSIS

Seismic motion of the earth may be treated as a member of a random process. Certain assumptions relative to the characteristics of typical earthquakes are made so that these characteristics can be easily employed in a dynamic response analysis. In general, the seismic motion of the surface of the earth occurs in three translational and three rotational modes. However, the significant response is in the horizontal plane of the earth's surface, and it is coupled with a lesser response in the vertical direction. Based on measured data, the major response is in the horizontal directions. The shear (S) and dilational (P) waves dissipate rapidly to leave a "Rayleigh" wave that is slightly dampened and produces mostly (2:1) horizontal motion.

Piping rarely if ever experiences the actual seismic motion of the earth since it is suspended on hangers and attached to vessels that are in turn attached to the ceilings and floors of the containment building. Although the earthquake motion contains a band of frequencies, the building itself acts as a filter to this environment and will effectively transmit only those frequencies corresponding to its own natural modes of vibration. The principles of structural design for dynamic loads are discussed in detail in Refs. 101, 102, and 103. The recommended approaches and structural parameters that should be included in a dynamic response analysis of piping subjected to forces derived from seismic excitation are discussed in this section, and a general procedure that may be used in the structural analysis of piping systems subjected to random earthquake excitations is presented in Appendix B of this manual.

9.1 Format of Environment Input

The forcing function input for the piping analysis is originally derived from a dynamic response analysis of the containment building subjected to seismic motion. Depending on the mathematical model and method of solution selected for analysis of the building, the piping input may

be in the form of

1. displacement, velocity, and acceleration time histories;
2. response spectra in terms of displacement, velocity, and acceleration;
or
3. power spectral densities.

If time histories are supplied for the piping analysis, the input can be described mathematically as a Fourier series in sines and cosines over a sufficiently long time interval. The use of this type of input allows the analyst some versatility in the choice of computer programs to be used in solving the equations of motion for the piping.

Response spectra inputs are usually presented in the form of displacement, velocity, or acceleration as a function of frequency, as is illustrated in Fig. 9.1. This input is obtained by plotting the maximum response of a single mass-spring-damper oscillator to a complex base motion time history, and a plot of the maximum response versus the natural frequency of the oscillator forms the response spectrum for that particular base motion input. The use of input in this form restricts the analyst to the use of the normal mode method in the solution of the equations of motion.

When the earthquake is considered as a random process acting on the containment building, the attachment points of the piping will also respond in a random fashion. This response is conveniently presented in terms of a power spectral density plot, which is the response parameter squared per cycle as a function of frequency. A typical power spectral density plot for a wide-band excitation is illustrated in Fig. 9.2. The use of this form of input allows the analyst to compute the steady-state response of the piping to a sinusoidal variation of the corresponding base motion parameter related to the input power spectral density. This analysis results in the transfer function (transmissibility) as a function of frequency, which when squared and multiplied by the input power spectral density is equal to the output power spectral density when the input is a stationary random signal.

Fig. 9.1. Idealized Response Spectrum for Seismic Environment.

Fig. 9.2. Typical Power Spectral Density Plot for a Wide-Band Excitation.

9.2 Mathematical Model

The first step in the development of a mathematical model is to reduce the piping system to a series of masses and interconnecting springs. The mass points are selected judiciously so that their locations will coincide with the locations of large valves and supporting hangers. The straight piping between these points should be divided into a large enough number of elements to obtain reasonable accuracy on the first few response modes. This is a somewhat arbitrary procedure that requires a certain amount of judgment. It must be remembered that a continuum, which has an infinite number of degrees of freedom, is being replaced with a finite number of discrete masses and springs. As a general rule, reasonable accuracy can be achieved if straight unsupported sections of pipe, including bends, are divided into five to ten elements. If any doubt about the number exists, the number of elements should be increased and the dynamic response of the system should be recalculated and checked for convergence. There is a number of elements beyond which the response does not change appreciably for a given forcing function.

The stiffness of each element is computed directly by generating the expression for the total strain energy in the element. The torsional energy must be included because the bending deflection of a right-angle bend in the pipe will produce a torque, tending to twist the pipe. The deflections as a function of the element loads can be computed by using any standard procedure such as unit-load, virtual-work, etc.

The element stiffness matrix is found by computing the load necessary to produce a unit deflection in the direction of one of the loads while all other deflections are zero. This is done for each deflection. The computed loads represent the direct and coupling stiffness coefficients. Once these loads are computed for each element, the stiffness matrix for the complete structure of physically coupled elements can be obtained.

Another approach is to compute the influence coefficients by assuming unit loads at the ends of the element and solving for the deflections. These deflections are the influence coefficients with dimensions of displacement per unit force as compared with stiffness coefficients with dimensions of force per unit displacement.

9.3 Methods of Solution

A definite pattern of terms emerges when the equations of motion are written for each mass point. The equation of motion is no more than a force summation for each coordinate or mass location. Such equations can be conveniently written in the following matrix equation form where capital letters are used to denote matrices and lower-case letters are used to denote vectors and scalars.

$$M\ddot{x} + C\dot{x} + Kx = f(x,\dot{x},t) , \quad (9.1)$$

where

- M = mass matrix,
- $\ddot{x}$ = acceleration vector,
- C = viscous damping matrix,
- $\dot{x}$ = velocity vector,
- K = stiffness matrix,

x = displacement vector, and

f = external forcing function vector in terms of the base motions

x or $\dot{x}$ or in terms of force with respect to time.

The mass matrix (M) is square with values along the diagonal only unless consistent mass matrices are used. The damping matrix (C) and the stiffness matrix (K) are also square and contain terms that are symmetric about the diagonal. The acceleration, velocity, displacement, and external forces are arranged as column vectors.

If the forcing function has been obtained as the result of a displacement, velocity, or acceleration time history, Eq. 9.1 may be solved by either direct integration or by the normal mode method. If the forcing function is in the form of a response spectrum (motion parameter as a function of frequency), the normal mode method is used. If the environmental input for the response of the containment building foundation is in the form of a power spectral density plot, the probabilistic approach is used. Each of these methods is discussed in the following subsections, and the use of each method is demonstrated in an analysis of the simple pipe element illustrated in Fig. 9.3. This pipe element consists of a straight length of 24-in. Schedule-80 steel pipe between two major components in the building. A valve is located 38 ft from the left-hand end of the pipe, and the weight of the pipe and valve is supported nearby.

Fig. 9.3. Simple Pipe Element Used to Demonstrate Seismic Environment Analysis Methods.

9.3.1 Time History Method

When the input for the piping environment is described as a displacement, velocity, or acceleration time history; the equations of motion can be solved by direct integration. The finite difference technique is used to solve the equations in a step-by-step procedure, the solution advancing with each time step. There are a number of schemes available for starting the solution and for carrying it out through the required time interval. When programmed for digital computers, some schemes are faster than others, but at the same time, they may be less stable. The choice of time interval is usually critical since the stability of the numerical solution depends upon the time interval used.

To illustrate the use of the time history method in an analysis of the pipe element shown in Fig. 9.3, it was assumed that the seismic environment input consisted of an acceleration history. The piping and valve were modeled as 17 discrete masses, as illustrated in Fig. 9.4.

Fig. 9.4. Discrete Element Model of Simple Pipe Element Used in Time History Method Involving Direct Integration.

The connecting pipe between these masses was assumed to act as a beam in only one plane. In actual practice, each pipe element would be assumed to possess six degrees of freedom and associated stiffnesses. However, the mechanics of the analysis can be completely demonstrated by assuming only two degrees of freedom per mass. This assumption was made to reduce the complexity of the example analysis. Thus, only transverse displacement and rotation about an axis normal to the plane of the paper of each end of the element were assumed as the degrees of freedom.

The mass matrix (M) was obtained from the weights of the pipe elements and their contents, the stiffness matrix (K) was calculated from the geometry and properties of the pipe, and a damping matrix (C) proportional to the mass matrix was assumed. The excitation was confined to the translation degree of freedom. Both ends of the pipe and the support spring were assumed to be excited in the vertical direction. The acceleration excitation was comprised of three harmonics, as given in Eqs. 9.2 and 9.3.

$$\ddot{x}_1 = 27.02(\sin 43.96t) + 15.44(\sin 75.36t) + 7.72(\sin 157.0t) \quad (9.2)$$

and

$$\ddot{x}_2 = 32.42(\sin 43.96t) + 19.30(\sin 75.36t) + 9.65(\sin 157.0t), \quad (9.3)$$

where

$$\ddot{x}_1 = \text{excitation at left-hand end of pipe and the support, in./sec}^2,$$

$$\ddot{x}_2 = \text{excitation at right-hand end of pipe, in./sec}^2.$$

The displacement excitation (x_1 , x_2) can be readily obtained from the expressions given in Eqs. 9.2 and 9.3, and the forcing functions on masses 1, 8, and 15 (shown in Fig. 9.4) can be obtained from the stiffnesses of the structure. The translation forcing function at mass 1 is illustrated in Fig. 9.5.

The equations of motion of the structure were written in the form of Eq. 9.1, and their solution was obtained by direct integration. The resulting response was then used to obtain the variation with time of the shear force and the bending moment throughout the structure. Examples of this response at particular nodes are illustrated in Figs. 9.6 and 9.7.

Fig. 9.5. Time Variation of Forcing Function at Left-Hand End of Pipe Analyzed by Time History Method Using Direct Integration.

Fig. 9.6. Time Variation of Shear Force at Mass 6 in Pipe Analyzed by Time History Method Using Direct Integration.

Fig. 9.7. Time Variation of Bending Moment at Mass 6 in Pipe Analyzed by Time History Method Using Direct Integration.

9.3.2 Response Spectra Method

When the forcing function is in the form of a response spectrum (motion parameter as a function of frequency), the normal mode approach is used to determine the maximum response of the pipe. In this approach, the response of each coordinate point is made up of a contribution from each independent mode of vibration. This has been a popular method of estimating structural response and is often offered as an acceptable substitute for the time history method because of the greater expense and sophistication of the latter.

To demonstrate application of the response spectra method, the response spectrum assumed for the simple pipe element to be analyzed is illustrated in Fig. 9.8. The first step in this application is to calculate the natural frequencies and mode shapes of the undamped homogeneous equations. It is assumed that a small amount of damping will not affect the frequencies and mode shapes. The simple pipe element illustrated in

Fig. 9.8. Response Spectrum for Pipe Element Analyzed by Response Spectra Method.

Fig. 9.4 was modeled as 17 discrete masses and 18 nodes, as is illustrated in Fig. 9.9.

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Fig. 9.9. Discrete Element Model of Pipe Element Used in Response Spectra Method of Analysis.

The second step is to transform the coordinates of the model into normal or generalized coordinates. Since the normal modes are independent of each other (orthogonal), the intent is to develop diagonal coefficient matrices so that each equation of motion is uncoupled and can be solved separately. For this analysis of the simple pipe element, the mass and stiffness matrices were calculated in a manner similar to that used for the time history method, and the natural frequencies and normal modes of the structure were obtained from equations of the form

$$M\ddot{x} + Kx = 0 \quad (9.4)$$

The final step is to solve each modal equation separately as a single degree of freedom. This is done by the method of variation of parameters more commonly known as Duhamel's integral. The maximum response for each modal frequency is directly related to the response spectrum. For load calculations, the maximum bending, shear, and torsional accelerations are basic outputs of the modal approach for each mass location. These accelerations are multiplied by their appropriate mass inertia terms, and maximum shear, moment, and torsion diagrams are constructed for stress evaluation. The actual time history of stress at each mass point is derived from the displacement response as a function of time and the associated stiffness coefficients. This time history of stress is used in a fatigue analysis to predict the service life of the piping.

In this example analysis, the peak acceleration responses for the natural frequencies were obtained from the response spectrum, and the upper bound of the generalized acceleration response for each mode was obtained by using the modal participation factors and the normal mode shapes. It has been suggested that the peak system response is the root square sum value of all contributing modes. However, if only three or fewer modes significantly contribute to the system response, they should be added directly. Otherwise, the root square sum system response is a reasonable estimate of the response. The upper bound and root square sum translation acceleration levels for this example analysis are given in Table 9.1, and the upper bound and root square sum rotation acceleration levels are given in Table 9.2.

The acceleration response may also be calculated as a time-varying function by combining the modal responses at their appropriate frequencies. These acceleration responses may then be used to obtain the bending moment and shear force distribution throughout the structure in a manner similar to that used in the time history method.

The usual way of presenting response spectrum curves is to plot a number of earthquake accelerometer traces and develop for each the maximum values of displacement, velocity, and acceleration as a function of the natural frequency of the system for a given damping value; normalize and average all of the results; and scale them to the design earthquake. When the designer is seeking the maximum response of a single-degree-of-freedom system, he can be reasonably certain that the results obtained in this manner will be fairly accurate and can be used for design purposes. Unfortunately, the majority of seismic analyses involve rather complex coupled multi-degree-of-freedom systems. While he may attempt to apply the response curves to the more complex piping system through the use of the modal participation factor, the designer has little assurance that the upper bound is conservative unless he actually adds up the maximum responses from each normal mode of vibration and performs a more accurate analysis to determine the adequacy of the upper bound. The true system response is a time-dependent combination of the various normal mode responses, and the response spectra method therefore suffers significantly when applied to a multi-degree-of-freedom system.

Table 9.1. Upper Bound and Root Square Sum Translation Acceleration Levels for Analysis of Pipe Element Using Response Spectra Method

Node Number	Upper Bound Acceleration (G's)	Root Square Sum Acceleration (G's)
2	0.0735	0.0404
3	0.2082	0.1195
4	0.3299	0.2112
5	0.4047	0.3091
6	0.5027	0.4087
7	0.5792	0.4964
8	0.6256	0.5595
10	0.609	0.5900
11	0.6305	0.5795
12	0.5689	0.5246
13	0.5453	0.4409
14	0.4540	0.3355
15	0.3366	0.2242
16	0.2332	0.1282
17	0.0906	0.0472

Table 9.2. Upper Bound and Root Square Sum Rotation Acceleration Levels for Analysis of Pipe Element Using Response Spectra Method

Node Number	Upper Bound Acceleration (radians/sec ²)	Root Square Sum Acceleration (radians/sec ²)
2	0.8106	0.4637
3	0.9650	0.6955
4	1.0808	0.8114
5	1.2352	0.8887
6	1.2738	0.8501
7	0.9650	0.6955
8	0.7720	0.4637
10	0.3088	0.1932
11	0.7334	0.4250
12	1.1194	0.6955
13	1.0808	0.8114
14	1.3124	0.9273
15	1.2352	0.9274
16	0.965	0.7342
17	0.8492	0.5023

9.3.3 Probabilistic Method

When the earthquake is considered as a stationary random process acting on the containment building and the environmental input for the response of the building foundation is in the form of a power spectral density plot, the steady-state solution of Eq. 9.1 for a unit input is required. This solution is obtained by assuming a sinusoidal variation of the parameter corresponding to the one presented in the power spectral density plot and by computing the particular solution of the equation of motion. For example, if the response at the attachment of a pipe to a vessel was described in terms of acceleration²/cps as a function of frequency, a sinusoidal acceleration function of 1.0G amplitude would be applied at that attachment point. The frequency of the forcing function would be varied throughout the same frequency range presented in the power spectral density plot. At each coordinate point (mass location), the transmissibility (T) is calculated as a function of frequency (ω). The random response power spectral density at each coordinate point is then computed by using the expression

$$S_{out} = T^2 S_{in} , \quad (9.5)$$

where S_{in} = the power spectral density of the foundation motion. If the input and output power spectral densities are in units of in.²/cps, the root-mean-square response in inches must be multiplied by the stiffness coefficients to obtain the root-mean-square forces, moments, and torques at each mass location. When the power spectral densities are in units of acceleration (G²/cps), the root-mean-square responses are multiplied by their appropriate inertia terms to obtain forces.

The simple pipe element illustrated in Fig. 9.3 was modeled as 17 discrete masses and 18 nodes, as illustrated in Fig. 9.9 (page 74) for analysis using the probabilistic method. A constant power spectral density function $f(\Omega)$, which is often referred to as a white noise, was used in this example analysis for simplicity. A value of $f(\Omega) = 0.00582 \text{ G}^2/\text{cps}$ was assumed. For a random excitation, the response of the structure may be characterized by its mean square value, and the mean square response is related to the mean square excitation by the square of the frequency

response function. The frequency response function for the structure of this analysis was obtained by using the normal mode method, the discrete element model illustrated in Fig. 9.9, and unit sinusoidal excitations at the same three points used in the time history method of analysis. A typical frequency response function for the translation of mass 6 is illustrated in Fig. 9.10.

Fig. 9.10. Frequency Response of Acceleration at Mass 6 for Sinusoidal Input of 1.0G Obtained in Analysis of Simple Pipe Element With Probabilistic Method.

The mean square response at each mass was then obtained as the integral over the frequency range of the product of the power spectral density and the square of the frequency response function. These values are given in Table 9.3. The mean square of the moments and shears throughout the pipe can be computed from the acceleration response in the same manner used for the time history method. Thus, the mean square stress levels can be obtained. When these levels have been computed, probability statements relative to the expected stress levels in the structure can be made.

Table 9.3. Pipe Response for Random Excitation Determined by Probabilistic Method

Node Number	Mean Square Response (G ²)	Root Mean Square Response (G)
2	13.719	3.704
3	76.141	8.726
4	125.46	11.201
5	136.16	11.66
6	175.77	13.25
7	232.68	15.25
8	258.95	16.09
10	276.41	16.63
11	262.14	16.19
12	215.94	14.69
13	218.33	14.78
14	154.07	12.41
15	97.004	9.849
16	90.522	9.514
17	25.796	5.079

10. THE NUCLEAR POWER PIPING DESIGN SPECIFICATION

The ANSI Code B31.7 and Subarticle NA-3250 of Section III require that the owner of a nuclear power plant prepare a specific type of design specification for each piping system in the plant. The information presented in this section of the manual is intended to provide specific guidance in the proper specification of the function, operating conditions, and design requirements for such systems. Guidelines for nuclear pressure vessels are discussed in Ref. 104 and in Appendix B of Section III. The required and supplementary information that is used in the design specification is described in Appendix C of this manual. Since the structural integrity of a piping system and compliance with the design specification must be established by the designer, suggested outlines and procedures for the required reports are given in Appendix D of this manual.

The discussion contained herein is addressed primarily to the user or owner of the nuclear power plant. The owner (he who intends that a power piping system be designed, constructed, tested, and certified to be a system which is in compliance with the Code) has the responsibility of providing or causing to be provided a design specification for each such system. Unfortunately, even after 8 years of operation under the rules of Section III, the design specifications produced are generally incomplete and confusing. This has contributed significantly to scheduling delays in obtaining nuclear vessels and the required documentation, particularly the stress report. A review of the intent of the Code requirements with respect to the design specification is provided here in an attempt to alleviate this situation.

The introduction to the Code provides an effective mechanism for inquiry to and communication with the Code Committee, and the reader is encouraged to use this mechanism. The Code is subject to a continuing review and may be explained by case interpretations or revised by the issuance of semiannual addenda. Therefore, any application of the Code should include a review of rule changes that have occurred since the last publication date.

10.1 System Classification

Selection of the system classification is the responsibility of the owner. This selection is based on the system and operational requirements and is subject to the rules and approval of the regulatory agencies. Note that it is permissible to construct a Class-2 or Class-3 system in accordance with Class-1 rules.

10.2 General Technical Considerations

The rules of the Code are written to be general enough to be applicable to all types of nuclear power plant systems. They do not provide for specialized problems of system operability. Similarly, the Code does not enter into areas that are properly contractual matters between the owner or his agent and the manufacturer.

Restricting the considerations to technical matters necessary to procure a proper piping system, the user or owner must specify

1. information that is required by the Code to be provided in order that the Code rules may be properly applied,
2. additional but not less restrictive requirements that modify the Code rules to make them complete for the specific systems or to provide more specific or restrictive requirements, and
3. supplementary technical requirements beyond the scope of the Code.

The document that contains all of this information is defined as the equipment specification. It may also contain nontechnical requirements.

The design specification required by Paragraph 700(c) of ANSI B31.7 and Subarticle NA-3250 of Section III covers only the first of the three items listed above. It is highly desirable that this point be recognized and that this information be provided as a specific portion of the equipment specification. In addition, each item should be identified as to the paragraph in the Code to which it pertains. Specific guidance for preparation of the design specification is contained in Appendix B of Section III.

Any additional requirements should also be identified with the corresponding paragraph in the Code where possible. The Code is divided so that each article pertains to a specific aspect of piping system construction. When additional requirements are not identified with the corresponding paragraph in the Code, the particular aspect to which the additional requirements are intended may not be clear. This is particularly true with respect to material and examination requirements. It is common for such additional requirements to fail to distinguish between examinations, such as those conducted by the manufacturer, and inspections, such as those conducted by the qualified inspector defined by the Code.

10.3 Code Requirements for Design Specification

That information required by the Code to be provided in the design specification is discussed in this subsection. The responsibility for and definition of the design specification are defined in Paragraph 700(c) of ANSI B31.7.

"The owner who requires that a piping system be designed, constructed, examined, tested, and certified in accordance with the rules of this Code shall provide or cause to be provided a design specification giving the function, design requirements, and classification for each such system. The design specification shall be related to operating conditions in sufficient detail to provide a complete basis for design, construction, examination, testing, and certification in accordance with these rules."

The reader is also referred to Subarticle NA-3250 of Section III for a more complete statement of this responsibility.

The items which must be considered in the design specification are discussed here in the order in which they are considered in the specification, and the paragraphs in the Code to which they pertain are noted in parentheses.

10.3.1 General

The following information is required by the Code to be included in the design specification.

1. The system classification must be stated (700.1.2)(NA-2130).
2. The enforcement authority at the location of system installation shall be identified [700(c)](NA-3256).
3. Certification of the design specification and provision of copies to the manufacturer and enforcement authority having jurisdiction at the location of system installation shall be explained [700(c)] (NA-3255 and NA-3256).
4. The procedure for review of the stress report by those responsible for the design specification and subsequent certification by the owner shall be provided [700(c)] (NA-3260).
5. The methods for handling the various data reports, particularly with respect to transmittal to enforcement authorities, shall be defined [700(c)] (NA-3270).
6. The method for continued maintenance of the quality control records for the life of the plant shall be established [700(c)] (NA-4910).

10.3.2 Function

The function [700(c)](NA-3252) of the piping system is defined by a written description and drawings. The nature of all operating equipment attached to the piping system, direction and mass rate of flow, and the purpose of the system should be indicated, even schematically.

1. Information shall be provided to define for each boundary of the Code jurisdiction (700.1.4)(NA-3254)
 - a. the specific dimensional location,
 - b. the allowable forces and moments at each boundary, and
 - c. the structural characteristics of the structure(s) excluded from Code jurisdiction or provided by another supplier when such a structure(s) provides constraints to the piping system.

This information and the information in items 2 and 3 will contribute to

a complete definition of the dimensional requirements which are required for functional reasons.

2. Any dimensions required for functional reasons shall be given. Other dimensions should be avoided or identified as reference dimensions.

3. The tolerances on important dimensions shall be provided for use as deformation limits [B-1224(b)(1)].

4. If any regions may be subjected to corrosion, erosion, or other environmental effects; the specific action to be taken by the manufacturer with respect to the condition shall be defined (1-702.4.1 and 1-704.4.1 through 1-704.4.3) [B-1224(b)(2)].

5. Any region in which the neutron fluence level may exceed 10^{19} nvt should be defined [B-1224(b)(3)].

6. The handling, storage, and delivery requirements should be defined.

10.3.3 Materials

1. Acceptable materials for each component and service must be specified. Materials must be selected from those permitted by the Code (NB-2160 and NB-2121).

2. Any material required for the surveillance program should be specified.

3. Any restrictions on cladding or coatings must be stated (1-723.2.7)(NB-4429).

4. The temperature limits imposed by hydrostatic testing and service temperatures must be stated (1-737.4.1) (NB-6212).

5. Any modifications to the allowable stresses or fatigue curves required by environmental conditions shall be defined (1-701.4.3)(NB-3124).

6. If user approval is required for new materials, this should be stated.

7. Any required rules related to cleanliness should be provided.

8. Impact testing requirements shall be specified for all materials for which they are needed (1-723.2.3) (NB-2300).

10.3.4 Design

With the exception of the design-related items covered under Subsection 10.3.2 on drawings, the major design items which must be treated in the design specification are those which define the design and operation loads. Since this portion of the design specification is seldom prepared in sufficient detail, a more narrative discussion of the items to be included follows.

(a) Design Loadings. In Division 1-701 of ANSI B31.7 (NB-3111), it is stated that the loadings that shall be taken into account in designing nuclear piping systems include but are not limited to

1. internal and external design pressure, including additional pressure resulting from the static and dynamic head of liquids;
2. design temperature;
3. weight of piping, normal contents, and insulation under operating and test conditions;
4. superimposed loads, such as those arising from attached operating equipment;
5. impact loads, earthquake loads, and vibration where specified;
6. reactions and displacements of supports and anchors; and
7. temperature effects with respect to Eqs. 10, 11, and 13 of Division 1-705 (Eqs. 10, 11, and 14 of NB-3653).

It is not sufficient to prescribe just one design value of each load. Instead, the operational history for which the owner desires to have the system designed must be described completely. The stress report will then indicate the adequacy of the piping for that particular operational history. Those responsible for system analysis are generally reluctant to predict the operational history to which a system will be subjected in such detail, but they must realize that this is an essential responsibility. Only the user or his agent can properly define the service conditions for which he wishes to purchase a system.

(b) Design Conditions. The design values given in the specification are equivalent to the similarly designated quantities used in the Code. These values are not for the actual operating conditions that the system

will undergo but are design values for the pressure, temperature, and mechanical loads.

Design pressure is defined in Paragraphs 1-701.2.2 (NB-3112.1) and 1-701.2.4 (NB-3133) of the Code. It may differ for different loops of the system, and it may be an internal pressure, an external pressure, or both an internal and external pressure.

Design temperature is defined in Paragraph 1-701.3.2 of ANSI B31.7 (NB-3112.2). It is the maximum temperature of the metal that will exist under the specified operating conditions for each area of the system considered. It may therefore differ from one area to another.

Design mechanical loads are those loads whose effects on the piping system must be combined with the effects of the design pressure for comparison with certain stress limits at the design temperature. The source of these design mechanical loads and their magnitudes must be included in the design specification since the rules in Subdivision 1-705.1 of ANSI B31.7 (NB-3652) require the use of only "specified mechanical loads". Consideration must be given to loadings which are associated with the specific system, such as the dynamic loads imposed by rapid closure of valves, and those resulting from postulated accident conditions.

It should be pointed out that two separate sets of design rules are presented in the Code. These two sets of rules are for a simplified analysis (1-705) (NB-3650) and a detailed analysis (Appendix F) (NB-3200). Both sets of design criteria are identical, with the only difference being in the ease of application. Application of the rules in Division 1-705 (NB-3650) provides more conservative results. If a component of the piping system does not meet the requirements of Division 1-705 (NB-3650), the designer has two alternatives. He can redesign the system to alleviate the situation which produced the excessive stress by the simplified method, or he can perform an analysis of that component by using the rules in Appendix F (NB-3200). Therefore, the design specification must include all the information required to perform both analyses.

The design temperature, design pressure, and design mechanical loads are used only with respect to the Code requirements for

1. pressure design of components (1-704) (NB-3640),
2. analysis of piping components,

3. limits of primary stress intensity (1-705.1) (NB-3652),
4. design criteria and alternate analysis (Appendix F) (NB-3200),
5. limits for general primary membrane stress intensity (F-104.1) (NB-3221.1),
6. limits for local primary membrane stress intensity (F-104.2) (NB-3221.2),
7. limits for primary membrane plus bending stress intensity (F-104.3) (NB-3221.3),
8. determination of bearing stresses (F-105.2.4)(NB-3227.1), and
9. determination of pure shear stresses (F-105.2.5) (NB-3227.2).

When checking compliance of items 8 and 9 above with the rules of Appendix F (NB-3200), the actual operating loads and temperatures must be considered in addition to the design pressure, temperature, and coincident mechanical loads.

(c) Operating Conditions. In addition to meeting the requirements for design pressure, design temperature, and design mechanical loads, as discussed in the preceding paragraph, the Code requires satisfaction of certain requirements relative to operating conditions. While not strictly within the scope of the Code, operational malfunction of the system may be prevented by supplying two categories of information outlined as follows.

1. The owner should prescribe a set of operational cycles and the number of occurrences of each to provide a basis for safe design, particularly against fatigue failure. The operational cycle is defined in Subdivision F-102.14 of ANSI B31.7 (NB-3213.15) as the initiation and establishment of new conditions followed by a return to the conditions that prevailed at the beginning of the cycle. It is perhaps a consequence of the simplicity of this definition that the basic concept of an operational cycle is so commonly misunderstood. One must realize that this definition leads to valid or conservative results only because of the detailed method of fatigue analysis with which it is used. The combination of the effects of various cycles is discussed in Division F-107 of ANSI B31.7 (NB-3222.4).

In defining the conditions to be considered for an operational cycle, it is essential that a complete description of the time variation of each of the loadings be given. If the time between the two halves of a cycle is indeterminate, such as the time at a given power, it should be stated that the time is long enough for equilibrium conditions to be established. Of great importance is the fact that the loadings should be defined in terms which are usable by the piping designer. The designer cannot make use of the fact that the power changes at the rate of 10% per minute unless he is responsible for the analysis of the entire system. He must be given the time variation in the quantities of the

- a. system pressure,
- b. system temperature,
- c. flow rate,
- d. internal heat generation in the piping system, and
- e. reactions of attached equipment.

2. It is also essential that the operating conditions be categorized in the manner described in ANSI B31 Code Case 70¹⁰⁵ (NB-3113). These categories are normal conditions, upset conditions, emergency conditions, and faulted conditions.

a. Normal conditions are defined as any condition in the course of system startup, operation in the design power range, and system shutdown that occurs in the absence of upset, emergency, or faulted conditions. The rules of Divisions 1-704 and 1-705 of ANSI B31.7 (NB-3640 and NB-3650) are applicable to the analysis of normal conditions.

b. Upset conditions are defined as any deviation from normal conditions expected to occur often enough to necessitate that the design include the capability to withstand the conditions without operational impairment. Upset conditions include those transients which result from any single operator error or control malfunction; transients caused by a fault in a system component, requiring its isolation from the system; transients resulting from the loss of load or power; and any system upset not resulting in a forced outage. Upset conditions include the effect of the specified earthquake for which the system must remain operational or must regain its operational status. The estimated duration of an upset condition shall be included in the design specification.

c. Emergency conditions are defined as any deviation from normal conditions which require a shutdown for correction of the conditions or repair of damage in the system. Such conditions have a low probability of occurrence but are included to provide assurance that no gross loss of structural integrity will result as a concomitant effect of any damage developed in the system.

d. Faulted conditions are defined as those combinations of conditions associated with extremely low probability postulated events whose consequences are such that the integrity and operability of the nuclear energy system may be impaired to the extent where considerations of public health and safety are involved. Such considerations require compliance with safety criteria as may be specified by jurisdictional authorities.

As with system classification, the decision as to what category should be used for a given operating cycle, transient, or event is a system consideration with which the Code is not involved. However, regulatory bodies may further define or restrict application of the procedures, such as requiring that the safe shutdown earthquake be treated as an emergency condition rather than a faulted condition. If this is done, the maximum hypothetical accident would be combined with the safe shutdown earthquake as the faulted condition. Special stress limits are established in ANSI B31 Case 70 (NB-3224 and NB-3225) for the last two categories: emergency conditions and faulted conditions.

(d) Stress Limitations. Quasi-static stress limitations are given in Subdivision F-104.4 (NB-3222.2) and in Eq. 10 of Division 1-705 in ANSI B31.7 (NB-3650) for primary plus secondary stresses and in Paragraph F-105.2.3 (NB-3227) for special situations. They are termed quasi-static because they in no way account for the fact that the situation may be changing with time. It is the general intent of these stress limitations to prevent distortions from occurring in a way which would invalidate the calculations made with respect to the other stress limits, particularly the fatigue analysis. Since the stress from a given source may either add to or subtract from the stress from another source, it is not necessarily conservative to use artificially high values, such as the design

values, for any given loading when performing the analysis. The actual operating values are therefore used, and all instances in time must be considered.

Fatigue stress limitations are given in Subdivisions 1-705.3 (NB-3653.2) and 1-705.4 (NB-3653.6) and in Division F-107 of ANSI B31.7 (NB-3222.4) for general materials of construction and in Division F-109 (NB-3232.3) for bolts. The major difference between the fatigue stress limitations and the other stress limitations of the Code is that localized thermal stresses [F-102.12(2)] [NB-3213.13(b)] and fatigue strength reduction factors (F-102.16) (NB-3213.17) are considered in performing the fatigue analysis.

The existence of steady-state operation is not recognized in the Code other than as a part of an operational cycle. Consequently, there are no specific stress limitations on steady-state operation, and any information given in the design specification for such conditions must be interpreted in such a way to make it applicable to the operational cycles. Such information might be of value in assisting the piping designer to establish end points for his thermal analysis.

10.4 Additional and Supplementary Requirements

Additional requirements have previously been introduced as those requirements which modify the Code rules to make them complete for the specific system or to provide more specific or restrictive requirements. Supplementary technical requirements have been previously introduced as those requirements which are beyond the scope of the Code. Since consideration of a specific system or requirements not governed by the Code is not within the scope of this manual, no specific discussion of these requirements is presented herein.

However, the term "requirements" as used here is intended to include both a statement relative to the situation which may exist and a statement relative to the action which must be taken because of the existence of the situation. Failure to state a criterion or criteria for such situations reduces the item from a specification requirement to an

informational item, and no action is required or should be expected as a result of the inclusion of informational items within a specification. This is not to imply that informational items should necessarily be avoided in the preparation of the equipment specification but that they must be recognized as such.

10.5 Design Specification Checklist

A checklist for the preparation of the design specification is presented here to provide further guidance with respect to the Code requirements. Those paragraphs in the Code that require the user to take specific action in preparing the design specification and those major paragraphs that contain information of which the user should be aware are identified in the following checklist.

<u>ANSI B31.7 Paragraph</u>	<u>Section III Paragraph</u>	<u>Item</u>
700(c)	NA-3200	<ol style="list-style-type: none"> 1. Note owner's responsibility to provide a certified copy of the design specification to the designer and to the enforcement authority having jurisdiction at the location of installation before the system is installed. The manufacturer is responsible for providing a certified copy to the authorized inspector at the manufacturing site. 2. Provide procedure for review of the stress report by those responsible for the design specification. Provide for subsequent certification by the user or his agent. Note owner's responsibility to have stress report filed with enforcement authorities having jurisdiction at installation site. 3. Identification of enforcement authority at the location of system installation must be stated.
700(e)	NA-3300	<ol style="list-style-type: none"> 1. Note the responsibilities of the manufacturer, fabricator, and erector with respect to Code compliance.
700(f)	NA-4910	<ol style="list-style-type: none"> 1. Note the responsibility of the owner with regard to retention of reports.
700.1.1	NA-1110	<ol style="list-style-type: none"> 1. Note the definition of nuclear piping in (a).
700.1.2	NA-2140	<ol style="list-style-type: none"> 1. Piping classification must be stated by the owner.
700.1.3	NA-2134	<ol style="list-style-type: none"> 1. Note that optional classification may be considered.
700.1.5		<ol style="list-style-type: none"> 1. Note the detailed information with regard to flexibility analysis that must be provided with respect to "Code Applications".
1-701.2.2 & 3	NB-3112.1	<ol style="list-style-type: none"> 1. Note the definition of design pressure.
1-701.3.2	NB-3112.2	<ol style="list-style-type: none"> 1. Note the definition of design temperature.

<u>ANSI B31.7 Paragraph</u>	<u>Section III Paragraph</u>	<u>Item</u>
1-701.4.3	NB-3124	1. Special requirements regarding deterioration which may occur in service as a result of radiation effects or instability of the material should be stated. Provisions for surveillance specimens should be made.
	B-1224(b)(3)	2. Fluence levels should be given, particularly where they are rather high and approach or exceed 10^{19} nvt.
1-701.5	NB-3112.3	1. All dynamic loadings that the system will experience, such as impact, earthquake loads, and vibration, must be stated in the specification. The designer is responsible for observing piping for excessive vibration.
1-701.6	NB-3111(b)	1. Note the weight effects. Both live and dead weight must be considered.
1-701.7	NB-3111(f)	1. Dimensions and material of attached equipment must be defined.
1-701.8	NB-3134 & NB-6215	1. Restriction on leak-tightness shall be specified.
1-701.9		1. State any restrictions on cladding material.
1-702.2.4	NB-7110	1. Paragraph NB-7000 in Section III establishes rules for protection against overpressure and for the preparation of an overpressure protection report by the user or his agent.
1-702.3.1 & 1-719.6	Appendix IV	1. Note the requirements for use of materials not covered by the Code.
1-702.4.1	NB-3121	1. Guidance should be given with respect to service conditions which may result in thinning by corrosion, erosion, mechanical abrasion, or other environmental effects.
1-704.3.2	NB-3643.2	1. Note the requirements placed upon attachment of branch connections in 1 and 2.
1-712	NB-3671.1	1. Note that the use of flanged joints shall be held to a minimum.
1-704.7.1		1. Note that expansion joints shall not be used in Class-1 nuclear piping systems.
1-716	NB-3671.5	1. Note that caulked or leaded joints shall not be used in Class-1 piping.
Chapter 1-II & F-100	NB-1120	1. Note maximum and minimum temperature limitations.
F-102.14	NB-3213.15	1. Note definition of operational cycle and of various types of operating conditions in B31 Case 70.
F-106	NB-3222.4	1. Guidance should be provided to the designer relative to the presence of any unusually corrosive environments which might accelerate fatigue failure.
F-109	NB-3232.3	1. Note that fatigue curves are not provided for non-ferrous bolting materials with tensile strengths in excess of 100,000 psi except in Section III.
1.723.1.2	NB-2130	1. Note the responsibility of the material manufacturer as to certification.
1-736.1	NA-5100	1. Note assignment of responsibility for inspection to the owner in B31.7 and to the inspector in Section III.

<u>ANSI B31.7 Paragraph</u>	<u>Section III Paragraph</u>	<u>Item</u>
1-737.1	NB-6111	1. Note the requirements of the hydrostatic tests.
1-737.1.1	NB-6212 & NB-6211	1. Note the temperature limits imposed by hydrostatic testing temperatures and service temperatures. 2. Note the requirement for vents in (c) (NB-6211).
1-737.1.2	NB-6112	1. Note the restrictions on substituting a pneumatic test for a hydrostatic test.
1-737.4	NB-6220	1. The test pressures must be stated.
	NA-8000	1. Note rules given for marking and stamping.
	NA-8230	1. Note that marking and stamping are required.
	NA-8233	1. Note the rules covering the method of marking.
	Appendix IX	1. Note that this appendix establishes quality control procedures.

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APPENDICES

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Appendix A

DETERMINATION OF LOADS IN A FLANGED CONNECTION

Presented herein are methods of analysis for determining the loadings that occur in a flanged connection as a function of bolt preload, internal pressure, temperature, and external bending moments. A free-body diagram of the typical flange joint discussed herein is illustrated in Fig. A.1.

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Fig. A.1. Free-Body Diagram of Typical Flange Joint.

The flanged joint assembly illustrated in Fig. A.1 consists of (1) an upper flange (H), (2) a lower flange (S), and (3) a stud (B). Of the various forces illustrated in Fig. A.1, the only ones that cannot be determined through static equilibrium are the redundant forces. There are four of these for the given arrangement, and they are designated V_{HC} , H_{HC} , M_{HC} , and H_{SA} . All of the other redundant forces can be expressed in terms of these four unknowns. To determine these four

quantities, four deflection components related to the four redundant loads through the compliance (4×4) matrix of the flange assembly are needed. The compliance is obtained as an influence coefficient matrix. The "free member" deflections provide the four deformations which occur with each type of loading condition (pressure, temperature, etc.) when all of the redundant forces are zero. The four independent deformations corresponding to the four redundant forces in the arrangement illustrated in Fig. A.1 are w_{HC} , μ_{HC} , β_{HC} , and μ_{SA} .

With the free member deformations and the compliance matrix, the redundant loads can be determined and will then be available for the solution of stresses in all members.

A.1 Compatibility

Examination of the compatibility of deformations at the points A, B, C, and D illustrated in Fig. A.1 results in the following relationships.

$$w_{HC} = w_{SD} + (w_{BC} - w_{BD}) \quad (A.1)$$

$$\mu_{HC} = \mu_{SD} + (\mu_{BC} - \mu_{BD} + L_B \beta_{SD}) \quad (A.2)$$

$$\beta_{HC} = \beta_{SD} + (\beta_{BC} - \beta_{BD}) \quad (A.3)$$

$$w_{HB} = w_{SA} \quad (A.4)$$

$$\mu_{HB} = \mu_{SA} \quad (A.5)$$

However, w_{HB} is not an independent quantity in Eq. A.4 since compatibility of deformation along two axial paths (the lower flange, upper flange path and the lower flange, stud, upper flange path) leads to the expression

$$w_{BC} - w_{BD} - w_{SD} - w_{SA} = w_{HC} - w_{HB} \quad (A.6)$$

But

$$w_{SA} = w_{HB}$$

and

$$w_{HC} = w_{BC} - w_{BD} \quad .$$

This means that there are only four independent deformations. Equations A.1 through A.5 are rewritten in matrix form as follows.

$$\begin{bmatrix} w_{HC} \\ \mu_{HC} \\ \beta_{HC} \\ w_{HB} \\ \mu_{HB} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & L_B & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_{SD} \\ \mu_{SD} \\ \beta_{SD} \\ w_{SA} \\ \mu_{SA} \end{bmatrix} + \begin{bmatrix} (w_{BC} - w_{BD}) \\ (\mu_{BC} - \mu_{BD}) \\ (\beta_{BC} - \beta_{BD}) \\ 0 \\ 0 \end{bmatrix} \quad (A.7)$$

The 5-by-5 matrix in Eq. A.7 is designated as T_c . Equation A.7 can be rearranged and rewritten to include the free member deformations resulting from pressure, temperature, etc. This results in an expression in the form

$$\text{Forces resulting from redundant loads} = \text{Free member deformations.} \quad (A.8)$$

$$\begin{bmatrix} w_{HC} \\ \mu_{HC} \\ \beta_{HC} \\ w_{HB} \\ \mu_{HB} \end{bmatrix} - [T_c] \begin{bmatrix} w_{SD} \\ \mu_{SD} \\ \beta_{SD} \\ w_{SA} \\ \mu_{SA} \end{bmatrix} - \begin{bmatrix} (w_{BC} - w_{BD}) \\ (\mu_{BC} - \mu_{BD}) \\ (\beta_{BC} - \beta_{BD}) \\ 0 \\ 0 \end{bmatrix} = - \begin{bmatrix} w_{HC} \\ \mu_{HC} \\ \beta_{HC} \\ w_{HB} \\ \mu_{HB} \end{bmatrix} + [T_c] \begin{bmatrix} w_{SD} \\ \mu_{SD} \\ \beta_{SD} \\ w_{SA} \\ \mu_{SA} \end{bmatrix} + \begin{bmatrix} (w_{BC} - w_{BD}) \\ (\mu_{BC} - \mu_{BD}) \\ (\beta_{BC} - \beta_{BD}) \\ 0 \\ 0 \end{bmatrix}.$$

A.2 Member Deflections

The deflections of each point on the members may be written in terms of the internal loads as follows. For the upper flange,

$$\begin{bmatrix} w_{HC} \\ \mu_{HC} \\ \beta_{HC} \\ w_{HB} \\ \mu_{HB} \end{bmatrix} = [C_H] \begin{bmatrix} r_c^V{}_{HC} \\ r_c^H{}_{HC} \\ r_c^M{}_{HC} \\ r_B^V{}_{HB} \\ r_B^H{}_{HB} \end{bmatrix} ; \quad (A.9)$$

for the lower flange,

$$\begin{bmatrix} w_{SD} \\ \mu_{SD} \\ \beta_{SD} \\ w_{SA} \\ \mu_{SA} \end{bmatrix} = [C_S] \begin{bmatrix} r_c^V{}_{SD} \\ r_c^H{}_{SD} \\ r_c^M{}_{SD} \\ r_B^V{}_{SA} \\ \mu_B^H{}_{SA} \end{bmatrix} ; \quad (A.10)$$

and for the stud,

$$\begin{bmatrix} (w_{BC} - w_{BD}) \\ (\mu_{BC} - \mu_{BD}) \\ (\beta_{BC} - \beta_{BD}) \end{bmatrix} = [C_B] \begin{bmatrix} r_c^V{}_{HC} \\ r_c^H{}_{HC} \\ r_c^M{}_{HC} \end{bmatrix} ; \quad (A.11)$$

where C_H , C_S , and C_B are the influence coefficients of the members.

A.3 Equilibrium

Loads such as V_{HB} , V_{SA} , etc., in Eqs. A.9 and A.10 evidently are not independent. These loads can be found in terms of the basic independent (redundant) loads through static equilibrium as follows.

$$\begin{aligned}
r_{cSD}^V &= -r_{cHC}^V = r_{BHB}^V = r_{BSA}^V \\
r_{cSD}^H &= -r_{cHC}^H \\
r_{cSD}^M &= -r_{cHC}^M - (r_{cHC}^H)L_B
\end{aligned} \tag{A.12}$$

Since each C_H , C_S , and T_C is a 5-by-5 matrix, it is convenient to write the terms of Eq. A.12 as a 5-by-5 matrix as follows.

$$\begin{bmatrix} r_{cSD}^V \\ r_{cSD}^H \\ r_{cSD}^M \\ r_{BSA}^V \\ r_{BSA}^H \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & -L_B & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_{cHC}^V \\ r_{cHC}^H \\ r_{cHC}^M \\ r_{BHB}^V \\ r_{BHB}^H \end{bmatrix} . \tag{A.13}$$

Obviously, $r_{BSA}^V = r_{BHB}^V$ has been introduced as a dummy equation to obtain a 5-by-5 matrix. Substitution from Eqs. A.9, A.10, A.11, and A.13 into Eq. A.12 results in the equation

$$\begin{bmatrix} C_H \end{bmatrix} - \begin{bmatrix} T_C \end{bmatrix} \begin{bmatrix} C_S \end{bmatrix} \begin{bmatrix} T_E \end{bmatrix} - \begin{bmatrix} C_B \end{bmatrix} \begin{bmatrix} r_{cHC}^V \\ r_{cHC}^H \\ r_{cHC}^M \\ r_{BHB}^V \\ r_{BHB}^H \end{bmatrix} = \begin{bmatrix} \text{Free Member} \\ \text{Deformations} \end{bmatrix} , \tag{A.14}$$

where each of the matrices C_H , T_C , etc., is a 5-by-5 matrix or

$$[K] \begin{bmatrix} r_c^V{}_{HC} \\ r_c^H{}_{HC} \\ r_c^M{}_{HC} \\ r_B^V{}_{HB} \\ r_B^H{}_{HB} \end{bmatrix} = \begin{bmatrix} \text{Free Member} \\ \text{Deformations} \end{bmatrix} \quad (A.15)$$

Since V_{HB} and the corresponding deformation w_{HB} are both dummies, Eq. A.15 is written in terms of the four basic redundant loads as follows.

$$[K] \begin{bmatrix} r_c^V{}_{HC} \\ r_c^H{}_{HC} \\ r_c^M{}_{HC} \\ r_B^H{}_{HB} \end{bmatrix} = \begin{bmatrix} \text{Free Member} \\ \text{Deformations} \end{bmatrix} \quad (A.16)$$

A.4 Influence Coefficients for Bolts

The geometry, loads, and deformations of flange bolting are defined by Zudans¹ in accordance with the sketch illustrated in Fig. A.2. The length Δ is the excess of the free grip length over the free bolt length; that is, the amount by which the bolt must stretch in order to match the unloaded flanges. The axial load N is that portion of the total axial load that is an unknown redundant quantity in the interaction analysis. The axial load S is the remainder of the total axial load. This definition permits a reasonably linear evaluation of the truly nonlinear behavior of bolts.

¹Z. Zudans, "Analysis of Axisymmetric Redundant Structures," Nuclear Structural Engineering, 1(2): 159-185 (February 1965).

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Fig. A.2. Definition of Geometry, Loads, and Deformations for Flange Bolting.

The values of S , N , and Q are expressed in units of pounds applied to the bolt, and M is the moment applied to the bolt in inch-pounds. It is assumed that S is greater than N . The axial deflection (w) is measured from the position of the top end of the bolt in the unloaded condition designated as point 1 in part B illustrated in Fig. A.3. The unloaded length of the bolt (distance from point 1 to point 2 in part B) is shorter by an amount Δ than is the unloaded length across the flanges (between point 1 on part C and point 2 on part A).

The corrected equation¹ for the axial deflection of the top of the bolt relative to the bottom of the bolt is

$$E_w = \frac{4NL}{\pi d^2} + E\alpha(T - T_r)L + E\Delta, \quad (\text{A.17})$$

Fig. A.3. Diagram of Loaded and Unloaded Lengths of Flange Bolt.

where

E = modulus of elasticity of the material,

α = coefficient of thermal expansion,

T = actual temperature, and

T_r = reference temperature.

For the discontinuity reaction (N) to be zero,

$$w = \alpha(T - T_r)L + \Delta . \quad (\text{A.18})$$

Since the total load on the flange bolts is $N + S$ and the general deflection equation for the bolt is

$$w = \frac{4(N + S)L}{\pi E d^2} + \alpha(T - T_r)L , \quad (\text{A.19})$$

for $N = 0$,

$$w = \frac{4SL}{\pi E d^2} + \alpha(T - T_r)L \quad (\text{A.20})$$

and S and Δ are related by

$$S = \frac{\pi d^2}{4L}(E\Delta) . \quad (\text{A.21})$$

To consider this relationship further, assume that the flanges are very still as compared with the bolts and that a preload force S_L has been applied to the bolts. This force would result from stretching the

bolts a distance $\Delta = \delta_0$. If an end load is applied to the flanged members, it tends to relieve the bearing load between the two flanges at the same rate as it tends to increase the bolt load. Therefore, until flange separation occurs, the bolt load remains constant at S_0 , and $N = 0$. When flange separation occurs, the bolt load must increase to equilibrate the end force. The effects of temperature have been neglected in this and the following discussion.

As a second example, consider the more practical case wherein the flange cannot be considered as rigid with respect to the bolts. Let the bolts be stretched a distance δ_0^1 by application of a load S_0^1 to the bolts only, as described by Eq. A.21. When the bolt tensioner is removed, each bolt attempts to apply a force S_0 to the flange. Since flexibility must be considered in this case, the flanges tend to deflect, permitting the bolt to attain some final value of w less δ_0^1 . The exact final length can be determined only by performing an interaction analysis between the bolts and the flange. If the final deflection of the bolt is δ_0 , the total bolt load (S_0) resulting from the preloading operation can be determined from Eq. A.19 or Eq. A.21 as

$$S_0 + (N_0 + S_0^1) = \frac{\pi d^2}{4L} E \delta_0 . \quad (A.22)$$

The same final bolt stretch obtained in this manner is identical to that obtained in the case where the flanges were assumed to be rigid. However, since δ_0 is less than δ_0^1 and S_0 is less than S_0^1 , the N_0 is negative.

Now consider the application of an end load on the flexible flanges. Again, the bearing stress between the flanges is reduced by the end force and the bolt load is increased accordingly. Let the total end force divided by the number of bolts be designated as S_F^1 , and let the deflection which will result from this force be designated as δ_F^1 . The total load on the bolt at this time is then $S_0 + S_F^1$, and the total axial deflection is $\delta_0 + \delta_F^1$. Application of the end force also results in deformation of the flanges. This deformation of the flanges must be calculated by assuming the end force to be equilibrated by a circumferential line load applied at the bolt circle. The total force (S_0)

multiplied by the number of bolts must be applied at the gasket face and at the bolt circle, so the total force applied at the bolt circle is proportional to $S_O + S_F^1$. Performance of an interaction analysis provides a final (loaded) bolt length and the total bolt forces (S_F). The total bolt force is obtained from the expression

$$S_F = S_O + S_F^1 + N_F , \quad (A.23)$$

where

S_O = the bolt preload given by Eq. A.22,

S_F^1 = the end load divided by the number of bolts, and

N_F = the bolt load obtained from the interaction analysis.

A.4.1 Alternate Procedure

A somewhat more direct procedure can be used to completely separate the analysis for preload and that for applied load effects. In this alternate procedure, the "free" flange analysis is performed by considering only the end load to be equilibrated by a circumferential line load applied at the bolt circle. Similarly, the only load considered to act on the bolts is the end load divided by the number of bolts (S_F^1) previously considered. Performance of an interaction analysis for this condition results in the determination of only the quantity N_F , as it appears in Eq. A.23. Therefore, only the value of $S_F^1 + N_F$ is determined by this alternate procedure.

However, the total bolt load (S_F), including the effects of bolt preload, may be determined by using the following expression as an alternate to Eq. A.17.

$$Ew = \frac{4NL}{\pi d^2} + E\alpha(T - T_r)L + \Delta_F + \delta_O , \quad (A.24)$$

where δ_O = the final bolt stretch applied during preload. This is the same quantity that appears in Eq. A.22. If Eq. A.24 is used in the interaction analysis in lieu of Eq. A.17, the value determined for N

will be $S_O + N_F$ and the preload will be considered as part of the total bolt load.

A.4.2 Influence Coefficients Neglecting Axial Load Stiffening

Both of the previously discussed methods give the same result when the stiffening effects of the axial bolt load are neglected; that is, when the deformation equations for the bolt are written in the following form.

$$\begin{bmatrix} Ew \\ E\mu \\ E_x \end{bmatrix} = \begin{bmatrix} \frac{4L}{\pi d^2} & 0 & 0 \\ 0 & \frac{L^3}{3I} & \frac{L^2}{2I} \\ 0 & \frac{L^2}{2I} & \frac{L}{I} \end{bmatrix} \begin{bmatrix} N \\ Q \\ M \end{bmatrix} + \begin{bmatrix} E\alpha L & E & (T - T_r) \\ 0 & 0 & \Delta \\ 0 & 0 & \end{bmatrix} \quad (A.25)$$

A.4.3 Influence Coefficients Considering Axial Load Stiffening

The equations for free-body analysis of bolts given by Zudans¹ and modified to suit the terms of this discussion are as follows.

$$Ew = \frac{4N}{\pi d^2} X + E\alpha X(T - T_r) + E\Delta \left(\frac{X}{L} \right), \quad (A.26)$$

where the terms are as illustrated in Fig. A.2 and as previously discussed.

$$E_\mu = C_1 e^{px} + C_2 e^{-px} + A + Bx \quad (A.27)$$

and

$$E_x = C_1 p e^{px} - C_2 p e^{-px} + B, \quad (A.28)$$

where

$$C_1 = -\frac{1}{2} \left(A + \frac{B}{p} \right),$$

$$p = (S/EI)^{1/2},$$

$$C_2 = -\frac{1}{2} \left(A - \frac{B}{P} \right) ,$$

$$A = Ef + F, \text{ and}$$

$$B = Q/Ip^2 .$$

The terms used in these definitions are defined as follows.

$$Ef = \frac{pF(1 - K) + B(pL - T)}{pK} ,$$

$$F = -\frac{QL + M}{Ip^2} ,$$

$$K = \frac{1}{2} (e^{pL} + e^{-pL}) ,$$

$$T = \frac{1}{2} (e^{pL} - e^{-pL}) , \text{ and}$$

$$I = \frac{1}{64} \pi d^4 .$$

The bending stress is given by the expression

$$\sigma_b = \pm \frac{32m}{\pi d^3} , \quad (\text{A.29})$$

where $m = -Q(L - X) - M + \frac{S}{E} (Ef - E\mu)$. When the configuration is known, the equations for deformations can be determined in terms of N , Q , and M if S is known. The larger the value of S , the stiffer the bolts in the radial direction.

Ideally, S should be chosen so that its final value $S_F = S_O + S_F^1 + N_F$, but this value is not known until after the computation has been completed unless flange separation occurs, in which case $S_F = S_F^1$. If the flanges are very stiff as compared with the bolts, $N_F = -S_F^1$ and $S_F = S_O$. In actual flanges, S_F is greater than S_O since N_F is greater than or equal to $-S_F^1$. The recommendation of Zudans¹ is to use $S = S_F^1$, which is probably a lower limit value since most preloads are such that S_O is greater than S_F^1 . Therefore,

$$S_F^1 \leq S_O \leq S_F . \quad (\text{A.30})$$

It therefore appears reasonable to

1. choose $S = S_0$,
2. determine the effect of $S_F^1 \leq S \leq S_F$ on the influence coefficients, and
3. iterate the results as required for reasonable accuracy.

In the analysis procedures discussed herein, reasonable accuracy is expected without the iteration and the influence coefficients obtained from Eqs. A.26, A.27, and A.28 can be used.

The bolt influence coefficients are found by using the techniques outlined by Zudans.¹ The sign convention set forth by Zudans is converted to agree with that illustrated in Fig. A.2 by comparing the terms as illustrated in Fig. A.4 and determining the equivalent quantities, which are multiplied by the applicable number of bolts where necessary.

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Fig. A.4. Comparison of Sign Conventions Used in Different Analyses.

The quantities illustrated in Fig. A.4(a) are thus equated to the equivalent terms for the interaction analysis shown in Fig. A.4(b) as follows.

$$w = w_z$$

$$\mu = -\mu_z$$

$$\beta = -x_z$$

$$R = a$$

$$z = x_z$$

$$V_N = \frac{nN_z}{2\pi}$$

$$V_z = \frac{nS_z}{2\pi}$$

$$M = -\frac{nM_z}{2\pi}$$

$$H = -\frac{nQ_z}{2\pi}$$

Appendix B

GENERAL PROCEDURE FOR SEISMIC ANALYSIS OF PIPING SYSTEMS

A general procedure for the structural analysis of piping systems subjected to random earthquake excitations is developed in this appendix. The material presented includes the development of the stiffness matrix for a single three-dimensional member, a discussion of the method by which single-member matrices are combined to give the overall stiffness property of a multi-degree-of-freedom system, and an introduction to the technique used to determine normal mode shapes and mode frequencies. The manner in which these mode shapes and frequencies are used with mathematical methods to estimate the dynamic structural response of the piping system to seismic forcing functions either independent of or as a function of time is outlined.

A piping structure may be classified as a one-dimensional, two-dimensional, or three-dimensional structure, depending upon the characteristics of its components. To a first approximation, a pipe or beam is generally represented by its centroidal axis and can be regarded as a one-dimensional structure even though the system itself may extend in three dimensions. The methods of analysis presented here are based on the assumption that a linear relationship exists between the applied loads and the resultant displacements. This assumption implies that the structural material is homogeneous, isotropic, and linearly elastic and that the geometrical changes in the structure under load are small enough to be neglected.

B.1 Lumped Parameter Mathematical Model

In addition to the spatial coordinates required for the static analysis, solution of the dynamic response problem involves time as a parameter. An explicit mathematical representation of the structure would lead to the solution of partial differential equations because the mass, energy dissipation, and stiffness are distributed continuously throughout

the structure. Such complexity is avoided for most structures by the substitution of an idealized structure or model that is better adapted to mathematical analysis and has approximately the same response as the real structure. This analytical model is designated as a mathematical model because it forms a bridge between the mathematical analysis and the physical system. It is important that the model be devised with care and good judgment if useful results are to be obtained.

The mathematical model comprised of elements of mass, elastic elements, and damping elements is referred to as a "lumped parameter model" because the distributed mass, elasticity, and energy dissipation of the real structure are lumped in discrete elements (the lumped mass elements are also referred to as joints) in such a manner that the dynamic characteristics of the structure are approximately preserved. Such models are widely used to represent structures ranging from the simple to the very complicated. It is possible to derive a set of differential equations of motion for the lumped parameter model in a straightforward manner directly from the laws of equilibrium. Forces acting on each element of mass can be associated with the deflections of the elastic elements and with appropriate motion parameters of the elements of damping.

The use of a complicated model is generally necessary for most piping systems in order to account for discontinuities in mass, stiffness, and geometry. Although it is difficult to state in general terms the degree of model complexity required to obtain a reliable dynamic response, the following comments may provide useful guidance.

1. The mass of structural members should be lumped at several carefully chosen locations (joints) along the member. The total number of joints will eventually be limited by the storage capacity of the computer, but increments should be made small enough to insure reasonably accurate results, particularly if high modal frequencies are to be investigated.

2. A joint should be positioned at each location where the system is restrained externally.

3. It is common practice to locate a joint at each position where a relatively large mass or external concentrated load acts. The center of the mass is assumed to be located along the neutral axis of the member. If the mass has relatively great rigidity over a finite length of the

supporting member and is attached in a manner which imparts additional rigidity to the member, the stiffness of the member should be adjusted accordingly.

4. Joints are always positioned at the junction between two or more elastic members or where there is a significant change in cross section along the length of one member.

5. Since most bend radii or piping systems constitute a significant percentage of the total length of each bent member, the circumference of the bent portion should be divided into a number of chord segments. The corresponding joints should be positioned approximately every 30° around the curvature.

B.2 Method of Analysis

The objective of the structural analysis is the determination of the dynamic joint displacements for each degree of freedom of the structure. By using the joint displacements, a structural analysis may be performed to determine the maximum loads in each structural member. The term "load" is used to denote forces and couples collectively, while the term "displacement" includes translations and rotations.

An important mathematical parameter in the performance of static and normal mode analyses is the stiffness (K) of the structure. Member loads are determined from a set of joint deflections (U), which in turn satisfy the relationship

$$U = K^{-1}F, \quad (B.1)$$

where F = the dynamic load matrix for the joints. The stiffness matrix method which in our experience has served most conveniently is developed in this procedure. However, other methods, such as the flexibility and moment distribution methods, should not be avoided. Whatever method selected to meet the specific problem requirements should

1. provide a complete analysis (deflections, loads, and normal modes),
2. permit the required input data (geometries and material properties) to be provided in simple form,

3. analyze statically-indeterminate structures with no additional effort on the part of the analyst,
4. be adaptable to any type of framework,
5. provide a solution whose accuracy is sufficient for engineering use, and
6. be of such a nature that an analytical program is relatively easy to write.

B.2.1 Axes

For consideration of the structural system as a whole, a single set of axes will be required, the origin of which is arbitrary. However, it is advisable to position the origin at the lower left-hand extreme of the structure to avoid possible input errors related to negative coordinates. Joint coordinates, deflections, and modal vectors are evaluated in terms of system axes; whereas member forces are generally a function of the member axes.

B.2.2 Displacements

Displacements for a member (p-q) are represented by the symbol "u". When the six components of displacement at end "p" of the member are referred to, translations in the system coordinate x_1 , x_2 , and x_3 directions are denoted by u_{p1} , u_{p2} , and u_{p3} , respectively. Positive displacements are taken to mean displacements in the positive direction of the system axes, as illustrated in Fig. B.1. Rotations around the x_1 , x_2 , and x_3 axes at end "p" are designated as u_{p4} , u_{p5} , and u_{p6} , respectively; the positive directions being in accordance with the right-hand rule, as indicated in Fig. B.1.

B.2.3 Single-Member Loads

Forces and couples applied to each end of a member are referred to as single-member loads. The number and type of member loads are dependent

Fig. B.1. Positive Displacement Directions.

upon the degrees of freedom of the associated joints. In the general case, each end of the member will be subjected to six component loads: forces in the x_1 , x_2 , and x_3 directions (f_1 , f_2 , and f_3 , respectively) and couples acting around the x_1 , x_2 , and x_3 axes (f_4 , f_5 , and f_6 , respectively).

These end loads for most of the structural members which form a part of a particular piping system can be determined in a straightforward manner by using relationships given in any structural handbook. For a member of variable cross section, it is generally more convenient to invert the flexibility coefficients which have been determined by the classical integration method.

B.3 Stiffness Matrix for a Single Member

A displacement of any joint in the structural system implies an identical displacement of all of the member ends connected to the joint. Furthermore, the force required to move the joint in a given direction is equal to the sum of the forces required to displace the member ends in

that direction. The stiffness of the joint is therefore equal to the sum of the member stiffness coefficients attached to it. For this reason, the stiffness properties of a single member will be evaluated as a preliminary to the determination of joint stiffness.

A typical fixed-end member (p-q) with equal cross-sectional moments of inertia and three-dimensional coordinates is illustrated in Fig. B.2.

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Fig. B.2. Single Member Illustrated in Three Dimensions.

Assuming for the present that end "q" is fully fixed and that end "p" is restrained in five directions but is free to displace in the sixth direction, a displacement in that direction will induce six component reactions at both "p" and "q". The same applies to the other five displacements at end "p", thereby resulting in the following relationships.

$$\begin{aligned}
 f_{p1} &= k_{11}u_{p1} + k_{12}u_{p2} + k_{13}u_{p3} + k_{14}u_{p4} + k_{15}u_{p5} + k_{16}u_{p6} \\
 f_{p2} &= k_{21}u_{p1} + k_{22}u_{p2} + k_{23}u_{p3} + k_{24}u_{p4} + k_{25}u_{p5} + k_{26}u_{p6} \\
 f_{p3} &= k_{31}u_{p1} + k_{32}u_{p2} + k_{33}u_{p3} + k_{34}u_{p4} + k_{35}u_{p5} + k_{36}u_{p6} \\
 f_{p4} &= k_{41}u_{p1} + k_{42}u_{p2} + k_{43}u_{p3} + k_{44}u_{p4} + k_{45}u_{p5} + k_{46}u_{p6} \\
 f_{p5} &= k_{51}u_{p1} + k_{52}u_{p2} + k_{53}u_{p3} + k_{54}u_{p4} + k_{55}u_{p5} + k_{56}u_{p6} \\
 f_{p6} &= k_{61}u_{p1} + k_{62}u_{p2} + k_{63}u_{p3} + k_{64}u_{p4} + k_{65}u_{p5} + k_{66}u_{p6}
 \end{aligned}
 \tag{B.2}$$

In the relationships of Eq. B.2, k_{ij} is the load induced along or around the i -th system coordinate (structure oriented) axis resulting from a unit displacement of joint p in the j -th direction, and u_{pi} (i, j) is zero.

There are 36 stiffness coefficients that relate the displacements at end p to the loads induced at point p . If F_{pp} represents the column matrix of end loads at end p resulting from displacements at joint p

$$F_{pp} = K_{pp} U_p, \quad (B.3)$$

where K_{pp} = the stiffness matrix at end p . The stiffness matrix K_{pp} is symmetric about the diagonal because $k_{ij} = k_{ji}$ (reciprocity).

The displacements at joint p will also cause loads to be induced at end q , and these loads are expressed by an additional set of six equations with different coefficients. These can be written in the matrix form

$$F_{qp} = K_{qp} U_p, \quad (B.4)$$

where

F_{qp} = column matrix of end loads at end q resulting from displacements at point p , and

K_{qp} = carry-over stiffness matrix.

If the member is now fully restrained at end p and unrestrained at end q , it will be possible to create six displacements at end q that can be represented by the column matrix U_q . The loads established at end q can be expressed in matrix form by the relationship

$$F_{qq} = K_{qq} U_q, \quad (B.5)$$

and the loads established at end p can be expressed by the relationship

$$F_{pq} = K_{pq} U_q. \quad (B.6)$$

If both ends of the member are now displaced at the same time, the total loads induced at ends p and q can be expressed as follows.

$$\begin{aligned} F_p &= F_{pp} + F_{pq} = K_{pp} U_p + K_{pq} U_q \\ F_q &= F_{qq} + F_{qp} = K_{qq} U_q + K_{qp} U_p \end{aligned} \quad (B.7)$$

The relationships expressed in Eq. B.7 are generally written as

$$\begin{Bmatrix} f_p \\ f_q \end{Bmatrix} = \begin{bmatrix} k_{pp} & k_{pq} \\ k_{qp} & k_{qq} \end{bmatrix} \begin{Bmatrix} u_p \\ u_q \end{Bmatrix}, \quad (\text{B.8})$$

where the vectors have components

$$\begin{Bmatrix} f_p \\ f_q \end{Bmatrix} = \begin{Bmatrix} f_{p1} \\ \vdots \\ f_{p6} \\ f_{q1} \\ \vdots \\ f_{q6} \end{Bmatrix}, \text{ which represents } \begin{Bmatrix} f_{p1} \\ f_{p2} \\ f_{p3} \\ f_{p4} \\ f_{p5} \\ f_{p6} \\ f_{q1} \\ f_{q2} \\ f_{q3} \\ f_{q4} \\ f_{q5} \\ f_{q6} \end{Bmatrix}, \text{ and}$$

$$\begin{Bmatrix} u_p \\ u_q \end{Bmatrix} = \begin{Bmatrix} u_{p1} \\ \vdots \\ u_{p6} \\ u_{q1} \\ \vdots \\ u_{q6} \end{Bmatrix}.$$

Equation B.8 defines a stiffness matrix for a particular member. This matrix includes the direct stiffness coefficients for both ends and the cross coefficients of coupling terms which relate the displacements at

one end to the loads induced at the other end. Generally, the matrix for a member will be of a 12-by-12 order and the submatrices will be of a 6-by-6 order.

A derivation of the partial stiffness matrix $[K_{pp} \quad K_{qp}]$ is presented here for illustrative purposes. For this derivation, the following quantities are treated as input.

1. Joint coordinates: $x_{p1}, x_{p2}, x_{p3}, x_{q1}, x_{q2}, x_{q3}$
2. Sectional properties, assumed constant over the length of the element:
 - E = modulus of elasticity
 - A = cross-sectional area
 - I = cross-sectional moment of inertia
 - K = torsional stiffness

Holding end q fixed, let joint p of Fig. B.2 be displaced a unit distance in the positive direction of the x_1 axis, as illustrated in Fig. B.3, where γ_1 represents the direction cosines.

Fig. B.3. Displacement of Single Member in Positive Direction.

$$\gamma_1 = \cos (pq, x_1) = \frac{x_{q1} - x_{p1}}{L}, \quad (B.9)$$

$$\gamma_2 = \cos (pq, x_2) = \frac{x_{q2} - x_{p2}}{L}, \quad (B.10)$$

$$\text{and} \quad \gamma_3 = \cos (pq, x_3) = \frac{x_{q3} - x_{p3}}{L}, \quad (B.11)$$

where $L = \text{length of } pq$

$$= \left[(x_{q1} - x_{p1})^2 + (x_{q2} - x_{p2})^2 + (x_{q3} - x_{p3})^2 \right]^{1/2}. \quad (B.12)$$

Consider a unit displacement of end p in the x_1 direction, as illustrated in Fig. B.3.

$$\begin{aligned} pp' &= u_{p1} = 1.0 \\ u_{pj} &= 0.0 \quad (j \neq 1) \\ u_{qj} &= 0.0 \quad (\text{for all } j). \end{aligned}$$

Construct a line from p' to p'' perpendicular to pq . The direction cosines of line $pp' = (1, 0, 0)$.

$$\cos (pp'', pp') = (1)\gamma_1 + (0)\gamma_2 + (0)\gamma_3 = \gamma_1.$$

$$\begin{aligned} \text{The length of } pp'' &= pp' \cos (pp'', pp') \\ &= (1.0)\gamma_1 = \gamma_1, \end{aligned}$$

$$\begin{aligned} \text{and the length of } p'p'' &= \left[1 - (pp'')^2 \right]^{1/2} \\ &= (1 - \gamma_1^2)^{1/2}. \end{aligned}$$

Drop a line $p''a$ from point p'' onto a plane parallel to plane x_1x_2 through pp' . Lines pa , ab , ap' , and pp' are in a plane parallel with plane x_1x_2 .

$$\begin{aligned} \text{The length of line } p''a &= pp'' \cos (pp'', x_3) \\ &= pp'' \cos (pq, x_3) \\ &= \gamma_1 \gamma_3. \end{aligned}$$

$$\begin{aligned} \text{The length of line } pb &= pp'' \cos (pp'', x_1) \\ &= pp'' \cos (pq, x_1) \\ &= \gamma_1 \gamma_1 = \gamma_1^2. \end{aligned}$$

The length of line bp' = pp' - pb

$$= 1.0 - \gamma_1^2 .$$

The length of line ab = $pp'' \cos (pp'', x_2)$

$$= pp'' \cos (pq, x_2)$$

$$= \gamma_1 \gamma_2 .$$

The length of line $p''b$ = $[(p''a)^2 + (ab)^2]^{1/2}$

$$= [(\gamma_1 \gamma_3)^2 + (\gamma_1 \gamma_2)^2]^{1/2}$$

$$= \gamma_1 (\gamma_2^2 + \gamma_3^2)^{1/2} .$$

Since

$$\gamma_1^2 + \gamma_2^2 + \gamma_3^2 = 1$$

and

$$\gamma_2^2 + \gamma_3^2 = 1 - \gamma_1^2 ,$$

the length of $p''b$ = $\gamma_1 (1 - \gamma_1^2)^{1/2}$. If desired, the lengths of pa and ap' can be found in a similar manner.

The unit deflection of joint p in the x_1 direction creates axial, torsional, bending, and shear loads at end p . The component forces resulting from axial and transverse deflections are illustrated as vectors in Fig. B.4.

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AXIAL DEFLECTION COMPONENTS

TRANSVERSE DEFLECTION COMPONENTS

Fig. B.4. Component Forces in the Axial and Transverse Directions, Resulting From the Unit Displacement of p in the x_1 Direction, Illustrated as Vectors.

Expressing the displacements as vectors,

$$\delta_1 = \gamma_1^2 \hat{x}_1 + \gamma_1 \gamma_2 \hat{x}_2 + \gamma_1 \gamma_3 \hat{x}_3 \quad (B.13)$$

for the axial direction and

$$\bar{\delta}_2 = (1 - \gamma_1^2) \hat{x}_1 - \gamma_1 \gamma_2 \hat{x}_2 - \gamma_1 \gamma_3 \hat{x}_3 \quad (B.14)$$

for the transverse direction. For the unit vector perpendicular to γ_1 and γ_2 ,

$$\begin{aligned} \hat{\delta}_3 &= \frac{\bar{\delta}_1 \times \bar{\delta}_2}{|\bar{\delta}_1| |\bar{\delta}_2|} \\ &= \frac{\gamma_1 \gamma_3 \hat{e}_2 - \gamma_1 \gamma_2 \hat{e}_3}{\gamma_1 (1 - \gamma_1^2)^{1/2}} \\ &= \frac{\gamma_3 \hat{e}_2 - \gamma_2 \hat{e}_3}{(1 - \gamma_1^2)^{1/2}} . \end{aligned} \quad (B.15)$$

The force vector exerted on joint p can be represented by the expression

$$\frac{AE}{L} \left(\bar{\delta}_1 \right) + \frac{12EI}{L^3} \left(\bar{\delta}_2 \right) , \quad (B.16)$$

and the moment vector can be represented by the expression

$$\frac{6EI}{L^2} (1 - \gamma_1^2)^{1/2} \hat{e}_{TR} . \quad (B.17)$$

The stiffness components (system oriented) at ends p and q that result from the unit deflection u_{p1} are as follows for end p.

$$\begin{aligned} k_{11} &= \text{force along } x_1 \text{ axis} = \frac{AE}{L} (\gamma_1^2) + \frac{12EI}{L^3} (1 - \gamma_1^2) \\ k_{21} &= \text{force along } x_2 \text{ axis} = \frac{AE}{L} (\gamma_1 \gamma_2) - \frac{12EI}{L^3} (\gamma_1 \gamma_2) \\ k_{31} &= \text{force along } x_3 \text{ axis} = \frac{AE}{L} (\gamma_1 \gamma_3) - \frac{12EI}{L^3} (\gamma_1 \gamma_3) \\ k_{41} &= \text{moment about } x_1 \text{ axis} = 0 \\ k_{51} &= \text{moment about } x_2 \text{ axis} = \frac{6EI}{L^2} (\gamma_3) \\ k_{61} &= \text{moment about } x_3 \text{ axis} = - \frac{6EI}{L^2} (\gamma_2) \end{aligned} \quad (B.18)$$

One column of the required general stiffness matrix that represents 12 loads created by the displacement u_{p1} has now been determined. The remainder of the matrix corresponding to the displacement of joint p is found by successively imposing deflections $u_{p2} = 1$, $u_{p3} = 1$, $u_{p4} = 1$, $u_{p5} = 1$, and $u_{p6} = 1$. The submatrices

$$\begin{bmatrix} k_{qq} \\ \vdots \\ k_{pq} \end{bmatrix}$$

may be easily determined by applying Maxwell's theorem of reciprocity with proper regard for sign convention.

B.4 Stiffness of a Structure

The three-member rigid frame illustrated in Fig. B.5 will serve as an example to illustrate the method by which the stiffness matrix for a structure is generated by superposition of single-member stiffness matrices. This structure can deform in its own plane as well as out of its plane. However, only in-plane deformations are considered here for simplicity of illustration. Accordingly, the structure is assumed to lie in the x_1 - x_2 plane, and

$$u_{p3} = u_{p4} = u_{p5} = u_{q3} = u_{q4} = u_{q5} = 0. \quad (\text{B.19})$$

The general three-dimensional stiffness matrix is therefore reduced to

Fig. B.5. Three-Member Rigid Frame.

the more simplified relationship which represents the single-member stiffness matrix for a two-dimensional rigid structure. This relationship is expressed in Eq. B.20 as follows.

$$\begin{Bmatrix} k_{pp} \\ k_{qp} \end{Bmatrix} = \begin{bmatrix} \frac{AE}{L}(\gamma_1^2) + \frac{12EI}{L^3}(1 - \gamma_1^2) & \left(\frac{AE}{L} - \frac{12EI}{L^3}\right)\gamma_1\gamma_2 & -\frac{6EI}{L^2}(\gamma_2) \\ \left(\frac{AE}{L} - \frac{12EI}{L^3}\right)\gamma_1\gamma_2 & \frac{AE}{L}(\gamma_2^2) + \frac{12EI}{L^3}(1 - \gamma_2^2) & \frac{6EI}{L^2}(\gamma_1) \\ -\frac{6EI}{L^2}(\gamma_2) & \frac{6EI}{L^2}(\gamma_1) & \frac{4EI}{L} \\ -\frac{AE}{L}(\gamma_1^2) - \frac{12EI}{L^3}(1 - \gamma_1^2) & \left(\frac{12EI}{L^3} - \frac{AE}{L}\right)\gamma_1\gamma_2 & \frac{6EI}{L^2}(\gamma_2) \\ \left(\frac{12EI}{L^3} - \frac{AE}{L}\right)\gamma_1\gamma_2 & -\frac{AE}{L}(\gamma_2^2) - \frac{12EI}{L^3}(1 - \gamma_2^2) & -\frac{6EI}{L^2}(\gamma_1) \\ -\frac{6EI}{L^2}(\gamma_2) & \frac{6EI}{L^2}(\gamma_1) & \frac{2EI}{L} \end{bmatrix}$$

The displacement vector for end p will be redefined to include only components

$$\begin{Bmatrix} u_{p1} \\ u_{p2} \\ u_{p3} \end{Bmatrix} \text{ which indicate } \begin{cases} \text{translation along } x_1 \text{ axis} \\ \text{translation along } x_2 \text{ axis} \\ \text{rotation about } x_3 \text{ axis} \end{cases} .$$

The force vector

$$f_p \text{ will include components } \begin{Bmatrix} f_{p1} \\ f_{p2} \\ f_{p3} \end{Bmatrix} .$$

The matrix for member 1-2, shown in Fig. B.5, at joint 1 is determined as given in Eq. B.21.

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = E \begin{bmatrix} 0.0069 & 0.0 & -0.0417 \\ 0.0 & 0.0833 & 0.0 \\ -0.0417 & 0.0 & 0.3333 \\ \hline -0.0069 & 0.0 & 0.0417 \\ 0.0 & -0.0833 & 0.0 \\ -0.0417 & 0.0 & 0.1667 \end{bmatrix} \begin{Bmatrix} u_1 \end{Bmatrix} \quad (B.21)$$

The matrix for member 1-2 at joint 2 is determined as

$$\begin{Bmatrix} f_1 \\ f_2 \end{Bmatrix} = E \begin{bmatrix} -0.0069 & 0.0 & -0.0417 \\ 0.0 & -0.0833 & 0.0 \\ +0.0417 & 0.0 & 0.1667 \\ \hline 0.0069 & 0.0 & 0.0417 \\ 0.0 & 0.0833 & 0.0 \\ 0.0417 & 0.0 & 0.3333 \end{bmatrix} \begin{Bmatrix} u_2 \end{Bmatrix} \quad (B.22)$$

The matrix for member 2-4 at joint 2 is determined as

$$\begin{Bmatrix} f_2 \\ f_4 \end{Bmatrix} = E \begin{bmatrix} 0.0500 & 0.0 & 0.0 \\ 0.0 & 0.0030 & 0.0300 \\ 0.0 & 0.0300 & 0.4000 \\ \hline -0.0500 & 0.0 & 0.0 \\ 0.0 & -0.003 & -0.0300 \\ 0.0 & 0.0300 & 0.2000 \end{bmatrix} \begin{Bmatrix} u_2 \end{Bmatrix} \quad (B.23)$$

The matrix for member 2-4 at joint 4 is determined as

$$\begin{Bmatrix} f_2 \\ f_4 \end{Bmatrix} = E \begin{bmatrix} -0.0500 & 0.0 & 0.0 \\ 0.0 & -0.0030 & 0.0300 \\ 0.0 & -0.0300 & 0.2000 \\ \hline 0.0500 & 0.0 & 0.0 \\ 0.0 & 0.0030 & -0.0300 \\ 0.0 & -0.0300 & 0.4000 \end{bmatrix} \begin{Bmatrix} u_4 \end{Bmatrix} \quad (B.24)$$

The matrix for member 3-4 at joint 4 is determined as given by Eq. B.25 on the following page.

$$\begin{Bmatrix} f_3 \\ - \\ f_4 \end{Bmatrix} = E \begin{bmatrix} -0.0069 & 0.0 & -0.0417 \\ 0.0 & 0.0833 & 0.0 \\ 0.0417 & 0.0 & 0.1667 \\ \hline 0.0069 & 0.0 & 0.0417 \\ 0.0 & 0.0833 & 0.0 \\ 0.0417 & 0.0 & 0.3333 \end{bmatrix} \begin{Bmatrix} u_4 \end{Bmatrix} \quad (B.25)$$

The matrix for member 3-4 at joint 3 is determined as

$$\begin{Bmatrix} f_3 \\ - \\ f_4 \end{Bmatrix} = E \begin{bmatrix} 0.0069 & 0.0 & -0.0417 \\ 0.0 & 0.0833 & 0.0 \\ -0.0417 & 0.0 & 0.3333 \\ \hline -0.0069 & 0.0 & 0.0417 \\ 0.0 & -0.0833 & 0.0 \\ -0.0417 & 0.0 & 0.1667 \end{bmatrix} \begin{Bmatrix} u_3 \end{Bmatrix} \quad (B.26)$$

The stiffness matrix for the structure will be a 12-by-12 array corresponding to 12 degrees of freedom. The matrix corresponding to the displacement components reacting at joint 2, for example, is determined by adding the matrices of members 1-2 and 2-4 as follows.

$$\begin{Bmatrix} f_1 \\ - \\ f_2 \\ - \\ f_3 \\ - \\ f_4 \end{Bmatrix} = E \begin{bmatrix} -0.0069 & 0.0 & -0.0417 \\ 0.0 & -0.0833 & 0.0 \\ 0.0417 & 0.0 & 0.1667 \\ \hline 0.0569 & 0.0 & 0.0417 \\ 0.0 & 0.0833 & 0.0300 \\ 0.0417 & 0.0300 & 0.7333 \\ \hline 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 \\ \hline -0.0500 & 0.0 & 0.0 \\ 0.0 & -0.003 & -0.0300 \\ 0.0 & 0.0300 & 0.2000 \end{bmatrix} \begin{Bmatrix} u_1 \\ - \\ u_2 \\ - \\ u_3 \\ - \\ u_4 \end{Bmatrix} \quad (B.27)$$

The final stiffness matrix for the structure, formulated by similar additions, is as given in Eq. B.28 on the following page.

$$\begin{Bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \\ f_{41} \\ f_{42} \\ f_{43} \end{Bmatrix} = E \begin{bmatrix} 0.0069 & 0.0 & -0.0417 & -0.0069 & 0.0 & -0.0417 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0833 & 0.0 & 0.0 & -0.0833 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.0417 & 0.0 & 0.3333 & 0.0417 & 0.0 & 0.1667 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\ -0.0069 & 0.0 & 0.0417 & 0.0569 & 0.0 & 0.0417 & 0.0 & 0.0 & 0.0 & 0.0 & -0.0500 & 0.0 & 0.0 \\ 0.0 & -0.0833 & 0.0 & 0.0 & 0.0833 & 0.0300 & 0.0 & 0.0 & 0.0 & 0.0 & -0.0030 & 0.0300 & 0.0 \\ -0.0417 & 0.0 & 0.1667 & 0.0417 & 0.0300 & 0.7333 & 0.0 & 0.0 & 0.0 & 0.0 & -0.0300 & 0.2000 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0069 & 0.0 & -0.0417 & -0.0069 & 0.0 & -0.0417 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0833 & 0.0 & 0.0 & -0.0833 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & -0.0417 & 0.0 & 0.3333 & 0.0417 & 0.0 & 0.1667 & 0.0 \\ 0.0 & 0.0 & 0.0 & -0.0500 & 0.0 & 0.0 & 0.0069 & 0.0 & 0.0417 & 0.0569 & 0.0 & 0.0417 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -0.0030 & -0.0300 & 0.0 & -0.0833 & 0.0 & 0.0 & 0.0863 & -0.0300 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & 0.0300 & 0.2000 & -0.0417 & 0.0 & 0.1667 & 0.0417 & -0.0300 & 0.7333 & 0.0 \end{bmatrix} \begin{Bmatrix} u_{11} \\ u_{12} \\ u_{13} \\ u_{21} \\ u_{22} \\ u_{23} \\ u_{31} \\ u_{32} \\ u_{33} \\ u_{41} \\ u_{42} \\ u_{43} \end{Bmatrix} \quad (B.28)$$

The general stiffness matrix can be simplified by recognizing that because of the presence of external restraints,

$$u_{11} = u_{12} = u_{13} = u_{31} = u_{32} = 0 . \quad (\text{B.29})$$

Column and row numbers 1, 2, 3, 7, and 8 in Eq. B.28 can be deleted since they make no contribution to the product. Forces f_{11} , f_{12} , f_{13} , f_{31} , and f_{32} are unknown reactions, and they can be determined from the unrestrained deflection components. The general stiffness matrix is therefore reduced to a 7-by-7 array which corresponds to the degrees of freedom of the structure. The expression for the determination of unknown displacements is thereby reduced to that given in Eq. B.30.

$$\begin{Bmatrix} f_{21} \\ f_{22} \\ f_{23} \\ f_{33} \\ f_{41} \\ f_{42} \\ f_{43} \end{Bmatrix} = E \begin{bmatrix} 0.0569 & 0.0 & 0.0417 & 0.0 & -0.0500 & 0.0 & 0.0 \\ 0.0 & 0.0833 & 0.0300 & 0.0 & 0.0 & -0.0030 & 0.0300 \\ 0.0417 & 0.0300 & 0.7333 & 0.0 & 0.0 & -0.0300 & 0.2000 \\ 0.0 & 0.0 & 0.0 & 0.3333 & 0.0417 & 0.0 & 0.1667 \\ -0.0500 & 0.0 & 0.0 & 0.0417 & 0.0569 & 0.0 & 0.0417 \\ 0.0 & -0.0030 & -0.0300 & 0.0 & 0.0 & 0.0863 & -0.0300 \\ 0.0 & 0.0300 & 0.2000 & 0.1667 & 0.0417 & 0.0300 & 0.7333 \end{bmatrix} \begin{Bmatrix} u_{21} \\ u_{22} \\ u_{23} \\ u_{33} \\ u_{41} \\ u_{42} \\ u_{43} \end{Bmatrix} . \quad (\text{B.30})$$

B.5 Multi-Degree-of-Freedom Structures

In general, the lumped parameter model that represents the physical piping system must include a number of joints with each joint being free to translate along three axes and rotate about three axes. The solution of a system of n masses, with each mass having six degrees of freedom, can be determined from $6n$ simultaneous differential equations. The dependency of the equations results from the presence of structural members that couple the n concentrated masses. Although the required equations may be easily written and a formal solution evaluated by the use of determinants, the analysis rapidly becomes involved from a mathematical viewpoint as the degrees of freedom are increased.

In theory, the solution which is sought is an expression that describes the manner in which each coordinate of the system varies with time. For most solutions to problems involving dynamic earthquake loads,

it is usually expedient to first determine the natural frequencies and corresponding mode shapes as numerical quantities. The dynamic response of the system can be determined by superimposing the proper contributions of various normal modes of vibration. This analytical approach is known as the normal mode method. An alternative method that involves the retention of algebraic expressions for the natural frequencies is significantly more time consuming. In any event, it is important that the natural frequencies be determined, and this part of the analysis is referred to as the Eigenvalue problem.

B.5.1 Solution of the Eigenvalue Problem

For a system with light damping, the normal modes and frequencies can be approximated satisfactorily by assuming zero damping. In the most general six-degrees-of-freedom case, free vibrations of the elastic structure may be expressed at any time by a set of ordinary differential equations of motion of the form

$$\begin{aligned}
 M_{11}\ddot{u}_{11} + k_{11}u_{11} + k_{12}u_{12} + \dots + k_{16}u_{16} + k_{17}u_{21} + k_{18}u_{22} + \dots + k_{1m}u_{n6} &= 0 \\
 M_{12}\ddot{u}_{12} + k_{21}u_{11} + k_{22}u_{12} + \dots + k_{26}u_{16} + k_{27}u_{21} + k_{28}u_{22} + \dots + k_{2m}u_{n6} &= 0 \\
 &\vdots \\
 M_{16}\ddot{u}_{16} + k_{61}u_{11} + k_{62}u_{12} + \dots + k_{66}u_{16} + k_{67}u_{21} + k_{68}u_{22} + \dots + k_{6m}u_{n6} &= 0 \\
 M_{21}\ddot{u}_{21} + k_{71}u_{11} + k_{72}u_{12} + \dots + k_{76}u_{16} + k_{77}u_{21} + k_{78}u_{22} + \dots + k_{7m}u_{n6} &= 0 \\
 &\vdots \\
 M_{n6}\ddot{u}_{n6} + k_{m1}u_{11} + k_{m2}u_{12} + \dots + k_{m6}u_{16} + k_{m7}u_{21} + k_{m8}u_{22} + \dots + k_{mm}u_{n6} &= 0
 \end{aligned}
 \tag{B.31}$$

where

m = number of degrees of freedom of the system,

n = number of joints with one or more degrees of freedom,

M_{m1}, M_{m2}, M_{m3} = translational inertial mass in the i -th coordinate direction,

M_{m4}, M_{m5}, M_{m6} = rotational mass moments of inertia about the i -th coordinate axis, and

k_{mi}, u_{nj} = stiffness and displacement elements as previously designated.

It is of interest to determine whether the solution corresponding to a normal mode of vibration will satisfy the preceding equations. When a structure is responding in a normal mode of vibration, it has three distinct characteristics. These are

1. the motion is periodic,
2. there is zero damping (hysteresis), and
3. all points in the system vibrate in phase with each other; at any time the deflected position of the structure is geometrically similar.

The above criteria are satisfied for any one of the structure's normal modes by the solution

$$u_{nj} = U_{nj} \sin \omega_n t, \quad (\text{B.32})$$

where

U_{nj} = amplitude of vibration for the j -th displacement of joint n and
 ω_n = circular frequency of the n -th mode of vibration.

Substitution of the assumed oscillatory motion into the set of equilibrium equations (Eq. B.31) yields a set of homogeneous linear algebraic equations of the following form.

$$\begin{aligned} k_{11}U_{11} + k_{12}U_{12} + \dots + k_{16}U_{16} + k_{17}U_{21} + k_{18}U_{22} + \dots + k_{1m}U_{n6} &= M_{11}\omega^2U_{11} \\ k_{21}U_{11} + k_{22}U_{12} + \dots + k_{26}U_{16} + k_{27}U_{21} + k_{28}U_{22} + \dots + k_{2m}U_{n6} &= M_{12}\omega^2U_{12} \\ \vdots & \\ k_{61}U_{11} + k_{62}U_{12} + \dots + k_{66}U_{16} + k_{67}U_{21} + k_{68}U_{22} + \dots + k_{6m}U_{n6} &= M_{16}\omega^2U_{16} \\ k_{71}U_{11} + k_{72}U_{12} + \dots + k_{76}U_{16} + k_{77}U_{21} + k_{78}U_{22} + \dots + k_{7m}U_{n6} &= M_{21}\omega^2U_{21} \\ \vdots & \\ k_{m1}U_{11} + k_{m2}U_{12} + \dots + k_{m6}U_{16} + k_{m7}U_{21} + k_{m8}U_{22} + \dots + k_{mm}U_{n6} &= M_{n6}\omega^2U_{n6} \end{aligned} \quad (\text{B.33})$$

By using matrix notation, the equations in this set (Eq. B.33) can be written in the form

$$KU = \omega^2 MU, \quad (\text{B.34})$$

where

K = a symmetric matrix,

U = an eigenvector, and

M = a diagonal matrix.

Equations written in the form of Eq. B.34 can be solved by using either direct or iterative methods.

(a) Direct Method. For the direct method of solution, the equations are written in the form

$$(K - \omega^2 M)U = 0 . \quad (B.35)$$

A non-trivial solution for U is possible only if

$$K - \omega^2 M = 0 . \quad (B.36)$$

Expansion of the determinant gives an m -th order polynomial in ω^2 that represents the frequency equation of the system. There are m real roots (m values of ω^2) to the frequency equations, with each root corresponding to a natural mode of vibration. Unless the initial conditions of the problem are defined, the relationship among the elements of $\{U\}$ corresponding to each natural frequency can only be determined to within an arbitrary additive constant. A particular set of constants, such as

$$U_{11}^{(r)}, U_{12}^{(r)}, \dots, U_{ne}^{(r)},$$

will be required for each natural frequency, $\omega^{(r)}$. When normalized, a given set of constants is referred to as an eigenvector or mode shape, $\phi^{(r)}$.

(b) Iterative Method. Although the roots of the frequency equation can theoretically be determined by expansion of the determinant, this procedure is not only tedious but it does not lend itself to computer application. However, the method of successive approximations does provide satisfactory answers, and it is the method by which most free vibration problems are solved in actual practice. Perhaps the most straightforward iterative procedure, although not the most rapidly convergent, is the Stodola method. This method gives the mode shape as well as the eigenvalues.

The system of homogeneous linear algebraic equations may be written in the form

$$CU = \omega^2 U , \quad (B.37)$$

where $C = M^{-1}K$. The iteration method converges to the highest eigenvalue of the matrix equation. In most problems, the lowest eigenvalue

is of physical interest. In this case, it is necessary to write the matrix equation in the form

$$\lambda U = DU, \quad (B.38)$$

where

$$\lambda = 1/\omega^2 \text{ and}$$

$$D = K^{-1}M.$$

Iteration then converges to the largest λ , from which the smallest ω^2 is computed.

The iteration procedure is begun by assuming U_1 , an approximation of the mode shape. The assumption is arbitrary, but for convenience, one of the elements of U_1 (say the first one) is taken as unity. Pre-multiplication of U_1 by D yields

$$DU_1 = \lambda_2 U_2, \quad (B.39)$$

where λ_2 and U_2 are obtained by factoring the constant λ_2 from the product DU_1 so that the first element of U_2 is unity. If U_1 is a true eigenfunction, $U_2 = U_1$ and λ_2 is the true eigenvalue associated with U_1 . However, it is unlikely that the assumption for U_1 will be a true eigenfunction. Nevertheless, λ_2 and U_2 may be considered first approximations of the highest eigenvalue and eigenfunction.

To obtain a second approximation, it is necessary to compute

$$DU_2 = \lambda_3 U_3, \quad (B.40)$$

where the first element of U_3 has been normalized to unity by factoring λ_3 from the product DU_2 . The process is repeated in accordance with the formula

$$DU_{n-1} = \lambda_n U_n \quad (B.41)$$

until U_{n-1} and U_n agree to the desired number of significant figures. The lowest eigenvalue is then $\omega^2 = 1/\lambda_n$ and the associated eigenfunction is U_n . The procedure by which the iteration method can be extended to give other eigenvalues in addition to the highest or lowest ones is described by S. H. Crandall.¹

¹S. H. Crandall, Engineering Analysis, McGraw-Hill Book Company, New York, 1956.

To illustrate the iterative procedure, the natural frequencies and mode shapes for the structure illustrated in Fig. B.5 are determined. The governing matrix equation, where the value of E was taken as 1.0×10^7 psi and the masses and mass moments of inertia (lb·sec²/ft and ft·lb·sec², respectively) were assumed to be equal to 10.0, is as follows.

$$\begin{bmatrix} 56,900.0 & 0.0 & 41,700.0 & 0.0 & -50,000.0 & 0.0 & 0.0 \\ 0.0 & 86,300.0 & 30,000.0 & 0.0 & 0.0 & -3,000.0 & 30,000.0 \\ 41,700.0 & 30,000.0 & 733,300.0 & 0.0 & 0.0 & -30,000.0 & 200,000.0 \\ 0.0 & 0.0 & 0.0 & 333,300.0 & 41,700.0 & 0.0 & 166,700.0 \\ -50,000.0 & 0.0 & 0.0 & 41,700.0 & 56,900.0 & 0.0 & 41,700.0 \\ 0.0 & -3,000.0 & -30,000.0 & 0.0 & 0.0 & 86,300.0 & -30,000.0 \\ 0.0 & 30,000.0 & 200,000.0 & 166,700.0 & 41,700.0 & -30,000.0 & 733,300.0 \end{bmatrix} \begin{Bmatrix} u_{21}^{(r)} \\ u_{22}^{(r)} \\ u_{23}^{(r)} \\ u_{33}^{(r)} \\ u_{41}^{(r)} \\ u_{42}^{(r)} \\ u_{43}^{(r)} \end{Bmatrix} = 10\omega^2(r) \begin{Bmatrix} u_{21}^{(r)} \\ u_{22}^{(r)} \\ u_{23}^{(r)} \\ u_{33}^{(r)} \\ u_{41}^{(r)} \\ u_{42}^{(r)} \\ u_{43}^{(r)} \end{Bmatrix} \quad (\text{B.42})$$

To simplify the presentation of the problem but not limit the generality of the method, it was assumed that the horizontal member (2-4) has infinite bending stiffness as compared with the two vertical members and that the axial deformation in the vertical members is negligible and will not affect the response of the system. Therefore, after deleting rows 2, 3, 6, and 7 and the corresponding columns of Eq. B.42, the modified matrix equation is of the form

$$\begin{bmatrix} 56,900.0 & 0.0 & -50,000.0 \\ 0.0 & 333,300.0 & 41,700.0 \\ -50,000.0 & 41,700.0 & 56,900.0 \end{bmatrix} \begin{Bmatrix} u_{21}^{(r)} \\ u_{33}^{(r)} \\ u_{41}^{(r)} \end{Bmatrix} = 10\omega^2(r) \begin{Bmatrix} u_{21}^{(r)} \\ u_{33}^{(r)} \\ u_{41}^{(r)} \end{Bmatrix} \quad (\text{B.43})$$

The iteration process for eight cycles is as follows.

$$\begin{aligned} \{U_1\} &= \{1.000, 1.000, 1.000\} \\ \{y_1\} &= 10^3 \{6.9, 375.0, 48.6\} & \omega^2 &= 690 \\ \{U_2\} &= \{1.000, 54.348, 7.043\} \\ \{y_2\} &= 10^3 \{-295.1, 18,794.0, 2620.0\} & \omega^2 &= 29,510 \\ \{U_3\} &= \{1.000, -63.504, -8.878\} \\ \{y_3\} &= 10^3 \{502.0, -21,571.0, -3216.0\} & \omega^2 &= 50,200 \end{aligned}$$

$$\begin{aligned}
\{U_4\} &= \{1.000, -42.970, -6.406\} \\
\{y_4\} &= 10^3 \{377.0, 14,567.0, 2204.0\} & \omega^2 &= 37,700 \\
\{U_5\} &= \{1.000, -38.912, -5.846\} \\
\{y_5\} &= 10^3 \{349.0, -13,104.0, -1993.0\} & \omega^2 &= 34,900 \\
\{U_6\} &= \{1.000, -37.547, -5.710\} \\
\{y_6\} &= 10^3 \{343.0, -12,698.0, -1951.0\} & \omega^2 &= 34,300 \\
\{U_7\} &= \{1.000, -37.020, -5.688\} \\
\{y_7\} &= 10^3 \{341.0, -12,587.0, -1917.0\} & \omega^2 &= 34,100 \\
\{U_8\} &= \{1.000, -36.912, -5.622\} \\
\{y_8\} &= 10^3 \{339.5, -12,535.0, -1906.0\} & \omega^2 &= 33,950
\end{aligned}$$

The normal mode frequency calculated in each of the iteration cycles is 82, 224, 194, 187, 184.5, and 184.1 radians/sec. It can be seen that the eigenvector converges much more slowly than the mode frequency. A frequency of 184.1 radians/sec represents an approximation of the third normal mode frequency, $\omega^{(3)}$, with a corresponding mode shape, $\phi^{(3)}$, equal to $\{1.000, -36.912, -5.622\}$. As a note of interest, the largest difference between the vector components of the seventh and eighth cycles is 0.066. Most computer programs have a built-in finite difference allowable between any two components of at least 1.0×10^{-6} . Although an appreciable amount of machine time is required to meet this criterion for all modes and for all components, this procedure is generally necessary to obtain reasonably accurate mode shapes for the higher frequencies.

B.5.2 Orthogonality

Orthogonality is a term which is applied to an important mathematical relationship among the amplitude components of any two normal modes of vibration. Considering any two modal vectors $\phi^{(1)}$ and $\phi^{(2)}$ where the vectors are composed of a number of modal amplitude components equal to the degrees of freedom of the system so that

$$\phi^{(1)} = \begin{Bmatrix} \phi_{11}^{(1)} \\ \phi_{12}^{(1)} \\ \vdots \\ \phi_{16}^{(1)} \\ \phi_{21}^{(1)} \\ \phi_{22}^{(1)} \\ \vdots \\ \phi_{n6}^{(1)} \end{Bmatrix} \quad \text{and} \quad \phi^{(2)} = \begin{Bmatrix} \phi_{11}^{(2)} \\ \phi_{12}^{(2)} \\ \vdots \\ \phi_{16}^{(2)} \\ \phi_{21}^{(2)} \\ \phi_{22}^{(2)} \\ \vdots \\ \phi_{n6}^{(2)} \end{Bmatrix},$$

the relationship between the two modes can be expressed as follows.

$$K\phi^{(1)} = \omega^2(1)M\phi^{(1)} \quad (\text{B.44})$$

and

$$K\phi^{(2)} = \omega^2(2)M\phi^{(2)}. \quad (\text{B.45})$$

Equation B.44 is transposed and post-multiplied by $\phi^{(2)}$, and Eq. B.45 is pre-multiplied by the transpose of $\phi^{(1)}$ to give

$$(\phi^{(1)})^T K \phi^{(2)} = \omega^2(1) (\phi^{(1)})^T M \phi^{(2)} \quad (\text{B.46})$$

and

$$(\phi^{(1)})^T K \phi^{(2)} = \omega^2(2) (\phi^{(1)})^T M \phi^{(2)} \quad (\text{B.47})$$

so that

$$(\phi^{(1)})^T K \phi^{(2)} = \omega^2(1) (\phi^{(1)})^T M \phi^{(2)} \quad (\text{B.48})$$

and

$$(\phi^{(1)})^T K \phi^{(2)} = \omega^2(2) (\phi^{(1)})^T M \phi^{(2)}, \quad (\text{B.49})$$

where M is equal to the transpose M^T since it is a diagonal matrix. In addition, the stiffness matrix K is equal to its transpose K^T because it is symmetric. The left-hand sides of Eqs. B.48 and B.49 are equal to each other, as are the right-hand sides. Therefore,

$$(\omega^2(2) - \omega^2(1)) (\phi^{(1)})^T M \phi^{(2)} = 0. \quad (\text{B.50})$$

In general, $\omega^{(1)}$ will not equal $\omega^{(2)}$ for piping structures.

$$(\phi^{(1)})^T M \phi^{(2)} = 0. \quad (\text{B.51})$$

Equation B.51 represents the mathematical relationship of orthogonality. Eigenvector $\phi^{(1)}$ is said to be orthogonal to eigenvector $\phi^{(2)}$ with respect

to the weighting matrix M. If the masses are equal,

$$(\phi(1))^\top \phi(2) = 0 \quad (\text{B.52})$$

and the eigenvectors are said to be orthogonal with respect to a unit matrix.

The principle of orthogonality is applied to the previous example of the three-member structure to find the first and second modes. Since the masses are equal, M need not be included. Therefore,

$$(\phi(3))^\top \phi(r) = 0, \quad (\text{B.53})$$

where r represents either the first or second mode eigenvectors. For the second mode,

$$(\phi(3))^\top \phi(2) = 0, \quad (\text{B.54})$$

$$\text{or} \quad 1.00U_{21}^{(r)} - 36.912U_{33}^{(r)} - 5.622U_{41}^{(r)} = 0. \quad (\text{B.55})$$

Equation B.55 is combined with the original frequency equation (Eq. B.43) to obtain a new relationship of the form

$$\begin{aligned} 333,000U_{33} + 41,700U_{41} &= 10\omega^2 U_{33} \\ -1,803,300U_{33} - 234,100U_{41} &= 10\omega^2 U_{41}. \end{aligned} \quad (\text{B.56})$$

Another 6-cycle iteration yields the following results.

$\{U_1\} = \{1, 1\}$	
$\{y_1\} = 10^3\{375.0, -2037.40\}$	$\omega_1^2 = 37,500$
$\{U_2\} = \{1, -5.430\}$	
$\{y_2\} = 10^3\{106.90, -532.10\}$	$\omega_2^2 = 10,690$
$\{U_3\} = \{1, -4.980\}$	
$\{y_3\} = 10^3\{125.60, -637.50\}$	$\omega_3^2 = 12,560$
$\{U_4\} = \{1, -5075\}$	
$\{y_4\} = 10^3\{121.70, -615.00\}$	$\omega_4^2 = 12,170$
$\{U_5\} = \{1, -5.060\}$	
$\{y_5\} = 10^3\{122.30, -618.80\}$	$\omega_5^2 = 12,230$
$\{U_6\} = \{1, -5059\}$	
$\{y_6\} = 10^3\{122.34, -619.00\}$	$\omega_6^2 = 12,234$

Using the orthogonality relationship to find the first component,

$$\begin{aligned} U_{21}^{(2)} &= 36.912(1.000) + 5.622(-5.059) \\ &= 8.47 \end{aligned} \quad (\text{B.57})$$

The second mode vector, normalized with respect to its first component, is as follows.

$$\begin{aligned} \phi^{(2)} &= \{U_{21}^{(2)}, U_{33}^{(2)}, U_{41}^{(2)}\} \\ &= \{1.000, 0.188, -0.597\} \end{aligned} \quad (\text{B.58})$$

with $\omega^{(2)} = \omega^2(2) = 12,234 = 110.6 \text{ radians/sec}$.

To determine the amplitude vector for the first or fundamental mode, the orthogonality property is applied to the two known vectors.

$$\begin{aligned} (\phi^{(3)})^T \phi^{(1)} &= 1.000U_{21}^{(1)} - 36.912U_{33}^{(1)} - 5.622U_{41}^{(1)} = 0 \\ (\phi^{(2)})^T \phi^{(1)} &= 1.000U_{21}^{(1)} - 0.118U_{33}^{(1)} - 0.597U_{41}^{(1)} = 0 \end{aligned} \quad (\text{B.59})$$

From this it can be determined that the first mode

$$U_{41} = -7.374U_{33} \quad (\text{B.60})$$

Substitution of this value in the original frequency equation yields

$$(25,804 - 10\omega^2)U_{33} = 0, \quad (\text{B.61})$$

and $\omega^{(1)} = \left(\omega^2(1)\right)^{1/2} = (2580.4)^{1/2} = 50.75 \text{ radians/sec}$.

Using $(\phi^{(3)})^T \phi^{(1)}$ again,

$$\begin{aligned} U_{21} &= 36.9U_{33} + 5.62(-7.374U_{33}) \\ &= 4.5U_{33} \end{aligned} \quad (\text{B.62})$$

If $U_{21} = 1.0$,

$U_{33} = -0.222$, and

$U_{41} = -7.374U_{33} = 1.637$;

the eigenvector for the first mode

$$\begin{aligned} \phi^{(1)} &= \{U_{21}^{(1)}, U_{33}^{(1)}, U_{41}^{(1)}\} \\ &= \{1.000, -0.222, 1.637\} \end{aligned} \quad (\text{B.63})$$

It should be noted that when the frequency is formulated on the basis of the stiffness matrix, the use of the iterative process results in the determination of the mode shapes and corresponding frequencies in descending order. Solutions corresponding to the smaller values of ω are of primary interest in structural analysis since larger deflections and larger loads generally occur in the lower modal frequencies. If higher modes need not be considered, convergence on the lower modes by the iterative method can be started if the reciprocal of the stiffness matrix, K^{-1} , was used in the basic frequency equation.

The relative deflected position of the three-member rigid frame for each of the three normal modes determined in this example is illustrated in Fig. B.6.

Fig. B.6. Relative Deflected Position of Example Three-Member Structure for Each of Three Normal Modes of Vibration.

For the mode illustrated in Fig. B.6(a),

$$\omega^{(1)} = 50.75 \text{ radians/sec}$$

and $\phi^{(1)} = \{1.000, -0.222, 1.637\}$.

For the mode illustrated in Fig. B.6(b),

$$\omega^{(2)} = 110.6 \text{ radians/sec}$$

and $\phi^{(2)} = \{1.00, 0.188, -0.597\}$.

For the mode illustrated in Fig. B.6(c),

$$\omega^{(3)} = 184.1 \text{ radians/sec}$$

and

$$\phi^{(3)} = \{1.00, -36.912, -5.622\} .$$

B.6 Use of Mode Shapes and Frequencies

Many studies of strong-motion earthquakes have been conducted, and structures in various environments have been instrumented with accelerometers and displacement meters to record the dynamic characteristics of seismic disturbances. One such seismograph, which is a record of a horizontal component of the strong-motion earthquake in El Centro, California, in 1940, is illustrated in Fig. B.7. Also illustrated are the ground

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Fig. B.7. Ground Acceleration, Velocity, and Displacement of the El Centro, California, Earthquake in 1940.

velocity and ground displacement histories that were obtained by integration of the acceleration data. The maximum excitation, consisting of an acceleration of approximately 0.33G (where G is the acceleration of gravity = 386.0 in./sec²), velocity of 13.7 in./sec, and a displacement of 8.3 in., is considered to be of such large magnitude that an earthquake of this severity is anticipated only once every 50 years.

Because of the dispersions, refractions, and reflections inherent in ground disturbances, the resulting excitations are so complex that they may be treated as random. Not only is the horizontal component random, but the horizontal resultant itself varies in direction with time. The vertical component is also similar to the horizontal components, but its intensity is somewhat less. Also, based on presently available data, two earthquake motions of approximately the same intensity do not exhibit horizontal components of essentially the same general characteristics because the excitations are random.

B.6.1 Excitations for Piping Systems

The most accurate analysis of the response of a piping system would encompass a model that not only included the system but the entire supporting structure as well. With respect to the duration and intensity for the location where the facility is constructed, the excitation pulse is then applied to the foundation of the supporting structure. If for some reason the facility and the piping system cannot both be included in the same model, the piping system can be analyzed satisfactorily as an isolated system if the dynamic response of the building is available at or near the support locations for the pipe. The response of the building at the pipe support points can be treated as the excitation for the pipe.

If the necessary building response is not available and if the piping system is supported by the rigid foundation and/or some other part of the building that is itself rigidly connected to the foundation, the piping structure may reasonably be considered to be an isolated system. The same ground motion selected to excite the building can be applied to the pipe supports.

It cannot be overemphasized that for the piping structure supported by a portion of the building that is capable of undergoing relatively large displacements and for which no response data are available, the excitations for the piping system will have to be estimated. The validity of a dynamic analysis associated with an estimated excitation is indeed questionable.

If the piping structure can be considered an isolated system and if a representative time history of ground motion is not available (or if the time history method of analysis is not used), the system can be analyzed in an approximate manner by using a suitable response spectrum. A suitable spectrum would indicate the maximum response to a representative earthquake for a particular location or an average response to many earthquakes.

B.6.2 Damping

Structural damping, the resistance to free motion, is present in all structural systems. Damping is caused in general by (1) the internal frictional resistance related to the molecular structure of the material of which the pipe is made and by (2) the frictional forces created by the sliding of one face of a member against another, such as at flanged connections.

The frictional resistance of most structures is generally treated as viscous (or velocity) damping and is expressed mathematically as a constant (C) which represents a percentage of critical damping. Damping can be determined explicitly only by an experiment in which the actual structure undergoes free or forced vibration. In free vibration, the fraction of critical damping is determined by noting the rate of decay of the system response. In forced vibration, damping is determined by establishing a relationship between the excitation and response amplitudes. Damping has been found to range from as little as 1% of critical for various girder systems to more than 15% for some buildings.

For the general piping system, an acceptable approximation can be made by determining the natural frequencies and associated mode shapes

by neglecting internal damping. However, when calculating the dynamic response of the system to impressed earthquake motions, amounts of damping may be included but should not be more than

1. $C = 0.5\%$ if the primary stress does not exceed the design stress limits of Division 1-705 of ANSI B31.7 (NB-3650) at operating conditions or
2. $C = 1.0\%$ for a primary stress between the design stress limits of Division 1-705 of ANSI B31.7 (NB-3650) and the yield stress.

B.6.3 Dynamic Response of Single-Degree-of-Freedom System by Response Spectra Method

Because of the time required and the analytical complexities involved in computing the dynamic response of a system as a function of time, the response spectra method is often used to determine engineering approximations of dynamic loads. In the response spectra method, it is assumed that (1) maximum responses for each mode occur simultaneously and that (2) all modal responses are in phase (in the same direction). Under these conditions, the maximum quantities for each mode are added to give an absolute maximum possible response. These maximum values will sometimes represent a reasonably good approximation, and at other times they will be quite conservative.

Although the maximum modal quantities of interest may be readily selected from a graph which represents the appropriate response spectrum, it is of interest to develop the mathematical response equations from which a general spectrum can be determined.

If a single-degree-of-freedom system such as that illustrated in Fig. B.8 is subjected to a ground motion (the acceleration pulse indicated in Fig. B.7, for example), it will be excited into motion and will respond in a vibratory manner.

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Fig. B.8. Single-Degree-of-Freedom System.

The nomenclature illustrated in Fig. B.8 is defined as follows.

M = mass (or mass moment of inertia for rotation),

C = damping coefficient,

k = axial or bending stiffness (or torsional stiffness for rotation),

$x(t')$ = time-varying displacement of base, and

$y(t')$ = time-varying absolute displacement of mass (M).

The equilibrium equation for this system is expressed as

$$M\ddot{y} + C\dot{u} + ku = 0, \quad (\text{B.64})$$

where $u(t')$ = the time-varying displacement of the mass relative to the base so that

$$y(t') = u(t') + x(t'), \quad (\text{B.65})$$

$$\dot{y}(t') = \dot{u}(t') + \dot{x}(t'), \quad (\text{B.66})$$

$$\text{and} \quad \ddot{y}(t') = \ddot{u}(t') + \ddot{x}(t'). \quad (\text{B.67})$$

Substitution of Eq. B.67 into Eq. B.64 yields the expression

$$M\ddot{u} + C\dot{u} + ku = -M\ddot{x}. \quad (\text{B.68})$$

Dividing Eq. B.68 by M and letting

$$\beta = C/2M \quad (\text{B.69})$$

$$\text{and} \quad \omega^2 = k/M, \quad (\text{B.70})$$

the equation of motion becomes

$$\ddot{u}(t') + 2\beta\dot{u}(t') + \omega^2 u(t') = -\ddot{x}(t'). \quad (\text{B.71})$$

To simplify the integration, the acceleration pulse, $\ddot{x}(t)$, is divided into a continuous series of straight lines, with each line terminated by a significant change in acceleration. The differential equation is then integrated throughout the time spans corresponding to each line segment. To develop a solution that will be general over any particular line segment, it is convenient to substitute the geometrical relationships illustrated in Fig. B.9.

Fig. B.9. General Coordinates for Acceleration as a Function of Time.

The relationships illustrated in Fig. B.9 are as follows.

$$t' = t - t_n \quad \text{for} \quad t_n \leq t \leq t_{n+1}, \quad (\text{B.72})$$

$$\frac{\Delta}{t_{n+1} - t_n} = \frac{\ddot{x}_{n+1} - \ddot{x}_n}{t_{n+1} - t_n}, \quad (\text{B.73})$$

and

$$\begin{aligned} \ddot{x}|_t &= \ddot{x}|_{t'} = \ddot{x}_n + \frac{\Delta}{t_{n+1} - t_n}(t') \\ &= \ddot{x}_n + \frac{\ddot{x}_{n+1} - \ddot{x}_n}{t_{n+1} - t_n}(t - t_n), \end{aligned} \quad (\text{B.74})$$

where $\ddot{x}_n$, $\ddot{x}_{n+1}$, t_n , and t_{n+1} are known quantities determined from the base excitation record. Letting

$$A_1 = \ddot{x}_n \quad (\text{B.75})$$

and
$$A_2 = \left(\frac{\ddot{x}_{n+1} - \ddot{x}_n}{t_{n+1} - t_n} \right), \quad (\text{B.76})$$

$$\ddot{x}|_t = \ddot{x}|_{t'} = A_1 + A_2 t' \quad (\text{B.77})$$

and
$$\ddot{u}(t') + 2\beta\dot{u}(t') + \omega^2 u(t') = -A_1 - A_2 t'. \quad (\text{B.78})$$

The complementary solution is

$$u_{\text{comp}} = e^{-\beta t'} (C_1 \sin \omega^* t' + C_2 \cos \omega^* t'), \quad (\text{B.79})$$

where $\omega^* = (\omega^2 - \beta^2)^{1/2}$. If the particular solution is

$$u_{\text{part}} = A + B t', \quad (\text{B.80})$$

$$2\beta B + \omega^2 A + \omega^2 B t' = -A_1 - A_2 t', \quad (\text{B.81})$$

where

$$\omega^2 B = -A_2, \quad (\text{B.82})$$

$$B = -A_2 / \omega^2, \quad (\text{B.83})$$

and
$$A = -(A_1 + 2\beta B) / \omega^2. \quad (\text{B.84})$$

$$u(t') = e^{-\beta t'} (C_1 \sin \omega^* t' + C_2 \cos \omega^* t') + A + B t', \quad (\text{B.85})$$

and

$$\begin{aligned} \dot{u}(t') = & -\beta e^{-\beta t'} (C_1 \sin \omega^* t' + C_2 \cos \omega^* t') \\ & + \omega^* e^{-\beta t'} (C_1 \cos \omega^* t' - C_2 \sin \omega^* t') + B. \end{aligned} \quad (\text{B.86})$$

The unknown coefficients C_1 and C_2 can be determined from the initial conditions on displacement and velocity at the beginning of each segment as follows.

$$u(t' = 0) = C_2 + A = u_n, \quad (\text{B.87})$$

$$C_2 = u_n - A, \quad (\text{B.88})$$

$$\dot{u}(t' = 0) = -\beta C_2 + \omega^* C_1 + B = \dot{u}_n. \quad (\text{B.89})$$

Substitution of $t' = t - t_n$ yields

$$u(t) = e^{-\beta(t-t_n)} [C_1 \sin \omega^*(t-t_n) + C_2 \cos \omega^*(t-t_n)] + A + B(t-t_n), \quad (B.90)$$

$$\dot{u}(t) = -\beta e^{-\beta(t-t_n)} [C_1 \sin \omega^*(t-t_n) + C_2 \cos \omega^*(t-t_n)] + \omega^* e^{-\beta(t-t_n)} [C_1 \cos \omega^*(t-t_n) - C_2 \sin \omega^*(t-t_n)] + B, \quad (B.91)$$

and

$$\ddot{u}(t) = (\beta^2 - \omega^{*2}) e^{-\beta(t-t_n)} [C_1 \sin \omega^*(t-t_n) + C_2 \cos \omega^*(t-t_n)] - 2\beta\omega^* e^{-\beta(t-t_n)} [C_1 \cos \omega^*(t-t_n) - C_2 \sin \omega^*(t-t_n)], \quad (B.92)$$

and finally,

$$\dot{y}(t) = \dot{u}(t) + \dot{x}(t). \quad (B.93)$$

So far, the solution has been obtained in a closed form for a specific time interval ($t_n \leq t \leq t_{n+1}$) by expressing the exciting acceleration as piece-wise linear. The solution can be carried out for the entire duration of the excitation by matching conditions for the adjacent intervals. There are numerical methods^{2,3} in which the solution can be obtained by numerical integration for the entire duration of the excitation. Irrespective of the method used to obtain the solution of the differential equation, absolute acceleration responses are determined for small increments of t for an assumed natural frequency ω over all or most of the duration of the excitation pulse. The maximum response is recorded and designated as S_x , and the maximum displacements relative to the base or the maximum velocities relative to the base are referred to as S_d and S_v , respectively. The same process is repeated for a different frequency, and another value of S_x is determined. The frequencies are varied over a range which is compatible with the natural frequencies of the structure. The graphical relationship between the maximum relative displacement (u), the maximum relative velocity ($\dot{u}$), and the maximum absolute acceleration

²C. H. Norris, R. J. Hansen, M. J. Holley, et al., Structural Design for Dynamic Loads, McGraw-Hill Book Company, New York, 1959.

³J. M. Biggs, Introduction to Structural Dynamics, McGraw-Hill Book Company, New York, 1964.

($\ddot{y}$) versus the natural circular frequency (ω) or the natural frequency (f) of the one-degree-of-freedom oscillator is called a response spectrum.

However, in actual practice, most spectra plot values do not precisely represent the maximum velocities or accelerations. These quantities are generated from the simple but reasonably accurate relationships

$$S_v = \omega S_d = 2\pi f S_d \quad (\text{B.94})$$

and

$$S_a = \omega^2 S_d = 4\pi^2 f^2 S_d, \quad (\text{B.95})$$

where S_v = spectral velocity and S_a = spectral acceleration. The spectra corresponding to the El Centro, California, earthquake in 1940 are illustrated in Fig. B.10.

Fig. B.10. Response Spectra for 1940 El Centro, California, Earthquake.

The shapes of the average velocity spectra for various values of damping, drawn to an arbitrary scale, are illustrated in Fig. B.11. The

Fig. B.11. Average Velocity Spectra for Various Values of Damping.⁴

ordinates of the average spectrum curves should be multiplied by the factors tabulated below to correspond to the intensities of the respective ground motions. These spectrum curves are useful in determining the maximum response of a single-degree-of-freedom system or of a mode of vibration of a more complex system.

	<u>Factor</u>
1. El Centro, California, May 18, 1940	2.7
2. El Centro, California, December 20, 1934	1.9
3. Olympia, Washington, April 13, 1949	1.9
4. Taft, California, July 21, 1952	1.6

⁴United Nuclear Corporation, "An Interpretive Review of Seismic Design Methods," USAEC Report ORNL-TM-2900, Oak Ridge National Laboratory, May 1970.

B.6.4 Dynamic Response of Multi-Degree-of-Freedom System by Response Spectra Method

As previously mentioned, in the response spectra method, it is assumed that maximum responses for each mode are in the same direction and occur simultaneously and that quantities can be added to give an absolute maximum possible response. This approach is based on the principle that equations of motion for a multi-degree-of-freedom structure can be transformed into normal coordinates where they become uncoupled or independent modes of vibration. Each mode then responds to a base excitation as an independent single-degree-of-freedom system. By proper definition of the modal factor, it is possible to write the following expressions for the response of a particular joint in the system.

$$u_{m,n} \leq \sum_r \Gamma_n^{(r)} \phi_{m,n}^{(r)} S_{d_n}^{(r)}, \quad (\text{B.96})$$

$$\dot{u}_{m,n} \leq \sum_r \Gamma_n^{(r)} \phi_{m,n}^{(r)} S_{v_n}^{(r)}, \quad (\text{B.97})$$

and

$$\ddot{u}_{m,n} \leq \sum_r \Gamma_n^{(r)} \phi_{m,n}^{(r)} S_{a_n}^{(r)}, \quad (\text{B.98})$$

where

m = a particular joint number,

n = direction of orthogonal axis,

r = normal mode number,

$\Gamma_n^{(r)}$ = modal participation factor, and

$\phi_{m,n}^{(r)}$ = normalized amplitude of displacement of joint m in direction n while vibrating in mode r .

The participation factor $\Gamma_n^{(r)}$ may be considered as a measure of the extent that the r -th normal mode contributes to the total inertial load on the structure, and it will depend on the manner by which the mass and stiffness are distributed over the structure. The participation factor is determined for each mode from the expression

$$\Gamma_n^{(r)} = \frac{\sum_{m=1}^{m=j} M_{m,n} \phi_{m,n}^{(r)}}{\sum_{m=1}^{m=j} M_{m,n} \left[\phi_{m,n}^{(r)} \right]^2}, \quad (\text{B.99})$$

where

$M_{m,n}$ = mass associated with deflection at joint m in the n direction
and

$\phi_{m,n}^{(r)}$ = translational amplitude of the mass at joint m in the n direction during vibration in the r -th mode.

If the rotations of the mass are of sufficient magnitude that they cannot be neglected, $\phi_{m,n}^{(r)}$ will represent the rotational amplitude of the mass about the n -th axis and $M_{m,n}$ will correspond to the mass moment of inertia about the n -th axis. However, only translational deflections need be considered for most structures.

Since the algebraic addition of the maximum modal responses without regard to sign gives an absolute upper limit of the total possible structural response, it is reasonable to assume for a structure of many degrees of freedom that all modes will not respond in phase. It is therefore suggested that the typical piping system response be based on the square root of the sums of the squares of the modal maxima.

$$u_{m,n} = \left[\sum \left(\Gamma_n^{(r)} \phi_{m,n}^{(r)} s_{d,n}^{(r)} \right)^2 \right]^{1/2}, \quad (\text{B.100})$$

$$\dot{u}_{m,n} = \left[\sum \left(\Gamma_n^{(r)} \phi_{m,n}^{(r)} s_{v,n}^{(r)} \right)^2 \right]^{1/2}, \quad (\text{B.101})$$

$$\ddot{u}_{m,n} = \left[\sum \left(\Gamma_n^{(r)} \phi_{m,n}^{(r)} s_{a,n}^{(r)} \right)^2 \right]^{1/2}, \quad (\text{B.102})$$

Although there is no rigorous mathematical proof of this concept of probable maximum response, it has been shown⁴ that this relationship arises from the consideration of equal probability of the modal responses in any mode and is in accord with perfectly random distributions of expected values for each of the modal components.

B.6.5 Response Analysis by Time-History Method

Although the time-history method of analysis is more complex and requires more time for examination of data than the response spectra method, it does provide the important capability to determine the exact magnitude and direction of the total modal response as a function of time. However, it is necessary to examine the time-changing response of the structure for critical loads which create maximum stresses in the various members. This search for maximum stresses generally involves more than a static structural analysis.

The equations of motion in matrix form for a base-excited, multi-degree-of-freedom, lumped-parameter system are of the form

$$[M]\{\ddot{y}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = 0, \quad (B.103)$$

and

$$\{y(t)\} = x(t)\{I\} + \{u(t)\}, \quad (B.104)$$

$$\{\dot{y}(t)\} = \dot{x}\{I\} + \{\dot{u}(t)\}, \quad (B.105)$$

$$\{\ddot{y}(t)\} = \ddot{x}\{I\} + \{\ddot{u}(t)\}, \quad (B.106)$$

where

- $[M]$ = diagonal matrix of lumped masses and mass moments of inertia,
- $\{\ddot{y}(t)\}$ = time-varying absolute acceleration of masses,
- $[C]$ = damping matrix,
- $[K]$ = stiffness matrix,
- $\ddot{x}\{I\}$ = time-varying base acceleration, and
- $\{\ddot{u}(t)\}$ = time-varying relative acceleration of masses.

Substitution yields

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = -[M]\ddot{x}(t)\{I\}. \quad (B.107)$$

The main reason for calculating the normal mode shapes and natural frequencies of vibration is that these parameters constitute a special class of coordinates which are most efficient in describing the dynamic response of the system. By using these parameters, the equations of motion for a multi-degree-of-freedom structure can be transformed into normal coordinates where they become uncoupled. Each modal response is

then excited as an independent single-degree-of-freedom structure. Therefore, after having determined the eigenvalues, $\omega^{(r)}$, and eigenvectors, $\phi^{(r)}$, the basic equations of motion are converted into uncoupled relationships by making the transformation

$$\{u(t)\} = [\phi]\{q(t)\}, \quad (\text{B.108})$$

where $q(t)$ represents the normal coordinates, and

$$\{\dot{u}(t)\} = [\phi]\{\dot{q}(t)\} \quad (\text{B.109})$$

$$\{\ddot{u}(t)\} = [\phi]\{\ddot{q}(t)\}. \quad (\text{B.110})$$

Substitution and pre-multiplication by the transpose of $[\phi]$ yields

$$\begin{aligned} [\phi]^T[M][\phi]\{\ddot{q}(t)\} + [\phi]^T[C][\phi]\{\dot{q}(t)\} \\ + [\phi]^T[K][\phi]\{q(t)\} = -[\phi]^T[M]\ddot{x}(t)\{I\}. \end{aligned} \quad (\text{B.111})$$

From the orthogonality relationship associated with normal modes

$$[\phi]^T[M][\phi] = [M^{(r)}] \quad (\text{generalized mass}) \quad (\text{B.112})$$

and

$$[\phi]^T[K][\phi] = [M^{(r)}][\omega^2(r)] = [K^{(r)}] \quad (\text{generalized stiffness}) \quad (\text{B.113})$$

For a damping matrix proportional to the mass matrix,

$$[\phi]^T[C][\phi] = 2\beta[M^{(r)}]. \quad (\text{B.114})$$

Substitution,

$$\begin{aligned} [M^{(r)}]\{\ddot{q}(t)\} + 2\beta[M^{(r)}]\{\dot{q}(t)\} + [M^{(r)}][\omega^2(r)]\{q(t)\} \\ = -[\phi]^T[M]\ddot{x}(t)\{I\}, \end{aligned}$$

and pre-multiplication by $[M^{(r)}]^{-1}$ yields

$$\{\ddot{q}(t)\} + 2\beta\{\dot{q}(t)\} + [\omega^2(r)]\{q(t)\} = -[M^{(r)}]^{-1}[\phi]^T[M]\ddot{x}(t)\{I\}. \quad (\text{B.115})$$

Defining the participation factor $\{\Gamma^{(r)}\}$ of the r -th uncoupled differential equation as

$$[\Gamma^{(r)}] = [M^{(r)}]^{-1}[\phi]^T[M], \quad (\text{B.116})$$

the equations of motion may be represented in matrix form as follows.

$$\{\ddot{q}(t)\} + 2\beta\{\dot{q}(t)\} + [\omega^2(r)]\{q(t)\} = -\{\Gamma^{(r)}\}\ddot{x}(t) . \quad (B.117)$$

These may be solved individually as

$$\ddot{q}^{(r)}(t) + 2\beta\dot{q}^{(r)}(t) + \omega^2(r)q^{(r)}(t) = -\Gamma^{(r)}\ddot{x}(t) . \quad (B.118)$$

This differential equation for the r -th mode is independent of those for all other modes. Therefore, it may be integrated directly to yield the normal displacement $q^{(r)}(t)$. This may be repeated independently for all r equations to determine r modal displacements

$$q^{(i)}(t) ,$$

where $i = 1, 2, \dots, r$. The time-varying absolute accelerations for a given direction at each joint are found from the relationship

$$\{\ddot{y}(t)\} = \{\ddot{u}(t)\} + \ddot{x}\{I\} = [\phi]\{\ddot{q}(t)\} + \ddot{x}\{I\} . \quad (B.119)$$

To determine the maximum loads for any particular member in the structure, it will generally be necessary to apply several different dynamic loads (each load corresponding to a particular instant of time) to the entire structure. This is so because the maximum stresses in any given member may not correspond to the maximum response of the joints which connect each end of the member.

Appendix C

INFORMATION TO BE INCLUDED IN
NUCLEAR POWER PIPING DESIGN SPECIFICATION

The emphasis in this appendix is directed toward pointing out the type of information that should be included in a nuclear power piping design specification rather than presenting a complete example design specification. Blank spaces _____ are used to indicate that information (words or numbers) is required. Notes within parentheses (_____) are sometimes provided to explain the nature of the information to be supplied.

C.1 General Information

(The general information contained in the piping design specification includes the scope of the document, description of the system covered, classification of the system, certification of the design specification, and the documentation requirements.)

C.1.1 Scope

This document constitutes the Design Specification (as required by Paragraph NA-3250 of Section III of the ASME Boiler and Pressure Vessel Code) for the piping system in the _____ (name of plant) Nuclear Power Plant of the _____ (owner) described as follows.

(Description of piping system, its function,
terminus points, and intermediate connections)

This is a Class-__ System and shall be designed and constructed in accordance with the rules of Section III of the ASME Boiler and Pressure Vessel Code, 1971 Edition with Addenda and Case Interpretations in effect as of the contract date (system classification required by Article NA-2000 of Section III).

C.1.2 Certification (required by NA-3250 of Section III)

I, the undersigned, being a registered Professional Engineer in the State of _____, competent in the design of piping and related nuclear energy system requirements, have examined this Design Specification and related drawings and specifications and certify that to the best of my knowledge and belief it is a complete and correct design specification with respect to the functions and operating conditions as required to provide a complete basis for design, construction, and inspection in accordance with the requirements of Subarticle NA-3250 of Section III of the ASME Boiler and Pressure Vessel Code.

Certified by _____ (signature) _____ P.E.
 Registration No. _____ State _____
 Date _____

C.1.3 Documentation Requirements

(a) The enforcement authority having jurisdiction at the location of the system installation is _____ (name) _____, whose address is _____ and whose telephone number is _____.

(b) Communications between the manufacturer and the enforcement authority will be through the Owner unless specifically required by the Code to be otherwise. In the latter case, copies of any correspondence shall be directed to the Owner.

(c) The Owner will provide a copy of the Design Specification to the enforcement authority before the system is installed.

(d) The manufacturer shall identify the authorized Inspector to the Owner on or before the contract date and shall provide the authorized inspection agency with all required documentation, including this Design Specification.

(e) The Stress Report [700(c)](NA-3352) shall be submitted to the Owner for review and certification [700(c)](NA-3260). Following certification that this review has been conducted, copies of the certification shall be attached to the Stress Report and it shall be filed with the enforcement authority at the installation site.

(f) Procedures for continued maintenance of the Quality Control Records [700(e)] (NA-4900) will be established on or before the contract date and shall be included in this Design Specification.

C.2 Design Information

C.2.1 General

The design, material, fabrication, inspection, and testing of the piping shall be in accordance with

- (a) this piping design specification with the attachments detailed herein and
- (b) the ASME Boiler and Pressure Vessel Code Section III, Nuclear Power Plant Components.

(c) The piping shall bear the ASME Code Stamp as required by Article 8000 of Section III.

(d) The piping shall be designed to meet the requirements of Section XI of the ASME Boiler and Pressure Vessel Code.

C.2.2 Radioactivity

The piping shall be designed to contain radioactive fluid with

- (a) N-16 activity of _____ ,
- (b) mixed fission-product activity of _____, and
- (c) mixed corrosion-product activity of _____ .

C.2.3 Water Chemistry

The piping shall be designed to operate with a water chemistry that has a

- (a) conductivity of _____ mho,
- (b) pH of _____ ,
- (c) boric acid content of _____ ppm,

- (d) suspended solid content of _____ ppm, and
- (e) dissolved oxygen content of _____ ppm.

C.2.4 Mechanical

The mechanical design of the piping shall be in accordance with the following requirements.

(a) All butt welds, nozzle welds, and boss welds shall be of a full-penetration design.

(b) End preparations for shop welds shall be machined either the same as for field welds or to an alternate weld preparation approved by the purchaser.

(c) Field weld preparations on the bypass lines and branch connection nozzles shall conform to attached drawing _____ for 3 to 24-in. nominal pipe sizes and to ANSI B16.11 for 2-in. and smaller nominal pipe lines.

(d) Installation of the special bosses for the thermal wells and the differential pressure taps shall be in accordance with the standards attached.

(e) Welded branch connections (nozzles, thermal sleeves, and sample connections) and welds shall conform to the following basic requirements as applicable or to an alternate design subject to approval by the purchaser.

(1) Design shall conform to the rules for reinforcement for internal pressure loading as required by Section III.

(2) Attachment of nozzles shall be accomplished by using a basic design specification.

(3) Thermal sleeves shall be in accordance with attachment _____ to this specification.

(4) Sample connection with scoop shall be in accordance with attached drawing _____ .

(5) The Shop Fabricator may use contour-forged integrally reinforced insert fittings with weld lip for welding into the wall of the pipe run.

(6) Socket weld half-couplings shall be 6000-lb ANSI rating with machine beveled ends to facilitate full penetration welds.

C.2.5 Pipe Bending

(a) Pipe bending shall be in conformance with Paragraphs 1-704.2.1 (NB-3642-1) and 1-729 (NB-4223). In addition, the following paragraphs shall apply.

(b) The bent portion of the piping system bypass lines shall be hot formed, given a post-bending solution anneal and quenching heat treatment, and given descaling and final cleaning operations.

(c) Any pipe that develops corrugations, wrinkles, locally excessive wall thinning, and excessive ovalization during the bending process shall be considered unacceptable and rejected accordingly.

C.2.6 Deformation Limits

The dimensions and tolerances that must be held during all conditions of operation are defined on drawing _____. These dimensions define the deformation limits for the system.

C.2.7 Environmental Effects

(a) Fluence Level (1-701.4.3) [B-1224(b)(3)]
The neutron fluence distribution is _____.

(b) Corrosion (1-702.4.1)(NB-3121)
No provisions need be made for corrosion of austenitic stainless steels.

(c) Erosion (1-702.4.1) (NB-3121)
To prevent significant erosion, flow velocities, including flow in local regions, shall be less than _____ fps.

(d) Ambient Temperature
The ambient temperature for the system is _____ °F.

C.2.8 Handling

_____ (only as affects design)

C.2.9 Storage

_____ (only as affects design)

C.2.10 Delivery

_____ (only as affects design)

C.3 MaterialsC.3.1 Surveillance Program Materials

_____ (Any material required for the surveillance program shall be specified.)

C.3.2 Fabrication Materials

Materials other than _____ steels may not be used without specific approval from the user.

C.3.3 Minimum Temperature Limits

The minimum temperature for testing or operation shall be _____ °F.

C.3.4 Cleanliness

In-process procedures and final cleaning shall satisfy the requirements of _____.

C.3.5 Impact Tests

Impact tests shall be performed on ferritic steels in accordance with Paragraph NB-2300 of Section III of the ASME Boiler and Pressure Vessel Code for the following.

<u>Main Coolant Loops</u>	Temperature <u>100 °F</u>
_____	_____

C.4 System Boundaries

C.4.1 General

The major loops that comprise the piping system are described schematically in drawings _____. Included are

- (a) the two main coolant loops, starting from the reactor vessel nozzle weld to the steam generator nozzle weld, similarly to the main coolant pump, and back to the reactor vessel;
- (b) the pressurizer surge line, from the main coolant piping connection to the pressurizer connection;
- (c) the main steam piping, from the two steam generator connections to the containment anchors;
- (d) the feedwater piping, from the two steam generator connections to the containment anchors;
- (e) the two safety injection lines, from the main coolant pumps discharge pipes to their first anchor points; and
- (f) the residual heat removal line, from the loop-no.-1 hot leg connection to the first anchor point.

C.4.2 Equipment Characteristics

(a) Reactor Vessel

Design pressure:

Design temperature:

Material of fabrication:

_____ psig
_____ °F

Allowable forces and moments:	_____ lb, _____ in.-lb			
Vessel support details:	_____			
(b) Steam Generators, primary side				
Design pressure:	_____ psig			
Design temperature:	_____ °F			
Material of fabrication:	_____			
Weight empty:	_____ lb			
Weight flooded with cold water:	_____ lb			
Weight during normal operating conditions:	_____ lb			
Allowable forces and moments:	_____ lb, _____ in.-lb			
Vessel support details:	_____			
(c) Reactor Coolant Pumps				
Design pressure:	_____ psig			
Design temperature:	_____ °F			
Weight empty:	_____ lb			
Weight flooded with cold water:	_____ lb			
Weight during normal operating conditions:	_____ lb			
Nominal flow:	_____ gpm			
Nominal rotational speed:	_____ rpm			
Number of vanes, rotor:	_____			
stator:	_____			
Frequency:	_____ cps			
Allowable forces and moments:	_____ lb, _____ in.-lb			
Pump support details:	_____			
(d) Pressurizer				
Design temperature:	_____ °F			
Design pressure:	_____ psig			
Material:	_____			
Allowable forces and moments:	_____ lb, _____ in.-lb			
Support details:	_____			
(e) Valves	No. 1 No. 2 No. 3 No. 4			
Design temperature, °F:	_____	_____	_____	_____
Design pressure, psig:	_____	_____	_____	_____
Material:	_____	_____	_____	_____
Weight empty, lb:	_____	_____	_____	_____
Weight during normal operation, lb:	_____	_____	_____	_____

C.5 Design Temperature and Pressure

Preliminary calculations indicate that the following values may be used. Should further calculations indicate that higher temperatures exist under normal or upset conditions, these higher values shall be used.

- (a) Loop Piping
 - Design pressure: _____ psig
 - Design temperature: _____ °F
- (b) Steam Line
 - Design pressure: _____ psig
 - Design temperature: _____ °F
- (c) Surge Line
 - Design pressure: _____ psig
 - Design temperature: _____ °F
- (d) Feedwater Line
 - Design pressure: _____ psig
 - Design temperature: _____ °F
- (e) Safety Injection Line
 - Design pressure: _____ psig
 - Design temperature: _____ °F

C.5 Operating Conditions

C.6.1 Normal Conditions

The conditions to be considered include the steady-state operational conditions in the design power range and the transient conditions that occur in the course of system startup, shutdown, and testing.

(a) Steady-State Temperature and Pressure Conditions. (The deformation and rotation of the piping connection points or some information leading to this data, such as temperature versus time curves for all attached equipment, must be given.)

Ambient temperature:	_____ °F
Main coolant	
pressure at pump discharge:	_____ psig
allowable heating and cooling rate:	_____ °F/hr
Coolant temperature	
leaving reactor vessel:	_____ °F
in main coolant pumps:	_____ °F
entering reactor vessel:	_____ °F
Steam generator	
primary side inlet temperature:	_____ °F
primary side outlet temperature:	_____ °F
Pressurizer normal operating temperature:	_____ °F

Secondary steam pressure (steam generators & lines)
 at 100% load: _____ psig
 at 0% load: _____ psig
 Secondary steam temperature
 at 100% load: _____ °F
 at 0% load: _____ °F
 Feedwater
 normal operating pressure: _____ psig
 maximum operating temperature: _____ °F

(b) Temperature and Pressure Transients. The main coolant piping shall be designed to be capable of withstanding the transient thermal and pressure stresses resulting from the transient conditions defined as follows and as illustrated by the referenced figures. (A written description should be given for each of the transient conditions.)

<u>Transient Conditions</u>	<u>Occurrences</u>	<u>Figure</u>
Plant heatup at 100°F per hour	_____	C.1
Plant cooldown at 100°F per hour	_____	C.1
Plant loading	_____	C.2
Plant unloading	_____	C.2
10% step load increase	_____	C.3
10% step load decrease	_____	C.3
100 to 5% step load decrease	_____	C.4
Scram from full power	_____	C.5
Loss of load	_____	C.6
Loss of flow, one pump	_____	C.7
Loss of secondary pressure	_____	C.8
Pressure tests	_____

Fig. C.1. Temperature and Pressure as a Function of Time for Plant Startup and Shutdown Over 150 Cycles.

ETR2-48

Fig. C.2. Temperature and Pressure as a Function of Time for Plant Loading and Unloading Over 11,000 Cycles.

ETR2-49

Fig. C.3. Temperature Variation and Pressure as a Function of Time for 10% Step Load Decrease and Increase Over 1500 Cycles.

Fig. C.4. Temperature and Pressure as a Function of Time for a Loss of Load Over 80 Cycles.

ETR2-51

Fig. C.5. Temperature and Primary Pressure as a Function of Time for Reactor Scram From Full Power.

ETR2-52

Fig. C.6. Temperature and Primary Pressure as a Function of Time for Loss-of-Load Accident.

ETR2-53

Fig. C.7. Primary Coolant Temperature as a Function of Time for Loss of Flow of One Pump.

ETR2-54

Fig. C.8. Temperature as a Function of Time for Loss of Secondary Pressure.

C.6.2 Upset Conditions

The upset condition to be considered is a seismic disturbance with operational status. An isometric diagram of the piping system is illustrated in Fig. C.9. The results of the system seismic analysis for the

Fig. C.9. Isometric Diagram of the Piping System.

earthquake for which the system must remain operational indicate that the displacements as a function of time at the building level where the piping is attached are as illustrated in Fig. C.10. (The vertical spectrum is two-thirds of the horizontal and the design basis earthquake is twice the oboe.) A dynamic analysis of the system shall be performed for each of the conditions prescribed in Fig. C.10. For this analysis, the mass and stiffness matrix for components not provided by the manufacturer shall be as shown in Fig. _____, in which the nature and location of these components are identified. Three such events shall be assumed to occur: one at 10% power, one at 100% power, and one at (shutdown) condition.

Fig. C.10. Envelope Response Spectrum for Multiple Spectra Analysis of Operating Basis Earthquake.

C.6.3 Emergency Conditions

The emergency conditions to be considered are an abnormal scram and recovery and the design basis earthquake.

(a) Abnormal Scram and Recovery. The temperature, pressure, and flow rate in the various loops as a function of time are given in Fig. _____. Note that this condition terminates in a condition identical to that steady-state condition which exists at the end of cooldown. The recovery mode and the number of cycles for each cycle of abnormal scram and recovery are as follows.

<u>Recovery Mode</u>	<u>Number of Cycles</u>
Normal heatup	_____
Drain, fill, normal heatup	_____

(b) Design Basis Earthquake. The design basis earthquake shall be considered to be a factor of ____ times as severe, in terms of imposed displacement and defined loadings, as was the operational-basis seismic event described in Subsection C.6.2.

C.6.4 Faulted Conditions

The faulted condition to be considered is the combination of a Design Basis Accident and the Design Basis Earthquake. The design criteria for this condition are _____

_____.

The Design Basis Accident is described as follows. _____

_____.

Appendix D

SUGGESTED OUTLINES AND PROCEDURES FOR REPORTS

The structural integrity of a piping system and compliance with the design specification must be established by the designer. Suggested outlines and procedures for presentation of interim reports and the design report are given in the following subsections.

D.1 Interim Report

An interim report shall be submitted to the owner for approval prior to the preparation of the design report. The interim report will provide both the owner and the designer the opportunity to agree on the general methods of analysis and loading conditions to be included in the design report. The benefit of this agreement is twofold: (1) the owner will know long before submittal of the design report the minimum amount of analysis to expect and the general analytical procedures to be used, and (2) the designer can perform the detailed calculations required for the design report with the knowledge that the general analytical procedures to be used have been agreed to and are acceptable.

D.1.1 Loading Conditions

The interim report shall include a description of the loading conditions which exist and a description of the loading conditions to be analyzed, such as steady-state and transient pressures and temperatures, external loadings, shock and vibration, etc. The lack of analysis of any existing loading conditions should be justified. If the loading is a transient, the description shall also include a discussion (with sketches and plots where required) of the progression of the transient through the system of component.

D.1.2 Assumptions

All assumptions and approximations to be used in the analysis shall be clearly stated and justified. This includes but is not limited to

1. identification and justification of the boundary conditions to be used in the flexibility analysis, such as deflections and rotations of attached equipment, equipment flexibilities, etc.; and
2. identification and justification or adequate referencing for the material properties used.

D.1.3 Analytical Models

A sketch of the piping system being analyzed must be provided. Sufficient dimensions to geometrically describe the run of the piping must be provided in the sketch. All branch lines shall be included, and those branch lines not included in the analysis shall be indicated and justification for not considering them shall be given.

The spring-mass model to be used in a shock analysis must also be presented.

When stress indices for a given component are not provided in the Code and a detailed analysis is therefore required, diagrams must be presented for interaction and thermal analyses. For an interaction analysis, a complete free-body diagram must clearly show the interaction forces, external loads, applicable dimensioning, and sign conventions. For a thermal analysis, the basic grid or subdivision network must be shown. This shall include a list of the heat transfer boundaries selected in which the data selected for each boundary (heat transfer coefficients, temperature, modulus of elasticity, coefficient of expansion, heat input, etc.) are specified. Whether the stipulated data are stationary or time (temperature) dependent must also be specified.

D.1.4 Methods of Analysis

The methods of analysis (flexibility, interaction, etc.) must be described in detail, and this description must include all formulas and derivations to be used. However, where the derivations are long, they should be incorporated in an appendix. Derivations readily available in open literature sources need not be duplicated in the report if adequate references are given.

D.2 Design Report

The purpose of the design report is to thoroughly demonstrate the adequacy of the piping system. The design report shall contain a complete analysis of the system and its components, where applicable. The design report shall be submitted after approval of the interim report, and it shall be comprised of

1. front cover and title page,
2. record of revisions page,
3. abstract,
4. list of contents,
5. introduction,
6. design specifications,
7. body of report,
8. list of references,
9. list of nomenclature,
10. design drawings,
11. appendices, and
12. certification.

D.2.1 Front Cover and Title Page

The title page for the design report shall contain the

1. report title,

2. name of piping system,
3. report number,
4. date of original report and all revision dates,
5. author's name(s) and title(s), and
6. approval signatures.

D.2.2 Record of Revisions Page

The Record of Revisions page shall contain the dates of all revisions, the numbers of the pages revised, and a short description of the revisions. Each entry shall also contain the name and initials of the author. Each entry in the Records of Revisions shall carry a revision symbol, such as "3", which shall also appear in the unbound margin opposite those paragraphs in the report that have been revised. If an entire page is revised, the corresponding revision symbol shall be placed next to the page number.

D.2.3 Abstract

A clear, concise indication of the objective and scope of the report and the results achieved shall be presented in the abstract. In addition, a statement of whether or not the design conforms completely to the specification requirements shall be included in the abstract.

D.2.4 List of Contents

All sections of the report, including the appendices, shall be identified in the list of contents.

D.2.5 Introduction

A brief explanation of the subject of the report (what the problem is), how the subject will be treated (how the problem will be solved),

and the necessary background information shall be provided in the introduction.

D.2.6 Design Specifications

The specifications to which the piping system was designed and is certified to meet shall be presented in the design report.

D.2.7 Body of Report

The piping system and its design requirements shall be described, the problem areas shall be defined, the methods of analysis shall be developed in a clear and logical manner, the computations shall be performed, and the performance and reliability of the piping system shall be evaluated in the body of the design report. Thus, the following information shall be presented in the body of the report.

(a) Description of System. The operation or function of the system to be analyzed shall be described. The configuration of the system shall also be described with appropriate references to drawings and with the aid of sketches.

(b) Loading Conditions. All loading conditions which exist and the loading conditions to be analyzed, such as steady-state and transient pressures and temperatures, external loads, shock and vibration, etc., must be described. The lack of analysis of any existing loading conditions must be justified. If the loading is a transient, the description shall also include a discussion (with sketches and plots where required) of the progression of the transient through the system or component.

(c) Assumptions. All assumptions and approximations to be used in the analysis shall be carefully stated and justified. This shall include the

- (1) identification and justification of the boundary conditions to be used in the flexibility analysis (deflections and rotations of attached equipment, equipment flexibilities, etc.),

- (2) stress concentration or strength reduction factors to be considered in the analysis for conditions for which indices are not given in the Code, and
- (3) identification and justification or adequate references for the material properties used.

(d) Analytical Models. A sketch of the piping system being analyzed shall be provided. This sketch shall be identifiable with the actual piping run and shall provide sufficient dimensions to geometrically describe the run of the piping. All branch lines shall be included in the sketch, and those branch lines not included in the analysis shall be indicated and justification for not considering them shall be given.

The spring-mass model to be used in a shock analysis shall be provided.

Where detailed interaction and thermal analyses are required, the computational models for these analyses shall be presented. For an interaction analysis, a complete free-body diagram must clearly show the interaction forces, external loads, applicable dimensioning, and sign conventions. For a thermal analysis, the basic grid or subdivision network must be shown. This shall include a list of the heat transfer boundaries selected in which the data selected for each boundary (heat transfer coefficients, temperature, modulus of elasticity, coefficient of expansion, heat input, etc.) are specified. Whether the stipulated data are stationary or time (temperature) dependent must also be specified.

(e) Description of Methods of Analysis. The methods of analysis (flexibility, interaction, etc.) must be described in detail, and this description must include all formulas and derivations used. However, where the derivations are long, they should be incorporated in an appendix. Derivations readily available in open literature sources need not be duplicated in the report if adequate references are given.

(f) Calculations and Presentation of Results. Where the calculations are relatively short and simple, they shall be given in their entirety. Where the calculations are long and complicated and/or are performed with a computer, the pertinent results of these calculations

shall be given in the body of the report and the calculations and/or computer printout sheets shall be appended. Where a sample calculation will adequately explain the method, repetitive calculations need not be reported if the results are given. The results shall be presented in tabular or graphical form, and their compliance with the allowable values shall be noted.

(g) Tables, Graphs, Sketches, and Illustrations. Tables, graphs, sketches, and illustrations may be included as part of the report. If not included within the text, they can be presented on separate pages which can either be distributed throughout the report or assembled at the end of the report. In any case, they must be consecutively numbered as Tables and Figures and cited in the report. Legends, captions, titles, dimensions, or headings must be fully noted.

D.2.8 List of References

References shall be numbered in the order of citation in the report and listed after the body of the report. Each reference shall be adequately identified as follows.

References to books shall include the name of the author (with initials), the title of the book, the publisher, edition number, year of publication, and specific page numbers.

References to journal articles shall include the name of the author (with initials), title of the article, name of the periodical, volume number, page numbers, and date of publication.

References to reports shall include the name of the author (with initials), name of report, report number, name of company or government agency responsible for publication, and the publication date.

References to specifications or codes shall include the specification or code number (with latest publication date), title of specification or code, and the name of the responsible agency or society.

References to drawings should include the name of the company or agency responsible for producing the drawing, the drawing number, title of drawing, and the latest revision date.

D.2.9 List of Nomenclature

A list of nomenclature shall be presented to define all mathematical symbols used throughout the report. For very extensive reports comprised of several self-contained sections and/or appendices, more than one list of nomenclature may be justified, even preferred. Any list of symbols used shall include appropriate subscripts, such as those used to differentiate between "inside" and "outside", "vessel" and "pipe", etc.

D.2.10 Design Drawings

The design report shall contain

1. layout drawings which pictorially describe the run of the piping and the components that make up a specific system;
2. detail drawings of any special fittings;
3. layout drawings depicting the locations of hangers, anchors, guides, and sway braces and indicating force reactions; and
4. detail drawings of hangers, anchors, guides, and sway braces that are not catalog items.

D.2.11 Appendices

In reports which are lengthy and/or highly mathematical, it may be advisable for reasons of continuity and clarity to limit the extent of the body of the report by incorporating certain self-contained areas of the analysis in appendices. Such areas might be mathematical derivations and descriptions of computer programs used with the program input and output data sheets.

D.2.12 Certification

The design report shall be certified as meeting the requirements of the Code for the conditions of the design specification by a Professional Engineer experienced in the design and analysis of piping systems.

D.3 Review of Reports

The owner of the nuclear power plant or his agent shall provide or cause to be provided a review of the interim report and the design report. The intent of these reviews is to provide assurance to the owner that adequate and correct interpretations of the conditions of design have been applied.

D.3.1 Interim Report

The review of the interim report will provide assurance with respect to the

1. applicability of the analytical methods employed,
2. acceptability of assumptions,
3. consideration of operating and design conditions as required by the design specification,
4. inclusion of all suspected critical areas in the analysis, and
5. applicability of analytical models.

D.3.2 Design Report

The review of the design report will provide assurance with respect to

1. agreement with the approved interim report with respect to methods of analysis, assumptions, analytical models, and operating and design conditions;
2. a spot review of the analytical calculations to confirm their adequacy; and
3. a comparison with similar analyses, if available, to determine the extent of agreement.