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NEUTRON ATTENUATION MECHANISMS IN CONCRETES

Prepared by: J. Greenborg
Research Engineer
Battelle-Northwest

MASTER

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Many nuclear reactions produce neutrons in sufficient quantity to require large protective shield structures to protect personnel and equipment located nearby. Concrete is one of the common materials employed as a bulk material for shielding because of its good physical properties and (as will be shown in this paper) desirable nuclear properties.

Since concrete is a mixture of materials one would expect concrete to have the neutron attenuating properties of its constituents, providing they are well mixed. This is indeed true, and the subject is reduced to a study of the neutron attenuation properties of the constituents with subsequent study of what happens to these properties when these materials are mixed.

To begin the discussion of neutron attenuation in concretes, a brief description is given of the neutron. This is followed by a discussion of the physics of interaction of a neutron with a nucleus and then extending this subject into the effect of neutron interaction with many nuclei in a small volume (as is found in common solids and liquids). Material mixing is discussed last with emphasis on estimation of neutron attenuation efficiency in relation to concrete composition. For a more complete treatment of neutron attenuation mechanisms than is given in this paper, the reader is referred to the shielding texts of Rockwell,¹ Goldstein,² Blizard,³ Grotenhuis,⁴ and Price,⁵ et al.

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THE NEUTRON

Neutrons are basic building blocks of atomic nuclei; all atomic nuclei having them except for ordinary hydrogen. Neutrons may be thought of as little hard impenetrable spheres weighing 1.00893 amu (atomic mass units) or 1.675×10^{-24} grams. Neutrons have no electric charge thus they may penetrate the electric fields of atoms without the requirement of possessing large amounts of kinetic energy. Since the nucleus is relatively small ($\sim 10^{-12}$ cm) in comparison with interatomic dimensions ($\sim 10^{-8}$ cm), the mean free path between neutron-nucleus collisions is very large when comparing this mean free path to the size of the nucleus or a neutron.

Common nuclear reactions which produce neutrons are fission (in a nuclear reactor), photoneutron reactions, and energetic charged particle reactions (both from laboratory neutron sources and accelerators). For every watt of heat produced in a nuclear reactor, about 3.1×10^{10} uranium atoms undergo fission every second. The average fission of a uranium²³⁵ nucleus yields 2.47 neutrons, one of which is required to react with another uranium nucleus to maintain the chain reaction. Thus, up to 1.47 neutrons per fission may leak out of the reactor and require shielding. In practice leakage is lower; less than 3% of the neutrons produced in large reactors and about 30% of the neutrons produced in small compact reactors (e.g. a space reactor). This is still a large number of neutrons. A large power reactor which produces 1000 Mw of electrical power will lose from the core approximately 10^{20} neutrons/sec which must be stopped by the reactor shields. Accelerators are commonly available which will produce neutrons up to the rate of 10^{12} per second. These too are impressive shielding problems. The

neutron energy may be very high, although quantity of neutrons is low compared to the nuclear reactor.

Of paramount importance to the discussion of neutron attenuation is an examination of the neutron source requiring shielding. The considerations are: physical size and shape of the source, intensity of the source (the quantity of neutrons being emitted), the kinetic energy, and the direction of the neutrons leaving the source. Size and shape may be generalized in that a reactor is usually a large sphere or cylinder (occasionally a cube) and the accelerator or laboratory source is usually small and usually can be considered to be a point source.

Fission Neutrons

Neutrons produced by fission are emitted with statistically random kinetic energy, the average energy being about 2 Mev. A significant fraction of these fission neutrons possess less than 0.25 Mev and a significant fraction possess more than 6 Mev (a small fraction has as much as 18 Mev).

The energy distribution of fission neutrons may be displayed as a plot (known as a differential energy spectrum) where the fraction of neutrons having energy within the energy bounds of one energy unit (surrounding the neutron energy) is plotted against neutron energy. Figure 1 is the differential energy spectrum of neutrons emitted by fission of U^{235} . This plot is normalized to a total emission of 1 neutron, that is

$$\int_0^{18 \text{ Mev}} \eta(E) dE = 1,$$

or the area under the curve of Figure 1 is equal to one.

FIGURE 1*

The Fraction of Neutrons Per Mev Energy Interval,
Emitted at Energy E, from Thermal Fission of U^{235}

The important conclusion to be drawn from Figure 1 is that neutrons produced by fission are energetic neutrons, having kinetic energies greater than 0.1 Mev. These are commonly called fast neutrons. The speed corresponding to their kinetic energy ranges from 4.36×10^6 m/sec for energy of 0.1 Mev to 5.85×10^7 m/sec for energy of 18 Mev. The speed of light is 3.0×10^8 m/sec.

Accelerator Sources

Neutrons are commonly produced with accelerators for use in neutron physics research. The reactions used are generally⁶

* From Reference 2, Page 49.

where the accelerated particle is a proton H^1 or a deuteron H^2 . By varying the energy of the accelerated particle one may obtain monoenergetic neutrons with kinetic energies in the range of 5 kev to 20 Mev. Higher energies are obtainable with other reactions.

Laboratory Neutron Sources

Common laboratory neutron sources make use of the reaction of the alpha particles with elements $[(\alpha, n) \text{ reactions}]$ and gamma rays with various elements [photoneutron, $(\gamma, n) \text{ reactions}]$. A common (α, n) source is the radium-beryllium source constructed by mixing one gram of radium (one curie) with several grams of beryllium and encasing it in a small sturdy metal can. This source will yield about 10^7 fast neutrons (1 Mev to 13 Mev) per second making use of the reaction:

where the He^4 is an alpha particle produced by radioactive disintegration of radium. A common photoneutron source is constructed with a small cylinder of Na^{24} (about 1-1/4 cm in diameter, 5 cm long) surrounded by a concentric cylinder (1 cm wall thickness) of Be. A source of this type would typically yield 10^6 to 10^7 neutrons per second with an average kinetic energy of 0.83 Mev.

NEUTRON INTERACTIONS WITH MATERIALS

Neutrons react with nuclei of matter in several common ways:

1. Neutrons are absorbed or captured by nuclei subsequently changing the atomic weight or mass number of the nuclei. The nuclei will subsequently be unstable (fission or eliminate energy by instantaneous gamma ray emission), and then be stable or partially stable (~~that is~~ radioactive with a delayed decay).

2. Neutrons are absorbed by nuclei and are re-emitted with a lower energy. This is commonly called inelastic scattering. This will only happen if the initial neutron energy is high enough, variable depending on the particular nucleus but roughly around 1 Mev. This reaction is characterized by the emission of one or more photons (gamma rays) from the nucleus in coincidence with emission of the neutron so that the residual nucleus is again in ground state. It is a general rule that a large fraction ~40 to 80% of the incident neutron energy is removed during an inelastic scattering event.
3. Neutrons undergo reaction with nuclei, which in simple analogy resemble billiard ball collisions and is called elastic scattering. In this reaction energy is exchanged depending on the angle of divergence of the contacting neutron and nucleus, and the weight of the nucleus as one would calculate using standard classical ~~ex~~ mechanics.

The Cross Section

The common term used in describing the probability for a neutron-nucleus interaction to occur is the cross section. The scattering cross section, inelastic scattering cross section and absorption scattering cross sections are the associated probabilities for these types of neutron-nuclei reactions to occur.

A microscopic cross section is defined which has the units "Barns/atom".

A barn is the unit of area 10^{-24} cm². If one places a nucleus randomly on the plane of one square centimeter with nothing else on this plane, then passes one neutron in random direction through this 1 square centimeter of area the probability of interacting is given by the magnitude of the cross

section (expressed in cm^2). Thus probability is expressed by the ratio of two areas, the apparent nucleus cross section area (not necessarily the geometric cross sectional area), and the unit area. The probability of a neutron reacting with a nucleus is very small since cross sections generally vary between 0.001 and 1000 barns, corresponding to probabilities between 10^{-27} and 10^{-21} . A neutron cross section is usually energy dependent; that is, dependent on the incident energy of the neutron (assuming the nucleus to be at rest).

Physics of Neutron-Nuclei Interaction*

The physics of neutron-nuclei interaction can be looked at in considerably more rigor; however, in order to do this one has to assume additional properties of the neutron and the nucleus. Neutrons have properties that are wave like and particle like. The neutron exhibits the deBroglie¹ wave properties, where the wave amplitude is everywhere proportional to the neutron density at that point, and whose wavelength λ is a function of neutron energy:

$$\lambda = \frac{h}{mv} = 2.86 \times 10^{-9} E^{-1/2} \text{ cm} \quad (1)$$

where

h = Planck's constant

m = mass of the neutron

v = velocity of the neutron

E = energy of the neutron, ev.

This property is closely tied to the neutron-nuclei interaction properties; for example for fast neutrons, the neutron has the interaction properties

* See Chapter 3, Reference 5.

of a particle with a physical radius $\chi = \frac{\lambda}{2\pi}$. A second neutron property is its intrinsic angular momentum, or spin. This has the magnitude $\hbar/2$ where $\hbar = h/2\pi$. This ^affects slow neutron cross sections because of the relative necessity to line up angular momentum to allow physical combination.

The nucleus exhibits properties as though it were a collection of protons and neutrons in constant agitation but restrained by strong short-range attractive forces. These short-range forces prevent the protons from immediately separating due to their electrostatic repulsion. ~~The presence of the neutrons obviously accounts for the short range attractive forces.~~ These short-range forces cease to be effective outside of the radius of the nucleus roughly given by $R = 1.46 A^{1/3} \times 10^{-13}$ cm, where A is the mass number of the nucleus.

If a neutron is captured by a nucleus, the resulting "compound" nucleus is lighter than the sum of the weights of the nucleus and neutron before interaction. This mass is given up as binding energy according to Einstein's equation $\Delta E = \Delta m c^2$ where Δm is the mass defect. The compound nucleus may only have an instantaneous life, the short life resulting from the excess energy because of the mass defect and the kinetic energy of the incident neutron (conservation of energy and momentum regulate the amount of neutron kinetic energy transferred as internal energy to the compound nucleus). This excess internal energy is known as "excitation energy". It is a matter of statistical preferential probability as to how the excitation energy is released from the compound nucleus. Possibilities for energy release from the compound nucleus are elimination of gamma rays, charged particles, and neutrons at energies equal to or less than that of the incident neutron.

Specific levels of excitation energy are preferred. This accounts for capture gamma ray spectra, resonances or peaks in the reaction cross sections, and the inelastic scattering process. The first few excitation levels of the compound nucleus above ground state are widely spaced, but as excitation energy increases the levels get closer together, and at high energies they overlap. Resonances in reaction cross sections occur when a resulting compound nucleus corresponds to exactly one of its possible excited states.

For medium weight nuclei and incident neutrons of low energy the cross section for the formation of a compound nucleus by neutron absorption, σ_{cpd} , is given by the Breit-Wigner* single-level formula:

$$\sigma_{\text{cpd}} = \pi \lambda^2 g \frac{\Gamma_n}{(E - E_r)^2 + \Gamma^2/4} \quad (2)$$

where

Γ = total width of the excited level = $\sum \Gamma_t$

Γ_n = the partial width for neutron emission $\approx h/2\pi\tau_n$

τ_n = the mean lifetime for neutron emission from the compound nucleus

E = kinetic energy of ingoing neutron

E_r = neutron energy corresponding to exact resonance

g = the statistical weight factor, a measure of the probability of the nucleus and neutron spins being properly aligned, a number lying between 0.5 and unity.

For high energy neutrons, excitation levels are close together and single level effects are not observable; however, it has shown⁷ that σ_{cpd} averaged over many resonances is given for a target nucleus of radius R by

* See Chapter 16, Reference 6.

$$\sigma_{cpd} \approx \pi (R + \lambda)^2 \cdot \zeta \quad (3)$$

The quantity ζ is a sticking probability, the probability that a neutron will penetrate the nucleus. Quick examination of Equation 3 shows that it is the cross sectional area of physical neutron nucleus contact corrected by an interaction or penetration probability.

Two types of elastic scattering occur; one of these being the same mechanism as inelastic scattering known as "capture elastic scattering", the other being similar to the billiard ball analogy mentioned earlier known as "potential scattering". ~~Capture elastic scattering and capture inelastic scattering are essentially the same mechanism.~~ For the inelastic capture, the target nucleus is left in one of its excited states after emission of the neutron. This excited state decays by gamma ray emission. To leave the target nucleus in an excited state, the neutron must have left the compound nucleus with less energy than it had prior to interaction. For capture elastic scattering, the neutron is usually captured at one of the low excitation levels and is ejected without loss of excitation energy through photon emission. If the captured neutron is not ejected prior to loss of excitation energy by photon emission to below the binding energy of the last neutron, then the reaction terminates as radioactive capture. Potential scattering may be regarded as reflections of the neutron wave by the potential jump at the nuclear surface. The potential scattering cross section for all neutron energies is given by approximately $4\pi R^2$, the surface an impenetrable sphere of radius R. The

observed scattering cross section does not closely follow $4\pi R^2$ due to numerous effects, some being capture elastic scattering in nearby resonances, chemical binding for light elements, and crystal diffraction effects.

From the above theory one can surmise that neutron capture, capture elastic scattering, and capture inelastic scattering are all competing types of energy loss of the excited compound nucleus. The incident neutron energy greatly varies the competitive routes for de-excitation because the type of de-excitation is closely related to amount of excitation energy in the compound nucleus. Other not so common reactions such as proton, alpha particle or double neutron emission are also competing processes for energy loss of the compound nucleus. In the case of charged particle emission the coulomb barrier must be overcome; thus, energy is required in excess of normal excitation energy. This may be made available by an exothermic reaction, one which yields products of lower mass than the mass of the incident neutron and the target nucleus.

A reaction of usually little importance, but perhaps of more importance to this paper since neutron shielding is being considered, is the $(n, 2n)$ reaction. The neutron threshold energy at which the $(n, 2n)$ reaction becomes possible is $(A + 1)/A$ times the binding energy of a neutron already

in the target nucleus (~ 7 Mev), A being the mass number of the target nucleus. The factor $(A + 1)/A$ allows for recoil energy given to the compound nucleus.

Interactions of Neutrons in Bulk Media

Of the three interactions, absorption, elastic and inelastic scattering described above, elastic and inelastic scattering are the most important for neutron shielding. This can be seen by examining the probability of these types of reactions to occur. Since neutrons are produced with a wide range of kinetic energies, and the process of scattering reduces neutron energy, the probabilities for neutron interaction with nuclei must be examined over a wide neutron energy range and a wide material range.

Table I presents microscopic cross sections for the three common reactions, absorption, inelastic scattering, and elastic scattering for several common elements ranging from light elements to heavy elements. The microscopic cross section does not provide an absolute measure for the gross attenuation properties of a material, but it does provide a measure of the relative importance of the various types of neutron reactions. Examining Table I, one can see that:

1. For neutron energies above 0.01 Mev the absorption cross section is very low compared to the other types of reaction. Since neutrons are almost always produced with more energy than 0.01 Mev, one would not expect reactions of "new" neutrons to be that of absorption.

TABLE I

SOME MICROSCOPIC NEUTRON CROSS SECTIONS* (BARNS) FOR COMMON ELEMENTS
(USUALLY FOUND IN CONCRETES) FOR SPECIFIC NEUTRON ENERGIES IN THE RANGE 0.025 EV TO 10 MEV

Neutron Energy		H ¹			C ¹²			O ¹⁶			Si		
		σ_{in}	σ_{el}	σ_a	σ_{in}	σ_{el}	σ_a	σ_{in}	σ_{el}	σ_a	σ_{in}	σ_{el}	σ_a
10	Mev		0.990	1.87x10 ⁻⁵	0.480	0.079		3.4x10 ⁻⁴	0.590	0.284(n, α)	0.701	0.75	
5			1.63		0.130	1.000			1.02	0.104	1.069	1.18	
					**								
2			2.93			1.78			1.60		0.116	3.02	
1			4.24			2.59			6.30		0.001	3.48	
0.5			6.00			3.70			7.30			8.04	
											**		
0.1			12.5			4.50			3.70	9.7x10 ⁻⁸		1.60	1.18x10 ⁻⁵
10	Kev		19.0			4.76						2.18	
1			19.5	0.002		4.62						1.75	
0.1			19.9	0.005		4.63						1.70	
10	ev		20.0	0.019		4.64	0.0003						0.005
1			20.0	0.049		4.68	0.0006		3.70				0.010
0.1			33.0	0.167		4.78	0.0019		4.00	0.00011			0.050
0.025			38.0	0.332		4.79	0.0034		4.20	0.00024		1.70	0.16
		Ca			Fe			Ba					
		σ_{in}	σ_{el}	σ_a	σ_{in}	σ_{el}	σ_a	σ_{in}	σ_{el}	σ_a	σ_{in}	σ_{el}	σ_a
10	Mev	1.18	1.25		1.35	1.78		0.547	2.35		* Data taken from the RBU Data Tape ^{8,9}		
5		1.16	2.23		1.41	2.19		0.565	3.34				
2		0.40	3.42		0.88	1.79		0.233	5.24		** Resonance cross section peaks occur in this range		
1			2.50		0.46	1.84		0.052	6.49				
0.5			1.90			2.94		0.001	6.67		*** Group averaged over resonances, (group width, 51 ev)		
					**								
0.1			2.03	0.0003			0.0002		7.36	0.099			
					**								
10	Kev		2.00			6.60			8.62	0.151			
1			3.00			8.97			9.48	0.252			
								**	**	**			
0.1						11.4	0.046		13.2***	6.11***			
								**	**	**			
10	ev					11.9	0.161		7.91	0.091			
1						11.9	0.402		7.95	0.229			
0.1						11.9	1.36		7.96	0.778			
0.025						11.5	2.69		7.96	1.27			

2. For high neutron energies, >1 Mev, inelastic scattering cross sections are relatively large compared to other types of reactions. These probabilities go to zero below about 1 Mev.
3. At intermediate neutron energies, 1 Mev down to 1 ev, elastic scattering probabilities are considerably more important than others.
4. The absorption cross section becomes quite large for very low neutron energies, <1 ev, and somewhat follows a neutron energy dependence proportional to $1/E$.

Thus, it is seen that for shielding bare neutron sources, the most important neutron reactions are inelastic and elastic scattering since these are the only reactions for which appreciable probability exists.

Energy Loss on Scattering

Inelastic scattering occurs when a neutron reacts with a target nucleus to form a compound nucleus which subsequently re-emits the neutron with a lower energy than it originally possessed. This leaves the original target nucleus in an excited state (a specific excitation level) which subsequently decays by γ -ray emission. As previously discussed, the excitation level spacing varies with atomic mass and with amount of excitation. Levels are far apart (~ 1 Mev) for the first few levels for the light elements, become closer (0.1 to 0.2 Mev) for the mid-range elements and (50-100 kev) for the heavy elements. Levels become increasingly close and begin to overlap with increasing excitation energy (~ 10 Mev). Since inelastic scattering begins at the threshold of the first excitation level and the level is significantly different in energy from the next few levels, inelastic

scattering of fission neutrons (neutrons of energy 1 to 2 Mev) will yield specific amounts of energy loss. As neutron energy increases the compound nucleus will preferably emit low energy neutrons because of the many high energy excitation levels at which the resulting nucleus may exist. This effect is offset by increased possibility with neutron energy for the neutron to leave the nucleus. This results in emission of neutrons with an energy distribution peaked between no energy loss and complete energy loss.

The problem has been treated with the statistical model of Weisskopf^{10,11} which describes the energy distribution of inelastically scattered neutrons by

$$N(E_2) = E_2 e^{-E_2/\theta} \quad (4)$$

where

$N(E_2)$ = probability the emitted particle will be in a unit interval at E_2

θ = temperature of the residual nucleus

The temperature of the residual nucleus has been given by¹²

$$\theta = \left(\frac{2}{B}\right) \sqrt{E_1} \quad (5)$$

where

E_1 = initial neutron energy

$B \approx 0.62 \sqrt{A}$

A = mass number of the scattering nucleus

The most probable energy of the inelastically scattered neutron is at $E_2 = \theta$, the average energy being somewhat less. The statistical theory

generally agrees well with experiments for high initial energy neutrons (10-14 Mev); however, does not well represent experimental spectra for low initial neutron energy, ~ 1 Mev, because this corresponds to leaving the nucleus in one of the first excitation levels where the statistical approximation is invalid. Table II indicates typical values for the most probable neutron emission energy for several nuclei and several initial neutron energies obtained from statistical theory, Equations 4 and 5.

TABLE II
MOST PROBABLE INELASTIC SCATTERING EMISSION ENERGY
FOR SEVERAL ELEMENTS AND SEVERAL NEUTRON ENERGIES

<u>Element</u>	<u>Initial Neutron Energy, E_1</u>	<u>Most Probable Emission Energy, E_2</u>
Na	1	.67
	5	1.50
	10	2.13
Fe	1	.43
	5	.96
	10	1.37
Zr	1	.34
	5	.75
	10	1.07
U	1	.21
	5	.46
	10	.66

Examination of Table II indicates the average functional energy loss for inelastic scattering may be taken to be about 0.8 for heavy isotopes and intermediate isotopes with high initial neutron energy ($\gtrsim 5$ Mev). For light isotopes and for intermediate isotopes at predominant fission neutron energies the average fractional energy loss is less, ~ 0.3 to 0.4 .

For analysis where inelastic scattering is taken into account, experimental data is usually used for light and intermediate nuclei at low initial neutron energies. Examination of existing inelastic scattering data¹³ for these nuclei also indicates that the average fractional energy loss of 0.3 to 0.4 is reasonable.

Energetic neutrons also have significant probability for elastic scattering reactions and these too cause energy loss of the neutron; kinetic energy of the neutron transferring to the target nucleus. For elastic scattering events with neutrons having energy in the Mev range, sufficient energy may be transferred to the nucleus to ionize it in addition to importing kinetic energy. The nucleus kinetic energy and ionization energy will dissipate to other nuclei through atomic collision and end up as molecular thermal energy. One can calculate the energy loss on elastic scattering related to the incident neutron energy. This may be given as a ratio of neutron energy before scattering to neutron energy after scattering:¹⁴

$$\frac{E_2}{E_1} = \frac{A^2 + 2A \cos \theta + 1}{(A + 1)^2} \quad (6)$$

where

E_2 = neutron energy after scattering

E_1 = initial neutron energy

A = mass number of the target nucleus

θ = the angle between the path of the incident neutron and scattered neutron, the angle of scatter.

For a head-on collision, the collision of maximum energy loss, it can be shown that

$$\frac{E_2}{E_1} = \left(\frac{A - 1}{A + 1} \right)^2 \quad (7)$$
$$\approx 1 - \frac{4}{A} \text{ for } A \geq 50$$

The above expressions indicate that collision with very light nuclei could cause very high energy loss and for heavy nuclei only a very small amount of energy is removed. In the case of collision with hydrogen all energy will be removed providing the collision is perfectly "head on". Since the elastic cross section is large for Mev neutrons and is the only significant cross section for mid energy range neutrons it is seen that a free neutron entering a shield will undergo perhaps an inelastic scattering reaction, lose much of its Mev energy, then undergo numerous elastic scattering reactions. The net effect of the elastic scattering collisions will be to dissipate the neutron kinetic energy.

The average logarithmic energy decrement (the logarithm of the ratio of energy after scattering to energy before scattering) per elastic scattering can be calculated and is given by

$$\xi = \ln \frac{E_1}{E_2} = 1 + \frac{(A - 1)^2}{2A} \ln \frac{A - 1}{A + 1} \quad (8)$$
$$\approx \frac{2}{A + 2/3}, \quad A > 2$$

for isotropic (spherically symmetric) scattering in the center of mass system.

Since ξ is dependent on the nucleus mass number and is independent of neutron energy this says that in collisions with specific scattering nuclei, the neutron loses with each collision the same fraction of initial energy. The average fractional neutron energy loss per elastic collision is a useful quantity for illustrative purposes and is given by the approximation

$$\bar{\xi} = \frac{2A}{(A + 1)^2}, \quad A > 2 \quad (9)$$

for isotropic scattering in the center of mass system.

For neutron energies greater than ~ 1 Mev and $A \gtrsim 5$ scattering tends toward anisotropy and Equation 9 overestimates average fractional energy loss per collision. For the case of hydrogen, the scattering process is isotropic up to 10 Mev. This scattering property, in addition to other properties, gives hydrogen unique distinction; the number of collisions with hydrogen to degrade neutron energy is least, and in the intermediate energy range the scattering cross section rises rapidly as neutron energy falls. This has the practical consequence that in homogeneous shields buildup of low-energy neutrons in equilibrium with fast neutrons is small.

The average number of collisions with nuclei of a particular moderator required to decrease the energy of a fission neutron of energy 2 Mev to the thermal energy of 0.025 ev may be obtained by dividing the total logarithmic energy decrement of 2 Mev to 0.025 ev by the average logarithmic energy decrement per collision.

$$\text{Average number of collisions to thermalize (2 Mev to 0.025 ev)} = \frac{\ln \frac{2 \times 10^6}{0.025}}{\bar{\xi}} = \frac{18.2}{\bar{\xi}} \quad (10)$$

Table III indicates the scattering properties of a number of elements.

TABLE III*
SCATTERING PROPERTIES OF NUCLEI

	<u>Mass No.</u>	<u>ξ</u>	<u>Collisions to Thermalize</u>
Hydrogen	1	1.000	18
Deuterium	2	0.725	25
Helium	4	0.425	43
Lithium	7	0.268	67
Beryllium	9	0.209	86
Carbon	12	0.158	114
Oxygen	16	0.120	150
Uranium	238	0.00838	2172

Since ξ gets smaller almost directly proportional to the nucleus mass number, the number of neutron collisions with a given nuclei increase with increasing mass number. This phenomena is readily observable from Table III. ξ then is a partial indication of a material's ability to slow neutrons.

To evaluate neutron interactions in physical materials, the microscopic cross section is extended to the macroscopic cross section. A macroscopic cross section is the product of the nuclear density and the microscopic cross section:

$$\Sigma_i = N\sigma_i \tag{11}$$

* From Reference 4, Page 145.

where

Σ_i = macroscopic cross section for the reaction of type i, cm^{-1}

N = number of nuclei of the reacting species per cm^3

σ_i = the macroscopic cross section, cm^2/atom or "barns/atom" for the reaction of type i.

The numbers of nuclei per cm^3 for common materials range from 10^{22} to 10^{23} thus macroscopic cross sections commonly run from 10^{-5} to 10^2 cm^{-1} . That is, the probability of a neutron undergoing a neutron-nucleus reaction of the type i for every centimeter of flight path through a material commonly runs from 10^{-5} to 10^2 .

This indication of a material's ability to slow neutrons is given by the neutron path length between elastic scattering collisions. This is a function of the macroscopic scattering cross section Σ_s : specifically the scattering mean free path, λ_s , the average distance a neutron travels between scattering collisions is:

$$\lambda_s = \frac{1}{\Sigma_s} \text{ cm} \quad (12)$$

The average distance a neutron travels between being born and being thermalized is given by

$$\begin{array}{l} \text{Average path length from} \\ \text{Production to thermal} \end{array} = \frac{\lambda_s (18.2)}{\xi}, \quad (13)$$

providing Σ_s is constant with respect to energy (a situation which in reality never occurs). A common term used in presentation of this theory is the slowing down power, $\xi \Sigma_s$, or the reciprocal of the average path length divided by 18.2. If an average scattering cross section for an

element is picked for neutron energies between fission and thermal the slowing down power may be calculated and the average neutron path length from fission to thermal may be calculated. Table IV indicates these parameters for some common materials:

TABLE IV
SLOWING DOWN PROPERTIES OF MODERATORS

<u>Moderator</u>	<u>Slowing Down Power*</u>	<u>Average Path Length From Fission to Thermal</u>
Water	1.53 cm ⁻¹	11.8 cm
Heavy Water	0.170	107.0 cm
Helium**	1.6 x 10 ⁻⁵	1.14 x 10 ⁶ cm
Beryllium	0.176	103.5 cm
Carbon	0.064	284.0 cm

* Reference 14, Page 146.

** At atmospheric pressure and temperature.

Examination of the probabilities (Table I) for neutron nucleus reactions for neutrons possessing kinetic energy less than 1 ev indicates reasonable probability for neutron capture as compared to neutron elastic scattering. However, for many materials this absorption cross section remains small as compared to elastic scattering for neutron energies the order of 0.025 ev, the average thermal energy of a room temperature neutron. Thus, one can see that a neutron will lose energy by elastic collision with nuclei until it reaches thermal equilibrium with the surroundings (each collision on the average, produces no energy exchange). Since all materials have a finite absorption cross section, once a neutron reaches thermal energy it is a matter of statistical chance until it undergoes absorption.

Neutron Attenuation in Concretes

In years 1955 to the present efforts at Hanford,^{15,16,17,18,19,20} Brookhaven, Oak Ridge,^{21,22,23} and AB Atomenergi, Sweden²⁴ have yielded considerable experimental neutron attenuation data for various concretes. A discussion of some of these experimental results with a comparison of what one would expect from a theoretical consideration will point out the desirable constituents of concretes for maximum neutron attenuation.

Figures II a, b, c, and d indicate, respectively, neutron distribution in ordinary concrete (as cured, $145\frac{\text{lb}}{\text{ft}^3}$), magnetite concrete (as cured, $205\frac{\text{lb}}{\text{ft}^3}$), limonite-steel concrete (as cured, $264\frac{\text{lb}}{\text{ft}^3}$), and limonite-steel concrete with the water baked out at various temperatures (100 C, 175 C, and 320 C). These data were obtained at Hanford^{15,16,18} and were selected because of the typical range in density of the shielding concretes.

Figure II a provides thermal neutron flux, epithermal neutron flux, and fast neutron flux through the as-cured ordinary concrete. Figures II b, c, and d present similar data for the other concretes. The fast flux is the total neutron flux for neutron energies greater than 1 Mev. The epithermal flux is the flux per unit lethargy ^(a logarithmic energy variable; see the energy variable on Fig. III) at 4.9 ev (the gold capture resonance) and can be assumed a constant between 1 ev and 0.1 Mev. The thermal flux is the total neutron flux below 0.4 ev (the cadmium cut-off) with an assumed average neutron energy of 0.025 ev. All fluxes are normalized to the same reactor power level and give a relative picture of the fast to thermal ratio.

From the nature of neutron attenuation, the process of slowing from fission energies down to near thermal then capture, a few simple correlations can be made to illustrate the neutron attenuation effectiveness of the concrete. Table V gives the composition of the concretes of Figure II in grams per cubic centimeter of each constituent element. Also given is the atomic density (e.g. the number of atoms/cm³) of each constituent element. The atomic density is of importance in evaluating the macroscopic cross section (Equation 11).

Light nuclei have high atomic densities in relation to the weight fraction of that element in a material, thus this contributes to neutron attenuation since the macroscopic cross section is directly proportional to atomic density. Light elements also are superior scatters in that they provide greater average energy degradation per scattering collision; however, they are poorer inelastic scatterers because of low inelastic cross section.

~~Thus, if we look at the macroscopic cross sections for concretes we can compare properties~~

Looking at the atomic densities of various elements in concrete, keeping in mind the general rules for cross section behavior and energy loss on scattering, attenuation in concretes can be discussed. Examining Table V it is seen that the atomic density of hydrogen and oxygen occur in the same order of magnitude in all of the concretes except for the heated ferro-phosphorous which has lost about 25% of its oxygen and 50 to 75% of the hydrogen. Intermediate elements between hydrogen and iron remain in the same order of importance for all concretes with exception of silicon in ordinary concrete for which it is a populous constituent. Iron shows the

FIGURE II

Fast, Epithermal and Thermal Neutron Flux Through
 (a) Ordinary (145 #/ft³), (b) Magnetite (205 #/ft³), (c) Limonite-Steel (264 #/ft³ as cured),
 and (d) Limonite-Steel Concrete Baked Out at Various Temperatures

marked change in density between concretes since this is purposely added to increase the concrete density.

Since a better measure of attenuation in concrete is given by the macroscopic cross section, Table VI presents macroscopic cross sections for the concretes of Table V. Given in Table VI are: the average fractional energy loss per elastic scattering collision for each element (calculated from Equation 9), microscopic cross section for the reactions inelastic scattering at 2 Mev (important for fast neutron slowing), elastic scattering at 2 Mev (for comparison to inelastic scattering), elastic scattering at 1 kev (the reaction important for neutron slowing), the capture cross section at thermal energy (energy at which most captures occur), the scattering cross section at thermal energy (for comparison to the capture cross section), and macroscopic cross sections for the same reactions for all constituents of the three concretes along with totals. These total reaction cross sections yield considerable insight for the concretes (except for the elastic scattering cross section, since the lighter constituents have more importance because of the higher fractional energy loss per collision). Examining Table VI several items should be noticed:

Microscopic Cross Sections

1. Fast Neutrons:

Silicon and iron have potentially good inelastic scattering cross sections for fast neutron attenuation. Other elements generally are not sufficiently populous to be important inelastic scatterers.

Hydrogen has a very high elastic scattering cross section and coupled with high atomic density and high energy loss per collision it is an important fast neutron attenuator. Oxygen is also significant for the same reason as hydrogen.

TABLE V

COMPOSITION (g/cc) AND ATOMIC DENSITIES (ATOMS/cc) OF ORDINARY (145#/ft³),
MAGNETITE (205#/ft³) AND LIMONITE-STEEL (264#/ft³) CONCRETES

Element	Ordinary Concrete		Magnetite Concrete	
	Composition (As Cured)	Atomic Density	Composition (As Cured)	Atomic Density
H	0.015	8.960 x 10 ²¹	0.015	8.960 x 10 ²¹
O	1.057	39.800	1.279	48.200
Na	0.041	1.060		
Mg	0.085	2.120	0.023	.571
Al	0.137	3.06	0.161	3.610
Si	0.487	10.600	0.129	2.770
P	0.002	.038	0.001	.019
S	0.002	.037	0.004	.075
K	0.015	.239		
Ca	0.295	4.350	0.220	3.250
Ti	0.011	.139	0.095	1.190
Mn	0.003	.033	0.002	.022
Fe	0.178	1.920	1.460	15.750
Total	2.328	72.356 x 10 ²¹	3.389	84.417 x 10 ²¹

Limonite-Steel Concretes

Element	Composition (As Cured)	Atomic Density	Composition (Baked 100C)	Atomic Density	Composition (Baked 175C)	Atomic Density
H	0.028	16.750 x 10 ²¹	0.018	10.78 x 10 ²¹	0.007	4.19 x 10 ²¹
O	0.806	30.250	0.726	27.250	0.638	24.04
Na						
Mg	0.007	.187	3.403	39.088	3.403	39.088
Al	0.032	.717				
Si	0.078	1.770				
P	0.001	.019				
S						
K						
Ca	0.250	3.690				
Ti						
Mn	0.005	.055				
Fe	3.030	32.650				
Total	4.237	86.088 x 10 ²¹	4.147	77.12 x 10 ²¹	4.048	67.32 x 10 ²¹

Element	Composition (Baked 320C)	Atomic Density
H	0.008	4.44 x 10 ²¹
O	0.646	24.24
Others	3.403	39.088
Total	4.057	67.77 x 10 ²¹

TABLE VI

MICROSCOPIC AND MACROSCOPIC REACTION CROSS SECTION (SELECTED NEUTRON ENERGIES)
FOR ELEMENTS IN ORDINARY (145#/ft³), MAGNETITE (205#/ft³) AND LIMONITE-STEEL (264#/ft³) CONCRETES

Element	f	Microscopic Cross Sections (Type Indicated, Barns)					Macroscopic Cross Sections Ordinary Concrete cm ⁻¹				
		σ_{in}	σ_s	σ_s	σ_s	σ_a	Σ_{in}	Σ_s	Σ_s	Σ_s	Σ_a
		2 Mev	2 Mev	1 kev	Thermal	Thermal*	2 Mev	2 Mev	1 kev	Thermal	Thermal
H	0.632	-	2.93	19.50	51.60	0.383	-	.0263	.1747	.4623	.00343
O	0.111	-	1.60	3.70	4.20	0.00024	-	.0636	.1472	.1671	-
Na	0.080	0.510	2.62	3.72	3.98	0.536	.00054	.0028	.0039	.0042	.00057
Mg	0.076	0.137	2.55	3.50	2.99	0.056	.00029	.0054	.0074	.0062	.00012
Al	0.069	0.300	2.70	1.40	1.40	0.297	.00091	.0082	.0042	.0042	.00091
Si	0.066	0.116	3.20	1.75	1.70	0.114	.00122	.0339	.0186	.0180	.00121
P	0.061	0.400	2.55	3.40	4.99	0.231	.00001	.0001	.0001	.0001	.00001
S	0.059	0.002	2.11	1.16	1.10	0.013	-	.0001	-	-	-
K	0.048	-	2.15	1.50	1.50	2.18	-	.0005	.0004	.0004	.00052
Ca	0.048	0.40	3.42	3.00	3.00	0.575	.00174	.0149	.0131	.0131	.00249
Ti	0.040	0.319	3.03	4.06	3.62	7.37	.00004	.0004	.0006	.0005	.00101
Mn	0.035	1.31	1.81	49.97**	2.30	15.35	.00004	.0001	.0016	.0001	.00051
Fe	0.035	0.88	1.79	8.97	12.50	2.69	.00168	.0034	.0174	.0242	.00522
Total							.00647	.1597	.3892	.7004	.01600

	Macroscopic Cross Sections Magnetite Concrete, cm ⁻¹					Macroscopic Cross Sections Limonite-Steel Concrete, cm ⁻¹				
H	-	.0263	.1747	.4623	.00343	-	.0491	.3266	.8643	.00641
O	-	.0771	.1783	.2024	-	-	.0484	.1119	.1271	-
Na	-	-	-	-	-	-	-	-	-	-
Mg	.00007	.0014	.0019	.0017	.00003	.00003	.0005	.0007	.0006	.00001
Al	.00108	.0097	.0051	.0051	.00107	.00022	.0019	.0010	.0010	.00021
Si	.00032	.0089	.0048	.0047	.00032	.00021	.0057	.0031	.0030	.00020
P	.00001	-	.0001	.0001	-	.00001	-	.0001	.0001	-
S	-	.0001	.0001	.0001	-	-	-	-	-	-
K	-	-	-	-	-	-	-	-	-	-
Ca	.00130	.0111	.0098	.0098	.00186	.00148	.0126	.0111	.0111	.00211
Ti	.00038	.0036	.0048	.0043	.00877	-	-	-	-	-
Mn	.00003	-	.0011	.0001	.00034	.00007	.0001	.0027	.0001	.00084
Fe	.01386	.0282	.1413	.1969	.04237	.02873	.0584	.2929	.4081	.08783
Total	.01705	.1164	.5220	.8875	.05819	.03075	.1767	.7501	1.4154	.09761

* Thermal cross section averaged over unit flux,
0.001 < E < 0.0759 ev (MAC 31st Group⁹).

** Cross section averaged over resonances.

σ_a = Absorption Cross Section

σ_{in} = Inelastic Scattering Cross Section

σ_s = Elastic Scattering Cross Section

TABLE VI, CONT'D

Macroscopic Cross Sections Limonite and Steel Concrete (Baked Out at 100 C)						Macroscopic Cross Sections Limonite and Steel Concrete (Baked Out at 175 C)				
Element	Σ_{in} 2 Mev	Σ_{el} 2 Mev	Σ_{el} 1 kev	Σ_{el} Thermal	Σ_a Thermal	Σ_{in} 2 Mev	Σ_{el} 2 Mev	Σ_{el} 1 kev	Σ_{el} Thermal	Σ_a Thermal
H	-	.0315	.2132	.5562	.00412	-	.0122	.0817	.2162	.00160
C	-	.0436	.1038	.1145	.00001	-	.0384	.0889	.1009	-
Na	}									
Mg										
Al										
Si										
F										
S										
K										
Ca										
Ti										
Mn										
Fe										
Total	.03075	.1543	.6226	1.0947	.09415	.03075	.1298	.4822	.7411	.09162

Macroscopic Cross Sections
Limonite and Steel Concrete (Baked Out at 320 C)

H	-	.0130	.0855	.2291	.00170
O	-	.0388	.0887	.1018	-
Others	.03075	.0792	.3116	.4240	.09002
Total	.03075	.1310	.4858	.7549	.09172

2. Epithermal Neutrons:

At intermediate energy hydrogen and oxygen have the only potential importance with perhaps some help from iron (in high density concrete).

3. Thermal Neutrons:

At thermal energies one expects considerable thermal neutron population because of the higher probability of scattering over capture. The heavier elements have scattering probabilities similar to the light elements but higher absorption cross sections.

Macroscopic Cross Sections

1. Comparing total reaction cross sections for the various concretes it is seen that the elastic scattering cross sections change in the order of 10 to 50% from ordinary to limonite-steel concretes. These changes in elastic cross section at 2 Mev and 1 kev occur by addition of heavy elements and have almost no additional effect in slowing down since the amount of hydrogen remains relatively constant. These heavy element scattering reactions do have some beneficial effect on attenuation by increasing the flight path.

Marked change occurs in the inelastic scattering cross section and absorption cross sections up by about a factor of 6 for limonite-steel compared to ordinary concrete.

2. The constituents hydrogen, oxygen, silicon, and iron produce the largest neutron attenuation effect in concrete except for the specialized concretes containing aggregate high in a specific element such as phosphorus in Ferro-phosphorus or barium in Barytes. Other elements

have usually less than 10% cumulative effect. These effects are readily observable by comparing the element macroscopic cross sections with the total concrete macroscopic cross sections.

3. The importance of hydrogen for fast neutron slowing in concretes is apparent since $f\Sigma_{sH}$ (2 Mev) for hydrogen for a concrete is similar in magnitude to $\sum f\Sigma_s + 0.4 \sum \Sigma_{in}$ (2 Mev) for all constituents in that concrete. For inelastic scattering at 2 Mev, 0.4 is taken as an average fractional energy loss typical for most iron aggregate concrete. For typical reactor shielding applications concrete shields are usually thick enough that a phenomena known as spectral hardening occurs (discussed in the next section) and the most penetrating neutrons are in the energy range 6 to 8 Mev. For 8 Mev the hydrogen cross section is about 0.4 of the value at 2 Mev. Inelastic cross sections increase by about a factor of 2.5 and the average fractional energy loss per inelastic scatter increases to about 0.8. Table VII provides values of $f\Sigma_{sH}$ and $\sum f\Sigma_s + f_{in}\sum \Sigma_{in}$ for the concretes of Figure II for 2 and 8 Mev neutrons.

TABLE VII

RELATIVE IMPORTANCE OF HYDROGEN IN CONCRETE FOR FAST NEUTRON SLOWING

Concrete	$f\Sigma_{sH}$ (2 Mev)	$\sum f\Sigma_s + .4 \sum \Sigma_{in}$ (2 Mev)	$\sum f\Sigma_{s+in}$ (2 Mev)	$f\Sigma_{sH}$ (8 Mev)	$\sum f\Sigma_s + .8 \sum \Sigma_{in}$ (8 Mev)	$\sum f\Sigma_{s+in}$ (8 Mev)
Ordinary (145#/ft ³)	.0166	.0139	.0305	.0066	.0217	.0283
Magnetite (205#/ft ³)	.0166	.0185	.0351	.0066	.0326	.0392
Limonite-Steel (264#/ft ³)	.0310	.0208	.0518	.0124	.0468	.0592
Limonite-Steel (100 C)	.0199	.0200	.0399	.0080	.0464	.0544
Limonite-Steel (175 C)	.0078	.0196	.0274	.0031	.0461	.0492
Limonite-Steel (320 C)	.0088	.0197	.0285	.0035	.0461	.0496

Examining Table VII it is seen that hydrogen provides roughly half of the 2 Mev neutron slowing power and about 1/3 to 1/4 of the slowing power at 8 Mev.

4. The importance of hydrogen in concrete for slowing epithermal neutrons is apparent since $f\Sigma_{sH}(1 \text{ kev}) \gg \sum f\Sigma_s(1 \text{ kev})$ for all as-cured concretes. This property of hydrogen in concrete is its most important. Examining Figure II d, it is seen that for heated limonite-steel concrete where a large fraction of the hydrogen has been removed (see Table V) the slow neutron flux increases rapidly with loss of hydrogen. The effect results in a large fraction of low and intermediate energy neutrons escaping through the concrete and can result in the biological dose at the exterior of the shield being controlled by leakage of the intermediate energy neutrons. Hydrogen content of as-cured concretes normally is sufficient to maintain the intermediate neutron leakage dose at a small fraction of the fast neutron leakage dose. Thus, up to a point hydrogen density is very important. Efforts to purposely increase hydrogen density beyond the normal density in concretes will somewhat aid fast neutron attenuation and will aid suppression of capture gamma radiation (by favoring quick slowing and capture of intermediate neutrons), but may not be justified economically and may not be consistent with gamma radiation attenuation requirements.
5. Thermal neutrons are predominantly captured by the iron in the heavy aggregate concretes.

The Removal Cross Section

The previous discussion on the macroscopic cross sections of concrete is instructive in that it gives good insight into the relative importance of the various materials in concretes. However, it provides no estimate of absolute attenuation effectiveness; only a relative measure of neutron slowing power. One important item concerning attenuation not emphasized in the previous discussion is that elastic scattering of fast and epithermal neutrons by elements heavier than hydrogen does have attenuating effect even though they produce almost insignificant energy loss. The elastic scattering cross sections for elements other than hydrogen are reasonably large (see Table VI). The attenuating effect exists because elastic scattering tends to deflect neutrons from the forward component of the beam. That is, elastic scatterings have the effect of greatly increasing the flight path of a neutron penetrating the shield (see the discussion of Equation 13 and Table IV). A cross section which takes the elastic scattering contribution into account is called the removal cross section. This theory developed as a result of experiments conducted in reactor shielding experimental facilities at Brookhaven and Oak Ridge National Laboratories. These shielding facilities were constructed by placing a large tank of water adjacent to a reactor. water served the purpose of allowing easy placement of shield samples detectors and providing local biological shielding. A fission plate (a plate of U^{235}) at the water's edge driven by thermal neutrons from the reactor provided a disk source of isotropic fission neutrons entering the water. It was observed that the neutron dose in water at various distances from the source plate could be described by integrating the removal kernel*

* The Albert and Welton Kernel²⁵ modified by Casper²⁶

$$G(r) = \frac{2.78 \times 10^{-5}}{4\pi r^2} e^{-[0.422 r^{0.698}]} e^{-0.0308r} \text{ rads/hr} \quad (14)$$

over the source distribution. The above kernel provides the dose in rads/hr at a distance r cm from a point isotropic source of fission neutrons yielding 1 neutron/sec. It was also found that if a slab of ^{Solid} material ~~other than~~ ~~water~~ was placed in the water between the fission source and the detector, providing a relatively large thickness of water still remained between slab and detector, the change in dose at the detector could be described by the attenuation provided by the remaining water (predicted by Equation 14) and the attenuation provided by the slab of material given as

$$e^{-\Sigma_T x} \quad (15)$$

(narrow beam geometry) where x is the thickness of the slab, and Σ_T is the effective removal cross section. Thus, Equation 14 modified by Equation 15 yields

$$G(r, x) = \frac{2.78 \times 10^{-5}}{4\pi r^2} e^{-[0.422(r-x)^{0.698}]} e^{-0.0308(r-x)} e^{-\Sigma_T x} \text{ rad/hr} \quad (16)$$

which gives the neutron dose from a point isotropic fission source of 1 neutron per second at r cm from the source through a slab of thickness x cm immersed in the water medium. It was found that from 45 to 60 cm of water between the slab of material and the detector was required to insure the validity of Equation 16.

The exponential nature of the removal cross section implies that the neutron collision process in the slab of material is one of absorption. It was previously shown that for neutrons of fission energies, the probability of absorption is almost negligible yet this is contrary to the phenomena observed experimentally. The experimental observation is explained by

the fact that neutron attenuation in water (hydrogen) is controlled by the penetration of high energy neutrons. Due to the high energy loss per collision and high collision cross section for intermediate energy neutrons, these neutrons are brought into equilibrium (after about 45 cm of penetration) with neutrons of fission energies undergoing their first collisions. The ratio of low energy neutrons to high energy neutrons incident on water has little effect on the ratio of low energy to high energy neutrons after about 45 to 60 cm of distance through water.

A slab of non-hydrogeneous material (for example, iron) will inelastically scatter fission energy neutrons into the kev energy range where these neutrons will become trapped because of insufficient mechanism to further energy loss (compare $f\Sigma_s$ (1 kev) to $0.4 \Sigma_{in}$ (2 Mev) for each material other than hydrogen and oxygen). This results in a buildup of intermediate energy neutrons which penetrate the non-hydrogeneous slab and substantially increases the dose that would be calculated using Equation 16. The 45 to 60 cm of required water following the non-hydrogeneous slab has the effect of filtering out the high flux of penetrating intermediate energy neutrons.

The removal cross section is therefore a probability that a fission energy neutron undergoes an effective first collision which removes it from the penetrating beam of uncollided neutrons. This will have the net effect of removing it into the intermediate energy group. It has been observed that an inelastic scatter practically always constitutes a removal and that an elastic scattering which significantly changes the direction of a penetrating neutron constitutes a removal.

The removal cross section has been applied with good results for calculating the penetration of fast neutrons in non-hydrogeneous (and hydrogeneous) materials since this is consistent with theory. Removal cross sections are obtained experimentally (effective removal cross sections) in the shielding facilities or are defined from microscopic cross section data. The experimentally determined fast neutron attenuation curves of Figures II a, b, c, and d were used to measure effective removal cross sections for the specified concretes. The removal cross sections for these concretes are given by the reciprocal of the fast neutron relaxation length, λ , (the thickness of material required to reduce the fast neutron flux by a factor of e) corrected by a geometry factor C.

$$\Sigma_r = C \left(\frac{1}{\lambda} \right) \quad (17)$$

The geometry factor corrects for the assumption the source is an infinite plane isotropic source rather than a collimated narrow beam source.

Figures II a, b, c, and d give the measured value of $1/\lambda$ and the value of Σ_r for the particular concretes. The value of Σ_r for these concretes may also be calculated from available values²⁷ of the effective removal cross sections for the concrete constituents using Equation 11. Agreement is generally good.

There are several ways of defining microscopic removal cross section σ_r from basic data. A simple approximation is*

$$\sigma_r = \frac{2}{3} \sigma_T \quad (18)$$

*Reference 3, Page 83.

where

σ_T = the average total cross section between 6 and 8 Mev

An energy dependent removal cross section has been defined^{5,9} as

$$\sigma_r(E) = \underset{\text{sub}}{\sigma_{in}}(E) + \underset{\text{sub}}{\sigma_a}(E) + (1 - \overline{\cos\theta}) \underset{\text{sub}}{\sigma_s}(E) \quad (19)$$

where

$\sigma_{in}(E)$ = microscopic inelastic scattering cross section at energy E

$\sigma_a(E)$ = microscopic absorption cross section at energy E

$\sigma_s(E)$ = microscopic elastic scattering cross section at energy E

$\overline{\cos\theta}$ = average cosine of the scattering angle in the laboratory system at energy E

This is the transport cross section of reactor theory which determines the net loss of forward momentum of the incident beam.

A more recent definition of this cross section has been given²⁴ as

$$\sigma_r(E) = \sigma_T(E) - 2\pi \int_{-1}^1 w(\mu) \sigma_a(E, \mu) d\mu \quad (20)$$

where

$w(\mu)$ = a weight function

$$w(\mu) = \begin{cases} 0 & , \mu < \mu_0 \\ 1 & , \mu \geq \mu_0 \end{cases}$$

μ_0 = a boundary cosine in the laboratory system defining the cone into which a neutron is scattered yet is not considered lost from the incident beam. (This parameter may be adjusted by experiment.)

It is interesting to compare the measured effective removal cross sections with those calculated from Equation 19 in the energy range 6 to 8 Mev.

These are given in Table VIII.

TABLE VIII
MICROSCOPIC REMOVAL CROSS SECTIONS FOR SOME ELEMENTS

<u>Element</u>	<u>Measured (AECD-3978)* Barns</u>	<u>Calculated** From Equation 19, Barns (Averaged 6-8 Mev)</u>
Li	1.01 ± .04	1.04
Be	1.07 ± .06	1.44
B	.97 ± .10	.76
C	.81 ± .05	.84
O	.99 ± .10	.75
F	1.29 ± .06	1.02
Al	1.31 ± .05	1.20
Cl	1.2 ± .8	1.40
Fe	1.98 ± .08	2.07
Ni	1.89 ± .10	2.14
Cu	2.04 ± .11	2.13
W	2.51 ± .55	3.03
Pb	3.53 ± .30	3.23
Bi	3.49 ± .35	3.33
U	3.6 ± .4	3.61

* Reference 27

** Reference 9

Six to eight Mev is considered to be the energy range of interest since it has been found that these neutrons are most likely to be the penetrating component. This is the phenomenon known as hardening. Only a small fraction of neutrons by fission are above 6 Mev; ~0.02%. However, neutrons produced by fission of lower energy tend to be removed sooner than the higher energy neutrons because of the generally higher interaction probability (compare the 2, 5 and 10 Mev reaction cross sections of Table I). Figure III shows neutron spectra calculated with the removal-diffusion theory computer program MAC²⁹ for a hydrous iron ore concrete (~200 #/ft³), shield of ^{one of the} a Hanford production reactor^{III}. Figure ^{III} a is the spectra ^{20"} 5.7" into the concrete from the inside shield edge. Figure ^{III} b is the spectra ^{30"} further into the shield, 37" from the inner

FIGURE III

edge. Hardening of the fast neutron flux is clearly visible. The epithermal flux is about constant per unit ^e_l^a energy and the thermal flux in relation to the fast flux is much lower at deeper penetrations.

Calculation of Neutron Attenuation

It has been shown that there is simple theory available to estimate attenuation of fast neutron flux through concrete. It has also been shown that simple removal theory handles the neutron dose reduction calculation of non-hydrogenous material followed by a thick slab of water. The problem of calculating dose behind a concrete shield or some other non-hydrogenous medium cannot be handled by removal theory alone.

There are a number of approaches to this problem. Removal theory has been coupled to diffusion theory where the removal collision density provides the source neutrons for the multigroup diffusion equation. The low energy neutron distributions calculated by diffusion theory may be used to evaluate capture rates (and subsequent gamma ray production rate) in the shield regions. Rigorous solution of the neutron penetration problem requires the solution of the neutron balance (the neutron balance over a differential shield volume element) equation which is commonly called the Boltzmann equation and is written as:

$$\frac{1}{v} \frac{\partial \phi(\underline{r}, \underline{E}, \underline{\Omega}, t)}{\partial t} + \Sigma_t(\underline{E})\phi(\underline{r}, \underline{E}, \underline{\Omega}, t) + \underline{\Omega} \cdot \nabla \phi(\underline{r}, \underline{E}, \underline{\Omega}, t) = S(\underline{r}, \underline{E}, \underline{\Omega}, t) + \int_{\underline{E}'}^{\infty} \int_{\underline{\Omega}'}^{\infty} \Sigma_s(\underline{E}')\phi(\underline{r}, \underline{E}', \underline{\Omega}', t)F(\underline{E}', \underline{\Omega}', \underline{E}, \underline{\Omega})d\underline{E}'d\underline{\Omega}' \quad (21)$$

$$\phi(\underline{r}, \underline{E}, \underline{\Omega}, t) = n(\underline{r}, \underline{E}, \underline{\Omega}, t)v \quad (22)$$

The terms in this equation are defined as:

$n(\underline{r}, \underline{E}, \underline{\Omega}, t) d^3r dE d\Omega$ = the expected number of neutrons in element of volume d^3r about $\underline{r}$, in energy increment dE about $\underline{E}$ and travelling in direction $d\Omega$ about $\underline{\Omega}$ at time t .

v = the neutron speed corresponding to energy E .

$\Sigma_{\alpha}(E)$ = the macroscopic cross section for a collision of type α

$\alpha = s$: scatter

$\alpha = a$: absorption

$\alpha = t = s + a$: total

$S(\underline{r}, \underline{E}, \underline{\Omega}, t) d^3r dE d\Omega$ = the expected number of neutrons born in element of volume d^3r about $\underline{r}$ in energy increment dE about $\underline{E}$ in direction $d\Omega$ about $\underline{\Omega}$ at time t .

$F(\underline{E}', \underline{\Omega}'; \underline{E}, \underline{\Omega}) dE d\Omega$ = the probability that a neutron travelling in direction $\underline{\Omega}'$ with energy E' , upon scattering have final energy in dE about E and direction motion in $d\Omega$ about Ω .

This is an integral-differential equation which can be solved rigorously by numerical integration (NIOBE)²⁹ for neutron shields in fairly complex design. It can be solved rigorously by combination of analytical techniques and numerical integration (Moments Method)² for idealized geometry and boundary conditions such as a plane source, infinite shield or point source-infinite shield. In all cases, rigorous solution of the Boltzmann equation is a difficult problem and normally only can be handled on a high speed electronic computer.

There are numerous other techniques for calculating the neutron attenuation properties of shields and all of these methods are approximate solutions of the Boltzman equations or solutions of a reduced Boltzman equation obtained by applying simplifying assumptions. Common solutions of these type are: removal-diffusion theory,^{5,28,30} Monte-Carlo, age-diffusion theory, and various ~~asymptotic~~^p approaches.

CONCRETE AS A RADIATION SHIELD

It has been shown that concretes are satisfactory neutron attenuators since the heavy elements provide reasonable fast neutron attenuation (by inelastic scattering) and good thermal neutron capture. The light elements (especially hydrogen) are sufficiently populous to prevent ^{excessive} buildup and leakage of intermediate energy neutrons. An additional factor and likely the most important one favoring the use of concrete is its gamma ray attenuating properties. For nuclear reactors the most penetrating radiation is usually gamma. That is, the controlling biological dose at the shield edge is ^{usually} gamma radiation, the gamma rays resulting from the fission process in the reactor fuel and from neutron capture in the inner shield regions.

Gamma rays interact with the atomic electrons of atoms; thus, attenuation of gamma rays is somewhat proportional to the atomic electron density and somewhat proportional to the density of the material. Attenuation efficiency also generally increases with atomic mass. Thus, for shields of least volume, one seeks dense materials and preferably materials containing heavy elements. Since this is not consistent^e with the need in materials for neutron shielding, a material is picked which fills the

gamma ray requirements and which also meets the neutron requirements.

Concrete has a rather unique mixture of heavy and light materials. If the concrete shield is designed to adequately reduce the biological dose due to gamma radiation arising from fission in the reactor core and neutron capture in the shield, the neutron leakage dose will usually be taken care of. This would not be the case if the light elements contained in the concrete were not present. Thus, concrete from a nuclear standpoint is advantageous for biological shielding of stationary reactors, and due to its physical properties it is usually found advantageous for all large volume stationary bulk shielding applications.

BIBLIOGRAPHY

1. Rockwell, Theodore, III, Editor, REACTOR SHIELDING DESIGN MANUAL, TID-7004, New York, McGraw-Hill Book Co., Inc., 1956.
2. Goldstein, Herbert, FUNDAMENTAL ASPECTS OF REACTOR SHIELDING, Reading, Mass., Addison-Wesley Publishing Co., Inc., 1959.
3. Blizard, E. P., Editor, REACTOR HANDBOOK, VOLUME III, PART B, SHIELDING, New York, Interscience Publishers, 1962.
4. Grotenhuis, M., LECTURE NOTES ON REACTOR SHIELDING, Argonne, Ill., ANL-6000, 1959.
5. Price, B. T., C. C. Horton, and K. T. Spinney, RADIATION SHIELDING, New York, Pergamon Press, 1957.
6. Kaplan, Irving, NUCLEAR PHYSICS, Reading, Mass., Addison-Wesley Publishing Co., Inc., 1959.
7. Field, B. T., "The Neutron", EXPERIMENTAL NUCLEAR PHYSICS, Volume II, John Wiley and Sons, Inc., 1953.
8. Liikala, R. C., UPDATED RBU BASIC LIBRARY, HW-75716, Volume I-III, May 20, 1963.
9. Greenberg, J., TWO CROSS SECTION LIBRARIES FOR USE WITH MAC SHIELDING CODE, HW-73381, Supplement 1, January, 1964.
10. Weisskopf, V., "Statistics and Nuclear Reactions", PHYS. REV., Volume 52, pp. 295-303, 1937.
11. Yiftah, S., E. Okrent, and P. A. Moldauer, FAST REACTOR CROSS SECTIONS, New York, Pergamon Press, p. 48, 1960.
12. Weinberg, A. M., and E. P. Wigner, THE PHYSICAL THEORY OF NEUTRON CHAIN REACTIONS, Chicago, Ill., University of Chicago Press, p. 104, 1958.

13. Soodak, H., Editor, REACTOR HANDBOOK, VOLUME II, PART A, PHYSICS, New York, Interscience Publishers, Table 1.33 (Also Table 3.4.1, Reference 5 above), 1962.
14. Gladstone, S., and M. C. Edlund, THE ELEMENTS OF NUCLEAR REACTOR THEORY, New York, D. Van Nostrand Co., Inc., 1952.
15. Bunch, W. L., ATTENUATION PROPERTIES OF HIGH DENSITY PORTLAND CEMENT CONCRETES AS A FUNCTION OF TEMPERATURE, HW-54656, January, 1958.
16. Wood, D. E., THE EFFECT OF TEMPERATURE ON THE NEUTRON ATTENUATION OF MAGNETITE CONCRETE, HW-58497, December, 1958.
17. Peterson, E. G., SHIELDING PROPERTIES OF FERROPHOSPHORUS CONCRETE AS A FUNCTION OF TEMPERATURE, HW-64774, July, 1960.
18. Peterson, E. G., SHIELDING PROPERTIES OF ORDINARY CONCRETE AS A FUNCTION OF TEMPERATURE, HW-65572, August, 1960.
19. Peterson, E. G., SHIELDING PROPERTIES OF IRON SERPENTINE CONCRETE, HW-73255, April, 1962.
20. Bennett, C. L., SHIELDING PROPERTIES OF AS-CURED BORITE CONCRETE, HW-71113, September, 1961.
21. Blizard, E. P., and J. M. Miller, RADIATION ATTENUATION CHARACTERISTICS OF STRUCTURAL CONCRETE, ORNL-2193, August, 1958.
22. Blosser, T. V., et al., A STUDY OF THE NUCLEAR AND PHYSICAL PROPERTIES OF THE ORNL GRAPHITE REACTOR SHIELD, ORNL-2195, September, 1958.
23. Grantham, W. J., Jr., BARYTES CONCRETE FOR RADIATION SHIELDING: MIX CRITERIA AND ATTENUATION CHARACTERISTICS, ORNL-3130, July, 1961.
24. Aalto, E., and R. Nilsson, MEASUREMENTS OF NEUTRON AND GAMMA ATTENUATION IN MASSIVE LAMINATED SHIELDS OF CONCRETE AND A STUDY OF THE ACCURACY OF SOME METHODS OF CALCULATION, AE-157, A.B. Atomenergi, Sweden, September, 1964.

25. Albert, R. D., and T. A. Welton, A SIMPLIFIED THEORY OF NEUTRON ATTENUATION AND ITS APPLICATION TO REACTOR SHIELD DESIGN, WAPD-15 (Del.), November, 1950.
26. Casper, A. W., MODIFIED FAST NEUTRON ATTENUATION FUNCTIONS, XDC 60-2-76, February, 1960.
27. Chapman, G. T., and C. L. Storrs, EFFECTIVE REMOVAL CROSS SECTIONS FOR SHIELDING, AECD-3978, September, 1955.
28. Peterson, E. G., MAC - A BULK SHIELDING CODE, HW-73381, April, 1962.
29. Preiser, S., G. Rabinowitz, and E. deOufour, A PROGRAM FOR THE NUMERICAL INTEGRATION OF THE BOLTZMANN TRANSPORT EQUATION - NIOBE, UNUCOR-632, March, 1963.
30. Avery, A. F., D. E. Bendall, J. Butler, and K. T. Spinney, METHOD OF CALCULATION FOR USE IN THE DESIGN OF SHIELDS FOR POWER REACTORS, AEERE-R-3216, May, 1960.

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