DEVELOPMENT OF A MODIFIED CHAPMAN-ENSKOG SOLUTION OF
THE LENARD-BALESCU EQUATION FOR TWO-TEMPERATURE PLASMAS¹

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Introduction

Fully ionized laboratory plasma systems are typically two temperature since the energy exchange mechanisms are such that one of the constituents is preferentially heated; and, by virtue of the large difference in mass between ions and electrons, the energy exchange between constituents is relatively ineffective. The determination of a mathematical description of the behavior of a two-temperature plasma is complicated by the fact that long-range collective interactions are important necessitating the use of the Lenard-Balescu kinetic equation, and by the fact that the "classical" Chapman-Enskog method is adequate only when both like- and unlike-particle interactions dominate the behavior, which is, of course, not the case for two-temperature systems. Numerous investigators [1, 2, 3, 4, 5] have attempted to develop methods which could adequately describe the behavior of this important class of systems; however, the mathematical methods utilized are justified only on the basis of intuitive arguments, and the studies carried out are incomplete.

In this paper a general rigorous method is developed to obtain solutions for the one-particle distribution functions for two-temperature plasmas. The "test particle" momentum and energy transfer frequencies computed for two-temperature plasmas in a companion paper being presented at this Conference are accepted as appropriate estimates of the corresponding frequencies associated with the various Lenard-Balescu interaction integrals used to describe the behavior of two-temperature plasmas. An expansion parameter, $\epsilon = (m_e/m_i)^{1/2}$, is identified as the appropriate expansion parameter for a two-temperature plasma. The Lenard-Balescu equation is ordered in ϵ and the resulting zeroth and first order equations solved by a rigorous mathematical procedure similar to the "classical" Chapman-Enskog approach. The formal solutions for the one-particle distribution functions are exact to $O(\epsilon)$ and exhibit a surprisingly simple form.

Development of the Modified Chapman-Enskog Method

Expanding the one-particle distribution function in ϵ in the usual manner, and using the previously discussed transfer frequencies as

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approximations to the corresponding frequencies associated with the Lenard-Balescu interaction integrals, the following zeroth and first order equations are obtained

$$J_{LB}(f_e^0, f_e^0) + J_L'(f_e^0, f_i^0) = 0 \quad (1a)$$

$$J_{LB}(f_i^0, f_i^0) = 0 \quad (1b)$$

$$\begin{aligned} \frac{Df_e^0}{Dt} - J_L(f_e^0, f_i^0) &= J_{LB}(f_e^0, f_e^0 \phi_e) + J_{LB}(f_e^0 \phi_e, f_e^0) \\ &+ J_L'(f_e^0 \phi_e, f_i^0) \end{aligned} \quad (2a)$$

$$\frac{Df_i^0}{Dt} - J_L(f_i^0, f_e^0) = J_L(f_i^0 \phi_i, f_i^0) + J_L(f_i^0, f_i^0 \phi_i) \quad (2b)$$

where

J_{LB} → Lenard-Balescu interaction integral

J_L → Landau interaction integral

J_L' → an approximation to the Landau interaction integral

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \vec{c} \cdot \frac{\partial}{\partial \vec{r}} + \frac{\vec{F}}{m} \cdot \frac{\partial}{\partial \vec{c}}$$

$$f_r = f_r^0(1 + \epsilon \phi_r)$$

The approximation to the Landau interaction integral J_L' is obtained from J_L by using the approximation

$$\delta(\mu - c'\mu'/c) \approx \delta(\mu)$$

where

$$\mu = \vec{k} \cdot \vec{c}/kc$$

$$\mu' = \vec{k} \cdot \vec{c}'/kc'$$

$\vec{k}$ → wave vector

$\vec{c}$ → electron velocity

$\vec{c}'$ → ion velocity

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The long-range collective effects are important only for electron-electron interactions, as exhibited in the companion paper referred to above; hence the Lenard-Balescu interaction integral is required only for these interactions while the simpler Landau form is used for all others. (It will be recalled that the Landau equation is equivalent to the Fokker-Planck equation.)

It should be noted that with the selection of this ordering procedure the energy coupling between electrons and ions first appears in the first order equations rather than in the zeroth order as is the case when the "classical" Chapman-Enskog approach is utilized.

The solutions to the zeroth order equations for electrons and for ions are Maxwellian at temperatures T_e and T_i , respectively. The solutions to the first order equations for electrons and for ions are

$$\phi_e = -A_e(W^2)W_\alpha \frac{\partial \ln T_e}{\partial r_\alpha} - B_e(W^2)(W_\alpha W_\beta - \frac{W^2}{3} \delta_{\alpha\beta}) \frac{\partial W_\alpha}{\partial r_\beta} - D_e(W^2)d_\alpha^e W_\alpha + \psi_e(W^2)$$

$$\phi_i = -A_i(W'^2)W'_\alpha \frac{\partial \ln T_i}{\partial r_\alpha} - B_i(W'^2)(W'_\alpha W'_\beta - \frac{W'^2}{3} \delta_{\alpha\beta}) \frac{\partial W'_\alpha}{\partial r_\beta} + \psi_i(W'^2)$$

where

$$W \equiv c(m_e/2kT_e)^{1/2}$$

$$W' \equiv c'(m_i/2kT_i)^{1/2}$$

$$\vec{d}^e \equiv \left(\frac{1}{n_e} \frac{\partial n_e}{\partial \vec{r}} - \frac{\vec{F}^e}{kT_e} + \frac{1}{T_e} \frac{\partial T_e}{\partial \vec{r}} \right)$$

The usual transport coefficients, electrical conductivity, thermal conductivity, and viscosity can be evaluated by the usual method of expanding the A_s , B_s , and D_e in Sonine polynomials and solving to the desired degree of accuracy. Since the terms ψ_e and ψ_i are scalars they make no contribution to the transport coefficients of interest.

Summary

In the "classical" Chapman-Enskog method of solution a very small expansion parameter ϵ is chosen, so small in fact that all collision terms appear in the kinetic equation to the same order in spite of the fact that these terms differ by as much as a factor $(m_e/m_i)^{1/2}$.

Although a very exact solution may be obtained, the solution is valid only for a small class of systems very close to equilibrium.

Two-temperature plasmas are such that unlike-particle energy transfer is not dominant; hence, the portion of the interaction integral corresponding to unlike-particle energy transfer should, to agree with physical reality, appear in a higher order equation. As exhibited in the previous section, it is found possible to split the electron-ion interaction integral into a portion which, when moments of the equation are taken, accounts for momentum transfer and another which accounts for energy transfer. It is thus possible to identify, without ambiguity, a parameter $\epsilon = (m_e/m_i)^{1/2}$ which is appropriate to this class of systems. We note that, at a small expense in accuracy (terms $O(\epsilon)^2$ are neglected), it is found possible to develop a rigorous method of solving the Lenard-Balescu kinetic equation for this important class of nonequilibrium fully ionized plasmas.

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