MASTED

ULTRASONIC INTERACTIONS AT
LIQUID-SOLID BOUNDARIES (U)

J. W. Garber

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE Y-12 PLANT

operated for the **ATOMIC ENERGY COMMISSION** under **U. S. GOVERNMENT** Contract W-7405 eng 26

**UNION
CARBIDE**

OAK RIDGE Y-12 PLANT
P. O. Box Y
OAK RIDGE, TENNESSEE 37831

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Printed in USA. Price \$3.00. Available from the Clearinghouse for Federal
Scientific and Technical Information, National Bureau of Standards,
U.S. Department of Commerce, Springfield, Virginia

LEGAL NOTICE

This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission:

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of any information, apparatus, method, or process disclosed in this report.

As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

H.C. \$ 2.00; MN .75

Date Issued: July 1, 1966

Report Number Y-1540Physics
TID-4500UNION CARBIDE CORPORATION
Nuclear Division

Y-12 PLANT

Contract W-7405-eng-26
With the US Atomic Energy Commission

ULTRASONIC INTERACTIONS AT LIQUID-SOLID BOUNDARIES

J. W. Garber

This report is based on a study by the author as partial fulfillment of requirements for the degree of Master of Science in Physics from The University of Tennessee.

RELEASED FOR ANNOUNCEMENT
IN NUCLEAR SCIENCE ABSTRACTS

Oak Ridge, Tennessee

Report Number Y-1540Physics
TID-4500

Distribution:

Ackerson, R. D.	(AFSC)	Knowles, C. M.	
Bailey, E. W.		Lewis, F. O.	(ORGDP)
Ballenger, H. F.	(2)	Long, P. J.	
Bell, B. B.		McLendon, J. D.	
Bernander, N. K.		Meeks, E. J.	(AFSC)
Breazeale, M. A.	(UT)(3)	Mitchel, G. W.	
Briscoe, O. W.		Rader, D. H.	
Burkhart, L. E.		Schweinler, H. C.	(ORNL)
Center, C. E.	(ORGDP)	Shelton, M. H.	
Christman, A. M.		Snow, S. G.	
Cofield, R. E.		Stoner, H. H.	
Cowen, D. D.	(ORNL)	Thompson, W. H.	
Deeds, W. E.	(UT)	Tilson, F. V.	
Evans, G. W.		Trotter, T. C.	
Fitzgerald, L. M.		UT Physics Dept	(2)
Garber, J. W.	(15)	Waters, J. L.	
Harwell, W. L.	(ORGDP)(5)	Whitson, W. K.	
Hemphill, L. F.		Williams, J. L.	
Huber, A. P.	(ORGDP)	Winkel, R. A.	(Paducah)
Jackson, V. C.		Yaggi, W. J.	
Jennings, D. A.		Zurcher, E.	
Keller, C. A.	(AEC-ORO)(4)	Y-12 Central Files	(5)
Kite, H. T.		Y-12 Central Files	(Y-12RC)

In addition, this report is distributed in accordance with the category Physics, as given in the "USAEC Standard Distribution Lists for Unclassified Scientific and Technical Reports", TID-4500.

ABSTRACT

A review is made of the theoretical equations predicting the relative energies and propagation directions of waves produced by a plane ultrasonic wave incident at different angles on a liquid-solid boundary. The following three cases are considered: a longitudinal wave incident from a liquid onto a solid; a longitudinal wave incident from a solid onto a liquid; and a shear wave incident from a solid onto a liquid. A computer program, written for making calculations of the energy ratios, is used to aid in predicting the general characteristics of ultrasonic interactions for many types of liquid-solid systems. Unusual predictions involving extreme points of the energy ratios are noted for extreme values of the Poisson's ratio of the solid.

Experimental measurements are reported for the energy reflected as a function of incident angle in water of a longitudinal wave incident on flat samples of aluminum and beryllium. The effects of surface waves are observed.

ACKNOWLEDGEMENTS

The author greatly appreciates the technical guidance given during the entire course of this investigation by Dr. M. A. Breazeale of The University of Tennessee. Also, he is indebted to personnel at the Atomic Energy Commission's Y-12 Plant for the support and encouragement in the performance of this work, and to Mr. S. G. Snow for his help with the computer programming. Finally, the author wishes to thank his wife for the preparation of the preliminary draft.

TABLE OF CONTENTS

CHAPTER	PAGE
I. INTRODUCTION	1
II. THEORY	3
Energy Ratio Equations.	3
Surface Waves	12
III. CHARACTERISTICS OF ENERGY-RATIO EQUATIONS	13
Longitudinal Wave Incident in a Liquid on a Solid	14
Longitudinal Wave Incident in a Solid on a Liquid	23
Shear Wave Incident in a Solid on a Liquid	29
IV. MEASUREMENT OF THE REFLECTION-ENERGY RATIO	38
Experimental Apparatus and Procedure	38
Experimental Results.	41
V. SUMMARY AND CONCLUSIONS	46
BIBLIOGRAPHY	49
APPENDIXES	51
APPENDIX A.	52
APPENDIX B.	54
SYMBOLS AND NOMECLATURE	69

LIST OF FIGURES

FIGURE	PAGE
1. Propagation Directions of Waves Generated at a Liquid-Solid Interface by a Longitudinal Wave Incident from the Liquid	5
2. Calculated Energy Ratios for a Reflected Longitudinal Wave, R , a Transmitted Longitudinal Wave, L , and a Transmitted Shear Wave, S , Which Are Produced When a Longitudinal Wave Is Incident from Water onto Aluminum	15
3. Calculated Values of the Reflected Longitudinal Wave Energy for a Longitudinal Wave in Water Incident on a Solid Which Has Different Values of Longitudinal Acoustical Impedance and Fixed Values of Poisson's Ratio and Shear Velocity	17
4. Calculated Values of the Reflected Longitudinal Wave Energy for a Longitudinal Wave in Water Incident on a Solid Which Has Different Values of Shear Velocity (or shear critical angle) and Fixed Values of Poisson's Ratio and Longitudinal Acoustical Impedance	18
5. Calculated Values of the Reflected Longitudinal Wave Energy for a Longitudinal Wave in Water Incident on a Solid Which Has Different Values of Poisson's Ratio and Fixed Values of Shear Wave Velocity and Longitudinal Acoustical Impedance	19

FIGURE

PAGE

6. Reflected Energy Peak Sharpness as a Function of Poisson's Ratio,
Shear Critical Angle, and Reflected Energy Ratio at Normal
Incidence 21
7. Calculated Energy Ratios for a Reflected Longitudinal Wave, R, a
Transmitted Longitudinal Wave, L, and a Transmitted Shear Wave,
S, Which Are Produced When a Longitudinal Wave Is Incident
from Water onto Beryllium. 22
8. Calculated Energy Ratios for a Reflected Longitudinal Wave, R, a
Transmitted Longitudinal Wave, L, and a Transmitted Shear Wave,
S, Which Are Produced When a Longitudinal Wave Is Incident at
a Narrow Range of Incident Angles from Water onto Beryllium . . . 24
9. Calculated Energy Ratios for a Reflected Longitudinal Wave, L, a
Reflected Shear Wave, S, and a Transmitted Longitudinal Wave,
T, Which Are Produced When a Longitudinal Wave Is Incident
from Aluminum onto Water 25
10. Calculated Values of the Reflected Longitudinal Wave Energy for a
Longitudinal Wave in a Solid Incident on Water for Different
Values of Poisson's Ratio for the Solid and Fixed Values of Shear
Wave Velocity and Longitudinal Acoustical Impedance 27
11. Calculated Values of the Reflected Shear Wave Energy for a
Longitudinal Wave in a Solid Incident on Water for Different

FIGURE

PAGE

	Values of Poisson's Ratio for the Solid and Fixed Values of Shear Wave Velocity and Longitudinal Acoustical Impedance	28
12.	Calculated Energy Ratios for a Reflected Longitudinal Wave, L , a Reflected Shear Wave, S , and a Transmitted Longitudinal Wave, T , Which Are Produced When a Longitudinal Wave Is Incident from Beryllium onto Water.	30
13.	Calculated Energy Ratios for a Reflected Longitudinal Wave, L , a Reflected Shear Wave, S , and a Transmitted Longitudinal Wave, T , Which Are Produced When a Longitudinal Wave Is Incident from Beryllium onto Water for Incident Angles Near 90 Degrees. . .	31
14.	Calculated Energy Ratios for a Reflected Longitudinal Wave, L , a Reflected Shear Wave, S , and a Transmitted Longitudinal Wave, T , Which Are Produced When a Shear Wave Is Incident from Aluminum onto Water	32
15.	Calculated Values of the Reflected Shear Wave Energy for a Shear Wave in a Solid Incident on Water for Different Values of Poisson's Ratio for the Solid and Fixed Values of Shear Wave Velocity and Longitudinal Acoustical Impedance	34
16.	Calculated Energy Ratios for a Reflected Shear Wave, S , a Reflected Longitudinal Wave, L , and a Transmitted Longitudinal Wave, T , Which Are Produced When a Shear Wave Is Incident	

FIGURE	PAGE
from Beryllium onto Water	36
17. Calculated Energy Ratios for a Reflected Shear Wave, S , a Reflected Longitudinal Wave, L , and a Transmitted Longitudinal Wave, T , Produced When a Shear Wave Is Incident from Beryllium onto Water for Angles Near the Critical Angle	37
18. Ultrasonic Goniometer	39
19. Calculated (Solid Line) and Measured (Dots) Values of the Reflected Energy Ratio for a Longitudinal Wave Incident from Water onto Type 1100 Aluminum	42
20. Calculated (Solid Line) and Measured (Dots) Values of the Reflected Energy Ratio for a Longitudinal Wave Incident from Water onto Beryllium	44
21. Benson-Lehner Electroplot of Energy Ratio for Waves Produced by a Longitudinal Wave in a Liquid Incident on a Solid	65
22. Benson-Lehner Electroplot of Energy Ratio for Waves Produced by a Longitudinal Wave in a Solid Incident on a Liquid	66
23. Benson-Lehner Electroplot of Energy Ratio for Waves Produced by a Shear Wave in a Solid Incident on a Liquid	67
24. Benson-Lehner Electroplot Relating the Refraction Angles for a Liquid-Solid System	68

CHAPTER I

INTRODUCTION

The energy ratios and propagation angles of mechanical waves produced by a plane wave incident on a material boundary are of basic importance to those studying ultrasonics, seismology, and nondestructive materials testing. The early predictions of the relative amplitudes of the various waves produced by a mechanical wave incident on a boundary at different angles have been credited to Knott (1899) and Zoeppritz (1919). Blut (1932) derived energy equations for the various waves in terms of their amplitudes and propagation directions. A unified presentation of the development of equations relating the energy in the various waves is given by Macelwane (1936).

Brekhovskikh (1960), Thurston (Mason, ed., 1964), and Ewing, Jardetzky, and Press (1957) present more elegant determinations of the interaction equations, and the subject of surface waves is discussed more thoroughly by them.

The unwieldiness of the equations relating energy ratios for various incident angles for a liquid-solid system has limited previous investigators. Ergin (1952) published a limited number of results, of particular interest to seismologists, from slide-rule calculations. Calculations reported by Mayer (1963, 1965) give useful curves for the interactions of a longitudinal wave for some common liquid-solid systems. Our development of a computer program for the calculation of energy ratios has led to some progress in predicting the characteristics of wave interactions

for a wider variety of solid materials and types of wave interactions. Computations and curves are made for waves produced by a longitudinal wave incident in a liquid onto a solid and for a longitudinal or shear wave incident in a solid onto a liquid at various angles of incidence. The general shape of the curves relating energy ratios and incident angles is discussed, and of particular interest are the drastic differences occurring between the curves for common solids and the curves for solids which have an unusually low value for Poisson's ratio, such as beryllium.

Experimental measurements are reported for the energy reflected as a function of the incident angle in water of a longitudinal wave incident on aluminum and beryllium.

CHAPTER II

THEORY

The problem of finding the relative amplitude of the mechanical waves produced at the boundary of two media by an incident wave is approached as a boundary value problem involving wave propagation in two semi-infinite media. The particle displacements, ϕ , due to the wave must satisfy the wave equation:

$$\nabla^2 \phi = \frac{1}{V^2} \frac{\partial^2 \phi}{\partial t^2}, \quad (1)$$

where:

V = the wave velocity.

The boundary conditions of continuity of stress and displacement at the interface permit the desired prediction of the relative amplitudes of waves in the system.

Energy Ratio Equations

Consider an undamped, longitudinal, plane mechanical wave traveling in a liquid and approaching a homogeneous, isotropic solid. If this wave has particle motion in the xy plane and the boundary occurs at $y = 0$ with Medium 1 (liquid) in space $y < 0$ and Medium 2 (solid) in space $y > 0$, then the particle displacements of the wave which satisfy the wave equation are:

$$\phi = A \exp \left[i\omega \left(t - \frac{x \sin \alpha + y \cos \alpha}{V} \right) \right], \quad (2)$$

where:

ϕ = the particle displacement in the xy plane in Medium 1 (liquid) due to the incident longitudinal wave,

A = the maximum amplitude of the displacement of the particles,

$\omega = 2\pi f = \frac{2\pi V}{\lambda}$ = the angular frequency of the wave,

V = the longitudinal wave velocity in Medium 1,

f = the frequency of the wave,

λ = the wave length of the wave,

t = time, and

α = the angle of incidence, defined as the angle between the propagation direction of the wave and the normal to the boundary.

In general, four ordinary waves could be generated at a boundary with the incidence of either a longitudinal or a shear wave. In the case of a longitudinal wave incident from a liquid onto a solid, a longitudinal and a shear wave are generated in the solid, but only a longitudinal wave is reflected into the liquid, because the liquid lacks the shear rigidity necessary to support shear-wave propagation. The assumed angles of propagation of the waves are shown in Figure 1.

Particle displacements for these three waves generated at the boundary may be represented in a manner similar to the representation of the incident longitudinal wave in Equation 1. A summary of the displacement equations is:

$$\phi_1 = B \exp \left[i\omega \left(t - \frac{x \sin \gamma - y \cos \gamma}{V} \right) \right], \quad (3)$$

FIGURE 1

PROPAGATION DIRECTIONS OF WAVES GENERATED AT A LIQUID-SOLID INTERFACE BY A LONGITUDINAL WAVE INCIDENT FROM THE LIQUID

$$\phi_2 = C \exp \left[i\omega \left(t - \frac{x \sin \eta + y \cos \eta}{V_l} \right) \right], \text{ and} \quad (4)$$

$$\phi_s = D \exp \left[i\omega \left(t - \frac{x \sin \zeta + y \cos \zeta}{V_s} \right) \right]. \quad (5)$$

The definitions involved in Equations 2, 3, 4, and 5 are given in the tabulation that follows.

Type of Wave	Incident Angle	Velocity	Amplitude of Displacement	Maximum Amplitude of Displacement
Incident Longitudinal	α	V	ϕ	A
Reflected Longitudinal	γ	V	ϕ_1	B
Transmitted Longitudinal	η	V_l	ϕ_2	C
Transmitted Shear	ζ	V_s	ϕ_s	D

By knowing simply that boundary conditions must apply at the interface, the very important angular relationships may be obtained by observing the phase factors of the waves. Since there are continuity requirements at $y = 0$, the phase of all the waves must be the same at this point, and thus the equations relating the angles of the resulting waves are:

$$\frac{\sin \alpha}{V} = \frac{\sin \gamma}{V} = \frac{\sin \eta}{V_l} = \frac{\sin \zeta}{V_s}. \quad (6)$$

The condition of continuity of displacement may be stated mathematically by resolving the displacements into normal and tangential components and setting the appropriate quantities equal. Since the liquid may slip along the boundary and

cause no disturbance in the solid, only the normal components of displacement are required to be equal, that is:

$$\phi \cos \alpha - \phi_1 \cos \gamma = \phi_2 \cos \eta + \phi_s \sin \zeta. \quad (7)$$

Since the exponentials in the equations for ϕ , ϕ_1 , ϕ_2 , and ϕ_s are equal at $y = 0$ and $\alpha = \gamma$ from Equation 6, then Equation 7 becomes:

$$A \cos \alpha - B \cos \alpha - C \cos \eta - D \sin \zeta = 0. \quad (8)$$

The stresses in two dimensions involve partial space derivatives of the normal and tangential particle displacements. The application of the continuity of normal stress condition, as outlined by Macelwane (1936) and others, results in the following equation:

$$A + B - C \frac{V_l \rho_2}{V \rho_1} \cos 2 \zeta - D \frac{V_s \rho_2}{V \rho_1} \sin 2 \zeta = 0, \quad (9)$$

where:

ρ_1 = the mass density of the liquid, and

ρ_2 = the mass density of the solid.

Since the liquid does not support shear stresses, the tangential stresses in the solid must vanish at the boundary. When this condition is imposed, the third equation relating the amplitudes is formed as:

$$C \sin 2 \eta - D \frac{V_l}{V_s} \cos 2 \zeta = 0. \quad (10)$$

Equations 8, 9, and 10 are the Zoeppritz (1919) equations for determining the relative amplitude of the four waves present when a longitudinal wave is incident from a liquid onto a solid.

A derivation of the energy equations by Blut (1932), stating the energy of each wave in terms of maximum amplitudes and propagation directions, is given by Macelwane (1936). The relative energies of the various waves are found by applying the theorem that the increase in energy in unit time within a surface, S , containing the area of interaction of the waves is equal to the work done in that time by the tractions acting across S . The energy ratios expressed in terms of true amplitudes are given by:

$$\frac{B^2}{A^2} + \frac{C^2}{A^2} \frac{\rho_2 \sin 2\eta}{\rho_1 \sin 2\alpha} + \frac{D^2}{A^2} \frac{\rho_2 \sin 2\zeta}{\rho_1 \sin 2\alpha} = 1, \quad (11)$$

$$b^2 = \frac{B^2}{A^2}, \quad (12)$$

$$c^2 = \frac{C^2}{A^2} \frac{\rho_2 \sin 2\eta}{\rho_1 \sin 2\alpha}, \text{ and} \quad (13)$$

$$d^2 = \frac{D^2}{A^2} \frac{\rho_2 \sin 2\zeta}{\rho_1 \sin 2\alpha}, \quad (14)$$

where:

b^2 = the energy ratio of the reflected longitudinal wave relative to the incident longitudinal wave,

c^2 = the energy ratio of the transmitted longitudinal wave relative to the incident longitudinal wave, and

d^2 = the energy ratio of the transmitted shear wave relative to the incident longitudinal wave.

Thus, the amplitude ratios may be obtained from Equations 8, 9, and 10 and these amplitude ratios may be substituted into Equations 12, 13, and 14 to find the energy ratios. When Equations 8, 9, and 10 are divided by A and the coefficients of A are transposed to the right hand side, the equations may be written in matrix form as:

$$\begin{vmatrix} \cos \alpha & \cos \eta & \sin \zeta \\ -1 & \frac{V_l \rho_2}{V \rho_1} \cos 2\zeta & \frac{V_s \rho_2}{V \rho_1} \sin 2\zeta \\ 0 & \sin 2\eta & -\frac{V_l}{V_s} \cos 2\zeta \end{vmatrix} \begin{vmatrix} B' \\ C' \\ D' \end{vmatrix} = \begin{vmatrix} \cos \alpha \\ 1 \\ 0 \end{vmatrix}, \quad (15)$$

where:

$$B' = \frac{B}{A}, \quad C' = \frac{C}{A}, \quad \text{and} \quad D' = \frac{D}{A}.$$

Thus, Equation 15 is a set of three linear algebraic equations which may be solved for the amplitude ratios. These ratios, along with Equations 12, 13, and 14, yield the following desired energy ratios for the waves leaving a liquid-solid boundary due to the incidence of a longitudinal wave in the liquid:

$$b = \frac{\cos \eta - \frac{V_l \rho_2}{V \rho_1} \cos \alpha \left[1 - 2 \sin \zeta \sin 2 \zeta \left(\cos \zeta - \frac{V_s}{V_l} \cos \eta \right) \right]}{\cos \eta + \frac{V_l \rho_2}{V \rho_1} \cos \alpha \left[1 - 2 \sin \zeta \sin 2 \zeta \left(\cos \zeta - \frac{V_s}{V_l} \cos \eta \right) \right]}, \quad (16)$$

$$c = \frac{2 \sqrt{\frac{V_l \rho_2}{V \rho_1} \cos \alpha \cos \eta \cos 2 \zeta}}{\cos \eta + \frac{V_l \rho_2}{V \rho_1} \cos \alpha \left[1 - 2 \sin \zeta \sin 2 \zeta \left(\cos \zeta - \frac{V_s}{V_l} \cos \eta \right) \right]}, \text{ and } (17)$$

$$d = \frac{2 \frac{V_s}{V} \sqrt{\frac{\rho_2}{\rho_1} \sin 2 \alpha \sin 2 \zeta \cos \eta}}{\cos \eta + \frac{V_l \rho_2}{V \rho_1} \cos \alpha \left[1 - 2 \sin \zeta \sin 2 \zeta \left(\cos \zeta - \frac{V_s}{V_l} \cos \eta \right) \right]}. \quad (18)$$

Equations 16, 17, and 18 are Ergin's equations for the square root of the energy ratios. Arguments similar to those just given will lead to similar equations for the energy ratios for the waves produced by a longitudinal or a shear wave incident from a solid onto a liquid. Ergin (1952) gives the square root of the energy ratios for the waves produced by a longitudinal wave incident from a solid onto a liquid as:

$$e = \frac{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2 \beta - \frac{V_s}{V_l} \sin 2 \alpha \sin 2 \beta \right) - \frac{V \rho_1}{V_s \rho_2} \cos \alpha}{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2 \beta + \frac{V_s}{V_l} \sin 2 \alpha \sin 2 \beta \right) + \frac{V \rho_1}{V_s \rho_2} \cos \alpha}, \quad (19)$$

$$g = \frac{2 \sqrt{\sin 2 \alpha \sin 2 \beta \cos 2 \beta \cos \eta}}{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2 \beta + \frac{V_s}{V_l} \sin 2 \alpha \sin 2 \beta \right) + \frac{V \rho_1}{V_s \rho_2} \cos \alpha}, \text{ and} \quad (20)$$

$$h = \frac{2 \frac{V_l}{V_s} \sqrt{\frac{V \rho_1}{V_l \rho_2}} \cos \alpha \cos \eta \cos 2 \beta}{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2 \beta + \frac{V_s}{V_l} \sin 2 \alpha \sin 2 \beta \right) + \frac{V \rho_1}{V_s \rho_2} \cos \alpha}, \quad (21)$$

where the symbols are those given in the tabulation that follows.

Type of Wave	Angle of Incidence	Energy Ratio
Incident Longitudinal	α	1
Reflected Longitudinal	α	e^2
Reflected Shear	β	g^2
Transmitted Longitudinal	η	h^2

The corresponding equations for the waves produced when a shear wave, whose particle displacement is in the xy plane, is incident from a solid onto a liquid are given by Ergin (1952) as:

$$i = \frac{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2 \beta - \frac{V_s}{V_l} \sin 2 \alpha \sin 2 \beta \right) + \frac{V \rho_1}{V_s \rho_2} \cos \alpha}{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2 \beta + \frac{V_s}{V_l} \sin 2 \alpha \sin 2 \beta \right) + \frac{V \rho_1}{V_s \rho_2} \cos \alpha}, \quad (22)$$

$$j = \frac{2 \sqrt{\sin 2 \alpha \sin 2 \beta \cos 2 \beta \cos \eta}}{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2 \beta + \frac{V_s}{V_l} \sin 2 \alpha \sin 2 \beta \right) + \frac{V \rho_1}{V_s \rho_2} \cos \alpha}, \text{ and} \quad (23)$$

$$k = \frac{2\sqrt{\frac{\rho_1}{\rho_2}} \sin 2\beta \sin 2\eta \cos \alpha}{\cos \eta \left(\frac{V_l}{V_s} \cos^2 2\beta + \frac{V_s}{V_l} \sin 2\alpha \sin 2\beta \right) + \frac{V\rho_1}{V_s \rho_2} \cos \alpha} . \quad (24)$$

Symbols for the incident shear wave interaction are given in the tabulation that follows.

<u>Type of Wave</u>	<u>Angle of Incidence</u>	<u>Energy Ratio</u>
Incident Shear	β	1
Reflected Shear	β	i^2
Reflected Longitudinal	α	i^2
Transmitted Longitudinal	η	k^2

Surface Waves

The existence of surface waves, or Rayleigh waves, at a free solid boundary has been discussed and verified by many investigators including Brekhovskikh (1960), Thurston (Mason, ed., 1964), and Ewing, Jardetzky, and Press (1957). However, discussions of surface waves at solid-liquid boundaries are less plentiful and the mathematical treatment of them becomes formidable. However, surface waves at a liquid-solid boundary have been experimentally observed. A discussion of an approach to this problem is presented in Appendix A, page 52.

CHAPTER III

CHARACTERISTICS OF ENERGY-RATIO EQUATIONS

The ultrasonic energy-ratio equations for a liquid-solid interface are rather complicated and many of their characteristics may not be revealed easily by inspection or by simple calculus-of-variations techniques. Ergin (1952) performed slide-rule calculations for energy ratios and a graphical solution to extreme-point equations for three solid-liquid systems of interest to seismologists (Poisson's ratio $= \sigma = 0.18$ to 0.277). Calculations reported by Mayer (1963, 1965) give useful curves for the interactions of a longitudinal wave for some common liquid-solid systems ($\sigma = 0.281$ to 0.350). The use of the computer program described in Appendix B, page 54, permits the calculation of energy ratios for many other materials of interest in pure and applied ultrasonic science.

The five independent quantities required to determine energies and refraction angles of ultrasonic wave interactions at an interface of a homogeneous and isotropic liquid and solid are the elastic constants of the solid, the elastic constant for the liquid, and the densities of both media. Even though the Lamé constants are generally considered to be the basic elastic constants, the use of the shear and longitudinal velocities as the independent elastic constants, along with the mass densities, is more convenient for studying the characteristics of the energy-ratio equations. This approach permits easier analysis in terms of such common acoustical quantities as acoustical impedance and critical angles.

Longitudinal Wave Incident in a Liquid on a Solid

The calculated energy ratios for a longitudinal wave in water incident on aluminum (shown in Figure 2) have many characteristics typical of most water-metal systems. Reference to the figure and to Equations 16, 17, and 18, page 10, reveal the following general characteristics worthy of note:

1. The two waves existing at normal incidence are both longitudinal in nature, and the reflected energy, R , is determined by:

$$R = \left(\frac{1 - r}{1 + r} \right)^2,$$

where:

$$r = \frac{V_l \rho_2}{V \rho_1},$$

which is the ratio of the longitudinal acoustical impedances.

2. Total reflection of the ultrasonic energy is predicted at the first critical angle, a_l , which is given by:

$$a_l = \sin^{-1} \frac{V}{V_l}.$$

3. Only a transmitted shear wave and reflected longitudinal wave are present for incident angles greater than a_l , and the partition of energy between the two modes is dependent upon the acoustical impedance ratio, s , which is given by:

$$s = \frac{V_s \rho_2}{V \rho_1}.$$

FIGURE 2

CALCULATED ENERGY RATIOS FOR A REFLECTED LONGITUDINAL WAVE, R, A TRANSMITTED LONGITUDINAL WAVE, L, AND A TRANSMITTED SHEAR WAVE, S, WHICH ARE PRODUCED WHEN A LONGITUDINAL WAVE IS INCIDENT FROM WATER ONTO ALUMINUM

4. Total reflection is predicted when the incident angle reaches the second critical angle, α_s , which is given by:

$$\alpha_s = \sin^{-1} \frac{V}{V_s}.$$

This total reflection is predicted for all angles greater than α_s , since surface wave phenomena are not considered in this simple analysis.

The independent quantities chosen for studying the characteristics of the energy-ratio equations are: the longitudinal acoustical impedance ratio, r ; the shear velocity, V_s ; and the Poisson's ratio, σ , of the solid. Figures 3, 4, and 5 show the effects of varying each one of these parameters separately while keeping the other two constant for a wave incident in water on a material whose elastic properties approximate those of a solid. The range of parameters chosen includes the values for most metals, and trends may be noted such that predictions of energy-ratio behavior for most materials may be made. Figures 3 and 4 show that longitudinal acoustical impedance primarily affects the amplitude of energy reflection, whereas shear velocity determines the second critical angle and thus the angular range of reduced reflection.

In Figure 5, some large increases in the peak sharpness at the first critical angle are noted for the smaller values of Poisson's ratio. Mayer (1963) suggests that the peak sharpness increases with decreasing R and α_s , but Figure 5 shows that Poisson's ratio has a much greater effect on the peak sharpness than does either R or α_s . In order that a quantitative measure may be ascribed to this effect, define

FIGURE 3

CALCULATED VALUES OF THE REFLECTED LONGITUDINAL WAVE ENERGY
 FOR A LONGITUDINAL WAVE IN WATER INCIDENT ON A SOLID
 WHICH HAS DIFFERENT VALUES OF LONGITUDINAL
 ACOUSTICAL IMPEDANCE AND FIXED VALUES
 OF POISSON'S RATIO AND
 SHEAR VELOCITY

FIGURE 4

CALCULATED VALUES OF THE REFLECTED LONGITUDINAL WAVE ENERGY
 FOR A LONGITUDINAL WAVE IN WATER INCIDENT ON A SOLID
 WHICH HAS DIFFERENT VALUES OF SHEAR VELOCITY (OR
 SHEAR CRITICAL ANGLE) AND FIXED VALUES OF
 POISSON'S RATIO AND LONGITUDINAL
 ACOUSTICAL IMPEDANCE

FIGURE 5

CALCULATED VALUES OF THE REFLECTED LONGITUDINAL WAVE ENERGY
 FOR A LONGITUDINAL WAVE IN WATER INCIDENT ON A SOLID
 WHICH HAS DIFFERENT VALUES OF POISSON'S RATIO AND
 FIXED VALUES OF SHEAR WAVE VELOCITY AND
 LONGITUDINAL ACOUSTICAL IMPEDANCE

the peak sharpness as the width of the reflection energy peak, in degrees, at a reflection energy of one-half of the difference between the total reflection and the average of the reflection energy minima immediately before and after the first critical angle. The plot of this value of peak sharpness in Figure 6 shows that a variation of Poisson's ratio over its possible range of values causes several orders of magnitude of change in peak sharpness, while that sharpness changes much less for variations in R and α_s .

The transmitted longitudinal and shear waves, of course, must possess the remainder of the energy not reflected in the system (cf. Figure 2, page 15). Thus, the shear wave has all the unreflected energy at angles greater than the first critical angle. The transmitted energy at angles less than the first critical angle is divided between the transmitted longitudinal and shear waves, with the shear-wave energy starting at zero for normal incidence and increasing to a maximum before it drops to zero again at the first critical angle. For water-solid systems similar to those of Figures 3, 4, and 5, the height of the maximum of transmitted shear energy increases for smaller values of Poisson's ratio but never exceeds the maximum obtained for angles greater than the first critical angle.

As an example of a system in which the solid has a low value of Poisson's ratio, the energy ratios are calculated and plotted for beryllium and water in Figure 7. It should be noted that the total reflection peak at 6.5 degrees is too narrow to resolve and it appears only as a line on the plot, and that the second critical angle occurs at an incident angle of only 9.5 degrees.

FIGURE 6

REFLECTED ENERGY PEAK SHARPNESS AS A
FUNCTION OF POISSON'S RATIO,
SHEAR CRITICAL ANGLE, AND
REFLECTED ENERGY RATIO
AT NORMAL INCIDENCE

FIGURE 7

CALCULATED ENERGY RATIOS FOR A REFLECTED LONGITUDINAL WAVE, R, A TRANSMITTED LONGITUDINAL WAVE, L, AND A TRANSMITTED SHEAR WAVE, S, WHICH ARE PRODUCED WHEN A LONGITUDINAL WAVE IS INCIDENT FROM WATER ONTO BERYLLIUM

The energy-ratio plot in Figure 8, whose angle scale has been expanded 2,000 times for angles near the critical angle, shows the width of the reflection energy peak. It should also be noted from Figure 8 that the transmitted longitudinal energy-ratio curve has experienced an inflection and is actually increasing slightly to a maximum before it drops to zero at the first critical angle. This slight maximum is typical for solid-water systems when the Poisson's ratio of the solid is less than about 0.24.

Longitudinal Wave Incident in a Solid on a Liquid

The energy ratios for the waves produced by a longitudinal wave incident in aluminum on water, as calculated from Equation 19, page 10, and Equations 20 and 21, page 11, are shown in Figure 9. Of interest is the prediction that the energy ratios for both the reflected longitudinal and shear waves vary widely with the incident angle.

Calculations similar to those made in the previous section, in which R , V_s , and σ are varied independently for a water-solid system, reveal some points of interest concerning these energy relations. Varying the shear velocity and maintaining constant R and σ does absolutely nothing to the energy ratio curves.

As would be expected, varying R changes the relative energy ratio levels, but it does nothing toward shifting or creating extreme points.

The interesting variations for this system of waves occur when Poisson's ratio is varied as R and V_s are kept constant. A plot of the calculated reflected

FIGURE 8

CALCULATED ENERGY RATIOS FOR A REFLECTED LONGITUDINAL WAVE, R, A TRANSMITTED LONGITUDINAL WAVE, L, AND A TRANSMITTED SHEAR WAVE, S, WHICH ARE PRODUCED WHEN A LONGITUDINAL WAVE IS INCIDENT AT A NARROW RANGE OF INCIDENT ANGLES FROM WATER ONTO BERYLLIUM

FIGURE 9

CALCULATED ENERGY RATIOS FOR A REFLECTED LONGITUDINAL WAVE, L, A REFLECTED SHEAR WAVE, S, AND A TRANSMITTED LONGITUDINAL WAVE, T, WHICH ARE PRODUCED WHEN A LONGITUDINAL WAVE IS INCIDENT FROM ALUMINUM ONTO WATER

longitudinal wave for several values of Poisson's ratio is presented in Figure 10. Different values of Poisson's ratio not only cause the energy level to change at many incident angles, but drastic changes in extreme points occur. For a Poisson's ratio greater than 0.28, the energy ratio goes through a nonzero minimum to total reflection at the grazing incidence. However, when Poisson's ratio is less than about 0.28, the reflected longitudinal energy is zero for two distinct incident angles, and the maximum of the reflected energy occurring between these angles increases sharply for smaller values of Poisson's ratio. That is, for low values of Poisson's ratio, there are two incident angles at which the incident longitudinal wave is totally converted to some combination of reflected shear and transmitted longitudinal waves. Arenberg (1948) reported a similar situation for reflection of a longitudinal wave at a free solid boundary. He showed that for a Poisson's ratio of less than about 0.26, total conversion from an incident longitudinal wave to a reflected shear wave occurs at two different incident angles on a free solid boundary.

Figure 11 is a plot of shear energy reflected for an incident longitudinal wave for different values of Poisson's ratio. Drastic changes in the energy ratio occur here also for a Poisson's ratio less than about 0.28; maxima occur at two different incident angles and the minimum between these angles dips greatly for smaller values of Poisson's ratio.

The reflected shear energy increases (or decreases) about the same amount as the reflected longitudinal energy decreases (or increases), resulting in only moderate changes in the transmitted longitudinal energy.

FIGURE 10

CALCULATED VALUES OF THE REFLECTED LONGITUDINAL WAVE ENERGY
 FOR A LONGITUDINAL WAVE IN A SOLID INCIDENT ON WATER
 FOR DIFFERENT VALUES OF POISSON'S RATIO FOR THE SOLID
 AND FIXED VALUES OF SHEAR WAVE VELOCITY AND
 LONGITUDINAL ACOUSTICAL IMPEDANCE

FIGURE 11

CALCULATED VALUES OF THE REFLECTED SHEAR WAVE ENERGY FOR A LONGITUDINAL WAVE IN A SOLID INCIDENT ON WATER FOR DIFFERENT VALUES OF POISSON'S RATIO FOR THE SOLID AND FIXED VALUES OF SHEAR WAVE VELOCITY AND LONGITUDINAL ACOUSTICAL IMPEDANCE

All three energy ratios plotted for beryllium and water in Figure 12 show the total picture for a system whose solid has a low value of Poisson's ratio. The total conversion of the incident longitudinal wave energy to reflected shear wave and transmitted longitudinal wave energy is noted for one value of the incident angle, but the second angle for total conversion is best seen in Figure 13, which is a plot expanded in the incident angle scale. It may be noted also in Figure 13 that the transmitted longitudinal wave energy approaches a maximum angle and falls to zero for angles very near the grazing incidence. This maximum is characteristic of water-solid systems whose solid has a value of Poisson's ratio less than about 0.28.

Shear Wave Incident in a Solid on a Liquid

Calculations of the energy ratios by Equations 22 and 23, page 11, and Equation 24, page 12, for the waves produced by a shear wave in a solid incident on a liquid are typified by those for water and aluminum in Figure 14. Total reflection of the shear wave is predicted for normal incidence, and the reflected shear energy decreases considerably for increasing angles less than the critical angle. A critical angle occurs at which total reflection of the shear wave is predicted and beyond which there should be no reflected longitudinal energy.

Again, calculations for various values of the longitudinal acoustical impedance, shear velocity, and Poisson's ratio are helpful in determining the characteristics of these energy-ratio equations. Variations of V_s with constant longitudinal acoustical impedance and Poisson's ratio causes only minor changes in

FIGURE 12

CALCULATED ENERGY RATIOS FOR A REFLECTED LONGITUDINAL WAVE,
 L, A REFLECTED SHEAR WAVE, S, AND A TRANSMITTED
 LONGITUDINAL WAVE, T, WHICH ARE PRODUCED
 WHEN A LONGITUDINAL WAVE IS
 INCIDENT FROM BERYLLIUM
 ONTO WATER

FIGURE 13

CALCULATED ENERGY RATIOS FOR A REFLECTED LONGITUDINAL WAVE,
 L, A REFLECTED SHEAR WAVE, S, AND A TRANSMITTED
 LONGITUDINAL WAVE, T, WHICH ARE PRODUCED
 WHEN A LONGITUDINAL WAVE IS INCIDENT
 FROM BERYLLIUM ONTO WATER
 FOR INCIDENT ANGLES
 NEAR 90 DEGREES

FIGURE 14

CALCULATED ENERGY RATIOS FOR A REFLECTED LONGITUDINAL WAVE, L, A REFLECTED SHEAR WAVE, S, AND A TRANSMITTED LONGITUDINAL WAVE, T, WHICH ARE PRODUCED WHEN A SHEAR WAVE IS INCIDENT FROM ALUMINUM ONTO WATER

the energy-ratio levels for angles greater than the critical angle and no other effects.

As before, variation in the longitudinal acoustical impedance causes only changes in the relative energy-ratio levels and produces no curve shape changes or extreme point abnormalities.

However, the variation of Poisson's ratio while the shear velocity and longitudinal acoustical impedance are kept constant, causes curve shape changes similar to those discussed for the previous two incident waves. Examination of Figure 15 shows that both the peak sharpness noted earlier for the longitudinal wave in liquid incident on a solid and the extreme point phenomena characteristic of a longitudinal wave incident from a solid onto a liquid are predicted for the incident shear-wave system.

For low values of Poisson's ratio ($\sigma < 0.25$), the energy-ratio equations predict two distinct angles of total conversion of incident shear wave to a combination of reflected and transmitted longitudinal waves. Arenberg (1948) reported that a shear wave is totally converted to a reflected longitudinal wave at two distinct incident angles at a free boundary of a solid whose Poisson's ratio is less than 0.26.

A reflected longitudinal wave exists only for angles less than the critical angle, a_c , given by:

$$a_c = \sin^{-1} \frac{V_s}{V_l} .$$

FIGURE 15

CALCULATED VALUES OF THE REFLECTED SHEAR WAVE ENERGY FOR A
 SHEAR WAVE IN A SOLID INCIDENT ON WATER FOR DIFFERENT
 VALUES OF POISSON'S RATIO FOR THE SOLID AND FIXED
 VALUES OF SHEAR WAVE VELOCITY AND
 LONGITUDINAL ACOUSTICAL
 IMPEDANCE

The energy ratios for the reflected longitudinal wave undergo wide variations and extreme point changes similar to those for the reflected shear wave in Figure 11, page 28. Maxima occur at two different incident angles for σ less than 0.25, and the minimum between the angles dips greatly for smaller values of Poisson's ratio.

For incident angles greater than the critical angle, α_c , the remaining energy not reflected in the shear mode according to Figure 15 is transmitted as a longitudinal wave.

Extremely sharp peaks in the reflected shear-wave energy for angles near the critical angle, α_c , similar to those discussed earlier for a longitudinal wave incident from a liquid to a solid, are noted also in Figure 15.

Calculated energy ratios plotted in Figure 16 for a beryllium-water system serve to illustrate the total energy distribution for a system whose solid has a low value of Poisson's ratio. Note that a minimum in the reflected longitudinal-wave energy and a maximum in the reflected shear-wave energy is predicted for angles near the critical angle. Actually both these energy ratios experience another extreme point before they reach the critical angle, as shown in the plot expanded 5,000 times in Figure 17. The reflected shear energy reaches a zero minimum at an angle amazingly close to the critical angle before total reflection in the shear mode is predicted at the critical angle. Conversely, the reflected longitudinal-wave energy ratio increases to about 0.93 and suddenly decreases to zero for angles near the critical angle.

FIGURE 16

CALCULATED ENERGY RATIOS FOR A REFLECTED SHEAR WAVE, S, A REFLECTED LONGITUDINAL WAVE, L, AND A TRANSMITTED LONGITUDINAL WAVE, T, WHICH ARE PRODUCED WHEN A SHEAR WAVE IS INCIDENT FROM BERYLLIUM ONTO WATER

FIGURE 17

CALCULATED ENERGY RATIOS FOR A REFLECTED SHEAR WAVE, S, A
 REFLECTED LONGITUDINAL WAVE, L, AND A TRANSMITTED
 LONGITUDINAL WAVE, T, PRODUCED WHEN A SHEAR
 WAVE IS INCIDENT FROM BERYLLIUM ONTO
 WATER FOR ANGLES NEAR THE
 CRITICAL ANGLE

CHAPTER IV

MEASUREMENT OF THE REFLECTION-ENERGY RATIO

Experimental investigation of the calculated reflection-energy ratio of a longitudinal wave incident at various angles on a solid in a liquid has been made. Measurements have been made for a water-aluminum and a water-beryllium system using 30 Mc./sec. ultrasonic waves. Of particular interest are the experimental results in support of the predicted total reflection and peak sharpness at the first critical angle. Total reflection after the second critical angle and surface-wave excitation have also been investigated.

Experimental Apparatus and Procedure

The basic mechanical fixture for positioning ultrasonic transducers such that the incident transmitting and receiving angles may be varied is accomplished by means of the goniometer shown in Figure 18. The goniometer provides for measurement of the incident angle by means of a scale and vernier graduated to two minutes of arc. Other major features of the fixture include an adjustable transducer angulation on each swinging arm, an adjustable sample holding stage, and small incident and receiving angle ($< 3^\circ$) capabilities, limited practically by the size of the transducers.

The half-inch-diameter transmitting transducer is a commercially mounted lithium sulfate piezoelectric element with a resonant frequency of 10 Mc./sec.

118722(U)

FIGURE 18
ULTRASONIC GONIOMETER

It is driven by the pulsed oscillator of a Sperry Ultrasonic Attenuation Comparator, Model 56A001, at its 30 Mc./sec. harmonic with pulses of two microseconds duration. The receiving transducer, similar to the transmitting transducer, has its signal fed directly into a Tektronix Type 545 oscilloscope which is triggered by a signal from the pulser.

Horizontal alignment of both transducers is accomplished by monitoring the reflected signal from an object at the axis of rotation of the swinging arm. The transducers are then swung to a position facing each other, and the signal transmitted between them is maximized with vertical angulation adjustments. The effects of absorption and beam divergence may be normalized out by considering this signal to be equivalent to total reflection from a surface. With the sample in the holder with its front surface coincident with the axis of rotation of the swinging arms, the transducers are positioned so that a signal reflected from the sample is obtained. A maximum reflected signal is obtained by adjusting slightly a screw-adjustable vertical angulation of the sample holder. The zero reading of the incident-angle indicator is set by maximizing the signal reflected by the sample back to the transmitting transducer at normal incidence.

Actual measurement of the reflection energy is accomplished by setting the transmitter at the desired incident angle and swinging the receiving transducer until a maximum signal is obtained. The voltage produced by the receiving transducer is proportional to the amplitude of the reflected wave or the square root of the reflected energy.

A number proportional to the receiver voltage is obtained by making a Polaroid photograph of the oscilloscope trace and measuring the magnified image on the photograph of the received pulses with an optical comparator. A measure of the precision of this experimental technique was found by making eleven measurements at the same incident angle (40 degrees) during the course of an experiment. The limit of error at the 95% confidence level for a single measurement of receiver voltage was found to be 1.5%, resulting in a 3% limit of error for the measured reflected energy.

A measure of the angular resolution was obtained by leaving the receiving transducer fixed and swinging the incident transducer to maximize the signal. The maximum range of the angular readings was two minutes of arc for ten such repeated trials.

It was determined that total reflection can be obtained from both aluminum and beryllium for angles a few degrees greater than the second critical angle. Thus, the average of the reflection-amplitude values obtained for these high angles is taken to be 100% and is subsequently used for normalization of the remainder of the measurements. This approach makes it possible to check the measurement stability during the experiment without removing the sample.

Experimental Results

A graphical display of the measured and theoretical reflected-energy ratio for Type 1100 aluminum in water is given in Figure 19. All the measurements for

FIGURE 19

CALCULATED (SOLID LINE) AND MEASURED (DOTS) VALUES OF THE REFLECTED ENERGY RATIO FOR A LONGITUDINAL WAVE INCIDENT FROM WATER ONTO TYPE 1100 ALUMINUM

angles less than the second critical angle seem to agree, within the 3% limit of error, with the theoretical predictions, with the possible exception of some measurements at angles near the first critical angle. The location of the first critical angle, α_1 , is determined by:

$$\alpha_1 = \sin^{-1} \frac{V}{V_1}.$$

Since V_1 was measured by pulse-echo techniques with accuracy no better than 0.5% and V was assumed to be 1.49×10^5 cm./sec., the agreement of critical-angle location within about 1.7% appears reasonable. The maximum reflected energy measured at the first critical angle was 96.7%. Although Rollins (1965) reports 100% reflection at the first critical angle for aluminum, it would seem reasonable not to expect fully 100% reflection if any beam spread is present. If any part of the beam is spread such that it is not incident at the critical angle, this portion of the beam would not be totally reflected. This condition would result in a lower measured reflection ratio.

At angles near the second critical angle, the measured reflection energy begins to increase, but drops sharply to a minimum of 58% at an incident angle of 28 degrees. This decrease is presumably due to energy absorption in the surface wave mode. Rollins (1965) reports a similar dip at 29.5 degrees for Type 6061-T6 aluminum.

The plot of the reflection-energy data for beryllium and water in Figure 20 reveals that the measured energy averages about 4% less than the predicted values

FIGURE 20

CALCULATED (SOLID LINE) AND MEASURED (DOTS) VALUES OF THE REFLECTED ENERGY RATIO FOR A LONGITUDINAL WAVE INCIDENT FROM WATER ONTO BERYLLIUM

for angles less than the second critical angle. The extremely sharp increase in reflection at the first critical angle could not be detected in the experimental measurements. From Figure 8, page 24, the predicted width of the reflection peak at the nominal energy ratio of 0.80 is only about 0.00165 degree. Thus, only the percentage of the beam which has divergence less than this amount should exhibit substantially greater reflection than that observed at angles on either side of this peak. Since the reflection occurs in the near field of the transmitter (transmitter-to-sample distance is 18 cm.), the amount of deviation from a plane wave front is difficult to predict, but it appears probable that a major portion of the beam is at an angle greater than ± 0.000825 degree from the beam axis. Many more measurements than those plotted were made for incident angles of 6 to 7 degrees, but no indication of increased reflectance was noted.

The experimental results predict a shear critical angle of about 2 to 4% higher than the calculated value. Again, the uncertainties in the pulse echo measurement of shear velocity in beryllium (9.01×10^5 cm./sec.) and the assumed velocity in water (1.49×10^5 cm./sec.) lower the confidence in the correct prediction of the second critical angle.

An extremely sharp dip in the reflection energy to about 25% was measured at an incident angle of 11 degrees. This change indicates that a large amount of energy presumably is converted into the surface wave mode at this angle.

CHAPTER V

SUMMARY AND CONCLUSIONS

The boundary-value-problem approach to the prediction of energy partition among the various waves produced by a wave incident at various angles on a liquid-solid boundary appears to give satisfactory results for ordinary shear and longitudinal-wave motion. Theoretical predictions agree reasonably well with experimental measurements for the case of a longitudinal wave incident in water on aluminum and beryllium. These experimental measurements enhance the confidence in several theoretical predictions including the nominal reflected-energy-ratio value, total reflection and peak sharpness at the first critical angle, and total reflection at the second critical angle.

The subject of surface-wave propagation at a liquid-solid interface appears to be deficient in theoretical explanation, but the existence of such waves has been documented experimentally. Strong energy absorption, presumably due to surface-wave propagation, is experimentally observed for the water-aluminum and water-beryllium systems studied.

The presentation of the characteristics of the energy-ratio equations, as determined by computerized calculations for many simulated water-solid systems, furnishes guides for predicting energy ratios for many liquid-solid systems. In general, the ratio of longitudinal acoustical impedance of the liquid and solid

determine the relative magnitudes of the emerging waves. Also, the most important contribution of the shear-wave velocity is that of determining the second critical angle and thus the angular range of reduced reflection for a longitudinal wave in liquid incident on a solid.

Very significant changes in the energy ratios are predicted when the solid of the system has various values of Poisson's ratio. An extremely sharp reflection energy peak at the first critical angle for a longitudinal wave in a liquid incident on a solid is predicted for solids such as beryllium which have low values of Poisson's ratio. The reflection peak is so sharp for beryllium that it was not observed experimentally with the technique used.

When the Poisson's ratio of the solid is less than about 0.28, drastic changes in the predicted energy ratios occur for the waves produced by a longitudinal wave in the solid incident on water. These changes include major differences in the extreme points of the energy ratios of the reflected shear and longitudinal waves, with predictions of total conversion of the incident longitudinal wave to a reflected shear wave and a transmitted longitudinal wave for two distinct incident angles.

The energy ratios for waves produced by a shear wave in the solid incident on water display both an unusual reflection-energy peak sharpness at the critical angle and significant extreme point changes when the solid has a low value of Poisson's ratio.

BIBLIOGRAPHY

BIBLIOGRAPHY

- Arenberg, David L. "Ultrasonic Solid Delay Lines," Journal of the Acoustical Society of America, 20:1-26, 1948.
- Blut, Heinrich. "Ein Beitrag zur Theorie der Reflexion und Brechung elastischer Wellen an Unstetigkeitsflächen," Zeitschrift für Geophysik, pp. 130-144, 305-322, Jahrgang 8, 1932, cited by J. B. Macelwane, Introduction to Theoretical Seismology. Part I. New York: John Wiley and Sons, Inc., 1936.
- Brekhovskikh, Leonid M. Waves in Layered Media. Trans. David Lieberman. New York: Academic Press, Inc., 1960.
- Ergin, Kazim. "Energy Ratio of Seismic Waves Reflected and Refracted at a Rock-Water Boundary," Bulletin of the Seismological Society of America, 42:349-372, 1952.
- Ewing, W. M., Jardetzky, W. S., and Press, F. Elastic Waves in Layered Media. New York: McGraw-Hill Book Company, Inc., 1957.
- Knott, C. G. "Reflection and Refraction of Elastic Waves, with Seismological Applications," Philosophical Magazine, 48:64-97, 1899.
- Macelwane, J. B. Introduction to Theoretical Seismology. Part I. New York: John Wiley and Sons, Inc., 1936.
- McMaster, Robert C. (ed.). Nondestructive Testing Handbook. Vol. II. New York: The Ronald Press Company, 1963.
- Mason, Warren P. (ed.). Physical Acoustics. Vol I, Part A. New York: Academic Press, Inc., 1964.
- Mayer, Walter G. "Reflection and Refraction of Mechanical Waves at Solid-Liquid Boundaries," Journal of Applied Physics, 34:909-911, 1963.
- _____. "Reflection and Refraction of Mechanical Waves at Solid-Liquid Boundaries. II," Journal of Applied Physics, 34:3286-3288, 1963.
- _____. "Energy Partition of Ultrasonic Waves at Flat Boundaries," Ultrasonics, pp. 62-68, April-June, 1965.

Rollins, F. R. Ultrasonic Wave Propagation and Interaction in Solid Materials.
Technical Documentary Report No. ML-TDR-64-21, Part II, Kansas City,
Missouri: Midwest Research Institute, April, 1965.

Zeoppritz, Karl. "Über Erdbebenwellen VII B: Über Reflexion and Durchgang
seismischer Wellen durch Unstetigkeitsflächen," Nachrichten der Akademische
Wissenschaften Goettingen, Mathematische-Physikalische Klasse, pp. 66-84,
Berlin, 1919, cited by J. B. Macelwane, Introduction to Theoretical
Seismology. Part I. New York: John Wiley and Sons, Inc., 1936.

APPENDIXES

APPENDIX A

A THEORETICAL APPROACH TO SURFACE WAVE PHENOMENA

Brekhovskikh (1960) and Ewing, Jardetzky, and Press (1957) maintain that when the denominator of the amplitude ratio equations is zero, the amplitude of all the waves leaving or approaching the boundary must approach zero. Thus, for these conditions, there exists a surface wave propagating along the boundary known as a generalized Rayleigh wave or Stoneley wave.

Requiring that the denominator of the amplitude-ratio equations vanish is equivalent to setting the denominator of the energy ratio equations (Equations 16, 17, and 18, page 10) equal to zero to get:

$$\cos \eta + \frac{V_1 \rho_2}{V \rho_1} \cos \alpha \left[1 - 2 \sin \zeta \sin 2 \zeta \left(\cos \zeta - \frac{V_s}{V_1} \cos \eta \right) \right] = 0. \quad (25)$$

One item of experimental and theoretical interest is the value of the incident angle, α , which satisfies Equation 25 and thus is required for surface-wave excitation. The use of Equation 6, page 6, permits the transformation of Equation 25 to:

$$\begin{aligned} & \sqrt{1 - \frac{V_1^2}{V^2} \sin^2 \alpha} + \frac{V_1 \rho_2}{V \rho_1} \cos \alpha \left[1 - 4 \frac{V_s^2}{V^2} \sin^2 \alpha \left(1 - \frac{V_s^2}{V^2} \sin^2 \alpha \right) + \right. \\ & \left. 4 \frac{V_s^3}{V^2 V_1} \sin^2 \alpha \sqrt{1 - \frac{V_s^2}{V^2} \sin^2 \alpha} \sqrt{1 - \frac{V_1^2}{V^2} \sin^2 \alpha} \right] = 0. \end{aligned} \quad (26)$$

For most practical systems, V is less than V_s which is, in turn, less than V_l , and there exists a critical angle, α_s , beyond which no shear or longitudinal-wave energy is transmitted into the solid. The critical angle, α_s , is given by: $\alpha_s = \sin^{-1}(V/V_s)$. The fact that the energy ratios may be calculated for all incident angles less than α_s , as has been verified by numerous calculations, means that the incident angle which satisfies Equation 26 must be greater than α_s .

Because of the relative values of the velocities, all the quantities under the square root symbols in Equation 26 are negative for values of α greater than α_s . Thus for α greater than α_s , the first term on the left hand side of Equation 26 is purely imaginary and the second term is real. If Equation 26 is valid, the imaginary and real parts of the left hand side must vanish separately for the same value of α . Of course, the real value of α which makes the imaginary first term vanish is the first critical angle, α_l , which is given by $\alpha_l = \sin^{-1}(V/V_l)$. Since α_l is less than α_s , there is a fallacy in the treatment just described because angles less than α_s were precluded as a solution to Equation 26 at the outset.

Thus, it may be seen that the simple approach using a real value of α for the solution to Equation 26 is inadequate. Rather, Equation 26 must require complex variable techniques for the determination of its solution and the interpretation of the results.

APPENDIX B

A COMPUTER PROGRAM FOR CALCULATING
ACOUSTICAL QUANTITIES

Numerous quantities of importance in ultrasonics and elasticity may be calculated from values of the mass density and shear and longitudinal wave acoustical velocities in materials. In addition to the energy ratios of reflected and transmitted ultrasonic waves at an interface, these densities and velocities permit calculations of the elastic constants of a solid, acoustical impedances, refraction angles, and critical angles for the system. A computer program has been written which calculates these quantities of interest for a liquid-solid system from the following input data: (1) mass density of the solid, ρ_2 , (2) longitudinal wave velocity for the solid, V_l , (3) shear wave velocity for the solid, V_s , (4) mass density of the liquid, ρ_1 , and (5) longitudinal wave velocity for the liquid, V .

The program directs calculation of the quantities from the formulas that follow, given by McMaster (1963), and the results are printed in both the English and centimeter-gram-second system of units

1. Poisson's ratio, σ :

$$\sigma = \frac{1 - 2 (V_s/V_l)^2}{2 - 2 (V_s/V_l)^2}.$$

2. Young's modulus of elasticity, Y :

$$Y = V_l^2 \rho_2 \frac{(1 + \sigma)(1 - 2\sigma)}{1 - \sigma}.$$

3. Shear modulus, μ :

$$\mu = V_s^2 \rho_2.$$

4. Bulk modulus, K :

$$K = \frac{Y}{3 - 6\sigma}.$$

5. Thin rod velocity, V_t :

$$V_t = V_l \sqrt{\frac{(1 + \sigma)(1 - 2\sigma)}{(1 - \sigma)}}.$$

6. Rayleigh surface wave velocity, V_r :

$$V_r = V_s \left(\frac{0.87 + 1.12\sigma}{1 + \sigma} \right) \text{ (Bergmann approximation).}$$

7. Longitudinal acoustical impedance of the solid, Z_s :

$$Z_s = V_l \rho_2.$$

8. Longitudinal acoustical impedance of the liquid, Z_l :

$$Z_l = V \rho_1.$$

9. First critical angle for a longitudinal wave in a liquid incident on a solid, α_l :

$$\alpha_l = \sin^{-1} (V/V_l), \quad (\text{if } V < V_l).$$

10. Second critical angle for a longitudinal wave in a liquid incident on a solid, α_s :

$$\alpha_s = \sin^{-1} (V/V_s), \quad (\text{if } V < V_s).$$

11. Critical angle for a shear wave in a solid incident on a liquid, α_c :

$$\alpha_c = \sin^{-1} (V_s/V_l).$$

The energy ratios and refraction angles are calculated from Equation 6, page 6, and Equations 16; 17, and 18, page 10, for the waves produced by a longitudinal wave in a liquid incident on a solid. The incident-angle interval from normal incidence to the second critical angle is divided into one hundred equally spaced angles, and the three energy ratios and two refraction angles are calculated for each of these angles. These data are printed in tabular form and are also graphically plotted by the Benson-Lehner Electroplotter. Special provisions are made to calculate the quantities for an incident angle exactly equal to the first critical angle even if it does not happen to be one of the one hundred evenly spaced angles.

The refraction angles and energy ratios for the waves produced by a longitudinal wave or a shear wave in a solid incident on a liquid are calculated from Equation 6, page 6, Equation 19, page 10, Equations 20, 21, 22, and 23, page 11, and Equation 24, page 12. Calculations of these quantities are made for every integral-degree incident angle from normal incidence to ninety degrees. Tabular and plotted presentation of these data is obtained also. The data sheets that follow and Figures 21 through 24 are reproductions of typical data tabulations and plots for the entire program, just as they are received from the Oak Ridge Central Data Processing Facility.

COMPUTER DATA PRINTOUT

MECHANICAL WAVES AT A LIQUID-SOLID INTERFACE

MATERIALS CONSIDERED
LIQUID - WATER
SOLID - ALUMINUM

SOURCE OF INPUT DATA - MEASURED AT Y-12 PLANT

INPUT DATA FOR THE SOLID

MASS DENSITY -	0.27000E C1 CM/CM**3	C.25266E-C3 LB-SEC**2/IN**4
LONGITUDINAL VELOCITY -	0.63100E C6 CM/SEC	C.24843E C6 IN/SEC
SHEAR VELOCITY -	0.31600E C6 CM/SEC	C.12441E C6 IN/SEC

INPUT DATA FOR THE LIQUID

MASS DENSITY -	0.10000E C1 CM/CM**3	C.93578E-C4 LB-SEC**2/IN**4
LONGITUDINAL VELOCITY -	0.14900E C6 CM/SEC	C.58661E C5 IN/SEC

CALCULATED VELOCITIES FOR THE SOLID

THIN ROD VELOCITY -	0.51589E C6 CM/SEC	C.2C311E C6 IN/SEC
RAYLEIGH SURFACE WAVE VELOCITY -	0.29464E C6 CM/SEC	C.11160E C6 IN/SEC

CALCULATED ELASTIC CONSTANTS FOR THE SOLID

POISSONS RATIO -	0.33263E C0	
YOUNGS MODULUS OF ELASTICITY -	0.71858E 12 DYNES/CM**2	C.1C423E C8 PSI
SHEAR MODULUS -	0.26961E 12 DYNES/CM**2	C.391C6E C7 PSI
BULK MODULUS -	0.71555E 12 DYNES/CM**2	C.1C379E C8 PSI

CALCULATED LONGITUDINAL ACOUSTIC IMPEDANCE

SOLID -	0.17C37E C7 CM/CM**2-SEC	C.62767E C2 LB-SEC/IN**3
LIQUID -	0.14900E C6 CM/CM**2-SEC	C.54894E C1 LB-SEC/IN**3

CALCULATED CRITICAL ANGLES FOR THE SYSTEM

LONGITUDINAL INCIDENT IN LIQUID		
FIRST CRITICAL ANGLE -	0.13658E C2 DEGREES	C.23838E C0 RADIANS
SECOND CRITICAL ANGLE -	0.28133E C2 DEGREES	C.491C1E C0 RADIANS
SHEAR INCIDENT IN SOLID -	0.30C52E C2 DEGREES	C.52451E C0 RADIANS

CALCULATED REFRACTION ANGLES AND ENERGY RATIOS FOR LONGITUDINAL WAVE IN LIQUID INCIDENT ON SOLID

INCIDENT ANGLE (DEGREES)	REFRACTION ANGLE (DEGREES)		ENERGY RATIO.....		
	LONGITUDINAL	SHEAR	REFLECTED	LONGITUDINAL	SHEAR
0.	0.	0.	C.7C42	C.2958	0.
0.285	1.205	0.603	C.7C42	C.2958	C.0001
0.569	2.411	1.207	C.7C42	C.2958	C.0003
0.854	3.617	1.811	C.7C42	C.2953	C.0006
1.138	4.826	2.414	C.7C41	C.2948	C.0011
1.423	6.036	3.018	C.7C41	C.2943	C.0016
1.707	7.249	3.623	C.7C41	C.2936	C.0024
1.992	8.464	4.227	C.7C40	C.2928	C.0032
2.276	9.684	4.832	C.7C40	C.2918	C.0042
2.561	10.908	5.438	C.7C39	C.2908	C.0053
2.846	12.136	6.043	C.7C39	C.2896	C.0065
3.130	13.370	6.650	C.7C38	C.2883	C.0079
3.415	14.610	7.257	C.7C38	C.2868	C.0094

COMPUTER DATA PRINTOUT (CONTINUED)

3.699	15.856	7.864	C.7C37	C.2853	C.0110
3.984	17.110	8.473	C.7C37	C.2836	C.0127
4.268	18.372	9.082	C.7C36	C.2818	C.0146
4.553	19.643	9.692	C.7C36	C.2798	C.0166
4.837	20.923	10.302	C.7C36	C.2778	C.0187
5.122	22.214	10.914	C.7C35	C.2756	C.0209
5.406	23.517	11.527	C.7C35	C.2733	C.0232
5.691	24.831	12.140	C.7C35	C.2709	C.0256
5.976	26.159	12.755	C.7C35	C.2683	C.0282
6.260	27.502	13.371	C.7C35	C.2657	C.0308
6.545	28.861	13.988	C.7C36	C.2629	C.0335
6.829	30.237	14.607	C.7C37	C.2600	C.0364
7.114	31.631	15.227	C.7C3E	C.2570	C.0393
7.398	33.046	15.848	C.7C39	C.2538	C.0422
7.683	34.483	16.471	C.7041	C.2506	C.0453
7.967	35.944	17.095	C.7044	C.2472	C.0484
8.252	37.432	17.722	C.7047	C.2437	C.0516
8.537	38.949	18.349	C.7051	C.2401	C.0547
8.821	40.498	18.979	C.7056	C.2364	C.0579
9.106	42.082	19.611	C.7062	C.2326	C.0611
9.390	43.706	20.244	C.707C	C.2287	C.0643
9.675	45.373	20.880	C.7079	C.2247	C.0674
9.959	47.089	21.518	C.709C	C.2205	C.0705
10.244	48.862	22.158	C.7104	C.2162	C.0733
10.528	50.697	22.800	C.712C	C.2119	C.0760
10.813	52.606	23.445	C.7141	C.2075	C.0785
11.097	54.600	24.092	C.7166	C.2029	C.0806
11.382	56.695	24.742	C.7197	C.1981	C.0822
11.667	58.911	25.395	C.7236	C.1933	C.0831
11.951	61.277	26.051	C.7287	C.1881	C.0832
12.236	63.833	26.710	C.7354	C.1826	C.0820
12.520	66.644	27.371	C.7445	C.1766	C.0790
12.805	69.814	28.036	C.7574	C.1693	C.0733
13.089	73.552	28.705	C.7776	C.1593	C.0630
13.374	78.393	29.377	C.8148	C.1409	C.0443
13.658	90.000	30.052	C.0000	C.0000	C.0000
13.943	0.	30.717	C.8979	0.	C.1021
14.215	0.	31.385	C.8019	0.	C.1961
14.493	0.	32.058	C.7203	0.	C.2797
14.772	0.	32.734	C.6567	0.	C.3433
15.050	0.	33.415	C.6106	0.	C.3894
15.329	0.	34.100	C.5795	0.	C.4205
15.607	0.	34.790	C.5599	0.	C.4401
15.885	0.	35.485	C.5490	0.	C.4510
16.164	0.	36.185	C.5440	0.	C.4560
16.442	0.	36.890	C.5431	0.	C.4569
16.720	0.	37.601	C.5446	0.	C.4552
16.999	0.	38.317	C.5483	0.	C.4517
17.277	0.	39.040	C.5527	0.	C.4473
17.555	0.	39.769	C.5577	0.	C.4423

COMPUTER DATA PRINTOUT (CONTINUED)

17.834	0.	40.505	C.5629	C.	0.4371
18.112	0.	41.247	C.5681	C.	0.4319
18.390	0.	41.997	C.5732	C.	0.4268
18.669	0.	42.755	C.5781	C.	0.4219
18.947	0.	43.521	C.5826	C.	0.4174
19.226	0.	44.295	C.5869	C.	0.4131
19.504	0.	45.079	C.5908	C.	0.4092
19.782	0.	45.871	C.5944	C.	0.4056
20.061	0.	46.674	C.5976	C.	0.4024
20.339	0.	47.488	C.6005	C.	0.3995
20.617	0.	48.313	C.6031	C.	0.3969
20.896	0.	49.150	C.6054	C.	0.3946
21.174	0.	50.000	C.6075	C.	0.3925
21.452	0.	50.863	C.6092	C.	0.3908
21.731	0.	51.741	C.6107	C.	0.3893
22.009	0.	52.635	C.6119	C.	0.3881
22.287	0.	53.545	C.6129	C.	0.3871
22.566	0.	54.474	C.6138	C.	0.3862
22.844	0.	55.422	C.6144	C.	0.3856
23.123	0.	56.391	C.6148	C.	0.3852
23.401	0.	57.384	C.6152	C.	0.3848
23.679	0.	58.402	C.6154	C.	0.3846
23.958	0.	59.449	C.6155	C.	0.3845
24.236	0.	60.526	C.6156	C.	0.3844
24.514	0.	61.638	C.6157	C.	0.3843
24.793	0.	62.789	C.6160	C.	0.3840
25.071	0.	63.984	C.6164	C.	0.3836
25.349	0.	65.230	C.6171	C.	0.3829
25.628	0.	66.534	C.6184	C.	0.3816
25.906	0.	67.907	C.6204	C.	0.3796
26.184	0.	69.363	C.6236	C.	0.3764
26.463	0.	70.921	C.6284	C.	0.3716
26.741	0.	72.607	C.6359	C.	0.3641
27.020	0.	74.465	C.6474	C.	0.3526
27.298	0.	76.566	C.6658	C.	0.3342
27.576	0.	79.046	C.6967	C.	0.3033
27.855	0.	82.265	C.7554	C.	0.2446
28.133	0.	90.000	1.0000	C.	0.0000

COMPUTER DATA PRINTOUT (CONTINUED)

CALCULATED REFRACTION ANGLES AND ENERGY RATIOS FOR LONGITUDINAL WAVE IN SOLID INCIDENT ON LIQUID

INCIDENT ANGLE (DEGREES)	ANGLE OF REFRACTED LONGITUDINAL (DEGREES)	ANGLE OF REFLECTED SHEAR (DEGREES)	ENERGY RATIO.....		
			REFLECTED LONGITUDINAL	REFLECTED SHEAR	REFRACTED LONGITUDINAL
0.	0.	0.	0.7042	0.	0.2958
1.000	0.236	0.501	0.7037	0.0005	0.2958
2.000	0.472	1.001	0.7023	0.0021	0.2956
3.000	0.708	1.502	0.6999	0.0046	0.2954
4.000	0.944	2.002	0.6966	0.0082	0.2951
5.000	1.179	2.502	0.6924	0.0128	0.2947
6.000	1.414	3.001	0.6872	0.0184	0.2943
7.000	1.649	3.499	0.6813	0.0250	0.2937
8.000	1.883	3.997	0.6744	0.0325	0.2931
9.000	2.117	4.493	0.6667	0.0409	0.2924
10.000	2.350	4.989	0.6582	0.0503	0.2916
11.000	2.582	5.483	0.6489	0.0605	0.2907
12.000	2.814	5.977	0.6388	0.0715	0.2897
13.000	3.045	6.468	0.6281	0.0833	0.2887
14.000	3.275	6.959	0.6167	0.0958	0.2875
15.000	3.504	7.447	0.6046	0.1091	0.2863
16.000	3.732	7.934	0.5920	0.1230	0.2851
17.000	3.959	8.419	0.5788	0.1375	0.2837
18.000	4.185	8.902	0.5651	0.1526	0.2823
19.000	4.409	9.384	0.5510	0.1682	0.2808
20.000	4.632	9.862	0.5364	0.1843	0.2793
21.000	4.854	10.339	0.5216	0.2008	0.2777
22.000	5.075	10.813	0.5064	0.2177	0.2760
23.000	5.294	11.284	0.4909	0.2348	0.2742
24.000	5.511	11.753	0.4753	0.2523	0.2724
25.000	5.727	12.219	0.4595	0.2700	0.2706
26.000	5.942	12.682	0.4436	0.2878	0.2687
27.000	6.154	13.141	0.4276	0.3057	0.2667
28.000	6.365	13.598	0.4116	0.3237	0.2647
29.000	6.574	14.051	0.3957	0.3417	0.2626
30.000	6.781	14.501	0.3799	0.3596	0.2605
31.000	6.985	14.947	0.3641	0.3775	0.2584
32.000	7.186	15.390	0.3486	0.3953	0.2562
33.000	7.389	15.828	0.3332	0.4128	0.2539
34.000	7.588	16.263	0.3181	0.4302	0.2517
35.000	7.784	16.693	0.3033	0.4473	0.2494
36.000	7.970	17.119	0.2888	0.4641	0.2471
37.000	8.170	17.541	0.2747	0.4806	0.2447
38.000	8.359	17.958	0.2609	0.4967	0.2424
39.000	8.546	18.370	0.2475	0.5125	0.2400
40.000	8.730	18.778	0.2346	0.5278	0.2376
41.000	8.912	19.181	0.2221	0.5427	0.2352
42.000	9.091	19.578	0.2101	0.5571	0.2328
43.000	9.267	19.971	0.1986	0.5710	0.2304
44.000	9.441	20.358	0.1876	0.5845	0.2280
45.000	9.612	20.739	0.1771	0.5974	0.2256

COMPUTER DATA PRINTOUT (CONTINUED)

46.000	9.780	21.115	0.1671	C.6C98	0.2232
47.000	9.945	21.485	0.1576	C.6216	0.2207
48.000	10.107	21.849	0.1487	0.6329	0.2183
49.000	10.266	22.207	0.1404	C.6437	0.2160
50.000	10.422	22.559	0.1325	C.6535	0.2136
51.000	10.574	22.904	0.1252	C.6635	0.2112
52.000	10.724	23.243	0.1185	C.6726	0.2089
53.000	10.870	23.575	0.1123	0.6812	0.2066
54.000	11.013	23.900	C.1C66	C.6891	0.2042
55.000	11.153	24.219	0.1015	C.6966	0.2020
56.000	11.289	24.530	C.0969	C.7034	0.1997
57.000	11.422	24.835	C.0928	C.7097	0.1975
58.000	11.552	25.131	0.0893	C.7155	0.1953
59.000	11.678	25.421	C.0863	C.7207	0.1931
60.000	11.800	25.703	C.0838	C.7253	0.1909
61.000	11.919	25.977	0.0819	C.7294	0.1887
62.000	12.034	26.243	C.0805	C.7329	0.1866
63.000	12.146	26.501	C.0797	C.7358	0.1844
64.000	12.253	26.751	0.0796	C.7381	0.1823
65.000	12.357	26.992	0.0801	C.7398	0.1801
66.000	12.458	27.226	C.0812	C.7408	0.1780
67.000	12.554	27.450	C.0832	C.7411	0.1758
68.000	12.647	27.667	0.0859	C.7406	0.1735
69.000	12.735	27.874	C.0896	C.7392	0.1712
70.000	12.820	28.073	0.0943	C.7366	0.1689
71.000	12.901	28.262	C.1C01	C.7335	0.1664
72.000	12.978	28.443	0.1072	0.7290	0.1638
73.000	13.051	28.614	C.1159	C.7231	0.1610
74.000	13.120	28.776	0.1263	C.7157	0.1580
75.000	13.184	28.929	0.1387	C.7066	0.1547
76.000	13.245	29.073	0.1534	C.6954	0.1511
77.000	13.302	29.206	C.1709	C.6819	0.1472
78.000	13.354	29.331	C.1915	C.6657	0.1428
79.000	13.403	29.445	C.2158	0.6463	0.1378
80.000	13.447	29.550	0.2445	C.6233	0.1322
81.000	13.497	29.645	C.2781	C.5960	0.1259
82.000	13.523	29.730	C.3177	C.5638	0.1185
83.000	13.555	29.806	C.3641	C.5258	0.1101
84.000	13.582	29.871	C.4184	0.4811	0.1005
85.000	13.605	29.926	C.4821	C.4286	0.0893
86.000	13.625	29.972	C.5566	C.3671	0.0763
87.000	13.639	30.007	C.6435	C.2952	0.0612
88.000	13.650	30.032	C.7449	C.2113	0.0438
89.000	13.656	30.047	C.8629	C.1136	0.0235
90.000	13.658	30.052	1.0000	C.0000	0.0000

COMPUTER DATA PRINTOUT (CONTINUED)

CALCULATED REFRACTION ANGLES AND ENERGY RATIOS FOR SHEAR WAVE IN SOLID INCIDENT ON LIQUID

SHEAR INCIDENT ANGLE (DEGREES)	ANGLE OF TRANSMITTED LONGITUDINAL (DEGREES)	ANGLE OF REFLECTED LONGITUDINAL (DEGREES)	ENERGY RATIO.....		
			REFLECTED SHEAR	REFLECTED LONGITUDINAL	TRANSMITTED LONGITUDINAL
0.	0.	0.	1.0000	0.	0.
1.000	0.472	1.997	0.9978	0.0021	0.0002
2.000	0.943	3.996	0.9911	0.0082	0.0007
3.000	1.414	5.999	0.9799	0.0184	0.0016
4.000	1.885	8.007	0.9646	0.0326	0.0029
5.000	2.355	10.023	0.9450	0.0505	0.0045
6.000	2.825	12.048	0.9216	0.0720	0.0064
7.000	3.294	14.085	0.8944	0.0969	0.0087
8.000	3.763	16.135	0.8637	0.1249	0.0114
9.000	4.230	18.202	0.8300	0.1557	0.0143
10.000	4.697	20.288	0.7934	0.1890	0.0176
11.000	5.162	22.397	0.7544	0.2244	0.0212
12.000	5.626	24.530	0.7133	0.2616	0.0251
13.000	6.089	26.692	0.6706	0.3002	0.0292
14.000	6.550	28.887	0.6268	0.3396	0.0336
15.000	7.010	31.119	0.5822	0.3796	0.0382
16.000	7.469	33.395	0.5373	0.4197	0.0430
17.000	7.924	35.720	0.4927	0.4594	0.0479
18.000	8.378	38.101	0.4487	0.4983	0.0530
19.000	8.830	40.550	0.4059	0.5360	0.0581
20.000	9.281	43.075	0.3649	0.5720	0.0631
21.000	9.728	45.693	0.3260	0.6060	0.0680
22.000	10.174	48.420	0.2898	0.6375	0.0727
23.000	10.617	51.281	0.2570	0.6662	0.0768
24.000	11.057	54.310	0.2282	0.6915	0.0803
25.000	11.494	57.554	0.2044	0.7130	0.0826
26.000	11.929	61.087	0.1871	0.7297	0.0832
27.000	12.361	65.032	0.1792	0.7398	0.0809
28.000	12.789	69.629	0.1885	0.7375	0.0737
29.000	13.214	75.486	0.2423	0.7014	0.0562
30.052	13.658	90.000	1.0000	0.0000	0.0000
31.000	14.055	0.	0.8559	0.	0.1441
32.000	14.470	0.	0.7266	0.	0.2734
33.000	14.881	0.	0.6307	0.	0.3833
34.000	15.288	0.	0.5832	0.	0.4168
35.000	15.691	0.	0.5558	0.	0.4442
36.000	16.090	0.	0.5442	0.	0.4552
37.000	16.485	0.	0.5432	0.	0.4568
38.000	16.876	0.	0.5466	0.	0.4534
39.000	17.262	0.	0.5525	0.	0.4475
40.000	17.643	0.	0.5593	0.	0.4407
41.000	18.020	0.	0.5664	0.	0.4336
42.000	18.391	0.	0.5732	0.	0.4268
43.000	18.758	0.	0.5795	0.	0.4205
44.000	19.120	0.	0.5853	0.	0.4147
45.000	19.476	0.	0.5904	0.	0.4096

COMPUTER DATA PRINTOUT (CONTINUED)

46.000	19.827	0.	C.5949	C.	0.4051
47.000	20.172	0.	C.5988	C.	C.4012
48.000	20.512	0.	C.6022	C.	C.3978
49.000	20.846	0.	C.6051	C.	C.3949
50.000	21.174	0.	C.6075	C.	C.3925
51.000	21.476	0.	C.6095	C.	C.3905
52.000	21.812	0.	C.6111	C.	C.3889
53.000	22.121	0.	C.6124	C.	C.3876
54.000	22.425	0.	C.6134	C.	C.3866
55.000	22.721	0.	C.6141	C.	C.3859
56.000	23.011	0.	C.6147	C.	C.3853
57.000	23.294	0.	C.6150	C.	C.3850
58.000	23.570	0.	C.6153	C.	C.3847
59.000	23.839	0.	C.6154	C.	C.3846
60.000	24.101	0.	C.6155	C.	C.3845
61.000	24.356	0.	C.6156	C.	C.3844
62.000	24.603	0.	C.6158	C.	C.3842
63.000	24.843	0.	C.6160	C.	C.3840
64.000	25.075	0.	C.6164	C.	C.3836
65.000	25.299	0.	C.6170	C.	C.3830
66.000	25.515	0.	C.6178	C.	C.3822
67.000	25.724	0.	C.6190	C.	C.3810
68.000	25.924	0.	C.6206	C.	C.3794
69.000	26.117	0.	C.6227	C.	C.3773
70.000	26.301	0.	C.6254	C.	C.3746
71.000	26.476	0.	C.6287	C.	C.3713
72.000	26.644	0.	C.6329	C.	C.3671
73.000	26.802	0.	C.6380	C.	C.3620
74.000	26.953	0.	C.6442	C.	C.3558
75.000	27.094	0.	C.6515	C.	C.3485
76.000	27.227	0.	C.6602	C.	C.3398
77.000	27.351	0.	C.6704	C.	C.3296
78.000	27.466	0.	C.6823	C.	C.3177
79.000	27.572	0.	C.6960	C.	C.3040
80.000	27.669	0.	C.7117	C.	C.2883
81.000	27.756	0.	C.7295	C.	C.2705
82.000	27.835	0.	C.7496	C.	C.2504
83.000	27.905	0.	C.7722	C.	C.2278
84.000	27.965	0.	C.7973	C.	C.2027
85.000	28.016	0.	C.8250	C.	C.1750
86.000	28.058	0.	C.8552	C.	C.1448
87.000	28.091	0.	C.8880	C.	C.1120
88.000	28.114	0.	C.9232	C.	C.0768
89.000	28.128	0.	C.9606	C.	C.0394
90.000	28.133	0.	1.0000	C.	C.0000

FIGURE 21

BENSON-LEHNER ELECTRO PLOT OF ENERGY RATIO FOR WAVES
PRODUCED BY A LONGITUDINAL WAVE IN A
LIQUID INCIDENT ON A SOLID

FIGURE 22

BENSON-LEHNER ELECTROPLOT OF ENERGY RATIO FOR WAVES
PRODUCED BY A LONGITUDINAL WAVE IN A
SOLID INCIDENT ON A LIQUID

FIGURE 23

BENSON-LEHNER ELECTROPLOT OF ENERGY RATIO FOR WAVES
PRODUCED BY A SHEAR WAVE IN A SOLID
INCIDENT ON A LIQUID

FIGURE 24

BENSON-LEHNER ELECTROPLOT RELATING THE REFRACTION
ANGLES FOR A LIQUID-SOLID SYSTEM

For some liquid-solid systems, the normal number of calculations is not sufficient to give the energy ratio details desired for a given interval of incident angles. Supplementary programs were written so that numerous calculations may be made for specified incident-angle intervals. These programs are necessary for giving energy-ratio details at angles near the critical angles for a longitudinal wave incident on a solid and a shear wave incident on a liquid, and for angles near ninety degrees for a longitudinal wave incident on a liquid.

SYMBOLS AND NOMENCLATURE

SYMBOL

- A Maximum particle displacement associated with a longitudinal wave incident in a liquid.
- B Maximum particle displacement associated with a reflected longitudinal wave in a liquid.
- b Square root of the energy ratio of the reflected longitudinal wave relative to the longitudinal wave in a liquid incident on a solid.
- C Maximum particle displacement associated with a longitudinal wave transmitted into a solid.
- c Square root of the energy ratio of the transmitted longitudinal wave relative to the longitudinal wave in a liquid incident on a solid.
- D Maximum particle displacement associated with a shear wave transmitted into a solid.
- d Square root of the energy ratio of the transmitted shear wave relative to the longitudinal wave in a liquid incident on a solid.
- e Square root of the energy ratio of the reflected longitudinal wave relative to the longitudinal wave in a solid incident on a liquid.
- f Frequency of a wave in cycles per second.

g	Square root of the energy ratio of the reflected shear wave relative to the longitudinal wave in a solid incident on a liquid.
h	Square root of the energy ratio of the transmitted longitudinal wave relative to the longitudinal wave in a solid incident on a liquid.
i	Square root of the energy ratio of the reflected shear wave relative to the shear wave in a solid incident on a liquid.
j	Square root of the energy ratio of the reflected longitudinal wave relative to the shear wave in a solid incident on a liquid.
k	Square root of the energy ratio of the transmitted longitudinal wave relative to the shear wave in a solid incident on a liquid.
R	Energy ratio at normal incidence of a reflected longitudinal wave relative to a longitudinal wave in a liquid incident on a solid.
r	Ratio of the longitudinal acoustical impedance of the solid to the longitudinal acoustical impedance of the liquid.
S	Ratio of the shear acoustical impedance of the solid to the longitudinal acoustical impedance of the liquid.
t	Time.
V	Longitudinal wave velocity in a liquid.
V_l	Longitudinal wave velocity in a solid.
V_s	Shear wave velocity in a solid.

x	Generalized displacement coordinate parallel to both the liquid-solid boundary and the component of particle displacement which is parallel to the boundary.
y	Generalized displacement coordinate perpendicular to the liquid-solid boundary.
α	Incident or reflected longitudinal wave incident angle.
α_c	Critical incident angle beyond which there will be no longitudinal wave reflected in the solid due to a shear wave incident in the solid.
α_l	Critical incident angle beyond which there will be no longitudinal wave transmitted into a solid due to a longitudinal wave incident in a liquid (if $V_l > V$).
α_s	Critical incident angle beyond which there will be no shear wave transmitted into a solid due to a longitudinal wave incident in a liquid (if $V_s > V$).
β	Incident or reflected shear wave incident angle.
γ	Incident angle of a reflected longitudinal wave due to an incident longitudinal wave.
ζ	Transmitted shear wave incident angle.
η	Transmitted longitudinal wave incident angle.
λ	Wavelength of a mechanical wave.

ρ_1	Mass density of a liquid.
ρ_2	Mass density of a solid.
σ	Poisson's ratio.
ϕ	Particle displacement associated with a longitudinal wave in a liquid incident on a solid.
ϕ_1	Particle displacement associated with a reflected longitudinal wave in a liquid.
ϕ_2	Particle displacement associated with a longitudinal wave transmitted into a solid.
ϕ_s	Particle displacement associated with a shear wave transmitted into a solid.