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INTERSECTING STORAGE ACCELERATOR NOTES

ENERGY AVAILABLE WITH COLLIDING BEAMS

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MASTER

When a particle of rest mass m_1 travelling with a velocity $\beta_1 c$ collides with a particle of mass m_2 travelling in the opposite direction with velocity $-\beta_2 c$, how much energy is available for creation of new particles?

The situation becomes much clearer when a transformation is made to the center-of-mass system, the moving system in which the vector sum of the particles' momenta is zero. In this system the maximum energy will be made available if the two particles are stopped in their tracks by their collision. The particle produced out of their energy will have the greatest possible mass if it has zero energy in the center-of-mass system.

It is not necessary to make an immediate evaluation of the quantities involved in the center-of-mass system. It is sufficient only to note that, in the center-of-mass system the two original particles and the particle produced have zero velocity and hence that, after the collision, all three particles have the same velocity, $\beta' c$, in the laboratory system.

We shall use the following symbols:

m_3 is the mass of the particle produced

$$\gamma_1 = (1 - \beta_1^2)^{-\frac{1}{2}}$$

$$\gamma_2 = (1 - \beta_2^2)^{-\frac{1}{2}}$$

$$\gamma' = (1 - \beta'^2)^{-\frac{1}{2}} .$$

Conservation of energy requires that

$$m_1 \gamma_1 + m_2 \gamma_2 = (m_1 + m_2 + m_3) \gamma' . \quad (1)$$

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From the conservation of momentum

$$m_1 \beta_1 \gamma_1 - m_2 \beta_2 \gamma_2 = (m_1 + m_2 + m_3) \beta' \gamma' \quad (2)$$

If we square (1) and (2) and subtract (2) from (1), recalling the fact that $\gamma^2(1 - \beta^2) = 1$, we obtain

$$\begin{aligned} (m_1 + m_2 + m_3)^2 &= (m_1 \gamma_1 + m_2 \gamma_2)^2 - (m_1 \beta_1 \gamma_1 - m_2 \beta_2 \gamma_2)^2 \\ &= m_1^2 + m_2^2 + 2m_1 m_2 \gamma_1 \gamma_2 (1 + \beta_1 \beta_2) \end{aligned}$$

whence

$$m_3 = \sqrt{m_1^2 + m_2^2 + 2m_1 m_2 \gamma_1 \gamma_2 (1 + \beta_1 \beta_2)} - (m_1 + m_2) \quad (3)$$

If all particles have high energies so β_1 and β_2 are both very close to unity, then Eq. (3) takes the form

$$m_3 \cong 2 \sqrt{m_1 m_2 \gamma_1 \gamma_2} \quad (4)$$

In this case the energy made available for particle production, namely $m_3 c^2$, is given by (4) in terms of the energies E_1 and E_2 of the original particles by

$$m_3 c^2 = 2 \sqrt{E_1 E_2} \quad (5)$$

As an example we consider the case of colliding beams of 200-GeV protons and 10-GeV electrons. The energy available for particle production, given by (5) is

$$2 \sqrt{2000} = 89 \text{ GeV} \quad (6)$$

As a second example we consider collisions between accelerated protons and protons at rest. From (3) the energy available for particle production is

$$m_0 c^2 \left\{ \sqrt{2 + 2E/(m_0 c^2)} - 2 \right\} \quad (7)$$

where m_0 is the rest mass of the proton and E is the total energy of the accelerated proton. For high energies, the available energy is approximately

$$\sqrt{2m_0 c^2 E} \quad (8)$$

To equal the 400 GeV of energy that will be available in ISABELLE, an accelerator bombarding protons at rest would require an energy E [from (8)] of 85,300 GeV.

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