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**TEMPERATURE OF A FLAT PLATE
HAVING ALTERNATING ADIABATIC AND
CONSTANT HEAT-FLUX STRIPS ON ONE SIDE**

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TEMPERATURE OF A FLAT PLATE HAVING ALTERNATING
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STRIPS ON ONE SIDE

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Temperature of a Flat Plate Having Alternating Adiabatic and Constant Heat Flux Strips on One Side

JAMES H. VANSANT¹

Nomenclature

A, B, C = constants

a = distance between heated and insulated strip centerlines

b = half-width of the heated strips

h = convection coefficient

k = thermal conductivity of the plate

Q = heat flux per unit area of the heated strips

$T = t - t_{\infty}$

T_m = maximum temperature variation on the cooled side of the plate, $T(0, 0) - T(0, a)$

t = temperature at x, y position in the plate

t_{∞} = free-stream temperature of the cooling medium

w = thickness of the plate

λ = characteristic value used in separation of variables

Introduction

Many engineering applications involve a flat plate heated on one side by attached strip heaters and cooled on the other side by convection, as illustrated in Fig. 1a. The presence of adiabatic regions between the heaters can cause

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a significant temperature variation on the cooled side of the plate. In this case it may be desirable to estimate the magnitude of the variation. This problem was resolved for a research project conducted recently by the author [1].² The method of solution is not a new one; however, computed results apparently have not been listed in any readily available publications.

Assumptions

Laplace's equation for steady-state heat conduction in two dimensions is assumed to be applicable. The plate thermal conductivity k , the convection coefficient h , and the temperature of the cooling medium t_{∞} , are assumed to be independent of position and to have finite values.

Analysis

An infinite plate with constant-flux heat-source strips and insulation backing, illustrated in Fig. 1a, is the geometry to be considered. The symmetry of heat flow about the centerlines passing through the insulated and heated strips allows use of the mathematical model shown in Fig. 1b. The required boundary conditions for this model are:

$$1. \quad \partial T(x, 0) / \partial y = 0$$

$$2. \quad \partial T(x, a) / \partial y = 0$$

$$3. \quad k \partial T(0, y) / \partial x = h T(0, y)$$

$$4. \quad k \partial T(w, y) / \partial x = \left\{ \begin{array}{ll} Q, & (0 < y < b) \\ 0, & (b < y < a) \end{array} \right\}$$

²Numbers in brackets designate References at end of paper.

Beginning with Laplace's equation in two dimensions and using the separation of variables technique [2], the following can be obtained:

$$T(x,y) = \left[C_1 \cosh(\lambda x) + C_2 \sinh(\lambda x) \right] \left[C_3 \cos(\lambda y) + C_4 \sin(\lambda y) \right] \quad (1)$$

When the first boundary condition is applied to Eq. (1), the result is

$$C_4 = 0 \quad (2)$$

If the second boundary condition is next applied to Eq. (1), one obtains

$$\sin(\lambda a) = 0$$

$$\lambda_n = n\pi/a, \quad n = 0, 1, 2, \dots \quad (3)$$

Equation (1) can now be rewritten as

$$T(x,y) = \sum_{n=0}^{\infty} \left[A_n \cosh(n\pi x/a) + B_n \sinh(n\pi x/a) \right] \cos(n\pi y/a) \quad (4)$$

The third boundary condition requires that

$$B_n = haA_n/n\pi k \quad (5)$$

This allows Eq. (4) to be changed to

$$T(x,y) = \sum_{n=0}^{\infty} A_n \left[\cosh(n\pi x/a) + (ha/n\pi k) \sinh(n\pi x/a) \right] \cos(n\pi y/a) \quad (6)$$

When L'Hospital's rule is applied to the first term of the foregoing series ($n = 0$) with n considered as the independent variable, the following can be obtained:

$$\lim_{n \rightarrow 0} \left\{ A_n \left[\cosh(n\pi x/a) + (ha/n\pi k) \sinh(n\pi x/a) \right] \cos(n\pi y/a) \right\} = A_0(1 + hx/k) \quad (7)$$

This allows one to change Eq. (6) to

$$T(x,y) = A_0(1 + hx/k) + \sum_{n=1}^{\infty} A_n \left[\cosh(n\pi x/a) + (ha/n\pi k) \sinh(n\pi x/a) \right] \cos(n\pi y/a) \quad (8)$$

After the fourth boundary condition has been applied to Eq. (8), the coefficients of the resulting series may then be equated to the coefficients of a Fourier series.

$$\left(A_n k\pi n/a \right) \left[\sinh(n\pi w/a) + (ha/n\pi k) \cosh(n\pi w/a) \right] = 2Q/a \int_0^b \cos(n\pi y/a) dy, \quad n > 0 \quad (9)$$

$$A_n = \frac{2aQ \sin(n\pi b/a)}{k(n\pi)^2 \left[\sinh(n\pi w/a) + (ha/n\pi k) \cosh(n\pi w/a) \right]}, \quad n > 0 \quad (10)$$

$$A_0 h = Q/a \int_0^b dy \quad (11)$$

$$A_0 = Qb/ha \quad (12)$$

After the values of A_n and A_0 (Eqs. (10) and (12)) have been substituted into Eq. (8) the general solution can be written as

$$kT(x,y)/aQ = \left(kb/ha^2 \right) (1 + hx/k) + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin(n\pi b/a) \cos(n\pi y/a)}{n^2 \left[\sinh(n\pi w/a) + (ha/n\pi k) \cosh(n\pi w/a) \right]} \times \left[\cosh(n\pi x/a) + (ha/n\pi k) \sinh(n\pi x/a) \right] \quad (13)$$

The temperature at the cooled side of the plate is thus

$$\frac{kT(0,y)}{aQ} = \frac{kb}{ha^2} + \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin(n\pi b/a) \cos(n\pi y/a)}{n^2 [\sinh(n\pi w/a) + (ha/n\pi k) \cosh(n\pi w/a)]} \quad (14)$$

Results

The temperature of the cooled side of the plate was determined by adding 100 terms of the series expression given in Eq. (14) with a high-speed digital computer. The dimensionless parameters ha/k , b/a , and w/a were varied to find their effect on the dimensionless parameter $kT(0,y)/aQ$. These parameters are presented in Fig. 2 and Table 1 with the temperature given as the maximum variation T_m (i. e., $T(0,0) - T(0,a)$). Listed in Table 1 are some values of the maximum temperature on the cooled surface, $T(0,0)$. The error for these results is estimated to be less than 0.1 percent.

It should be noted that T_m reaches a maximum value at $b/a = 0.5$. This can be established by finding the maxima of the function $T_m(b/a)$. Also, $T_m(b/a) = T_m(1 - b/a)$. These results were borne out by the computer calculations.

Acknowledgments

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References

1. James H. VanSant, "Convection Heat Transfer in Separated Regions — Subsonic Diffusers." Ph.D. Thesis, Oregon State Univ. Aug. 1964, 144 numb. leaves.
2. Dunham Jackson, "Fourier Series and Orthogonal Polynomials," The Mathematical Assoc. of America, Univ. of Buffalo, 1961, p. 90.

Table 1. Values of kT_m/aQ and $kT(0,0)/aQ - 100$ (kT_m/aQ is the upper value of each pair of values)

$b/a \backslash ha/k$		10^{-2}	10^0	10^2
$w/a = 0.05$				
0.5		2.4412	0.7838	0.01019
		1.2206	0.3912	0.00509
0.7		2.0509	0.6929	0.01022
		0.8876	0.2582	0.00305
0.9		0.8769	0.3435	0.00948
		0.3217	0.0891	0.00101
$w/a = 0.05$				
0.5		0.1753	0.1300	0.00444
		0.0874	0.0650	0.00243
0.7		0.1422	0.1060	0.00401
		0.0670	0.0494	0.00176
0.9		0.0549	0.0410	0.00158
		0.0301	0.0228	0.00063
$w/a = 1.5$				
0.5		0.0723	0.0055	0.00022
		0.0036	0.0028	0.00011
0.7		0.0059	0.0048	0.00018
		0.0029	0.0022	0.00009
0.9		0.0023	0.0017	0.00007
		0.0011	0.0009	0.00003

Fig. 1. (a) Flat plate with strip heaters. (b) Mathematical model.

Fig. 2. Maximum temperature variation of the cooled surface of a flat plate having alternating adiabatic and constant heat-flux strips on the opposite surface.

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