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In this Letter, the empirical evidence available on the non-mesic decay of Λ-hypernuclei will be analyzed in terms of the spin- and isospin-dependence of the weak interactions

$$\Lambda + p \rightarrow n + p, \tag{1}$$

$$\Lambda + n \rightarrow n + n. \tag{2}$$

Recently, measurements on the non-mesic decay processes of ${}_{\Lambda}H^{4}$ and ${}_{\Lambda}He^{4}$ hypernuclei produced in K^{-} - He^{4} reactions have been reported by Block et al.[1] For ${}_{\Lambda}He^{4}$, they have obtained

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1. M. M. Block, R. Gessaroli, J. Kopelman, S. Ratti, M. Schneeberger, L. Grimellini, T. Kikuchi, L. Lendinara, L. Monari, W. Becker, and E. Harth, Proc. Intl. Conf. on Hyperfragments at St. Cergue, Switzerland (CERN, 1963), to be published.

$Q({}_\Lambda He^5) = 0.52 \pm 0.10$ and $C({}_\Lambda He^4) = 2.2 \pm 0.8$, where Q denotes the ratio of non-mesic to π^- decay modes and C denotes the ratio of reactions (1) to (2), determined from the energy spectrum of the final-state protons. For ${}_\Lambda H^4$, they have obtained $Q({}_\Lambda H^4) = 0.26 \pm 0.13$. Using theoretical estimates[2] for the π^--mesic decay rates,

2. R. H. Dalitz and G. Rajasekharan, Phys. Letters 1, 58 (1962).

we may deduce the non-mesic decay rates for these hypernuclei,

$$\Gamma_{nm}({}_\Lambda He^4) = (0.14 \pm 0.03)\Gamma_\Lambda \quad , \tag{3a}$$

$$\Gamma_{nm}({}_\Lambda H^4) = (0.29 \pm 0.14)\Gamma_\Lambda \quad , \tag{3b}$$

where $\Gamma_\Lambda = \tau_\Lambda^{-1} = (4.25 \pm 0.1) \times 10^9$ Sec.$^{-1}$ denotes the free Λ decay rate.[3] We note first that these rates are just compatible

3. M. M. Block, R. Gessaroli, S. Ratti, L. Grimellini, T. Kikuchi, L. Lendinara, L. Monari, E. Harth, W. Becker, W. M. Bugg, and H. Cohn, Phys. Rev. 130, 766 (1963).

with the inequality

$$\Gamma_{nm}({}_\Lambda H^4) \leq 2\, \Gamma_{nm}({}_\Lambda He^4) \tag{4}$$

required by the $\Delta I = \frac{1}{2}$ rule.[4]

4. See Appendix, R. H. Dalitz and L. Liu, Phys. Rev. 116, 1312 (1959).

We shall base our detailed analysis on a simplified calculation[2] for the non-mesic decay rates, which treats the Λ de-excitation by different nucleons as incoherent. This procedure neglects final state interactions for the two fast out-going nucleons, and neglects the interference effects which usually arise from antisymmetrization of the final state, corrections which are not expected to be important here, because of the large energy release. In this model, these rates are expressed in terms of the elementary rates R_{NS} for non-mesic de-excitation of a ΛN system with total spin S, for unit density of nucleon N at the Λ position. In light hypernuclei, the initial ΛN states are s-wave, ^1S_o and $^3S_1$. The $\Lambda N \rightarrow NN$ transitions then possible are listed in Table I, together with the spin-dependence of their corresponding matrix element. Thus, for ^1S_o capture, the failure of parity conservation for weak interactions allows both the transitions $^1S_o \rightarrow {}^1S_o$ and $^1S_o \rightarrow {}^3P_o$. We note that ^1S_o capture leads only to I = 1 final states, whereas both I = 0 and I = 1 final states are available for $^3S_1$ capture. From this, it follows that the validity of the $\Delta I = \frac{1}{2}$ rule for the non-mesic decay interactions would require[5]

$$R_{no} = 2R_{po} , \qquad (5a)$$

$$R_{n1} \leq 2R_{p1} , \qquad (5b)$$

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5. The $\Delta I = 1/2$ rule requires the following values for the (Λn) amplitudes: $a_n = 2a$, $b_n = 2b$, $c_n = d_n = e_n = 0$, $f_n = 2f$. The (Λn) reaction rates are only twice those for the (Λp) reactions to the same $I = 1$ final states, because the phase space available for the nn system is only half that for the np system.

With this model,[2] the non-mesic decay rate for hypernucleus ${}_{\Lambda}Z^{A}$ is given by $\rho_A \bar{R}({}_{\Lambda}Z^{A})$, where $\bar{R}$ denotes the spin and charge average of the R_{NS} appropriate to this hypernucleus, and ρ_A denotes the mean nucleon density at the Λ position, given by $\rho_A = (A-1) \times \int \rho_N(\underline{r})\ \psi_{\Lambda}^2(\underline{r})\ d_3r$, ρ_N being the nucleon density (normalized to unity) and ψ_{Λ} the wavefunction for the relative motion between the Λ particle and the nuclear core. For the $J = 0$ hypernuclei ${}_{\Lambda}H^4$ and ${}_{\Lambda}He^4$, we then have the non-mesic rates

$$\Gamma_{nm}({}_{\Lambda}He^4) = \rho_4(3R_{p1} + R_{po} + 2R_{no})/6, \qquad (6a)$$

$$\Gamma_{nm}({}_{\Lambda}H^4) = \rho_4(3R_{n1} + R_{no} + 2R_{po})/6, \qquad (6b)$$

where ρ_4 has the value[2] $0.019\ f^{-3}$, with

$$C({}_{\Lambda}He^4) = (3R_{p1} + R_{po})/2R_{no}\ , \qquad (7a)$$

$$C({}_{\Lambda}H^4) = 2R_{po}/(3R_{n1} + R_{no})\ . \qquad (7b)$$

From (6a), (7a), and the data on ${}_{\Lambda}He^4$ alone, we deduce directly $R_{no} = (6.6 \pm 2.1)\ \Gamma_{\Lambda} f^3$, and $(3R_{p1} + R_{po})/4 = (7.3 \pm 1.8)\ \Gamma_{\Lambda} f^3$.

In order to carry this analysis further, we shall now assume the $\Delta I = \frac{1}{2}$ rule to be valid for these interactions. With the equality (5a), this allows the determination of all the R_{NS} from these data. As remarked above, the non-mesic rates (3) are compatible with the inequality (4); however, with Eqs. (6) and (7) (which introduce the additional information that J = 0), the relations (5) lead to the stronger inequality,[6]

$$\Gamma_{nm}({}_{\Lambda}H^4) \leqslant 2\,\Gamma_{nm}({}_{\Lambda}He^4)\left\{1 - \frac{3}{4(1 + C({}_{\Lambda}He^4))}\right\}, \qquad (8)$$

6. These relations also lead to the inequality $1 \geqslant C({}_{\Lambda}H^4) \geqslant (4C({}_{\Lambda}He^4))^{-1}$.

with which the data is compatible, within experimental error. However, in order to obtain consistent equations, we now adopt the value

$$\Gamma_{nm}({}_{\Lambda}H^4) = (0.21^{+0.05}_{-0.06})\Gamma_{\Lambda}, \qquad (9)$$

where the median value and upper limit are fixed by the upper limit of (8) and the lower limit corresponds to one standard deviation below the measured rate. From the ${}_{\Lambda}He^4$ data, the equality (5a) leads to $R_{p1} = (8.6 \pm 3.0)\Gamma_{\Lambda}f^3$, with $R_{po} = \frac{1}{2}R_{no} = (3.3 \pm 1.1)\Gamma_{\Lambda}f^3$. Finally, the ${}_{\Lambda}H^4$ rate (9) then leads to $R_{n1} = (17.2 \pm 6.0)\Gamma_{\Lambda}f^3$. We note that the equality $R_{n1} = 2R_{p1}$ is required (within experimental errors) by the data; this reflects the fact that the value (9) corresponds to the equality in Eq. (8). Consequently, the $\Lambda N \rightarrow NN$ transitions take place dominantly to I = 1 final states; further, $R_{p1} = 2.6\ R_{po}$, so that the triplet interaction rate is the stronger.[7]

7. These values for R_{NS} lead to the following predictions for He^5 decay: $Q({}_{\Lambda}He^5) = 1.68 \pm 0.36$, $C({}_{\Lambda}He^5) = 0.5$. The first leads to good agreement with the estimates available in the literature (cf. P. Schlein, Phys. Rev. Letters 2, 220 (1959), and earlier references cited there) for $Q({}_{\Lambda}He)$ in emulsion, where the events correspond to a mixture of ${}_{\Lambda}He^4$ and ${}_{\Lambda}He^5$ in a ratio about

1:4. This agreement is not to be regarded as strong support for the $\Delta I = \frac{1}{2}$ rule, however, since the input data and positive definiteness for the R_{NS} already constrain $Q({}_\Lambda He^5)$ to lie between 1.2 ± 1.0 and 2.3 ± 1.0. No measurement of $C({}_\Lambda He^5)$ is yet available; an equivalent estimate may possibly be made by comparing the proton spectrum observed from non-mesic decay of heavy hyperfragments with the proton spectra computed for elementary (Λp) and (Λn) de-excitation processes.

Indeed, from Table I, we can say that the dominant transition is ${}^3S_1 \to {}^3P_1$, together with some ${}^1S_o \to {}^1S_o$ and ${}^3P_o$ transitions, corresponding to a matrix-element of the form:[8]

$$M(\Lambda p \to np) = \frac{f\sqrt{6}}{4M}(\underline{\sigma}_Y + \underline{\sigma}_N)\cdot\underline{q} + \left(a + \frac{b}{2M}(\underline{\sigma}_Y - \underline{\sigma}_N)\cdot\underline{q}\right)\left(\frac{1 - \underline{\sigma}_Y\cdot\underline{\sigma}_N}{4}\right). \tag{10}$$

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8. If time-reversal invariance holds for these interactions, the phases of a,b and f are given by the nucleon-nucleon scattering phases in the corresponding final states.

A measurement of $C({}_\Lambda H^4)$ would provide a sensitive test of the $\Delta I = \frac{1}{2}$ assumption. Since $C({}_\Lambda H^4) = 2R_{po}/(3R_{n1} + R_{no})$, this would determine R_{po} directly and test the significant equality (5a). The value expected from the R_{NS} given above is $C({}_\Lambda H^4) = 1/(8.8 \pm 3.2)$. No empirical estimate of this ratio is available at present.

Finally, we discuss briefly the possibilities for a simple interpretation of these values R_{NS}. With the $\Delta I = \frac{1}{2}$ rule, these four-fermion interactions have form limited to

$$\sum_i \left\{ f_\Lambda (\bar{N} K_i \Lambda_s)(\bar{N} K_i' N) + g_\Lambda (\bar{N} L_i \underline{\tau} \Lambda_s)\cdot(\bar{N} L_i' \underline{\tau} N)\right\}, \tag{11}$$

where Λ_s denotes the spurion ($I = \frac{1}{2}$, $I_3 = -\frac{1}{2}$) wavefunction for the Λ particle. Rewriting this interaction, we have the general form

$$\sum_i \left\{ f_\Lambda (\bar{n} K_i \Lambda)(\bar{p} K_i' p + \bar{n} K_i' n) + g_\Lambda \left[-(\bar{n} L_i \Lambda)(\bar{p} L_i' p - \bar{n} L_i' n) + 2(\bar{p} L_i \Lambda)(\bar{n} L_i' p) \right] \right\}. \qquad (12)$$

The primary four-fermion interaction must also be of this general form; its contributions to the process $\Lambda + p \to n + p$ are illustrated in Figs. 1(a,b). However, with four strongly-interacting particles, the primary form may be distorted by mesonic corrections in quite a complicated way. For example, Karplus and Ruderman[9] have discussed

9. R. Karplus and M. Ruderman, Phys. Rev. 76, 1458 (1949). See also F. Cerulus, Nuovo Cimento 5, 1685 (1957), and S. B. Treiman, Proc. 1958 Intl. Conf. on High Energy Physics (CERN, Geneva, 1958) p. 276.

a class of mesonic corrections directly related with the $\Lambda \to N + \pi$ interaction, illustrated in Figs. 1(c,d), although there is no reason at present to believe that these necessarily represent the dominant corrections.

As the simplest possibility, we consider a general (V,A) four-fermion interaction (11), with

$$K_i = L_i = \gamma_\mu + \lambda \gamma_\mu \gamma_5 \quad , \quad K_i' = L_i' = \gamma_\mu + \eta \gamma_\mu \gamma_5 \quad , \qquad (13)$$

for which the individual transition amplitudes have been listed in Table I. In order to suppress both the I = 0 transitions and the $^1S_o \to {}^1S_o$ transition, it is necessary to choose $(A - 3B) \approx 0$ and $(1 + 3\lambda\eta) \approx 0$, quite closely. The parameters λ, η can then be obtained to fit the ratio of the $^3S_1 \to {}^3P_1$ and $^1S_o \to {}^3P_o$ transition rates; only an upper limit is known for the latter rate, since we know only the sum R_{NO} of the $^1S_o \to {}^1S_o$ and $^1S_o \to {}^3P_o$ rates. The data allows four possible regions for (λ, η), (i) $0.6 \leqslant \eta \leqslant 0.75$ with

λ from -0.6 to -0.45, (ii) $-0.6 \geqslant \eta \geqslant -0.75$ with λ from 0.6 to 0.45, (iii) $0.6 \geqslant \eta \geqslant 0.45$ with λ from -0.6 to -0.75, and (iv) $-0.6 \leqslant \eta \leqslant -0.45$ with λ from 0.6 to 0.75; in each case, the right hand limit corresponds to zero rate for the ${}^1S_o \to {}^1S_o$ transition, the left hand limit to zero rate for the ${}^1S_o \to {}^3P_o$ transition. We note explicitly that these interactions are quite different in character from the (V-A) interaction, for which $\eta = \lambda = +1$ holds and which allows only the transitions ${}^1S_o \to {}^1S_o$ and ${}^3P_o$, contrary to observation. We have no interpretation to offer for the $(3 + \underline{\tau}_Y \cdot \underline{\tau}_N)$ form required by the data; all simple models of weak interactions considered at present lead naturally to a form $\underline{\tau}_Y \cdot \underline{\tau}_N$, since the strangeness-conserving weak current necessarily has an isovector component and is usually assumed to be pure isovector. With these sets (λ, η), it is of interest to note that the value required for g_Λ to fit the total transition rate is $g_\Lambda = 0.35(\pm 0.05) \times 10^{-5}/M^2$, which may be compared with the beta-decay coupling parameter $g_\beta = 1.02 \times 10^{-5}/M^2$.

The Karplus-Ruderman terms provide a second possibility of particular interest. For the process $\Lambda + p \to n + p$, these have the form

$$D\left\{(s_o + p_o \underline{\sigma}_Y \cdot \underline{q}/q_\Lambda)(-\underline{\sigma}_N \cdot \underline{q}/M) - P^\sigma_{YN}\sqrt{2}(s - p\underline{\sigma}_Y \cdot \underline{q}/q_\Lambda)(\underline{\sigma}_N \cdot \underline{q}/M)\right\}, \qquad (14)$$

where (s_o, p_o) and (s,p) denote the $\Lambda \to n + \pi^0$ and $\Lambda \to p + \pi^-$ decay amplitudes, $\underline{\sigma}_Y$ denotes the spin of the Λ particle or the final neutron, P^σ_{YN} is the spin-exchange operator, and the coefficient $D = G/2(M(M_\Lambda - M) + m_\pi^2)$, where $G^2/4\pi \approx 13.5$ is the pion-nucleon coupling constant. The transition amplitudes corresponding to (14) are given in Table I. With the $\Delta I = \frac{1}{2}$ rule, $s_o/s = p_o/p = -1/\sqrt{2}$, and the dominant transitions are those from the $^3S_1$ state to final I = 0 states, quite contrary to the observations;[10]

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10. With the Karplus-Ruderman terms alone, (Λp) de-excitation would be the dominant process. With $x = (pq/sq_\Lambda)^2 \approx 2.2$, the ratio $C({}_\Lambda He^5)$ would be $(6 + 14x)/(3 + x) \approx 7.1$.

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further, the $^1S_o \rightarrow {}^3P_o$ and $^3S_1 \rightarrow {}^3P_1$ and $^1P_1$ transitions contribute comparably to the rates R_{NS}, in the ratio 3:2:9. We note that a linear combination of a (V-A) interaction with the Karplus-Ruderman terms cannot fit the data, since this necessarily gives strong I = 0 transitions; in particular, any linear combination of a (V,A) interaction (13) with the Karplus-Ruderman terms (14) necessarily leaves a strong $^3S_1 \rightarrow {}^1P_1$ amplitude. The only conclusion which can be drawn at this stage is that, if the $\Delta I = \frac{1}{2}$ rule holds for the $\Lambda N \rightarrow NN$ weak interaction and our present notions about the current-current nature of weak interactions are valid, then it must be that the higher-order mesonic corrections to this primary four-fermion interaction are sufficiently large to mask the simplicity of their primary form.

Figure Caption

Fig. 1 Graphs (a),(b) show the primary processes contributing to the $\Lambda + p \rightarrow n + p$ interaction. Graphs (c),(d) show the mesonic correction terms discussed by Karplus and Ruderman.

Allowed Transitions		Matrix-element	Transition rate	(V,A) Interaction	K-R terms (Eq. 19)
${}^1S_0 \to {}^1S_0$	$I=1$	$\frac{a}{4}(1-\underline{\sigma}_Y\cdot\underline{\sigma}_N)$	$\lvert a\rvert^2$	$a=(f_\Lambda+g_\Lambda)(1+3\lambda\eta)$	$a=(p_0+p\sqrt{2})q^2/Mq_\Lambda=(pq^2/\sqrt{2}Mq_\Lambda)$
$\to {}^3P_0$		$\frac{b}{8M}(\underline{\sigma}_Y-\underline{\sigma}_N)\cdot\underline{q}(1-\underline{\sigma}_Y\cdot\underline{\sigma}_N)$	$\lvert b\rvert^2\left(\frac{q}{M}\right)^2$	$b=-2(f_\Lambda+g_\Lambda)(\lambda+\eta)$	$b=(s_0+s\sqrt{2})=s/\sqrt{2}.$
${}^3S_1 \to {}^3S_1$	$I=0$	$\frac{c}{4}(3+\underline{\sigma}_Y\cdot\underline{\sigma}_N)$	$\lvert c\rvert^2$	$c=(f_\Lambda-3g_\Lambda)(1-\lambda\eta)$	$c=(p\sqrt{2}-p_0)q^2/3Mq_\Lambda=(pq^2/\sqrt{2}Mq_\Lambda)$
$\to {}^3D_1$		$\frac{3d}{\sqrt{2}M^2}(\underline{\sigma}_Y\cdot\underline{q}\,\underline{\sigma}_N\cdot\underline{q}-\frac{1}{3}\underline{\sigma}_Y\cdot\underline{\sigma}_N q^2)$	$\lvert d\rvert^2\left(\frac{q}{M}\right)^4$	$d=(f_\Lambda-3g_\Lambda)(1-\lambda\eta)/6\sqrt{2}$	$d=(p\sqrt{2}-p_0)M\sqrt{2}/3q_\Lambda=pM/q_\Lambda.$
$\to {}^1P_1$		$\frac{e\sqrt{3}}{8M}(\underline{\sigma}_Y-\underline{\sigma}_N)\cdot\underline{q}(3+\underline{\sigma}_Y\cdot\underline{\sigma}_N)$	$\lvert e\rvert^2\left(\frac{q}{M}\right)^2$	$e=0$	$e=(s_0-\sqrt{2}s)/\sqrt{3}=-s\sqrt{(3/2)}$
$\to {}^3P_1$	$I=1$	$\frac{f\sqrt{6}}{4M}(\underline{\sigma}_Y+\underline{\sigma}_N)\cdot\underline{q}$	$\lvert f\rvert^2\left(\frac{q}{M}\right)^2$	$f=\sqrt{\frac{2}{3}}(f_\Lambda+g_\Lambda)(\eta-\lambda)$	$f=-2(\sqrt{2}s+s_0)/\sqrt{6}=-s/\sqrt{3}$

Table I. Summary of the properties of the transitions $\Lambda p \to np$ from initial ${}^1S_0$ and ${}^3S_1$ states. The spin $\underline{\sigma}_Y$ denotes that for the Λ particle or the final neutron, and $\underline{q}$ denotes the final neutron momentum ($(q/M)^2 \simeq 0.20$). The rates R_{NS} are given by $R_{p0} = \lvert a\rvert^2 + \lvert b\rvert^2(q/M)^2$, $R_{p1} = \lvert c\rvert^2 + \lvert d\rvert^2(q/M)^4 + \lvert e\rvert^2(q/M)^2 + \lvert f\rvert^2(q/M)^2$, and by $R_{n0} = \lvert a_n\rvert^2 + \lvert b_n\rvert^2(q/M)^2$, $R_{n1} = \lvert f_n\rvert^2(q/M)^2$. The coefficients a,...f are given for (Λp) de-excitation through the (V,A) interaction of Eqs.(17) and (18), and through the Karplus-Ruderman processes of Figs. 1(c,d). The last entry is given for $(s_0,p_0) = -(s,p)/\sqrt{2}$, corresponding to the $\Delta I = 1/2$ rule; the value of (pq/sq_Λ) is about 1.45.

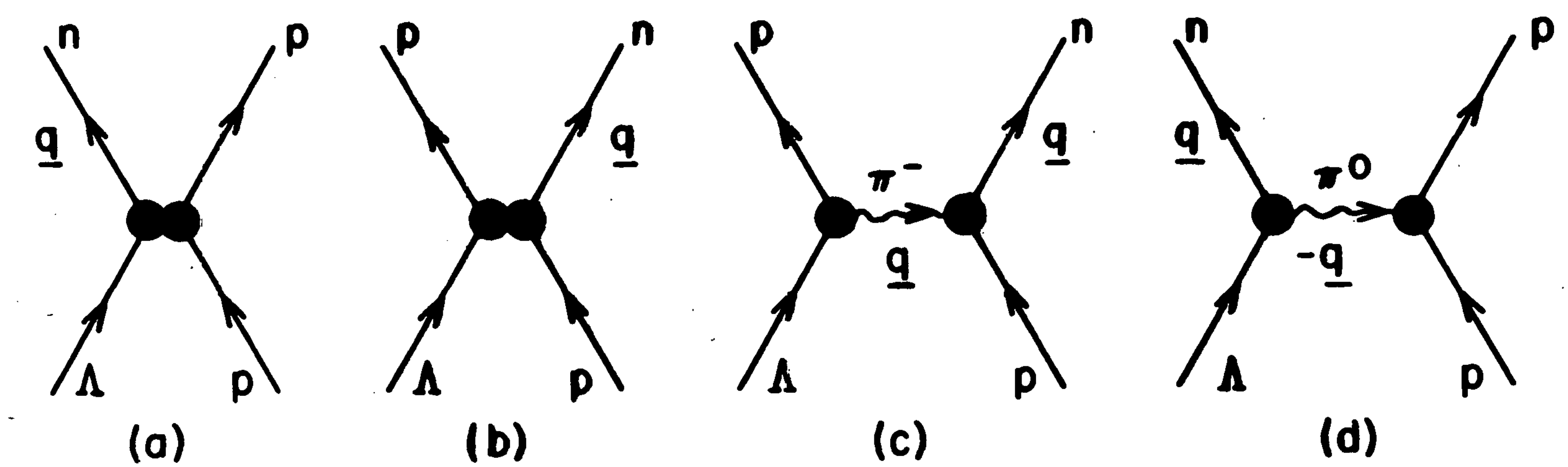

Fig. 1