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A COMPUTER PROGRAM TO EVALUATE
STRAIN HARDENING EXPONENT (SEX)

Stanley Beitscher

Joe H. Doyle

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A COMPUTER PROGRAM TO EVALUATE STRAIN HARDENING EXPONENT (SEX)

Stanley Beitscher and Joe H. Doyle

Abstract. A basic language computer program (SEX) has been devised for the calculation of strain hardening exponent and strength coefficient from raw data obtained from a tensile or compression test. The program yields tabular and graphical displays of both nominal (engineering) stress and strain and true stress and strain, as well as performing a linear regression analysis of the correlation between log true stress and log true plastic strain. A limited number of punched tapes of the program file are available upon request from the authors.

INTRODUCTION

Uniaxial stress tests, either tension or compression, are commonly used to evaluate the plastic flow characteristics of a metal. The results of these tests which are most commonly reported (yield strength, ultimate strength, fracture elongation or contraction, and reduction of area) require little mathematical manipulation and can be efficiently computed with a desk-top calculator or slide rule. However, other data such as strain hardening exponent and strength coefficient can be gleaned from these tests which are of significant interest for general material characterization, in design, and in the evaluation of the material's formability. Strain hardening exponent and strength coefficient involve the relationship between the true stress and the true plastic strain. Calculations of true stress and true strain and the determination of the strain hardening exponent and strength coefficient are tedious, and a computer program will be described which can be conveniently used for these calculations.

COMPUTATIONAL PROCEDURE

Development of Required Equations

We will assume that the material is elastically and plastically isotropic and that the available raw data consist of loads and corresponding nominal longitudinal plastic strains, the original sample cross-sectional area, the modulus of elasticity, and Poisson's ratio. A review of methods to measure plastic strain is given in Appendix I. A review of elasticity and plasticity may be obtained elsewhere.^{1,2,3}

The first step in the procedure is to convert load (F) into true stress (S). True stress is defined as load divided by instantaneous sample cross-sectional area (A),

$$S = F/A, \quad (1)$$

as contrasted to nominal stress (S'), which is load divided by original area (A_0). The area of the specimen under load is dependent upon the original area, the plastic strain, and the elastic strain. The condition of constant volume, which states that the permanent (plastic) volume of a metal does not change because of plastic deformation, can be used to calculate the portion of the change in the cross-sectional area as a result of plastic strain:

$$\text{Volume} = A_0 L_0 = AL = \text{constant}, \quad (2)$$

where L = gage length and the subscripts refer to the original or unstrained condition.

From the definition of nominal strain and from Equation (2),

$$\epsilon' = \frac{L - L_0}{L_0} = \frac{A_0}{A} - 1 \quad (3)$$

where ϵ' = nominal plastic strain.

The change in the area caused by plastic strain may be expressed as:

$$(\Delta A)_{\text{plastic}} = -A_0 \left(\frac{\epsilon'}{\epsilon' + 1} \right) \quad (4)$$

Hooke's law will be used to calculate the change in area by elastic deformation.

The elastic strain in the longitudinal direction (e) is

$$e = S'/E \quad (5)$$

where E = modulus of elasticity.

The elastic strains in the two transverse directions (e_t) are

$$e_t = -\mu S'/E, \quad (6)$$

where μ = Poisson's Ratio.

The change in the area caused by elastic strain is

$$(\Delta A)_{\text{elastic}} = A_0 \left[\left(1 - \frac{\mu S'^2}{E^2} \right) - 1 \right] \quad (7)$$

The area ratio from both elastic and plastic deformation may be found by summing the individual changes in area and rearranging to

$$\left(\frac{A_0}{A}\right)_{\text{total}} = \left[\frac{1}{\epsilon' + 1} + \left(1 - \frac{\mu S'}{E}\right)^2 - 1 \right]^{-1} \quad (8)$$

If the strains are small, the equation may be simplified by binomial approximations to

$$\left(\frac{A_0}{A}\right)_{\text{total}} \simeq 1 + \epsilon' + 2\mu \frac{S'}{E} \quad (9)$$

Another possible simplification is to consider that the constant volume condition is applicable to the total strain instead of to just the plastic strain as before. The resulting equation is

$$\left(\frac{A_0}{A}\right)_{\text{total}} = 1 + \epsilon' + e = 1 + \epsilon' + \frac{S'}{E}, \quad (10)$$

which is the same as Equation (9) if $\mu = 1/2$. The usual range of values of μ is from 0.03 (beryllium) to 0.35 (most metals),⁴ so that Equation (10) does not appear to be an accurate approximation to Equation (9). The ultimate simplification is to ignore the effect of the elastic strain entirely which results in

$$\left(\frac{A_0}{A}\right)_{\text{total}} = 1 + \epsilon' \quad (11)$$

Equations (8), (9), (10), and (11) will be compared using sample data in the following section. True stress may now be written as nominal stress times the area ratio:

$$S = F/A = \left(\frac{F}{A_0}\right) \left(\frac{A_0}{A}\right)_{\text{total}} = S' \left(\frac{A_0}{A}\right)_{\text{total}} \quad (12)$$

True plastic strain is defined in terms of its differential,

$$d\epsilon = \frac{dL}{L} \quad (13)$$

Integration between the limits of the original length (L_0) and the final length (L) yields

$$\epsilon = \ln \frac{L}{L_0} = \ln (1 + \epsilon') \quad (14)$$

Calculations of Sample Data

Calculations of true stress and true plastic strain from Equations (12) and (14) with sample data obtained from a compression test are outlined in Table I. True stress was calculated by substitution of Equation (8), the exact expression for the area ratio, into Equation (12). A comparison of the results from the different equations for the calculation of true stress will be shown later. The determination of the strain hardening exponent and the strength coefficient is predicated on the observation that the relation between true stress and true plastic strain for many metals, for a wide range of testing conditions can be approximately described by the following equation:⁵

$$|S| = A|\epsilon|^m, \quad (15)$$

where A = strength coefficient (value of $|S|$ at $|\epsilon| = 1$)
 m = strain hardening exponent.

It is convenient to take the logarithm of Equation (15),

$$\log |S| = \log A + m \log |\epsilon|, \quad (16)$$

and refer to m as the slope of the best fit line to the $\log |S| - \log |\epsilon|$ plot and A as the antilog of the intercept of the line ($|S|$ at $\epsilon = 1$). A graph of the sample data plotted

Table I. Manual Calculations of S and ϵ .*

Raw Data		Intermediate Computations			Results	
Load	Nominal Plastic Strain	$1/(1 + \epsilon')$	$(1 - \mu S'/E)^2$	$(B + C - 1)^{-1}$	True Stress DXF/A_0	True Pl. Strain $\ln (1 + \epsilon')$
F	ϵ'	B	C	D	S	ϵ
-2175	-.002	1.002004	1.001697	.996313	-28,167	-.002002
-2475	-.004	1.004016	1.001931	.994088	-31,981	-.004008
-2860	-.008	1.008064	1.002231	.989810	-36,798	-.008032
-3190	-.016	1.016260	1.002490	.981596	-40,702	-.016129
-3410	-.032	1.033057	1.002662	.965513	-42,797	-.032523
-3460	-.040	1.041660	1.002700	.957524	-43,066	-.040822

*Elastic modulus (E) = 10×10^6 psi
 Poisson's Ratio (μ) = 0.3
 Original area (A_0) = 0.07693 in.²

as $\log |S|$ and $\log |\epsilon|$ is shown in Figure 1 together with estimated values of A and m calculated from a crude visual fit to the data.

Figure 1. Manual Plot of Sample Compression Data.

It is obvious that a curve rather than a straight line is required to precisely describe the data. Plots of this type are often significantly nonlinear, but it should be realized that a straight line often provides the necessary accuracy to describe the data for many purposes. When greater precision of fit is required, two or more straight lines can be fitted to the data; this has the effect of treating m in Equation (15) as a function of ϵ . At least one equation relating S and ϵ of different form than (15) has been proposed,⁶ but it is more complex and not generally used. As previously mentioned, the sample data were obtained from a compression test, and buckling of the specimen may be the cause of some of the nonlinearity observed between $\log |S|$ and $\log |\epsilon|$.

Equation (15) is not proposed as a universal equation. Each time it is used to characterize a material, a judgement is required as to whether the equation provides the necessary accuracy to properly describe the data. Values of A and m should be reported with: (1) the range of strain in which the values were determined, (2) the precision to which the line describes the data, and (3) the variation of the values determined by repeated testing.

Least Squares Analysis

Linear regression analysis of $\log |S|$ and $\log |\epsilon|$ will be used to accurately determine the values of strength coefficient (A) and strain hardening exponent (m). The method of least squares^{7,8} will be used to obtain the best estimate of A and m ; defined as those values which minimize the sum of squares of the differences between the observed values of $\log |S|$ and the values that are predicted from Equation (16). Hand calculation of the linear regression analysis to determine A and m is tedious, therefore a time sharing computer program (available at Rocky Flats from the computer service as COLINR***) was used. The results listed in Table II were derived from

Table II. Linear Regression Analysis of Sample Compression Data Using COLINR***.

Equation Number used to Calculate True Stress	m	$\log A$ (log psi)	A (psi)	Index of Determination
(8)	0.1426	4.847	70440	0.96369
(9)	0.1425	4.847	70410	0.96346
(10)	0.1422	4.846	70220	0.96336
(11)	0.1429	4.849	70700	0.96360

the sample data and from the four equations: (8), (9), (10), and (11). The index of determination (R^2) is a parameter between 0 and 1 which indicates how well the data fit the assumed strain hardening expression, Equation (16). If the index is 0, no linear functional relation exists between $\log |S|$ and $\log |\epsilon|$. If the index is 1, a perfect linear fit exists. The regression analysis also yields confidence limits on A and m , but these have little meaning for the problem described here. Confidence limits that are valid estimates of the range in which the true values of the parameters fall should be determined from A and m values obtained from replicated tests.

The values of A and m obtained by the least squares analysis (Table II) differ considerably (>10 percent) from those obtained by the crude visual fit (Figure 1). It is felt that the improvement in accuracy of the estimates of A and m obtained by the least squares analysis outweighs the possible benefit of an intuitive weighting of data points in the visual approach. Of course, this assumes that the data for the least squares analysis were properly obtained.

The values of A and m listed in Table II show that for the sample data the choice of the equation used to calculate true stress is unimportant. For example, the maximum deviation of m from the value obtained from the exact Equation (8) is 0.0004, which is insignificant compared to

the expected variation of m that would be obtained from replicate testing (about 0.005). Therefore, if μ and E are unknown, the elastic strains may be set equal to zero [which modifies Equation(8) to Equation (11)] and only a slight loss in accuracy will result for data obtained at low stresses where the elastic strains are small. However, it should be mentioned that for some materials (for example, ultra-high strength alloys), ignoring the elastic contribution to the change in area would result in an error in the true stress of about 1%, and therefore may not be justified.

COMPUTER CALCULATIONS

Following is a detailed explanation of the program usage. All statements entered by the user must be followed by a carriage return. Reference should be made to the time-sharing computer operations in the general manual.⁹

The computer program for the calculation of m and A is listed in Appendix II, and is available from the authors on punched tape. True stress is calculated using the exact expression, Equation(8). In addition to the calculation of m and A , the program will:

1. Tabulate nominal stress, nominal plastic strain, true stress, true plastic strain, log absolute true stress, and log absolute true plastic strain.
2. Plot nominal stress-nominal plastic strain points, true stress-true plastic strain points, and log absolute true stress-log absolute true plastic strain points.
3. Tabulate the linear correlation coefficient of log absolute true stress and log absolute true plastic strain.
4. Plot the best fit equation as a line of log absolute true stress-log absolute true strain.

Option (4) must be performed subsequent to Options (2) and (3). The plotting operations can be performed using a Hewlett Packard 7200A Graphic Plotter (available on the time-sharing computer terminal in Building 779, Rocky Flats). The use of other plotters or other computer systems may require modifications to the program. Absence of a plotter will not affect the tabulation and least squares analysis.

The start of a sample run on the compression data used previously is given in Figure 2. Data which the user provides are underlined. The data must be entered on lines beginning with 100 and may continue to line 109. Type the line number, DATA, and then the first load, the corresponding strain, the second load, the corresponding strain and so on. When the end of the line is reached, additional data can be entered on a new line by typing a higher line number,

DATA, and then continuing the data input. Instructions on the method of data entry will be printed out if the user commands the program to run before data are entered. Both tension and compression loads and strains are entered as positive numbers. After being commanded to run, the computer will ask whether the test is tension or compression, and will make all necessary corrections and print negative numbers for stress and strain if the test is compression. The computer will next ask for the specimen number, the original cross sectional area of the specimen, the elastic modulus, and Poisson's ratio. If the elastic strains are to be ignored, enter zero for Poisson's ratio. Never enter zero for the elastic modulus. The computer will next ask whether a table, plot, or least squares analysis is wanted or whether the program is to be ended. If a table is requested, the computer will print a table similar to that shown on the bottom of Figure 2 and then ask whether a table, plot, or least squares analysis is wanted or whether the program is to be ended. If a plot is requested, as in Figure 3, the computer will ask for the type of plot; nominal stress-strain, true stress-true plastic strain, or log absolute true stress-log absolute true plastic strain.

Figure 3 shows the method of obtaining a plot of nominal stress-strain. After receiving an instruction of what to plot, the computer will ask for the ranges of the X and Y axes of the plot. Enter the desired value of the strain or log strain of the original (X1), the value for the right hand limit (X2), then the stress or log stress for the original (Y1), and the value for the upper limit (Y2). The plotter must be previously prepared; that is, graph paper inserted and aligned, power on, hold-down on, pen inserted and primed, lower left (origin) point established, and upper right point established.

Some forethought is required in choosing the stress-function and ranges so that the intermediate lines on the graph paper fall on even values. After receiving the plotting ranges, the computer will print numbers (unless the teletyper is muted) which are of no concern to the user, and it will also plot the required points on the graph. Examples of the graphs that can be obtained are shown in Figures 4, 5, and 6. The computer prints the data points as dots; all other information on the graphs is either the result of another subroutine or was manually drawn on these figures. The computer will ask what analysis is to be performed after each plot is completed. If another plot is requested, (for example, of true stress-strain) the process is repeated.

If a least squares analysis is requested, as in Figure 7, the computer will ask for the range of the log absolute true plastic strain which is to be analyzed. This option in the program allows the user to discard some of the data. If all the data are to be included in the analysis, choose limits such as -3, -1 for the sample data. The resultant print out

OLD:SEX

READY
RUN

SEX 11:13 CTC THURS. 07/13/72

ENTER ABSOLUTE VALUES OF LOAD AND NOMINAL PLASTIC STRAIN
STARTING ON LINE 100 AND CONTINUING NO FURTHER THAN 109

TIME: 3.1 UNITS

READY

100DATA 2175,.002,2475,.004,2860,.008,3190,.016,3410,.032,3460,.04
RUN

SEX 11:14 CTC THURS. 07/13/72

TENSION=1, COMPRESSION=-1 ?-1
SPECIMEN NUMBER, AREA, MODULUS, POISSON'S RATIO
?2TRT,.07693,10E6,.3

TABLE=1, PLOT=2, LEAST SQUARES=3, END=4 ?1

SPECIMEN # 2TRT

LOAD	NOM STRESS	NOM PL STRAIN	TRUE STRESS	TRUE PL STRAIN	LOG TRUE STRESS	LOG TRUE PL STRAIN
-2175.0	-28272	-0.00200	-28168	-0.00200	4.44976	-2.69854
-2475.0	-32172	-0.00400	-31982	-0.00401	4.50490	-2.39707
-2860.0	-37177	-0.00800	-36798	-0.00803	4.56582	-2.09517
-3190.0	-41466	-0.01600	-40703	-0.01613	4.60963	-1.79238
-3410.0	-44326	-0.03200	-42797	-0.03252	4.63142	-1.48781
-3460.0	-44976	-0.04000	-43065	-0.04082	4.63413	-1.38911

Figure 2. Computer Tabulation of Sample Data.

Figure 3. Computer Entries for a Plot of Nominal Stress-Strain.

TABLE=1, PLOT=2, LEAST SQUARES=3, END=4 ?2
NOM STRESS-STRAIN=1, TRUE STRESS-STRAIN=2, LOG TRUE STRESS-STRAIN=3 ?1
ENTER X1,X2,Y1,Y2 ?0,-.05,-25000,-45000
PLTP
399 1636
799 3585
1599 6087
3199 8232
6399 9662
7999 9986
1 2 3
PLTT

Figure 4. Computer Plot of Nominal Stress-Strain.

Figure 6. Computer Plot of Log True Stress-Strain.

Figure 5. Computer Plot of True Stress-Strain.

is shown in Figure 7. The computer will next ask whether a line corresponding to the parameters obtained in the least squares analysis is to be drawn. If an affirmative answer is given as in Figure 7 and if the graph of the log $|S|$ -log $|e|$ points (Figure 6) is on the plotter, the best-fit line will be drawn on the graph (as shown in Figure 6).

If instead of including all the data, the user prefers to drop the last data point (for example, because he has seen from the log $|S|$ -log $|e|$ plot that the last point reduces the linearity of the plot), he would choose limits of -3, -1.4 as in Figure 8. The resultant print out of the least squares analysis is given in Figure 8, and the line which is produced with the last data point deleted is shown on the plot in Figure 6. Note that for the sample data, dropping the last point had an insignificant effect on A and m , but significantly decreased the difference between the index of determination and unity. It is not necessary to obtain a plot of the data before the least squares analysis, but it is necessary to go through the plotting routine to obtain the best fit line which is optioned after the regression analysis. When the analysis of a group of data is complete, end the program.

If a new group of data is to be entered, the previously entered data lines must be removed, either by reusing the line number to enter new data or by deleting the line number. The program will remind the user to do this

TABLE=1, PLOT=2, LEAST SQUARES=3, END=4 ?2
 NOM STRESS-STRAIN=1, TRUE STRESS-STRAIN=2, LOG TRUE STRESS-STRAIN=3 ?3
 ENTER X1,X2,Y1,Y2 ?-3,-1,4.4,4.8
 PLTP

1507	1243
3014	2622
4523	4145
6037	5240
7560	5784
8053	5852
1	2
	3

PLTT

TABLE=1, PLOT=2, LEAST SQUARES=3, END=4 ?3
 LOG TRUE STRAIN MIN AND MAX FOR LEAST SQUARES ANALYSIS ?-3,-1

LOG STRENGTH COEF= 4.84782

STRENGTH COEF= 70439.6

STRAIN HARDENING EXP= .1426

INDEX OF DETERMINATION(R²)= .963574
 NO OF DATA POINTS= 6

LINE ON GRAPH YES=1 NO=2 ?1

PLTL

0	500
1999	1926
3999	3352
5999	4777
7999	6203
9999	7629

PLTT

Figure 7. Computer Print Out of Least Squares Analysis of all Sample Data.

TABLE=1, PLOT=2, LEAST SQUARES=3, END=4 ? 3
 LOG TRUE STRAIN MIN AND MAX FOR LEAST SQUARES ANALYSIS ? -3,-1.4
 LOG STRENGTH COEF= 4.8761

STRENGTH COEF= 75179.

STRAIN HARDENING EXP= .154614

INDEX OF DETERMINATION(R²)= .973406
 NO OF DATA POINTS= 5

LINE ON GRAPH YES=1 NO=2 ? 1

PLTL

0 306

1999 1852

3999 3398

5999 4944

7999 6490

9999 8036

PLTT

TABLE=1, PLOT=2, LEAST SQUARES=3, END=4 ? 4
 REMEMBER TO DELETE PREVIOUSLY USED DATA LINES 100-109
 BEFORE ENTERING NEW DATA

Figure 8. Computer Print Out of Least Squares Analysis Excluding Last Point.

before printing END. If no additional use of the program is required, type BYE which will remove the data lines and restore the initial data entry instructions if the program has been saved.

CONCLUSIONS

The computer program SEX is an efficient and accurate method of analyzing tension or compression data to obtain the strain hardening exponent and strength coefficient as well as to obtain tabulations and graphs of various stress and strain parameters.

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APPENDIX I

MEASURING PLASTIC STRAINS FROM TENSILE OR COMPRESSION TEST RECORDS

This section will deal with two techniques that may be used to obtain the raw data for the determination of plastic strain from a tensile or compression test. The most commonly used method is to attach an extensometer to the ends of the uniform-diameter gage length of the specimen, and obtain an autographic record of load-versus-nominal total strain. Recommended testing practices and a general description of the "Offset Method" of determining plastic strain may be found in the ASTM Standards.¹⁻¹ However, the example of the test record in Reference I-1 is idealized and a more practical description will be given. Furthermore, the recommended practices do not include a description of the determination of plastic strain when an extensometer is not attached to the specimen, which is the second method that will be discussed.

A typical tensile test record of load versus nominal plastic strain is shown schematically in Figure I-1. Note that the initial portion of the curve is not linear. This effect may be the result of several common minor testing errors such as an improperly applied extensometer or an initially bent specimen. The key to the analysis of the

curve is the accurate drawing of the nominal elastic line (AD), and the determination of plastic strain from the offset of the curve from the elastic line (CD), as shown in Figure I-1. This method inherently eliminates the errors that might be caused by the initial nonlinear and erroneous portion of the curve. When there is significant nonlinearity in the beginning of the test record, it is not correct to determine the plastic strain at C by subtracting from the indicated total strain (EC) the elastic strain computed by dividing the stress by the elastic modulus. When there is significant nonlinearity in the beginning of the test record, plastic strains are correctly measured only by differences between the drawn-in elastic line (AD) and points on the load-strain curve. The elastic strains are correctly measured only by differences in strain along the drawn-in elastic line.

Although the determination of plastic strain without the use of an extensometer is not recommended by ASTM, it can be done accurately in many tests. In this method, an autographic record of load-versus-time (if a machine with an accurately controlled constant velocity crosshead is used) or load-versus-crosshead deflection is obtained. The initial loading line will, for most specimens, be even more nonlinear than in Figure I-1 because of the take-up of slack in the loading train. Furthermore, the subsequent linear portion of the curve will reflect the elastic deflection of the entire stressed structure, not just that of the specimen. Therefore, the initial loading line has little significance to the stiffness of the specimen, and cannot be used to determine the elastic modulus of the specimen if no other information is available. However, with a good specimen configuration in a well-designed testing machine, any deviation from linear behavior of the load-crosshead deflection record, after the slack is taken up, will be the result of plastic deformation in the gage length of the specimen. It is for this reason that the offset method shown in Figure I-1 can be used to determine plastic strain when an extensometer is not used. The offsets (which correspond to crosshead deflection) are converted to plastic strains by dividing the corresponding crosshead movement by the effective gage length of the specimen.

For long specimens, the effective gage length is well approximated by the length of the uniform portion of the specimen. For short specimens, the effective gage length may be experimentally determined by use of gage marks on the uniform portion of the gage length. The effective gage length is calculated by dividing the change in the overall

Figure I-1. Schematic of Load-Strain or Load-Time Curve.

length of the specimen by the strain determined from the gage marks. Accurate measurements are required for small strains and it should also be mentioned that the effective gage length may vary significantly with plastic strain. It is strongly suggested that the crosshead-deflection method be carefully calibrated against tests performed with an extensometer, and not be used to measure plastic strains that are much smaller than the corresponding elastic strains. The authors' experience has indicated that plastic strains much smaller than 0.001 are below the limit of resolution of the method.

It should be emphasized that these methods involve the assumption of uniform straining along the gage length of

the specimen. Therefore, the methods apply in tensile testing only to the portion of the load-strain or load-time curve prior to the point of maximum load at which necking usually starts. Analysis of the tensile test curve after necking initiates is outside the scope of this report.

REFERENCE

- I-1. 1967 Book of ASTM Standards, Part 31, Physical and Mechanical Testing of Metals, Tension Testing of Metallic Materials (E8-66), Compression Testing of Metals (E9-67), American Society for Testing and Materials, Philadelphia, Pennsylvania, 1967, pp 201-220, 221-229.

APPENDIX II.

SEX: A COMPUTER PROGRAM TO CALCULATE STRAIN HARDENING EXPONENT

SEX

```

100 GO TO 120
110 GO TO 150
120 PRINT "ENTER ABSOLUTE VALUES OF LOAD AND NOMINAL PLASTIC STRAIN"
130 PRINT "STARTING ON LINE 100 AND CONTINUING NO FURTHER THAN 109"
140 GO TO 1310
150 DATA 999,0
160 PRINT "TENSION=1, COMPRESSION=-1";
170 INPUT Y
180 PRINT "SPECIMEN NUMBER, AREA, MODULUS, POISSON'S RATIO"
190 INPUT A$,A0,E,Z
200 IF E>0 THEN 230
210 PRINT "MODULUS MUST BE POSITIVE AND NONZERO"
220 GO TO 180
230 PRINT
240 PRINT
250 PRINT "TABLE=1, PLOT=2, LEAST SQUARES=3, END=4";
260 INPUT X
270 IF X=1 THEN 410
280 IF X=3 THEN 370
290 IF X=4 THEN 1290
300 PRINT "NOM STRESS-STRAIN=1, TRUE STRESS-STRAIN=2, LOG TRUE ";
310 PRINT "STRESS-STRAIN=3";
320 INPUT Q
330 PRINT "ENTER X1,X2,Y1,Y2";
340 INPUT X1,X2,Y1,Y2
350 PRINT "PLTP"
360 GO TO 490
370 PRINT "LOG TRUE STRAIN MIN AND MAX FOR LEAST SQUARES ANALYSIS";
380 INPUT X3,X4
390 LET U1=U2=U3=V1=V2=V3=N2=0
400 GO TO 490
410 PRINT
420 PRINT "                                SPECIMEN # " A$
430 PRINT
440 PRINT"                NOM        NOM PL        TRUE        TRUE PL        LOG TRUE"
450 PRINT"  LOG TRUE PL"
460 PRINT"  LOAD        STRESS    STRAIN        STRESS    STRAIN        STRESS";
470 PRINT "        STRAIN"
480 PRINT
490 READ A1,B1
500 IF A1=999 THEN 770
510 LET A2=Y*A1/A0
520 LET B2=A2/E
530 LET A3=A2*(1+Y*B1+B2)
540 LET B3=LOG(1+Y*B1)
550 LET A3=A2/(((1+Y*B1)+1)+(1-(Z*A2)/E)+2-1)
560 LET A4=CLG(ABS(A3))
570 LET B4=CLG(ABS(B3))
580 IF X=2 THEN 630
590 IF X=3 THEN 830

```

SEX CONTINUED

```

600 PRINT USING 610, A1*Y,A2,B1*Y,A3,B3,A4,B4
610:#####.## ##### ##.##### ##### ##.##### #.##### ##.#####
620 GO TO 490
630 IF Q=1 THEN 680
640 IF Q=2 THEN 710
650 LET X8=B4
660 LET Y8=A4
670 GO TO 730
680 LET X8=B1*Y
690 LET Y8=A2
700 GO TO 730
710 LET X8=B3
720 LET Y8=A3
730 LET Y9=((Y8-Y1)/(Y2-Y1))*9999
740 LET X9=((X8-X1)/(X2-X1))*9999
750 PRINT INT(X9); INT(Y9)
760 GO TO 490
770 IF X=3 THEN 970
780 RESTORE
790 IF X=1 THEN 230
800 PRINT 1;2;3
810 PRINT "PLTT"
820 GO TO 230
830 IF B4<X3 THEN 490
840 IF B4>X4 THEN 490
850 LET S=B4
860 LET A=A4
870 LET U1=U1+S
880 LET V1=V1+A
890 LET U2=U2+S*2
900 LET V2=V2+A*2
910 LET U3=U3+A*S
920 LET W=W+1
930 DIM S(100),A(100)
940 LET S(W)=S
950 LET A(W)=A
960 GO TO 490
970 IF W=2 THEN 780
980 LET U4=(U2*W)-U1*2
990 LET V4=(V2*W)-V1*2
1000 LET U5=U3*W-U1*V1
1010 LET W2=U5/U4
1020 LET W3=(V1-(U1*W2))/W
1030 LET U6=U5*2/U4
1040 LET W4=U6/V4
1050 LET M7=(W3)-T*SQR(M2)
1060 PRINT
1070 PRINT" LOG STRENGTH COEF="W3
1080 PRINT
1090 PRINT" STRENGTH COEF="10*W3

```

SEX CONTINUED

```
1100 PRINT
1110 PRINT
1120 PRINT " STRAIN HARDENING EXP="W2
1130 PRINT
1140 PRINT "INDEX OF DETERMINATION(R2)="W4
1150 PRINT " NO OF DATA POINTS="W
1160 PRINT
1170 PRINT "LINE ON GRAPH YES=1 NO=2";
1180 INPUT W6
1190 IF W6=2 THEN 1270
1200 PRINT "PLTL"
1210 FOR X8=X1 TO X2+.01 STEP(X2-X1)/5
1220 LET Y9=((W3+(X8*W2)-Y1)/(Y2-Y1))*9999
1230 LET X9=((X8-X1)/(X2-X1))*9999
1240 PRINT INT(X9); INT(Y9)
1250 NEXT X8
1260 PRINT "PLTT"
1270 RESTORE
1280 GO TO 230
1290 PRINT "REMEMBER TO DELETE PREVIOUSLY USED DATA LINES 100-109"
1300 PRINT "BEFORE ENTERING NEW DATA"
1310 END
```