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SPACE VEHICLE SHIELDING STUDIES
CALCULATIONS OF THE ATTENUATION OF A
MODEL SOLAR FLARE AND MONOENERGETIC
PROTON BEAMS BY ALUMINUM SHIELDS

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OAK RIDGE NATIONAL LABORATORY

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U.S. ATOMIC ENERGY COMMISSION

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Neutron Physics Division

Space Vehicle Shielding Studies*

CALCULATIONS OF THE ATTENUATION OF A MODEL SOLAR FLARE AND
MONOENERGETIC PROTON BEAMS BY ALUMINUM SHIELDS

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**Central Data Processing Facility of the Oak Ridge Gaseous Diffusion Plant.

OAK RIDGE NATIONAL LABORATORY
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Abstract

Using the straight-ahead approximation, nucleon-meson cascade calculations have been carried out for a typical proton flare spectrum incident on a shield and also for approximately monoenergetic incident proton beams. The shield material considered has approximately the properties of aluminum.

The results from the monoenergetic beams may be used to obtain shielding information for any incident proton spectrum.

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Table of Contents

	<u>Page No.</u>
Abstract	iii
Table of Notation	1
I. Introduction	3
II. Details of the Calculation	4
III. Dose Calculation	8
IV. Cascade Results from a Model Solar Flare	9
V. Importance Functions	20
VI. Conclusions	55
Appendix I	59
Appendix II	68
Appendix III	74

Table of Notation

x = normal distance through a slab shield.

r = distance through a slab shield measured in the direction of the incident beam.

E = kinetic energy.

E_i = kinetic energy of monoenergetic incident beam.

u = lethargy variable defined by $u = \ln(E_0/E)$.

E_0 = energy which defines the zero of lethargy.

$P(E, r) = P_i(E, r) + P_s(E, r)$.

$P_i(E, r)$ = primary proton flux per unit energy range.

$P_s(E, r)$ = secondary proton flux per unit energy range.

$N(E, r)$ = neutron flux per unit energy range. The neutron flux in this report is always the secondary neutron flux since there is no primary neutron flux.

$\pi(E, r)$ = pion flux per unit energy range. The pion flux in this report is always the secondary pion flux since there is no primary pion flux.

$\mu(E, r)$ = muon flux per unit energy range. The muon flux in this report is always the secondary muon flux since there is no primary muon flux.

$P(u, r)$ = proton flux per unit lethargy range defined by

$$P(u, r) du = - P(E, r) dE.$$

There are, of course, similar definitions for the other fluxes.

Γ = lowest energy used in computing a given dose; i.e., when Γ is specified in conjunction with the dose only those particles with energy greater than or equal to Γ have been used in computing the dose. In particular, $\Gamma = 0$ is used to indicate a dose which contains an extrapolated contribution from low-energy particles.

$d(r)$ = total dose from all particles
 $= d_p + d_N + d_\pi + d_\mu$.

$d_s(r)$ = dose from secondary cascade particles. [When dealing with monoenergetic beams $d(E_i, r)$ and $d_s(E_i, r)$ are used for the total dose and secondary dose respectively.]

$D(x)$ = total dose from an isotropic incident beam. This symbol is used to indicate that the Gross Transformation has been carried out.

$$\begin{aligned} d_P(r) &= \text{dose from protons} \\ &= d_{P_i}(r) + d_{P_s}(r). \end{aligned}$$

$$d_{P_i}(r) = \text{dose from primary protons.}$$

$$d_{P_s}(r) = \text{dose from secondary protons.}$$

$$d_N(r) = \text{dose from neutrons.}$$

$$d_\pi(r) = \text{dose from pions.}$$

$$d_\mu(r) = \text{dose from muons.}$$

$$d_{P_iS}(r) = \text{dose from primary protons due to atomic stopping.}$$

$$d_{P_iA}(r) = \text{dose from primary protons due to nonelastic collisions.}$$

$$d_{P_sS}(r) = \text{dose from secondary protons due to atomic stopping.}$$

$$d_{P_sA}(r) = \text{dose from secondary protons due to nonelastic collisions.}$$

$$d_{NE}(r) = \text{dose from neutrons due to elastic neutron proton scattering.}$$

$$d_{NA}(r) = \text{dose from neutrons due to nonelastic collisions.}$$

$$d_{\pi S}(r) = \text{dose from pions due to atomic stopping.}$$

$$d_{\pi A}(r) = \text{dose from pions due to nonelastic scattering.}$$

$$I_1(E_i, r) = \text{first importance function.}$$

$$I_2(E_i, r) = \text{second importance function.}$$

I. Introduction

In order to design shielding experiments for manned space vehicles, such as those being conducted at the Oak Ridge National Laboratory, most efficiently it is necessary to know what energy regions are most important in the shielding problem. The present calculations were undertaken to give a preliminary answer to this question.

Using the straight-ahead approximation nucleon-meson cascade calculations have been carried out for various proton beams incident on a slab shield. The equations and the physical data concerning high-energy interactions have been discussed in detail in a previous report¹ (hereinafter referred to as 1).^{*} Because many of the approximations which are used, particularly the straight-ahead approximation, are invalid at low energies the calculations are carried out only for energies greater than 30.4 Mev. Also, all gamma rays are neglected throughout the calculation.^{**}

The various particle fluxes have been converted to dose in rads by a method introduced by W. A. Gibson.³ Since the fluxes are calculated only at the higher energies (> 30.4 Mev) we can, strictly speaking, calculate the dose only from the higher energy particles. However, there will be particles at the lower energies and it is desirable to estimate their contribution to the dose. Such an estimate has been obtained by use of a crude extrapolation procedure and is included.

Section II contains some details concerning the calculations. In particular, the approximation used to simulate monoenergetic proton beams is discussed. Section III contains a discussion of the flux-to-dose conversion and the dose extrapolation to include low-energy particles.

^{*}The parts of 1 which are relevant to the present work are reproduced in Appendix III of this report.

^{**}The neglect of the gamma rays may not be justified.²

1. R. G. Alsmiller, Jr., F. S. Alsmiller, and J. E. Murphy, Transverse Shielding for a 45-Gev Electron Accelerator, ORNL-3289 (1962).
2. R. Madey, A. G. Duerer, Jr., and T. J. Kreiger, Gamma Dose from Solar Flare Protons Incident on an Aluminum Shield, Annual Meeting of the American Nuclear Society June 18-21, 1962, Boston, Mass.
3. W. A. Gibson, Energy Removed from Primary Proton and Neutron Beams by Tissue, ORNL-3260 (1962).

In Section IV results are given and discussed for shielding against a typical solar flare. In Section V results for monoenergetic proton beams are given, importance functions are defined, and their implication with respect to the design of shielding experiments discussed. In Section VI the conclusions are given.

Appendix I contains a comparison between calculations as carried out in this report and similar calculations by the Lockheed group.⁴ Appendix II contains some numerical results associated with the dose calculations. In Appendix III the parts of 1 which are relevant to the present work are reproduced verbatim.

II. Details of the Calculation

In 1 the calculations were to apply to earth but it was found that to a reasonable approximation earth could be replaced by aluminum in all respects except density. All of the physical data used in 1 was taken to be that for aluminum insofar as this was possible.

In the present work we wish to do calculations in aluminum and thus we use the same data as in 1, with one exception. A theoretical proton nonelastic cross section for protons on aluminum which is expected to be more accurate than the cross section used in 1 is now available and is used here.* This cross section is shown in Fig. 1. We again arbitrarily take the pion nonelastic cross section to be equal to the proton cross section.

The multiplicities and secondary energy distributions which were used in 1 and are used in the present calculation were taken from cosmic-ray measurements in emulsions, and thus are not precisely those for aluminum. However, in 1 they were compared with the calculation of Metropolis and his co-workers⁵ and were found to be approximately correct for aluminum.

*For energies of 550 Mev or less this cross section is taken from the calculations of H. W. Bertini; above this energy the cross section was assumed to be constant. We wish to thank Mr. Bertini for use of his results prior to their publication.

4. R. I. Allen, J. F. Perkins et al., Shielding Problems in Manned Space Vehicles, NR-140, Lockheed Nuclear Products, Marietta, Georgia.
5. N. Metropolis, R. Bivens, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevich, Phys. Rev. 110, 204 (1958).

Fig. 1. Proton Nonelastic Cross Section on Aluminum in Units of the Geometric Cross Section vs. Energy.

The incident beam considered in 1 contained a continuum of energies, and the IBM code which was written to solve the cascade equations numerically contained no provision for treating monoenergetic beams. In the present work monoenergetic beams are considered, but in order to utilize the previous code without extensive recoding they are approximated by a suitable continuum.*

To understand this approximation it must be recalled that the calculations are done using as independent variable the lethargy, u , defined by

$$u = \ln \frac{E_0}{E} \quad (2.1)$$

where

E_0 = an arbitrary energy which is greater than the source energy,
 E = kinetic energy of the cascade particle,
 and all fluxes must be specified as a function of lethargy at $x = 0$.
 Since only incident proton beams will be considered, all fluxes are zero at $x = 0$ except the proton flux, $P(u,0)$. The numerical calculations are done by using a finite difference approximation to the equations; this requires $P(u,0)$ be specified at the discrete points, u_j , which are the mesh points of the finite difference calculation.

To approximate a monoenergetic beam, $P(u,0)$ is specified to be

$$\begin{aligned} P(u_j, 0) &= \frac{1}{\Delta} & (i = j) \\ P(u_j, 0) &= 0 & (i \neq j) \end{aligned} \quad (2.2)$$

where

u_1 = lethargy corresponding to the energy of the monoenergetic beam, E_1 ,
 $\Delta = u_{j+1} - u_j$.
 = the difference between two successive lethargy mesh points.

*A code which will treat monoenergetic beams rigorously is in preparation.

Since linear interpolation is used the initial flux of protons is

$$\begin{aligned}
 P(u, 0) &= \frac{1}{\Delta} \left[1 - \frac{u - u_i}{u_{i+1} - u_i} \right] & (u_i < u < u_{i+1}) \\
 &= \frac{1}{\Delta} \left[1 + \frac{u - u_i}{u_i - u_{i-1}} \right] & (u_{i-1} < u < u_i)
 \end{aligned} \quad (2.3)$$

To convert to the proton flux per unit energy range, $P(E, 0)$, one must utilize the fact that by definition,

$$P(u, x) du = - P(E, x) dE \quad (2.4)$$

so

$$\begin{aligned}
 P(E, 0) &= \frac{1}{\Delta} \frac{1}{E} \left[1 - \frac{\ln \left(\frac{E_i}{E} \right)}{\Delta} \right] & (E_i > E > E_i e^{-\Delta}) \\
 &= \frac{1}{\Delta} \frac{1}{E} \left[1 - \frac{\ln \left(\frac{E}{E_i} \right)}{\Delta} \right] & (E_i < E < E_i e^{\Delta})
 \end{aligned} \quad (2.5)$$

Note that the normalization is such that

$$\int_{E_i e^{-\Delta}}^{E_i e^{\Delta}} P(E, 0) dE = 1 \quad (2.6)$$

since

$$\begin{aligned}
 \int_{E_i e^{-\Delta}}^{E_i e^{\Delta}} P(E, 0) dE &= \int_{u_{i-1}}^{u_{i+1}} P(u) du \\
 &= 1.
 \end{aligned} \quad (2.7)$$

Equation (2.5), with a suitable choice of Δ , is the approximation which is used for $\delta(E - E_i)$. In general, Δ is chosen to have a different value for every E_i . In giving the results a plot of $P(E, 0)$ is shown for each E_i used.

III. Dose Calculation

If the calculated fluxes are to be useful in determining shielding requirements it is necessary that they be converted into dose. There is at present no entirely satisfactory method of doing this for high-energy particles. In the present work we content ourselves with calculating the surface dose, i.e., the dose received by a very thin slab of tissue placed in the beam, by an approximate method introduced by W. A. Gibson.³ We shall always give the dose in rads rather than in rem because at the present time there seems to be no generally accepted way of making this conversion.

An extensive discussion of the flux-to-dose conversion has been given previously.^{1,3*}

In this report as in 1 the fluxes are calculated only at the higher energies (> 30.4 Mev) and therefore the dose can be obtained only from these high-energy particles. However, there will be particles at the lower energies, and it is desirable to obtain some estimate of their contribution to the dose.

To estimate this low-energy dose an extrapolation has been used. Let $G(E, r)$ be the flux per unit energy range for some specific type of particle. Then the dose from those particles which have energy greater than some energy Γ may be written

$$d_G(r, \Gamma) = C \int_{\Gamma}^{\infty} G(E, r) S(E) dE \quad (3.1)$$

*A slight difference exists between the methods used in 1 and 3. When a particle undergoes a nonelastic collision it is assumed to deposit all of its energy in 3 and only one-half of its energy in 1 (and in this report). Thus, the dose obtained from 1 will necessarily be smaller than the dose obtained from 3.

where

$S(E)$ = energy loss per unit distance in tissue for the type of particle involved,

C = constant which converts energy loss per unit volume into rads.

Equation (3.1) defines d_G as a function of Γ . For values of Γ greater than 30.4 Mev d_G can be calculated since for these values of Γ the flux is known. The procedure used is to calculate $d_G(r, \Gamma)$ for a number of values of Γ and to extrapolate to Γ equal zero. The extrapolation is done by passing a quadratic curve through the calculated points by the method of least squares. Throughout this report Γ equal zero will be used to indicate the extrapolated value.

There is no very good physical justification for this extrapolation procedure. The results are given only for what they are worth and could be quite wrong. In the body of the paper the extrapolated results are used, but the dose results of Γ equal to 32 Mev which can be obtained from the calculated fluxes without extrapolation are given in Appendix II. Also given in this appendix are the individual contributions to the dose from the various physical processes involved.

IV. Cascade Results from a Model Solar Flare

Using a model solar flare the fluxes and dose have been calculated assuming this flare is incident on a slab shield.

The flare spectrum is shown in Fig. 2 (Ref. 6). This spectrum has been obtained by integrating over time, so the results which are given must be considered to be the total results for the duration of the flare.

Since the flare is incident isotropically on the shield it will be necessary to apply the Gross transformation to obtain the scalar flux as a function of distance through the shield.⁷ Let

6. W. L. Gill, Statement on the Approach to the Radiation Problem for Apollo (Submitted to National Academy of Sciences, Space Science Board, Working Group on Radiation Problems in Space Flight, Jan. 12-13, 1962), NASA Manned Spacecraft Center, Life Systems Division.
7. B. Rossi, High-Energy Particles, Prentice-Hall, Inc., New Jersey (1956), p. 440.

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Fig. 2. Time Integrated Differential Energy Spectrum for May 10, 1959, Event.

x = normal distance through the shield,

r = slant distance through the shield in the direction θ

(see Fig. 3),

$P(E, r)$ = the number of protons per Mev per cm^2 per steradian in the direction θ .

Fig. 3

The angular flux, $P(E, r)$, is the quantity which is obtained directly from the cascade equations. To obtain the scalar flux, $J_P(E, x)$, we must integrate over solid angle holding x fixed. That is

$$J_P(E, x) = 2\pi \int_0^{\pi/2} P[E, (x/\cos\theta)] \sin\theta \, d\theta \quad (4.1)$$

and, changing variables,

$$J_P(E, x) = 2\pi x \int_x^\infty P(E, r) \frac{1}{r^2} \, dr. \quad (4.2)$$

Furthermore, since the dose is nothing more than an integral over energy we have

$$D_P(x) = 2\pi x \int_x^\infty d_P(r) \frac{1}{r^2} \, dr \quad (4.3)$$

where

$d_p(r)$ = proton dose from an isotropic incident beam, at a slant distance r (in units rads/steradian),

$D_p(x)$ = proton dose at a depth x from an isotropic incident beam.

There are, of course, completely analogous relations for the other particle species and for the other dose quantities.

The results of the calculations are shown in Figs. 4 to 10. It is important to note that the results shown in Figs. 4 to 9 have not been transformed by Eqs. (4.2); i.e., the angular fluxes are shown.

In Fig. 4 the primary proton flux is plotted as a function of energy for various slant distances through the shield. Note particularly the very steep rise at low energy of the initial proton spectrum and note the rapid decrease of these low-energy protons as the beam goes through the shield. This rapid decrease of the low-energy protons will have a decisive effect on the dose as we shall see later.

The cutoff at high energy as the beam traverses the shield is due to the fact that the incident beam was arbitrarily cut off at 1 Gev. After the primary beam has traversed a distance Δr , the highest energy incident particles ($E = 1$ Gev) will have lost an energy ΔE and there are then no primary particles with energy greater than 1 Gev minus ΔE . The fact that the cutoff is shown to be the same for $r = 0.1, 0.3$, and 0.5 is a defect of the finite difference calculation. In such a calculation one obtains the value of the flux only at the mesh points of the calculation. When the cutoff occurs between mesh points the only information one has is that the flux has a value at one mesh point and is zero at the next. For $r = 0.1, 0.3$, and 0.5 the cutoff occurs between 916 Mev and 1 Gev and thus on a log plot appears to occur at the same energy for all three values of r . In practice, of course, the mesh spacing must be chosen sufficiently small so that the error caused by the uncertainty in this cutoff is negligible.

Fig. 4. Primary Proton Flux vs. Energy. Flare proton spectrum incident isotropically. r is measured in collision lengths ($= 34.74 \text{ cm} = 93.80 \text{ g/cm}^2$).

Fig. 5. Secondary Proton Flux vs. Energy. Flare proton spectrum incident isotropically.

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Fig. 6. Neutron Flux vs. Energy. Flare proton spectrum incident isotropically.

Fig. 7. Pion Flux vs. Energy. Flare proton spectrum incident isotropically.

Fig. 8. Muon Flux vs. Energy. Flare proton spectrum incident isotropically.

Fig. 9. Dose vs. Slant Distance. Flare proton spectrum incident isotropically.

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Fig. 10. Dose vs. Distance. Flare proton spectrum incident isotropically.

The secondary proton, neutron, pion, and muon fluxes are shown in Figs. 5 to 8, respectively. Note the very slow attenuation of the neutron flux with distance compared to the attenuation of the charged-particle fluxes.

The various contributions to the dose are shown in Fig. 9. For thin shields ($< 50 \text{ gm/cm}^2$) the total dose is due almost entirely to primary protons. Very thick shields ($\sim 120 \text{ gm/cm}^2$) are required before the dose from secondaries becomes comparable with the dose from primaries. The pion and muon contributions can be neglected over the entire range of thicknesses considered.

In Fig. 10 the total dose transformed by Eq. (4.3) is shown, i.e., the total dose as a function of depth for an isotropically incident flare. The two curves given correspond to including the estimated dose from low-energy particles ($\Gamma = 0$) and to not including this estimate ($\Gamma = 32$). The curves of Fig. 10 indicate that an aluminum slab shield of the order of 12 to 15 gm/cm^2 will reduce the total dose from the flare to 50 rad.*

V. Importance Functions

The results of the preceding section are instructive, but they are dependent on the particular flare spectrum used and they cannot readily be used for the design of experiments. In this section somewhat more general results are obtained by treating nearly monoenergetic proton beams.

Let us assume that the total dose has been obtained for various monoenergetic beams. Denoting the total dose per unit incident flux by $d(E_i, r)$, where E_i = energy of monoenergetic beam, we can define an importance function, I_1 , by

$$I_1(E_i, r) = d(E_i, r) I(E_i)$$

*Although the discussion has been carried out for slab shields, it is clear that, since we use the straight-ahead approximation, the results can also be applied to spherical shell shields. For example, the doses at the center of a spherical shell shield of thickness r when the flare spectrum of Fig. 2 is incident isotropically on the shell may be obtained by multiplying the doses in Fig. 9 by 4π steradians.

where

$I(E_i)$ = any arbitrary incident proton flux which is to be shielded against.

It is clear that the total dose from the incident spectrum, $I(E_i)$, at a given depth can be obtained by integration, i.e.,*

$$d(r) = \int_{\text{All } E_i} I_1(E_i, r) dE_i.$$

Furthermore, the largest contribution to the total dose comes from the values of E_i for which I_1 is largest so the magnitude of I_1 is a measure of the importance of a particular energy region of the incident spectrum.

Of course, not all information one might want can be obtained from I_1 . A different importance function can be defined for each of the various doses. A second importance function, I_2 , which shall be of interest is defined by

$$I_2(E_i, r) = d_s(E_i, r) I(E_i)$$

where

$d_s(E_i, r)$ = dose per unit incident flux from all secondary cascade particles.

These importance functions are of direct interest in designing shielding experiments. Since the experiments will usually be carried out with mono-energetic beams it is necessary to know which energy region of an incident spectrum is of the most importance. This information can, of course, be obtained directly from the importance functions.

Of course, the functions $d(E_i, r)$ and $d_s(E_i, r)$ are of general interest, since with these functions the importance functions for an arbitrary incident spectrum can easily be constructed.

*If I_1 is an isotropic angular flux incident on a slab shield, $d(r)$ must be transformed by the Gross transformation to obtain the total dose at a shield depth x . If I_1 is an isotropic angular flux incident on a spherical shell shield of thickness r , the dose at the center of the shield is obtained by multiplying $d(r)$ by 4π steradians.

The results of the various calculations will now be given. Before discussing the importance functions the fluxes and doses from the various monoenergetic proton beams will be given.

The results for a proton beam with incident energy of approximately 1 Gev are given in Figs. 11 to 17. In Fig. 11 the initial proton flux which was used to approximate a 1-GeV beam is shown. In Fig. 12 the proton flux is plotted as a function of energy for various distances through the shield, measured in the direction of the incident beam. (All distances are in units of 93.8 gm/cm^2 .) In each of the curves the large peaks represent primary protons while the rest of the curves represent secondary protons. In showing the primary proton peaks all points have been plotted and joined by straight lines. This means, since the trapezoidal integration rule is used throughout the calculation, that the peak as shown gives the actual numerical values used in the calculation. As the somewhat jagged nature of these peaks indicate, the numerical calculations are only approximate. The finite difference calculation does a reasonable, but by no means perfect, job of giving the primary peak correctly.

The neutron, pion, and muon fluxes are shown in Figs. 13 to 15, respectively. Note that the neutron flux is an increasing function of distance through the shield. Also note that the pion and muon fluxes are small compared with the proton and neutron fluxes.

The various contributions to the total dose as well as the total dose are shown in Fig. 16. For all of these curves Γ is equal to zero so they include an extrapolation contribution from the low-energy particles.*

The total dose is not a monotonic function of distance but shows a slight peak. Therefore, in this particular case of 1000-Mev incident particles, the first few gm/cm^2 is detrimental. For very thin shields the dose is primarily due to primary protons while for very thick shields

*In Appendix II tables are given of the various doses without the extrapolation contribution.

Fig. 11. Initial Proton Flux vs. Energy.

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Fig. 12. Proton Flux vs. Energy.

Fig. 13. Neutron Flux vs. Energy.

Fig. 14. Pion Flux vs. Energy.

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Fig. 15. Muon Intensity vs. Energy.

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Fig. 16. Dose vs. Distance.

Fig. 17. Dose vs. Γ .

the secondaries contribute appreciably to the dose. The pion dose is almost negligible and the muon dose is completely negligible.

In Fig. 17 the total dose is plotted as a function of Γ (see Section III) at various distances. For Γ greater or equal to 32 Mev the points were obtained by direct calculation. For Γ less than 32 Mev the curve was obtained by extrapolation. Since the dose is a slowly varying function of Γ only a small contribution to the total dose is obtained from the low-energy particles (extrapolated contribution); however, it must be emphasized that there is very little justification for the extrapolation procedure and the answer obtained may be quite wrong. Also, the slow variation of the dose with Γ means that a very appreciable part of the dose is coming from the high-energy particles, i.e., from particles with energy greater than 100 Mev.

The results for an incident proton beam of approximately 600 Mev are shown in Figs. 18 to 22. The pion and muon fluxes have not been plotted in this and the subsequent cases since they are too small to be of interest. The curves for this case are very similar to those for the 1-GeV case and seem to require no further discussion.

The results for an incident proton beam of approximately 400 Mev are shown in Figs. 23 to 27. The proton flux is shown in Fig. 24. The range of the primary proton beam is approximately 1.1 in the units we are using so the primary proton peak is not present in the curve for $r = 1.5$; i.e., the curve at $r = 1.5$ represents entirely secondary protons. It is interesting to note that at those distances where the primary peak is present the secondary protons are considerably larger at energies below the peak than at energies above the peak.

The various contributions to the dose are shown in Fig. 26. The dashed lines indicate portions of the curves which are not known and cover the distance over which the primary proton beam slows down from an energy of just greater than 30 Mev to zero. Since only particles above 30 Mev are considered the dose from the primary beam when it is at an energy of less than 30 Mev but greater than zero is not calculated. Also, when the primary proton beam is at an energy of less than 30 Mev

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Fig. 18. Initial Proton Flux vs. Energy.

Fig. 19. Proton Flux vs. Energy.

Fig. 20. Neutron Flux vs. Energy.

Fig. 21. Dose vs. Distance.

Fig. 22. Dose vs. Γ .

Fig. 23. Initial Proton Flux vs. Energy.

Fig. 24. Proton Flux vs. Energy.

Fig. 25. Neutron Flux vs. Energy.

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Fig. 26. Dose vs. Distance.

Fig. 27. Dose vs. Γ .

most of the secondary protons have energy less than 30 Mev, so the secondary proton dose cannot be calculated.

As long as the primary beam is present most of the dose comes from this primary beam. When the primary beam has been stopped the total dose is primarily due to secondary neutrons. Note that the dose from primary protons begins to increase in the vicinity of $x = 0.8$. This increase is due to an increase in the stopping power at the lower energies.

The results from incident proton beams of approximately 300 Mev and 200 Mev are shown in Figs. 28 to 32b and 33 to 37, respectively. The curves are completely analogous to those for a 400-Mev proton beam. Note, however, from Figs. 32a, 32b, and 37, that for these low-energy beams an appreciable part of the total dose comes from particles of energy less than 32 Mev. This low-energy dose (obtained by extrapolation at values of x for which the primary beam is either above 30 Mev or totally absent) is uncertain; and thus, the total dose is considerably more uncertain for the low-energy proton beams than for the high-energy beams.

In Fig. 38 the total dose, $d(E_i, r)$, is plotted against incident energy, E_i , for various distances through the shield. All plotted points are shown. The dotted portions are to be representative, but not precise. For each curve, a discontinuity is shown at the initial energy for which r is the total proton range. To the right of the discontinuity the primary beam is present while to the left of the discontinuity the primary beam is absent. Physically, of course, there is no discontinuity, but rather the function is very rapidly varying in the vicinity of the discontinuity.

In Fig. 39 the dose from secondary cascade particles, $d_s(E_i, r)$, is plotted against incident energy, E_i , for various distances through the shield.

The functions shown in Figs. 38 and 39 are those which are needed to calculate the first and second importance functions.* These importance

*Since only approximately monoenergetic beams were used in calculating $d(E_i, r)$ and $d_s(E_i, r)$ the importance functions obtained from these functions are only approximate.

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Fig. 28. Initial Proton Flux vs. Energy.

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Fig. 29. Proton Flux vs. Energy.

Fig. 30. Neutron Flux vs. Energy.

Fig. 31. Dose vs. Distance.

Fig. 32a. Dose vs. Γ .

Fig. 32b. Dose vs. Γ .

Fig. 33. Initial Proton Flux vs. Energy.

Fig. 34. Proton Flux vs. Energy.

Fig. 35. Neutron Flux vs. Energy.

Fig. 36. Dose vs. Distance.

Fig. 37. Dose vs. Γ .

Fig. 38. Dose vs. Incident Energy.

Fig. 39. Dose from All Secondaries vs. Incident Energy.

functions, calculated using the flare spectrum of Fig. 2, are shown in Figs. 40 and 41.

From Fig. 40 it follows that the primary beam contributes an appreciable part of the total dose at all thicknesses considered. Furthermore, it is clear that one cannot specify any energy region as being most important without specifying the shield thickness to be considered. For a given shield thickness the most important energy is that such that the proton range at this energy just exceeds the shield thickness.

The second importance function, Fig. 41, is slightly larger at the lower energies (~ 300 Mev) but does not vary appreciably over the entire energy range. This means that from the point of view of producing secondaries no one energy range in the incident flare spectrum predominates.

VI. Conclusions

On the basis of the calculations a few tentative conclusions can be drawn. The tentative nature of these conclusions must be emphasized since the calculations in their present form leave much to be desired. It must be remembered that only one flare has been considered. It must also be remembered that the calculations are very approximate (straight-ahead approximations), are for only one element, yield essentially no information below 30 Mev, and employ a crude flux-to-dose conversion. Also, no contribution from gamma rays is included.

In shielding against solar flares of the general shape of that given in Fig. 2 it seems that secondaries will not be a problem unless thick shields (> 50 gm/cm² of aluminum) are used.* The most characteristic feature of such a flare is the large number of very low-energy protons which will be stopped by even thin shields.

In doing experiments using monoenergetic beams to obtain information for shielding against the flare of Fig. 2 there is no single energy which is most interesting.

*It must be understood that this statement is based on the dose in rads. When the dose in rems is considered it may no longer be correct.

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Fig. 40. First Importance Function vs. Incident Energy. Flare spectrum of Fig. 2 incident.

Fig. 41. Second Importance Function vs. Incident Energy. Flare spectrum of Fig. 2 incident.

Rather one should use an experimental shield thickness and proton energy such that the proton range at this energy is slightly greater than the shield thickness. This insures that the measurement will be made where the dose is largest; however, since most of this dose will be from the primary beam (and can thus readily be calculated) such measurements will be relatively uninteresting.

In doing experiments to obtain information for checking shielding calculations the secondary fluxes are necessarily of the most interest. Then -- always having in mind the flare spectrum of Fig. 2 -- all energies (> 200 Mev) are of roughly the same interest and the shield thickness used must be such that the primary beam is stopped in the shield.

Appendix I

In this appendix calculations as done in this report are compared with similar calculations by R. I. Allen and his coworkers.^{4*} The calculations differ in that the Lockheed calculation does not include pions and muons and the physical data concerning high-energy interactions were derived from quite different sources. Allen et al. rely entirely on the calculations of Metropolis and his coworkers for the high-energy data while the high-energy data used in this report were obtained from cosmic-ray measurements. This means, of course, that the Lockheed results apply directly to an aluminum shield while the calculations in this report apply only approximately to an aluminum shield.

The initial proton spectrum which is used in making the comparison is the proton spectrum of the Van Allen belt as determined by Freden and White.^{8**} The initial neutron, pion, and muon fluxes are taken to be zero.

The results of our calculations as well as the comparison between the two calculations are shown in Figs. A-1 to A-7. In Fig. A-1 the primary proton flux is plotted as a function of energy for several shield thicknesses. The curve at $r = 0$ is, of course, the initial proton spectrum. The only appreciable difference between the two calculations occurs at the high energies where the primary proton flux is rigorously zero (for the assumed incident spectrum).***

The secondary proton flux is shown in Fig. A-2. The two calculations are in reasonable agreement but our calculation is somewhat lower than Lockheed's at the low energies (< 300 Mev). The bump in the $r = 98.5$ gm/cm² curve at $E \sim 900$ Mev is real and is due to the presence of the cutoff in the primary proton flux.

*We wish to thank the members of the Lockheed group, particularly C. W. Hill, for making some of their unpublished results available to us.

**See also Ref. 4, p. 42.

***This cutoff in the primary proton flux is discussed on p. 12 of this report.

8. S. C. Freden and R. S. White, Particle Fluxes in the Inner Radiation Belt, J. Geophys. Res. 65, 1377-1383 (1960). The initial spectrum is the integral of the angular flux over 4π steradians, so if one assumes that the protons in the Van Allen belt are isotropic, the dose given in Fig. A-7 is the dose at the center of a spherical shell shield of thickness r .

Fig. A-1. Primary Proton Flux vs. Energy. Freden-White spectrum incident.⁸

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Fig. A-2. Secondary Proton Flux vs. Energy. Freden-White spectrum incident.⁸

Fig. A-3. Neutron Flux vs. Energy. Freden-White spectrum incident.⁸

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ORNL-LR-DWG 72299 Fig. A-4. Neutron Flux vs. Energy. Freden-White spectrum incident.⁸

Fig. A-5. Pion Flux vs. Energy. Freden-White proton spectrum incident.⁸

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ORNL-LR-DWG 72302 Fig. A-6. Muon Flux vs. Energy. Freden-White spectrum incident.⁸

Fig. A-7. Dose vs. Distance. Freden-White spectrum incident.⁸

The secondary neutron flux is shown in Figs. A-3 and A-4. In the case of these neutrons there are appreciable differences over most of the energy range. The differences are not so large that they cannot reasonably be accounted for by the different multiplicities used in the calculations.

The secondary pion and muon fluxes are shown in Figs. A-5 and A-6. The secondary pion and muon fluxes are small compared with the proton and neutron fluxes and can be neglected.

The total dose is shown as a function of distance in Fig. A-7. In calculating the dose no attempt has been made to make the calculations equivalent. That is, the methods used to estimate the dose in the two calculations are different, so even if all fluxes were the same the dose results would be different.* Considering this, the discrepancy between the two dose calculations is reasonably satisfactory.

*In addition to the fact that the dose from low-energy particles is estimated quite differently a difference also exists in the method used to obtain the dose from high-energy particles. In the Lockheed calculation the method of Ref. 3 is used while in our calculation the method of Ref. 1 is used (see footnote on page 8).

Appendix II

In this appendix some numerical results from the dose calculation are given. All results given are for $\Gamma = 32$ Mev; that is, all of the results of this appendix are obtained from the calculated fluxes without extrapolation.

In order to show how much each physical process contributes to the total dose the contributions from nonelastic scattering, atomic stopping, etc. are shown separately. All the symbols used are defined at the beginning of this report.

Proton Flare Spectrum Incident Isotropically (see Fig. 2)

r, in Collision Lengths	$d_{P_i S}$	$d_{P_i A}$	$d_{P_s S}$	$d_{P_s A}$	d_{NE}	d_{NA}	$d_{\pi S}$	$d_{\pi A}$	d_{μ}
Dose in rad/steradian									
0	8.00×10^2	1.97×10	0	0	0	0	0	0	0
0.1	2.79×10^1	1.43×10^0	1.74×10^{-1}	9.64×10^{-3}	1.66×10^{-2}	2.38×10^{-2}	2.58×10^{-3}	6.67×10^{-4}	4.09×10^{-6}
0.2	7.44×10^0	5.64×10^{-1}	1.40×10^{-1}	1.00×10^{-2}	2.16×10^{-2}	3.25×10^{-2}	3.65×10^{-3}	9.93×10^{-4}	1.03×10^{-5}
0.3	3.18×10^0	3.11×10^{-1}	1.11×10^{-1}	9.42×10^{-3}	2.35×10^{-2}	3.63×10^{-2}	4.04×10^{-3}	1.14×10^{-3}	1.63×10^{-5}
0.4	1.67×10^0	1.97×10^{-1}	8.98×10^{-2}	8.54×10^{-3}	2.40×10^{-2}	3.77×10^{-2}	4.07×10^{-3}	1.19×10^{-3}	2.12×10^{-5}
0.5	9.81×10^{-1}	1.34×10^{-1}	7.37×10^{-2}	7.66×10^{-3}	2.38×10^{-2}	3.81×10^{-2}	3.92×10^{-3}	1.18×10^{-3}	2.47×10^{-5}
0.6	6.24×10^{-1}	9.35×10^{-2}	6.10×10^{-2}	6.71×10^{-3}	2.31×10^{-2}	3.74×10^{-2}	3.65×10^{-3}	1.12×10^{-3}	2.70×10^{-5}
0.7	4.18×10^{-1}	6.86×10^{-2}	5.11×10^{-2}	5.90×10^{-3}	2.21×10^{-2}	3.62×10^{-2}	3.33×10^{-3}	1.04×10^{-3}	2.83×10^{-5}
0.8	2.91×10^{-1}	5.17×10^{-2}	4.31×10^{-2}	5.13×10^{-3}	2.10×10^{-2}	3.48×10^{-2}	3.02×10^{-3}	9.60×10^{-4}	2.88×10^{-5}
0.9	2.09×10^{-1}	3.96×10^{-2}	3.67×10^{-2}	4.47×10^{-3}	1.99×10^{-2}	3.32×10^{-2}	2.72×10^{-3}	8.76×10^{-4}	2.87×10^{-5}
1.0	1.52×10^{-1}	3.01×10^{-2}	3.13×10^{-2}	3.88×10^{-3}	1.87×10^{-2}	3.15×10^{-2}	2.44×10^{-3}	7.92×10^{-4}	2.83×10^{-5}
1.1	1.14×10^{-1}	2.37×10^{-2}	2.68×10^{-2}	3.35×10^{-3}	1.75×10^{-2}	2.97×10^{-2}	2.16×10^{-3}	7.07×10^{-4}	2.75×10^{-5}
1.2	8.67×10^{-2}	1.88×10^{-2}	2.30×10^{-2}	2.90×10^{-3}	1.64×10^{-2}	2.80×10^{-2}	1.92×10^{-3}	6.30×10^{-4}	2.66×10^{-5}
1.3	6.64×10^{-2}	1.51×10^{-2}	1.98×10^{-2}	2.50×10^{-3}	1.53×10^{-2}	2.63×10^{-2}	1.71×10^{-3}	5.64×10^{-4}	2.54×10^{-5}
1.4	5.09×10^{-2}	1.13×10^{-2}	1.71×10^{-2}	2.15×10^{-3}	1.42×10^{-2}	2.46×10^{-2}	1.52×10^{-3}	4.98×10^{-4}	2.42×10^{-5}
1.5	3.98×10^{-2}	9.16×10^{-3}	1.48×10^{-2}	1.85×10^{-3}	1.32×10^{-2}	2.30×10^{-2}	1.34×10^{-3}	4.40×10^{-4}	2.29×10^{-5}
Dose in (rad/hr)/(proton/cm ² sec), $E_i = 1000$ Mev									
0	1.28×10^{-4}	2.16×10^{-4}	0	0	0	0	0	0	0
0.1	1.17×10^{-4}	1.94×10^{-4}	3.73×10^{-5}	6.80×10^{-6}	3.33×10^{-6}	8.07×10^{-6}	3.24×10^{-6}	1.69×10^{-6}	3.58×10^{-9}

r, in Collision Lengths	d_{PiS}	d_{PiA}	d_{PsS}	d_{PsA}	d_{NE}	d_{NA}	$d_{\pi\text{S}}$	$d_{\pi\text{A}}$	d_{μ}
0.2	1.08×10^{-4}	1.74×10^{-4}	5.73×10^{-5}	1.17×10^{-5}	6.22×10^{-6}	1.50×10^{-5}	5.63×10^{-6}	2.97×10^{-6}	1.13×10^{-8}
0.3	9.87×10^{-5}	1.56×10^{-4}	7.04×10^{-5}	1.52×10^{-5}	8.75×10^{-6}	2.10×10^{-5}	8.07×10^{-6}	3.60×10^{-6}	2.12×10^{-8}
0.4	9.04×10^{-5}	1.39×10^{-4}	7.93×10^{-5}	1.76×10^{-5}	1.09×10^{-5}	2.60×10^{-5}	8.68×10^{-6}	4.61×10^{-6}	3.22×10^{-8}
0.5	8.41×10^{-5}	1.27×10^{-4}	8.48×10^{-5}	1.92×10^{-5}	1.27×10^{-5}	3.02×10^{-5}	9.55×10^{-6}	5.05×10^{-6}	4.35×10^{-8}
0.6	7.53×10^{-5}	1.11×10^{-4}	8.81×10^{-5}	2.02×10^{-5}	1.42×10^{-5}	3.37×10^{-5}	1.01×10^{-5}	5.30×10^{-6}	5.46×10^{-8}
0.7	6.88×10^{-5}	9.96×10^{-5}	8.97×10^{-5}	2.07×10^{-5}	1.54×10^{-5}	3.65×10^{-5}	1.04×10^{-5}	5.49×10^{-6}	6.52×10^{-8}
0.8	6.27×10^{-5}	8.85×10^{-5}	9.00×10^{-5}	2.07×10^{-5}	1.64×10^{-5}	3.86×10^{-5}	1.05×10^{-5}	5.49×10^{-6}	7.49×10^{-8}
0.9	5.72×10^{-5}	7.87×10^{-5}	8.92×10^{-5}	2.04×10^{-5}	1.71×10^{-5}	4.03×10^{-5}	1.05×10^{-5}	5.44×10^{-6}	8.37×10^{-8}
1.0	5.21×10^{-5}	7.00×10^{-5}	8.75×10^{-5}	1.99×10^{-5}	1.77×10^{-5}	3.17×10^{-5}	1.03×10^{-5}	5.30×10^{-6}	9.15×10^{-8}
1.1	4.90×10^{-5}	6.37×10^{-5}	8.52×10^{-5}	1.92×10^{-5}	1.80×10^{-5}	4.22×10^{-5}	1.00×10^{-5}	5.10×10^{-6}	9.81×10^{-8}
1.2	4.62×10^{-5}	5.88×10^{-5}	8.23×10^{-5}	1.83×10^{-5}	1.82×10^{-5}	4.25×10^{-5}	9.70×10^{-6}	4.86×10^{-6}	1.04×10^{-7}
1.3	4.28×10^{-5}	5.30×10^{-5}	7.89×10^{-5}	1.74×10^{-5}	1.83×10^{-5}	4.26×10^{-5}	9.29×10^{-6}	4.62×10^{-6}	1.08×10^{-7}
1.4	3.90×10^{-5}	4.69×10^{-5}	7.53×10^{-5}	1.64×10^{-5}	1.83×10^{-5}	4.23×10^{-5}	8.85×10^{-6}	4.35×10^{-6}	1.12×10^{-7}
1.5	3.50×10^{-5}	4.08×10^{-5}	7.15×10^{-5}	1.53×10^{-5}	1.81×10^{-5}	4.18×10^{-5}	8.39×10^{-6}	4.07×10^{-6}	1.14×10^{-7}

Dose in (rad/hr)/(proton/cm² sec), $E_i = 600$ Mev

0	1.48×10^{-4}	1.29×10^{-4}	0	0	0	0	0	0	0
0.1	1.36×10^{-4}	1.14×10^{-4}	2.59×10^{-5}	3.43×10^{-6}	1.96×10^{-6}	4.31×10^{-6}	1.21×10^{-6}	4.44×10^{-7}	1.50×10^{-9}
0.2	1.13×10^{-4}	1.00×10^{-4}	3.84×10^{-5}	5.49×10^{-6}	3.60×10^{-6}	7.87×10^{-6}	2.03×10^{-6}	7.48×10^{-7}	4.64×10^{-9}
0.3	1.16×10^{-4}	8.75×10^{-5}	4.53×10^{-5}	6.71×10^{-6}	4.91×10^{-6}	1.07×10^{-5}	2.58×10^{-6}	9.48×10^{-7}	8.52×10^{-9}
0.4	1.06×10^{-4}	7.63×10^{-5}	4.88×10^{-5}	7.34×10^{-6}	5.98×10^{-6}	1.29×10^{-5}	2.92×10^{-6}	1.06×10^{-6}	1.26×10^{-8}
0.5	9.65×10^{-5}	6.56×10^{-5}	5.01×10^{-5}	7.53×10^{-6}	6.80×10^{-6}	1.46×10^{-5}	3.08×10^{-6}	1.11×10^{-6}	1.66×10^{-8}

r, in Collision Lengths	d_{P_iS}	d_{P_iA}	$d_{P_S S}$	$d_{P_S A}$	d_{NE}	d_{NA}	$d_{\pi S}$	$d_{\pi A}$	d_{μ}
0.6	8.78×10^{-5}	5.64×10^{-5}	4.96×10^{-5}	7.34×10^{-6}	7.39×10^{-6}	1.58×10^{-5}	3.12×10^{-6}	1.10×10^{-6}	2.01×10^{-8}
0.7	9.72×10^{-5}	5.88×10^{-5}	4.84×10^{-5}	7.00×10^{-6}	7.82×10^{-6}	1.67×10^{-5}	3.06×10^{-6}	1.07×10^{-6}	2.32×10^{-8}
0.8	7.58×10^{-5}	4.27×10^{-5}	4.57×10^{-5}	6.42×10^{-6}	8.07×10^{-6}	1.71×10^{-5}	2.92×10^{-6}	1.00×10^{-6}	2.58×10^{-8}
0.9	7.94×10^{-5}	4.16×10^{-5}	4.28×10^{-5}	5.83×10^{-6}	8.16×10^{-6}	1.73×10^{-5}	2.74×10^{-6}	9.19×10^{-7}	2.77×10^{-8}
1.0	7.53×10^{-5}	3.64×10^{-5}	3.96×10^{-5}	5.15×10^{-6}	8.21×10^{-6}	1.72×10^{-5}	2.52×10^{-6}	8.26×10^{-7}	2.90×10^{-8}
1.1	6.97×10^{-5}	3.08×10^{-5}	3.62×10^{-5}	4.47×10^{-6}	8.12×10^{-6}	1.69×10^{-5}	2.30×10^{-6}	7.39×10^{-7}	2.98×10^{-8}
1.2	6.54×10^{-5}	2.62×10^{-5}	3.28×10^{-5}	3.82×10^{-6}	7.97×10^{-6}	1.65×10^{-5}	2.09×10^{-6}	6.51×10^{-7}	3.00×10^{-8}
1.3	6.14×10^{-5}	2.20×10^{-5}	2.94×10^{-5}	3.21×10^{-6}	7.78×10^{-6}	1.60×10^{-5}	1.89×10^{-6}	5.73×10^{-7}	2.99×10^{-8}
1.4	6.12×10^{-5}	1.93×10^{-5}	2.60×10^{-5}	2.64×10^{-6}	7.48×10^{-6}	1.55×10^{-5}	1.70×10^{-6}	5.01×10^{-7}	2.94×10^{-8}
1.5	5.90×10^{-5}	1.61×10^{-5}	2.27×10^{-5}	2.12×10^{-6}	7.24×10^{-6}	1.48×10^{-5}	1.53×10^{-6}	4.36×10^{-7}	2.80×10^{-8}

Dose in (rad/hr)/(proton/cm² sec), $E_i = 400$ Mev

0	1.76×10^{-4}	8.65×10^{-5}	0	0	0	0	0	0	0
0.1	1.66×10^{-4}	7.44×10^{-5}	1.61×10^{-5}	1.50×10^{-6}	1.06×10^{-6}	2.07×10^{-6}	3.59×10^{-7}	9.77×10^{-8}	4.90×10^{-10}
0.2	1.54×10^{-4}	6.27×10^{-5}	2.31×10^{-5}	2.23×10^{-6}	1.93×10^{-6}	3.73×10^{-6}	6.04×10^{-7}	1.62×10^{-7}	1.50×10^{-9}
0.3	1.61×10^{-4}	5.88×10^{-5}	2.59×10^{-5}	2.48×10^{-6}	2.59×10^{-6}	4.96×10^{-6}	7.53×10^{-7}	1.97×10^{-7}	2.73×10^{-9}
0.4	1.48×10^{-4}	4.51×10^{-5}	2.64×10^{-5}	2.42×10^{-6}	3.06×10^{-6}	5.83×10^{-6}	8.32×10^{-7}	2.12×10^{-7}	3.97×10^{-9}
0.5	1.38×10^{-4}	3.87×10^{-5}	2.53×10^{-5}	2.18×10^{-6}	3.39×10^{-6}	6.37×10^{-6}	8.56×10^{-7}	2.11×10^{-7}	5.10×10^{-9}
0.6	1.37×10^{-4}	3.27×10^{-5}	2.32×10^{-5}	1.83×10^{-6}	3.58×10^{-6}	6.70×10^{-6}	8.41×10^{-7}	1.99×10^{-7}	6.05×10^{-9}
0.7	1.36×10^{-4}	2.68×10^{-5}	2.01×10^{-5}	1.41×10^{-6}	3.66×10^{-6}	6.76×10^{-6}	7.88×10^{-7}	1.78×10^{-7}	6.76×10^{-9}
0.8	1.37×10^{-4}	2.13×10^{-5}	1.64×10^{-5}	9.87×10^{-7}	3.64×10^{-6}	6.66×10^{-6}	7.08×10^{-7}	1.53×10^{-7}	7.18×10^{-9}
0.9	1.48×10^{-4}	1.64×10^{-5}	1.23×10^{-5}	6.08×10^{-7}	3.54×10^{-6}	6.42×10^{-6}	5.99×10^{-7}	1.23×10^{-7}	7.29×10^{-9}
1.0	1.73×10^{-4}	1.11×10^{-5}	7.36×10^{-6}	2.80×10^{-7}	3.38×10^{-6}	6.12×10^{-6}	4.79×10^{-7}	9.53×10^{-8}	7.06×10^{-9}

r, in Collision Lengths	d_{PiS}	d_{PiA}	d_{PsS}	d_{PsA}	d_{NE}	d_{NA}	$d_{\pi\text{S}}$	$d_{\pi\text{A}}$	d_{μ}
1.1	1.88×10^{-4}	3.72×10^{-6}	1.75×10^{-6}	7.92×10^{-8}	3.14×10^{-6}	5.69×10^{-6}	3.83×10^{-7}	7.29×10^{-8}	6.56×10^{-9}
1.2	0	0	1.06×10^{-6}	5.88×10^{-8}	2.90×10^{-6}	5.25×10^{-6}	3.04×10^{-7}	5.54×10^{-8}	5.87×10^{-9}
1.3	0	0	9.55×10^{-7}	5.39×10^{-8}	2.67×10^{-6}	4.84×10^{-6}	2.41×10^{-7}	4.15×10^{-8}	5.04×10^{-9}
1.4	0	0	8.81×10^{-7}	4.96×10^{-8}	2.46×10^{-6}	4.47×10^{-6}	1.88×10^{-7}	3.08×10^{-8}	4.12×10^{-9}
1.5	0	0	8.14×10^{-7}	4.60×10^{-8}	2.27×10^{-6}	4.14×10^{-6}	1.44×10^{-7}	2.26×10^{-8}	3.18×10^{-9}

Dose in (rad/hr)/(proton/cm² sec), $E_i = 300$ Mev

0	2.05×10^{-4}	6.46×10^{-5}	0	0	0	0	0	0	0	N
0.1	1.95×10^{-4}	5.35×10^{-5}	1.21×10^{-5}	8.51×10^{-7}	7.39×10^{-7}	1.31×10^{-6}	1.22×10^{-7}	5.49×10^{-8}	1.80×10^{-10}	
0.2	2.14×10^{-4}	4.96×10^{-5}	1.66×10^{-5}	1.14×10^{-7}	1.32×10^{-7}	2.30×10^{-6}	1.97×10^{-7}	8.55×10^{-8}	5.30×10^{-10}	
0.3	2.02×10^{-4}	3.86×10^{-5}	1.66×10^{-5}	1.05×10^{-7}	1.69×10^{-7}	2.92×10^{-6}	2.26×10^{-7}	9.38×10^{-8}	9.19×10^{-10}	
0.4	1.95×10^{-4}	2.89×10^{-5}	1.45×10^{-5}	7.97×10^{-7}	1.90×10^{-7}	3.05×10^{-6}	2.21×10^{-7}	8.70×10^{-8}	1.24×10^{-9}	
0.5	2.14×10^{-4}	2.21×10^{-5}	1.12×10^{-5}	4.96×10^{-7}	1.99×10^{-7}	3.33×10^{-6}	1.86×10^{-7}	7.00×10^{-8}	1.44×10^{-9}	
0.6	2.57×10^{-4}	1.46×10^{-5}	6.10×10^{-6}	1.91×10^{-8}	1.96×10^{-7}	3.24×10^{-6}	1.42×10^{-7}	5.10×10^{-8}	1.48×10^{-9}	
0.7	1.29×10^{-5}	1.99×10^{-7}	4.66×10^{-7}	1.88×10^{-8}	1.83×10^{-7}	3.01×10^{-6}	1.08×10^{-7}	3.69×10^{-8}	1.40×10^{-9}	
0.8	0	0	3.64×10^{-7}	1.60×10^{-8}	1.68×10^{-7}	2.77×10^{-6}	8.21×10^{-8}	2.61×10^{-8}	1.24×10^{-9}	
0.9	0	0	3.34×10^{-7}	1.47×10^{-8}	1.54×10^{-7}	2.55×10^{-6}	6.12×10^{-8}	1.81×10^{-8}	1.02×10^{-9}	
1.0	0	0	3.08×10^{-7}	1.36×10^{-8}	1.41×10^{-7}	2.34×10^{-6}	4.42×10^{-8}	1.21×10^{-8}	7.58×10^{-10}	
1.1	0	0	2.84×10^{-7}	1.25×10^{-8}	1.29×10^{-7}	2.15×10^{-6}	3.02×10^{-8}	7.68×10^{-9}	5.01×10^{-10}	
1.2	0	0	2.62×10^{-7}	1.15×10^{-8}	1.19×10^{-7}	1.98×10^{-6}	1.92×10^{-8}	4.66×10^{-9}	2.89×10^{-10}	
1.3	0	0	2.42×10^{-7}	1.06×10^{-8}	1.09×10^{-7}	1.82×10^{-6}	1.13×10^{-8}	2.72×10^{-9}	1.44×10^{-10}	
1.4	0	0	2.23×10^{-7}	9.82×10^{-9}	1.00×10^{-7}	1.68×10^{-6}	6.27×10^{-9}	1.61×10^{-9}	6.56×10^{-11}	
1.5	0	0	2.06×10^{-7}	8.94×10^{-9}	9.19×10^{-8}	1.55×10^{-6}	3.77×10^{-9}	1.10×10^{-9}	3.28×10^{-11}	

r, in Collision Lengths	d_{P_iS}	d_{P_iA}	$d_{P_S S}$	$d_{P_S A}$	d_{NE}	d_{NA}	$d_{\pi S}$	$d_{\pi A}$	d_{μ}
Dose in (rad/hr)/(proton/cm ² sec), $E_i = 200$ Mev									
0	2.61×10^{-4}	4.47×10^{-5}	0	0	0	0	0	0	0
0.1	2.80×10^{-4}	3.57×10^{-5}	6.75×10^{-6}	3.03×10^{-7}	3.92×10^{-7}	5.93×10^{-6}	9.47×10^{-8}	1.50×10^{-8}	1.63×10^{-10}
0.2	3.01×10^{-4}	2.49×10^{-5}	6.62×10^{-6}	2.40×10^{-7}	6.22×10^{-7}	9.14×10^{-6}	8.06×10^{-8}	1.22×10^{-8}	3.38×10^{-10}
0.3	4.26×10^{-4}	1.52×10^{-5}	2.26×10^{-6}	5.10×10^{-7}	6.76×10^{-7}	9.67×10^{-6}	5.68×10^{-8}	8.02×10^{-9}	3.92×10^{-10}
0.4	0	0	5.62×10^{-8}	1.79×10^{-9}	6.17×10^{-7}	8.89×10^{-6}	3.97×10^{-8}	5.15×10^{-9}	3.70×10^{-10}
0.5	0	0	5.15×10^{-8}	1.65×10^{-9}	5.64×10^{-7}	8.12×10^{-6}	2.69×10^{-8}	3.16×10^{-9}	2.94×10^{-10}
0.6	0	0	4.73×10^{-8}	1.51×10^{-9}	5.15×10^{-7}	7.44×10^{-6}	1.68×10^{-8}	1.75×10^{-9}	1.85×10^{-10}
0.7	0	0	4.35×10^{-8}	1.39×10^{-9}	4.69×10^{-7}	6.80×10^{-6}	8.73×10^{-9}	7.92×10^{-10}	7.92×10^{-11}
0.8	0	0	4.00×10^{-8}	1.28×10^{-9}	4.27×10^{-7}	6.17×10^{-6}	2.72×10^{-9}	2.27×10^{-10}	1.80×10^{-11}
0.9	0	0	3.68×10^{-8}	1.18×10^{-9}	3.89×10^{-7}	5.64×10^{-6}	4.23×10^{-10}	5.01×10^{-11}	2.97×10^{-12}
1.0	0	0	3.38×10^{-8}	1.08×10^{-9}	3.54×10^{-7}	5.15×10^{-6}	2.99×10^{-10}	3.99×10^{-11}	1.68×10^{-12}
1.1	0	0	3.11×10^{-8}	9.96×10^{-10}	3.23×10^{-7}	4.73×10^{-6}	2.75×10^{-10}	3.67×10^{-11}	1.50×10^{-12}
1.2	0	0	2.86×10^{-8}	9.09×10^{-10}	2.95×10^{-7}	4.32×10^{-6}	2.53×10^{-10}	3.38×10^{-11}	1.37×10^{-12}
1.3	0	0	2.64×10^{-8}	8.46×10^{-10}	2.68×10^{-7}	3.95×10^{-6}	2.33×10^{-10}	3.11×10^{-11}	1.26×10^{-12}
1.4	0	0	2.42×10^{-8}	7.78×10^{-10}	2.45×10^{-7}	3.62×10^{-6}	2.14×10^{-10}	2.86×10^{-11}	1.16×10^{-12}
1.5	0	0	2.23×10^{-8}	7.14×10^{-10}	2.23×10^{-7}	3.30×10^{-6}	1.97×10^{-10}	2.63×10^{-11}	1.07×10^{-12}

Appendix III

In this appendix those parts of 1 (Sections I - IV, pp. 1-38, of ORNL-3289, Nucleon-Meson Cascade Calculations: Transverse Shielding for a 45-Gev Electron Accelerator (Part I), by R. G. Alsmiller, Jr., F. S. Alsmiller, and J. E. Murphy) which are relevant to the present work are reproduced verbatim. The reader must, of course, refer to 1 itself to find Sections V and VI and the Appendices I - III referred to here.

I. INTRODUCTION

In the proposed Stanford 45-Gev electron accelerator¹ it is expected that some electrons will strike the walls of the accelerator tube and give rise to penetrating photonucleons and photopions. The accelerator will be surrounded by an earth shield; estimates of the shielding thickness required have been made by the Stanford group.^{1,2}

In this report an approximate nucleon-meson cascade calculation is carried out in order that more reliable estimates of the shielding thickness may be obtained. No attempt has been made to estimate the shielding thickness, but, rather, results are presented which we hope will enable the Stanford group to obtain such estimates.

Using the straight-ahead approximation, i.e., assuming that a particle and all of the secondaries, tertiaries, etc. produced by this particle travel in the same straight line, a set of coupled integro-differential transport equations which give the neutron, proton, charged pion, and muon fluxes as a function of energy and distance is obtained. Because there is very little experimental information about high-energy interactions the equations include many quantities, e.g., cross sections, multiplicities, etc., which are known only very approximately. In order to obtain the necessary information to make the calculation we have found it necessary to introduce many ad hoc assumptions. Consequently, the results presented in this report must be considered to be of a very preliminary nature.

The equations have been solved numerically using an IBM-7090 code which was written for this purpose.* The code is in no way special to the Stanford problem and may be used for other shielding calculations. Numerical results are given for several cases of interest. Because of the approximations involved, particularly the straight-ahead approximation,

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1. Proposal for a Two-Mile Linear Electron Accelerator, Stanford University, Stanford, California (1957).
 2. H. C. DeStaebler, Jr., A Review of Transverse Shielding Requirements for the Stanford Two-Mile Accelerator, W. W. Hansen Laboratories of Physics, M-262 (1962).

*The coding and numerical computations were executed by R. G. Mashburn, I. R. Parsley, R. G. Edwards, and C. S. Hodge of the Central Data Processing Facility of the Oak Ridge Gaseous Diffusion Plant.

the equations are valid only at reasonably high energies; the results are given only at the higher energies.

Using an approximate method introduced by W. A. Gibson,³ the calculated fluxes are converted to dose in rads. In the dose calculations an estimate of the contribution from low-energy neutrons, but not from low-energy charged particles, is included.

In Section II a description is given of the nucleon-meson cascade equations. In Section III the methods used to obtain the physical information required in the calculation are discussed. In Section IV the method used to convert the various fluxes to dose is described. In Section V the Stanford problem is considered briefly and initial values used in the calculation are discussed. In Section VI the results are given.

In Appendix I some results are shown which are obtained by using physical data different from that used in the body of the paper, in order to test the sensitivity of the calculation to this input data. In Appendix II some of the detailed numerical results from the dose calculation are given. Appendix III includes a discussion of the effect of the air gap between the accelerator tube and the earth shield, and also a more rigorous treatment of the muon production.

II. NUCLEON-MESON CASCADE EQUATIONS

We restrict consideration to the four cascade components: neutrons, protons, charged pions, and muons arising from pi-mu decay. No distinction is made between positively and negatively charged pions, or between positive and negative muons. Neutral pions decay rapidly into two gamma rays and can affect the cascade only through the subsequent photo-production of lower energy nucleons and mesons; these we neglect.

It is convenient for study of the Stanford problem to consider a point source* on the inside surface of the shield from which each component may be emitted at a constant rate, with arbitrary distribution in energy and angle. The equations used to describe the cascade developing in each

3. W. A. Gibson, Energy Removal from Primary Proton and Neutron Beams by Tissue, ORNL-3260 (1962).

*Although the discussion will be carried out for a point source, the equations will be equally valid - with some reinterpretation of the functions involved - for a plane parallel beam incident on a shield.

direction are appropriate versions of transport equations employed by many authors; e.g., Messel,⁴ Rossi,⁵ Puppi,⁶ among others.

In all cases, E will refer to the kinetic energy in Mev and r to the distance into the shield measured from the source in a given direction. We denote by λ a general "collision length," in units of which r will be measured. Normally, λ^{-1} will be taken proportional to the density, ρ . Nevertheless, the results will not be independent of ρ , or of the value of λ used, because λ enters into the meson equations explicitly.

Subscripts N , P , π , and μ on cross sections and the like will refer to neutrons, protons, pions, and muons, respectively.

All particles are assumed to travel in straight line paths in their initial directions. Neutron moderation, e.g., by elastic nuclear collisions, is omitted;* but the charged particles are assumed to have a continuous rate of energy loss per collision length:

$$S_j(E) = - \left. \frac{dE}{dr} \right|_j = \lambda \sum_i n_i \epsilon_{ij}(E): \quad (j = P, \pi, \mu) \quad (2.1)$$

where

n_i = number density of the i^{th} element nuclei in the shielding medium,

$\epsilon_{ij}(E)$ = the stopping cross section of the i^{th} element for the j^{th} particle species.

The stopping at energies less than 45 Gev is primarily due to atomic excitation and ionization; this is the only process included in the present calculation. At higher energies, it may be necessary to include

4. H. Messel, Progress in Cosmic Ray Physics, Vol. 11, Interscience Publishers, Inc., New York (1954), p. 197.

5. B. Rossi, High-Energy Particles, Prentice-Hall, Inc., New Jersey (1956), p. 232.

6. G. Puppi, Progress in Cosmic Ray Physics, Vol. 111, Interscience Publishers, Inc., New York (1956), p. 351.

*This is clearly not valid for hydrogenous materials in which both the neutron slowing-down and the generation of energetic recoil protons are important. A modification to include these effects is under consideration.

losses from radiation and electron pair production.*

The mesons are further attenuated by spontaneous decay. If t_j ($j = \pi, \mu$) is the lifetime in the meson rest system, the apparent lifetime in the lab system is $t_j(1 - \beta^2)^{-1/2}$, where $\beta = v/c$; and the average distance traveled before decay is

$$\Lambda_j = \beta c t_j (1 - \beta^2)^{-1/2}. \quad (2.2)$$

The decay probability per meson per collision length is λ/Λ_j ; one can use

$$E + m_j c^2 = m_j c^2 (1 - \beta^2)^{-1/2}$$

to obtain

$$\frac{\lambda}{\Lambda_j} = \frac{\lambda m_j c}{t_j} [E(E + 2m_j c^2)]^{-1/2} \quad (2.3)$$

where m_j is the meson mass.

Nucleons and pions are also absorbed in nonelastic nuclear collisions; the total absorption probability per collision length per particle of energy E is represented by:

$$Q_j(E) = \lambda \sum_i n_i \sigma_{ij}(E) \quad (j = N, P, \pi) \quad (2.4)$$

where

σ_{ij} = the nonelastic collision cross section for j -type particles with i^{th} element nuclei.

We neglect the relatively weak interaction of muons with nuclei.

Each such nonelastic collision is accompanied by the emergence of secondary nucleons and pions, all, by assumption, in the direction of the incident particle in the lab system. One can represent by the product:

*Reference 5, Sections 2.14 and 2.20.

$$\nu_{ijk}(E') f_{ijk}(E', E) dE \quad (j, k = N, P, \pi)$$

the average number of k-type secondaries, with energies between E and E + dE, produced per collision between a j-type incident particle of energy E' and an ith element nucleus.

The energy distribution functions, $f_{ijk}(E', E)$, are normalized such that

$$\int_{E_L}^{E'} f_{ijk}(E', E) dE = 1 \quad (2.5)$$

where E_L is a lower limit ($E_L \geq 0$) we may wish to put on the energy of the secondaries. Then the multiplicity, $\nu_{ijk}(E')$, is the total number of the specified secondaries produced per collision with energies greater than E_L .

The average distribution functions and multiplicities for mixtures may be formally defined:

$$f_{jk}(E', E) = \frac{\sum_i n_i \sigma_{ij}(E') \nu_{ijk}(E') f_{ijk}(E', E)}{\sum_i n_i \sigma_{ij}(E') \nu_{ijk}(E')} \quad (2.6)$$

$$\nu_{jk}(E') = \frac{\sum_i n_i \sigma_{ij}(E') \nu_{ijk}(E')}{\sum_i n_i \sigma_{ij}(E')} \quad (2.7)$$

At present, the use of these expressions is impractical since the necessary data is missing.

For purposes of simplicity we have assumed that both the energy distribution and the multiplicity of each kind of secondary particle are independent of the nature of the incident particle.* Hence, f_{jk} and

*This restriction is probably more serious for the multiplicities than for the distribution functions.

v_{jk} of Eqs. (2.6) and (2.7) are replaced by f_k and v_k , where k continues to refer to the particle produced.

For a particular direction of emission into the solid angle element, $d\Omega$, about the vector $\vec{r}$, let $N(E,r)dE d\Omega$ represent the average number of neutrons at r , with energies between E and $E + dE$ and with velocity directions in $d\Omega$, which cross the area element, $r^2 d\Omega$ (normal to $\vec{r}$), per second. We call $N(E,r)$ the differential neutron intensity at E and r (in units of neutrons per Mev-second-steradian); the corresponding neutron flux (in units of neutrons per Mev-second-[collision length]²) is obtained after division by r^2 . Similarly, let $P(E,r)$, $\pi(E,r)$, and $\mu(E,r)$ be the respective proton, pion, and muon intensities.

The intensities are commonly separated into "primary" and "secondary" components, which will be denoted by subscripts i and s , respectively. Hence,

$$N(E,r) = N_i(E,r) + N_s(E,r)$$

with similar sums for the other particles. Analytic solutions can be obtained for the primaries, which satisfy the uni-directional equations without sources:

$$\frac{\partial N_i}{\partial r} + Q_N N_i = 0$$

$$\frac{\partial P_i}{\partial r} + Q_P P_i - \frac{\partial}{\partial E} [S_P P_i] = 0$$

$$\frac{\partial \pi_i}{\partial r} + \left[Q_\pi + \frac{\lambda}{\Lambda_\pi} \right] \pi_i - \frac{\partial}{\partial E} [S_\pi \pi_i] = 0$$

$$\frac{\partial \mu_i}{\partial r} + \frac{\lambda}{\Lambda_\mu} \mu_i - \frac{\partial}{\partial E} [S_\mu \mu_i] = 0. \quad (2.8)$$

The primary intensities at $r = 0$ are the time-independent emission rates from the point source: $N(E,0)$, $P(E,0)$, etc.; these may be specified arbitrarily for each problem and serve as boundary conditions for Eqs. (2.8).

The energy E_j ($j = p, \pi, \mu$), which a charged particle must possess initially if it is to be slowed down to energy E in the distance r is defined by the relation:

$$r = \int_E^{E_j} \frac{dE'}{S_j(E')} \quad (2.9)$$

Using the energies $E_p(E, r)$, $E_\pi(E, r)$, and $E_\mu(E, r)$, the solutions of Eqs. (2.8) are:

$$\begin{aligned} N_i(E, r) &= N(E, 0) e^{-Q_n(E)r} \\ P_i(E, r) &= P(E_p, 0) \left[\frac{S_p(E_p)}{S_p(E)} \right] e^{-\int_E^{E_p} dE' Q_p(E') \left[S_p(E') \right]^{-1}} \\ \pi_i(E, r) &= \pi(E_\pi, 0) \left[\frac{S_\pi(E_\pi)}{S_\pi(E)} \right] e^{-\int_E^{E_\pi} dE' \left[Q_\pi(E') + \frac{\lambda}{\Lambda_\pi(E')} \right] \left[S_\pi(E') \right]^{-1}} \\ \mu_i(E, r) &= \mu(E_\mu, 0) \left[\frac{S_\mu(E_\mu)}{S_\mu(E)} \right] e^{-\int_E^{E_\mu} dE' \lambda \left[\Lambda_\mu(E') S_\mu(E') \right]^{-1}} \end{aligned} \quad (2.10)$$

The secondary components of the cascade particle intensities satisfy equations similar to those for the primaries, but with added source terms, and with the boundary conditions:

$$N_s(E, 0) = P_s(E, 0) = \pi_s(E, 0) = \mu_s(E, 0) = 0. \quad (2.11)$$

The equations are:

$$\frac{\partial N_s}{\partial r} + Q_N N_s = \int_E^{E_0} dE' f_N(E', E) v_N(E') \bar{\phi}(E', r)$$

$$\frac{\partial P_s}{\partial r} + Q_P P_s - \frac{\partial}{\partial E} [S_P P_s] = \int_E^{E_0} dE' f_P(E', E) v_P(E') \bar{\phi}(E', r)$$

$$\frac{\partial \pi_s}{\partial r} + \left[Q_\pi + \frac{\lambda}{\Lambda_\pi} \right] \pi_s - \frac{\partial}{\partial E} [S_\pi \pi_s] = \int_E^{E_0} dE' f_\pi(E', E) v_\pi(E') \bar{\phi}(E', r)$$

$$\frac{\partial \mu_s}{\partial r} + \frac{\lambda}{\Lambda_\mu} \mu_s - \frac{\partial}{\partial E} [S_\mu \mu_s] = T_\mu(E, r) \quad (2.12)$$

where

E_0 = upper limit on kinetic energy, ($E_0 < \infty$);

$$\bar{\phi}(E', r) = \left[Q_N(N_i + N_s) + Q_P(P_i + P_s) + Q_\pi(\pi_i + \pi_s) \right]_{E', r};$$

$T_\mu(E, r)$ = intensity of muons produced per collision length.

The source terms for the nucleon and pion equations represent the intensity of particles produced per collision length and are fairly self-evident. In particular, they incorporate the approximation mentioned below Eq. (2.7).

For the production rate of muon intensity in the differential energy interval E to $E + dE$, we have used the decay rate of pion intensity in the energy interval E_π to $E_\pi + dE_\pi$, where E_π is taken proportional to E ; i.e.,

$$E_\pi = KE \quad (2.13)$$

where

$$K = \frac{m_{\pi} c^2}{m_{\mu} c^2 + E_1} = 1.27,$$

$$E_1 = 4.18 \text{ Mev}$$

= muon kinetic energy in rest system of decaying pion;

and, hence,

$$T_{\mu}(E, r) dE = \frac{\lambda K}{\Lambda_{\pi}} \frac{\pi(KE, r)}{(KE)} dE. \quad (2.14)$$

The assumption that in the pion rest system the muons are emitted isotropically, all with kinetic energy E_1 , leads to a relation between the pion energy and the average energy, E , of the muons produced:*

$$E_{\pi} + m_{\pi} c^2 = K(E + m_{\mu} c^2)$$

or

$$E_{\pi} = K(E - E_1) \approx KE.$$

Hence, Eq. (2.14), an approximation chosen for its simplicity in numerical computation, is roughly justified. (It is essentially the same as that used by Puppi.**) A more rigorous formulation is given in Appendix III.

Since the energy range under consideration is too large for convenient numerical integration, we change variables from energy E to lethargy u defined by

$$u = \ln \frac{E_0}{E}. \quad (2.15)$$

The neutron intensity per unit lethargy, $N(u, r)$, is defined by

$$N(u, r) du = - N(E, r) dE \quad (2.16)$$

or

*Reference 5, pp. 191-192.

**Reference 6, p. 351.

$$N(u, r) = E N(E, r).$$

Similarly,

$$P(u, r) = E P(E, r)$$

$$\pi(u, r) = E \pi(E, r)$$

$$\mu(u, r) = E \mu(E, r)$$

$$T_{\mu}(u, r) = E T_{\mu}(E, r) \quad (2.17)$$

In the same manner, the lethargy distribution functions per unit lethargy are defined by

$$f_j(u', u) = E f_j(E', E) \quad (j = N, P, \pi) \quad (2.18)$$

so that

$$f_j(u', u) du = - f_j(E', E) dE.$$

The stopping power functions, $S_j(u)$, are defined in the following manner:

$$S_j(u) = \frac{1}{E} S_j(E) \quad (j = P, \pi, \mu) \quad (2.19)$$

so that

$$\frac{du}{S_j(u)} = - \frac{dE}{S_j(E)};$$

$$\frac{\partial}{\partial u} \left[S_P(u) P(u, r) \right] du = \frac{\partial}{\partial E} \left[S_P(E) P(E, r) \right] dE; \text{ etc.}$$

Finally, we make the identifications:

$$Q_j(u) = Q_j(E) \quad (j = N, P, \pi) \quad (2.20)$$

$$v_j(u) = v_j(E) \quad (j = N, P, \pi) \quad (2.21)$$

$$\Lambda_j(u) = \Lambda_j(E) \quad (j = \pi, \mu) \quad (2.22)$$

Equation (2.10) for the primary components of the particle intensities become:

$$\begin{aligned} N_i(u, r) &= N(u, 0) e^{-Q_N(u)r} \\ P_i(u, r) &= P(u_P, 0) \left[\frac{S_P(u_P)}{S_P(u)} \right] e^{-\int_{u_P}^u du' Q_P(u') \left[S_P(u') \right]^{-1}} \\ \pi_i(u, r) &= \pi(u_\pi, 0) \left[\frac{S_\pi(u_\pi)}{S_\pi(u)} \right] e^{-\int_{u_\pi}^u du' \left[Q_\pi(u') + \frac{\lambda}{\Lambda_\pi(u')} \right] \left[S_\pi(u') \right]^{-1}} \\ \mu_i(u, r) &= \mu(u_\mu, 0) \left[\frac{S_\mu(u_\mu)}{S_\mu(u)} \right] e^{-\int_{u_\mu}^u du' \lambda \left[\Lambda_\mu(u') S_\mu(u') \right]^{-1}} \end{aligned} \quad (2.23)$$

with u_j , ($j = P, \pi, \mu$), defined by

$$r = \int_{u_j}^u \frac{du'}{S_j(u')} . \quad (2.24)$$

Equations (2.13) for the secondary components become:

$$\frac{\partial N_s}{\partial r} + Q_N N_s = \int_0^u du' f_N(u', u) \nu_N(u') \left[Q_N N + Q_P P + Q_\pi \pi \right]_{u', r}$$

$$\frac{\partial P_s}{\partial r} + Q_P P_s + \frac{\partial}{\partial u} \left[S_P P_s \right] = \int_0^u du' f_P(u', u) \nu_P(u') \left[Q_N N + Q_P P + Q_\pi \pi \right]_{u', r}$$

$$\frac{\partial \pi_s}{\partial r} + \left[Q_\pi + \frac{\lambda}{\Lambda_\pi} \right] \pi_s + \frac{\partial}{\partial u} \left[S_\pi \pi_s \right] = \int_0^u du' f_\pi(u', u) \nu_\pi(u') \left[Q_N N + Q_P P + Q_\pi \pi \right]_{u', r}$$

$$\frac{\partial \mu_s}{\partial r} + \frac{\lambda}{\Lambda_\mu} \mu_s + \frac{\partial}{\partial u} \left[S_\mu \mu_s \right] = \frac{\lambda \pi(u - \alpha, r)}{\Lambda_\pi(u - \alpha)}$$

$$= T_\mu(u, r), \quad (2.25)$$

where

$$u - \alpha = \ln \left[\frac{E_0}{KE} \right] = u - \ln K,$$

$$N(u', r) = N_i(u', r) + N_s(u', r), \text{etc.}$$

and

$$N_s(u, 0) = P_s(u, 0) = \pi_s(u, 0) = \mu_s(u, 0) = 0. \quad (2.26)$$

Equations (2.23) to (2.26) are solved by numerical integration. The functions $Q_j(u)$, $\nu_j(u)$, $S_j(u)$, and $f_j(u', u)$, as well as the initial spectra, $N(u, 0)$, $P(u, 0)$, $\pi(u, 0)$, and $\mu(u, 0)$ may be specified as desired. In particular, note that $f_j(u', u)$ may have an arbitrary dependence on u' and u ; for the Stanford problem we have assumed a product of a function of u and a normalizing factor depending on u' , but this is by no means necessary (see page 21).

It should be noted that the equations are just those which would be written for the cascade fluxes in a plane slab of material if $N(u,0)$, $P(u,0)$, etc., are interpreted as the number of particles per second per unit area per unit lethargy incident in a given direction, and r denotes distance into the slab in the same direction.

III. PHYSICAL DATA

In this section we describe the methods used to obtain the physical data required for the calculation. It must be understood at the outset that the information available concerning high-energy reactions is not adequate to determine the various functions we require with any certainty. In what follows we introduce many ad hoc assumptions; and thus, there is considerable arbitrariness in the functions we ultimately use.

We are primarily interested in doing calculations in earth since this is the shielding material which will be used at Stanford. Therefore, the physical data we use will be chosen to be representative of earth insofar as this is possible.

Let us first consider the quantity

$$\sum_i n_i \sigma_{i,j}$$

where $j = N, P$, or π which occurs in the definition of the Q 's. Let

$$\begin{aligned} \sigma_{iG} &= \text{geometric cross section of } i^{\text{th}} \text{ element nucleus} \\ &= \pi r_0^2 A_i^{2/3}; \end{aligned} \tag{3.1}$$

and

$$n_i = \frac{N_{O_i}}{A_i}; \tag{3.2}$$

where

A_i = atomic weight of the i^{th} element,

$r_0 = \text{constant}$

$= 1.3 \times 10^{-13} \text{ cm},$

$N_0 = \text{Avogadro's number},$

$\rho_i = \text{mass density of } i^{\text{th}} \text{ element in earth.}$

We have

$$\sum_i n_i \sigma_{ij} = N_0 \rho \pi r_0^2 \sum_i \frac{\rho_i}{\rho} \left(\frac{\sigma_{ij}}{\sigma_{iG}} \right) \frac{1}{A_i^{1/3}} \quad (3.3)$$

where

$\rho = \text{density of earth } (1.8 \text{ g/cm}^3).$

Since the σ_{ij} 's are not known for all elements we now assume that the quantity $(\sigma_{ij}/\sigma_{iG})$ may be taken out of the sum and evaluated at some average atomic weight $\bar{A}$. Thus

$$\sum_i n_i \sigma_{ij} = N_0 \rho \pi r_0^2 \left[\frac{\sigma_{ij}}{\sigma_{iG}} \right] \bar{A} \sum_i \frac{\rho_i}{\rho} \frac{1}{A_i^{1/3}} \quad (3.4)$$

and, using an average composition of earth,⁷

$$\sum_i \frac{\rho_i}{\rho} \frac{1}{A_i^{1/3}} \approx \frac{1}{(27)^{1/3}}. \quad (3.5)$$

In our calculations we take

$$\bar{A} = A_{Al} = 27,$$

$A_{Al} = \text{atomic mass number of aluminum, so we have}$

$$\sum_i n_i \sigma_{ij} = N_0 \rho \pi r_0^2 \frac{1}{A_{Al}^{1/3}} \left(\frac{\sigma_{ij}}{\sigma_{iG}} \right)_{Al}. \quad (3.6)$$

For the parameter λ in which we measure lengths we use

7. Handbook of Chemistry and Physics, The Chemical Rubber Publishing Co., Cleveland, Ohio (1961).

$$\frac{1}{\lambda} = N_0 \rho \pi \frac{r_0^2}{A_{Al}^{1/3}} \quad (3.7)$$

so we have

$$Q_N = \frac{\sigma_{Al,N}}{\sigma_{Al,G}}$$

$$Q_P = \frac{\sigma_{Al,P}}{\sigma_{Al,G}}$$

$$Q_\pi = \frac{\sigma_{Al,\pi}}{\sigma_{Al,G}} \quad (3.8)$$

where

$$\sigma_{Al,G} = \pi r_0^2 A_{Al}^{2/3},$$

= geometric cross section of aluminum,

$\sigma_{Al,N}$ = neutron absorption cross section of aluminum,

$\sigma_{Al,P}$ = proton absorption cross section of aluminum,

$\sigma_{Al,\pi}$ = pion absorption cross section of aluminum.

The neutron cross section is taken from the measurements of Voss and Wilson⁸ and is extrapolated to higher energies as a constant. The proton cross section is taken from the calculation of Metropolis and his collaborators^{9*} and is extrapolated to the higher energies as a

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8. R. G. P. Voss and R. Wilson, Proc. Roy. Soc. (London) A236, 41 (1956).
 9. N. Metropolis, R. Bivens, M. Storm, J. M. Miller, G. Friedlander, and A. Turkevich, Phys. Rev. 110, 204 (1958).

*Metropolis and his coworkers have calculated the cross section only for energies greater than 80 Mev. To obtain the cross section at lower energies we have extrapolated the cross section they give. This extrapolation is unjustified since one knows that the cross section should be increasing rapidly in this energy region. The only justification for our procedure is that it is conservative in the sense that it will give a higher, rather than a lower, proton intensity at energies greater than 30 Mev.

constant. The pion cross section is arbitrarily taken to equal the proton cross section. The Q's used in the calculation are shown in Fig. 1.

For the stopping cross section we use the formula given by Sternheimer:¹⁰

$$\epsilon_{ij}(E) = \frac{2\pi e^4}{m_e c^2} \frac{Z_i}{\beta^2} \left\{ \ln \left[2 \frac{m_e c^2 \beta^2 W_{\max}}{I_i^2 (1 - \beta^2)} \right] - 2 \beta^2 - \delta_i \right\} \quad (3.10)$$

where

$m_e c^2$ = rest energy of electron,

$\beta = v/c$

$= [E(E + 2m_j c^2)]^{1/2} (E + m_j c^2)^{-1}$,

v = velocity of incident charged particle (j-type),

m_j = mass of incident particle,

Z_i = nuclear charge of the i^{th} element;

I_i = average ionization potential of the i^{th} element;

$$W_{\max} = \frac{\beta^2}{1 - \beta^2} m_j c^2 \left[\frac{m_j}{2m_e} + \frac{m_e}{2m_j} + \frac{1}{\sqrt{1 - \beta^2}} \right]^{-1} \quad (3.11)$$

δ_i = density effect correction for the i^{th} element

$= 4.606 x + c_i + a_i (x_{ij} - x)^{m_i} \quad x_{oi} < x < x_{li}$

$= 4.606 x + c_i \quad x > x_{li}$

$x = \log_{10} [\beta(1 - \beta^2)^{-1/2}]$.

The constants c_i , a_i , m_i , x_{oi} , x_{li} are the same constants, without subscript i , defined in Sternheimer's paper.¹⁰

In the calculation we need the quantities

10. R. M. Sternheimer, Phys. Rev. 115, 137 (1959).

Fig. 1. Nonelastic Cross Section vs. Energy.

$$S_j = \lambda \sum_i n_i \epsilon_{ij} \quad (j = P, \pi, \mu)$$

which can in principle be obtained once the composition of the earth is known. Rather than try to use this complicated sum in the calculation we shall approximate earth by a single element as we have done previously. If we assume that the bracketed term in ϵ_{ij} may be taken out of the sum and evaluated at some average value, we have

$$\sum_i n_i \epsilon_{ij} \propto \sum_i n_i Z_i; \quad (3.12)$$

and using Eq. (3.3)

$$\sum_i n_i \epsilon_{ij} \propto N_o \rho \sum_i \frac{\rho_i}{\rho} \frac{Z_i}{A_i};$$

so the sum is proportional to the average value of (Z_i/A_i) . Introducing the average composition of earth,⁷

$$\sum_i \frac{\rho_i}{\rho} \frac{Z_i}{A_i} \simeq 0.48 \quad (3.13)$$

so the average value of (Z_i/A_i) is nearly the same as the value of (Z/A) for aluminum. Thus in our calculation we shall use the approximation

$$\sum_i n_i \epsilon_{ij} = \frac{N_o \rho}{A_{Al}} \epsilon_{Alj} \quad (3.14)$$

where

ρ = density of earth,

A_{Al} = atomic weight of aluminum,

ϵ_{Alj} = stopping cross section of aluminum for j-type particles.

The parameters in the stopping cross section of aluminum were taken from Sternheimer.¹⁰

Using the value of λ given by Eq. (3.7), we have

$$S_j(E) = \frac{\epsilon_{Al,j}(E)}{\sigma_{Al,G}} \quad (j = P, \pi, \mu) \quad (3.15)$$

for the energy loss per collision length.

Let us now consider the average multiplicities ν_N , ν_P , and ν_π . Most of the information about these quantities at high incident energies comes from measurements on cosmic-ray reactions in emulsions. These measurements refer only to charged particle secondaries and thus we have no experimental information about ν_N . To overcome this difficulty we make the assumption

$$\nu_N = \nu_P. \quad (3.16)$$

Camerini, et al., have measured, as a function of the energy of the incident proton or pion, the average number of secondary protons and charged pions which are emitted per collision in emulsions.¹¹ The data given are the number of protons with energy below 30 Mev (black tracks); the number of protons with energies between 30 and 500 Mev and, possibly, some charged pions with energies less than 80 Mev (grey tracks); and the sum of the protons with energies greater than 500 Mev and the charged pions with energies greater than 80 Mev (thin tracks).

As was stated earlier, many of our approximations become inaccurate at low energies and we shall not attempt to compute below the energy E_L . In order to use the above data we now take E_L to be 30 Mev. However, it must be understood that this choice is dictated primarily by the data available and is somewhat lower than the energies at which our equations are probably trustworthy.

With this choice of E_L , ν_P is to be the number of secondary protons emitted with energies greater than 30 Mev and ν_π is to be the number of secondary charged pions emitted with energies greater than 30 Mev.

11. U. Camerini, W. O. Lock, and D. H. Perkins, Progress in Cosmic Ray Physics, Vol. 1, Chapt. 1, North Holland Publishing Company, 1952, p. 11, Fig. IV.

No way has been found to estimate the number of grey tracks which correspond to pions, so in our calculation we have assumed that all grey tracks correspond to protons. It has been estimated that one fourth of the thin tracks are due to protons so this is the number we use.¹² With these assumptions and with some extrapolation at the higher incident energies the multiplicities we require may be obtained from Camerini.¹¹

We can obtain some check on the validity of our proton multiplicities by comparing with the calculations of Metropolis and his coworkers.⁹ They have calculated the number of cascade nucleons (neutrons plus protons) which arise when protons with energies between 80 Mev and 2 Gev interact with aluminum. Unfortunately their results include some nucleons with energies below 30 Mev and are thus not directly comparable to our estimates. If we use the energy distribution of secondary protons which they give to find the fraction of protons emitted above 30 Mev, and assume that equal numbers of neutrons and protons are emitted we obtain from their results:

1.14 protons emitted with $E > 30$ Mev when a 460-Mev proton interacts with aluminum.

2.53 protons emitted with $E > 30$ Mev with a 1,840-Mev proton interacts with aluminum.

These values are to be compared with our values:

$$\nu_P(460) = 1.03$$

$$\nu_P(1,840) = 2.86.$$

The agreement is not very good. To some extent this is to be expected since the multiplicities we use are for emulsions and not for aluminum. On the other hand, we would like to have the multiplicities for earth, which are probably different from those for either emulsions or aluminum. In our calculations we shall always use the multiplicities obtained directly from the cosmic-ray data, i.e., we have not attempted to make any corrections for the difference between emulsions and earth.

12. C. C. Butler and K. H. Baker, Proc. Phys. Soc. (London) A64, 4 (1951).

The multiplicities we use are shown in Fig. 2.

The energy distribution functions, f_N , f_P , and f_π are not available, either experimentally or theoretically, at all energies of interest to us. At the lower energies there are some calculations by Metropolis⁹ and his coworkers but at the higher energies we must again rely on cosmic-ray data. However, the cosmic-ray measurements are related to the functions we require in a very complicated way and can be used to determine these functions only very approximately.

As with the multiplicities we have information only about charged particle secondaries, so we assume

$$f_N(E', E) = f_P(E', E). \quad (3.17)$$

Let us assume that the energy distributions are of a product form; that is

$$\begin{aligned} f_P(E', E) &= f_P(E) A_P(E') & E \leq E' \\ &= 0 & E > E' \end{aligned} \quad (3.18)$$

$$\begin{aligned} f_\pi(E', E) &= f_\pi(E) A_\pi(E') & E \leq E' \\ &= 0 & E > E' \end{aligned} \quad (3.19)$$

and let us further assume that the A's are to be determined by normalization. We have

$$\begin{aligned} 1 &= \int_{E_L}^{E'} f_P(E', E) dE = A_P(E') \int_{E_L}^{E'} f_P(E) dE \\ 1 &= \int_{E_L}^{E'} f_\pi(E', E) dE = A_\pi(E') \int_{E_L}^{E'} f_\pi(E) dE \end{aligned} \quad (3.20)$$

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Fig. 2. Multiplicity vs. Incident Energy.

so

$$A_P(E') = \frac{1}{\int_{E_L}^{E'} f_P(E) dE}$$

$$A_\pi(E') = \frac{1}{\int_{E_L}^{E'} f_\pi(E) dE} . \quad (3.21)$$

The fact that the A's diverge causes no difficulty in practice because we simply cut off the calculation before the energy E_L is reached.

Camerini and his coworkers* have made measurements of the differential energy distribution of the secondary protons and pions produced per star by cosmic rays in emulsion. Unfortunately, they do not measure the secondary energy spectra as functions of the energy of the incident particle; but, rather, they measure secondary spectra which have been averaged over an incident cosmic-ray energy spectrum. That is, they measure $F_P(E)$ and $F_\pi(E)$ defined by

$$F_P(E) = \frac{\int_{\bar{E}}^{\infty} f_P(E', E) v_P(E') \bar{\Phi}(E') dE'}{\int_{\bar{E}}^{\infty} \bar{\Phi} dE'} ;$$

$$F_\pi(E) = \frac{\int_{\bar{E}}^{\infty} f_\pi(E', E) v_\pi(E') \bar{\Phi}(E') dE'}{\int_{\bar{E}}^{\infty} \bar{\Phi} dE'} ; \quad (3.22)$$

*Reference 11, p. 24, Fig. 10.

where

$\phi(E)$ = cosmic-ray energy spectrum (protons, pions, and neutrons)
divided by the interaction mean free path,
 $E, \bar{E}$ = the greater of E and $\bar{E}$.

The cutoff energy, $\bar{E}$, must be introduced because the measurements include only those stars with one or more thin tracks. The energy spectra and multiplicities in the equations above are those for an emulsion rather than for earth, but we shall neglect this distinction as before.

Let us again introduce the product assumption to obtain

$$F_P(E) = f_P(E) \frac{\int_{\bar{E}}^{\infty} A_P(E') \nu_P(E') \phi(E') dE'}{\int_{\bar{E}}^{\infty} \phi(E') dE'} ;$$

$$F_{\pi}(E) = f_{\pi}(E) \frac{\int_{\bar{E}}^{\infty} A_{\pi}(E') \nu_{\pi}(E') \phi(E') dE'}{\int_{\bar{E}}^{\infty} \phi(E') dE'} ;$$

where f_{π} and f_P are the functions we require. One would like simply to make the small f 's and the capital F 's proportional; however, since the integrals go to zero as E becomes large it is clear that this assumption is not valid at all energies. We can, however, check this assumption at low energies by comparison with the calculations of Metropolis.⁹

In Figs. 3 and 4 are shown the energy spectra of secondary protons obtained by Metropolis et al., when 450-Mev and 1,840-Mev protons interact with aluminum. Each of these curves has been normalized to unity by dividing by the integral from 30 Mev to the incident energy. Also shown in the figures is $F_P(E)$, similarly normalized. The agreement between the curves is quite good. At the lower incident energy the situation is somewhat confused by a bump which the aluminum curve shows around 400 Mev,

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Fig. 3. Secondary Energy Distributions.

Fig. 4. Secondary Energy Distributions.

but this is of little concern to us here. Thus there is some justification both for the product assumption and for the use of

$$f_P = F_P. \quad (3.24)$$

Of course, we are again comparing data from emulsions with calculations for aluminum so there is no reason why they should be the same. However, since they are nearly the same we can assume that the secondary energy distribution is not strongly dependent on atomic weight and use

$$f_P = F_P \quad (3.25)$$

for earth at low incident energies. We also make the obvious extrapolation and use

$$f_\pi = F_\pi \quad (3.26)$$

at low incident energies.

Still to be answered, of course, is at what energies Eqs. (3.25) and (3.26) break down.

As yet no consideration has been given to energy conservation. When an individual reaction occurs we may write the condition of energy conservation as

$$E' - C(E') = 2\nu_P \int_{E_L}^{E'} f_P(E', E) E \, dE + \frac{3}{2} \nu_\pi \int_{E_L}^{E'} f_\pi(E', E) (E + m_\pi c^2) \, dE \quad (3.27)$$

where we have assumed that a neutral pion is produced for every two charged pions and

$m_\pi c^2$ = rest energy of pion (assumed the same for all charge states),

$C(E')$ = an energy loss term which is introduced to account for the various energy losses which we do not consider explicitly; e.g., particles emitted below 30 Mev. In the case of a pion-induced reaction the rest energy of the pion must be

included in $C(E')$ so for this case it is not clear that $C(E')$ will necessarily be positive.

Introducing again the product assumption we have

$$E' - C(E') = 2\nu_P A_P \int_{E_L}^{E'} f_P(E) E dE + \frac{3}{2} \nu_\pi A_\pi \int_{E_L}^{E'} f_\pi(E) (E + m_\pi c^2) dE \quad (3.28)$$

which may be considered to be a relation between the ν 's and the f 's. We do not have any precise information about $C(E')$ but we do know that at sufficiently high energy, E' , it will be negligible compared with E' .

In Fig. 5 we show the fractional energy loss, $[C(E')/E']$, which one obtains if one uses the multiplicities of Fig. 1 in conjunction with the assumptions*

$$f_P = F_P$$

$$f_\pi = F_\pi. \quad (3.29)$$

For very large E' it is apparent that $C(E')$ is much too large. This is, of course, due to the fact that F_P and F_π fall much too rapidly as E increases. On the other hand, for smaller E' ($\lesssim 2$ Gev), $C(E')$ is less than is believable. Since in this energy region we believe that the energy distributions are approximately correct this must mean that our multiplicities are too large. However, because of our inexact knowledge of $C(E')$ in this energy region we do not feel that we can get a meaningful correction. Thus we make no corrections in this energy region ($E' \lesssim 2$ Gev).

On the other hand, at the high energies some correction must be made but in this region the true $C(E')$ is negligible compared with E' so we need not be unduly concerned about its value or its form. With $C(E')$ considered known, Eq. (3.28) may be considered to be an integral equation to determine either f_P or f_π if the other is known.

*The functions F_P and F_π given by Camerini¹¹ extend only to something of the order of 5 Gev and have been extrapolated in order to carry out the calculations.

Fig. 5. Fractional Energy Loss Per Collision Using Uncorrected Energy Distributions vs. Incident Energy.

Of course, we still have the difficulty that we have only one equation to determine two functions. We make the assumption

$$f_p(E) = g(E) F_p(E)$$

$$E \gtrsim 2 \text{ Gev} \quad (3.30)$$

$$f_\pi(E) = g(E) F_\pi(E);$$

i.e., we apply the same correction factor to f_p and f_π . From Eq. (3.23) it appears that corrections should not be the same for both F_p and F_π so this assumption is very unsatisfactory.*

Using Eq. (3.30) in Eq. (3.28) we obtain an integral equation for the single function $g(E)$. This integral equation has been solved numerically. Or, more precisely, because we do not know the exact form of $C(E')$ we have, more or less by trial and error, arrived at a numerical solution for $g(E)$ which makes $C(E')$ small compared with E' over the energy range 2 to 45 Gev.

The actual functions f_p and f_π which we find by this procedure and which we use in the calculation are shown in Fig. 6. Also shown in the figure for purposes of comparison are the functions F_p and F_π used in obtaining the corrected functions.

Finally in Fig. 7 we show the fractional energy loss, $[C(E')/E']$, obtained from Eq. (3.27) when one uses our corrected f_p and f_π .

Thus we have obtained all the functions we require; it is apparent that many of them are quite unsatisfactory. By introducing many assumptions we have arrived at estimates of these functions, but we do not wish to imply that these are the only or even the best assumptions that can be made in order to utilize the available data.

*As an alternative to this procedure one could attempt to obtain f_p and f_π from Eq. (3.23) by introducing information about the form of the cosmic-ray flux. This method is being attempted but has not as yet been fruitful.

Fig. 6. Secondary Energy Distributions.

Fig. 7. Fractional Energy Loss Per Collision Using Corrected Energy Distributions vs. Incident Energy.

IV. DOSE CALCULATION

In order that the calculated fluxes be useful in determining the shielding thickness it is necessary that they be converted into dose. There is at present no entirely satisfactory method of converting from flux to dose for high-energy particles. In the present work we content ourselves with calculating the surface dose; i.e., the dose received by a very thin slab of tissue placed in the beam, by an approximate method introduced by W. A. Gibson.³ We shall give the dose in rads rather than the more meaningful dose in rems because at the present time there seems to be no generally accepted way of making the conversion.

Let $G(E)$ be the flux per unit energy range of some specific type of particle. Then the dose from this flux may be written

$$d_G = C \int_0^{\infty} G(E) S(E) dE \quad (4.1)$$

where

$S(E)$ = energy loss per unit distance in tissue for the type of particle involved,

C = constant which converts energy/unit volume into rads.

In the case when the incident particles are neutrons there are two mechanisms which contribute to the energy loss: the neutron may interact with oxygen, producing neutrons and protons as secondaries; and the neutron may be elastically scattered, producing a recoil proton. As an estimate of the energy loss per unit distance when a neutron of energy E is absorbed we use

$$n_O \sigma_A \frac{E}{2} \quad (4.2)$$

where

n_O = number density of oxygen atoms in water,*

σ_A = neutron absorption cross section of oxygen.³

*Water is used as an approximation for tissue throughout the calculation.

That is, we assume that when a reaction takes place half of the energy is deposited locally.

The factor of $1/2$ is very arbitrary. When a reaction occurs not all of the energy is deposited locally. However, the energy emitted in the form of charged particles will be deposited reasonably locally, and if we assume that half of the energy goes into charged particles we get the factor $1/2$.

As an estimate of the energy loss per unit distance when a neutron is scattered elastically by a proton we use

$$n_P \sigma_E \frac{E}{2} \quad (4.3)$$

where

n_P = number density of protons in water,

σ_E = elastic neutron-proton scattering cross section.¹³ The elastic cross section was taken to be the same as the total cross section below 1 Gev and was extrapolated as a constant above this energy.

The factor of $1/2$ is introduced because on the average in an elastic n-p collision the neutron gives half of its energy to the proton and it is the proton energy which we assume to be deposited.

Thus for neutrons we introduce the two quantities

$$d_{NA} = c_1 \int_{\Gamma}^{\infty} \frac{N(E,r)}{r^2} \frac{n_O \sigma_A}{2} E dE \quad (4.4)$$

$$d_{NE} = c_1 \int_{\Gamma}^{\infty} \frac{N(E,r)}{r^2} \frac{n_P \sigma_E}{2} E dE \quad (4.5)$$

where

$N(E,r)$ = neutron intensity defined previously. [The $(1/r^2)$ is introduced to convert intensity to flux.]

13. S. J. Lindenbaum, Ann. Rev. Nucl. Sci. 7, 317 (Annual Reviews, Inc., Palo Alto, California, 1957).

- Γ = lower limit on energy which will be specified shortly,
 c_1 = constant needed to convert the assumed energy deposition
 into any desired units,
 d_{NA} = dose due to neutron absorption (energy $> \Gamma$),
 d_{NE} = dose due to neutron elastic scattering (energy $> \Gamma$).

Here and elsewhere the units of dose will be (rad/hr)/(source electron/sec).

Even though we do not calculate the neutron flux at low energy there will certainly be such a flux and it is necessary to estimate the dose from it if our dose calculation is to be meaningful. The equilibrium neutron flux spectrum in the atmosphere has been measured.¹⁴ We make the simple assumption that at low energies the neutron spectrum in the earth shield will have the same shape as the equilibrium cosmic-ray neutron spectrum in the atmosphere. Then we normalize this measured spectrum to the calculated spectrum at energy Γ and use this renormalized spectrum to calculate the low-energy dose.

For the neutron flux below 10 Mev there is a known flux-to-dose conversion factor which we use,¹⁵ while for the flux above 10 Mev we use the method described above. Thus we introduce

$$d_{NC} = c_1 \int_{10}^{\Gamma} \frac{N_C(E)}{r^2} \left[\frac{n_O \sigma_A}{2} + \frac{n_P \sigma_E}{2} \right] E dE + \int_0^{10} \frac{N_C(E)}{r^2} (E) dE \quad (4.6)$$

where

- $N_C(E)$ = renormalized cosmic-ray neutron intensity,
 (E) = neutron flux-to-dose conversion factor,¹⁴
 d_{NC} = dose from low-energy neutrons estimated from cosmic-ray spectrum.

Finally, we have

$$d_N = d_{NA} + d_{NE} + d_{NC} \quad (4.7)$$

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14. W. N. Hess, H. W. Patterson, R. Wallace, and E. L. Chupp, Phys. Rev. 116, 445 (1959).
 15. C. D. Zerby, A Monte Carlo Calculation of Air-Scattered Neutrons, ORNL-2227 (195), p. 31.

where

d_N = total neutron dose.

Since in general the energy spectrum which we calculate does not have the same shape as the cosmic-ray spectrum in the energy region above 30 Mev the dose we calculate will be a function of Γ and to this extent will be unsatisfactory. In practice, we choose Γ to be the point where the two energy spectra have approximately the same tangent.

In the case when the incident particles are protons or charged pions there are again two mechanisms which contribute to the energy loss. The particles will undergo the usual stopping due to atomic ionization, and they will be absorbed by the oxygen to produce neutron and proton secondaries.

The dose due to absorption is obtained in the same way as in the case of neutrons so we have - assuming that the oxygen absorption cross section is the same for protons and pions and for neutrons -

$$d_{PA} = c_1 \int_L^{\infty} \frac{P(E, r)}{r^2} \frac{n_O \sigma_A}{2} E dE \quad (4.8)$$

$$d_{\pi A} = c_1 \int_L^{\infty} \frac{\pi(E, r)}{r^2} \frac{n_O \sigma_A}{2} E dE \quad (4.9)$$

where

d_{PA} = dose due to proton absorption (energy $> L$),

$d_{\pi A}$ = dose due to pion absorption (energy $> L$).

The dose due to atomic stopping may be written

$$d_{PS} = c_1 \int_L^{\infty} \frac{P(E, r)}{r^2} S_P dE \quad (4.10)$$

$$d_{\pi S} = c_1 \int_L^{\infty} \frac{\pi(E, r)}{r^2} S_{\pi} dE \quad (4.11)$$

where

S_P, S_π = stopping power of water for protons and pions, respectively,*

d_{PS} = dose due to atomic stopping of protons (energy $> L$),

$d_{\pi S}$ = dose due to atomic stopping of pions (energy $> L$).

The total proton dose, d_P , is given by

$$d_P = d_{PA} + d_{PS} \quad (4.12)$$

and the total pion dose, d_π , by

$$d_\pi = d_{\pi A} + d_{\pi S} \quad (4.13)$$

In the case of muons the only mechanism for energy loss is atomic stopping so

$$d_\mu = c_1 \int_L^\infty \frac{\mu(E, r)}{r^2} S_\mu dE \quad (4.14)$$

where

d_μ = dose due to muons (energy $> L$)

S_μ = stopping power of water for muons.

The dose from the charged particles as defined above includes only the contribution from those particles with energy greater than L because we have not calculated the charged-particle fluxes below this energy.**

*The stopping powers used in the dose calculation are obtained from the Bethe-Block formula;¹⁶ i.e., the corrections to this formula used previously were not used in the dose calculation.

**In all of the calculations L is arbitrarily taken to be 32 Mev. This energy is slightly larger than necessary since we calculate the fluxes at all energies greater than 30.4 Mev. This larger energy was chosen to avoid any difficulties associated with the singularities in A_P and A_π (see Eq. 3.20).

16. H. A. Bethe and J. Ashkin, Experimental Nuclear Physics, Vol. 1, Part II, p. 223 (John Wiley and Sons, Inc., New York, 1953).

Of course, this low-energy flux will exist and should in principle be included. We have not, however, found any satisfactory way of estimating the dose contribution from these low-energy charged particles and hence do not include such an estimate in this report.

The total dose, d , is of course the sum of all the above doses so we have

$$d = d_N + d_P + d_\pi + d_\mu. \quad (4.15)$$

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