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DAMPING IN A COMPOSITE BEAM MODEL

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UNION CARBIDE CORPORATION
NUCLEAR DIVISION
OAK RIDGE GASEOUS DIFFUSION PLANT

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CARBIDE**

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DAMPING IN A COMPOSITE BEAM MODEL

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A B S T R A C T

The damping characteristics of a steel-concrete composite floor beam have been determined with the use of a 1/5-scale model. The critical damping ratio, β , at the first bending mode for this construction was found to be 0.0332 with a standard deviation of 0.00375. By loading the beam with lead bricks, a curve of the first bending frequency as a function of load was obtained and from this, using model predictive equations, a fundamental frequency for the prototype floor beam was estimated to be 11.6 Hz. A technique was developed which evaluates the parameters of natural frequency and damping ratio, using a nonlinear least squares curve fit routine of a transmissibility curve.

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DAMPING IN A COMPOSITE BEAM MODEL

INTRODUCTION

In analyzing the dynamic response of the K-33 process building of the Oak Ridge Gaseous Diffusion Plant (ORGDP), a knowledge of the damping or energy absorbing characteristics of the floor was needed to evaluate its dynamic stress levels. Limited information applicable to the composite steel-concrete type construction used in the building was available because of the complex nature of damping forces, which depend upon materials, geometry, and methods of construction. Therefore, a program was undertaken to experimentally determine the required damping characteristics. To accomplish this objective, a scale model of a typical floor beam was constructed and tested. In the mathematical representation of this floor, for which the damping data were required, the structure was assumed to be a system of interconnected beams with dimensions such that their strength and stiffness would approximate that of the actual floor. For this reason, the scale model was considered to be a beam with construction details identical to those of an actual floor beam and dimensioned so that its stiffness was proportional to that of the floor. Due to the difficulty of adequately modeling the damping effects, testing of the actual floor is preferred; however, size limitations of available excitation equipment have precluded this approach. Full-scale testing will be done when the equipment becomes available.

SUMMARY

A 1/5-scale model of a typical K-33 floor beam was tested to determine the damping and frequency characteristics which would be found in the actual floor. An average critical damping ratio, β , of 0.0336 is recommended for use in the mathematical modeling of the floor. For sensitivity studies, a range for β of 0.02 to 0.07 is recommended. A value of the fundamental frequency of a floor beam is estimated to be approximately 11.6 Hz; this figure is only approximate owing to the simplifications made in representing the floor as a beam.

Based upon a comparison between the experimental and analytically determined values for the first and second bending mode frequencies, it is concluded that the mathematical model used to predict these frequencies produces results that are approximately 25% high for the first mode and approximately 18% high for the second mode.

THEORY

An engineering model can accurately simulate the behavior of a prototype system provided all pertinent variables in the problem are grouped into dimensionless terms,

$$\pi_1, \pi_2, \pi_3, \text{-----}$$

and that the model is constructed and tested so that

$$\pi_{1m} = \pi_{1p}, \pi_{2m} = \pi_{2p}, \text{-----} \quad (1)$$

where

m = the model, and

p = the prototype.

In order to construct the model to yield information on the damping characteristics and the system response of a floor beam, the following variables were considered to be important: ℓ , a characteristic length; F, the imposed loads; ρ , the density; ω , the frequencies of vibration; C, the internal damping; E, the modulus of elasticity; I, the area moment of inertia; and a_i , the linear dimensions of the beam. From these variables, the following set of dimensionless π terms can be obtained:

$$\pi_1 = \frac{a_i}{\ell}, \quad (2)$$

$$\pi_2 = I/\ell^4, \quad (3)$$

$$\pi_3 = F/E\ell^2, \quad (4)$$

$$\pi_4 = \omega \ell^3 / \sqrt{EI/\rho}, \text{ and} \quad (5)$$

$$\pi_5 = C/\omega\rho\ell^3. \quad (6)$$

The scale factor, λ , is

$$\lambda = \frac{\ell_p}{\ell_m}. \quad (7)$$

The design conditions for the model can be determined from the dimensionless π terms according to equation 1.

Thus, considering equation 2,

$$\left(\frac{a_i}{\ell}\right)_m = \left(\frac{a_i}{\ell}\right)_p$$

or

$$a_{im} = \frac{a_i}{\lambda}. \quad (8)$$

This condition requires that the model be geometrically similar to the prototype. From equation 3,

$$(I/\ell^4)_m = (I/\ell^4)_p.$$

Therefore,

$$I_m = \frac{I_p}{\lambda^4}. \quad (9)$$

From equation 4,

$$(F/E\ell^2)_m = (F/E\ell^2)_p.$$

By constructing the model of the same material as the prototype, $E_m = E_p$ and the preceding equation becomes:

$$F_m = \frac{F_p}{\lambda^2}. \quad (10)$$

This requirement specifies the magnitude of the model loads. The prediction equations used to relate the vibrational frequencies and damping characteristics in the model to those in the prototype are obtained in a similar manner. Thus, from equation 5,

$$(\omega \ell^3 / \sqrt{EI/\rho})_p = (\omega \ell^3 / \sqrt{EI/\rho})_m.$$

Considering identical construction materials as before, $E_m = E_p$ and $\rho_m = \rho_p$ and using equation 9, the above equation reduces to

$$\omega_p = \frac{\omega_m}{\lambda}. \quad (11)$$

The equation relates frequencies in the model to those in the prototype. Define C_c , the critical damping coefficient, to be

$$C_c = 2 \omega_p \rho \ell^3$$

and β , the critical damping ratio to be

$$\beta = \frac{C}{C_c}.$$

Then from equation 6,

$$\beta = 2\pi\zeta. \quad (12)$$

A more detailed discussion of the theory of models can be found in reference 1.

EXPERIMENTATION

Model Construction

Several factors require consideration to select the scale factor by which to construct a model. These factors, which have conflicting requirements, include materials selection, construction techniques, loading methods, handling ease, and model measurement techniques. The problems of model cost, loading, and handling require that the model be as small as possible; yet, because of the difficulties encountered in reproducing detail with the required accuracy and in measuring the small displacements associated with a small scale model, the model needs to be large. For these reasons, a compromise was made in selecting a scale factor of 5.

In attempting to model damping effects, great care must be taken to model not only the macroscopic details of construction, but also the microscopic. Owing to the importance of internal damping of the materials, the same basic materials were used in the model as in the prototype. As shown in figure 1, the model was constructed of reinforced concrete coupled with a spiral shear connector to a steel I-beam. The aggregate used in the cement was graded from No. 100 to a maximum of $3/8$ in. to approximate the scaled diameter used in the prototype. Likewise, the diameters of the reinforcing rods and shear connectors were scaled appropriately. In considering the steel I-beam for the model, the scale factor of 5 resulted in a web thickness of 0.08 in. which would have been difficult to fabricate. Therefore, the web thickness was increased to 0.119 in. and the depth of the model I-beam was adjusted to maintain the same area moment of inertia, I , as required by design equation 9. Although this resulted in a somewhat distorted model due to the requirements of equation 8, the effects were thought to be small. The material used for the I-beam was 11 gage SM ASTM A-245 grade A steel. Because of the relatively thin web of the I-beam, a transverse web was attached at the support points to increase the lateral stability of the beam. Since a support point acts as a mode of the beam, with no movement at this point, the effects of the web on the dynamics and damping properties were considered to be negligible. The beam was supported by concrete blocks bolted to the floor; it was tack welded to these blocks in order to reduce chatter or bounce. The tack welds greatly improved the quality of the data and added no detectable stiffness to the system. The beam model with exciter is shown in figure 2.

Instrumentation

The beam was excited by an electromechanical shaker, Model C-31, produced by the MB Manufacturing Company. This machine had a frequency range of 2 to 20,000 Hz and a force rating of 9 lb per 1.0 amp rms of alternating current applied to the driver coil.

Figure 1
SCHEMATIC DIAGRAM OF MODEL BEAM

Figure 2
MODEL BEAM AND EXCITATION EQUIPMENT

The exciter was coupled to the beam through a force gage which was used to maintain a constant input force to the beam during a test. The gage used was a model No. 2103-100 force gage manufactured by the Endevco Corporation and had a voltage sensitivity of 430 mv/lb at an external capacitance of 100 pF.

The beam's acceleration and displacement responses were measured with accelerometers attached to the steel I-beam with magnets. These accelerometers, Model 302-6, were manufactured by the Columbia Research Laboratories, Inc., and had a sensitivity of 100 mv/g. They were used in conjunction with a General Radio Corporation vibration meter, type 1553-A, which converted the pickup output to either displacement, velocity, or the time derivative of acceleration known as jerk. A cathode ray oscilloscope was used to determine the phase relationship between the input force and the resultant displacement of the beam.

Testing Procedure

During testing, the exciter was attached to the beam, through the force gage, at either the midpoint or at a quarter point, depending upon the vibrational mode to be excited. The output from the force gage was displayed on an oscilloscope and was maintained at a constant value of 0.0116 lb throughout a test by adjusting the alternating current through the field coil of the exciter. The accelerometers were attached to the underside of the I-beam with magnets and the desired output, either displacement or acceleration, was read on the vibration meter. The frequency of vibration was varied from approximately 30 to 500 Hz in increments of 2 to 5 Hz.

To determine the effects of frequency on the damping ratio, β , the beam was loaded with lead bricks, thereby lowering its fundamental frequency. These bricks were weighed to an accuracy of 2 oz and averaged 32.8 lb each. For a test, the bricks were stacked symmetrically about the center of the beam. The assembly was then excited as described earlier, and the response was measured. Three loadings were tested: 130.8, 456.6, and 852.5 lb, which covered the range of interest for loads in the prototype beam. As the load scales as the square of the scale factor, these three loadings represented loads in the prototype of from 3270 to 21312.5 lb, excluding the weight of the beam.

RESULTS

Acceleration response curves are presented in figures 3 through 7 for a variety of loads and forcing conditions. Figure 3 is for a no-load condition with forcing at the midpoint, and figure 4 is for a no-load condition with forcing at a quarter point. As can be seen, forcing at the quarter point greatly increases the response of the second bending mode and seems to shift the frequency from approximately 436 to 455 Hz. Figures 5, 6, and 7 show the beam responses when loaded with weights of 130.8, 456.6, and 852.5 lb, respectively. As expected, the first bending mode is shifted downward with increasing load.

DWG. NO. G-69-343

Figure 3
BEAM RESPONSE WITH MIDPOINT FORCING
No-Load Condition

Figure 4
BEAM RESPONSE WITH QUARTER POINT FORCING
No-Load Condition

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Figure 5

BEAM RESPONSE WITH 130.8-lb LOAD

Figure 6
BEAM RESPONSE WITH 456.6-lb LOAD

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Figure 7

BEAM RESPONSE WITH 852.5-lb LOAD

In addition to the first and second modes, the beam had numerous other modes that could be excited in the range of frequencies that were tested. These modes, which include torsion and rocking, resulted in additional peaks in the response curves as seen in the figures. While these modes were minimized by appropriate restraints, such as tack welding the beam to the base, they could not be eliminated without affecting the response of the first and second bending modes.

DISCUSSION

The vibrational parameters of primary interest in the experiment are the fundamental frequency, ω_0 , and the critical damping ratio, β . There are several methods available to determine these parameters, and they usually depend on the use of only a few data points; e.g., determining ω_0 from the frequency at the maximum amplitude of the response curve and determining β from the "half-power point" or the value of the response equal to the maximum response divided by $\sqrt{2}$.

Depending on where these points were taken, some interpolation of the data may be required. In addition, no estimate can be made of the accuracy of the results. Therefore, a new technique to evaluate ω_0 and β was developed, based upon a least squares curve fit of the data to the form of a transmissibility curve or the ratio of the transmitted force to the applied force. This is discussed in more detail in appendix A. This technique utilizes all the data points of the amplitude peak in determining the "best fit" values for the natural frequency, ω_0 , and the critical damping ratio, β , and yields a statistical estimate to be made of the precision of these values. Numerical stability problems were encountered for the second mode when forcing at the quarter point due to the sharpness of the peak (i.e., small damping) and the curve fit routine did not converge. In these cases estimates were made of the frequency and damping coefficient. A comparison is shown in figure 8 between the actual data and the resulting curve fit equation. As seen in the figure, the transmissibility curve fits the data very well over most of the range, deviating from the measured values only near the beginning and end of the peak. This deviation can, in general, be explained by the presence of adjacent amplitude peaks and in the error of measurement at the low amplitude levels. A summary of the test results and analysis of the data are given in table 1. A plot of the critical damping ratio, β , versus frequency, as shown in figure 9, indicates a general reduction of damping ratio with increasing frequency. However, the relatively large values of β at the lower frequencies may not be an inherent property of the beam but might be due to the relative motion between the lead bricks which were used for loading the beam. At the higher frequencies, e.g., at the second bending mode, the small values of the damping ratio β might indicate a displacement amplitude dependence of β . At the higher frequencies, the displacement amplitude of the beam was very small, on the order of 0.0001 in. However, this aspect of the problem was not pursued here. Because of the uncertainty of determining a frequency dependent value for β , the average value for β for the unloaded beam as determined at 100 Hz is recommended for use

Figure 8
COMPARISON BETWEEN CURVE FIT ROUTINE AND
MEASURED TRANSMISSIBILITY CURVE

Table 1
SUMMARY OF TEST RESULTS

Test Number	Load, lb	Vibrational Mode	Natural Frequency (ω_0), Hz	Standard Deviation for ω_0 , Hz	Critical Damping Ratio (β)	Standard Deviation for β	Forcing Point
1	0	1	109.4	0.188	0.0272	0.0019	1/2
1	0	2	436.6	0.555	0.0054	0.0020	1/2
2	0	1	108.5	0.150	0.0304	0.0018	1/2
2	0	2	435.9	1.05	0.0126	0.0029	1/2
3	0	1	106.4	0.208	0.0438	0.0024	1/4
3	0	2	455.0	--	< 0.01	--	1/4
4	0	1	107.2	0.1063	0.0293	0.0013	1/4
4	0	2	455	--	< 0.01	--	1/4
5	130.8	1	65.0	0.059	0.0229	0.0010	1/2
6	456.6	1	46.3	0.098	0.0341	0.0025	1/2
7	852.5	1	32.1	0.196	0.0583	0.0071	1/2

Figure 9
VARIATION OF CRITICAL DAMPING RATIO WITH FREQUENCY

in the mathematical analysis of the prototype. Thus, from the data in table 1, this value for β is estimated to be 0.0332 with a standard deviation of 0.00375. For sensitivity studies, a range of β from 0.02 to 0.07 is recommended.

In comparing the measured value of the fundamental frequency, 108 Hz, with the calculated value, a difference of approximately 25% was found. The theoretical value of 134 Hz for the fundamental frequency indicates the mathematical beam model is stiffer than the actual beam; i.e., the actual area moment of inertia, I , is less than the calculated value. Also, the second bending mode of 455 Hz was approximately 18% lower than predicted. This discrepancy was not pursued further, but there were indications that there was not full composite action between the steel and the concrete.

To predict the fundamental frequency of the prototype beam, an equivalent load of 9700 lb was used. Scaling this figure to the model by the use of equation 10, the required load on the model is 389 lb. Considering the model to weigh 150 lb, the net load is 539 lb, and from the curve in figure 10, which presents the fundamental frequency versus the weight of the model and load, the resultant beam frequency is 58 Hz. From the predictive equation 11, the prototype beam would have a fundamental frequency of 11.6 Hz. As no attempt was made to properly locate the loads on the model, the value of ω_0 given above is subject to some error.

Full-scale testing on the actual floor will be reported at a later date.

Figure 10
MODEL BEAM NATURAL FREQUENCY VS TOTAL BEAM WEIGHT

ACKNOWLEDGMENTS

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REFERENCE

1. Hudson, Donald E., "Scale Model Principles," *Shock and Vibration Handbook*, C. M. Harris and C. E. Crede, ed., New York, McGraw-Hill Book Company, 1961.

APPENDIX A

TRANSMISSIBILITY CURVE FIT PROCEDURE

To determine the vibrational parameters of natural frequency, ω_0 , and the damping ratio, β , for a system, a method has been utilized which relates the acceleration response curve for the system to a force transmissibility curve T . T is defined as the ratio of the transmitted force F_T to the applied force F_0 and is given by the equation:

$$\frac{F_T}{F_0} = T = \sqrt{\frac{1 + (2 \beta \omega / \omega_0)^2}{(1 - \omega^2 / \omega_0^2) + (2 \beta \omega / \omega_0)^2}} \quad (A-1)$$

where

β = critical damping ratio,

ω = forcing frequency, and

ω_0 = natural frequency.

It is reasoned that since the acceleration, A , of a system is proportional to the transmitted force, from Newton's second and third laws, and since the applied force can be maintained constant during a test, the acceleration of a system is proportional to the transmissibility, or

$$A = \alpha T = \alpha \sqrt{\frac{1 + (2 \beta \omega / \omega_0)^2}{(1 - \omega^2 / \omega_0^2) + (2 \beta \omega / \omega_0)^2}} \quad (A-2)$$

where α is the constant of proportionality. Therefore by measuring A as a function of ω , the parameters α , β , and ω_0 might be obtained which would best represent the acceleration response curve as a force transmissibility curve. This has been accomplished utilizing a nonlinear least squares curve fit computer code which has been programmed for use on an IBM 7090 computer. Input to the program is a table of acceleration and forcing frequencies together with an estimate of the parameters α , β , and ω_0 . The program output is the "best fit" values for α , β , and ω_0 , a statistical estimate of their precision and the calculated values for the acceleration. An example of the comparison between the measured acceleration and the computed transmissibility curve is shown in figure 8. As seen in the figure, there is excellent agreement between the two quantities. It should be remembered, however, that equation A-1 is only valid for values of ω in the vicinity of ω_0 for multi-degrees of freedom systems. Therefore, care should be taken in selecting values of A and ω so that the amplitudes are not influenced by adjacent modes.

The value of the transmissibility curve fit method of obtaining vibrational parameters is that many data points contribute to the results rather than a few, thus minimizing the effects of an experimental error in one or two points. Also, a statistical estimate can be made as to the precision of the results.