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A SIX-DEGREE-OF-FREEDOM INVESTIGATION OF THE MOTION
OF TWO SNAP FUEL CAPSULE CONFIGURATIONS RE-ENTERING
THE ATMOSPHERE WITH VARIOUS INITIAL CONDITIONS

D. H. Johnson, 9321

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D. H. Johnson, 9321
Sandia Laboratory, Albuquerque

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ABSTRACT

The aerodynamic characteristics of two SNAP fuel capsule configurations were determined by using Newtonian theory. For one of these configurations (a cylindrical body with hemispherical end caps), several six-degree-of-freedom trajectories were run using the NASA Six-Degree-of-Freedom Program on the IBM 790 machine in order to study its motion as it re-entered the atmosphere from orbit around the earth. Particular attention was paid to the tumbling motion of the capsule. Also, a literature search was made to determine what has been done in the way of measuring the aerodynamic characteristics of right circular cylinders. A summary of the available experimental data is given, and a comprehensive bibliography is included in the report.

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LIST OF SYMBOLS

A	= Axial force
C_A	= Axial force coefficient
C_M	= Pitching moment coefficient
C_{M_q}	= Damping in pitch coefficient due to pure rotational motion
$C_{M_{\dot{\alpha}}}$	= Damping in pitch coefficient due to rotation of the free stream vector
C_N	= Normal force coefficient
D	= Maximum diameter of body
F	= Force
i, j, k	= Unit vectors along the Cartesian coordinate axis x, y, z
L	= Length of the cylindrical portion of a body
M	= Pitching moment
N	= Normal force
n	= Unit vector normal to a surface and directed inward
q	= Dynamic pressure
R	= Radius of a body
S	= Surface vector
V	= Velocity
x, y, z	= Cartesian coordinates
α	= Angle of attack
α_i	= Induced angle of attack
β	= $90^\circ - \alpha$
θ, ϕ, R	= Spherical coordinates
ρ	= Density

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Introduction

The aerodynamic characteristics of two SNAP fuel capsule configurations were determined by using Newtonian theory. For one of these configurations (a cylindrical body with hemispherical end caps), several six-degree-of-freedom trajectories were run using the NASA Six-Degree-of-Freedom Program on the IBM 790 machine in order to study its motion as it re-entered the atmosphere from orbit around the earth. Particular attention was paid to the tumbling motion of the capsule. Also, a literature search was made to determine what has been done in the way of measuring the aerodynamic characteristics of right circular cylinders. A summary of the available experimental data is given, and a comprehensive bibliography is included in the report.

This study was conducted by Division 9321 at the request of Division 9312.

Statement of the Problem

Two fuel capsules with the following physical characteristics were to be studied:

Configuration 1

Shape: Right circular cylinder with hemispherical end caps

Center of Gravity: On center line at midpoint of capsule

Weight: 1.3 pounds

Pitch and Yaw Moments of Inertia: 3.128 lb-in^2

Roll Moment of Inertia: 0.176 lb-in^2

Configuration 2

Shape: Right circular cylinder with indented end caps

Center of Gravity:

Case 1: On center line at midpoint of capsule length

Case 2: 0.061 in off-center line at midpoint of capsule length (center of gravity to be in tumble plane)

Case 3: On center line 1.4035 in from midpoint of capsule length

Weight: 1.3 pounds.

Configurations 1 and 2 are shown in Figure 1.

Configuration 1

Configuration 2

Figure 1. Dimensions of the Two SNAP Fuel Rod Configurations

The aerodynamic coefficients of Configuration 1 were determined using the best theory and all available experimental data to compute three six-degree-of-freedom trajectories with the following initial conditions.

General Initial Conditions

Time 0 sec
 Height 265,595 ft
 Latitudes 36.285 deg
 Longitude 179.3 deg
 Velocity 25,161 ft/sec
 Euler Angles (ψ , ϕ , θ) 92.78 deg, 0 deg, -0.788 deg.

Trajectory 1

Orientation Capsule axis aligned with flight path
 Yaw Rate 0 deg/sec
 Pitch Rate: 4 deg/sec
 Roll Rate 0 deg/sec

Trajectory 2

Orientation Capsule axis aligned with flight path
Yaw Rate 0 deg/sec
Pitch Rate 115 deg/sec
Roll Rate 0 deg/sec.

Trajectory 3

Orientation
Yaw Rate 0 deg/sec
Pitch Rate 20 deg/sec
Roll Rate 0 deg/sec.

The aerodynamic coefficients of Configuration 2 were determined using the most expedient method and compared with those of Configuration 1. No trajectories were computed for Configuration 2.

Theoretical Analysis

The requested six-degree-of-freedom trajectories have been run, and in Figures 2, 3, and 4, the Euler angle of pitch is plotted versus time for each of the three sets of initial conditions. Also, the aerodynamic coefficients for Configuration 2 have been computed and are compared with those of Configuration 1 in Figure 5. The rest of this section will be devoted to outlining the analytical methods used to determine the aerodynamic coefficients of these two configurations.

First, the flight regimes through which the capsules pass must be determined. Figure 6 shows the combinations of Mach number, Reynolds number, and Knudsen number which fall within the various regimes of free molecular, intermediate, slip, and continuum flow.¹ The given initial conditions for Configuration 1 all fall within the slip flow regime. The best theory available for this region, short of exact numerical solutions, is Newtonian theory.² Thus Configuration 1 was analyzed using Newtonian theory. The hemispherical caps were considered separately from the cylindrical body.

Analysis of Configuration 1 - The Hemispherical Caps

Figure 7a shows the spherical coordinate system used to analyze the caps, and Figure 7b shows the relation of this system to the cylindrical body and the rectangular coordinate systems along which all aerodynamic forces were resolved. The unit vector normal to the surface of a cap and directed inward is

$$\mathbf{n} = -\sin \phi \cos \theta \mathbf{i} - \sin \phi \sin \theta \mathbf{j} - \cos \phi \mathbf{k}.$$

The velocity vector of the flow relative to the cap is

$$\mathbf{V} = v \mathbf{i}$$

Thus, the force caused by an infinitesimal stream-tube of air hitting a cap and losing its normal component of momentum is

$$d\mathbf{F} = -\rho(\mathbf{V} \cdot \mathbf{n})^2 d\mathbf{S}$$

$$d\mathbf{F} = -\rho V^2 R^2 \sin^2 \phi \cos^2 \theta [\sin \phi \cos \theta \mathbf{i} + \sin \phi \sin \theta \mathbf{j} + \cos \phi \mathbf{k}] d\phi d\theta.$$

Figure 2. Euler Angle of Pitch versus Time for Configuration 1, Case 1

Figure 3. Euler Angle of Pitch versus Time for Configuration 1, Case 2

Figure 4. Euler Angle of Pitch versus Time for Configuration 1, Case 3

Figure 5. Aerodynamic Coefficients of Configurations 1 and 2 versus Angle of Attack

Figure 6. Aerodynamic Flight Regimes

Figure 7. Coordinate System Used to Analyze the Hemispherical End Caps of Configuration 1

The total force on the caps in the x direction is the integral of the x component of the above expression over that portion of the hemispherical surfaces exposed to the flow.

$$\begin{aligned}
 F_x &= \iint_{\text{exposed surface}} dF_x \\
 &= -\rho V^2 R^2 \left[\int_{\pi-\beta}^{3\pi/2} \int_0^\pi \sin^3 \phi \cos^3 \theta d\phi d\theta + \int_{\pi/2}^{\pi-\beta} \int_0^\pi \sin^3 \phi \cos^3 \theta d\phi d\theta \right] \\
 F_x &= \frac{4\rho V^2 R^2}{3} \left[\int_{\pi}^{3\pi/2} \cos^3 \theta d\theta + \int_{\pi/2}^{\pi-\beta} \cos^3 \theta d\theta \right] \\
 F_x &= \frac{\rho V^2 R^2 16}{9}
 \end{aligned}$$

The limits on the integrals can be determined by inspection of Figure 7. Also by inspection of Figure 7, the total force on the caps in the y direction is zero because of symmetry.

$$F_y = 0$$

The total force on the caps acting normal to the axis of the cylinder is:

$$\begin{aligned}
 N &= F_x \cos \beta + F_y \sin \beta \\
 \therefore N &= \left(\frac{\rho V^2 R^2 16}{9} \right) \cos \beta
 \end{aligned}$$

and the normal force coefficient of the caps is:

$$\begin{aligned}
 C_N &= \frac{N}{(1/2)\rho V^2 R^2} \\
 C_N &= 1.16 \cos \beta
 \end{aligned}$$

The pitching moment about the center of the cylinder due to the forces on the end caps is:

$$M = -F_{yA} L \sin \beta + F_{xB} \frac{L}{2} \cos \beta,$$

where the Regions A and B are as marked on Figure 7. The above expression becomes obvious upon inspection of Figure 7. The forces in the y direction acting on Region B of the hemispherical end cap are equal and opposite by symmetry, and thus cancel each other. Similarly, the forces in the x direction acting on Regions A of the hemispherical end caps create equal but opposite moments about the center of the cylinder and thus cancel each other. This leaves only the forces in the y direction acting on Region A and the forces in the x direction acting on Region B to contribute to the pitching moment.

$$\begin{aligned}
F_{x_B} &= \iint_{\text{over surface B}} dF_x \\
&= -\rho V^2 R^2 \int_{\pi-\beta}^{\pi+\beta} \int_0^\pi \sin^3 \phi \cos^3 \theta \, d\phi d\theta \\
&= \frac{-\rho V^2 R^2 4}{3} \int_{\pi-\beta}^{\pi+\beta} \cos^3 \theta d\theta
\end{aligned}$$

$$F_{x_B} = \left(\frac{\rho V^2 R^2 8}{9} \right) \sin \beta (\cos^2 \beta + 2)$$

$$\begin{aligned}
F_{y_A} &= \iint_{\text{over surface A}} dF_y \\
&= -\rho V^2 R^2 \int_{\pi+\beta}^{3\pi/2} \int_0^\pi \sin^3 \phi \cos^2 \theta \sin \theta \, d\phi d\theta \\
&= \frac{-\rho V^2 R^2 4}{3} \int_{\pi+\beta}^{3\pi/2} \cos^2 \theta \sin \theta \, d\theta
\end{aligned}$$

$$\begin{aligned}
F_{y_A} &= \left(\frac{\rho V^2 R^2 4}{9} \right) \cos^3 \beta \\
M &= \frac{\rho V^2 R^2 4L}{9} \left[-\cos^3 \beta \sin \beta + \sin \beta (\cos^3 \beta + 2 \cos \beta) \right] \\
M &= \left(\frac{\rho V^2 R^2 4L}{9} \right) 4 \sin 2\beta
\end{aligned}$$

and the pitching moment coefficient due to the end caps is:

$$C_M = \frac{M}{(1/2)\rho V^2 \pi R^2 D}$$

$$C_M = 0.282 \frac{L}{D} \sin 2\beta$$

The total force on the caps acting parallel to the axis of the cylinder is:

$$A = F_x \sin \beta + F_y \cos \beta$$

F_x and F_y have been determined above. It was found that

$$\begin{aligned} F_x &= \frac{16\rho V^2 R^2}{9} \\ F_y &= 0 \\ \therefore A &= \left(\frac{16\rho V^2 R^2}{9} \right) \sin \beta \end{aligned}$$

and the axial force coefficient due to the end caps is

$$\begin{aligned} C_A &= \frac{A}{(1/2)\rho V^2 R^2} \\ C_A &= 1.16 \sin \beta \end{aligned}$$

Analysis of Configuration 1 - The Cylindrical Body

Figure 8 shows the cylindrical coordinate system used to analyze the body. The unit vector normal to the surface of the body and directed inward is

$$\mathbf{n} = -\cos \theta \mathbf{i} - \sin \theta \mathbf{j}$$

The velocity vector of the flow relative to the body is

$$\mathbf{V} = -v \sin \alpha \mathbf{i} - v \cos \alpha \mathbf{k}$$

Thus the force caused by an infinitesimal stream-tube of air hitting the body and losing its normal component of momentum is

$$\begin{aligned} d\mathbf{F} &= -\rho(\mathbf{V} \cdot \mathbf{n})^2 dS \\ d\mathbf{F} &= -\rho R V^2 \sin^2 \alpha \cos^2 \theta (\cos \theta \mathbf{i} + \sin \theta \mathbf{j}) d\theta dz \end{aligned}$$

The total force on the body in the x direction is the integral of the x component of the above expression over that portion of the cylindrical surface exposed to the flow.

$$\begin{aligned} F_x &= \iint_{\text{exposed surface}} dF_x \\ &= -\rho V^2 R \sin^2 \alpha \int_{-L/2}^{L/2} \int_{\pi/2}^{-\pi/2} \cos^3 \theta d\theta dz \\ F_x &= \frac{4\rho R V^2 L \sin^2 \alpha}{3} \end{aligned}$$

The limits on the integral can be determined by inspection of Figure 8. Also by inspection of Figure 8 the total force on the body in the y direction is zero because of symmetry.

$$F_y = 0$$

Figure 8. Coordinate System Used to Analyze the Cylindrical Body of Configurations 1 and 2

The total force on the body acting normal to the axis of the cylinder is

$$\mathbf{N} = F_x \mathbf{i} + F_y \mathbf{j}$$

$$\mathbf{N} = \frac{4\rho R V^2 L \sin^2 \alpha_1}{3}$$

and the normal force coefficient of the body is

$$C_N = \frac{N}{1/2 \rho V^2 R^2}$$

$$C_N = 1.70 \frac{L}{D} \sin^2 \alpha$$

It is obvious upon inspection of Figure 7 that because of symmetry the forces acting on the cylindrical body of the capsule do not contribute to the pitching moment coefficient or the axial force coefficient.

Analysis of Configuration 1 - Damping in Pitch

The total damping in pitch coefficient was determined following the analysis of Reference 3. In this analysis, the effects of the hemispherical end caps were neglected and it was assumed that there is no difference between the pitch angle θ , the pitch rate $\dot{\theta}$, and the corresponding parameters, the angle of attack α , and the angle of attack rate $\dot{\alpha}$. In other words, it is assumed that the motion of the center of gravity of the capsule is horizontal and rectilinear. Also, any damping moments due to downwash were neglected. These assumptions imply that the total damping in pitch is approximately due only to the rotational pitching rate of the body about its own axis. Referring to Figure 9, the tangent of the induced pitch angle due to a steady pitch rate is

$$\tan \alpha_1 = \frac{\dot{\alpha} x \cos \alpha}{V + \alpha x \sin \alpha}$$

Assuming that α_1 is small ($\dot{\alpha} x \ll V$), this becomes

$$\alpha_1 = \frac{\dot{\alpha} x \cos \alpha}{V}$$

It was shown above that for the cylindrical body,

$$C_N = 1.70 \frac{L}{D} \sin^2 \alpha$$

$$\frac{dC_N}{dx} = \frac{1.70 \sin^2 \alpha}{D}$$

The restoring moment per unit length is then

$$\frac{dM}{dx} = \frac{-x dN}{dx}$$

$$= \frac{-x q A dC_N}{dx}$$

$$= -1.70 \frac{A q}{D} x \sin^2 \alpha_1$$

$$= -1.70 \frac{A q}{D} x \left[\frac{\alpha x \cos \alpha}{V} \right]^2$$

$$\frac{dM}{dx} = -1.70 \frac{A q}{D} \left[\frac{\alpha \cos \alpha}{V} \right]^2 x^3$$

Figure 9. Geometrical Relationships Needed to Determine the Damping in Pitch Coefficient

Therefore, within the small angle approximation:

$$dM = -1.70 \frac{Aq}{D} \left[\frac{\dot{\alpha} \cos \alpha}{V} \right]^2 x^3 dx$$

$$M = -3.40 \frac{Aq}{D} \left[\frac{\dot{\alpha} \cos \alpha}{V} \right]^2 \int_0^{L/2} x^3 dx$$

$$M = -3.40 \frac{Aq}{D} \left[\frac{\alpha \cos \alpha}{V} \right]^2 \frac{L^4}{64}$$

so that, by definition:

$$C_{M_{\dot{\alpha}}} = \frac{M}{qAD} \left/ \frac{\dot{\alpha} D}{2V} \right. = 0.106 \frac{L^4}{D^3} \frac{\cos^2 \alpha}{V} \dot{\alpha}$$

Analysis of Configuration 1 - The Total Configuration

The aerodynamic coefficients of the total configuration were determined by adding together those due to the component parts as determined above. Therefore, the total normal force coefficient is:

$$C_N = 1.16 \cos \beta + 1.70 \frac{L}{D} \sin^2 \alpha$$

or since $\beta = 90^\circ - \alpha$

$$C_N = 1.16 \sin \alpha + 1.70 \frac{L}{D} \sin^2 \alpha$$

The total pitching moment coefficient is

$$C_M = 0.282 \frac{L}{D} \sin^2 \beta$$

$$C_M = 0.282 \frac{L}{D} \sin^2 \alpha$$

The total axial force coefficient is

$$C_A = 1.16 \sin \beta$$

$$C_A = 1.16 \cos \alpha$$

and the total damping in pitch coefficient is

$$C_{M_q} = 0.106 \frac{L^4}{D^3} \frac{\cos^2 \alpha}{V} \dot{\alpha}$$

Analysis of Configuration 2

Configuration 2 was also analyzed using Newtonian theory, however, since this theory is incapable of predicting pressure forces on concave surface, it was treated as a simple right circular cylinder. In this case the normal force coefficient is that derived above for a cylindrical body, referring to Figure 8.

$$C_N = 1.70 \frac{L}{D} \sin^2 \alpha$$

The unit vector normal to the windward face of the cylinder and directed inward is

$$\mathbf{n} = -\mathbf{k}$$

The velocity vector of the flow relative to this face is

$$\mathbf{V} = -v \sin \alpha \mathbf{i} - v \cos \alpha \mathbf{k}$$

Thus, the force caused by an infinitesimal stream-tube of air hitting the face and losing its normal component of momentum is

$$d\mathbf{F} = -\rho(\mathbf{V} \cdot \mathbf{n})^2 d\mathbf{S}$$

$$d\mathbf{F} = -\rho V^2 \cos^2 \alpha R dr d\theta \mathbf{k}$$

The total axial force on the face is then

$$F = -\rho V^2 R \cos^2 \alpha \int_0^R \int_0^{2\pi} dr d\theta$$

$$F = V^2 \rho R^2 2\pi \cos^2 \alpha$$

and the axial force coefficient is

$$C_A = \frac{F}{(1/2)\rho V^2 \pi R^2}$$

$$C_A = 2 \cos^2 \alpha$$

The damping in pitch coefficient for Configuration 2 is the same as that derived above for a cylindrical body.

$$C_{M_q} = 0.106 \frac{L^4}{D^3 V} \cos^2 \alpha \dot{\alpha}$$

For a completely symmetrical right circular cylinder the pitching moment is zero, however, as described above, in two cases Configuration 2 is asymmetrical in that its center of gravity is offset from the geometric center. Therefore, for Case 1

$$C_M = 0$$

for Case 2

$$C_M = \left(\frac{x_{CG}}{D} \right) C_A$$

$$C_M = \frac{0.061}{0.981} (2 \cos^2 \alpha)$$

$$C_M = 0.124 \cos^2 \alpha$$

for Case 3

$$C_M = \left(\frac{X_{CG}}{D} \right) C_N$$

$$C_M = \frac{1.4035}{0.981} (1.70) \frac{L}{D} \sin^2 \alpha$$

$$C_M = 2.44 \frac{L}{D} \sin^2 \alpha$$

Survey of Available Experimental Data

A literature search was made to determine what has been done in the way of measuring the aerodynamic coefficients of a right circular cylinder at all angles of attack and at hypersonic Mach numbers. It was discovered that surprisingly little experimental work has been done on such a fundamental shape. In general, the normal force coefficient and the drag have been measured in supersonic flow at angles of attack of zero or 90 degrees. Very little data exists on static pressure distributions and virtually no dynamic measurements have been made. Most of the experimental work has been done at supersonic Mach numbers and high Reynolds numbers, although some work has been done at low Reynolds numbers. Also, most of the data has been accumulated at angles of attack of zero or 90 degrees, although several papers have investigated the full range of angle of attack. A summary of the experimental data discussed above is given in Figures 10 to 14 and a comprehensive bibliography is given at the end of this report.

Conclusions

This study yields two results concerning the tumbling motion of Configuration 1

1. At least for the initial tumbling rates given, which are believed to be representative of the actual tumbling rates encountered upon re-entry, the tumbling motion of a cylinder with hemispherical end caps is rather rapidly damped. Even for Case 2 with an initial rate of 115 degrees per second, the tumbling motion is damped in 9 cycles.
2. The stable position of a cylinder with hemispherical end caps is normal to the flow, as can be seen by examining Figures 2, 3, and 4.

These results support previous theoretical work done on a less comprehensive basis.

In regard to the amount of experimental data available for right circular cylinders, this study has disclosed that surprisingly little is known about such a fundamental shape. Dynamic force data and both static and dynamic stability data are especially needed.

Figure 10. Experimentally Determined Normal Force Coefficient versus Mach Number for Right Circular Cylinders at Angles of Attack of 90 Degrees

Figure 11. Experimentally Determined Axial Force Coefficient versus Mach Number for Right Circular Cylinders at Angles of Attack of 0 Degrees

Figure 12. Experimentally Determined Normal Force Coefficient Slope versus Mach Number for Right Circular Cylinders at Angles of Attack of 0 Degrees

Figure 13. Experimentally Determined Center of Pressure (X_{cp}/L) Measured from the Base of a Right Circular Cylinder at Zero Angle of Attack versus Mach Number

Figure 14. Experimentally Determined Axial Force Coefficient versus Angle of Attack for a Right Circular Cylinder

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