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AN INFORMAL REPORT

A Bifilar Helical Multiwire Proportional
Chamber for Position Sensitive Detection of
Minimum Ionizing Particles

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A Bifilar Helical Multiwire Proportional Chamber for Position Sensitive Detection of Minimum Ionizing Particles

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A BIFILAR HELICAL MULTIWIRE PROPORTIONAL CHAMBER FOR POSITION SENSITIVE DETECTION
OF MINIMUM IONIZING PARTICLES

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ABSTRACT

A new type of multiwire proportional chamber with a helical cathode for delay line readout has been developed and studied in detail. It gives cathode delay line pulses nearly as large as the anode pulses, hence is well-suited for minimum ionizing particles. Measurements on such a chamber yielded spatial resolution better than 0.33 mm fwhm for ^{90}Sr β particles, a resolving time less than 30 nsec, and maximum counting rate greater than 2 MHz. The most probable pulse height induced on the delay line by minimum ionizing particles in this chamber is approximately 8 mV.

I. INTRODUCTION

In the past few years, the multiwire proportional chamber has been developed into a useful position sensitive detector for medium and high energy physics applications. Such counters are particularly useful because of their high count rate capability ($\approx 10^6$ particles per second per wire), their good resolving time (30 nsec), their low mass, their high multitrack efficiency, and the fact that they are continuously active and do not require external trigger counters. Charpak¹ has given an excellent review of the properties of these detectors and their applications.

For many purposes the type of chamber usually used, that is, with an amplifier on each anode wire, cannot achieve adequate spatial resolution. In such chambers the fwhm spatial resolution for particles incident normal to the chamber is very nearly equal to the wire spacing. Most groups have found it impractical to work with wire spacing smaller than 2 mm, although some groups have reported success with 1 mm spacing.² The spatial resolution required in the focal plane detectors of many modern spectrometers is on the order of 0.25 mm fwhm. In this application it is possible to sacrifice high count rate capability and high multitrack efficiency in order

to achieve the necessary resolution. Since we intend to use our detectors in the focal planes of the LAMPF spectrometers, we have concentrated our efforts on achieving the best possible spatial resolution.

In addition to reducing the wire spacing, two other methods have been used to obtain improved spatial resolution. One method was to measure the electron drift time to determine the initial location of the ion pairs. A best resolution of 0.47 mm fwhm is expected with this technique.³ The other method consists of capacitively coupling to a delay line readout the signal induced on the cathode. This technique has been highly developed by Grove, et al.,⁴ and recent results with a ^{55}Fe x-ray source indicated that 0.3 mm fwhm resolution can be obtained.

In addition to the excellent spatial resolution, the delay line technique of Grove, et al.,⁴ has two important useful features. First, the fact that only two amplifiers are needed for one coordinate means that a large reduction in complexity and cost of readout electronics can be achieved. Second, x-y readout from a single plane is possible which reduces the mass of the detector and makes possible

two-dimensional readout of the location of low energy γ rays. The disadvantages of this technique are that very small signals must be processed (0.1 - 1.0 mV for minimum ionizing particles and 2 mm anode wire spacing) and that the system is dead for as long as the signals are remembered by the delay line (2.5 μ sec for a 35 cm line with 70 ns/cm delay). These properties may make the chambers difficult to use under real experimental conditions.

We have developed a method of inducing the pulses directly into the delay line, thus increasing the amplitude of the pulses by a factor on the order of 10. In addition, our delay lines have a clean pulse response, and a memory time that can be reduced to 500 nsec or less. We believe that these developments have resulted in a practical high resolution detector which has applications in many areas of research in addition to our own. In the sections which follow, we report the considerations which led to our developments and document the results of measurements made with a prototype chamber.

II. CONSTRUCTION AND PROPERTIES OF HELICAL DELAY LINES

A. Principles of Helical Cathodes

After an avalanche is formed near an anode wire, the motion of electrons and positive ions induces a positive pulse in the cathode and a negative pulse in the anode. The spatial extent of the cathode signal is comparable to the anode-cathode gap spacing (see Appendix). If the cathode is fabricated in the form of a delay line with axis parallel to the anode wires, the cathode pulse will propagate in both directions away from the avalanche position. The avalanche position along the anode wires can then be measured by measuring the difference in times of arrival of the cathode pulse at the two ends of the delay line. Since the cathode is its own delay line, there are no coupling losses and the cathode signal will be comparable to the anode signal.

A schematic view of a chamber with a helical delay line cathode is shown in Fig. 1. The cathode is a simple helical winding around the entire array of anode wires. The signal is taken from the two ends of the helix. We have constructed chambers with such a design and have obtained high accuracy in the determination of the spatial location of the cathode pulses. The principle disadvantages of such

Fig. 1. Schematic view of helical multiwire proportional chamber.

chambers are the long memory time of the delay line resulting in long dead times, and the difficulty of termination at the ends due to the dispersion. The details of our results on simple helical chambers will be the subject of a future publication.

In order to overcome the difficulties with simple helical cathodes we have turned to cathodes which consist of a bifilar helical winding, one winding of which is grounded at every turn. The result can be described as being a parallel-wire transmission line wound helically around the chamber with one wire grounded. The delay line has all of the properties of a parallel wire transmission line, namely little dispersion, and a characteristic impedance which is low and can be calculated from well-known formulas. The line can be terminated easily with a pure resistance resulting in no significant reflections. The remainder of this paper details our findings on bifilar helical chambers.

B. Mechanical Construction

A chamber was constructed with an active area of 10 cm x 25 cm. The supporting frame was made from two sheets of polystyrene, each 20 cm x 40 cm x 0.32 cm with a central cutout of approximately 12 cm x 25 cm. The anode wires, made from 20 μ m diam gold plated tungsten, were stretched on one piece of the frame parallel to the long axis of the chamber, and with 2 mm spacing. Then the second frame member was fastened on top of the first with the anode wires

sandwiched between. The cathode was constructed by winding two separate helical windings each of 50 μm diam copper wire. The first helix was wound with 1 mm spacing and each turn was in electrical contact with a copper rod placed along one edge of the frame before winding was begun. Thin insulating tape was then placed over the copper rod and the second winding with 1 mm spacing was laid midway between the turns of the first winding. The assembly was placed in an aluminum box with windows the size of the active area, and the copper bar was grounded to the box. Because of the high attenuation of the helix, it was not possible to measure the signal at only the ends of the helix. Electrical taps were connected to the helix at intervals of 3.65 cm, and signals were measured at these points. The small distance between the taps reduces the maximum delay before the signals are received. The large attenuation resulting from the low characteristic impedance results in a small memory time, hence a small dead time.

C. Electrical Properties of the Bifilar Helix

We analyze the bifilar helix as essentially a parallel wire transmission line, the properties of which are well-known and are tabulated in many places.⁵ When tested with a pulser, the leading edge of the pulse undergoes only a small change in rise time after traversing a distance in which the pulse is attenuated by a factor of two. These properties are clearly displayed in Fig. 2. The relevant properties of this delay line are: 1) the delay per unit length is 20 nsec/cm; 2) the line impedance is 300 Ω ; and 3) the delay for 1/e attenuation is approximately 120 nsec for a pulse rise time of 10 nsec. We are able to cleanly terminate this line with a resistor at each end.

D. Comparison of Performance with System of Grove, et al.⁴

The delay lines used by Grove, et al.,⁴ have a characteristic impedance of $\approx 620 \Omega$, a delay of 40-200 nsec/cm, and a minimum rise time of 100 nsec. There is a coupling loss of a factor of 10-30 between the cathode wires and the delay line. Our bifilar helix has a characteristic impedance of 300 Ω , delay of 20 nsec/cm, and a minimum rise time of ≈ 10 nsec. The bifilar helix can be terminated easily with a pure resistance. The attenuation time is shorter for the bifilar helix because of its smaller

Fig. 2. Pulse response of bifilar helical delay line;

- a) Input pulse and output pulse 2.54 cm from input (10 ns/division);
- b) Input pulse and output pulse 7.62 cm from input (20 ns/division);
- c) Attenuation factor as a function of delay.

characteristic impedance. This property is actually advantageous in reducing the memory time while affecting the pulse rise time very little.

III. PULSE SHAPES IN THE BIFILAR HELICAL CHAMBER

A. Observation of Fast Anode Pulse

We have observed a very fast rising and falling pulse in a 50 Ω load at the anode of our chambers. A sample of such pulses is shown in Fig. 3. An upper limit for the gain can be obtained by assuming that a single electron is responsible for initiating the avalanche. With this assumption, we find that the charge contained in the first 15 nsec of the pulse corresponds to a gain of 0.6×10^7 . (Similar fast pulses have been seen by Grunberg, et al.,⁶ in a different gas mixture from ours.) We observe such a pulse shape over the voltage range from 3000 to 4000 V in our 3.18 mm gap chamber. No apparent change occurs when we add a small amount (up to 1%) of freon 13 B1 to our 50% argon - 50% isobutane gas mixture.

The observed pulse shape appears to be inconsistent with previous explanations of the mechanism of pulse formation. The fast fall time of the

Fig. 3. Anode pulse measured across 50 Ω (10 ns/cm).

observed current pulse cannot be explained by the motion of positive ions toward the cathode. We believe that since we are operating in the region of limited proportionality, space charge limits the buildup of the avalanche. As in a Geiger-Muller counter, the positive ions build a sheath of charge which shields the anode while the avalanche is still a finite distance from the anode. If the avalanche action is stopped at a distance on the order of 5 wire diameters from the center of the anode, then the electrons will be collected from a point at which the potential difference from the anode is 25% of the applied voltage. Using the theory of Rose and Korff,⁷ we have calculated the gain for an avalanche which starts far from the anode and terminates at 5 wire diameters. This calculation gives a gain which is even greater than the observed gain indicating that it is reasonable to assume that avalanche action terminates at least this far from the anode.

B. Cathode Pulse Formation in the Bifilar Helix

The motion of the electrons and positive ions resulting from the avalanche induces a current pulse directly in our delay line. This current pulse is spread out over a finite region in space. We have computed the spatial dependence of the induced current, $j(z)$ for simplified models of our chamber in the Appendix. If we assume that $j(z,t)$ can be written in the form of a product of a function of z and a function of t , we may write the following expression for the voltage pulse observed on the delay line:

$$v(z,t) =$$

$$\frac{Z_0}{2} \int_0^t F(t-t') [j(z+v_z t') + j(z-v_z t')] \exp(-t'/\tau) dt' \quad (1)$$

In this expression, Z_0 is the impedance of the delay line, $F(t)$ is the time dependence of the induced current, and τ is the attenuation time for the delay line. This form represents waves propagating in both the positive and negative z -direction from an event at $z = 0$ and is correct in the limit of small attenuation and no dispersion. In practice, for large delays there will be distortion produced by resistance and skin effect.

We have computed $v(z,t)$ for an assumed pulse shape $F(t) = t^2 \exp(-t/t_0)$, with $v_z = 0.06$ cm/nsec, $t_0 = 2.5$ nsec, $\tau = 300$ nsec, and $j(z)$ calculated for cylindrical geometry with radius 3.18 mm. The result is shown in Fig. 4. This figure clearly indicates the existence of a far zone in which the pulse arrives at a time proportional to the distance from the source to the observer and a near zone in which the proportionality is destroyed. In order to maintain good spatial resolution in a system with a tapped helix, it is necessary to ignore the information received from the tap closest to the event.

Fig. 4. Calculated voltage as a function of time for several source locations. The earliest arriving pulse is calculated for a source located at the receiver, later arriving pulses are for sources located at intervals of 2 mm up to 18 mm from the receiver.

The electronic system we shall use can accomplish this task (see Section V). Photograph of the pulses observed on the bifilar helix is shown in Fig. 5. The rise and fall times are somewhat slower than predicted by Eq. (1), but the differences may be explained by the finite frequency response of the helix.

Fig. 5. Cathode pulse from bifilar helical chamber observed in 300 Ω (5 mV/division, 20 ns/cm).

C. Leading Edge Timing

From the curves of Fig. 4 we have derived the time at which the helix pulse crosses a fixed threshold as a function of z . A family of such curves is shown in Fig. 6, where we have used thresholds from 0.12 to 0.95 of the nominal pulse height at 10 mm from the event. These results show that the time is a linear function of distance as long as the observation point is more than 3 mm from the source. Since all curves are parallel to an accuracy better than 1 nsec, two leading edge discriminators observing the same event (at different points on the helix) would trigger at a time which is independent of the amplitude over a wide range of amplitudes. In our test setup with a tap spacing of 7.3 cm, we have changed the discrimination level by a factor of 5 with no noticeable shift in the time difference and no degradation of the spatial resolution. For the same reasons, we expect that the spatial resolution will be independent of pileup effects. Thus, we have chosen to use leading edge timing in our electronic system.

Fig. 6. Calculated time difference vs source location for several discriminator thresholds. The input data were taken from Fig. 4. The largest time difference corresponds to a threshold of 8.0 in the units of Fig. 4, and the remaining curves are for thresholds stepped by 1 unit down to a threshold of 1.0.

IV. TESTS WITH ^{90}Sr SOURCE

A. Test Setup

A ^{90}Sr source was used to produce minimum ionizing particles in our chambers. The electrons from this source undergo considerable multiple Coulomb scattering in our chambers, so we used several collimators in order to measure the spatial resolution. The source consisted of 0.7 mc of ^{90}Sr deposited on the end of a 0.13 mm diam wire. The first collimator was a 0.25 mm x 25.4 mm slit of brass, the second and third were 0.25 mm x 25.4 mm slits in aluminum. The chamber was placed between the first and second collimators, and a plastic scintillator coincidence detector was placed beyond the third collimator as shown in Fig. 7. With only the first collimator in place, we observed a coincidence rate of approximately 3000 counts/sec; with all collimators in place we observed 0.5 counts/sec.

The electronics used for all resolution tests are shown in Fig. 8. On each tap we used an SG3401 integrated circuit amplifier⁸ followed by an emitter

Fig. 7. Test setup for measurement of helical chamber using a ^{90}Sr β source.

Fig. 8. Electronics block diagram for test setup.

follower. The output from the emitter follows a 50 Ω cables to commercial modules in NIM packaging. The input impedance of the SG3401 was measured to be approximately 1.5 $\text{K}\Omega$. Neither this circuit nor the Ortec 434 amplifier which followed it was fast enough to pass the observed pulses without increasing the rise times. We had an equivalent noise at the input of approximately 400 μV ; most of this noise resulted from the Ortec amplifier. Although considerable optimization of this system is possible, it proved adequate for the tests we report here.

All measurements were made with a gas mixture of 50% argon and 50% isobutane except where noted.

B. Resolution and Linearity

The time difference as a function of source position was measured with the ^{90}Sr source in order to measure the linearity of the response. For this test the only collimator required was a 1 mm diam hole at the source. The results are shown in Fig. 9. Two curves are shown. One was measured with a pair of taps placed 7.30 cm apart on the helix; the other was measured with a simulated amplifier placed on a tap halfway between the taps which were used for timing measurements. The curves are linear within 0.1 mm except when the source was within a few mm of one of the outside taps. Furthermore, the results indicate that there is no significant difficulty from reflections at an intermediate tap.

Fig. 9. Time difference vs source location for a tapped bifilar helical chamber. The non-linearity at a source position greater than 3 cm results from approaching too close to a tap being used in the time difference measurement (located at 3.8 cm for this test).

The three-slit arrangement was used to measure the spatial resolution of the system. The result is shown in Fig. 10. For this figure, measurements were made at spatial intervals of 2.54 mm. The counting time for one peak was made longer than the others in order to get a better measure of the resolution. The width of the peak was 0.33 mm fwhm and was independent of source position. Since finite collimator size and multiple scattering are included in this result we believe that our resolution is on the order of 0.25 mm. However, we quote the observed width to avoid the possibility of error.

Fig. 10. Spatial resolution of bifilar helical chamber measured with a collimated ^{90}Sr source. The upper figure shows the observed timing spectrum for a source moved in 2.54 mm steps, the lower curve shows the observed linearity of the time difference vs source position. The measured width of each peak is 0.33 mm fwhm which includes the resolution of the chamber, the finite size of the collimator, and the effects of multiple scattering.

It is important to stress the reproducibility of these results. We have obtained identical results for chambers with a magic gas mixture (1% freon 13B1 added) for both 2 mm and 3 mm anode wire spacing. In addition, no special efforts were made in optimizing or standardizing accurately the amplifier gains and the discrimination levels. In fact, the same results were achieved when the amplifier gains at each tap were increased by a factor of 2 and also by a factor of 5. This confirms the earlier conclusion that the discriminator thresholds are

noncritical and can be set anywhere in a wide range without degrading the performance of the system.

C. Pulse Height and Resolving Time

The pulse height distribution for cathode pulses from ^{90}Sr electrons was measured in the same setup. The result is shown in Fig. 11. The most probable pulse height was approximately 8 mV for a ^{90}Sr source located 3.65 cm from a tap. The flag indicates the 2 mV threshold that was used for all measurements. A coincidence resolving time spectrum is shown in Fig. 12. For this spectrum, the time-to-amplitude converter was started with the scintillator and stopped with the anode pulse from the helical chamber. Both the resolving time spectrum and the amplitude spectrum are similar to those observed by other authors.¹

Fig. 11. Cathode pulse height spectrum for bifilar helical proportional chamber measured with a ^{90}Sr β source. The most probable pulse height is 8 mV. The flag is set for the 2 mV threshold used in all tests.

Fig. 12. Spectrum of time difference between anode pulse and scintillator pulse in ^{90}Sr coincidence setup. Two peaks are shown which have been arbitrarily separated by 64 ns additional delay.

We have measured the amplitude of the pulse induced in the helix for a ^{55}Fe source as a function of the applied voltage. Also recorded was the amplitude of the anode pulse in a load of $50\ \Omega$. The result is shown in Fig. 13. The amplitude varies a factor of 2 for each 400 V change in the applied voltage. The maximum safe operating voltage was 4000 V for our 50% argon, 50% isobutane mixture. The addition of 1% freon 13 B1 allowed us to raise the voltage by approximately 200 V with a corresponding amplitude increase of a factor of 1.5.

Fig. 13. Most probable cathode pulse height vs applied voltage for bifilar helical chamber and ^{59}Fe source located 2.81 cm from tap. Also shown is the anode pulse height observed in 50 Ω .

We are designing the electronics for our chambers to have a dead time of approximately 500 nsec. The prototype chamber has a helix length of approximately 25 cm which gives a maximum delay time of 500 nsec. The attenuation of the delay line reduces the amplitude to about 2% in that time, and the ends of the delay line can be terminated in order to eliminate reflections. An appropriate spacing for taps on the helix is approximately 2.5 cm, or 50 nsec. We will have three time-to-digital converters (TDC) which are started from the "OR" of the anode pulses. The taps will be divided into

A time resolution of 1.0 nsec fwhm is necessary to accomplish the design goal of 0.25 mm fwhm resolution. Either emitter coupled logic or Schottky

Fig. 14. Simplified block diagram of electronics proposed for bifilar helical chamber.

clamped TTL logic⁹ is available in integrated circuits convenient for this job. Because of the relative ease of interfacing to CAMAC and the larger selection of circuit types, we have tentatively chosen TTL for the bulk of our system. The amplifier/discriminator used on each tap will be developed from discrete components. Since we require only one such circuit for each 2.5 cm of chamber length, we can afford to use a higher quality design than is commonly used with MWPC's.

VI. SUMMARY AND CONCLUSIONS

We have demonstrated the feasibility of using the bifilar helical multiwire proportional chamber for detection of minimum ionizing particles. We have measured the spatial resolution and found that it is better than 0.33 mm fwhm. We have shown that the chamber and electronics can recover and be ready to count again after a time of 500 nsec or less. Because of the similarity to more conventional chambers, many of the properties of our chambers may be safely assumed to be the same as those reviewed by Charpak.¹ We believe that these chambers will perform well in applications in which two-dimensional position readout with very good resolution (0.3 mm) in one direction is required. These applications include, but are not limited to, the focal plane detectors for modern spectrometers such as those at LAMPF.

It is interesting to speculate on future variations of our technique which may prove to be useful. First, the chamber thickness can be reduced in order to increase the pulse height on the delay line. In addition, it should be possible to fabricate chambers up to 1 m in length, although the required supports for the anode wires may cause dead spots to appear.¹

For applications in which multitrack efficiency and spatial resolution may be compromised in favor of simplicity, it may be possible to make chambers as large as 1 m by 1 m with a coarse helix spacing. For a bifilar helix constructed of 125 μ m diam copper wire with a pitch of 4 mm/turn, it should then be possible to use a single pair of amplifier/discriminators located at the ends of a 1 m by 1 m chamber and achieve a spatial resolution of 2 mm. The single winding helix with its larger attenuation time may prove advantageous in such applications.

Either system should result in a significant cost reduction when compared with amplifier-per-wire systems.

Other possibilities include using a helical winding for only one cathode and replacing the other with a very thin conducting window. They may allow the detection of very low energy particles without difficulties caused by particles stopping or scattering in the entrance cathode wires. Finally, it may be possible to use a delay line for both the anode and the cathode, although we must emphasize that there will be serious construction difficulties to be overcome before this variation would become generally useful.

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APPENDIX
SPATIAL DISTRIBUTION OF INDUCED CURRENT PULSE

I. RECTANGULAR COORDINATES

Define a coordinate system with the z-axis parallel to the anode wires, and the x-axis parallel to the cathode wires. We wish to compute the field resulting from a point charge located at $x = 0$, $y = c$, $z = 0$ between two conducting plates located at $y = 0$ and $y = b$. Since in our helix the wires encircle the chamber in the x-direction, we want the induced current distribution integrated over x from $-\infty$ to $+\infty$. In this situation we can use two-dimensional solutions to Laplace's equation. For the geometry defined above, we have

$$v = \sum A_n \sin \frac{n\pi y}{b} \exp - \left| \frac{n\pi z}{b} \right|.$$

For a unit charge located at $y = c$, we have

$$-\frac{dv}{dz} \Big|_{z=0+} = \delta(y-c) = \sum A_n \frac{n\pi}{b} \sin \frac{n\pi y}{b}.$$

This results in coefficients

$$A_n = \frac{2}{n\pi} \sin \frac{n\pi c}{b}.$$

The induced charge is given by

$$E_y(b) = \left| \frac{dv}{dy} \right|_{y=b} = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2}{b} \sin \frac{n\pi c}{b} \exp - \left| \frac{n\pi z}{b} \right|.$$

This series may be summed to give the result

$$E_y(b) = \frac{1}{b} \frac{\sin \frac{\pi c}{b}}{\cosh \frac{\pi z}{b} + \cos \frac{\pi c}{b}}.$$

The induced current pulse is given by the relation

$$j(z,t) = \frac{d}{dt} E_y(b) = \frac{\pi}{b} \frac{dc}{dt} \frac{\cos \frac{\pi c}{b}}{\cosh \frac{\pi z}{b} + \cos \frac{\pi c}{b}} + \frac{\sin^2 \frac{\pi c}{b}}{\left(\cosh \frac{\pi z}{b} + \cos \frac{\pi c}{b} \right)^2}.$$

For $c \ll b$

$$j(z,t) = \frac{\pi}{2b^2} \frac{dc}{dt} \left[\frac{2}{\cosh \frac{z}{b} + 1} \right].$$

The term within the brackets in the above equation has been plotted in Fig. 15 for $b = 3.18$. It is

Fig. 15. Calculated z-dependence of induced current pulse for rectangular geometry (dashed curve) and cylindrical geometry (solid curve). The anode-cathode separation is 3.18 mm for both cases.

important to note the occurrence of the term $1/b^2$. This indicates that the peak amplitude of the induced pulse can be increased dramatically if the distance between anode and cathode is minimized. For this reason, we have concentrated our efforts on chambers with anode to cathode separation comparable to the anode wire spacing.

II. RESULTS FOR CYLINDRICAL GEOMETRY

We assume a ring of charge located at $r = c$, $z = 0$ between two concentric cylinders of $r = a$, b . In this case, the solutions to Laplace's equation are:

$$v = \sum A_n F_n(k_n r) \exp - |k_n z|$$

where the F_n 's are

$$F_n(k_n r) = J_0(k_n r) - \frac{J_0(k_n a)}{Y_0(k_n a)} Y_0(k_n r)$$

where J_0 and Y_0 are the Bessel functions of the first and second kind and the k_n 's are chosen such that

$$F_n(k_n b) = 0.$$

For a ring located at $r = c$ containing a unit charge, we find that

$$A_n = \frac{F_n(k_n c)}{k_n \{b^2 [F_n'(k_n b)]^2 - a^2 [F_n'(k_n a)]^2\}}.$$

The field at $r = b$ is given by the relation

$$E_r(z) = \sum_n \frac{F_n(k_n c) F_b'(k_n b)}{b^2 [F_n'(k_n b)]^2 - a^2 [F_n'(k_n a)]^2} \exp - |k_n z|.$$

As in the rectangular case, we want the derivative with respect to c of the above expression

$$j(z,t) = \frac{dc}{dt} \sum_n \frac{k_n F_n'(k_n c) f_n'(k_n b)}{b^2 [F_n'(k_n b)]^2 - a^2 [F_n'(k_n a)]^2} \exp - |k_n z|.$$

We have computed the sum in the above expression numerically (for $a = 0.01$ mm, $b = 3.18$ and $c = 0.01$ mm), normalized it to unity at $z = 0$, and plotted the result in Fig. 15. As in the rectangular geometry case, $j(z)$ is approximately independent of c for $c \ll b$, and we have checked this result for $0.01 \leq c \leq 0.1$ mm. Thus, $j(z,t)$ can be approximately factored into a function of time and a function of position. This property is required in order that we be able to write $v(z,t)$ in the form given in Eq. (1).