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Nuclear Wave Functions from Particle Transfer Data

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Nuclear Wave Functions from Particle Transfer Data*

by

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*Work supported in part by US AEC Division of Research

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ABSTRACT

Using the analytic properties of scattering amplitudes in the $\cos \theta$ plane, along with the conformal mapping technique to improve the convergence, nuclear properties are extracted directly from reaction data. At low energies one obtains spectroscopic factors--the amplitude of the asymptotic part of the wave function for a particular channel. At high energies short-range aspects of the wave functions are studied by including form factors at the vertices.

Our present knowledge of nuclear wave functions is based on nuclear models and fits to weak and electromagnetic transition rates and reaction data. Some of the most detailed information about the particle structure is based on the analysis of particle transfer reactions with distorted wave Born Approximation calculations. With that method the magnitude of the cross section depends upon the "spectroscopic factors",

$$|S|^2 = |\langle \phi_{A-n} \phi_n | \phi_A \rangle|^2,$$

the square of the overlap integral of the A -particle wave function ϕ_A with the products of the wave function for the n particles being transferred, ϕ_n , and the wave function ϕ_{A-n} for the residual nucleus. This quantity contains valuable spectroscopic information about the probability of breakup of $A \rightarrow n + (A-n)$, and thus about the particle structure of the nucleus.

In considering this breakup, it is useful to recognize that the wave function has the form

$$\phi_A = \phi_A(\text{short-range}) + N_{A,n} \phi_{A-n} \phi_n e^{-\kappa r}/r, \quad (1)$$

in which the second term is the asymptotic part of the wave function for breakup into the n and $(A-n)$ systems, with κ determined by the binding energy of these subsystems in A . The first term, ϕ_A (short range), vanishes asymptotically with r , the distance

between the n and $A-n$ systems, faster than $e^{-\kappa r}/r$.

Note that the normalization of the asymptotic part, $N_{A,n}$, contains the same spectroscopic information as "S" without explicit dependence on the internal wave functions ϕ_n and ϕ_{A-n} . We will refer to $N_{A,n}^2$ as the spectroscopic factor.

This leads us to consider the analytic properties of the scattering amplitudes, since the residues of the poles corresponding to particle transfer are entirely determined by the spectroscopic factors (along with known properties, such as masses). This was exploited in the very early days in a study of neutron-deuteron scattering with reasonable success.¹ At that time it was suggested by Amado² that one use an extrapolation to the pole to determine spectroscopic factors for pickup and stripping reactions. From the work that followed³ it became apparent that the straightforward application of pole approximations would not provide a satisfactory program for nuclear structure studies due to the presence of nearby cuts.

One approach to this problem is to try to explicitly calculate the contribution from the branch cuts. However, there is great difficulty in doing this accurately enough to calculate the pole residues.⁴ A most promising recent approach has been to seek improved representations for the scattering amplitude in order to include cut corrections in a more nearly optimal manner.⁵ Cutkosky and his co-workers have applied these methods to various problems in particle physics. Here we discuss application to low energy nuclear physics for the calculation of spectroscopic factors, and an extension to

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high energy to determine high momentum components of nuclear wave functions.

II. Low Energy

At low energy the poles are sufficiently close to the physical region for us to try to separate the poles from all of the cuts, and to determine the residues of the poles. Let us first take the example of neutron - ${}^4\text{He}$ scattering. The only pole in the scattering amplitude occurs at lab kinetic energy $E = E_0 \approx -15$ MeV and corresponds to ${}^3\text{He}$ transfer (Fig. 1(a)). The once-subtracted forward dispersion relation for this process is

$$\Delta(E) \equiv \text{Re}[f(E) - f(0)] - \frac{k^2}{2\pi^2} P \int_0^\infty dk' \frac{\sigma_{\text{tot}}(k')}{k'^2 - k^2}$$

$$= \frac{gE}{2\mu E_0(E_0 - E)} + \frac{E}{\pi} P \int_{-\infty}^{E_c} dE' \frac{\text{Im}[f(E')]}{E'(E' - E)} \quad (2)$$

Where μ is the reduced mass, $E_c \approx -20$ MeV is the branch point for the nearest left-hand cut (corresponding to $p+d$ exchange (Fig. 1(b)), and $E = k^2 + M^2$ is the laboratory kinetic energy. The left-hand side of (2), $\Delta(E)$, can be obtained from experiments. If one neglects the last term in (2), the contribution from the left-hand cut, then the theoretical quantity $E/\Delta(E)$ is a straight line with slope $2\mu E_0/g$ and zero at E_0 . Locher and Ericson have shown that although the experimental $E/\Delta(E)$ has approximately the form of a straight line at low energy, the intercept is incorrect by several MeV. Phenomenologically including a cut contribution can change the residue by an order of magnitude. It appears that the method cannot provide spectroscopic factors with sufficient accuracy to be useful in practice.

Fig. 1. Diagrams for closest singularities in n - ${}^4\text{He}$ scattering.

Fig. 2. Diagrams for closest singularities for n - d scattering.

It is interesting to compare this with the application of forward dispersion relations for n - d scattering. In this case there are two poles, a nucleon exchange (Fig. 2(a)) pole at $E_n \approx -1$ MeV and a triton s -channel pole (Fig. 2(b)) at $E_t \approx -9$ MeV. The nearest left-hand cut corresponds to the π -triangle diagram (Fig. 2(c)). Due to the anomalous threshold, this cut occurs fairly close to the physical region at $E_b \approx -18$ MeV. The forward dispersion relation now has the form

$$\Delta(E) = \frac{g_n E}{2\mu E_n(E_n - E)} + \frac{g_t E}{2\mu E_t(E_t - E)}$$

$$+ \frac{E}{\pi} P \int_{-\infty}^E dE' \frac{\text{Im}[f(E')]}{E'(E' - E)} \quad (3)$$

If one neglects the left-hand cut contribution and determines the $d+n+p$ and $t+n+d$ spectroscopic factors by fitting to the experimental $\Delta(e)$, the results are in reasonable agreement with known wave functions.^{1,6} In order to understand the difference between the n - d and n - ${}^4\text{He}$ calculations, a model for the left-hand cut corresponding to diagram 1(c) was used to estimate its importance.⁴ It was found that in the model for the one pion part the left-hand cut contribution is quite small, and thus the two pole approximation is not bad. However, it

would not be surprising if n-d scattering is the only case for nucleon-nuclear scattering where the left-hand cut can be neglected without disastrous results, for in all other nuclei branch cuts of the type represented in Fig. 1(b) will be most important.

One should note here that the entire program of extracting nuclear wave functions directly from the data is feasible only if there are specific effects in the data which reflect the particle transfer mechanism. For the elastic scattering being discussed here the phenomenon is peaking in the angular distribution at large angles. For this reason we consider the angular distribution as a function of the center-of-mass scattering angle θ , and the analytic properties in the $\cos\theta$ plane.⁷ For n-³He the singularity structure is shown in Fig. 3. The only pole arising from ³He exchange (Fig. 1(a)) occurs at

$$\cos \theta_p \approx (1 - 0.0574 + 3.73E)/k^2 \quad (4)$$

where E is the laboratory energy (GeV) and k the center of mass momentum (GeV/c). The left-hand and right-hand branch cuts arising from processes illustrated in Figs. 1(b) and 1(c), respectively, have respective branch points given by

$$\cos \theta_u = 1 - (0.073 + 3.73E)/k^2, \quad (5a)$$

and

$$\cos \theta_t = 1 + 0.0992/k^2. \quad (5b)$$

Fig. 3. Singularities in the $\cos\theta$ -plane for n-³He scattering.

Since we are mainly interested in the residue of the pole at $\cos\theta = \cos\theta_p$, i.e.,

$$\begin{aligned} [2k^2(\cos\theta - \cos\theta_p)f(\theta)]_{\cos\theta = \cos\theta_p} \\ = \text{const.} \times (\text{spectroscopic factor}) \end{aligned} \quad (6)$$

we make a search in the form

$$\begin{aligned} 4k^4(\cos\theta - \cos\theta_p)^2 \frac{d\sigma}{d\Omega} \\ = \sum_{i=1}^{N_p} A(i) P_i(\cos\theta) \end{aligned} \quad (7)$$

In Eq. (7) the P_i are polynomials orthogonalized over the data:

$$\begin{aligned} \sum_J P_i(\cos\theta_J) P_j(\cos\theta_J)/E(J)^2 \\ = 0, \quad i \neq j \end{aligned} \quad (8)$$

where $\cos\theta_J$ are the experimental points and $E(J)$ is the error in the experimental quantity being considered at the point $\cos\theta_J$. Using data at 10 and 15 MeV the parameters $A(i)$ were fit by the MINUIT routine, with chi-square per degree of freedom ≈ 1 . One finds the optimal solution (for $N_p \approx 6$) to be the same at both energies, and that the magnitude of the spectroscopic factor is very nearly the one obtained using the result of the one pole fit in ref.6.

The same procedure is followed using the variable $Z(\theta)$, which is obtained using the mapping of ref.5, one obtains a value for the spectroscopic factor which is about three times larger, consistent with the attempts to explicitly include the cut contribution in the forward dispersion relation. The spectroscopic factor appears to be determined to within about 20%.

Fig. 4. Singularities in the $\cos\theta$ -plane for n-d scattering.

For the n-d scattering the singularity structure in the $\cos\theta$ plane is illustrated in Fig. 4. The nearest left-hand cut arises from the π -triangle (Fig. 2(c)), and the nearest right-hand cut from the form factor (Fig. 2(d)). An analysis of the same type as that described above was carried out using n-d data at several energies (this is described in detail in ref. 7). It is found that with the use of the conformal mapping the spectroscopic factor $N_{d,n}^2$ is in excellent agreement with the value obtained from modern deuteron wave functions. Without the mappings, the results are some 20% poorer, about the error in the spectroscopic factor itself. This is consistent with the cuts being not so important in the n-d case.

These results can be understood as the use of the mapped variable providing an efficient method to include the cuts. Since the cuts are effectively placed farther away from the poles, a rapidly converging expansion is obtained, and only a few parameters are needed to represent the cuts. It is expected that this feature will provide many uses for the method in low energy nuclear physics.

III. Intermediate and High Energies

At high energy the momentum transfer at the vertices in the exchange processes becomes quite large. Physically, this means that the asymptotic part of the wave function is playing a less important role compared to the short distance part (Eq. (1)). In standard nuclear treatments, the Born Approximation corresponding to a pole diagram such as Fig. 2(a) would involve high Fourier components of the wave function. This is expressed mathematically by recognizing that the cuts, such as represented in Fig. 2(c), are becoming increasingly important as the energy increases. For this reason we

wish to include them more directly in the calculation; i.e., we employ form factors in the pole diagrams. By this means we hope to study the high momentum components of the wave function without the dynamic calculations which are extremely difficult at high energies. Only the p-d scattering at energies of 0.6 - 1.5 GeV is discussed here. Only the data¹⁰ at large angles is used and Coulomb effects are ignored.

In order to introduce form factors we must decide on an explicit model. We shall assume that it is possible to describe a nucleus as a collection of baryons - i.e., we do not explicitly include meson degrees of freedom. From a variety of arguments⁹ one knows that it is important to include baryon resonance contributions. Physically, by allowing higher intrinsic spin one can introduce higher angular momentum components and thus high momentum components. Therefore, in spite of the large masses of the baryon resonances, they can dominate the high energy backward scattering. This leads us to consider the baryon exchange poles of nucleons, N, and baryon resonances, N^* , illustrated in Figs. 5(a) and 5(b).

The meson exchange diagram such as Fig. 2(c) are included in the form factors represented by blobs in Figs. 5(a) and (b). For example, it has been pointed out¹¹ that the process represented by Fig. 2(c) can give an important contribution to backward p-d scattering especially at the energy at which there is a $\Delta(1236)$ pion-nucleon resonance in a sub-diagram - which is an energy in the region being discussed here. One could explicitly calculate this contribution to the form factors by looking at the resulting contributions in the crossed-(u-) channel in the isospin one-half state of appropriate spin (spin 1/2 for the $d \rightarrow n + p$ form factor, spin 5/2 for the $d \rightarrow N(1688) + p$ form factor, and so forth). However, rather than attempting such

Fig. 5. Diagrams used for n-d high energy scattering.

calculations which are at best incomplete (as was seen above⁴), we shall attempt to extract the form factors directly from the data. The fact that the form factors will depend on s as well as u will indicate the presence of processes other than baryon transfer.

Typical cuts that remain are depicted in Fig. 5(c) and 5(d). Processes corresponding to Fig. 5(c) and more complicated diagrams of that type are the impulse series, which dominate in the forward direction and give a rather small contribution in the backward direction at high energy. Distortions of the Born diagram are provided by processes like the one represented in Fig. 5(d). The method of this work is to place the entire right-hand cut (impulsive processes) on the ellipse, and to try sample mappings for the left-hand cut.

As a guide to the form factors we use the previous study of backward p - d scattering with known standard deuteron wave functions and perturbation estimates of the $N(1688)$ contribution — which we feel are the most important exchanged baryons at these energies. Note the following important qualitative features: 1) the s -wave component of the deuteron wave function is a negligible part of the deuteron vertex function for momenta $> 2 F^{-1}$. Thus it can be neglected for laboratory energies > 600 MeV; 2) The D state has a zero at $\sim 5 F^{-1}$ (as well as at 0) for all known potentials fitting the two-body data; 3) The D^* wave function has a zero at a momentum component somewhat larger than the D -state. We thus choose for our amplitude the form

$$f(\theta) = A(1) \left\{ (1-\Delta)/(1-\Delta_0) \right\} \exp \left\{ -A(2) (\Delta-\Delta_0) \right\} / (u-M_N^2)^2 \\ + A(3) \left\{ (1-b\Delta)/(1-b\Delta_0) \right\} \exp \left\{ -A(4) (\Delta-\Delta_0) \right\} / (u-M_N^*)^2 \\ + \sum_{j=5}^{N_p} A(j) P_j(\cos \theta). \quad (9)$$

In (9) $u = (p-d')^2$, $\Delta = (p-d'/2)^2$, and Δ_0 is the value of Δ at the pole. The three-momentum transfer variable Δ and the four-momentum transfer variable u are closely related. The Minuit program was used to find the parameters $A(i)$ which fit backward p - d data¹⁰ at .590, 1.0, 1.3, and 1.5 GeV. The parameter b was varied to improve the fit, and various

left-hand branch cuts were used. The most significant results of this preliminary investigation were that with the mapping the D^*/D contribution went from about 0.3 at 590 MeV to about 1 at 1 GeV to about 2-3 at 1.5 GeV, consistent with our expectations.⁹ One needs much better data (the best fits were with 7-8 parameters, while there were only 10-15 useful data points in the backward direction). Without the mapping the results were quite unreasonable, with far too much D^* contribution needed to fit the data. It should be noted that there is some sensitivity to the nature of the mapping of the left-hand cut, and to the form of the parameterization, and that more theoretical study as well as experimental data is needed. However, it is clear that this is a most promising approach to determining the particle structure of nuclei at momenta for which dynamical calculations are not reliable.

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