W BOSON PAIRS, THEIR PHOTOPRODUCTION AND DECAY
TO LEPTONS

A. P. Sathe
Department of Physics
Columbia University, New York, N. Y.

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ABSTRACT

We have studied the photoproduction of W pairs, obtaining explicitly the amplitudes for various spin states of the W 's. We find that total cross sections grow rapidly with energy of incident photon. Also for an assumed value of the anomalous moment of the W to be equal to one, the W bosons tend to be transversely polarized. Further, the leptonic decay of the W pair is calculated including the interference from different spin states of the two W bosons. The differential cross section with respect to the transverse component of the sum of the laboratory momenta of the two decay charged leptons is obtained. It is found that even for high values of this momentum component, the differential cross section decreases with increasing assumed mass of the W boson.

Photoproduction of W pairs has been studied in the past ¹. Now, with the promise of higher energy photons from the new accelerators, interest in these calculations has been revived. In this paper we calculate contributions to the pair production cross sections from both elastic and inelastic final hadron states. Experiments are planned in which W bosons are to be detected by the characteristic momentum spectrum of the charged leptons into which they can decay. It is therefore interesting to calculate the angular distribution of the decay leptons of the W pairs including the effects of interference between different spin states of the W bosons. The calculation of this process, which we call "coherent decay" of the photoproduced W pairs, has been done here.

We find first of all that total cross sections for production of W pairs (unlike those for spin half pairs) increase strongly with the incident energy of the photons. Also, for the case anomalous moment of the W equal to one, the W pairs produced tend to be transversely polarized. The differential cross section with respect to the transverse (to the direction of the incident photon) component of the sum of the laboratory momenta of the two decay charged leptons is obtained. It is found that even for high values of this momentum component, the differential cross section decreases with increasing assumed mass of the W boson.

In studying W pair production and decay we have explicitly evaluated amplitudes for the various spin states of the two W bosons and the incident and exchanged photons. We find this method of calculation simpler and more convenient than the usual method of working with covariant expressions of absolute squares of amplitudes summed over the

spins of these particles.

In section 1 we obtain expressions which give us the amplitudes for W pair production and formulae giving differential and total cross sections; and we discuss our results in section 2 . The process "coherent decay" (referred to earlier) is handled in section 3 ; the resultant distribution in momentum space of the decay leptons is looked at in section 4 . Discussion of the actual variables used in numerical computation and other details can be found in the appendices.

1 Calculation of pair production.

Using the Feynman rules for vector boson electrodynamics³ we have^{4, 5}

$$\langle \begin{matrix} p_1, p_2, p_N \\ \epsilon_1, \epsilon_2, p_N \end{matrix} | S | k, p \rangle = i (2\pi)^4 \delta^4(p + k - p_1 - p_2 - p_N) \times \\ \sqrt{\frac{1}{V^3 8 \omega E_1 E_2}} M_\nu \langle N | j_\nu(0) | p \rangle. \quad (1.1)$$

We will include contributions only from the diagrams in Fig.(1.1). In that case

$$M_\nu = \frac{e^3}{q^2} \epsilon_{1\alpha} \epsilon_{2\beta} \epsilon_\mu [A + B + C]_{\alpha\beta\mu\nu} \quad (1.2)$$

and

$$A_{\alpha\beta\mu\nu} =$$

$$\begin{aligned} & \left[(2p_1 - k)_\mu \delta_{\alpha\gamma} - \delta_{\mu\gamma} \{ (\kappa+1)(p_1 - k) - \kappa p_1 \}_\alpha - \delta_{\mu\alpha} \{ (\kappa+1)p_1 - \kappa(p_1 - k) \}_\gamma \right] \\ & \times \left[\delta_{\gamma\delta} + \frac{(p_1 - k)_\gamma (p_1 - k)_\delta}{m^2} \right] \times \left[\frac{1}{(p_1 - k)^2 + m^2} \right] \\ & \times \left[(p_1 - p_2 - k)_\nu \delta_{\beta\delta} - \delta_{\nu\beta} \{ (\kappa+1)(-p_2) - \kappa(p_1 - k) \}_\delta - \delta_{\nu\delta} \{ (\kappa+1)(p_1 - k) + \kappa p_2 \}_\beta \right]. \end{aligned} \quad (1.3)$$

$B_{\alpha\beta\mu\nu}$ is obtained from $A_{\alpha\beta\mu\nu}$ by the interchange $(p_1, \alpha) \leftrightarrow (p_2, \beta)$

$$\text{and } C = -2 \delta_{\alpha\beta} \delta_{\mu\nu} + \delta_{\alpha\nu} \delta_{\mu\beta} + \delta_{\alpha\mu} \delta_{\nu\beta}. \quad (1.4)$$

κ is the anomalous moment of W. The notation is given in Table 1.

We evaluate the amplitudes in the rest frame of the W pair. First we choose an orthonormal basis (Fig. 1.2). The plane containing the spatial components of the momenta k, q, p_1, p_2 , is called the production plane. The plane of the paper coincides with this. The "z" axis is taken to be parallel to k , the "x" axis is taken to be perpendicular to the plane of the paper and comes out of it.

In this frame we have

$$k = (0, 0, k, ik)$$

$$q = (0, 0, -k, iE)$$

$$\epsilon'^1 = \epsilon_1^1 = \epsilon_2^1 = \epsilon^1 = (1, 0, 0, 0).$$

$$\text{Now consider } M_\nu \langle N | j_\nu(0) | p \rangle = M_\mu \delta_{\mu\nu} \langle N | j_\nu(0) | p \rangle. \quad (1.5)$$

We can replace $\delta_{\mu\nu}$ by $\sum_{\lambda=1}^4 \epsilon_\mu'^\lambda \epsilon_\nu'^\lambda$ where ϵ'^λ are a set of four

orthonormal vectors which we choose to be

$$\epsilon'^1 = (1, 0, 0, 0)$$

$$\epsilon'^2 = (0, -1, 0, 0)$$

$$\epsilon'^3 = (0, 0, E, -ik) \frac{1}{\sqrt{q^2}}$$

$$\epsilon'^4 = (0, 0, -k, iE) \frac{1}{\sqrt{q^2}}.$$

Note that ϵ'^4 is proportional to q . A vector orthogonal to $\epsilon'^1, \epsilon'^2, \epsilon'^4$ would be time like and hence ϵ'^3 has an overall factor of "i" in order to make $\epsilon'^3 \cdot \epsilon'^3 = 1$.

Now ϵ'' is proportional to q and hence by current conservation $\epsilon''_\nu \langle N | j_\nu(0) | p \rangle = 0$.

Thus (1.5) reduces to

$$\sum_{\lambda=1}^3 M_\mu \epsilon_\mu^{\lambda} \epsilon_\nu^{\lambda} \langle N | j_\nu(0) | p \rangle \quad (1.6)$$

Let us use the orthogonality of k, q, p_1, p_2 and $\epsilon, \epsilon', \epsilon_1, \epsilon_2$ respectively, and four momentum conservation to write

$$\begin{aligned} & \epsilon_{1\alpha} \epsilon_{2\beta} \epsilon_\mu \epsilon'_\nu A_{\alpha\beta\mu\nu} \\ &= \left\{ 2(p_1 \cdot \epsilon) \epsilon_{1\gamma} + (\vec{k} \cdot \epsilon_1)(\kappa+1) \epsilon_\gamma - (\epsilon \cdot \epsilon_1)(p_1 + \kappa \vec{k})_\gamma \right\} \\ & \times \left\{ \delta_{\gamma\delta} - \frac{(p_1 - \vec{k})_\gamma (p_2 - q)_\delta}{m^2} \right\} \times \left\{ \frac{1}{-2\vec{k} \cdot p_1} \right\} \\ & \times (-) \left\{ 2(p_2 \cdot \epsilon') \epsilon_{2\delta} + (q \cdot \epsilon_2)(\kappa+1) \epsilon'_\delta - (\epsilon' \cdot \epsilon_2)(p_2 + q\kappa)_\delta \right\} \\ &= \left(-V_1 \cdot V_2 + \frac{1}{m^2} S_1 S_2 \right) \frac{1}{-2\vec{k} \cdot p_1} \quad (1.7) \end{aligned}$$

where V_1 and V_2 are four vectors and S_1 and S_2 scalars.

$$V_1 = 2(p_1 \cdot \epsilon) \epsilon_1 + (\vec{k} \cdot \epsilon_1)(\kappa+1) \epsilon - (\epsilon \cdot \epsilon_1)(p_1 + \kappa \vec{k}) \quad (1.8)$$

$$V_2 = 2(p_2 \cdot \epsilon') \epsilon_2 + (q \cdot \epsilon_2)(\kappa+1) \epsilon' - (\epsilon' \cdot \epsilon_2)(p_2 + q\kappa) \quad (1.9)$$

$$S_1 = V_1 \cdot (p_1 - \vec{k}) \quad S_2 = V_2 \cdot (p_2 - q) \quad (1.10)$$

$$(1.11)$$

Observe that we have simplified our work greatly. If we had chosen to

evaluate all possible amplitudes (i.e. of various spins) we would have had $(2 \times 3 \times 3 \times 3)$ (real photon, virtual photon, W^- , W^+ respectively), that is, 54 rather complicated amplitudes (of which 27 would have been zero). Instead we have achieved a factorization and we need evaluate only the four vectors V_1 , V_2 , and the scalars S_1 , S_2 . Further since each depends on the spins of only two of the particles, there are only six different S_1 's and V_1 's and only nine V_2 's and S_2 's, some of which are very simple indeed.

The contribution from the other diagram can be obtained by a suitable interchange,

$$\left. \begin{matrix} p_1 \\ \epsilon_1 \end{matrix} \right\} \rightarrow \left\{ \begin{matrix} p_2 \\ \epsilon_2 \end{matrix} \right.$$

We note that, if $\theta \rightarrow \theta + \pi$,
 $p_1 \rightarrow p_2$
 $\epsilon_1^\lambda \rightarrow \epsilon_2^\lambda$

We can now write (1.6) as
$$\frac{\epsilon^3}{q^2} S^{\mu\nu\lambda\psi} R^\nu \quad (1.12)$$

where $R^\nu = \epsilon'^\nu_\alpha \langle N | j_\alpha(0) | P \rangle \quad (1.13)$

and $S^{\mu\nu\lambda\psi} = \epsilon^\lambda_{1\alpha} \epsilon^\psi_{2\beta} \epsilon^\mu_\gamma \epsilon'^\nu_\delta [A + B + C]_{\alpha\beta\gamma\delta} \quad (1.14)$

(N. B. μ, ν, λ, ψ as superscripts indicate the state of polarization of the real photon, virtual photon, W^- , W^+ respectively. The expression $M_\nu \langle N | j_\nu(0) | P \rangle$ of course, all along, had indices describing these polarizations, but we simply chose not to display them earlier.

We now need to take the absolute square of (1.12) and sum over spins.

$$\sum_{\mu\lambda\psi} \left| \sum_{\nu} S^{\mu\nu\lambda\psi} R^{\nu} \right|^2$$

$$= \sum_{\mu\lambda\psi} \sum_{\nu_1\nu_2} S^{\mu\nu_1\lambda\psi} (S^{\mu\nu_2\lambda\psi})^* R^{\nu_1} (R^{\nu_2})^*$$

Let us treat $R^{\nu_1} R^{\nu_2*}$ in the usual manner, average over initial proton polarization and sum over the spins and quantum numbers of whatever becomes of the initial proton, introduce the form factors and finally obtain the quantity

$$\left\{ \frac{W_2}{M^2} p \cdot \epsilon'^{\nu_1} (p \cdot \epsilon'^{\nu_2})^* + W_1 \bar{\delta}_{\nu_1\nu_2} \right\} \quad \text{which}$$

we choose to call $W^{\nu_1\nu_2}$. (1.17)

(Please see footnote 6, appendix A1, and footnote 7.)

In obtaining an expression for the cross section for W pair production we have to integrate over the four momentum p_N (i.e. whatever becomes of the proton).

We change variables here and write $\int d^4 p_N = \frac{\pi}{2\omega} \int dq^2 d\nu d(-P^2)$ (1.18)

where $-P^2$ is the square of the energy of the two W's in their rest system.

In the case where the target proton is not excited by the collision we have

$$\int d^3 p' \frac{M}{E'} = \frac{\pi}{2\omega} \int dq^2 d(-P^2) \quad (1.19)$$

To integrate over the momenta of the W's, we start with

$$\int \frac{d^3 p_1}{2E_1} \frac{d^3 p_2}{2E_2} \delta^4(p + k - p_1 - p_2 - p_N) \quad (1.20)$$

In the usual fashion this is written as
$$\int d^4 p_1 d^4 p_2 \delta(p_1^2 + m^2) \delta(p_2^2 + m^2) \delta^4(\dots) . \quad (1.21)$$

We change variables to $P = p_1 + p_2$ and $q' = (p_1 - p_2)/2$

and get
$$\int d^4 P d^4 q' \delta(p_1^2 + m^2) \delta(p_2^2 + m^2) \delta^4(\dots) . \quad (1.22)$$

$\int d^4 P$ is done using the $\delta^4(p + k - p_N - P)$.

The remaining expression
$$\int \delta(p_1^2 + m^2) \delta(p_2^2 + m^2) d^4 q' = \frac{1}{2} \int \delta\left(\frac{P^2}{4} + m^2 + q'^2\right) \delta(P \cdot q') |\vec{q}'|^2 d|\vec{q}'| dq'_0 d\theta_{q'} d\phi_{q'} \sin \theta_{q'} . \quad (1.22)$$

We use the first delta function to do $\int d|\vec{q}'|$ and the second to do $\int dq'_0$ and obtain

$$\frac{|\vec{q}'|}{4 P_0} \int d\theta_{q'} d\phi_{q'} \sin \theta_{q'} , \quad (1.23)$$

where $\theta_{q'}$ and $\phi_{q'}$ are angular variables defined below.

$\theta_{q'}$ is taken to be the same as the θ defined in Fig. (1.2). To define $\phi_{q'}$,

we first choose an orthonormal basis obtained from that defined at the beginning of this section

by a rotation about the z axis. $\hat{x}_p, \hat{y}_p, \hat{z}_p$ are the new basis vectors. The $\hat{x}_p, \hat{z}_p$ plane

is chosen to be that containing the initial momentum of the proton (which is not at

rest in the rest system of the W pair). Then ϵ^i , the polarization vector of

the incident photon perpendicular to the production plane is given by

$$\epsilon^i = \hat{x}_p \cos \phi_{q'} + \hat{y}_p \sin \phi_{q'} . \quad (1.24)$$

Having completed all these parts we are in a position to put everything

together and write

$$d\sigma_{inel} = \frac{e^6}{q^4} \frac{1}{64\pi^3} \frac{2\pi}{4\pi^2\omega^2} \frac{1}{4} \frac{|\vec{q}'|}{P_0} d\nu d(-P^2) dq^2 []_{inel} d\theta_{q'} \sin\theta_{q'} d\phi_{q'} \quad (1.25)$$

where $d\sigma_{inel}$ is the contribution to the differential cross section from the final states in which the target proton is excited

and

$$d\sigma_{el} = \frac{e^6}{q^4} \frac{1}{64\pi^3} \frac{2\pi}{4\pi^2\omega^2} \frac{1}{4} \frac{|\vec{q}'|}{P_0} d(-P^2) dq^2 []_{el} d\theta_{q'} \sin\theta_{q'} d\phi_{q'} \quad (1.26)$$

where $d\sigma_{el}$ is the contribution to the cross section from those final states in which the proton is not excited by the collision;

where $[]_{inel}$

$$= S^{\mu\nu\lambda\psi} (S^{\mu\nu\lambda\psi})^* \left\{ \frac{p \cdot \epsilon'^{\nu 1} (p \cdot \epsilon'^{\nu 2})^*}{M^2} W_2 + \delta^{\nu 1 \nu 2} W_1 \right\} \quad (1.27)$$

and $[]_{el}$ is the same expression but with W_2 replaced by $\frac{G_E^2 + \frac{q^2}{4M^2} G_M^2}{1 + q^2/4M^2}$ and W_1 by $\frac{G_M^2 q^2}{4M^2}$, G_E, G_M being the form factors of the proton.

Let us now do $\int d\phi_{q'}$. Note S is independent of $\phi_{q'}$ and so is $p \cdot \epsilon'^3$

This gives rise to the simplification that cross terms between unequal ν 's vanish.

$$\int (p \cdot \epsilon'^{\nu})^2 d\phi_{q'} = 2\pi \frac{p_\perp^2}{2}, \quad \nu = 1, 2 \quad (1.29)$$

where $p_\perp$ is the transverse component of the momentum p as seen in the

rest system of the W pair. ("Transverse" means perpendicular to the momentum of the incident photon which is along the z axis .) (Fig. 1.2)

Notice that we have had as yet no need to sum over the polarizations of the W 's . Our method thus allows us to investigate quite simply the contributions of the longitudinal and transverse² polarizations of the W 's .

We now write expressions for the total cross section for W pair production

$$\text{as } \sigma_{\text{inel}} = \frac{\alpha^3}{m^2} \int d\nu \, dq^2 \, d(-P^2) \, \frac{1}{4m^2} \frac{|\vec{q}'|}{P_0} \int d\theta' \sin\theta_{q'} \times$$

$$\times \sum \frac{m^4}{q^4 \omega^2} \left[\frac{W_2}{M^2} \left\{ \frac{P_T^2}{2} \sum_{\nu=1}^2 |S^{\mu\nu\lambda\psi}|^2 + |p \cdot \epsilon'^3|^2 |S^{\mu 3 \lambda \psi}|^2 \right\} + W_1 \left(\sum_{\nu=1}^2 |S^{\mu\nu\lambda\psi}|^2 - |S^{\mu 3 \lambda \psi}|^2 \right) \right] . \quad (1.30)$$

$\sum$ means averaged over incident photon spin and summed over W spins
(1.31)

A similar expression_A can be written for σ_{el} , the only difference being that there is no $\int d\nu$ and W_2 and W_1 are replaced by expressions (1.28) .

$$\sigma_t = \sigma_{\text{el}} + \sigma_{\text{inel}} . \quad (1.32)$$

Section 2 . Results of W pair production

Let us look at the results of our numerical integration. (Some discussion of the procedure and details is given in Appendices A2 and A3.) First of all we mention that we compared our results with those available in the literature. We computed what we have called σ_{el} and which corresponds to the cross section in Table 1 of the paper by Berman and Tsai¹. The agreement with the figures quoted there was very good ($\sim 1\%$). We also compared our results with those of Wu, Wu, and Fuchel¹: The agreement here was not as good ($\sim 2 - 3\%$), but we have some doubts about the exact form factors they used.

The total cross section (i. e. $\sigma_t = \sigma_{el} + \sigma_{inel}$) is given in Table 2 and Fig. 2. 1 for mass of W equal to 3 GeV and 7 GeV, anomalous moment zero and one and various values of incident photon energy. We note that cross sections rise rapidly with incident photon energy quite unlike cross sections for production of spin half pairs, which rise only logarithmically. Some attempt is made to pin down this difference in Appendix A2.

Next we see the results from a study of the polarizations of W's . (Table 3) Here " transversely polarized" W means a W whose polarization vector is perpendicular to its momentum in the rest frame of the W pair. We have tabulated σ_{tr}, σ_{ln} which are the cross sections for producing W's which are both transverse and both longitudinal respectively. Note that,

for anomalous moment of the W equal to one, the W 's tend to be transversely polarized. Also at the higher energies, for anomalous moment zero, the W 's are mostly either both longitudinal or both transverse.

Finally we have Table 4 which tells us what fraction of the contribution to the total cross section comes from the inelastic final states of the hadron. We observe that their contribution becomes more important as the energy of the incident photon rises. Also, it is more important for the case anomalous moment of the W equal to one than for the case anomalous moment zero.

Section 3 Calculation of W pair decay

Having studied the cross section for W pair production we turn to the problem of calculating the angular distribution of the decay leptons including the interference effects from different spin states of the W's, or what we earlier called "coherent decay" of the W pair.

We shall start by studying the decay of a single W^- with polarization vector ϵ_λ^λ . We wish to obtain the amplitude K^λ for it to decay with the charged lepton going in the θ_1, ϕ_1 direction in the rest frame of the W.

$$K^\lambda = \kappa \epsilon_\mu^\lambda \bar{u} \gamma_\mu (1 + \gamma_5) v \quad (3.1)$$

where κ is a constant which will be later determined to be $\frac{1}{2} \sqrt{\frac{3}{8\pi}}$. We evaluate

the amplitude in the rotated frame in which the charged lepton goes in the $+z$

direction and the antineutrino in the $-z$ direction. Since $m \gg m_e$ or m_μ ,

it is a good approximation to treat electron and muon as if they were massless.

This leads to the important simplification that, of the four helicity amplitudes corresponding to the two helicities each of the charged lepton and the neutrino/antineutrino, only one amplitude is non zero — the one in which the particle is left handed and the antiparticle is right handed.

In terms of the angles θ_1, ϕ_1 , the vector K is obtained as

$$K^1 = 2\kappa (-\sin\phi_1 - i \cos\theta_1 \cos\phi_1) \quad (3.2)$$

$$K^2 = 2\kappa (\cos\phi_1 - i \cos\theta_1 \sin\phi_1) \quad (3.3)$$

$$K^3 = 2\kappa i \sin\theta_1 \quad (3.4)$$

$$\kappa = \frac{1}{2} \sqrt{\frac{3}{8\pi}} \quad (3.5)$$

is obtained by demanding that $\int |K^\lambda|^2 d\Omega = 1 \quad (3.6)$

for any λ .

Similarly we can write out the matrix element for the decay of W^+ , noting that now u is the spinor for the neutrino and v for the charged lepton which is now an antiparticle.

We are now ready to consider the coherent decay of the W pair. To do this we contract the tensor $S^{\mu\nu\lambda\psi}$ (it is a tensor in spin space) with the decay tensor for the two W 's, $K^\lambda(\theta_1, \phi_1) K^\psi(\theta_2, \phi_2)$. Thus the amplitude for the combined process of production and decay is

$$\frac{e^3}{q^2} S^{\mu\nu\lambda\psi} K^\lambda(\theta_1, \phi_1) K^\psi(\theta_2, \phi_2) R^\nu \quad (3.7)$$

$$\text{We can define } Z^{\mu\nu} = S^{\mu\nu\lambda\psi} K^\lambda K^\psi \quad (3.8)$$

and the absolute square of (3.7) becomes

$$\frac{e^6}{q^4} Z^{\mu\nu_1} (Z^{\mu\nu_2})^* R^{\nu_1} R^{\nu_2 *} \quad (3.9)$$

The expression for the differential cross section for the decay products

is

$$\frac{d\sigma_{inel}}{B^2} = \frac{\alpha^3}{16\pi^2} \cdot \frac{1}{\omega^2} \cdot 2\pi d\nu dq^2 d(-p^2) \frac{1}{q^4} \frac{|\hat{q}'|^2}{P_0} d\phi_{q'} d\theta_{q'} \sin\theta_{q'}$$

$$\times \frac{1}{2} \sum_{\mu\nu 1}^2 Z^{\mu\nu 1} (Z^{\mu\nu 2})^* W^{\nu 1 \nu 2} \left(\frac{3}{8\pi}\right)^2 d\theta_1 d\phi_1 \sin\theta_1 d\theta_2 d\phi_2 \sin\theta_2$$

(3.10)

where B is the branching ratio for a single W to decay into a muon and a neutrino.

$d\sigma_{el}$ is as usual the same expression but with the $\int d\nu$ omitted and the substitution used in (1.28).

$$d\sigma = d\sigma_{el} + d\sigma_{inel} \quad (3.11)$$

Let us now approach our final task, that of studying the distribution of the decay products in momentum space. As mentioned earlier we are particularly interested in the distribution in $(p_3 + p_4)_t^{lab}$. ("Transverse" here means perpendicular to the incident photon direction in the lab.) We do not have an analytical expression for $\frac{d\sigma}{d(p_3 + p_4)_t^{lab}}$. Instead we consider the possible range $(p_3 + p_4)_t^{lab}$. It certainly has to be less than $\frac{\sqrt{s}}{2}$ where $\sqrt{s}$ is the total energy in the rest system of the incident photon and target proton. We divide this region into a certain number of bins (say thirty or forty) and obtain the contribution to the cross section coming from the final states belonging to each bin. We thus use a histogram to describe $\frac{d\sigma}{d(p_3 + p_4)_t^{lab}}$. Inspiration to do so comes from the experimentalists.

One remark : Table 1 indicates that we have chosen one of the charged decay leptons to be an electron and the other to be a muon. The choice is a deliberate one — we wish to calculate cross sections for a process in which the final detectable particles cannot arise from a simpler process like electron pair production or muon pair production.

Section 4. Results : Momentum space distribution of decay products

Obtaining $\frac{d\sigma}{d(p_3 + p_4)_t^{lab}}$ numerically is a formidable task. It involves an integration over eight or nine dimensions. This is discussed in Appendix A4.

Let us look at the results. (Figures 4.1 to 4.4)

We note that $\frac{d\sigma}{d(p_3 + p_4)_t^{lab}}$ begins to fall quite rapidly around $(p_3 + p_4)_t^{lab}$ equal to m . This is what one would expect. Also we notice something important.

For a given energy of incident photon and assumed anomalous moment of W , let

us look at $\frac{d\sigma}{d(p_3 + p_4)_t^{lab}}$ for different assumed values of W mass, 3 GeV and 7 GeV .

The differential cross section for mass 3 GeV is everywhere greater than that corresponding to mass 7 GeV.

For the anomalous moment of the W equal to one we got the following result for $\frac{d\sigma}{d(p_3 + p_4)_t^{lab}}$ for the higher values of $(p_3 + p_4)_t^{lab}$. For the case in which the W mass is assumed to be 3 GeV, the main contribution to the differential cross section comes from inelastic final hadron states, and for the case W mass equal to 7 GeV, the contributions to the differential cross section from elastic and inelastic final states are roughly equal. (Of course, there is a small region near the highest possible $(p_3 + p_4)_t^{lab}$ to which only the elastic final state of the hadron can contribute ; and the above remark does not obviously apply to that region and to the region near it .)

We have here computed only $\frac{d\sigma}{d(p_3 + p_4)_t^{\text{lab}}}$ but our approach is sufficiently general and very little more work would be required to study distributions in some other variables e. g., $(p_3)_t^{\text{lab}}$ alone or say the invariant mass of the two charged leptons, etc.

It is our hope that this work will be useful in studying the results of experiments now contemplated at N. A. L. to search for the elusive W bosons. ⁸

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Appendix A1

The expression (1.17) looks somewhat ugly because of the presence of $(p \cdot \epsilon')^*$. This happened because ϵ'^3 had an overall factor of i in order to have $\epsilon'^3 \cdot \epsilon'^3 = 1$ though ϵ'^3 is time like.

In our work we found it convenient to define $\tilde{\epsilon}$, $\bar{\epsilon}$ such that

$$\tilde{\epsilon}^\lambda = \bar{\epsilon}^\lambda = \epsilon' \text{ for } \lambda = 1, 2 \text{ \& } i\tilde{\epsilon}^3 = \frac{\bar{\epsilon}^3}{i} = \epsilon'^3. \quad (\text{A1.1})$$

$$\text{Then (1.6)} \rightarrow \sum M_\mu \tilde{\epsilon}_\mu^\lambda \bar{\epsilon}_\nu^\lambda \langle N | j_\nu(0) | p \rangle \quad (\text{A1.2})$$

$$(1.14) \rightarrow S^{\mu\nu\lambda\psi} = \epsilon_{1\alpha}^\lambda \epsilon_{2\beta}^\psi \epsilon_\gamma^\mu \tilde{\epsilon}_\delta^\nu [A + B + C]_{\alpha\beta\gamma\delta} \quad (\text{A1.3})$$

$$\text{and (1.30)} \quad \sigma_{\text{inel}} = \frac{\alpha^3}{m^2} \int d\nu dq^2 d(-P^2) \frac{1}{4m^2} \frac{|\vec{q}'|}{P_0} \int d\theta' \sin\theta_{q'}.$$

$$\begin{aligned} & \sum \frac{m^4}{q^4 \omega^2} \left[\frac{W_2}{M^2} \left\{ \frac{P_t^2}{2} \sum_{\nu=1}^2 (S^{\mu\nu\lambda\psi})^2 + (p \cdot \bar{\epsilon}^3)^2 (S^{\mu 3 \lambda \psi})^2 \right\} \right. \\ & \left. + W_1 \left\{ \sum_{\lambda=1}^2 (S^{\mu\nu\lambda\psi})^2 - (S^{\mu 3 \lambda \psi})^2 \right\} \right]. \quad (\text{A1.4}) \end{aligned}$$

Similarly we can do a little more work on equation (3.10). We start with (3.7). We note that $K^\lambda K^\psi$ is complex and we break it up into its real and imaginary parts.

$$K^\lambda(\theta_1, \phi_1) K^\psi(\theta_2, \phi_2) = L^{\lambda\psi}(\theta_1, \phi_1, \theta_2, \phi_2) + iN^{\lambda\psi}(\theta_1, \phi_1, \theta_2, \phi_2). \quad (\text{A1.5})$$

Then (3.9) $\rightarrow$

$$\frac{e^6}{q^4} \left\{ S^{\mu\nu 1 \lambda 1 \psi 1} S^{\mu\nu 2 \lambda 2 \psi 2} R^{\nu 1} R^{\nu 2 *} \right\} (L^{\lambda 1 \psi 1} + iN^{\lambda 1 \psi 1})(L^{\lambda 2 \psi 2} - iN^{\lambda 2 \psi 2}). \quad (\text{A1.6})$$

After we have subjected $R^{\nu 1} R^{\nu 2 *}$ to the manipulation (1.17), introduced

W_2, W_1 etc., the entire expression in the curly brackets in (A1.6) is now symmetric under the interchange $\{\lambda_1 \psi_1\} \leftrightarrow \{\lambda_2 \psi_2\}$. The cross terms between

L and N now vanish. Define

$$H^{\mu\nu} = S^{\mu\nu\lambda\psi} L^{\lambda\psi} \quad (\text{A1.7})$$

$$I^{\mu\nu} = S^{\mu\nu\lambda\psi} N^{\lambda\psi} \quad (\text{A1.8})$$

$$(3.9) \rightarrow \frac{e^6}{q^4} \{ H^{\mu\nu_1} H^{\mu\nu_2} + I^{\mu\nu_1} I^{\mu\nu_2} \} R^{\nu_1} R^{\nu_2} \quad (\text{A1.9})$$

and (3.10) $\rightarrow$

$$\begin{aligned} & \frac{\alpha^3}{16\pi^2} \frac{1}{\omega^2} 2\pi d\nu dq^2 d(-P^2) \frac{1}{q^4} \frac{|\vec{q}'|}{P_0} d\phi_{q'} d\theta_{q'} \sin\theta_{q'} \\ & \times \{ H^{\mu\nu_1} H^{\mu\nu_2} + I^{\mu\nu_1} I^{\mu\nu_2} \} \{ W^{\nu_1\nu_2} \} \\ & \times \frac{1}{2} \left(\frac{3}{8\pi} \right)^2 d\theta_1 d\phi_1 \sin\theta_1 d\theta_2 d\phi_2 \sin\theta_2 \quad (\text{A1.10}) \end{aligned}$$

Appendix A2

Let us study equations (1.30) and (1.31). It turns out that the variables $-p^2$ and q^2 are very convenient to work with.

Define $2W = \sqrt{-p^2}$, that is W is the energy of each W boson

in the rest system of the W pair

$$k + E = 2W \quad (\text{A2.1})$$

$$k = W + q^2/4W \quad (\text{A2.3})$$

$$k^2 - E^2 = q^2 \quad (\text{A2.2})$$

$$E = W - q^2/4W. \quad (\text{A2.4})$$

Let the momentum of the proton in this system be $(\vec{p}_t, p_z, ip_0)$.

We can write down the scalar products $p \cdot k$ and $p \cdot q$ in the lab frame and the

and the rest system of the W pair and thus obtain

$$p_0 = \frac{M(\omega - \nu)}{k + E} \quad (\text{A2.5})$$

$$p_z = p_0 - \frac{M}{k} \omega \quad (\text{A2.6})$$

Then p_t of equation (1.29) can be obtained from the known $p^2 = -M^2$.

We get

$$\frac{p_t^2}{M^2 \omega^2} = \frac{k - E - (2\nu k/\omega)}{k^2 (k + E)} - \frac{1}{\omega^2} \quad (\text{A2.7})$$

$$= \frac{4q^2}{(q^2 - p^2)^2} \left\{ 1 - \frac{\nu}{\omega} \frac{q^2 - p^2}{q^2} - \frac{1}{4} \frac{(q^2 - p^2)^2}{\omega^2 q^2} \right\} \quad (\text{A2.8})$$

$$\frac{|p \cdot \epsilon'^3|^2}{M^2 \omega^2} = \frac{1}{q^2} \left(1 - \frac{E}{k} - \frac{\nu}{\omega} \right)^2 = \frac{4q^2}{(q^2 - p^2)^2} \left[1 - \frac{\nu}{\omega} \frac{q^2 - p^2}{2q^2} \right]^2. \quad (\text{A2.9})$$

$$(\text{A2.10})$$

We have taken the trouble to write out all these detailed expressions as

they both show the dominant behaviour $\frac{q^2}{(q^2 - p^2)^2}$. The factors multiplying this

are of the order of 1 . They drop as the available phase space becomes limited. The expressions for the cross sections have thus been factored. We have isolated (i) the part containing the amplitude for pair production by the interaction between two photons and (i i) the part which deals with the interaction between the exchanged photon and the proton. We can then compare the photoproduction of spin one pairs to that of spin half or spin zero pairs. Part (i i) is common to all three processes. Part (i) integrated over θ_q rises spectacularly with $-P^2$ and q^2 for spin one particles quite unlike that for spin half particles.

Appendix A3

We have now a formula for cross section as an integral over $-P^2$ and q^2 .

$$(1.30) \quad \text{The limits of integration for } -P^2 \text{ are } 4m^2 \text{ to } (\sqrt{s} - M)^2$$

$$\text{where } s = M^2 + 2M\omega. \quad (A3.1)$$

For a given value of $-P^2$, the limits on q^2 can be found as follows.

$$\text{We have } p_t'^2 = q^2 - p_l'^2 + \nu^2 \quad (A3.2)$$

where p' is the lab. momentum of the recoiling proton and p_t' and p_l' are components perpendicular and parallel respectively to the incident photon

direction in the lab. We also have

$$p_l' - \nu = \frac{q^2 - P^2}{2\omega} \quad (A3.3) \quad \nu = \frac{q^2}{2M}. \quad (A3.4)$$

Now putting $p_t'^2 = 0$ yields a quadratic equation whose roots are the upper and lower limits of q^2 . This will do for σ_{el} . In σ_{inel} , $\nu \neq \frac{q^2}{2M}$. However we retain the same limits of integration on q^2 . Now the domain of q^2 will include regions which are kinematically forbidden. We arrange to have the integrand set equal to zero there, as will be made clear below.

First we wish to obtain the upper and lower limits on ν for a fixed q^2 and $-P^2$. We can obtain a lower limit by demanding that enough energy be transferred to the proton to produce the lowest possible inelastic state.

The upper limit is found by putting $p_t'^2 = 0$. (This time we know $-P^2$ and q^2

and therefore y is the only unknown.) It can happen that for q^2 near $q_{\max}^2$ or $q_{\min}^2$ that $y_{\max}$ is less than $y_{\min}$. When this happens we know that we are in a forbidden region and we set the integrand zero as mentioned above.

Only a small fraction of the final states are thus rejected as kinematically forbidden. The great advantage of this procedure is that the amplitudes S have already been evaluated in obtaining the integrand for σ_{el} and there is consequently a great saving of computer time. Another advantage is that we know that the elastic contribution is significant only for low q^2 whereas the inelastic contribution becomes significant beyond q^2 around 0.2 or so. Then since we are interested primarily in $\sigma_{el} + \sigma_{inel}$ we are effectively integrating a function which is somewhat more smooth in q^2 than that entering either σ_{el} or σ_{inel} .

The integrations are done using a Monte Carlo method with a sequence of pseudo-random numbers.

Earlier we noted the presence of the dominant term

$$q^2/(q^2 - p^2)^2 \quad \text{from } p_t^2 \quad \text{and } |p \cdot \epsilon^{13}|^2 \quad (\text{A2.8}) \text{ and } (\text{A2.10})$$

and of course we have $1/q^4$ from the photon propagator. We choose the variables of integration so that this dominant behaviour is cared for. Instead of q^2 take y where $q_{\min}^2 e^y = q^2$ and $z = 4m^2/(-P^2)$ instead of $-P^2$. This gives us extra factors of q^2 and $(-P^2)^2$ in the numerator which will smooth out the integrand.

Notice that we have ignored the dependence of S^2 on q^2 and $-p^2$. We do know that it does rise considerably with q^2 and $-p^2$ but we are investigating cross sections with photon energies not too far from the threshold for producing the W pairs. Thus the contribution to the cross section from high values of $-p^2$ and q^2 will be rapidly cut down by the unavailability of phase space. If we investigated ω such that $2M\omega \gg m^2$ we would have to reconsider our choice of variables for integration.

For the ν integration we used a variable x , $\nu = \nu_{\min} e^x$. This introduces a needed factor of ν . (We know that for high q^2 νW_2 is smoother than W_2 alone.)

Appendix A4

The integrals involved in obtaining $\frac{d\sigma}{d(p_3 + p_4)_t}^{\text{lab}}$ are indeed formidable.

We need more variables to describe the decay than we needed in W pair production.

These extra variables are chosen to be $\theta_1, \phi_1, \theta_2, \phi_2$. The reason for our choice is that the integrand as a function of these variables does not peak sharply anywhere. Also the dependence of $(p_3 + p_4)_t^{\text{lab}}$ on these variables is mild. Their ranges are simple too.

We are interested in the region $(p_3 + p_4)_t^{\text{lab}}$ large. Contributions to this come from $-p^2$ large. This region does not contribute much to the total cross section. We suppressed it by using the variable z instead of $-p^2$. Here therefore we use the variable $-p^2$ itself.

With the variables chosen thus we obtained reasonable results without using an excessive amount of computer time. It took about 15 minutes of computer time on the IBM 360/91 (for 2.5×10^5 points of integration) for each of the seven histograms displayed in figures 4.1 to 4.4. The accuracy is around 1-5% for about 60% of the bins. It deteriorates with increasing $(p_3 + p_4)_t^{\text{lab}}$ and for the last 6 bins the accuracy is 20% or even worse.

Footnotes

1. S. M. Berman and Y. S. Tsai , Phys. Rev. Letters 11, 483 (1963) .
A.C.T. Wu , T. T. Wu and K. Fuchel Phys. Rev. Letters 11, 390 (1963) .
2. This term is explained in Section 2 .
3. T. D. Lee and C. N. Yang , Phys. Rev. 128 , 885 (1962) .
4. We have used the Pauli metric , a four vector momentum p is represented as (p_x, p_y, p_z, ip_0) .
Lorentz indices appear only as subscripts. Superscripts are reserved for indicating what spin states the particles have or what Lorentz frame is used.
5. Our notation is generally that of David Lurie , "Particles and Fields " , Interscience Pub. (1968) .
6. $\bar{\delta}^{\nu_1\nu_2} = \delta^{\nu_1\nu_2}$ for $\nu_1 = 1, 2$
 $\bar{\delta}^{\nu_1\nu_2} = -\delta^{\nu_1\nu_2}$ for $\nu_1 = 3$
We get $\bar{\delta}^{\nu_1\nu_2}$ instead of $\delta^{\nu_1\nu_2}$ because ϵ'^3 had an overall factor i , which made its fourth component real and the first three components imaginary , unlike normal vectors in our metric in which the first three components are real and the fourth is imaginary .
7. Form factors W_2 and W_1 : for $q^2 \leq 1.0$ we have used interpolations from the preliminary working data from SLAC-MIT collaboration (experiment described in the doctoral theses of Guthrie Miller and Martin Briedenbach). For $q^2 > 1.0$ we have

used a parametric fit given in equation 6 of Miller et al., Phys. Rev. D 5, 528 (1972).

A dipole fit is used for G_E and G_M .

$$G_E = \frac{1}{\left(1 + \frac{q^2}{0.71}\right)^2} \quad (q^2, \text{in GeV})$$

$$G_M = 2.79278 G_E$$

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Table 1

Particle/ system	Four momentum	Energy in lab.	Polarization vector	Invariant mass
incident photon	k	ω	ϵ	
W^-	p_1	E_1	ϵ_1	m
W^+	p_2	E_2	ϵ_2	m
exchanged virtual photon	q	$-\nu$	ϵ'	
target proton	p	M		M
final inelastic hadron state	p_N	$M + \nu$		
final elastic hadron state	p'	E'		M
decay electron	p_3			
decay muon (μ^+)	p_4			
W pair	P			$2W$

Table 2.

ω in GeV	σ_t in units of 10^{-34} cm^2			
	$\kappa = 0$		$\kappa = 1$	
	$m = 7 \text{ GeV}$	$m = 3 \text{ GeV}$	$m = 7 \text{ GeV}$	$m = 3 \text{ GeV}$
50		0.00591		0.0438
70.7		0.0283		0.191
100		0.0888		0.529
141.4	1.76×10^{-6}	0.223	1.34×10^{-5}	1.15
200	1.79×10^{-4}	0.485	0.00136	2.17
282.8	0.00168	0.956	0.0121	3.67
400	0.00721	1.75	0.0474	5.76
565.7	0.0215	3.05	0.125	8.55
800	0.0524	5.07	0.265	12.1
1600	0.217	12.8	0.813	22.1
3200	0.678	29.5	1.85	37.3
6400	1.79	64.7	3.53	60.8

Table 3.

Anomalous moment	ω in GeV	Cross sections in units of 10^{-34} cm^2 for W boson mass = 3 GeV		
		σ_{tr}	σ_{in}	σ_{t}
0	50	0.00288	0.00168	0.00591
	200	0.204	0.241	0.485
	800	1.76	3.03	5.07
	1600	4.07	8.05	12.8
	6400	18.3	44.0	64.7
1	50	0.0268	0.00537	0.0438
	200	1.61	0.162	2.17
	800	9.41	0.665	12.1
	1600	16.8	1.18	22.1
	6400	39.4	4.38	60.8

Table 4

ω in GeV	$\frac{g_{\kappa}}{g_{\pi}}$			
	m = 3 GeV		m = 7 GeV	
	$\kappa = 0$	$\kappa = 1$	$\kappa = 0$	$\kappa = 1$
50	0.268	0.279		
70.7	0.327	0.343		
100	0.367	0.388		
141	0.396	0.421	0.136	0.138
200	0.418	0.443	0.315	0.318
283	0.438	0.463	0.382	0.392
400	0.453	0.479	0.427	0.441
566	0.467	0.492	0.457	0.474
800	0.477	0.508	0.479	0.501
1600	0.501	0.542	0.512	0.532
3200	0.523	0.586	0.535	0.553
6400	0.541	0.636	0.549	0.579

Figure Captions

Figure 1.1 Feynman Diagrams for pair production.

Figure 1.2 Momenta of the two W 's, and the real and virtual photons in the rest system of the W pair.

Figure 2.1 Plot of σ_t (cross section for photoproduction of W pairs including contributions from both elastic and inelastic final states) against ω , energy of the incident photon in the lab. for different assumed values of anomalous moment, κ , and mass of the W , m .

----- for $\kappa = 1$

————— for $\kappa = 0$

Figures 4.1 4.4 . Histograms for $\frac{d\sigma}{d(p_3 + p_4)_t^{lab}} \times \frac{1}{B^2}$, the differential cross sections with respect to the transverse component of the sum of the laboratory momenta of the two charged leptons into which the W 's decay, against $(p_3 + p_4)_t^{lab}$, this momentum component, for various ω , incident photon energy, and assumed values κ , and m , the anomalous moment and mass of W respectively.

————— for $m = 3$ GeV

..... for $m = 7$ GeV

Fig. 4.1 $\kappa = 1$, $\omega = 100$ GeV

Fig. 4.2 $\kappa = 1$, $\omega = 300$ GeV

Fig. 4.3 $\kappa = 1$, $\omega = 500$ GeV

Fig. 4.4 $\kappa = 0$, $\omega = 300$ GeV

Fig. 1.1

Fig. 1.2

Fig. 2.1

Fig. 4.1

Fig. 4.2

Fig. 4.3

Fig. 4.4