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AMBIGUITIES IN R-MATRIX PARAMETERS FOR INTERFERING RESONANCES WITH
TWO FISSION CHANNELS.*

MASTER

D. B. Adler and F. T. Adler

Department of Physics

University of Illinois

Urbana-Champaign

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AMBIGUITIES IN R-MATRIX PARAMETERS FOR INTERFERING RESONANCES WITH TWO FISSION CHANNELS.

The parametrization of fissile element cross sections has been studied widely, lately also with regard to uniqueness of the R-matrix representations.^{1/} Because the cross section expansions contain non-linearly the partial width amplitudes of R-matrix theory, non-uniqueness of these parameters may be expected when analyzing cross section curves.

On the other hand, the same cross section curves are equivalently represented by multilevel parameters,^{2/} which are nonlinear functions of the R-matrix parameters, but enter linearly in the cross section expansions. Once the poles of the collision matrix are specified, these parameters are thus uniquely related to the cross section curves.

The same relations between the two types of parameters can be used to investigate whether the "false vectors" of Auchampaugh^{1/} are an effect of least squares analysis, or whether they represent an inherent non-uniqueness of the R-matrix parameters.

(The results summarized here, as those of Auchampaugh,^{1/} are obtained assuming N well-spaced interfering resonances (of the same spin state), having constant Γ_γ , with two fission channels.) For this case, only $N-1$ independent channel space angles define the interferences through the matrix $\Gamma_{\lambda\lambda}^f$, which is thus of rank 2. Because for well-spaced resonances $|E_\lambda - E_{\lambda'}| \gg |\Gamma_{\lambda\lambda}^f|$ a perturbation procedure can be applied to simplify the problem.^{3,4/}

Within perturbation, it is possible to transform the original matrix Γ^f , leaving unchanged reaction widths $\Gamma_\lambda^{(x)}$ and resonance energies E_λ , so that the N-1 angles in channel space are the only parameters undergoing transformation. Within perturbation, also, the symmetrical line shape parameters α , G , depending only on $\Gamma_{\lambda\lambda}^f$, in second order, can be treated as constants, while the asymmetrical parameters β, H^F vary linearly with $\Gamma_{\lambda\lambda}^f$:

$$\beta_k = 2 \sqrt{\Gamma_{kn}^o} \sum_{\lambda \neq k} \frac{\sqrt{\Gamma_{\lambda n}^o} (\Gamma_{\lambda\lambda}^f + \sqrt{\Gamma_{\lambda n} \Gamma_{kn}})}{E_\lambda - E_k} \quad (1)$$

$k = 1, N$

Solutions for $\Gamma_{\lambda\lambda}^f$, different from the original, are found imposing the condition that each β_k remains constant while varying $\Gamma_{\lambda\lambda}^f$. Rigid body rotations in channel space and reflections (minor-solutions^{1/}) do not change $\Gamma_{\lambda\lambda}^f$, and are therefore discarded.

Numerically, the solutions are obtained by constructing "iso-beta" surfaces (of equal β_k) in the N-1-phase space and determining their intersections. For N=3 the problem reduces to define the intersections of iso-beta curves in a plane. Three numerical examples are given in Table I showing one angle set independent from the initial one in each case.

Aside from the conclusions reported by Auchampaugh^{1/} the non-uniqueness of the fission width amplitudes may have consequences in the statistical study of their distribution, but does not affect the distributions of the reaction widths, nor the statistical implications obtained from them. In conclusion, the apparent non-uniqueness of the R-matrix parameters seems to be inherent to formalism and not a consequence of least squares procedures.)

Table I

Results for R-Matrix Parameters From a Study of Three Resonances with Two Fission Channels Corresponding to a Prescribed Set of Multilevel Parameters.

Input Parameters ^(a)								Calculated Solutions ^(b)		
Problem	λ	E_λ (eV)	$\Gamma_{\lambda\gamma}$ (eV)	$2g\Gamma_{\lambda n}^0$ (eV) ^{1/2}	$\Gamma_{\lambda 1}$ (eV)	$\Gamma_{\lambda 2}$ (eV)	θ_λ (°)	$\theta_\lambda^{(c)}$	β_λ (eV) ^{1/2}	
Problem 1	1	141.726	.04	3.562 10^{-3}	.0363	.2732	214°	257°	-.528	10^{-4}
	2	149.155	.04	.344 10^{-3}	.4959	.0206	-88°	134°	-.290	10^{-4}
	3	155.322	.04	1.866 10^{-3}	.0524	.9310	0.0	0.0	.818	10^{-4}
Problem 2	1	141.726	.04	3.562 10^{-3}	-.0363	.2732	33°.4	82°.5	.610	10^{-4}
	2	149.155	.04	.344 10^{-3}	.4959	-.0206	272°.0	35°.0	.341	10^{-4}
	3	155.322	.04	1.866 10^{-3}	.0524	.9310	0.	0.	-.951	10^{-4}
Problem 3	1	141.7	.04	3.66 10^{-3}	-.0363	.2732	-8°.47	277°.7	.768	10^{-4}
	2	149.1	.04	.340 10^{-3}	.4959	-.0206	230°	290°.5	-.274	10^{-4}
	3	155.3	.04	1.86 10^{-3}	-.2222	.7539	0.0	0.0	-.494	10^{-4}

(a) The input parameter values used here correspond to some of those used by Auchampaugh; "mirror image values" ^{1/} of the original angles were found but are not listed.

(b) The "mirror image values" of these solutions were found but are not listed here.

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