Accelerator Department
BROOKHAVEN NATIONAL LABORATORY
Associated Universities, Inc.
Upton, New York

CRISP 71-18
ISABELLE PROJECT

INTERSECTING STORAGE ACCELERATOR NOTES

ON DISPERSION IN CIRCULAR MACHINES

MASTER

J. Claus

July 27, 1971

In circular accelerators or storage rings the equilibrium orbits for particles of different energies are physically different. This dispersion is a function of azimuthal position. The machine structure may be chosen either to keep its maximum value low by distributing it more or less uniformly around the ring or to produce areas where it is zero. In storage rings it may be of interest to have zero dispersion in some of the interaction areas because this increases the effective beam brightness.

In this note we show that the dispersion caused by any linear mirror symmetric system may be removed to first order by having a quadrupole of proper strength in the plane of symmetry. Of course this quadrupole will also change the betatron phase shifts of the original structure but the focusing elements already present and the quadrupole may be chosen together to yield both the desired phase shifts and freedom of dispersion.

If we take for the transfer matrix of one half of the system

$$[M_{1,2}] = \begin{bmatrix} a & b & p \\ c & d & q \\ 0 & 0 & 1 \end{bmatrix}$$

with $ad-bc = 1$, we have for the transfer matrix of its mirror symmetric counterpart

$$[M_{3,4}] = \begin{bmatrix} d & b & -dp+bq \\ c & a & -cp+aq \\ 0 & 0 & 1 \end{bmatrix}$$

The matrix for the complete system inclusive of the quadrupole Q becomes

$$[M] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = [M_{3,4}] \begin{bmatrix} 1 & 0 & 0 \\ Q & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} [M_{1,2}] .$$

As was to be expected, $a_{31} = 0$, $a_{32} = 0$, $a_{33} = 1$. For the dispersive elements we find

$$\begin{aligned} a_{13} &= b[pQ + 2q] \\ a_{23} &= a[pQ + 2q] . \end{aligned}$$

Obviously both elements become zero if $Q = -2q/p$, in which case we have freedom of dispersion.

It follows that the structure of a storage ring may be chosen to yield any mixture of interaction areas without dispersion and interaction areas with large dispersion that seems desirable. In the dispersion-free areas one would have bright beams with a relatively large uncertainty about the particle energy; in areas with large dispersion the beam would be less bright but there would be a known correlation between particle energy and transverse position.

We note that the maximum beam width due to dispersion tends to decrease with increasing number of mirror symmetric cells.

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Atomic Energy Commission, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

Distr.: HE S&P at Physics
S&P at AD
Outside Users
M. Goldhaber, L.J. Haworth, G.H. Vineyard

$$[M] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = [M_{3,4}] \begin{bmatrix} 1 & 0 & 0 \\ Q & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} [M_{1,2}]$$

As was to be expected, $a_{31} = 0$, $a_{32} = 0$, $a_{33} = 1$. For the dispersive elements we find

$$a_{13} = b[pQ + 2q]$$

$$a_{23} = a[pQ + 2q]$$

Obviously both elements become zero if $Q = -2q/p$, in which case we have freedom of dispersion.

It follows that the structure of a storage ring may be chosen to yield any mixture of interaction areas without dispersion and interaction areas with large dispersion that seems desirable. In the dispersion-free areas one would have bright beams with a relatively large uncertainty about the particle energy; in areas with large dispersion the beam would be less bright but there would be a known correlation between particle energy and transverse position.

We note that the maximum beam width due to dispersion tends to decrease with increasing number of mirror symmetric cells.

Distr.: HE S&P at Physics
S&P at AD
Outside Users
M. Goldhaber, L.J. Haworth, G.H. Vineyard